

2002

# Water-drive gas reservoir: sensitivity analysis and simplified prediction

Junpeng Yue

*Louisiana State University and Agricultural and Mechanical College, jyue1@lsu.edu*

Follow this and additional works at: [https://digitalcommons.lsu.edu/gradschool\\_theses](https://digitalcommons.lsu.edu/gradschool_theses)



Part of the [Petroleum Engineering Commons](#)

---

## Recommended Citation

Yue, Junpeng, "Water-drive gas reservoir: sensitivity analysis and simplified prediction" (2002). *LSU Master's Theses*. 1171.  
[https://digitalcommons.lsu.edu/gradschool\\_theses/1171](https://digitalcommons.lsu.edu/gradschool_theses/1171)

This Thesis is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Master's Theses by an authorized graduate school editor of LSU Digital Commons. For more information, please contact [gradetd@lsu.edu](mailto:gradetd@lsu.edu).

**WATER-DRIVE GAS RESERVOIR:  
SENSITIVITY ANALYSIS AND SIMPLIFIED PREDICTION**

A Thesis  
Submitted to the Graduate Faculty of the  
Louisiana State University and  
Agricultural and Mechanical College  
in partial fulfillment of the  
requirements for the degree of  
Master of Science in Petroleum Engineering

in

The Department of Petroleum Engineering

by  
Junpeng Yue  
B.S., University of Petroleum (East China), 1994  
M.S., University of Petroleum (East China), 1997  
December 2002

# **DEDICATION**

To my parents.

## **ACKNOWLEDGEMENTS**

I would like to give particular thanks to Dr. Christopher White for his patience, guidance and mentoring. I benefited from each discussion with him in the past two years. Without his direction, suggestions and encouragements, this work would have been impossible.

To Dr. Zaki Bassiouni, for his serve on the examining committee and giving good suggestions.

To Dr. Andrew Wojtanowicz, for his serve on the examining committee and giving good suggestions.

# TABLE OF CONTENTS

<b>ACKNOWLEDGEMENTS</b> .....	<b>iii</b>
<b>ABSTRACT</b> .....	<b>vi</b>
<b>CHAPTER 1. INTRODUCTION</b> .....	<b>1</b>
<b>CHAPETR 2. RESPONSE SURFACE METHODOLOGY</b> .....	<b>4</b>
<b>2.1 Approximating Response Functions</b> .....	<b>4</b>
<b>2.2 Building Empirical Models</b> .....	<b>7</b>
<b>2.2.1 Linear Regression Models</b> .....	<b>7</b>
<b>2.2.2 Model Fitting</b> .....	<b>8</b>
<b>2.3 Hypothesis Testing in Multiple Regression</b> .....	<b>11</b>
<b>2.3.1 Test for Significance of Regression</b> .....	<b>11</b>
<b>2.3.2 Tests on Individual Regression Coefficients</b> .....	<b>13</b>
<b>2.4 Experimental Design</b> .....	<b>14</b>
<b>2.4.1 Two-level Factorial Designs</b> .....	<b>15</b>
<b>2.4.2 Confounding</b> .....	<b>15</b>
<b>2.4.3 Two-Level Fractional Factorial Designs</b> .....	<b>16</b>
<b>2.4.4 Design Resolution</b> .....	<b>17</b>
<b>2.5 The Box-Behnken Design</b> .....	<b>19</b>
<b>CHAPETR 3. MATERIAL BALANCE</b> .....	<b>21</b>
<b>3.1 Water-drive Gas Reservoir Material Balance</b> .....	<b>21</b>
<b>3.2 Fetkovitch Aquifer Model</b> .....	<b>25</b>
<b>3.3 Need for A Well Flow Model</b> .....	<b>29</b>
<b>3.4 The Al-Hussainy, Ramey, Crawford Solution Technique</b> .....	<b>30</b>
<b>CHAPTER 4. SENSITIVITY ANALYSIS</b> .....	<b>34</b>
<b>4.1 Defining Responses</b> .....	<b>34</b>
<b>4.1.1 Aquifer Productivity Index</b> .....	<b>35</b>
<b>4.1.2 Water Produced</b> .....	<b>35</b>
<b>4.1.3 Cumulative Gas Production</b> .....	<b>36</b>
<b>4.1.4 Cut-off line</b> .....	<b>38</b>
<b>4.2 Model Description</b> .....	<b>38</b>
<b>4.2.1 Reservoir Geometry and Properties</b> .....	<b>39</b>
<b>4.2.2 Grid Description</b> .....	<b>40</b>
<b>4.2.3 Reservoir Factor Ranges</b> .....	<b>41</b>
<b>4.3 Simulation Design</b> .....	<b>42</b>
<b>4.4 Sensitivity Analysis</b> .....	<b>43</b>
<b>4.4.1 Matching Aquifer Productivity Index</b> .....	<b>43</b>
<b>4.4.2 Gas Production Factor</b> .....	<b>48</b>
<b>4.4.3 Gas Recovery</b> .....	<b>49</b>
<b>4.4.4 Sweep efficiency</b> .....	<b>50</b>

4.4.5 Water Breakthrough .....	50
4.4.6 Other Responses .....	51
<b>CHAPTER 5. APPLICATIONS .....</b>	<b>55</b>
5.1 Simplified Prediction Using Response Models.....	55
5.2 Simplified Prediction for p/z Curves.....	56
<b>CHAPTER 6. DISCUSSIONS AND CONCLUSIONS.....</b>	<b>58</b>
6.1 Discussions.....	58
6.2 Conclusions.....	59
<b>REFERENCES .....</b>	<b>61</b>
<b>APPENDIX: RESPONSE MODELS.....</b>	<b>66</b>
<b>VITA .....</b>	<b>67</b>

## **ABSTRACT**

Water influx and well completions affect recovery from water-drive gas reservoir. Material balance, aquifer models and well inflow equations are used to examine and predict the pressure depletion, water influx, and production rates of water-drive gas reservoirs. The parameters of these simple, lumped models are estimated from simulation results using response surfaces and experimental designs for eight varying geologic and engineering factors. Eleven simulated responses (including maximum gas rate, aquifer and well constants, and water breakthrough) are analyzed using ANOVA and response models.

A sensitivity analysis of aquifer productivity index, gas production factor, and sweep efficiency reveals that permeability is the dominating factor. In contrast to earlier investigations, this study indicates that water-drive gas recovery is often higher for higher permeability water-drive gas reservoirs. The high gas mobility more than offsets the high aquifer mobility. The other seven factors are statistically significant for many responses, but much less important in determining reservoir behavior.

The proposed approach combines simple analytic expressions with more complete but difficult-to-use reservoir simulation models. The response models can be used to make quick, accurate predictions of water-drive gas reservoirs that include the effects of changing geologic and engineering variables. These simple, approximate models are appropriate for prospect screening, sensitivity analysis and uncertainty analysis.

## CHAPTER 1. INTRODUCTION

Prediction of gas production is an important part of reservoir development and management, pipeline and distribution management, and economic evaluation. The production of gas reservoirs that have no associated aquifers is relatively simple to predict and recovery efficiency is usually high (Lee et al., 1996). However, gas recovery from water-drive reservoirs may decrease because water influx may trap gas. The gas is trapped as an immobile, immiscible phase within the portion of the reservoir invaded by water. At higher abandonment pressure, the amount of trapped within the water-invaded pore space is higher. Efforts to predict water-drive gas reservoir performance have focused on material balances.

Material balances are a fundamental reservoir engineering tool that describe and predict the relation between fluid withdrawal, expansion, influx and pressure. Material balances provide a simple but effective alternative to volumetric methods based on isopach maps. Material balances can predict original gas in place and gas reserves at any stage of reservoir depletion (Craft and Hawkins, 1959). For a constant-volume (or volumetric) gas reservoir without water influx, the  $\frac{P}{z}$  versus cumulative gas production plot can predict the gas reservoir behavior. If the rock and water compressibility are small, the  $\frac{P}{z}$  versus cumulative gas production  $G_p$  plot is a straight line (Craft and Hawkins, 1959). For a water-drive gas reservoir, the aquifer affects the reservoir behavior. The  $\frac{P}{z}$  vs.  $G_p$  plots for these water-drive gas reservoirs are no longer straight lines (Bruns, 1965). The deviation from a straight

line is determined by the aquifer properties, size and the production means. Material balance and related models are discussed in Chapter 3.

Water-drive gas material balances include aquifer models. The aquifer water influx can be estimated using the Schilthuis (1936) steady-state method, Hurst modified steady-state method (Pirson, 1958), and various unsteady-state methods such as those of van Everdingen and Hurst (1949), Hurst (1958), and Carter and Tracy (1960). The unsteady state influx theory of Hurst and van Everdingen is the most rigorous method for radial and linear aquifers. Unfortunately, this method requires awkward, time-consuming superposition calculations. This drawback is exacerbated by the repetition in most influx calculations when history matching. Because of this, engineers have sought a more direct method of water influx calculation that duplicates results obtained with the Hurst and van Everdingen method without requiring superposition (Dake, 1978). The most successful of the methods was proposed by Fetkovitch (1971). Chapter 3 details the Fetkovitch method.

The aquifer productivity index in the Fetkovitch approach is one important parameter used to predict the water influx. It is determined by the reservoir properties, reservoir geometry, and fluid properties. The simple mechanistic model for the relationship between aquifer productivity index and those factors is available (Dake, 1978). But for specific cases, when there exists a dip or the reservoir is in special shape or more complex, how these factors interact in the model make it difficult to use those simple models. In this situation, the researchers can approximate the mechanistic model with an empirical model. This empirical model

is called a response surface model. Response surface methodology is often realized in combination with experimental design method. In this study, the 2-level fractional factorial designs were used. Chapter 2 discusses response surface methodology and experimental designs.

In addition to the aquifer productivity index, the empirical model for sweep efficiency and gas production factor were also derived using response surface methodology. These derived responses are discussed in Chapter 4.

A simple rectangular reservoir model was used to study the water influx. The model is described in Chapter 4. Eight factors, including initial reservoir pressure, permeability, reservoir width, reservoir thickness, aquifer size, tubing size, tubing head pressure and reservoir dip, were selected to do 2-level half-fraction factorial design; 128 simulation runs are required compared to 256 runs for a two-level full factorial design. A first-order response surface model with two-term interactions was derived to do sensitivity analysis. These models were also used to do simplified prediction.

This study provides response-surface based methods for quick reserve estimates and performance prediction. The objectives of this study are to understand production sensitivities and to formulate the aquifer productivity index, gas production factor, initial maximum gas production and sweep efficiency.

## CHAPTER 2. RESPONSE SURFACE METHODOLOGY

In this chapter, the Response Surface Methodology and experimental design are introduced. Response surface methodology (RSM) is a collection of statistical and mathematical techniques to develop, improve, and optimize processes (Myers and Montgomery, 1995). The most extensive applications of RSM are in the industrial world, particularly in situations in which several input variables potentially influence some performance measure or quality characteristics of products or processes. This performance measure or quality characteristics is called the response. The input variables are called independent variables or factors.

### 2.1 Approximating Response Functions

In some systems the nature of the relationship between response  $y$  and the input variables  $\xi_1, \xi_2, \xi_3, \dots, \xi_k$  might be known “exactly”, based on the underlying engineering, chemical, or physical principles. Then we could write a model of the form  $y = g(\xi_1, \xi_2, \dots, \xi_k) + \varepsilon$ , where the term  $\varepsilon$  in this model represents the “error” in the system. This type is often called a *mechanistic model*. If the underlying mechanism is not fully understood, the experimenter must approximate unknown function  $g$  with an approximate empirical model  $y = f(\xi_1, \xi_2, \dots, \xi_k) + \varepsilon$ . Such empirical models are called a response surface model (Myers and Montgomery, 1995).

In some situations, the *mechanistic model* exists, but it is difficult to compute or use. Researchers can also use empirical response models to approximate the *mechanistic model*.

Usually the function  $f$  is a first-order or second order polynomial, and  $\varepsilon$  is the term that represents other sources of variability not accounted for in  $f$ . Thus  $\varepsilon$  includes effects such as measurement error on the response, other sources of variation that are inherent in the process or system, the effect of other variables, and so on.  $\varepsilon$  is treated as a statistical error, and often it is assumed to have a normal distribution with mean zero and variance  $\sigma^2$ . If the mean of  $\varepsilon$  is zero, then (Myers and Montgomery, 1995)

$$E(y) = \eta = E[f(\xi_1, \xi_2, \dots, \xi_k)] + E(\varepsilon) \quad (2.1)$$

$$\eta = f(\xi_1, \xi_2, \dots, \xi_k) \quad (2.2)$$

The variables  $\xi_1, \xi_2, \dots, \xi_k$  in Equation (2.1) and (2.2) are usually called natural variables, because they are expressed in the natural units of measurement. In much RSM work it is convenient to transform the natural variables to coded variables  $x_1, x_2, \dots, x_k$ , where these coded variables are usually defined to be dimensionless with center point value zero and the same spreads around the center point, usually expressed with  $-1$  and  $1$ . Chapter 4 discusses some coding functions. In terms of the coded variables, the true response function (2.2) is now written as (Myers and Montgomery, 1995)

$$\eta = f(x_1, x_2, \dots, x_k) \quad (2.3)$$

Because the form of the true response function  $f$  is unknown, it needs to be approximated. RSM depends upon a suitable approximation for  $f$ . Usually, a low-order polynomial in some relatively small region of the independent variable space

is appropriate. In many cases, either a first-order or a second-order model is used. In general, the first-order model is (Myers and Montgomery, 1995)

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k \quad (2.4)$$

and the second-order model is (Myers and Montgomery, 1995)

$$\eta = \beta_0 + \sum_{j=1}^k \beta_j x_j + \sum_{j=1}^k \beta_{jj} x_j^2 + \sum_{i=1}^{k-1} \sum_{j>i}^k \beta_{ij} x_i x_j \quad (2.5)$$

The form of the first-order model in Equation (2.4) is sometimes called a main effect model, because it includes only the main effects of the variables  $x_1, x_2, \dots, x_k$ .

The interaction between variables  $x_i x_j$  can be added to the model.

The first-order model (even with interaction term included) cannot describe the curvature in responses. A second-order model is required in these situations.

The second-order model is widely used in RSM for several reasons. First, the second-order model is very flexible. It can take on a wide variety of functional forms, so it will often work well as an approximation to the true response surface. Contour plots are good application of the second-order model. Second, it is easy to estimate the parameters in the second-order model. The method of least squares discussed later in this chapter can be used. Third, there is considerable practical experience indicating that second-order models work well in solving real response surface problems (Narayanan, 1999). Fourth, the second-order model can be used for optimization whereas first-order models cannot.

In some situations, approximating polynomials of order greater than two are used. The general motivation for a polynomial approximation for the true response

function  $f$  is based on the Taylor series expansion around the point  $x_{10}, x_{20}, \dots, x_{k0}$ .

For example, the first-order model is developed from the first-order Taylor series expansion (Myers and Montgomery, 1995)

$$f \cong f(x_{10}, x_{20}, \dots, x_{k0}) + \left. \frac{\partial f}{\partial x_1} \right|_{x=x_0} + \left. \frac{\partial f}{\partial x_2} \right|_{x=x_0} + \dots + \left. \frac{\partial f}{\partial x_k} \right|_{x=x_0} \quad (2.6)$$

and the higher-order models can be derived similarly.

