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Project explorations and student learning in geometry

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PROJECT EXPLORATIONS AND STUDENT LEARNING IN GEOMETRY

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Words cannot express how grateful I am of the assistance I received to accomplish this honorable milestone. This has, indeed, marked a pivotal point in my professional career and has opened a new horizon.

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ABSTRACT

The purpose of this study was to examine the structural framework of the EBRPSS 8th Grade Mathematics Comprehensive Curriculum and to compare its effectiveness to a set of project-based lessons that I created for "Unit 4: Measurement and Geometry". Two classes participated in this study. Pretest and Posttest scores were analyzed to determine if there was a significant advantage to using my supplements. Results of the analysis revealed that there was an advantage to using the supplements, in spite of the time shortage. Hopefully, the supplements implemented in this study will serve as a model for teaching "Unit 4: Measurement and Geometry".

INTRODUCTION

In this thesis, I chose a specific unit within the EBRPSS 8^{th} Grade Mathematics Comprehensive Curriculum and attempted to test new strategies for delivering it. Given that measurement is one of life's most essential skills and that it tends to be the area in which 8^{th} grade students are weakest, I chose to concentrate on the Geometry and Measurement unit (Unit 4). I compared the effectiveness of the lessons, strategies, and activities from the Comprehensive Curriculum to a set of lessons that I developed. The first week within this four-week unit was dedicated to an in depth review of the prerequisite skills. I was pleased that within a three-week period, students within my experiment group had higher average learning gains upon finishing the unit.

CHAPTER 1 - THE EBRPSS 8TH GRADE COMPREHENSIVE CURRICULUM

Defining the Comprehensive Curriculum

The Louisiana Comprehensive Curriculum is an organized, coherent framework provided by the Louisiana State Department of Education to all school districts in Louisiana (LDE 2008). Its primary purposes are to establish curriculum consistency and to support professional development for teachers by providing model lessons and activities. The goal is to ensure that all students have equitable access to lessons and activities that are in alignment with the state standards. The state standards themselves are known as the Grade Level Expectations (or GLEs). They stipulate what should be taught as well as what students should know by the end of an academic school year (LDE 2010).

The EBRPSS Comprehensive Curriculum is an adaptation of the state comprehensive curriculum, which was prepared by the district. From here on, "Comprehensive Curriculum" refers to the EBRPSS Comprehensive Curriculum.

Its Conceptual Framework

As implemented, the Comprehensive Curriculum may be conceptualized in three divisions, of which the first is the written, the second is the taught, and the third is the tested.

The written curriculum is the document that bears the title "EBRPSS Comprehensive Curriculum". It outlines unit objectives and defines desired learning outcomes. Each unit consists of activities that support the unit objectives. The district regulates the implementation of the curriculum, enforcing the following rules. First, units may not be reordered, but activities within units may be. Second, all activities must be implemented, but can be modified to accommodate learners' needs. Third, all units must be taught within their stipulated calendar dates. Last, teachers must allow time for activities as well as for reviewing prerequisite knowledge and skills.

The taught curriculum refers to what actually occurs within the classroom under the teacher's direction. The qualities of the taught curriculum depend upon the teacher's expertise and the manner in which he or she responds to the students' learning needs. The Comprehensive Curriculum emphatically stresses that teachers must ensure proficiency in prerequisite skills prior to introducing new content. So clearly, in some classes, more teaching will be devoted to this, thus modifying the taught curriculum.

The tested curriculum consists of benchmarked assessments that are administered at the end of each unit. Presumably, these can be used to determine students' mastery of content. Typically, many assessment items occur in quizzes and teacher-prepared tests. The tested curriculum has a core consisting of state and district-sponsored test, and a fringe of other types of assessments.

The Curriculum's Structure

The Comprehensive Curriculum, as a written document, comprises eight units. Each unit contains text specifying the following:

- Time frame. The suggested amount of time for teaching each unit.
- Unit description. Summary of the unit's focus, learning objectives and performance outcomes.
- Grade Level Expectations. State standards.
- Guiding questions. Questions that drive instruction as well as gauge student understanding.
- Prerequisite knowledge/skills. What students should know before being introduced to new material.
- Vocabulary. Terms students should understand and utilize.
- Materials/Resources. Instructional resources.
- Intervention Strategies. Instructional approaches to assist struggling learners.
- Unit Notes. Information of what to expect or consider.
- Activities. Assignments that provide practice in reinforcing concepts.

The units and their intended content are as follows:

1. Rational Numbers, Measures, and Models
2. Rates, Ratios, and Proportions
3. Geometry and Measurement
4. Measurement and Geometry
5. Algebra, Integers and Graphing
6. Growth and Patterns
7. What Are the Data?
8. Examining Chances

Unit 1 focuses on fraction and decimal notation for rational numbers, procedures for converting from fraction to decimal notation and vice versa, and arithmetic. The following is a list of skills that the learner should acquire: compare rational numbers, arrange them in order on a number line and perform arithmetic with them in real-life situations; use order of operations to correctly interpret expressions; compute with numbers expressed in scientific notation; measure angles and arcs and understand their relationships.

Unit 2 focuses on applications of rates, ratios, percents and proportions. Objectives for this unit are as follows: understand the meaning of percents; evaluate percentages less than 1% and greater than 100%; determine percents of change; solve proportional word problems; identify similar figures and their corresponding parts; set up proportions and determine unknown side lengths given similar figures.

Unit 3 focuses on transformations and constructions of geometric figures, applications of angle relationships in transversals, and the Pythagorean Theorem and its converse. Students should acquire the following skills: reflect, translate, and rotate objects in a coordinate system; use distance measurements in geometry problems; identify the applications of angle bisectors and perpendicular bisectors; compare and contrast the meaning of similarity and congruence;

apply the concept of ratios to similar figures; scale figures given scale factors; apply the Pythagorean Theorem and its converse to real-life applications.

Unit 4 focuses on the properties and characteristics of three-dimensional objects. Upon finishing this unit, students should be able to do the following: conceptualize and discuss surface area and volume; use nets to describe the surfaces of solids; calculate the surface area and volume of three-dimensional objects; understand how scaling an object affects its volume; convert units within the same system. Density and probability are also included in this unit, but these topics are not tied in strongly to the main objectives within this unit.

Unit 5 is a collection of several themes, which are stated in the unit description below.

> This unit focus is on determining relationships of patterns. Representations of these relationships are made using tables, graphs, and equations. Equation solutions and descriptions of how rates of change in one variable affect the rate of change in the other variable are also explored as graphs are analyzed and slopes are discussed. The unit focus is also on simplifying variable expressions including monomial and polynomial operations. (p. 211)

During my participation in teacher team meetings at my school, this unit was often discussed. Some experienced teachers expressed difficulty in determining the main focus. It appears, however, that a central focus is for students to understand linear expressions in real-life situations and to create visual displays that represent them.

Unit 6, an extension of Unit 5, is similarly structured. It has a stronger emphasis on arithmetic and geometric sequences. The unit description for Unit 6 quotes the following.

> This unit examines the nature of changes to the input variables in function settings through the use of tables and sequences. There is emphasis on recognizing and differentiating between linear and exponential change and developing the expression for the *n*th term for a given arithmetic or geometric sequence. (p. 275)

Unit 7 focuses on data displays. Objectives for this unit are as follows: identify the most appropriate graphical representation and make inferences from those graphs; determine the

advantages and disadvantages to using bar graphs, histograms, circle graphs, stem-and-leaf plots, line plots, and box-and-whisker plots given certain situations.

Unit 8 is a relatively short unit that focuses on the application of combinations, permutations, and probability. Objectives for this unit are as follows: identify mutually inclusive and exclusive events and non-mutually inclusive and exclusive events; determining orderings or possible outcomes using permutations or combinations; calculate single and multiple event probabilities.

In considering how the other units relate to the unit of my focus, I note the following. Introducing the Pythagorean Theorem in Unit 3 readies students for its applications in Unit 4. The section, "Operations with Integers" in Unit 1, provides students with the prerequisite skills for evaluating formulas for area, surface area and volume with ease. In Unit 3, however, scaling is applied to two-dimensional figures. In Unit 4, students struggle with scaling objects of three-dimensions. Perhaps, including in Unit 3 some scaling of three-dimensional bodies (without volume concentrations) would help students make the connection. This would provide a smoother transition between Units 3 and 4. Last but not least, in Unit 4, students might be introduced to the numerical patterns that represent the relationships between dimensional changes and volume changes. This could provide students a smoother transition into numerical patterns, which are studied in Units 5 and 6.

CHAPTER 2 – ANALYZING UNIT 4: MEASUREMENT AND GEOMETRY

Unit 4 concerns two main themes: measurement and geometry. The central focus is on three-dimensional bodies, their characteristics, how their structure may be visualized and how their features (area and volume) can be measured. Objectives of primary importance are as follows. Students must be able to:

- Demonstrate an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures).
- Demonstrate the connection of measurement to the other strands and to real-life situations.
- Construct two- and three-dimensional models.
- Identify, describe, compare, construct, and classify geometric figures and concepts.
- Apply the concepts of length, area, surface area, volume, and rate to real-world experiences.

This unit contains active investigations that provide students opportunities to explore problems and to communicate their solutions. I will now discuss the mathematical structures interwoven into the activities that I developed.

Conceptualizing Units of Measure

What is a unit? A unit is a fixed quantity of a given kind by means of which we measure other quantities of the same kind. (This is not to be confused with the meaning of the term in abstract algebra, where "unit" means "has a multiplicative inverse" (Weisstein 2010)). Given a particular unit, a unit count can be used to describe the size of a body. Numbers have a meaning in measurement as a tally of units or parts of units. Therefore, numbers and units are not one and the same but are, rather, distinct entities.

Measurement makes the connection between numbers and units. It is the process of "identifying a unit of measure[,] subdividing . . . [an object's measurable attribute] by that unit,

[and translating the unit successively] alongside the [measured attribute, until no more fit]" (Clements 2003). To grasp this, students must first understand "length as a distance that can be subdivided" into congruent parts (Clements 2003). They must also understand that fitting units successively into a measured attribute, without gaps or overlaps, is a process of division (Clements 2003). Figure 2.1 illustrates this process.

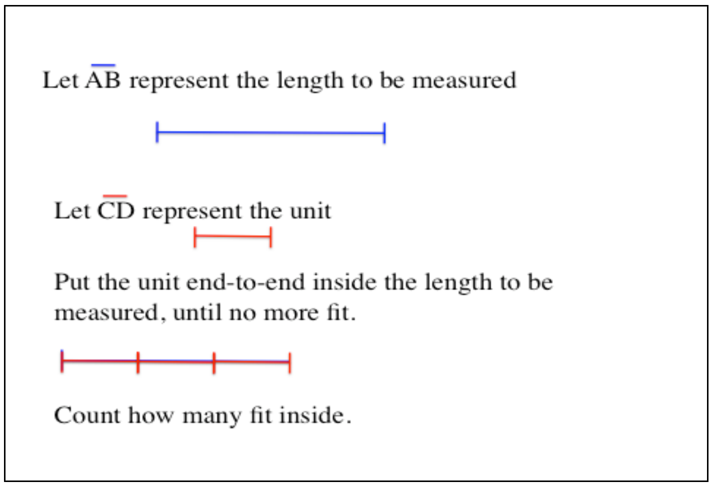


Figure 2.1 – The Concept of Measuring

In situations where a particular unit cannot equally subdivide a measured length, smaller units are used for better approximations.

Conceptualizing Area

Area is a quantity that represents the size or magnitude of an enclosed part of a two-dimensional surface (Lebesgue 1966). Magnitude is a property of an object's size that it retains when it is cut into pieces and/or moved around. Magnitude itself is not a number, but it can be measured, and a number assigned to it, after a choice of unit. Geometric objects of each dimension have magnitude in their dimensions. Lines have length, planar regions have area, and three-dimensional bodies have volume. Just as linear comparisons created a need for quantifying lengths to describe the magnitudes of distinct one-dimensional figures, similar means of

quantifying an object's area are applied to figures of two-dimensions, and bodies of three-dimensions have volumes that are measured (Lebesgue 1966).

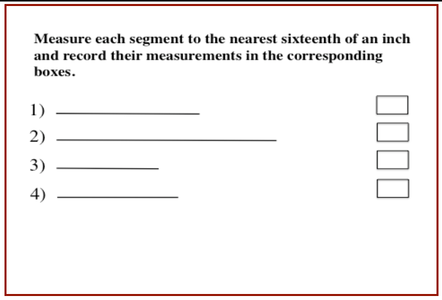


Figure 2.2 - Lengths of Line Segments

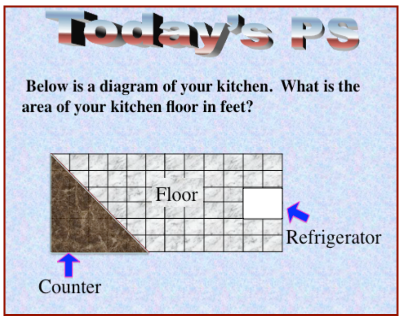


Figure 2.3– Areas of a Polygons

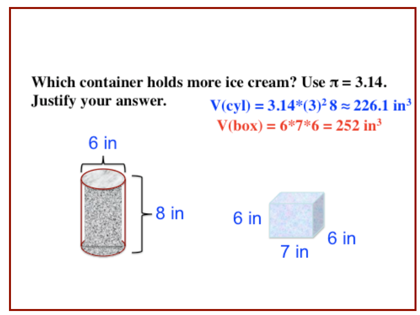


Figure 2.4 – Volumes of a Polyhedrons

Area measurement is the process of successively iterating square units to fill a plane region without gaps or overlaps, until no more fit (Clements 1999). This is called tiling. Area can also be conceptualized as the subdivision of two-dimensional regions into congruent area units (usually square units). However, some regions (for example, curved regions) cannot be completely covered by a subdivision of square units. In this case, smaller units are used to provide better approximations of area.

In my classrooms, students began Unit 4 by recalling the following properties of area, which I have adopted from Lebesgue.

1. The area numbers assigned when we measure areas are completely determined by the choice of unit for measuring and the size of the region being measured.

2. Once a unit of area has been chosen, to each polygon corresponds a positive number, which we call its area.

From Figure 2.5, students could see the visual properties of this idea, as it relates to the two-dimensional figures studied in this unit. Each circle represents a specific family, category or domain of polygons that enclose two-dimensional regions, which we can quantify using "positive real numbers" to represent their sizes or magnitudes.

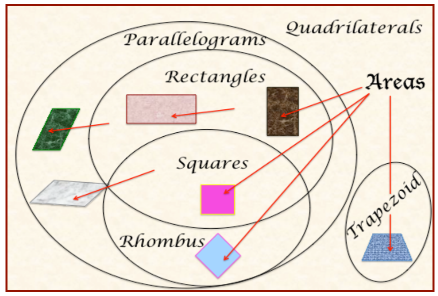


Figure 2.5 – The Domain of Quadrilaterals

3. The area number assigned to a domain formed by the union of two disjoint domains is the sum of the two area numbers of the two domains.

4. Any two congruent regions have equal area numbers.

More concretely, area depends on size and shape and not where an object is located in space. In one Unit 4 exercise (see Figure 2.6), students applied these ideas. Students found the area (number) of the star by performing the following steps: First, they cut it into seven pieces, namely one hexagon and six congruent triangles. Second, they divided the hexagon into two congruent trapezoids. Third, they calculated the area of each trapezoid. Fourth, they determined the area of the hexagon by summing the areas of the trapezoids. Finally, they added to areas of the six triangles together with the area of the hexagon to find the total area of the diagram.

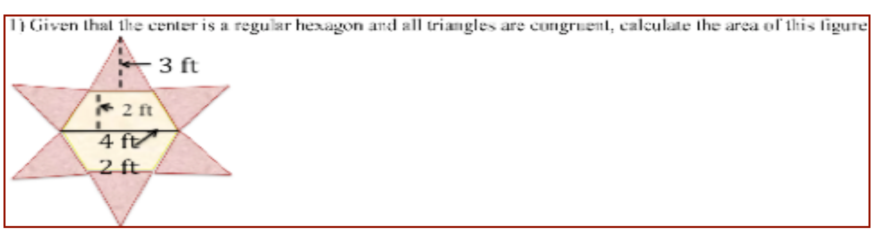


Figure 2.6 – Determining Area using Partitions

Conceptualizing Volume

What is volume? How is it measured? Volume is a feature of a body, reflecting the amount of three-dimensional space it consumes. It remains unchanged if the body is divided, provided we keep all the pieces. Volume is approximated by fitting copies of a three-dimensional unit (for example, a cube) inside a body and counting the number of units (or cubes) to yield a volume number. This process is illustrated in Figure 2.7.