## 2.2 Building Empirical Models

Multiple regression is a collection of statistical techniques useful for building the types of empirical models required in response surface methodology.

### 2.2.1 Linear Regression Models

A first-order response surface model described as (Myers and Montgomery, 1995)

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

is a multiple linear regression model with two variables or regressors.

In general, the response variable  $y$  may be related to  $k$  regressor variables.

The model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \varepsilon$$

is called a multiple linear regression model with  $k$  regressor variables. The parameters  $\beta_j, j = 0, 1, \dots, k$ , are called the regression coefficients. This model describes a hyperplane in the  $k$ -dimensional space of the regressor variables  $x_j, j = 0, 1, \dots, k$ . The parameter  $\beta_j$  represents the expected change in response  $y$

per unit change in  $x_j$  when all the remaining independent variables  $x_i (i \neq j)$  are held constant.

The variable  $x_j$  could be a function of other variables, such as  $x_1x_2$ ,  $x_3^2$  or other forms. At this situation, we can let  $x_j = x_1x_2$  or  $x_j = x_3^2$ . In general, any regression model that is linear in the parameters  $\beta_j, j = 0, 1, \dots, k$  is a linear model, regardless of the shape of the response surface that it generates.

### 2.2.2 Model Fitting

The methods for estimating the parameters in multiple linear regression models are often called model fitting. The least squares method is typically used to estimate the regression coefficients from multiple linear regression. Suppose that  $n > k$  observations on the response variable are available, say  $y_1, y_2, \dots, y_n$ . Along with each observed response  $y_i$ , we will have an observation on each regressor variable, and let  $x_{ij}$  denote the  $i^{\text{th}}$  observation of  $x_j$ . The data will appear as in Table 2.1. Classically, we assume that the error term  $\varepsilon$  in the model has  $E(\varepsilon) = 0$  and  $Var(\varepsilon) = \sigma^2$  and that the  $\{\varepsilon_i\}$  are uncorrelated random variables.

**Table 2.1 Data for Multiple Linear Regression**

$y$	$x_1$	$x_2$	$\dots$	$x_k$
$y_1$	$x_{11}$	$x_{12}$	$\dots$	$x_{1k}$
$y_2$	$x_{21}$	$x_{22}$	$\dots$	$x_{2k}$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
$y_n$	$x_{n1}$	$x_{n2}$	$\dots$	$x_{nk}$

In terms of the observations in Table 2.1 the model equation may be written as

$$y_i = \beta_0 + \beta_1x_{i1} + \beta_2x_{i2} + \dots + \beta_kx_{ik} + \varepsilon_i$$

$$= \beta_0 + \sum_{j=1}^k \beta_j x_{ij} + \varepsilon_i, \quad i = 1, 2, \dots, n \quad (2.7)$$

The method of least squares chooses the  $\beta$ 's in Equation (2.7) so that the sum of the squares of the errors,  $\varepsilon_i$ , are minimized. The least squares function is (Myers and Montgomery, 1995)

$$\begin{aligned} L &= \sum_{i=1}^n \varepsilon_i^2 \\ &= \sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^k \beta_j x_{ij} \right)^2 \end{aligned} \quad (2.8)$$

The function  $L$  is to be minimized with respect to  $\beta_0, \beta_1, \dots, \beta_k$ . The least squares estimators, say  $b_0, b_1, \dots, b_k$ , must satisfy

$$\left. \frac{\partial L}{\partial \beta_0} \right|_{b_0, b_1, \dots, b_k} = -2 \sum_{i=1}^n \left( y_i - b_0 - \sum_{j=1}^k b_j x_{ij} \right) = 0 \quad (2.9a)$$

and

$$\left. \frac{\partial L}{\partial \beta_j} \right|_{b_0, b_1, \dots, b_k} = -2 \sum_{i=1}^n \left( y_i - b_0 - \sum_{j=1}^k b_j x_{ij} \right) x_{ij} = 0, \quad j = 1, 2, \dots, k \quad (2.9b)$$

Equation (2.7) may be written in matrix notation as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

where  $\beta_0$  is a constant (or mean) term. In general,  $\mathbf{y}$  is an  $(n \times 1)$  vector of the observed responses,  $\mathbf{X}$  is an  $(n \times p)$  matrix of the levels of the independent variables,  $\boldsymbol{\beta}$  is a  $(p \times 1)$  vector of the regression coefficients, and  $\boldsymbol{\varepsilon}$  is an  $(n \times 1)$  vector of random errors.

To find the vector of least squares estimators,  $\mathbf{b}$ , that minimizes

$$\begin{aligned} L &= \sum_{i=1}^n \varepsilon_i^2 = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \\ &= \mathbf{y}'\mathbf{y} - \boldsymbol{\beta}'\mathbf{X}'\mathbf{y} - \mathbf{y}'\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta} \\ &= \mathbf{y}'\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta} \end{aligned} \quad (2.10)$$

since  $\boldsymbol{\beta}'\mathbf{X}'\mathbf{y}$  is a  $(1 \times 1)$  matrix, or a scalar, and its transpose  $(\boldsymbol{\beta}'\mathbf{X}'\mathbf{y})' = \mathbf{y}'\mathbf{X}\boldsymbol{\beta}$  is the same scalar. The least squares estimators must satisfy

$$\left. \frac{\partial L}{\partial \boldsymbol{\beta}} \right|_{\mathbf{b}} = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X}\mathbf{b} = \mathbf{0}$$

which simplifies to

$$\mathbf{X}'\mathbf{X}\mathbf{b} = \mathbf{X}'\mathbf{y} \quad (2.11)$$

Equation (2.11) is the set of least squares normal equations in matrix form. To solve the normal equations, multiply both sides of Equation (2.11) by the inverse of  $\mathbf{X}'\mathbf{X}$ . Thus, the least squares estimator of  $\boldsymbol{\beta}$  is

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y} \quad (2.12)$$

The fitted regression model is

$$\hat{\mathbf{y}} = \mathbf{X}\mathbf{b} \quad (2.13)$$

In scalar notion, the fitted model is

$$\hat{y}_i = b_0 + \sum_{j=1}^k b_j x_{ij}, \quad i = 1, 2, \dots, n$$

The difference between the observations  $y_i$  and the fitted value  $\hat{y}_i$  is a residual, say

$e_i = y_i - \hat{y}_i$ . The  $(n \times 1)$  vector of residuals is denoted by

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} \quad (2.14)$$

### 2.3 Hypothesis Testing in Multiple Regression

In multiple linear regression problems, certain tests of hypothesis about the model parameters help in measure the usefulness and significance of the model. In this section, several hypothesis-testing procedures are described. These procedures require that the errors  $\varepsilon_i$  in the model be normally and independently distributed with mean zero and variance  $\sigma^2$ , abbreviated  $\varepsilon \sim \text{NID}(0, \sigma^2)$ . As a result of this assumption, each observation  $y_i$  is normally and independently distributed with mean  $\beta_0 + \sum_{j=1}^k \beta_j x_{ij}$  and variance  $\sigma^2$ .

#### 2.3.1 Test for Significance of Regression

The test for significance of regression is a test to determine if there is linear relationship between the response  $y$  variable and a subset of the regressor variables  $x_1, x_2, \dots, x_k$ . The appropriate hypotheses are

$$\begin{aligned} H_0 : \beta_1 = \beta_2 = \dots = \beta_k = 0 \\ H_1 : \beta_j \neq 0 \quad \text{for at least one } j \end{aligned} \quad (2.15)$$

Rejection of  $H_0$  : in (2.15) implies that at least one of the regressor variables  $x_1, x_2, \dots, x_k$  contributes significantly to the model. The test procedure involves partitioning the total sum of squares  $S_{yy}$  into a sum of squares due to the model (or to the regression) and a sum of squares due to residual (or error), say

$$S_{yy} = SS_R + SS_E \quad (2.16)$$

The regression sum of squares is

$$SS_R = \mathbf{b}'\mathbf{X}'\mathbf{y} - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n} \quad (2.17)$$

and the error sum of squares is

$$SS_E = \mathbf{y}'\mathbf{y} - \mathbf{b}'\mathbf{X}'\mathbf{y} \quad (2.18)$$

The test procedure for  $H_0 : \beta_1 = \beta_2 = \dots = \beta_k = 0$  is to compute

$$F_0 = \frac{SS_R / k}{SS_E / (n - k - 1)} = \frac{MS_R}{MS_E} \quad (2.19)$$

and to reject  $H_0$  if  $F_0$  exceeds  $F_{\alpha, k, n-k-1}$  where  $\alpha$  is the confidence level.

Alternatively, one could use the  $P$ -value approach to hypothesis testing and reject  $H_0$  if the  $P$ -value for the statistic  $F_0$  is less than  $\alpha$ . The test procedure is called an analysis of variance or  $F$ -test.

The coefficient of multiple determination  $R^2$  is defined as

$$R^2 = \frac{SS_R}{S_{yy}} = 1 - \frac{SS_E}{S_{yy}} \quad (2.20)$$

$R^2$  is a measure of the amount of reduction in the variability of  $y$  obtained by using the regressor variables  $x_1, x_2, \dots, x_k$  in the model.  $0 \leq R^2 \leq 1$ . However, a large value of  $R^2$  does not necessarily imply that the regressions model is a good one. Adding a variable to the model will always increase  $R^2$ , regardless of whether the additional variable is statistically significant or not. Because  $R^2$  always increases as terms are added to the model, some regression model builders prefer to use an adjusted statistic defined as

$$R_{adj}^2 = 1 - \left( \frac{n-1}{n-k-1} \right) (1 - R^2) \quad (2.21)$$

In general, the adjusted  $R_{adj}^2$  statistic will not always increase as variables are added to the model: if unnecessary terms are added, the value of  $R_{adj}^2$  may decrease.

### 2.3.2 Tests on Individual Regression Coefficients

Individual regression coefficients may be tested to determine the importance of the regressor variables in the regression model. For example, the model might be more effective with the inclusion of additional variables, or perhaps with the deletion of one or more of the variables already in the model. If some regressor variable  $x_j$  is not important and deleted, it is not necessary to measure  $x_j$  any more, which can make the experiments less expensive. In the context of reservoir modeling, this may be a very important result: expensive core measurements might be suspended if their results were shown not to affect important process measures.

The hypotheses for testing the significance of any individual regression coefficient, say  $\beta_j$ , are

$$H_0 : \beta_j = 0$$

$$H_1 : \beta_j \neq 0$$

If  $H_0 : \beta_j = 0$  is not rejected, then this indicates that  $x_j$  can be deleted from the model. The test statistic for this hypothesis is

$$t_0 = \frac{b_j}{\sqrt{\hat{\sigma}^2 C_{jj}}} \quad (2.22)$$

where  $C_{jj}$  is the diagonal element of  $(\mathbf{X}'\mathbf{X})^{-1}$  corresponding to  $b_j$ . The null hypothesis  $H_0 : \beta_j = 0$  is rejected if  $|t_0| > t_{\alpha/2, n-k-1}$ . Note that this is really a partial or marginal test, because the regression coefficient  $b_j$  depends on all the other regressor variables  $x_i (i \neq j)$  that are in the model.

Response surface methodology is often used with experimental design. Experimental design allows us to select a small set of simulations to run from the large set that we could run. By choosing an appropriate design, we minimize the number of runs that need to be made to obtain the required results.

## 2.4 Experimental Design

Experimental design has been used in reservoir engineering applications including performance prediction (Chu, 1990), uncertainty modeling (Damsleth et al., 1991, van Elk et al., 2000, Friedmann et al., 2001), sensitivity studies (Willis and White, 2000), upscaling (Narayanan and White, 1999), history matching (Eide et al., 1994) and development optimization (Dejean and Blanc, 1999).

The simplest experimental designs are factorials. The most common designs are two-level design.

### **2.4.1 Two-level Factorial Designs**

Factorial designs are widely used in experiments involving several factors to investigate joint effects of factors on a response. Joint factor effects are main effects and interactions. A case of the factorial design is that where each of the  $k$  factors of interest has only two levels. Because such a design has exactly  $2^k$  experimental trials or runs, these designs are called  $2^k$  factorial designs.

### **2.4.2 Confounding**

The  $2^k$  factorial designs are simple to use. However, in many situations, it is impossible to perform a complete factorial design in one block (usually in agriculture and industry). Also, when there are many factors the number of experiments required becomes large (for 20 factors, more than one million experiments would be needed). To reduce the number of experiments required, statisticians have formulated a number of strategies including partial factorials or confounding. Confounding is a design technique for arranging a complete factorial experiment in blocks, where the block size is smaller than the number of treatment combinations in a complete factorial. The technique causes information about certain treatment effects (usually high-order interactions) to be indistinguishable from, or confounded with, blocks. Confounding reduces the power or resolution of the design but greatly decreases the cost. Below, we will discuss how the ideas of confounding and blocking can be used to create two-level factorial designs that require fewer experiments.

### 2.4.3 Two-Level Fractional Factorial Designs

A complete replicate of the  $2^6$  designs requires 64 runs. In this design only 6 of 63 degrees of freedom are used to estimate the main effects, and only 15 degrees of freedom are used to estimate the main two-factor interactions. The remaining 41 degrees of freedom (one is used to estimate the mean) are associated with three-factor and higher interactions. If the experimenters can assume that these high-order interactions are negligible, then the main effects and low-order interactions may be estimated from only a fraction of the complete factorial experiment. These fractional factorial designs are among the most widely used types of design.

Consider the situation in which three factors are of interest, but the experimenters do not wish to run all  $2^3=8$  treatment combinations. Suppose they consider a design with four runs. This suggests a one-half fraction of the  $2^3$  designs. Because the design contains  $2^{3-1}=4$  treatment combinations, a one-half fraction of the  $2^3$  designs is often called a  $2^{3-1}$  design.

The table of  $-1$  and  $+1$  signs for  $2^3$  designs is shown in Table 2.2. Suppose to select the four treatment combinations  $a, b, c$  and  $abc$  as the one-half fraction. These runs are shown in the top half of Table 2.2.

**Table 2.2 +1 and  $-1$  Signs for the  $2^3$  Factorial Design**

Treatment Combination	Factorial Effect							
	<i>I</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>AB</i>	<i>AC</i>	<i>BC</i>	<i>ABC</i>
<i>A</i>	+1	+1	-1	-1	-1	-1	+1	+1
<i>B</i>	+1	-1	+1	-1	-1	+1	-1	+1
<i>C</i>	+1	-1	-1	+1	+1	-1	-1	+1
<i>ABC</i>	+1	+1	+1	+1	+1	+1	+1	+1
<i>AB</i>	+1	+1	+1	-1	+1	-1	-1	-1
<i>AC</i>	+1	+1	-1	+1	-1	+1	-1	-1
<i>BC</i>	+1	-1	+1	-1	-1	-1	+1	-1
(1)	+1	-1	-1	+1	+1	+1	+1	-1

Notice that  $2^{3-1}$  designs are formed by selecting only those treatment combinations that have a plus in the  $ABC$  column. Thus,  $ABC$  is called the generator of this particular fraction. Furthermore, the identity column  $I$  is also always plus, so

$$I=ABC$$

is called the defining relation of the design. In general, the defining relation for a fractional factorial will always be the set of all columns that equal to the identity column  $I$ .

#### 2.4.4 Design Resolution

The preceding  $2^{3-1}$  design is called a resolution III design. In such a design, main effects are aliased with two-factor interactions. The alias structure for this design may be easily determined by using the defining relation  $I=ABC$ . Multiplying any column by the defining relation yields the aliases for that effect. In this example, this yields as the alias of  $A$

$$A \cdot I = A \cdot ABC = A^2 BC$$

or, because the square of any column is just the identity  $I$ ,

$$A = BC$$

Similarly, the aliases of B and C as

$$B \cdot I = B \cdot ABC$$

$$B = AB^2 C = AC$$

and

$$C \cdot I = C \cdot ABC$$

$$C = ABC^2 = AB$$

Consequently, it is impossible to differentiate between  $A$  and  $BC$ ,  $B$  and  $AC$  and  $C$  and  $AB$ . In fact, when we estimate  $A$ ,  $B$ , and  $C$ , we are really estimating  $A + BC$ ,  $B + AC$ , and  $C + AB$ . These designs are often described using a notation such as  $2_R^{k-p}$  where  $k$  is the number of factors,  $p$  is the fraction of the factorial, and  $R$  is the resolution. The number of runs required in a fractional factorial is smaller than a full factorial by  $2^p$ . Usually a Roman numeral subscript is employed to denote design resolution, thus the one-half fraction of the  $2^3$  design with the defining relation  $I=ABC$  is a  $2_{III}^{3-1}$  design.

Designs of resolution III, IV, V are widely used. Table 2.3 gives definitions of these resolutions.

**Table 2.3. Definitions of Resolution III, IV, and V**

<b>Resolution</b>	<b>Main Effects</b>	<b>Two-factor Interactions</b>	<b>Examples</b>
<i>III</i>	Not aliased with each other, but are aliased with two-factor interaction.	May be aliased with each other.	$2_{III}^{3-1} : I=ABC,$ $A=BC, B=AC, C=AB.$
<i>IV</i>	Not aliased with each other or two-factor interactions.	May be aliased with each other.	$2_{IV}^{4-1} : I=ABCD,$ $A=BCD, AB=CD,$ $AC=BD, AD=BC.$
<i>V</i>	Not aliased with each other, two-factor, or three factor interactions.	Not aliased with each other; may be aliased with three-factor interactions.	$2_{V}^{5-1} : I=ABCDE,$ $A=BCDE, AB=CDE,$ $AC=BDE, AD=BCE.$

These two-level designs can estimate first-order effects and interaction only. To consider quadratic effects, a third level must be introduced into the design. The most straightforward way to do this is with a three-level factorial, in which factors are set to minimum, center, or maximum values. Full three-level designs require  $3^k$  experiments. It is very expensive if the factor number becomes large.

## **2.5 The Box-Behnken Design**

Box and Behnken (1960) developed a family of efficient three-level designs for fitting second-order responses. The Box-Behnken design is a fractional design with additional runs on the edges of the faces of the hypercube and at the center.

Compared with three-level full factorial design, a Box-Behnken design reduces the number of required experiments by confounding higher-order interactions. This reduction become more significant as the number of factors increases. For 7 factors a Box-Behnken design requires 57 experiments compared to 2187 experiments required for a full 3-level factorial and 128 for a full 2-level factorial. Box-Behnken designs have the desirable qualities of being nearly orthogonal and rotatable for many cases (Box and Behnken, 1960). Box-Behnken designs allow estimation of quadratic terms and do not imply constant sensitivities of responses to factors. Most two-level designs do not include experiments at the design centerpoint. By including the center point, Box-Behnken designs reduce estimation error for the most likely responses.

The preceding discussed the response surface methodology and experimental design. Researchers have applied the RSM in oil industry. Wang (2001) illustrated some researchers' applications. Beside those applications, Gerbacia et al. (1980) conducted experiments to study the effects of the fraction of high-equivalent-weight sulfonate, the cosurfactant HLB (Hydrophile-lipophile balance) and the weight ratio of cosurfactant to sulfonate on oil recovery and interfacial tension. He evaluated the data statistically, obtaining optimal formulation for this data space and developed a high crude oil recovery formulation for that crude oil recovery. Aanonsen et al.

(1995) optimized well location under reservoir geometry and petrophysical parameter uncertainties. Wang and White (2002) approximated the relationship between gas recovery responses and reservoir and production parameters, and generated quality maps to choose optimal well locations for production.

In this study, experimental design and response surfaces were used in predicting the material balance shape, approximating water influx, and inflow gas performance. Chapter 3 introduces the gas reservoir material balance method, water influx prediction method and gas inflow performance.

## CHAPTER 3. MATERIAL BALANCE

Water-drive gas recovery increases with decreasing permeability, trapped gas saturation, and increasing withdrawal rates (Agarwal et al., 1965). Gas recovery decreases with increasing aquifer size (Al-Hashim et al., 1988). Gas recovery under water drive depends on geologic uncertainties and engineering factors, which are all interrelated and complicate the analysis. These parameters determine the shape of the  $\frac{p}{z}$  performance curves for the reservoir. The  $\frac{p}{z}$  method (volumetric material balance) is a common procedure used in an attempt to describe and predict the behavior of a petroleum reservoir. It can be used to predict the ultimate gas recovery.

### 3.1 Water-drive Gas Reservoir Material Balance

Agarwal (1965) demonstrated the effect water influx has on  $\frac{p}{z}$  versus cumulative gas produced for a gas reservoir using a material balance equation for the reservoir and a water influx equation for the aquifer. Simultaneous solution provides the cumulative water influx and reservoir pressure.

If water and rock compressibility are neglected, a general form of the material balance for a water-drive dry gas reservoir is (Agarwal, 1965)

$$GB_{gi} = (G - G_p)B_g + W_e - W_p B_w \quad (3.1)$$

Equation (3.1) can be rearranged to

$$\frac{p}{z} = \frac{\frac{p_i}{z_i} \left(1 - \frac{G_p}{G}\right)}{1 - \frac{1}{G} \frac{p_i}{p_{sc}} \frac{z_{sc}}{z_i} \frac{T_{sc}}{T} (W_e - W_p B_w)} \quad (3.2)$$

where

$G$  = original gas in place

$G_p$  = cumulative gas produced

$B_{gi}$  = gas formation volume factor at initial reservoir pressure  $p_i$

$B$  = gas formation volume factor at reservoir pressure  $p$

$B_w$  = water formation volume factor

$W_e$  = cumulative water influx

$W_p$  = cumulative water produced

$p_i$  = initial reservoir pressure

$p_{sc}$  = standard condition pressure

$p$  = reservoir pressure

$T$  = reservoir temperature

$T_{sc}$  = standard condition temperature

$z$  = gas deviation factor at pressure  $p$

$z_{sc}$  = gas deviation factor at pressure  $p_{sc}$

Agarwal (1965) used the Carter-Tracy method (Carter and Tracy, 1960) to approximate water influx. Then  $\frac{p}{z}$  is related to the gas produced  $G_p$  at any time.

Agarwal (1965) derived one further equation to set the end point, or abandonment condition – a material balance which states that the maximum gas recovery is equal to the initial gas in place, less gas trapped as residual gas in the watered region, less gas regions not swept by water, but unavailable to production because of breakthrough of water into all existing producing wells. The end-point equation is (Agarwal, 1965)

$$G_p = G \left[ 1 - E_p \left( \frac{S_{gr}}{S_g} + \frac{(1 - E_p)}{E_p} \right) \frac{p}{z} \frac{z_i}{p_i} \right] \quad (3.3)$$

where,

$S_{gr}$  = residual gas saturation

$E_p$  = volumetric invasion efficiency (also sweep efficiency)

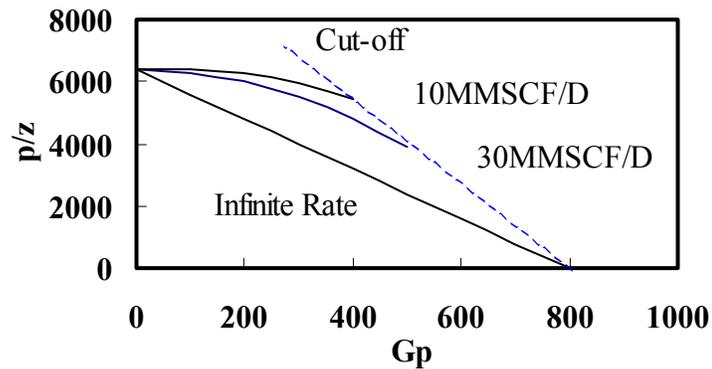
$S_g$  = initial gas saturation

Equation (3.3) can be rearranged to

$$\frac{p}{z} = \frac{\frac{p_i}{z_i} \left( 1 - \frac{G_p}{G} \right)}{E_p \left[ \frac{S_{gr}}{S_g} + \frac{(1 - E_p)}{E_p} \right]} \quad (3.4)$$

Equation (3.4) expresses the end-point  $\frac{p}{z}$  as a linear function of the ultimate gas recovery, and that the line passes through the point  $G$ , initial gas in place, at a zero value of  $\frac{p}{z}$ . The line in Equation (3.4) is referred as to cut-off line.

Equations (3.2) and (3.4) suggest a graphical solution of the water flux gas reservoir performance problem (Agarwal, 1965). If  $\frac{p}{z}$  vs.  $G_p$  can be estimated, the intersection of  $\frac{p}{z}$  vs.  $G_p$  (Equation (3.2)) and Equation (3.4) is the estimated ultimate gas recovery (Agarwal, 1965, Figure 3.1).



**Figure 3.1**  $p/z$  vs.  $G_p$  (from Agarwal, 1965)

Agarwal (1965) estimated the reservoir performance for ranges of aquifer permeabilities, reservoir production rates, initial formation pressures, residual gas saturations, and water influx reservoir efficiencies. Performance for a water-drive gas reservoir was computed for a reservoir of 5000 acres in area surrounded by an infinitely large aquifer. Agarwal did not vary other parameters including reservoir dip, thickness, width, aquifer compressibility.

Gas recovery for Agarwal's case depends upon production practices. A high production rate draws down reservoir pressure before water influx completely engulfs the reservoir (Figure 3.1). Gas recovery efficiency is lower at a given production rate for high-pressure reservoirs. Gas recovery is less sensitive to production rates as aquifer permeability increases. Water influx responds to pressure

changes in high-permeability gas reservoir so quickly that there is no benefit from increased production rate. In the limit, aquifer performance approaches a full water drive as permeability increases for sufficiently large aquifers (Agarwal, 1965).

Al-Hashim et al. (1988) researched the effect of aquifer size on partial water-drive gas reservoirs. They concluded that if  $r_a / r_g < 2$ , the effect of the aquifer on the performance of the gas reservoir can be neglected. Gas recovery is sensitive to initial reservoir pressure and the aquifer size if  $r_a / r_g > 2$ . As  $r_a / r_g$  and the initial reservoir pressure increase, gas recovery decreases. Saleh (1988) established a model for development and analysis of gas reservoirs with partial water drive. Hower et al. (1991) established an analytical model to predict the performance of gas reservoirs producing under water-drive conditions. All these studies used particular methods to calculate the water encroachment (Chapter 1). The theory of Fetkovitch (1971) for finite aquifers to approximate water influx is used in this study.

### 3.2 Fetkovitch Aquifer Model

In this approach the flow of aquifer water into a hydrocarbon reservoir is modeled in precisely the same way as the pseudosteady flow of oil from a reservoir into a well. An inflow equation of the form

$$q_w = \frac{dW_e}{dt} = J_w (\bar{p}_a - p) \quad (3.5)$$

is used where

$J_w$  = aquifer productivity index

$q_w$  = water influx rate

$p$  = reservoir pressure, i.e. pressure at the oil or gas water contact

$\bar{p}_a$  = average pressure in the aquifer

$W_e$  = water influx

The latter is evaluated using the simple aquifer material balance

$$W_e = \bar{c}W_i(p_i - \bar{p}_a) \quad (3.6)$$

where

$W_i$  = initial volume of water in the aquifer and is therefore dependent upon aquifer geometry

$\bar{c}$  = total aquifer compressibility

in which  $p_i$  is the initial pressure in the aquifer and reservoir. This balance can be alternatively expressed as

$$\bar{p}_a = p_i \left( 1 - \frac{W_e}{\bar{c}W_i p_i} \right) = p_i \left( 1 - \frac{W_e}{W_{ei}} \right) \quad (3.7)$$

where  $W_{ei} = \bar{c}W_i p_i$  is defined as the initial amount of encroachable water and represents the maximum possible expansion of the aquifer. Differentiating equation (3.7) with respect to time gives

$$\frac{dW_e}{dt} = -\frac{W_{ei}}{p_i} \frac{d\bar{p}_a}{dt} \quad (3.8)$$

and substituting equation (3.8) into equation (3.5) and separating the variables gives

$$\frac{d\bar{p}_a}{\bar{p}_a - p} = -\frac{J_w p_i}{W_{ei}} dt$$

this equation can be integrated for the initial condition that at  $t = 0, W_e = 0$  and  $\bar{p}_a = p_i$ . There is a pressure drop  $\Delta p = p_i - p$  imposed at the

reservoir boundary. Furthermore, the boundary pressure  $p$  remains constant during the period of interest so that

$$\ln(\bar{p}_a - p) = -\frac{J_w p_i t}{W_{ei}} + C1$$

where  $C1$  is an arbitrary constant of integration which can be evaluated from the initial conditions as  $C1 = \ln(p_i - p)$ , and therefore

$$\bar{p}_a - p = (p_i - p)e^{-J_w p_i t / W_{ei}} \quad (3.9)$$

which on substituting in the inflow equation (3.5) gives

$$\frac{dW_e}{dt} = J_w (p_i - p)e^{-J_w p_i t / W_{ei}} \quad (3.10)$$

Finally, integrating equation (3.10) for the stated initial conditions yields the following expression for the cumulative water influx

$$W_e = \frac{W_{ei}}{p_i} (p_i - p) (1 - e^{-J_w p_i t / W_{ei}}) \quad (3.11)$$

As  $t$  tends to infinity, then

$$W_e = \frac{W_{ei}}{p_i} (p_i - p) = \bar{c} W_i (p_i - p)$$

which is the maximum amount of water influx that could occur once the pressure drop  $p_i - p$  has been transmitted throughout the aquifer.

As it stands, equation (3.11) is not particularly useful since it was derived for a constant inner boundary pressure. To use this solution in the practical case, in which the boundary pressure is varying continuously as a function of time, it should again to apply the superposition theorem. Fetkovitch has shown, however, that a difference form of equation (3.11) can be used which eliminates the need for

superposition. That is, for influx during the first time step  $\Delta t_1$ , equation (3.11) can be expressed as

$$\Delta W_{e1} = \frac{W_{ei}}{p_i} (p_i - \bar{p}_1) \left(1 - e^{-J_w p_i \Delta t_1 / W_{ei}}\right) \quad (3.12)$$

where  $\bar{p}_1$  is the average reservoir boundary pressure during the first time interval.