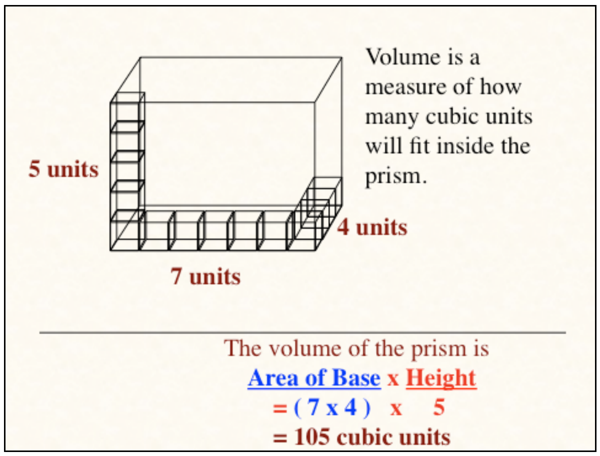


Figure 2.7 – Conceptualizing Volume

Just as linear measurements are iterations of one-dimensional units (u) and area measurements are iterations of two-dimensional or square units (u^2), volume measurements are iterations of cubic units (u^3).

Evaluating Strategies for Teaching Area and Volume

The Comprehensive Curriculum recommends rote memorization to help students familiarize themselves with area and volume formulas. I implemented this approach with students in my control group. They used index cards, called "formula cards", to learn formulas for area, perimeter, surface area, and volume. In compliance with the guidelines for this activity, these cards provided only a brief summary of learned content and did not to include concrete examples.

As a result of this approach, students acquired proficiency in evaluating surface area and

volume formulas. However, they struggled for weeks to determine which formulas to use, given real-life situations. They were unable to conceptualize the meaning of surface area and volume and were, therefore, unable to apply them in a practical sense.

On the other hand, one of the intervention strategies that I used was very effective in helping my students visualize surface area. Students traced the faces of a rectangular prism, creating its two-dimensional layout (or net). Next, they measured the dimensions and calculated the area of each face. Finally, they added the areas of the faces to determine the surface area of the object.

Shared Concerns

My colleagues and I shared a few concerns about the Comprehensive Curriculum. First, a common concern was that it does not provide sufficient time to effectively review with students prerequisite knowledge and skills, given that they come exceptionally deficient in these areas. The table below is the "Suggested Pacing for Unit 4".

Table 2.1 – Suggested Pacing for Unit 4

Day	**Lesson**	**Resources**
1	Review: Areas of Triangles, Quadrilaterals, and Circles	10.1, 10.2
2	Review: Areas of Triangles, Quadrilaterals, and Circles	10.1, 10.2
3	Introduction to Solids (Terminologies, Characteristics, & Parts of Solids)	10.3
4	Nets (Activity 9: The Net & Activity 13: Netting the Cubes *optional*)	10.4
5	Surface Area of Prisms	10.4
6	Volume of Prisms (Activity 1: Volume and Surface Area)	10.6
7	Volume of Prisms	10.6
8	Surface Area of Cylinders	10.4
9	Volume of Cylinders (Activity 5: Cylinders optional)	10.6
10	Volume Relationships (Activity 2: Rectangular Prism)	
11	Changes in Volume & Dimensions (Activity 10: Changing Areas and Volumes)	
12	Changes in Volume & Dimensions (Activity 15: Dimensions and Surface Area)	

Table continued on next page

13	Review: Common Units (metric & customary)	
14	Volume Unit Conversions & Estimation (Activity 8: Common Containers & Activity 14: Packaging Costs)	
15	Surface Area of Pyramid & Cones	10.5
16	Volumes of Pyramid & Cones (Activity 6: Pyramids and Cones & Activity 7: Comparing Cones optional)	10.7
17	Probability (Activity 3: What's the Probability &Activity 4: Odd Volume, Is it Fair? Optional)	12.7, 12.8
18	Density (Activity 11: Density & Activity 12: Different Densities optional)	
19	Review & Catch-Up	
20	Unit 4 Assessment (Introduction to Unit 5)	

The "Suggested Pacing" from Unit 4 allows only two days for students to refresh their acquaintance with computing areas, perimeters, and circumferences of two-dimensional shapes. It does not include the time for reviewing customary and metric units. Quoted below are some items from its "Prerequisite Skills/Knowledge" and its "Intervention Strategies":

> Teachers need to ensure these concepts are mastered before going into the curriculum activities or review these concepts as needed. A review during the warm-up may be sufficient. (p. 150)
>
> In prior courses (6th & 7th grade Math), students should have practiced converting customary and metric units within the same system for area. Mastery of this concept will make conversions for volume (this unit) easier to grasp. If necessary, teachers may want to review the units in both system, linear conversions (i.e. inches to feet) and then go into square conversions (i.e. cubic inches to cubic feet) before moving on to cubic conversions. (p. 151)
>
> A day or two (at most) can be spent on reviewing finding areas of 2-dimensional shapes. Students must know this skill in order to find the surface area. (p. 151)

Speaking for myself as well as for my colleagues, most of the students we teach come extremely deficient in these skills, which usually means that we need about a week to help them reach an acceptable level of proficiency. Considering this, it would be beneficial for the Comprehensive Curriculum to incorporate model lessons and activities that address unit conversions as well as those skills that are crucial to helping students deal with more complex

applications. If students are not provided the opportunity to explore the conceptual meaning of measuring objects in one-dimension, they are not likely to understand what it means to measure figures in two-dimensions, let alone, three-dimensions. Therefore, I judged that it would be useful to incorporate lessons or activities that provide students with visual tools to help them gain an intuitive sense of what measurement is and how to employ measurement techniques.

The second concern that my colleagues and I had was that some of the GLEs are nonrelated to the central theme of this unit. The following GLEs were labeled as relevant, irrelevant, or general in the introduction to Unit 4. The bold text represents relevant skills of higher conceptual involvement.

Table 2.2 – GLE's for Unit 4

(Relevant Skills/Concepts)
20. Identify and select appropriate units for measuring volume
27. Construct polyhedra using 2-dimensional patterns (nets)
17. Determine the volume and surface area of prisms and cylinders
19. Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating
21. Compare and estimate measurements of volume and capacity within and between the U.S. and metric systems
22. Convert units of volume/capacity within systems for U.S. and metric units
32. Model and explain the relationship between the dimensions of a rectangular prism and its volume (i.e., how scale change in linear dimension(s) affects volume)
48. Illustrate patterns of change in dimension(s) and corresponding changes in volumes of rectangular solids.
(Irrelevant Skills/Concepts)
18. Apply rate of change in real-life problems, including density, velocity, and international monetary conversions
45. Calculate, illustrate, and apply single- and multiple-event probabilities, including mutually exclusive, independent events and non-mutually exclusive, dependent events
(General Skills/Concepts)
33. Graph solutions to real-life problems on the coordinate plane
39. Analyze and make predictions from discovered data patterns

Third, the random listing of the GLEs does not provide structure for sequencing lessons. When my colleagues and I (the 8th grade Math team) gather for lesson planning, we always discussed the sequence in which the GLE(s) should be taught. This usually consumes a considerable amount of planning time. Therefore, I felt that it would be beneficial to list the GLEs in the order that we settled upon.

Table 2.3 – Arranged GLE's for Unit 4

19. Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating (M-2-M) (G-1-M)
20. Identify and select appropriate units for measuring volume (M-3-M)
27. Construct polyhedra using 2-dimensional patterns (nets) (G-4-M) *(Visualize the surface of a polyhedron.)*
17. Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M)
32. Model and explain the relationship between the dimensions of a rectangular prism–solid and its volume (i.e., how scale change in linear dimension(s) affects volume) (G-5-M)
48. Illustrate patterns of change in dimension(s) and corresponding changes in volumes of rectangular solids (P-3-M)
21. Compare and estimate measurements of volume and capacity within and between the U.S. and metric systems (M-4-M) (G-1-M)
22. Convert units of volume/capacity within systems for U.S. and metric units (M-5-M)

In planning my own lessons, I also included supporting GLEs to help tie in related skills from previous units. This helped students see the connection between the units.