$\bar{p}_1 = \frac{p_i + p_1}{2}$ .  $p_1$  is the reservoir boundary pressure at the end of the first time

interval. For the second interval  $\Delta t_2$

$$\Delta W_{e2} = \frac{W_{ei}}{p_i} (\bar{p}_{a1} - \bar{p}_2) \left(1 - e^{-J_w p_i \Delta t_2 / W_{ei}}\right) \quad (3.13)$$

where  $\bar{p}_{a1}$  is the average aquifer pressure at the end of the first time interval and is evaluated using equation (3.7) as

$$\bar{p}_{a1} = p_i \left(1 - \frac{\Delta W_{e1}}{W_{ei}}\right) \quad (3.14)$$

In general for the  $n^{\text{th}}$  time period,

$$\Delta W_{en} = \frac{W_{ei}}{p_i} (\bar{p}_{an-1} - \bar{p}_n) \left(1 - e^{-J_w p_i \Delta t_n / W_{ei}}\right) \quad (3.15)$$

where

$$\bar{p}_{an-1} = p_i \left(1 - \frac{\sum_{j=1}^{n-1} \Delta W_{ej}}{W_{ei}}\right) \quad (3.16)$$

The values of  $\bar{p}_n$ , the average reservoir boundary pressure, are calculated as

$$\bar{p}_n = \frac{p_{n-1} + p_n}{2} \quad (3.17)$$

Fetkovitch has demonstrated that using equations (3.16) and (3.17), in stepwise fashion, the water influx calculated for a variety of different aquifer geometries matches closely the results obtained using the unsteady state influx theory of Hurst and van Everdingen (1949) for finite aquifers.

Values of the aquifer productivity index  $J_w$  depend both on the geometry and flowing conditions, and are tabulated in the book *Fundamentals of Reservoir Engineering* (Dake, 1978).

Material balance Equation (3.2) and water influx Equation (3.15) can be jointly used to predict the reservoir performance. However all these depend on the depletion performance of the well.

### 3.3 Need for a Well Inflow Model

Cumulative gas production  $G_p$  at any time needs be calculated. Two methods can be used to express gas flow approximately. One is the Russell, Goodrich et al.  $p^2$  formulation (Russell et al., 1966), the other is the Al-Hussainny, Ramey and Crawford real gas pseudo-pressure  $m(p)$  formulation (1966). This study uses real gas pseudo pressure. The reasons for adopting this approach are (Dake, 1978):

1. It is theoretically the better method and in using it one does not have to be concerned about the pressure ranges in which it is applicable, as is the case when using the  $p^2$  formulation.
2. It is technically the more simple method to use because the basic relationship for  $m(p)$  as a function has been available.

3. The necessity for iteration in solving the inflow equation for bottom hole pressure  $p_{wf}$  is avoided.

### 3.4 The Al-Hussainy, Ramey, Crawford Solution Technique

The basic equation for the radial flow of fluid in a homogeneous porous medium is derived as

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k\rho}{\mu} r \frac{\partial p}{\partial r} \right) = \phi c \rho \frac{\partial p}{\partial t} \quad (3.18)$$

For real gas flow, this equation is non-linear because the coefficients on both sides are themselves functions of the dependent variable pressure. Al-Hussainy et al. linearized the equation (3.18) using an integral transformation (Dake, 1978)

$$m(p) = 2 \int_{p_b}^p \frac{p dp}{\mu Z} \quad (3.19)$$

which is the real gas pseudo pressure. The limits of integration are between a base pressure  $p_b$  and the pressure of interest  $p$ . The value of the base pressure is arbitrary since in using the transportation only differences in pseudo pressures are considered i.e.

$$m(\bar{p}) - m(p_{wf}) = 2 \int_{p_b}^{\bar{p}} \frac{p dp}{\mu Z} - 2 \int_{p_b}^{p_{wf}} \frac{p dp}{\mu Z} = 2 \int_{p_{wf}}^{\bar{p}} \frac{p dp}{\mu Z}$$

Al-Hussainy et al. replaced the dependent variable  $p$  by the real gas pseudo pressure  $m(p)$  in the following manner. Because

$$\frac{\partial m(p)}{\partial r} = \frac{\partial m(p)}{\partial p} \frac{\partial p}{\partial r}$$

and

$$\frac{\partial m(p)}{\partial p} = \frac{2p}{\mu Z}$$

then

$$\frac{\partial m(p)}{\partial r} = \frac{2p}{\mu Z} \frac{\partial p}{\partial r} \quad (3.20)$$

and similarly

$$\frac{\partial m(p)}{\partial t} = \frac{2p}{\mu Z} \frac{\partial p}{\partial t} \quad (3.21)$$

Substituting for  $\frac{\partial p}{\partial r}$  and  $\frac{\partial p}{\partial t}$  in Equation (3.18) and using Equation (3.20) and

(3.21) gives

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k\rho}{\mu} r \frac{\mu Z}{2p} \frac{\partial m(p)}{\partial r} \right) = \phi c \rho \frac{\mu Z}{2p} \frac{\partial m(p)}{\partial t} \quad (3.22)$$

Finally, using the equation of state for a real gas

$$\rho = \frac{Mp}{ZRT}$$

and substituting this expression for  $\rho$  in Equation (3.22) and canceling some terms gives the simplified expression

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m(p)}{\partial r} \right) = \frac{\phi \mu c}{k} \frac{\partial m(p)}{\partial t} \quad (3.23)$$

Equation (3.23) has precisely the same form as the diffusivity equation

$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{\phi \mu c}{k} \frac{\partial p}{\partial t}$  (Dake, 1978) except that the dependent variable has been

replaced by  $m(p)$ .

Note that in reaching this stage it has not been necessary to make any restrictive assumptions about the viscosity being independent of pressure.

The diffusivity  $\eta = \frac{\phi\mu c}{k}$  in Equation (3.23) is not a constant since for a real gas both viscosity and compressibility are highly pressure dependent. Equation (3.23) is therefore, a non-linear form of the diffusivity equation.

Continuing with the above procedure, in order to derive an inflow equation under semi-steady state flow conditions, then applying the simple material balance for a well draining a bounded part of the reservoir at a constant rate

$$cV \frac{\partial p}{\partial t} = -\frac{\partial V}{\partial t} = -q \quad (3.24)$$

and for the drainage of radial volume element

$$\frac{\partial p}{\partial t} = -\frac{q}{\pi r_e^2 h \phi c} \quad (3.25)$$

Also, using Equation (3.21)

$$\frac{\partial m(p)}{\partial t} = \frac{2p}{\mu Z} \frac{\partial p}{\partial t} = -\frac{2p}{\mu Z} \frac{q}{\pi r_e^2 h \phi c} \quad (3.26)$$

and substituting Equation (3.26) in (3.23) gives

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m(p)}{\partial r} \right) = -\frac{\phi\mu c}{k} \frac{2p}{\mu Z} \frac{q}{\pi r_e^2 h \phi c}$$

or

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m(p)}{\partial r} \right) = -\frac{2}{\pi r_e^2 k h} \left( \frac{pq}{Z} \right)_{reservoir} \quad (3.27)$$

Furthermore, using the real gas equation of state,

$$\left(\frac{pq}{Z}\right)_{reservoir} = p_{sc} q_{sc} \frac{T}{T_{sc}}$$

Equation (3.27) can be expressed as

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m(p)}{\partial r} \right) = - \frac{2 p_{sc} q_{sc} T}{\pi r_e^2 kh T_{sc}} \quad (3.28)$$

For isothermal reservoir depletion, the right hand side of equation (3.28) is a constant, and the differential equation has been linearized. A solution can now be obtained using precisely the same technique applied for liquid flow. If in addition, field units are employed then the resulting semi-steady state inflow equation is

$$m(\bar{p}) - m(p_{wf}) = \frac{1422qT}{kh} \left( \ln \frac{r_e}{r_w} - \frac{3}{4} + S \right) \quad (3.29)$$

A generalized expression considering reservoir geometry and well asymmetry is

$$m(\bar{p}) - m(p_{wf}) = \frac{1422qT}{kh} \left( \frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right) \quad (3.30a)$$

Equation (3.30a) can be rearranged to

$$q = \frac{kh}{1422T \left( \frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right)} \left[ m(\bar{p}) - m(p_{wf}) \right] \quad (3.30b)$$

The preceding discussed the material balance, water influx predicting approach and gas inflow performance method. These approaches can be combined to use to predict the reservoir behavior. Their applications are discussed in the next chapter.

## CHAPTER 4. SENSITIVITY ANALYSIS

Experimental designs and response surface methods are applied in a sensitivity analysis of water-drive gas reservoirs. Responses analyzed include aquifer productivity index, water sweep efficiency, gas production factor, total water influx, initial maximum gas production, and gas recovery.

As discussed in Chapter 2, all eight factors span a range (maximum to minimum). They are transformed to (-1, 1) using coding functions (Reservoir Factor Ranges, below). A  $2^{8-1}$  factorial design was used to reduce the number of simulation runs. Reservoir simulations were used to estimate the aquifer productivity index, gas production factor and sweep efficiency. These responses were related to the eight factors. Multiple regressions fit empirical models including main effect and two-term interactions. These are the response models.

Simulation can be used to model complex reservoir models, such as heterogeneous or irregularly shaped reservoirs. In this study, it was used to study a simple rectangular reservoir model.

### 4.1 Defining Responses

Analytic water influx predicting methods are discussed in chapter 3. From Equation (3.2), if  $G_p, W_e, W_p$  can be estimated, the reservoir performance  $\frac{p}{z}$  can be estimated at any time.

$$\frac{p}{z} = \frac{\frac{p_i}{z_i} \left( 1 - \frac{G_p}{G} \right)}{1 - \frac{1}{G} \frac{p_i}{p_{sc}} \frac{z_{sc}}{z_i} \frac{T_{sc}}{T} (W_e - W_p B_w)} \quad (3.2)$$

### 4.1.1 Aquifer Productivity Index

Aquifer productivity index in the theory of Fetkovitch for finite aquifers to approximate water influx is determined by the fluid viscosity, reservoir permeability, and reservoir geometry. It is defined as the first response.

$$J_w = f_1(x_1, x_2, x_3, x_4, \dots) \quad (4.1)$$

Referring to Equations (3.10) to (3.15),

$$q_w = f_1 \times (p_i - p) e^{-f_1 p_i t / W_{ei}} \quad (4.2)$$

$$\Delta W_e = \int q_w dt = \int f_1 \times (p_i - p) e^{-f_1 p_i t / W_{ei}} dt$$

$$\Delta W_e = \frac{W_{ei}}{p_i} (p_i - p) (1 - e^{-f_1 p_i \Delta t / W_{ei}}) \quad (4.3)$$

In the above equation  $x_1, x_2, x_3, x_4, \dots$  are coded variables for independent variables including initial pressure gradient, permeability, reservoir width, and aquifer size. These factors will be discussed in Section 4.2.

### 4.1.2 Water Produced

Equation (3.2) includes water produced,  $W_p$ . Before water breakthrough, the water production is zero. After breakthrough, the water production is increasing with the gas production. Water breakthrough time  $t_{bt}$  is controlled by production means, reservoir properties and reservoir geometry. It is defined as the second response and,

$$t_{bt} = f_2(x_1, x_2, x_3, x_4, \dots) \quad (4.4)$$

This research assumes the water production rate after breakthrough can be approximated using a second-order polynomial,  $q_w = a(t - t_{bt}) + b(t - t_{bt})^2$ . The parameters  $a$  and  $b$  are defined the third and fourth response.

$$a = f_3(x_1, x_2, x_3, x_4, \dots) \quad (4.5)$$

$$b = f_4(x_1, x_2, x_3, x_4, \dots) \quad (4.6)$$

$$W_p = \int [f_3 \times (t - t_{bt}) + f_4 \times (t - t_{bt})^2] dt \quad (4.7)$$

#### 4.1.3 Cumulative Gas Production

This study stipulates that the gas is produced at a constant tubing head pressure. Thus, gas production will decrease with the reservoir pressure depletion, and the bottom hole pressure also changes with gas production. To use Equation (3.29) and (3.30), semisteady flow and constant gas production rate are assumed in a short time interval; that is, the reservoir is assumed to pass through a succession of semisteady states. The cumulative gas production can be approximated using Al-Hussainy, Ramey, Crawford Solution Technique and the inflow performance can be expressed as

$$q = \frac{kh}{1422T \left( \frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right)} [m(\bar{p}) - m(p_{wf})] \quad (3.30b)$$

where non-Darcy effects are neglected and semi-steady state flow is assumed in the above model. Equation (3.30b) can be revised to

$$q = C [m(\bar{p}) - m(p_{wf})] \quad (4.8)$$

where

$$C = \frac{kh}{1422T \left( \frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} + S \right)}$$

$$G_p = \int Q dt = \int C [m(\bar{p}) - m(p_{wf})] dt \quad (4.9)$$

$C$  is determined by reservoir properties, geometry and skin.  $C$  is referred to as the gas production factor. Also,  $C$  is defined the fifth response and

$$C = f_5(x_1, x_2, x_3, x_4, \dots) \quad (4.10)$$

For a volumetric reservoir without water influx,  $C$  is a constant. Unfortunately, aquifer water will flow into the gas zone and therefore the gas effective permeability decreases with the water influx. Hence  $C$  will usually decrease with the water influx and thus vary in time. This complication is neglected in the current study.

Once these responses are derived, they can be used to predict the reservoir pressure at any time for similar reservoirs. Then the material balance plot described using these responses is as follows.

In general for the  $n^{\text{th}}$  time period,

before water breakthrough:

$$\frac{p_n}{z_n} = \frac{\frac{p_i}{z_i} \left\{ 1 - \frac{\sum_{n=1}^n \int_{t_{n-1}}^{t_n} \{ f_5 \times [m(p_{n-1}) - m(p_{wf, n-1})] \} dt}{G} \right\}}{1 - \frac{1}{G} \frac{p_i}{p_{sc}} \frac{z_{sc}}{z_i} \frac{T_{sc}}{T} \left\{ \sum_{n=1}^n \frac{W_{ei}}{p_i} (\bar{p}_{an-1} - p_{n-1}) (1 - e^{-f_1 p_i \Delta t_n / W_{ei}}) \right\}}$$
(4.11)

$p_{wf}$  is the flowing bottom hole pressure and can be estimated using vertical flow performance curves.

after water breakthrough:

$$\frac{p_n}{z_n} = \frac{\frac{p_i}{z_i} \left\{ 1 - \frac{\sum_{n=1}^n \int_{t_{n-1}}^{t_n} \{f_5 \times [m(p_{n-1}) - m(p_{wf,n-1})]\} dt}{G} \right\}}{1 - \frac{1}{G} \frac{p_i}{p_{sc}} \frac{z_{sc}}{z_i} \frac{T_{sc}}{T} \left\{ \sum_{n=1}^n \frac{W_{ei}}{p_i} (\bar{p}_{a,n-1} - p_{n-1}) (1 - e^{-f_1 p_i \Delta t_n / W_{ei}}) - W_{pn} \right\}} \quad (4.12)$$

where

$$W_{pn} = \int [f_3 \times (t_n - t_{bt}) + f_4 \times (t_n - t_{bt})^2] dt$$

#### 4.1.4 Cut-off line

The sweep efficiency is determined by reservoir properties and production conditions. From equation (3.4), the cut-off line is a straight line, and the slope and intercept are determined by the residual gas saturation and sweep efficiency. In this research, the sweep efficiency is defined as the sixth response.

$$E_p = f_6(x_1, x_2, x_3, x_4, \dots) \quad (4.13)$$

Once the response of efficiency is derived, the cut-off line at any cases is determined as

$$\frac{p}{z} = \frac{\frac{p_i}{z_i} \left( 1 - \frac{G_p}{G} \right)}{f_6 \left[ \frac{S_{gr}}{S_g} + \frac{(1 - f_6)}{f_6} \right]} \quad (4.14)$$

## 4.2 Model Description

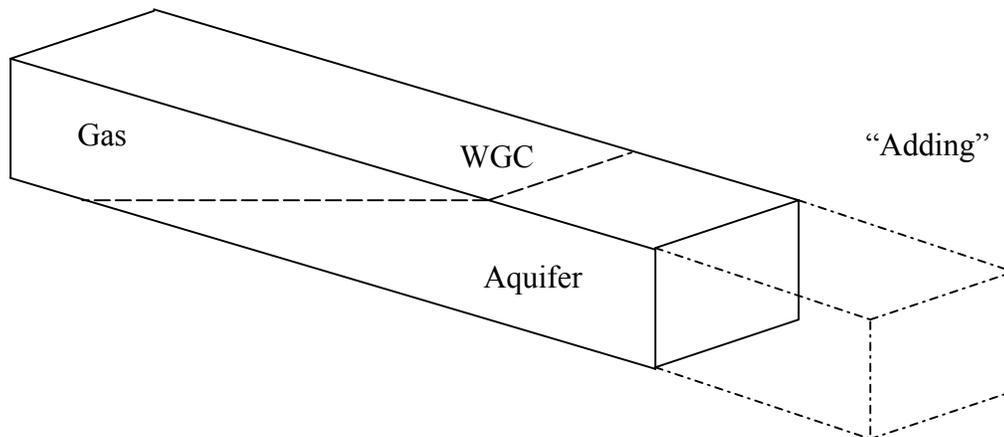
Reservoir simulation was used to model water influx into gas reservoirs. The reservoir can be divided into two regions, the gas zone and the aquifer. The water volume change in the region aquifer is calculated with the production of gas.

#### 4.2.1 Reservoir Geometry and Properties

In this study, a simple rectangular reservoir model is used. The reservoir length, width and thickness can be varied to different levels for the simulation designs. In this study, the gas zone length was set to 5750 feet (when the reservoir dip was zero), and was not one of the eight factors examined.

The reservoir dip can be modeled by rotating the reservoir, keeping the reservoir thickness unchanged. The gas zone pore volume was kept constant before and after the rotating.

The aquifer size can be varied through “adding” to the gross model length. The gross model length is expanded and the expanded zone contains only water. A sketch of the simple gas-water system is illustrated in Figure 4.1.



**Figure 4.1 Sketch of the Simple Rectangular Reservoir Model**

The center elevation of the gas zone was set at 5000 feet. The ground surface temperature was set to 60°F and the temperature gradient was set to 1.2°F per 100 feet. The reservoir temperature is 120°F.

The porosity was set to 25%. The irreducible water saturation was set to 30% and the residual gas saturation was set to 20%. The vertical permeability was set to 10% of the horizontal permeability.

The gas and water properties were estimated using correlations. The gas specific gravity was set to 0.65 without considering CO<sub>2</sub>, H<sub>2</sub>S and N<sub>2</sub>. The gas viscosity was estimated using the correlation developed by Lee et al. (1966) and extended by Gonzalez et al. (1968). The gas deviation factor was estimated using correlations presented by Dranchuk (Dranchuk et al., 1974). The water specific gravity was set to 1 and the water viscosity was estimated using correlations published by Numbere et al. (1977). McMullan (2000) also cited these methods.

The gas-water two-phase relative permeabilities presented by McMullan (2000) were used. Capillary pressure was ignored in this study.

This study considered only one producing well. The well was drilled at the center of the gas zone.

This study stipulated the well produces at constant tubing head pressure. The tubing head pressure was related to the bottom hole pressure using Gray method (Eclipse Reference Manual, 2000A).

#### **4.2.2 Grid Description**

Block-centered grid would have worked fine for this constant-thickness rectangular reservoir. At the beginning of this study, considering the aquifer zone could be gridded into wedge-shaped zone, cornerpoints grid was selected. Simulation runs using cornerpoints grids have demonstrated the importance of accurately representing the geometry of rock property variations. Corner point grid

was used in this study. The reservoir was gridded into five layers in the vertical direction.

### 4.2.3 Reservoir Factor Ranges

Preliminary work determined ranges of initial reservoir pressure, aquifer permeability, and gas production rate to the material balance (Agarwal, 1965). In this study, eight factors were selected with varying levels. The eight factors were illustrated in Table 4.1.

The initial reservoir pressure gradient  $PIDZ = p_i / Depth$  ( $Depth$  is the center elevation of the reservoir.) was used to parameterize the initial reservoir pressure. Its ranges were set to 0.7 to 0.9. It is transformed using coding function  $\frac{PIDZ - 0.8}{0.1}$ .

Permeability  $k$  was set to varying from 10 to 1000 md. It is transformed using function  $\log k - 2$ .

**Table 4.1 Factors Considered for Material Balance**

Factors	Variables	Coded Variables	Levels		
			-1	0	1
Reservoir pressure gradient	$PIDZ$	$x_1$	0.7	0.8	0.9
Permeability	$K$	$x_2$	10	100	1000
Reservoir width	$W$	$x_3$	4500	5500	6500
Aquifer size	$AQ$	$x_4$	35	60	85
Reservoir thickness	$H$	$x_5$	75	125	175
Tubing size	$DT$	$x_6$	2.5	3.25	4
Tubing head pressure	$PTH$	$x_7$	750	1000	1250
Reservoir dip	$DIP$	$x_8$	7.5	10	12.5

The aquifer size  $AQ = L_a / L_g$  ( $L_g = 5750$ ) were varied from 35 to 85, where  $L_a$  is the effective aquifer zone length. It is defined as

$$L_a = \frac{V_a}{WH}$$

where  $V_a$  is the bulk volume of the aquifer zone. Aquifer size is transformed using function  $\frac{AQ - 60}{25}$ .

Those five other factors were transformed using the same format with reservoir pressure gradient and aquifer size.

### 4.3 Simulation Design

A full two-level factorial design for eight factors will have  $2^8$  (256) runs. The partial factorial design is an effective method to reduce the number of runs. In this study, a  $2^{8-1}$  partial factorial design was used, then only 128 runs are required in this kind of design. One design generator specifies a  $2^{8-1}$  partial factorial design. The generator is  $ABCDEFGH$  and the defining relation is  $I=ABCDEFGH$ . The results after confounding will be: The seven-factor interactions are aliased with the main effect; The six-factor interactions are aliased with two-factor interactions; The five-factor interactions are aliased with three-factor interactions; The four-factor interaction are aliased with each other. They are shown as follows.

$$A \cdot I = A \cdot ABCDEFGH$$

$$A = BCDEFGH$$

$$AB \cdot I = AB \cdot ABCDEFGH$$

$$AB = CDEFGH$$

$$ABC \cdot I = ABC \cdot ABCDEFGH$$

$$ABC = DEFGH$$

$$ABCD = ABCD \cdot ABCDEFGH$$

$$ABCD = EFGH$$

#### 4.4 Sensitivity Analysis

128 simulations were run using Eclipse 100, and the necessary outputs including reservoir pressure and water change in the aquifer were written into summary files. These summary files were used to calculate the aquifer productivity index, gas production factor, sweep efficiency and gas recovery.

##### 4.4.1 Matching Aquifer Productivity Index

Equation (3.15) was used to calculate the water productivity index. One-half year was set as the difference time step. Average pressures were calculated using Equation (3.16) and (3.17). For each run, the aquifer productivity index was calculated using nonlinear regression (Microsoft Excel 2000, solver) to match the water influx calculated using Fetkovitch theory with the simulation results.

The water productivity index for all 128 simulations was then set as the dependent variable or response. A multiple linear regression was run to relate the water productivity index to the eight factors. The first-order polynomial model considering the two-term interaction between the eight factors was used in the multiple regressions.

This regression results are illustrated in Table 4.3.

**Table 4.3a Analysis of Variance for Aquifer Productivity Index**

Source	df	SS	MS	F	Significance F
Model	36	54337.00	1509.37	633.60	<0.0001
Error	91	2176.81	2.38		
Total	127	54554.00			

**Table 4.3b Regression Statistics  
- Aquifer Productivity Index**

Regression Statistics	
R Square	0.9960
Adjusted R Square	0.9945

**Table 4.3c Parameter Estimates for Water Productivity Index  
(For Significant Terms Only)**

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	20.36	0.13	149.22	<.0001
<i>K</i>	1	16.92	0.13	124.00	<.0001
<i>W</i>	1	3.70	0.13	27.10	<.0001
<i>H</i>	1	8.17	0.13	59.85	<.0001
<i>DIP</i>	1	-0.48	0.13	-3.49	0.0007
<i>K*W</i>	1	3.08	0.13	22.61	<.0001
<i>K*H</i>	1	6.76	0.13	49.52	<.0001
<i>K*DIP</i>	1	-0.41	0.13	-2.99	0.0036
<i>W*H</i>	1	1.51	0.13	11.03	<.0001
<i>H*DIP</i>	1	-0.23	0.13	-1.67	0.0988

A 10% significance level was set. The coefficients statistically significantly different from zero are illustrated in Table 4.3c. Of the  $p_{\max} = 1 + \frac{(n+1)n}{2} = 37$  possible terms in the linear model with two-term interactions, only 10 are significant. Figure 4.2 shows which terms are significant and which are not. Insignificant terms can be deleted from the first order model. Stepwise regression was further done to these significant factors. The stepwise regression results will be used as the response model. The derived first-order model with interaction for aquifer productivity index is

$$\begin{aligned}
 J_w = & 20.36 + 16.92x_2 + 3.70x_3 + 8.17x_5 \\
 & - 0.48x_8 + 3.08x_2x_3 + 6.76x_2x_5 \\
 & - 0.41x_2x_8 + 1.51x_3x_5 - 0.23x_5x_8
 \end{aligned} \tag{4.15}$$

Aquifer productivity index is dominated by the permeability (referring to Equation (4.15)). The reservoir width, thickness and dip effects are also sensible. Dake (1978) listed some equations of aquifer productivity index for linear aquifers. They are shown in Table 4.4.

**Table 4.4. Aquifer Productivity Index (from Dake, 1978)**

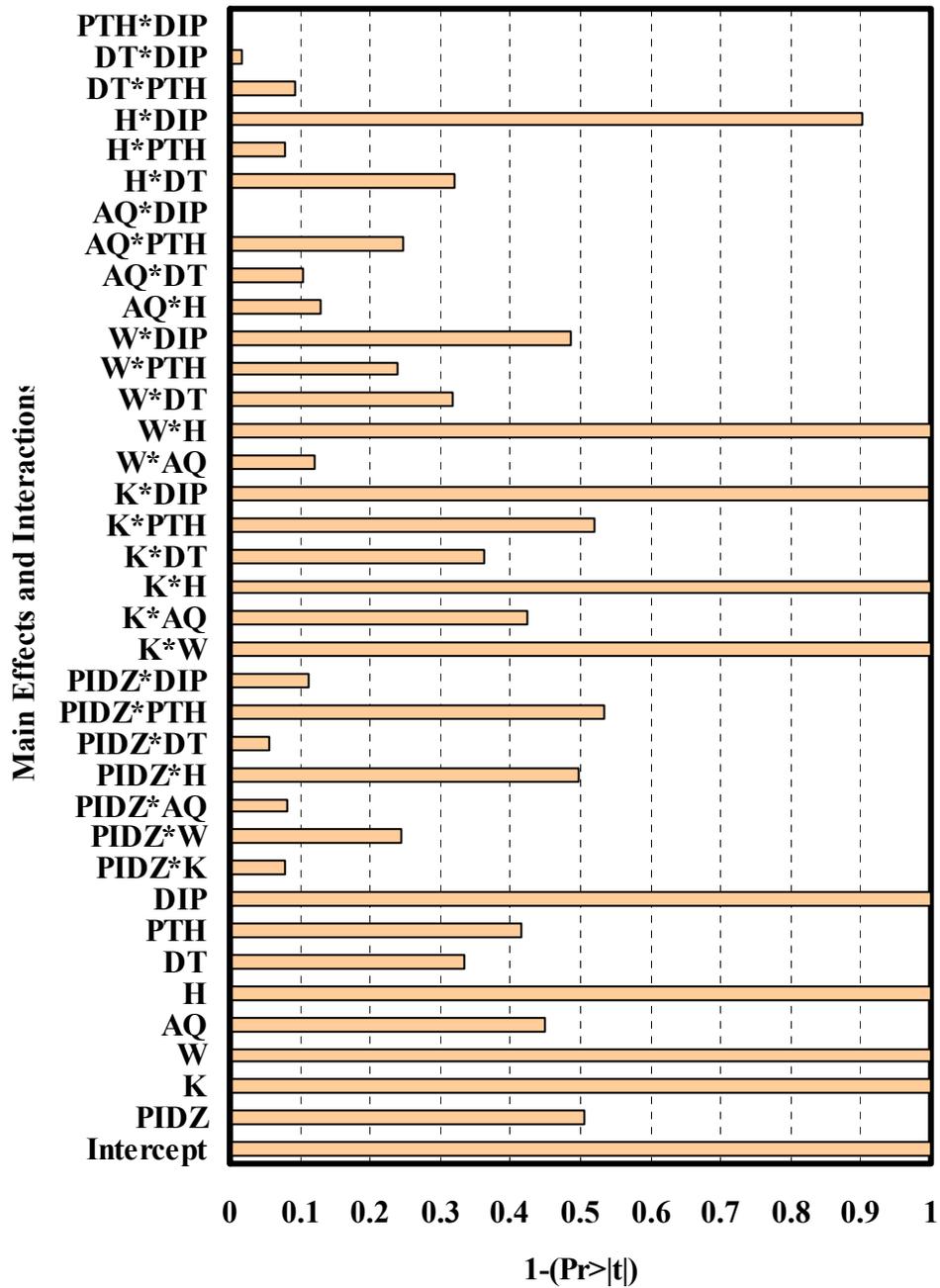
Flowing Condition	Linear Aquifer
Semi Steady	$3 \frac{khw}{\mu L}$
Steady State	$\frac{khw}{\mu L}$

Unfortunately, the reservoir length (aquifer size) does not appear in this model. First, the aquifer size is relatively large (201,250 to 488,750 feet in length). This makes the aquifer water a transient flow and we cannot determine the aquifer length. The water flows into the gas zone from the aquifer is mainly caused by the compressibility. In fact, pressure at the lower end of the aquifer does not vary too much for such large and dip reservoirs. It may not flow really.