Table 2.4 – Supporting GLE's for Unit 4

10. Write real-life meanings of expressions and equations involving rational numbers and variables (A-1-M) (A-5-M)
16. Explain and formulate generalizations about how a change in one variable results in a change in another variable (A-4-M)
18. Apply rate of change in real-life problems, including density, velocity, and international monetary conversions (M-1-M) (N-8-M) (M-6-M)
33. Graph solutions to real-life problems on the coordinate plane (G-6-M)
39. Analyze and make predictions from discovered data patterns (D-2-M)

Last but not least, probability and density are thematically distinct from the main focus. Extraneous elements such as these were often spotted throughout the units and have been frequently addressed amongst my colleagues. They have reasoned that, perhaps, this is to ensure that students are exposed to all pertinent content prior to LEAP testing.

In summary, I believe the following ideas may be useful in improving the effectiveness of this unit: First, allow more time for reviewing prerequisite knowledge and/or skills prior to introducing nets, surface area and volume. Second, include measurement in the beginning of the unit together with model lessons and activities for measuring in customary and metric units. Third, incorporate unit conversions across different systems of measure. Fourth, list only the GLEs that support the unit's focus. Fifth, include supporting GLEs to help tie in concepts from previous units. Sixth, arrange the GLEs in the order in which they need to be taught. Last but not least, teach the activities in a reasonable order.

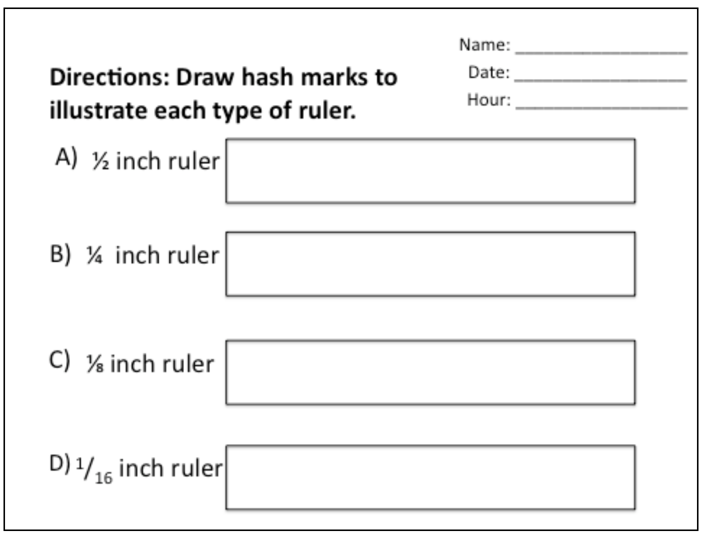
CHAPTER 3 – MY SUPPLEMENTS

Mathematical Structures in the Activities I Implemented and Why I Made Them

The following supplements were created to develop students' conceptual understanding of measurement and geometry.

Activities for Prerequisite Knowledge and Skills

1) I designed and implemented the "Ruler Construction" activity (shown in Figure 3.1) to help students conceptualize units, partial units and their applications. Students constructed the following types of rulers by repeatedly partitioning subunits: a ½-inch, a ¼-inch, an $^1/_8$-inch and a $^1/_{16}$-inch ruler. Upon completion of this activity, they were able to iterate units and partial units.


Name: ________________
Date: ________________
Hour: ________________

Directions: Draw hash marks to illustrate each type of ruler.

A) ½ inch ruler

B) ¼ inch ruler

C) ⅛ inch ruler

D) $^1/_{16}$ inch ruler

Figure 3.1 – The "Ruler Construction" Activity

2) A set of rectangles and triangles were constructed to provide students the opportunity to explore their relationships. Students used these objects to demonstrate how their area formulas were derived. Figure 3.2 illustrates the process of how students arranged the triangles in such a way that the smaller triangles covered the largest one. Viewing this enabled students to understand that the areas of the smaller triangles add together to give the area of the largest one.

Therefore, the area of a triangle, that has an interior altitude, is half the area of the rectangle drawn around it.

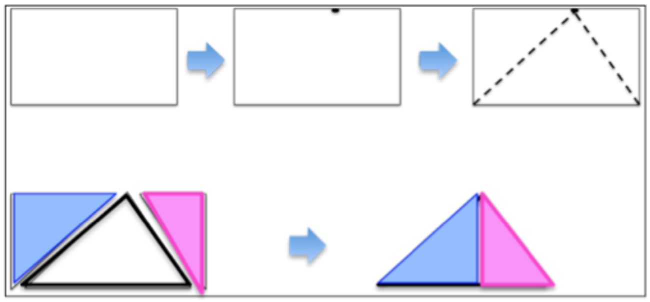


Figure 3.2 – Visualizing the Area Formula of a Triangle

3) The following problems were borrowed from NCTM's Illuminations website and were presented to challenge students' conceptual understanding of area.

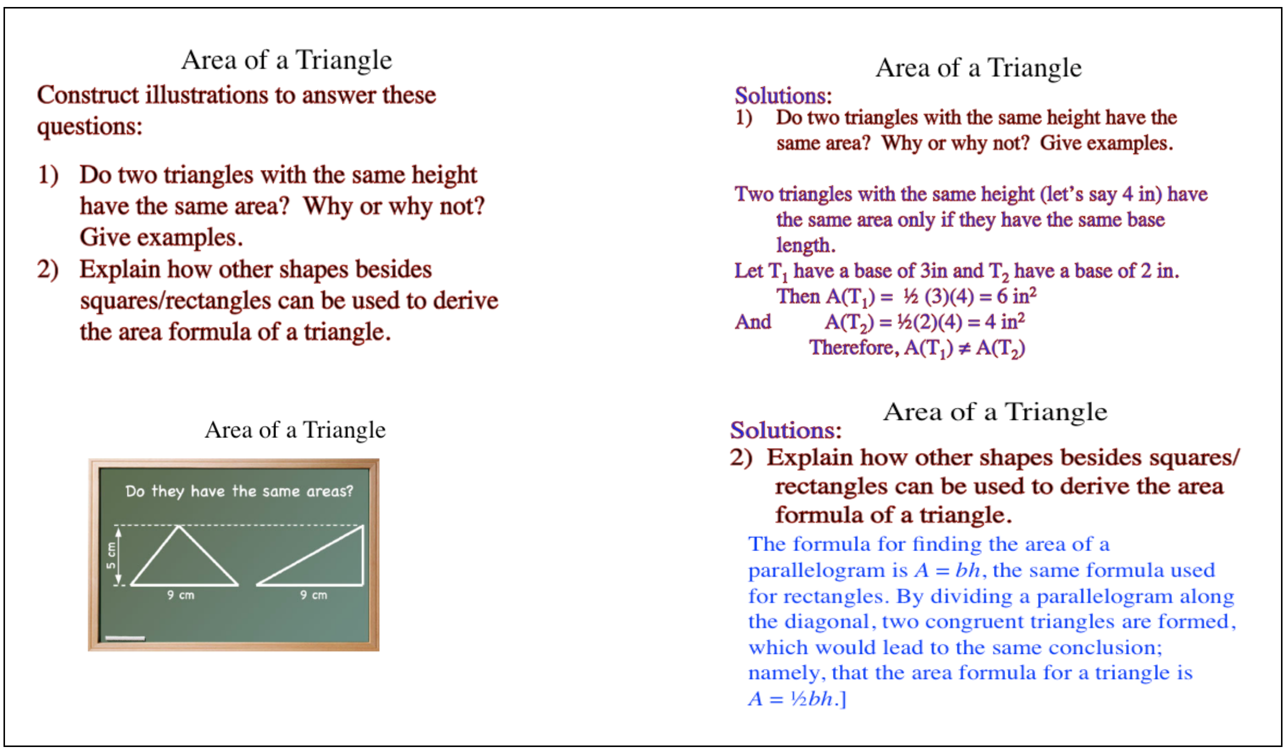


Figure 3.3 – Analyzing How Changing Dimensions Affect Area

4) A set of rectangles (illustrated in Figure 3.4) was constructed to provide students the opportunity to explore the relationships between area and perimeter. Students measured the dimensions of each rectangle, calculated the area and perimeter, recorded the data in their "Area and Perimeter" charts, and from the data, derived numerical patterns. Working with the charts

helped them quickly realize that the rectangles had the same perimeter (40 square inches) but different areas. From this, they concluded that objects can have the same perimeter but different areas.