The aquifer productivity index in Equation (4.15) is the initial aquifer productivity index value when the reservoir begins to produce. The actual water productivity index is changing with time. This is caused by the transient flow. The aquifer productivity index is proportional to the inverse of the square root of time. It is used in the following form in this study.

$$J_{w,t_D} = \frac{J_w}{\sqrt{t_D}}$$

where  $J_w$  is the initial aquifer productivity index and  $t_D$  is dimensionless time.



**Figure 4.2. Bar Chart For Probabilities**  
 A 10% significance level was set. Any parameter has  $(Pr>|t)<0.1$  is significant. In the above bar chart, any cross the line  $1-(Pr>|t)=0.1$  is significant.

How much will these significant factors contribute to the water productivity index model? In Equation (4.15),  $b_0 = 20.36$  and  $b_1 = 16.92$ .  $\frac{b_1}{b_0}$  gives a relative factor and states the relative importance for the term  $x_2$ . Figure 4.3 illustrates the relative importance for all significant factors. It is sensible that the six terms  $K$ ,  $W$ ,  $H$ ,  $K*W$ ,  $K*H$ , and  $W*H$  control the response.  $DIP$ ,  $K*DIP$  and  $H*DIP$  are STATISTICALLY significant but not PRACTICALLY significant.

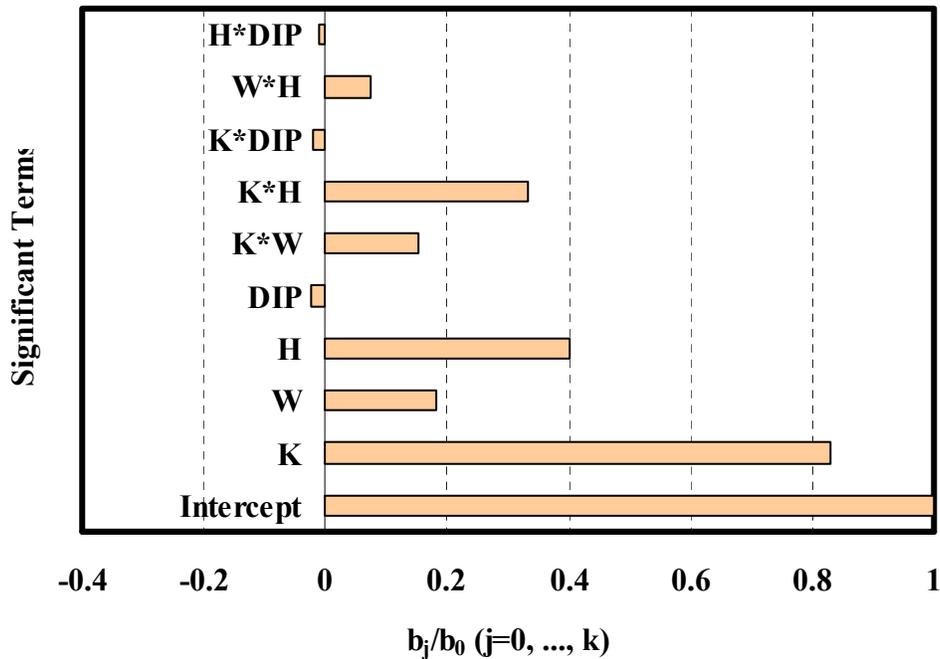
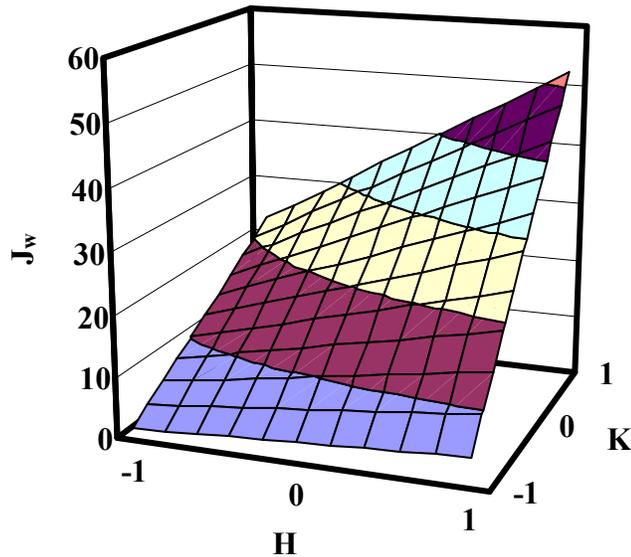


Figure 4.3. Bar Chart for  $b_j/b_0$  ( $j=0, \dots, k$ )

This kind of regression gives the first order response surface. The first-order response surfaces can be used to do the sensitivity analysis. The sensitivity of aquifer productivity index ( $J_w$ ) to the two factors permeability and reservoir thickness is illustrated in Figure 4.4.

The permeability is the dominating factor affecting the water productivity index. The reservoir thickness is the secondary. The parameters for permeability  $k$  and reservoir thickness illustrated in Equation (4.15) proved this.



**Figure 4. 4. Aquifer Productivity Index ( $J_w$ ) Sensitivity to Factors  $K$  and  $H$   
The other factors are set zero**

#### 4.4.2 Gas Production Factor

The gas production factor  $C$  was calculated using Equation (4.8). The gas pseudo pressure was calculated using the gas deviation factor and gas viscosity correlations to numerically integrate the quantity  $\frac{2p}{\mu z}$  from a base of zero to the pressure  $p$ . In this study, the effects of water influx on the gas production factor were neglected at early time to calculate the initial gas production factor.

The same stepwise regression procedure was used to gas production factor. The derived model is

$$C = 4.586 + 4.474x_2 + 1.410x_5 + 0.428x_6 + 1.368x_2x_5 + 0.426x_2x_6 + 0.336x_5x_6 \quad (4.16)$$

The permeability (coded variable  $x_2$ ), reservoir thickness ( $x_5$ ) and their interaction ( $x_2x_5$ ) are really important ones that control the gas production factor. The other three (tubing size  $x_6$ , tubing size and permeability interaction  $x_2x_6$ , and tubing size and thickness interaction  $x_5x_6$ ) are less than 10 percent of the mean value 4.586. Permeability and reservoir thickness effects are sensible and reasonable, referring to Equation (4.16). Also, tubing size is significant to this model. For some production range, the larger is the tubing size, the lower is bottom hole pressure for a given gas production rate. So, the tubing size affects the flow behavior of the gas. The gas flow is not really semisteady, which contradicts with our assumption. This is a possible explanation why the tubing size is a significant factor in the response model.

#### 4.4.3 Gas Recovery

In this study, the gas reservoir was abandoned when the gas production rate was lower than 10 percent of the initial maximum gas production rate and the ultimate gas recovery was calculated.

The derived model is

$$\begin{aligned} \text{Recovery} = & 0.67028 + 0.00186x_1 + 0.00631x_2 - 0.00394x_3 \\ & - 0.00523x_5 + 0.01266x_6 - 0.04350x_7 \\ & - 0.02142x_1x_2 + 0.00689x_1x_7 - 0.00666x_2x_3 \\ & - 0.01870x_2x_5 + 0.03347x_2x_6 + 0.02269x_2x_7 \\ & + 0.00198x_5x_7 - 0.00297x_6x_7 \end{aligned} \quad (4.17)$$

Fourteen factors are statistically significant, but half of them are small, less than 2 percent of the mean 0.67028. Tubing size, tubing head pressure, initial pressure and permeability interaction, permeability and thickness interaction, permeability and tubing size, and permeability and tubing head pressure are more important comparing to other seven factors.

#### 4.4.4 Sweep Efficiency

The derived model is

$$\begin{aligned}
 E_p = & 0.20846 + 0.01337x_1 + 0.12504x_2 \\
 & + 0.00982x_3 + 0.01479x_5 - 0.01712x_6 \\
 & - 0.01063x_7 - 0.00582x_8 + 0.00232x_1x_2 \\
 & + 0.00263x_1x_5 + 0.00568x_2x_5 + 0.00238x_6x_8
 \end{aligned} \tag{4.18}$$

The permeability is the dominating factor controls the sweep efficiency. The initial reservoir pressure, reservoir thickness, tubing size, and tubing head pressure are secondary. The others are very small compared to the mean 0.20846, less than 5 percent of it.

#### 4.4.5 Water Breakthrough

Twenty STB/DAY is set as the water breakthrough limit. If the water production rate is larger than it, the water breakthrough happens. In the 128 simulations, only 62 cases met water breakthrough criteria. Here for the no water breakthrough cases, we can assume the water breakthrough time is infinity. Its inverse will be zero. A regression for the inverse of breakthrough is run and the derived model is

$$\begin{aligned}
\frac{1}{t_{bt}} = & 0.00827 + 0.00182x_1 + 0.00827x_2 - 0.00059x_3 \\
& - 0.00175x_5 + 0.00280x_6 - 0.00077x_7 + 0.00182x_1x_2 \\
& - 0.00052x_1x_3 - 0.00084x_1x_5 + 0.00098x_1x_6 - 0.00175x_2x_5 \\
& + 0.00280x_2x_6 - 0.00077x_2x_7 + 0.00055x_5x_7 - 0.00058x_6x_7
\end{aligned} \tag{4.19}$$

Sixteen terms were statistically significant. Only six terms initial reservoir pressure, permeability, reservoir thickness, tubing size, initial reservoir pressure and permeability interaction, permeability and reservoir thickness interaction are really important. They are above 21 percent of the mean 0.00807. The others are below 12 percent of it.

Reference to Equation (4.5) and (4.6). For the simulations without water breakthrough, the factor  $a$  and  $b$  are treated as zero. The derived models for  $a$  and  $b$  are

$$\begin{aligned}
a = & 112 + 74x_1 + 113x_2 + 30x_5 - 32x_6 \\
& - 53x_7 + 74x_1x_2 + 26x_1x_6 - 38x_1x_7 \\
& + 30x_2x_5 + 32x_2x_6 - 53x_2x_7 + 27x_4x_8 \\
& + 55x_5x_6
\end{aligned} \tag{4.20}$$

$$\begin{aligned}
b = & 13 + 6x_1 + 13x_2 - 7x_3 - 12x_5 + 10x_6 \\
& + 6x_1x_2 - 8x_1x_5 + 7x_1x_6 - 7x_2x_3 \\
& - 12x_2x_5 + 10x_2x_6 - 8x_4x_8 - 10x_5x_6
\end{aligned} \tag{4.21}$$

#### 4.4.6 Other Responses

Except the above analysis, the original gas in place, field life, water influx and initial maximum gas production sensitivities to factors are also conducted and the models are derived. These models are illustrated in the APPENDIX.

Eleven responses were discussed above. A significance level 10% was set to all the regression analysis. Through examining the estimated coefficients for all these

eight factors pressure gradient  $PIDZ$ , permeability  $K$ , aquifer size  $AQ$ , tubing head pressure  $PTH$ , tubing size  $DT$ , reservoir thickness  $H$ , reservoir width  $W$  and  $DIP$  were significantly different from zero to these eleven responses. The regression results for all the response is summarized in Table 4.5.

Chapter 1 through chapter 4 discussed the research approaches. Chapter 5 will illustrate how those approaches will be used to do simplified prediction.

**Table 4.5 Response Surface Model Regression Results**

<b>Response</b>	<b>OGIP</b>	<b>C</b>	<b><math>J_w</math></b>	<b><math>Q_{gmax}</math></b>	<b>Recovery</b>	<b>Life</b>
$R^2$	0.9998	0.9949	0.9960	0.9947	0.9792	0.9850
Adjusted $R^2$	0.9997	0.9928	0.9945	0.9926	0.9710	0.9790
Pr(F>F0)	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
<i>Intercept</i>	<b>193.3594</b>	<b>4.5858</b>	<b>20.3590</b>	<b>78.0364</b>	<b>0.6703</b>	<b>158.3672</b>
<i>PIDZ</i>	<b>16.6647</b>	0.0202	-0.0935	<b>10.6349</b>	<b>0.0019</b>	<b>-5.4141</b>
<i>K</i>	-0.0810	<b>4.4744</b>	<b>16.9178</b>	<b>25.2968</b>	<b>0.0063</b>	<b>-71.6328</b>
<i>W</i>	<b>35.2514</b>	-0.0346	<b>3.6971</b>	0.4464	<b>-0.0039</b>	<b>27.9297</b>
<i>AQ</i>	-0.1186	0.0131	-0.0820	-0.1000	-0.0008	-0.0391
<i>H</i>	<b>77.3263</b>	<b>1.4103</b>	<b>8.1652</b>	<b>6.4751</b>	<b>-0.0052</b>	<b>42.1953</b>
<i>DT</i>	-0.0776	<b>0.4279</b>	-0.0591	<b>36.1255</b>	<b>0.0127</b>	<b>-55.8203</b>
<i>PTH</i>	-0.0810	-0.0055	-0.0747	<b>-2.4151</b>	<b>-0.0435</b>	<b>-7.0391</b>
<i>DIP</i>	-0.0633	-0.0078	<b>-0.4768</b>	0.0667	-0.0012	-0.8047
<i>PIDZ*K</i>	0.0810	0.0219	0.0136	<b>2.7873</b>	<b>-0.0214</b>	<b>-3.1641</b>
<i>PIDZ*W</i>	<b>2.9348</b>	0.0115	-0.0425	0.1693	0.0000	-0.6328
<i>PIDZ*AQ</i>	0.0738	-0.0118	-0.0138	-0.0976	0.0002	0.3984
<i>PIDZ*H</i>	<b>6.7153</b>	0.0140	-0.0917	<b>0.9722</b>	-0.0009	-0.6797
<i>PIDZ*DT</i>	0.0776	-0.0258	0.0096	<b>4.8935</b>	0.0017	0.8672
<i>PIDZ*PTH</i>	0.0810	-0.0065	0.1001	0.2019	<b>0.0069</b>	-0.0703
<i>PIDZ*DIP</i>	0.0845	-0.0093	0.0194	0.0974	-0.0004	-0.5234
<i>K*W</i>	0.0776	-0.0343	<b>3.0846</b>	0.2091	<b>-0.0067</b>	<b>-14.7578</b>
<i>K*AQ</i>	-0.0810	0.0134	-0.0767	0.0971	-0.0008	-0.5703
<i>K*H</i>	-0.0776	<b>1.3679</b>	<b>6.7564</b>	<b>-4.5955</b>	<b>-0.0187</b>	<b>-14.3359</b>
<i>K*DT</i>	-0.0810	<b>0.4264</b>	-0.0646	<b>18.6426</b>	<b>0.0335</b>	<b>19.8984</b>
<i>K*PTH</i>	-0.0776	-0.0048	-0.0966	0.0240	<b>0.0227</b>	<b>7.7734</b>
<i>K*DIP</i>	-0.0810	-0.0079	<b>-0.4076</b>	-0.0777	-0.0007	-0.5234
<i>W*AQ</i>	0.0736	-0.0111	-0.0210	0.1003	0.0000	-0.2266
<i>W*H</i>	<b>14.1517</b>	-0.0163	<b>1.5052</b>	-0.2222	0.0005	<b>7.8828</b>
<i>W*DT</i>	0.0810	-0.0151	0.0561	0.3850	-0.0001	<b>-10.2891</b>
<i>W*PTH</i>	0.0776	-0.0112	-0.0418	0.0999	0.0012	-1.6016
<i>W*DIP</i>	0.0837	-0.0119	-0.0893	-0.0970	-0.0004	0.0078
<i>AQ*H</i>	-0.0973	0.0124	-0.0221	0.0980	-0.0003	-0.4609
<i>AQ*DT</i>	-0.0776	0.0106	0.0178	-0.1002	-0.0001	0.2109
<i>AQ*PTH</i>	-0.0810	0.0112	0.0431	-0.1002	-0.0001	0.1172
<i>AQ*DIP</i>	-0.0601	0.0107	0.0007	0.0978	0.0005	0.0391
<i>H*DT</i>	-0.0810	<b>0.3357</b>	0.0564	<b>4.2164</b>	-0.0009	<b>-22.6484</b>
<i>H*PTH</i>	-0.0776	-0.0010	-0.0135	0.0424	<b>0.0020</b>	<b>-2.8672</b>
<i>H*DIP</i>	-0.0393	-0.0001	<b>-0.2276</b>	-0.0969	-0.0013	-0.4453
<i>DT*PTH</i>	-0.0810	0.0169	-0.0160	<b>-1.3180</b>	<b>-0.0030</b>	<b>3.9922</b>
<i>DT*DIP</i>	-0.0776	0.0157	0.0028	0.0772	0.0006	0.2891
<i>PTH*DIP</i>	-0.0810	0.0115	0.0002	0.0979	-0.0002	-0.2734

Note: Significant coefficients are shown in bold type.