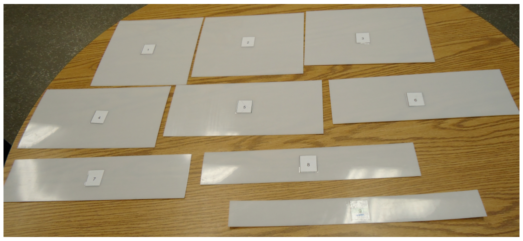


Figure 3.4 – A Set of Rectangles with the Same Perimeter but Different Areas

Below is the "Area and Perimeter Chart" used to analyze the data.

Table 3.1 – The "Area and Perimeter" Chart

Rectangle #	Dimensions	Perimeter	Area
1	10 in x 10 in	40 in	100 in^2
2	9 in x 11 in	40 in	99 in^2
3	8 in x 12 in	40 in	96 in^2
4	7 in x 13 in	40 in	91 in^2
5	6 in x 14 in	40 in	84 in^2
6	5 in x 15 in	40 in	75 in^2
7	4 in x 16 in	40 in	48 in^2
8	3 in x 17 in	40 in	51 in^2
9	2 in x 18 in	40 in	36 in^2

Figure 3.5 illustrates how the rectangles were arranged to help students visualize a numerical pattern resulting from the difference of their areas, which are consecutive squares.

Figure 3.5 – A Set of Rectangles Arranged to Illustrate a Numerical Pattern

5) The activity "Am I True or False?" required students to provide examples that supported their opinion of whether or not they believed the following statements are true or false.

Statement #1: Rectangles must have the same dimensions to have the same area.
Statement #2: Rectangles with the same area have the same perimeter.

Figure 3.6 is a sample of student work.

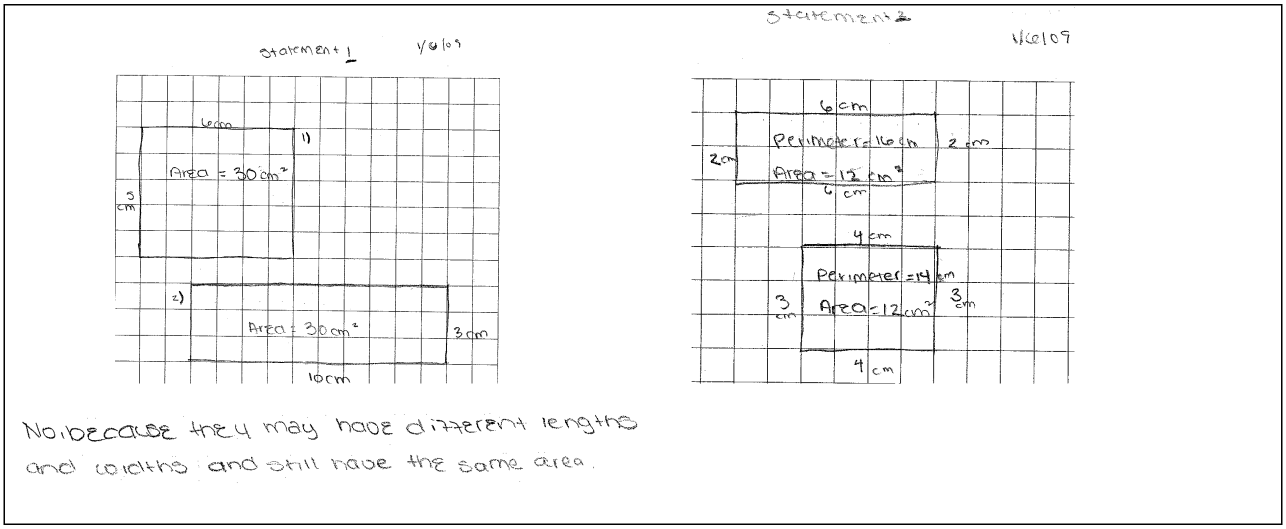


Figure 3.6 – A Sample of Student Work

6) Figure 3.7 illustrates an animated Powerpoint slideshow that I used to help students visualize the relationship between rectangles and parallelograms. From this, students learned the conservation property of area, which states that rearranging the parts of a figure's does not change its area. Therefore, the area of the parallelogram is equal to the area of its respective rectangle.

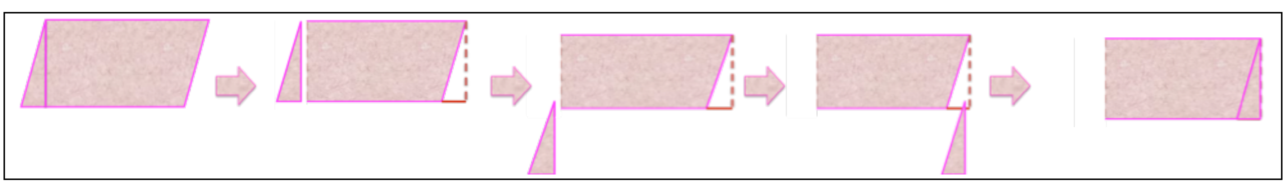


Figure 3.7 – Visualizing the Area Formula of a Parallelogram

Activities for Conceptualizing Surface Area and Volume

1) Nets of prisms, cylinders and cones were used to help students conceptualize surface area. In one demonstration, I flattened a cubic cardboard box of edge length one foot to create a large net of unit squares. In another demonstration, six distinct rectangular prism nets were created out of square wooden pieces. Presenting these variations provided students the opportunity to make comparisons and helped them describe the characteristics of rectangular prisms from their nets. In yet another demonstration, cylinder nets helped students see that the surface of its lateral part is a rectangle. Finally, triangular prism nets helped students realize that a triangular prism consists of three rectangular faces and a pair of parallel congruent triangular bases. Students determined the surface areas of prisms, pyramids, and cylinders by measuring the dimensions of each face, calculating their areas, and then, adding the areas to find the total area (or surface area). Figure 3.8 contains samples of student work.

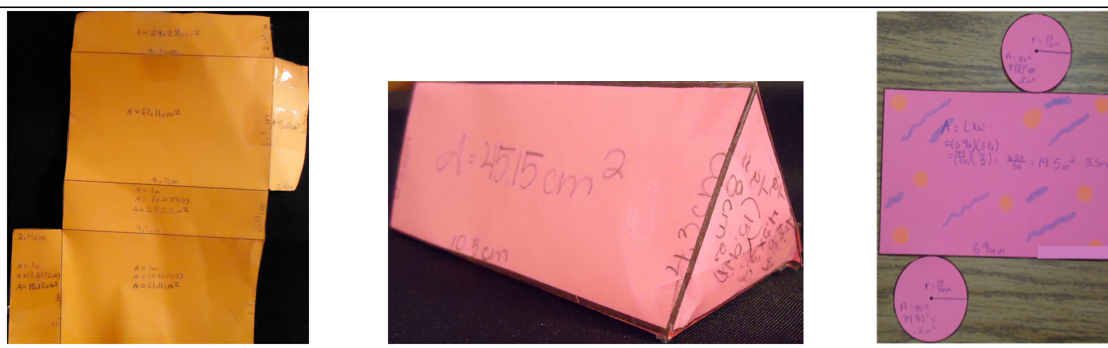


Figure 3.8 – Determining the Surface Area of Polyhedrons

2) To help students conceptualize volume and its applications, I constructed a set of cubes. In this activity, students iterated the one-inch cubes to determine how many would fill a three-inch cube or and a six-inch cube. Iterating volume units helped them realize the application of adding, subtracting, or dividing cubic units.

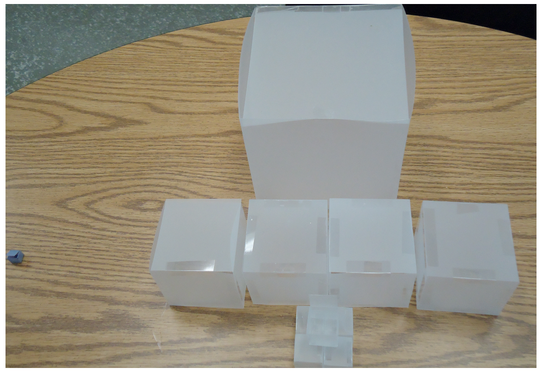


Figure 3.9 - A Set of Cubes

My Goals

My goal for this unit was to provide students the opportunity to explore geometric and measurement concepts through experimental learning. Given that many middle schoolers have difficulty visualizing three-dimensional figures, unless they are given objects to explore and feel, I felt that it would be beneficial to provide them the opportunity to learn concepts by seeing, manipulating and measuring real things.

My objectives for this unit were as follows. Students must demonstrate an intuitive understanding of what units are and how to apply them. They must be well acquainted with units of measure, subdividing them, and performing unit conversions within the same system of measure. They must be able to determine which mathematical operations take place when determining linear, area and volume measures from data about figures. They must be able to demonstrate an intuitive sense of surface area and volume and how they are used in today's society.

To accomplish these objectives, students took part in an array of activities that made learning experience meaningful and concrete. The activities provided students the tools to explore and to help them understand a part of the world in which they live.

Testing

Prior to the introduction of Unit 4, students were administered the "Unit 4 Pretest". Upon finishing the unit, students were administered the Unit 4 Edusoft exam as their posttest. I constructed the pretest using the Enhanced Assessment of Grade-Level Expectations, formally known as the Eagle System. This online assessment tool was developed by the Louisiana State Department of Education to provide primary and secondary educators access to resources that are specifically aligned to our state's GLEs. I selected questions that reflected the main objectives for this unit. This was to ensure that the pretest was in close alignment with the main GLEs for this unit (LDE 2010).

The Edusoft Unit 4 exam was used as their Posttest because it is primarily used to assess whether or not students are meeting state standards within a particular unit (LDE 2010). Edusoft exams assess specific areas in which students are deficient to help teachers pinpoint the content that needs to be retaught.

CHAPTER 4 – REFLECTIONS ON IMPLEMENTING THE MODIFIED APPROACH

The following are some qualitative observations concerning my interactions with the students in my experimental group, and some comparisons with the control group.

Lesson A: Developing the Concept of Measurement

The purpose of this activity was to help students conceptualize units and partial units by repeatedly subdividing units and subunits into equally sized partitions. I assisted the students in constructing their ½-inch, ¼-inch, $^{1}/_{8}$-inch and $^{1}/_{16}$-inch rulers. Upon finishing the activity, they were able to conceptualize how one unit (a particular length) can be partitioned into congruent parts. Additionally, they iterated these units and their fractional parts.

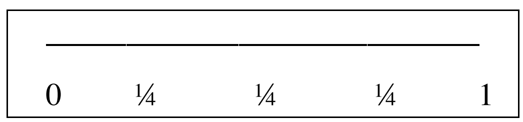


Figure 4.1 - A Sample of Student A's work

According to Figure 4.1, Student A knew that a quarter inch ruler was divided into four equal parts. However, the numbers assigned to each of the parts were not consecutive. I asked him the following questions:

> **Teacher**: "If the distance between zero and the first hash mark is ¼ of an inch, is the distance between zero and the second hash mark also ¼?"
> **Student A**: "No." [He quickly changed his numeration to 0, ¼, $^{2}/_{4}$, $^{3}/_{4}$, 1.]
>
> **Teacher**: "Which number represents my halfway mark?"
>
> **Student A**: [puzzled]
>
> **Teacher**: [Relating to his prior knowledge] "If I simplify $^{2}/_{4}$, what is it?"
>
> **Student A**: "½"
>
> **Teacher**: "Therefore, which is my halfway mark?"
>
> **Student A**: [He pointed to $^{2}/_{4}$, and then, wrote ½ under it.]

> **Teacher**: "Therefore, this mark represents the distance halfway between 0 and 1."

Next, I instructed the students to construct an eighth inch ruler.

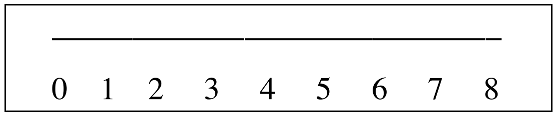


Figure 4.2 - A Sample of Student B's work

> **Teacher**: "Do these numbers represent the distance between 0 to 1 inch? It's good you know that there are 8 equally spaced sections. But remember, the distance is between 0 and 1 inch and not between 1 and 8 inches."
>
> **Student B**: [thinking]
>
> **Teacher**: "What if I asked you to divide a pie into 8 equal parts, what does one slice of the pie represent?"
>
> **Student B**: "One eighth of a pie"
>
> **Teacher**: "Therefore, if the distance is between 0 and 1 inch and we divided the distance into 8 equal sections, then how would that look like to you?"
>
> **Student B**: [She renumbered each division as $0, {}^{1}/_{8}, {}^{2}/_{8}, \ldots, {}^{7}/_{8}, {}^{8}/_{8}$.]

At this point, students were able to demonstrate their understanding of how partitioning can be applied to any standard unit (be it inches, feet, yards or a mile). My next question was, "How well can students employ units given the knowledge they currently own?"

I set up stations, whereby each station had at least two polyethylene rectangles. At the stations, students measured rectangles with the following dimensions: 10 inch by 10 inch, 9 inch by 11 inch, 8 inch by 12 inch, 7 inch by 13 inch, 6 in by 14 inch, 5 inch by 15 inch, 4 inch by 16 inch, 3 inch by 17 inch, 2 inch by 18 inch. They were given twelve-inch rulers to measure the dimensions of each rectangle. The time constraint was two minutes prior to group rotation. Some rectangles had side lengths longer than twelve inches. This provided me an opportunity to identify those students who knew when to iterate their rulers. Most of the students successfully

performed this process without my assistance. However, a few requested, "I need another ruler." For those, I demonstrated to them how to iterate the ruler along the desired attribute.

After the students recorded the dimensions of each rectangle, they computed their areas and analyzed the data. Some students made the general assertion that as the heights of the rectangles were decreasing by one unit, their bases were increasing by one unit, thus, generating the following algebraic sequence.

$$
\begin{gathered}
10 * 10 \\
(10-1)(10+1) \\
(10-2)(10+2) \\
\vdots
\end{gathered}
$$

Additionally, they mentioned that the areas were decreasing as follows: 100 in^2, 99 in^2, 96 in^2, etc. When I asked them to provide a numerical pattern that represents the area changes, one student generated the pattern 1, 3, 5, etc. He explained that the difference between 100 square units and 99 square units (from a 10 inch by 10 inch square and a 9 inch by 11 inch rectangle respectively) is 1 square unit. Likewise, the difference between 99 square units and 96 square units (from a 9 inch by 11 inch rectangle and an 8 inch by 12 inch respectively) is 3 square units, and so on. Algebraically, he realized that the area differences can be rewritten as a sum of consecutive odd numbers:

$$
\begin{aligned}
&A(1^{\text{st}}\text{ rectangle}) - A(2^{\text{nd}}\text{ rectangle}) = 100 - 99 = 1 \\
&A(1^{\text{st}}\text{ rectangle}) - A(3^{\text{rd}}\text{ rectangle}) = 100 - 96 = 4 = 1 + 3 \\
&A(1^{\text{st}}\text{ rectangle}) - A(4^{\text{th}}\text{ rectangle}) = 100 - 91 = 9 = 1 + 3 + 5 \\
&\qquad\vdots
\end{aligned}
$$

The equation for this sequence is $(n + 1)^2 - n^2 = 2n + 1$. As we continued our discussion, I asked him if he could determine another pattern that would represent the decrease in area. My "what if" question was, "If we keep 100 square units as our reference number, then, by what

pattern would the areas be decreasing?" One student asserted, "1, 4, 9, 16, ..." I asked, "What kinds of numbers are these?" The students replied, "square numbers!" From this, we concluded that as one dimension decreased by one unit and the other increased by one unit, the difference in their areas were consecutive square numbers. Also, they noted that these rectangles had the same perimeter even though they had different areas. From this, they concluded that even though shapes may have the same perimeter, they may not have the same area and vice versa.