**Table 4.5 (continued)**

Response	<i>We</i>	<i>Ep</i>	<i>l/Tbt</i>	<i>a</i>	<i>b</i>
$R^2$	0.9951	0.9896	0.9259	0.7044	0.5192
Adjusted $R^2$	0.9932	0.9855	0.8966	0.5874	0.3290
Pr(F>F0)	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
<i>Intercept</i>	<b>30.3158</b>	<b>0.2085</b>	<b>0.0083</b>	<b>112.7131</b>	<b>12.6305</b>
<i>PIDZ</i>	<b>2.1730</b>	<b>0.0134</b>	<b>0.0018</b>	<b>74.1516</b>	<b>5.9143</b>
<i>K</i>	<b>18.0609</b>	<b>0.1250</b>	<b>0.0083</b>	<b>112.7131</b>	<b>12.6305</b>
<i>W</i>	<b>6.7255</b>	<b>0.0098</b>	<b>-0.0006</b>	16.0426	<b>-6.5131</b>
<i>AQ</i>	-0.1023	0.0004	0.0000	-11.6673	0.4053
<i>H</i>	<b>14.0298</b>	<b>0.0148</b>	<b>-0.0018</b>	<b>30.2935</b>	<b>-11.7332</b>
<i>DT</i>	<b>-2.3309</b>	<b>-0.0171</b>	<b>0.0028</b>	<b>32.1344</b>	<b>10.0598</b>
<i>PTH</i>	<b>-1.4020</b>	<b>-0.0106</b>	<b>-0.0008</b>	<b>-53.4258</b>	0.5659
<i>DIP</i>	<b>-0.7855</b>	<b>-0.0058</b>	-0.0001	-12.1843	3.3220
<i>PIDZ*K</i>	0.1497	<b>0.0023</b>	<b>0.0018</b>	<b>74.1516</b>	<b>5.9143</b>
<i>PIDZ*W</i>	<b>0.6325</b>	0.0004	<b>-0.0005</b>	8.4239	-4.6286
<i>PIDZ*AQ</i>	0.0242	-0.0010	0.0000	-13.6281	2.3704
<i>PIDZ*H</i>	<b>1.0565</b>	<b>0.0026</b>	<b>-0.0008</b>	22.6065	<b>-8.2498</b>
<i>PIDZ*DT</i>	-0.2442	-0.0002	<b>0.0010</b>	<b>26.1558</b>	<b>7.1112</b>
<i>PIDZ*PTH</i>	-0.1095	0.0009	0.0005	<b>-37.7783</b>	5.3423
<i>PIDZ*DIP</i>	-0.1672	0.0002	0.0000	-0.3844	1.1625
<i>K*W</i>	<b>3.4387</b>	0.0001	-0.0006	16.0426	<b>-6.5131</b>
<i>K*AQ</i>	-0.1101	-0.0020	0.0000	-11.6673	0.4053
<i>K*H</i>	<b>7.7543</b>	<b>0.0057</b>	<b>-0.0018</b>	<b>30.2935</b>	<b>-11.7332</b>
<i>K*DT</i>	0.0152	-0.0002	<b>0.0028</b>	<b>32.1344</b>	<b>10.0598</b>
<i>K*PTH</i>	-0.0419	0.0006	<b>-0.0008</b>	<b>-53.4258</b>	0.5659
<i>K*DIP</i>	<b>-0.5727</b>	-0.0022	-0.0001	-12.1843	3.3220
<i>W*AQ</i>	0.0072	0.0011	0.0000	-12.9080	3.4556
<i>W*H</i>	<b>3.1139</b>	-0.0004	-0.0002	-18.7803	4.7557
<i>W*DT</i>	<b>-0.4621</b>	-0.0015	0.0001	11.7166	-3.2917
<i>W*PTH</i>	<b>-0.3421</b>	-0.0017	0.0004	6.4299	3.1490
<i>W*DIP</i>	-0.1978	-0.0017	0.0000	-12.6436	1.8644
<i>AQ*H</i>	-0.0375	-0.0013	0.0000	-1.5491	0.4756
<i>AQ*DT</i>	-0.0990	-0.0017	0.0000	-15.9537	2.2729
<i>AQ*PTH</i>	-0.0145	-0.0012	0.0000	10.4053	-1.7472
<i>AQ*DIP</i>	0.0102	-0.0006	0.0004	<b>26.7081</b>	<b>-7.6595</b>
<i>H*DT</i>	<b>-1.0860</b>	-0.0002	-0.0002	<b>55.3116</b>	<b>-9.5549</b>
<i>H*PTH</i>	<b>-0.6843</b>	0.0002	<b>0.0005</b>	1.9401	1.2036
<i>H*DIP</i>	<b>-0.5638</b>	-0.0010	0.0000	9.6965	-1.6318
<i>DT*PTH</i>	0.1148	0.0019	<b>-0.0006</b>	-19.2629	-2.2515
<i>DT*DIP</i>	0.1974	<b>0.0024</b>	0.0000	-8.4536	1.1365
<i>PTH*DIP</i>	0.0377	0.0014	0.0000	8.3623	-1.8990

Note: Significant coefficients are shown in bold type.

## CHAPTER 5. APPLICATIONS

Most of the derived models have high adjusted R-Square values which proved these models have a good fitness. They are illustrated in the Table 4.5.

The  $R^2$  values for  $1/T_{bt}$ ,  $a$  and  $b$  are relatively low, possibly because 66 simulations did not have water breakthrough and we assumed the infinite breakthrough time value and zero  $a$  and  $b$  values.

### 5.1 Simplified Prediction Using Response Models

Here we give 3 random points and 1 point with all the factors are at the medium level. All these four points are not the design points used in the 128 simulations. The eight factor values for these four points are illustrated in the Table 5.2.

**Table 5.2 Non-design Points Used for Prediction Testing**

Point	<i>PIDZ</i>	<i>K</i>	<i>W</i>	<i>AQ</i>	<i>H</i>	<i>DT</i>	<i>PT</i>	<i>DIP</i>
1	0.75	90	5500	45	150	3.25	900	11
2	0.85	800	5500	70	100	3.25	1100	8.5
3	0.79	20	5500	81	80	3.25	1200	9
4	0.8	100	5500	60	125	3.25	1000	10

The water productivity index, gas production factor and nine other responses can be calculated using the derived response models. These results were compared with the simulation results. Table 5.3 illustrated the difference.

These RSM results are not ideal. However, the RSM results for original gas in place, sweep efficiency are good. In other side, the first-order model with two term interaction can not be used to describe a curvature, and those responses  $J_w$ ,  $C$ ,  $Q_{gmax}$ , *Recovery*, *Life*,  $1/T_{bt}$ ,  $a$ , and  $b$  are not necessarily in a linear relationship with those eight factors.

**Table 5.3 RSM vs. Simulation**

	P1		P2	
	RSM	Simulation	RSM	Simulation
<i>OGIP</i>	222.02	223.03	161.35	161.90
<i>C</i>	5.055	1.155	7.303	6.640
<i>J<sub>w</sub></i>	23.28	12.44	28.94	27.03
<i>Q<sub>gmax</sub></i>	75.70	85.84	105.09	100.124
<i>RECOVERY</i>	0.69	0.77	0.67	0.69
<i>LIFE</i>	189	153	75	61
<i>W<sub>e</sub></i>	35.27	33.26	37.11	36.03
<i>E<sub>p</sub></i>	0.205	0.197	0.318	0.320
<i>1/T<sub>bt</sub></i>	0.00658	0.00000	0.01865	0.01786
<i>a</i>	92	0	201	152
<i>b</i>	8	0	46	30

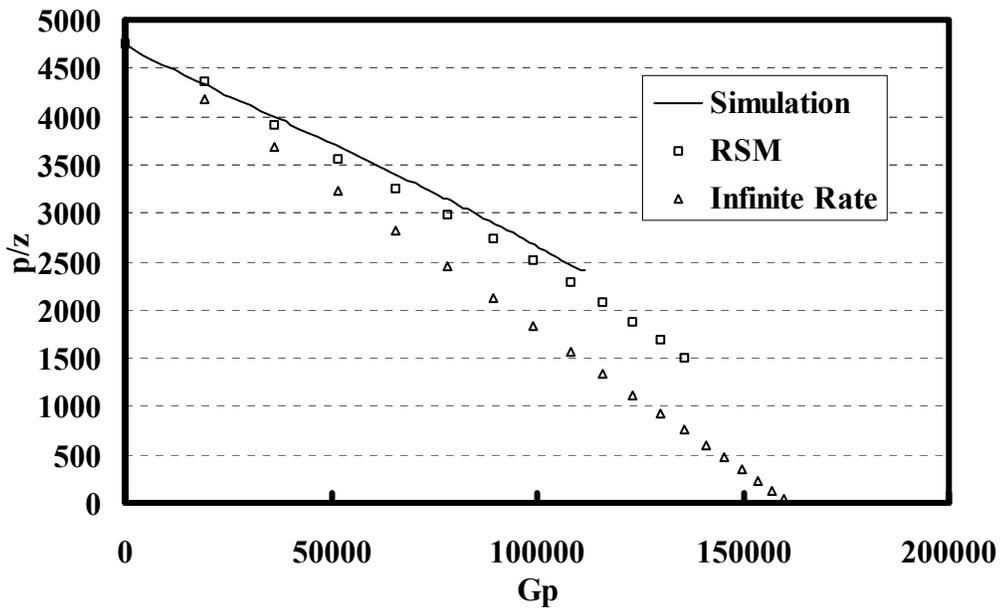
  

	P3		P4	
	RSM	Simulation	RSM	Simulation
<i>OGIP</i>	122.70	123.49	193.36	194.71
<i>C</i>	1.050	0.143	4.586	1.082
<i>J<sub>w</sub></i>	5.43	3.21	20.36	10.80
<i>Q<sub>gmax</sub></i>	48.91	60.03	78.04	90.64
<i>RECOVERY</i>	0.61	0.62	0.67	0.76
<i>LIFE</i>	154	108	158	124
<i>W<sub>e</sub></i>	9.12	6.77	30.32	27.70
<i>E<sub>p</sub></i>	0.104	0.075	0.208	0.197
<i>1/T<sub>bt</sub></i>	0.00225	0.00000	0.00827	0.00000
<i>a</i>	4	0	112	0
<i>b</i>	9	0	13	0

## 5.2 Simplified Prediction for p/z Curves

Given point 2, we can use those RSM models to do simplified prediction for

the  $\frac{P}{z}$  curve. Figure 5.1 shows the  $\frac{P}{z}$  curve calculated using the RSM models.



**Figure 5.1  $p/z$  Curves from Simulation and RSM**

The ultimate gas reservoir recovery and reservoir pressure at abandonment is illustrated in Table 5.4.

**Table 5.4 Ultimate Recovery and Abandonment Pressure**

	RSM	Simulation
<i>OGIP</i>	161.35	161.90
<i>RECOVERY</i>	0.67	0.69
Gas Recovered	108.10	111.71
$p/z$ at abandonment	2282.49	2407.00

From Figure 5.1,  $\frac{p}{z}$  curve from the RSM models is very close to the simulation results.

## CHAPTER 6. DISCUSSIONS AND CONCLUSIONS

Eight factors were selected to do the sensitivity analysis. Two-level simulation designs were done and the first-order polynomial models were derived. Eleven responses were researched and analyzed. The R-Square values for these models have been illustrated in Chapter 4.

A 10% significance level was set. The factor significant to any of the eleven responses was selected. All these eight factors were important in at least some of these eleven responses. The following discussion and conclusion is limited to the design range of these eight factors.

### 6.1 Discussions

**Simulation Designs.** For the two-level eight-factor simulation design, only 128 simulation runs was required using the confounding technique, comparing to 256 runs required by a full factorial two-level eight-factor design. Experimental design can reduce the number of simulations (or cost) significantly. Also, response surface methodology can be easily used to do sensitivity analysis.

**Recovery.** Gas recovery is significantly affected by initial pressure, permeability, reservoir geometry, and engineering factors (tubing size and tubing head pressure), referring to Equation (4.17). In common senses, the larger tubing size or lower tubing head pressure mean larger gas production. This model shows that the larger tubing size and lower tubing head pressure is helpful to gas recovery, which is consistent with Agarwal's conclusion (1965). Larger initial reservoir pressure and permeability increase the gas recovery in this model. The production means (constant tubing head pressure is stipulated) will affect the gas recovery. If

the reservoir pressure is close to the tubing head pressure, the gas recovery will be zero. For the high permeability without water breakthrough, the reservoir with high permeability is depleted more quickly than the low permeability reservoir, which causes the higher recovery for high permeability reservoir. These results are at odds with Agarwal's conclusions that the high permeability and initial reservoir pressure will cause low gas recovery.

**Water Breakthrough.** Because water breakthrough does not happen in more than half of the simulations, this complicates the analysis. But the derived model can still provide some insights [Equation (4.19)]. High reservoir pressure and permeability will make the water break through early. As discussed earlier, the larger tubing size and lower tubing head pressure mean larger gas production, which will cause water break through at earlier time, but perhaps at higher recovery efficiency. The aquifer size appears in the model Equation (4.20) and (4.21). It interacts with dip. Because  $a$  and  $b$  values for all simulations without water breakthrough were assumed zero, these two models is not really meaningful, which can be verified by the lower R-Square values 0.6032 and 0.3881.