The next activity presented the following statement: "Rectangles with the same area have the same perimeter. Prove this statement false by creating two rectangles that have the same area but with different perimeters". The following are a couple of examples they produced:

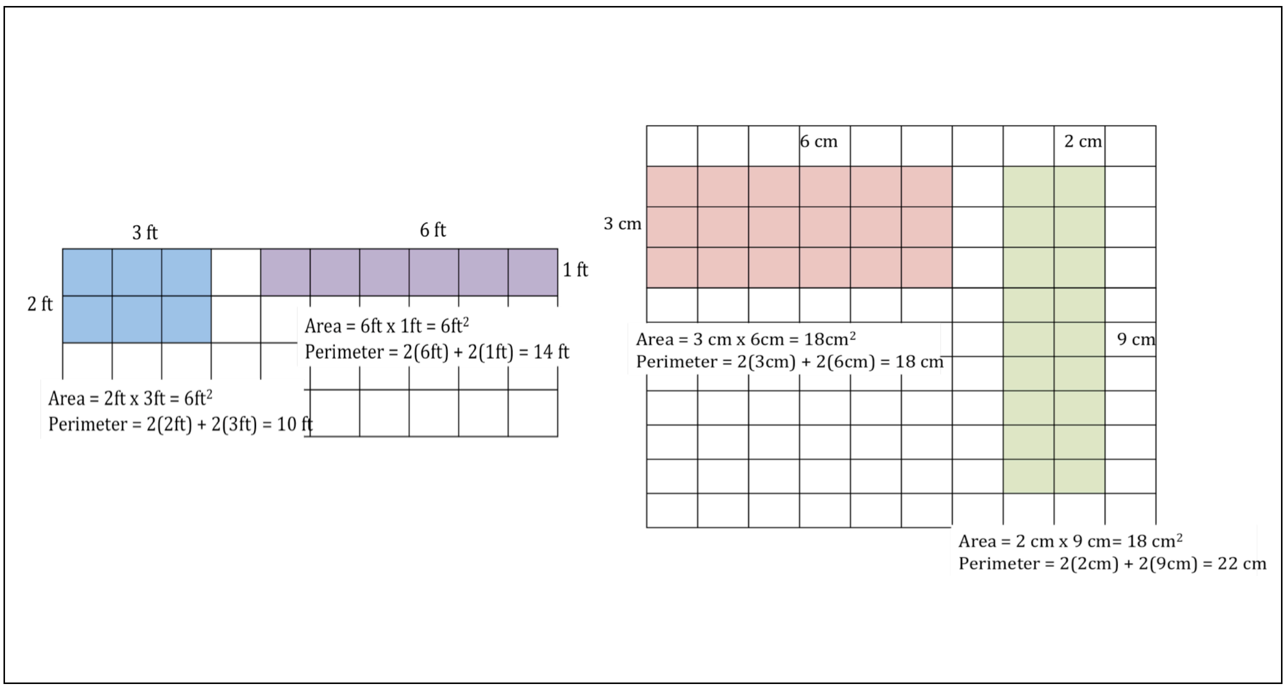


Figure 4.3 - A Sample of Student C's Work (left) and Student D's Work (right)

Lesson B: Seeing the Relationship between Rectangles and Triangles

In this activity, students were guided through clues and questioning as they used index cards to create a visual proof that the area of a triangle is half the area of a rectangle. They were given the following instructions: 1) With the longest side serving as the base of your rectangle, mark a point anywhere along the top of your note card; 2) Draw a line segment from the dot to

the bottom left hand corner and another to the bottom right hand corner of your card; 3) Cut along the lines (you should have three distinct triangles); 4) Position the triangles in such a way that would prove that the area of a triangle is half the area of its respective rectangle (all triangles must be used). Figure 4.4 below illustrates the process.

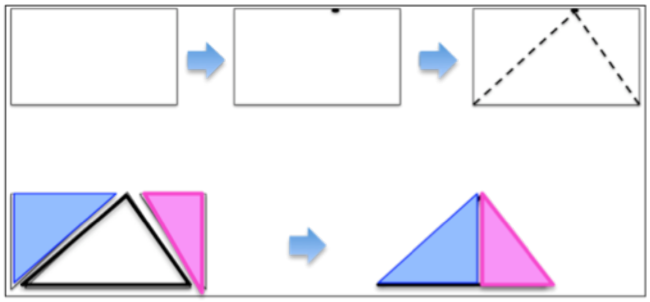


Figure 4.4 - A Sample of Student E's Work

As I circulated the room, I observed that some students used only two triangles to form a rectangle (as shown in Figure 4.5 below), arguing that one of those triangles would be half its area. I asked them if both triangles were congruent and if the newly formed shape was a rectangle. They quickly replied, "No," and rearranged the three shapes.

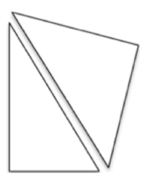


Figure 4.5 - A Sample of a Student's Work

> **Student A**: "Look, this is it!" [He recreated the original rectangle by placing the triangles back in their original position]
>
> **Teacher**: "Even though all three pieces create the original rectangle, this does not show me how the area formula of a triangle is derived from a given rectangle. How would you position all three pieces to show that the area of a triangle is half the area of its respective rectangle?"
>
> **Student B**: "Miss. Richard, look!" [He positioned the smaller triangles in such a way that they covered the largest triangle]
>
> **Teacher**: "Very good." [I addressed the class]. "So, what can we conclude about this demonstration?"

> The students replied, “The area of the big triangle is half the area of the rectangle.”
>
> **Teacher**: “That holds true for all triangles. Notice that your triangles may look different from you neighbor’s, but regardless of their difference in shape, you can still position the pieces in such a way that the two smaller pieces match the size of the largest piece, meaning that the area of the largest triangle is exactly half the area of the rectangle.”

Upon finishing the activity, students were able to explain how the area formula of a triangle was derived. They were able to visualize the relationship between rectangles and triangles.

By the end of the week, I was able to advance to nets with my 4th period, while my 2nd period struggled to differentiate the area formulas of the two-dimensional shapes we studied.

Lesson C: Developing the Concept of Surface Area

The purpose of this activity was to help students visualize the surface area of a three-dimensional object. I showed the students the net of a cube with dimensions 2 cm x 2 cm x 2 cm. I used four tiles to show them that the surface area of one face was exactly 4 square centimeters. Seeing this, some of my students reacted, “ohhhh . . . ”. Then, I presented the problem, “If it takes exactly four square tiles to cover one face, how many tiles would it take to cover all six faces, provided all the faces are congruent?” My students quickly answered, “24.”

Following this demonstration, my students selected two rectangular wooden blocks that had different dimensions. For each block, I assisted them in tracing all six faces on lineless paper. They measured the dimensions of each face in centimeters, and then, calculated the area of each face using the area formula $A = lw$. Some students continued to ask, “How do I find the surface area?” I explained that surface area is the sum of all the areas of each surface. There were many “Ahhh” and “Ah ha” moments. They, finally, made the connection.

Lesson D: Developing the Concept of Volume

In this activity, students compared the sizes of a one-inch cube to a three-inch cube. I, then, asked them to estimate how many of the one-inch cubes would it take to fill the three-inch cube. The students circulated the cubes around the room, placing the one-inch cube inside of the three-inch cube. One student said, “I think 27”. I asked her why. She replied: “Three times three times three is twenty-seven. Three times three is nine [which is the bottom layer of the three-inch cube and you need 3 layers of nine to fill the three-inch cube]. Therefore, twenty-seven of the smaller cubes makes twenty-seven cubic inches”.

Allowing the students to compare the cubes gave them an opportunity to take ownership of their discovery. They iterated cubic units to determine the volume of a larger cube. Additionally, students could visualize the relationship between the dimensions of a cube and its volume. “Three times three times three is 27,” one student replied. In abstract form, that is $V = l*w*h$.

Student Feedback

At the closing of Unit 4, I asked students in my 2nd period (the control group) to write an honest brief reflection about their learning experiences. Their responses were as follows: 1) “I found the measurement parts a little difficult”; 2) “I found the probability part kind of hard”; 3) “measurement” was difficult; 4) “Volume was easy [but] surface area was the hardest”; 5) “The part I found difficult is the metric and conversion unit. It’s confusing.”; 6) “The difficult part was converting”; 6) “The part I found difficult was the part when you have to convert the gallons to pints and others.” Students in my 4th period, on the other hand, expressed that they had a good grasp the content and that the lessons were “fun”. One student said that the Edusoft Unit 4 exam was a “piece of cake”. In fact, he scored one point shy of “Mastery”. I guess so since they made the scores to prove it!

CHAPTER 5 - ANALYZING PRETEST AND POSTTEST RESULTS

Analyzing Pretest Results

Prior to the introduction of Unit 4, students were given the "Unit 4 Pretest". The test results of both groups are illustrated in Figure 5.1, where the control group is the top plot and the experimental group is the bottom plot. The pretest results of both groups are similar.