## 6.2 Conclusions

Sensitivity analysis was conducted using experimental design and RSM techniques. The first order models with two-term interactions were also derived. These models can be applied in the reservoir behavior prediction, as discussed in Chapter 5. Although the prediction is not very accurate, they are verified that they can be used as approximations, at least for the  $p/z$  curves. A second-order could be considered for use in future work, and might yield more representative predictions.

Considering that many simulations did not have water breakthrough, the model proposed in Chapter 4 is not a good approach [Equations (4.4) to (4.7)]. Other forms of models that can include the cases without breakthrough need be considered.

## REFERENCES

- Agarwal, R. G.: "The Importance of Water influx in Gas Reservoirs", paper SPE 1244, SPE Annual Fall Meeting held in Denver, Colorado, 3-6 October, 1965.
- Al-Hussainy, R., Ramey Jr., H. J., and Crawford, P. B.: "The Flow of Real Gases through Porous Media", *JPT*, May 1966 624-636. Trans. AIME.
- Anonsen S. I., Eide A. L., and Holden L., Aasen, J. O.: "Optimizing reservoir Performance Under Uncertainty with Application to Well Location", paper SPE 30710, SPE Annual Technical Conference & Exhibition held in Dallas, U. S. A., 22-25, October, 1995.
- Armenta, M. and Wojtanowicz, A.: "Severity of Water Coning in gas Wells", SPE 75720, SPE Gas Technology Symposium held in Calgary, Alberta, Canada, 30 April-2 May 2002.
- Al-Hashim, H. S.: "Effect of Aquifer Size on the Performance of Partial Water-drive Gas reservoirs", SPE 13233, SPE Reservoir Engineering p. 380-386, May 1998.
- Beggs, H. D. and Brill, J. P.: "A Study of Two Phase Flow in Inclined Pipes", SPE 4007, May 1973 JPT p. 607-617.
- Box, G. E. P. and Behnken, D. W.: "Some New Three Level Designs for the Study of Quantitative Variables", *Technometrics* (November 1960) 2, No. 4, p. 455-475.
- Bruns, J. R., Fetkovich, M. J., and Meitzen, V. C.: "The Effect of Water Influx  $p/z$ -cumulative Gas Production Curves", paper SPE 898, SPE 38<sup>th</sup> Annual Fall Meeting held in Houston, Texas, 11-14 October 1964.
- Carter, R. D.: "Type Curves for Finite Radial and Linear Gas-flow Systems: Constant-Terminal-Pressure Case", SPE 12917, October 1985 SPEJ p. 719-728.
- Carter, R. D. and Tracy, G. W.: "An Improved Method for Calculating Water influx", Trans., AIME (1960) 219, 415.
- Chewaroungroj, J., Varela, O. J., and Lake, L. W.: "An Evaluation of Procedures to Estimate Uncertainty in Hydrocarbon Recovery Predictions", paper SPE 59449, 2000 SPE Asia Pacific Conference on Integrated Modelling for Asset Management held in Yokohama, Japan, 25-26 April 2000.
- Chu, C.: "Prediction of Steamflood Performance in Heavy Oil Reservoirs Using Correlations Developed by Factorial Design Method", paper SPE 20020, 60<sup>th</sup> California Regional Meeting held in Ventura, California, 4-6 April 1990.

Chu, C.: “Optimal Choice of Completion Intervals for Injectors and Producers in Steamfloods”, paper SPE 25787, International Thermal Operations Symposium held in Bakersfield, CA, U. S. A., 8-10 February 1993.

Cohen, M. F.: “Recovery Optimization in a Bottom/Edge Water-drive Gas reservoir, Soehlingen Schneverdingen”, SPE 19068, SPE Gas Technology Symposium held in Dallas, Texas, 7-9 June 1989.

Craft, B. C. and Hawkins, M. F.: “*Applied Petroleum Reservoir Engineering*”, Prentice-Hall PTR, Englewood Cliffs, NJ (1959), second edition, revised by Terry, R. E., 431 p.

Da Cruz, P. S., Horne, R. N. and Deutsch, C. V.: “The Quality Map: A Tool for Reservoir Uncertainty Quantification and Decision Making”, SPE 56578, SPE Annual Technical Conference and Exhibition held in Houston, Texas, 3-6 October 1999.

Dake, L. P.: “*Fundamentals of Reservoir Engineering*”, 1978, Elsevier Scientific Publishing Company, Amsterdam-Oxford-New York, 443 p.

Damsleth, E., Hage, A., and Volden, R.: “Maximum Information at Minimum Cost: A North Sea Field Development Study Using Experimental Design”, paper SPE 23139, Offshore Europe Conference held in Aberdeen, 3-6 September 1991.

Dejean, J. P. and Blanc, G.: “Managing Uncertainties on Production Predictions Using Integrated Statistical Methods”, paper SPE 56696, SPE Annual Technical Conference and Exhibition held in Houston, Texas, 3-6 October 1999.

Dranchuk, P. M., Purvis, R. A., and Robison, D. B., “Computer Calculations of Natural Gas Compressibility Factors Using the Standing and Katz Correlation”, Institute of Petroleum Technical Series, No. IP 74-008, 1974.

Egeland, T., Hatlebakk E., and Hoden L., and Larsen, E. A.: “Designing Better Decisions”, paper SPE 24275, SPE European Petroleum Computer Conference held in Stavanger, Norway, 25-27 May 1992..

Fetkovich, M. J.: “A simplified Approach to Water Influx Calculations-Finite Aquifer Systems”, SPE 2603, July 1971, JPT p. 814-828.

Firoozabadi, A. and et al: “Residual Gas Saturation in Water-drive gas reservoirs”, SPE 16355, SPE California Regional Meeting held in Ventura, California, 8-10 April 1987.

Friedmann, F., Chawathe, A. and Larue, D. K.: “Assessing Uncertainty in Channelized Reservoirs Using Experimental Designs”, paper SPE 71622, SPE

Annual Technical Conference and Exhibition held in New Orleans, Louisiana, 30 September-3 October 2001.

Gerbacia, W.: "The Evaluation of Surfactant Systems for Oil recovery Using Statistical Design Principles and Analysis", paper SPE 7070, Fifth Symposium on Improved Methods for Oil Recovery of the Society of Petroleum Engineers of AIME held in Tulsa, Oklahoma, 16-19 April 1978.

Gerbacia, W. and Mcmillen T. J.: "Oil recovery Surfactant Formulation Development Using Surface Design Experiments", paper SPE 8878, 1982 SPEJ p. 237.

Guehria, F. M.: "A New Approach to P/Z analysis in Abnormally Pressured Reservoirs", SPE 36703, SPE Annual Technical Conference and Exhibition held in Denver, Colorado, U. S. A., 6-9 October 1996.

Hagedorn, A. R. and Brown K. E.: "Experimental Study of Pressure Gradients Occuring during Continuous Two Phase Flow in Small Diameter Vertical Conduits", SPE 940, SPE 89<sup>th</sup> Annual Fall Meeting held in Houston, 11-14 October 1964.

Havlena, D. and Odeh A. S.: "The Material Balance as an Equation of a Straight Line", SPE 559, SPE Production Research Symposium, in Norman, Okla, 29-30 April 1963.

Havlena, D. and Odeh A. S.: "The Material Balance as an Equation of a Straight Line-Part II, Field Cases", SPE 869, July 1964 JPT p. 815-822.

Hower, T. L. and Jones, R. E.: "Predicting recovery of gas reservoirs under Water-drive Conditions", SPE 22937, SPE 66<sup>th</sup> Annual Conference and Exhibition held in Dallas, TX, 6-9 October 1991.

Hubbard, R. M., Elenbase, J. R.: "Determining Gas-filled Pore Volume in a Water-drive Gas Storage reservoir", SPE 685, SPE Annual Fall Meeting held in New Orleans, LA, 6-9 October 1963.

Hurst, W.: "The Simplification of the Material Balance Formulas by the Laplace Transformation", Trans., AIME (1958) 213, 292.

Katz, D. L. and Tek M. R.: "A Generalized Model for Predicting the Performance of Gas reservoirs Subject to Water Drive", SPE 428, 37<sup>th</sup> Annual Fall Meeting of the Society of Petroleum Engineers of AIME in Los Angeles, 7-10 October 1962.

Katz, D. L. and Shah, D. P.: "Establishing the Effective Aquifer Pressure Controlling Water-drive for Gas Storage Cycles", SPE 13234, 59<sup>th</sup> Annual Conference and Exhibition held in Houston, Texas, 16-19 September 1984.

Layne, M. A., Numbere, D. T. and Koederitz, L. F.: “Future Performance Prediction for Water Drive Gas Reservoirs”, SPE 26669, Annual Technical Conference and Exhibition of the Society of Petroleum Engineers held in Houston, Texas, 3-6 October 1993.

Lee, W. J. and Wattenbarger, R. A.: *Gas Reservoir Engineering*, SPE Textbook Series Volume 5 (1996).

Lutes, J. L., Chiang, C. P., Rossen, R. H. and Brady, M. M.: “Accelerated Blowdown of a Strong Waer-drive Gas reservoir”, SPE 6166, December 1977 JPT p. 1533-1538.

Myers, R. H. and Montgomery, D. C.: *Response Surface Methodology: Process and Product optimization Using Designed Experiments*, Wiley, New York (1995), 700 p.

McMullan, J. H. and Bassiouni, Z.: “Optimization of Gas-Well Completion and Production Practices”, SPE 58983, 2000 SPE International Petroleum Conference and Exhibition in Villahermosa, Mexico, 1-3 February 2000.

Narayanan, K.: “Applications for Reponse Surfaces in Reservoir Engineering”, Thesis, University of Texas at Austin, May 1999.

Narayanan, K., White, C. D., Lake, L. and Willis, B. J.: “Response Surface Methods for Upscaling Heterogeneous Geologic Models”, SPE 51923, 1999 SPE reservoir Simulation Symposium held in Houston, Texas, 14-17 February 1999.

Orkiszewski, J.: “Predicting Two-phase Pressure Drops in vertical Pipe”, SPE 1546, 41<sup>st</sup> Annual Fall Meeting held in Dallas, Texas, 2-5 October 1966.

Pirson, S. J.: *Elements of Oil and Reservoir Energy*, 2<sup>nd</sup> Ed., McGraw-Hill, Inc., N. Y. (1958) 608.

Pletcher, J. L.: “Improvements to reservoir material balance Methods”, SPE 62882, 2000 SPE Annual Technical Conference and Exhibition held in Dallas, TX, 1-4 October 2000.

Reinicke, K. M., Remer R. J. and Hueni, G.: “ Comparison of measured and Predicted Pressure Drops in Tubing for High Water Cut Gas Wells”, SPE 13279, August 1987 SPE Production Engineering p. 165-177.

Russell, D. G., Goodrich, J. H., Perry, G. E. and Bruskotter, J. F.: “Methods of Predicting Gas Well Performance”, paper SPE 1242, SPE Annual Fall Meeting held in Denver, Colorado, 3-6 October 1965.

Saleh, S. T.: "A Model for Development and Analysis of Gas reservoirs with partial Water Drive", SPE 18289, 63<sup>rd</sup> Annual Technical Conference and Exhibition of the Society of Petroleum Engineers held in Houston, TX, 2-5 October 1988.

Schilthuis, R. J.: "Active Oil and Reservoir Energy", Trans., AIME (1936) 118, 37.

Turner, R. G., Hubbard, M. G. and Dukler, A. E.: "Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from gas Wells", November 1969 JPT p. 1475-1482.

Van Elk, J. F., Guerrero, L. and Gupta, R.: "Improved Uncertainty Management in Field Development Studies through the Application of the Experimental Design Method to the Multiple Realizations Approach", paper SPE 64462, 2000 SPE Annual Technical Conference and Exhibition held in Dallas, Texas, 1-4 October 2000.

Van Everdingen, A. F. and Hurst, W.: "The application of Laplace Transformation to flow Problems in Reservoirs", Trans., AIME (1949) 186, 305.

Wang, F.: "Designed Simulation for Turbidite Reservoirs Using The Bell Canyon 3D Data Set", MS Thesis, Louisiana State University, 2001.

Wang, F. and White, C. D.: "Designed Simulation for a Detailed 3D Turbidite Reservoir Model", paper SPE 75515, SPE Gas Technology Symposium held in Calgary, Alberta, Canada, 30 April-2 May 2002.

White, C. D., Willis, B. J., Narayanan, K. and Dutton, S. P.: "Identifying Controls on Reservoir Behavior Using Designed Simulations", paper SPE 62791, 2000 SPE Annual Technical Conference and Exhibition held in Dallas, Texas, 1-4 October 2000.

## APPENDIX: RESPONSE MODELS

The following surfaces were derived using the response surface methodology.

### Original Gas In Place *OGIP*

$$\begin{aligned} OGIP = & 193.36 + 16.66x_1 + 35.25x_3 + 77.33x_5 \\ & + 2.93x_1x_3 + 6.72x_1x_5 + 14.15x_3x_5 \end{aligned} \quad (A.1)$$

### Initial Maximum Gas Production $Q_{g\max}$

$$\begin{aligned} Q_{g\max} = & 78.04 + 10.63x_1 + 25.30x_2 + 6.48x_5 \\ & + 36.13x_6 - 2.42x_7 + 2.79x_1x_2 + 0.97x_1x_5 \\ & + 4.89x_1x_6 - 4.60x_2x_5 + 18.64x_2x_6 + 4.22x_5x_6 \\ & - 1.32x_6x_7 \end{aligned} \quad (A.2)$$

### Life

$$\begin{aligned} Life = & 158 - 5x_1 - 72x_2 + 28x_3 + 42x_5 - 56x_6 \\ & - 7x_7 - 3x_1x_2 - 15x_2x_3 - 14x_2x_5 + 20x_2x_6 \\ & + 8x_2x_7 + 8x_3x_5 - 10x_3x_6 - 23x_5x_6 \\ & - 3x_5x_7 + 4x_6x_7 \end{aligned} \quad (A.3)$$

### Total Water Influx $W_e$

$$\begin{aligned} W_e = & 30.32 + 2.17x_1 + 18.06x_2 + 6.73x_3 + 14.03x_5 \\ & - 2.33x_6 - 1.44x_7 - 0.79x_8 + 0.63x_1x_3 + 1.06x_1x_5 \\ & + 3.44x_2x_3 + 7.75x_2x_5 - 0.57x_2x_8 + 3.11x_3x_5 - 0.46x_3x_6 \\ & - 0.34x_3x_7 - 1.09x_5x_6 - 0.68x_5x_7 - 0.56x_5x_8 \end{aligned} \quad (A.4)$$

## **VITA**

Junpeng Yue was born in Donging, Shandong, People's Republic of China. He attended No. 1 Kenli High School, graduating in 1990. He enrolled at University of Petroleum (East China) and received Bachelor of Science and Master of Science degrees in petroleum engineering in 1994 and 1997. He worked with China Offshore Oil Donghai Corporation for three years in Shanghai since 1997. He enrolled at Louisiana State University in 2000. At the 2002 December commencement, he will receive the degree of Master of Science in Petroleum Engineering.