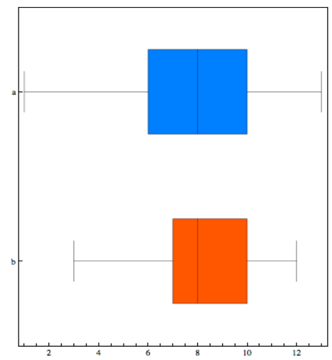


Figure 5.1 – Pretest Results (Box-and-Whisker Plots)

As seen in Figure 5.1, both groups had a median score of 8 and an upper quartile score of 10. This means that the top 50% of students made a score of at least an 8 in each group. Also, third quartile ranges are the same. In my control group, the top 25% of students scored in the range of 10 and 13. The top 25% of students in my experimental group scored in the range of 10 and 12. However, the lowest score from the control group was 1, whereas the lowest score from the experimental group was 3. The highest score from the control group was 14, whereas the highest score from the experimental group was 12.

Analyzing Posttest Results

Figure 5.2 shows the Posttest results. Once again, the top plot represents the control group and the bottom plot represents the experimental group.

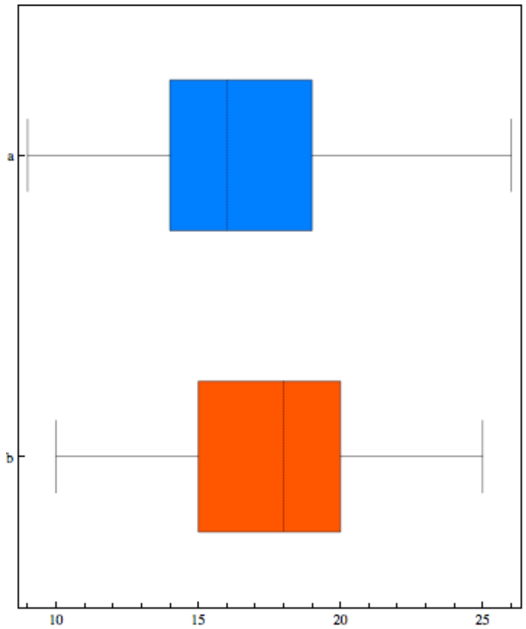


Figure 5.2 – Posttest Results (Box-and-Whisker Plots)

The scores from both groups fall within a similar range. However, the median in the top plot is higher. More than 50% of the experimental group scored at or above the proficient level of 17. If we take a closer look, we can see that their distribution of test scores is different. The distribution of scores within my control group is positively skewed, and in fact, there are more students who scored below the level of proficiency (a score of 17) than above it.

To visualize the distribution of the Posttest scores in more detail, I constructed histograms for both groups: the first graph represents the control group and the second graph represents the experimental group.

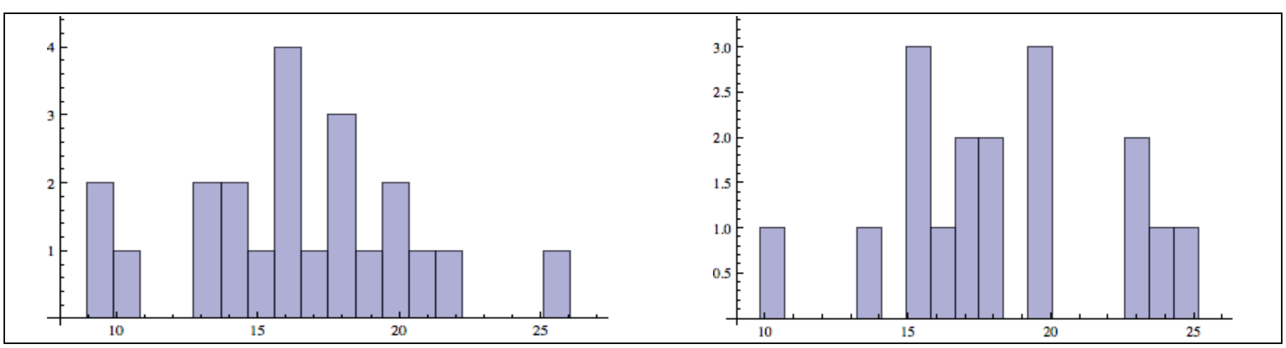


Figure 5.3 – Posttest Results for the Control Group (left) and the Experiment Group (right)

The histograms clearly show that a larger proportion of students in the control group scored below “Basic” (a score of 17). In fact, nine out of twenty-two (or 41%) scored “Basic”. Only one scored “Mastery”. In my experimental group, 9 out of 17 (or 53%) scored “Basic” and two scored “Mastery”. Considering that both groups had rather similar Pretest results, the differences in the posttest scores merit investigation.

The scatter plot below was constructed to help me analyze the relationship between the Pretest and Post-test scores of both groups. The *x*-axis represents the Pretest scores, and the *y*-axis represents the Posttest scores. The red squares represent my experimental group and the blue diamonds represent my control group. The purple figures represent score pairs that occurred in both groups.

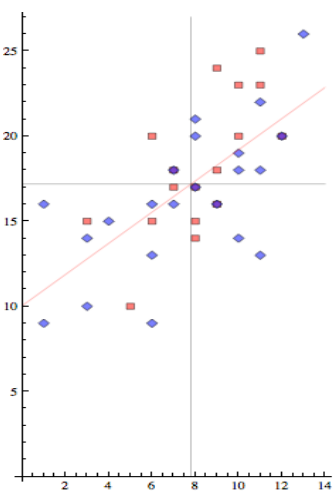


Figure 5.4 – Pretest and Posttest Results (Scatter Plot)

There is a positive correlation between the Pretest and Posttest scores. The regression line was $y = 10 + 0.92x$. The sum of the residuals for my control group was negative (-12.003), which indicates that there were more students who fell below the line of prediction than above it. On the contrary, the sum of the residuals for my experiment group was positive (12.003), which indicates that there were more students who scored above the line of prediction than below it.

My next question was, “How likely is it that a similar advantage for the experimental group would occur by chance, if in fact the curriculum I tried was no better? See the next section.

Significance Testing

I conducted the permutation test to determine the likelihood that a similar advantage for the experimental group would occur by chance. A permutation test, also called “an exact test” or a “randomization test”, is a statistical significance test that considers all possible outcomes (Wikipedia 2010). It re-shuffles “observed data [values] to determine how unusual an observed outcome is” (Wikipedia 2010). It is also used to determine the strength of the evidence when trying to decide if it supports the conclusion of an effect from a treatment.

For example, suppose two populations have been identified and random samples of sizes n_A and n_B have been chosen from each. Also, suppose that the sample means are m_A and m_B. A statistical test, such as the t-test, when applied to this data, will tell me the strength of the evidence that we have here for the hypothesis that the original groups have different means. The t-test requires certain assumptions about the populations, as well as truly random samples. The permutation test does not require these assumptions. It simply tells me, given the same students as I had in both groups and the scores they actually had, how likely would it be for a difference as great as the one I observed to arise by a random reclassification in control and experimental groups. The p-value that I obtained from the permutation test was about 0.10 or 10%. A p-value of 0.10 indicates some evidence against the hypothesis that my curriculum produced no advantages. Considering the time shortage and the motivation that the experimental group exhibited, I was pleased that within a three-week period, test results suggested a possible advantage to using the experimental supplements.

SUMMARY

The EBRPSS 8^{th} Grade Mathematics Comprehensive Curriculum has been undergoing revisions since its creation. This framework embodies a wealth of state standards. However, some of its structures and content can be modified to better accommodate both teachers and students. For example, the units need to be more coherent. Unit 4's content is independent from the others. Second, more time needs be allowed for reviewing prerequisite knowledge and skills, considering students' level of deficiency. Third, units should contain only the GLEs that reflect the unit's focus. Fourth, including model lessons and arranging the activities in the order of which they should be taught would be of great help to teachers.

Reflecting upon my observations in Unit 4, my 2^{nd} period (who received lessons and activities from the Comprehensive Curriculum) primarily struggled with measurement and unit conversions. They found it "difficult". Students in my 4^{th} period, on the other hand, demonstrated an intuitive understanding of measurement. They exhibited more motivation and excitement throughout this unit than my 2^{nd} period. Pretest and Posttest results of my 2^{nd} period (the control group) and my 4^{th} period (the experiment group) show an advantage for students who used the experimental materials. The permutation test was used to determine if the advantage was significant. The p-value was about 0.10. This is not as small as customarily demanded (by journal editors, for example), but it does suggest some evidence that the experimental materials merit study. I was pleased that within a three-week period, test results indicated some advantage to using the experimental supplements that I created. However, I cannot help but wonder if more than three weeks had been granted for content mastery, would there have been a greater difference in the average learning gains of both groups.

I will perform another trial of this experiment because the results merit further investigation. I would like to determine to what extend time shortage affected the outcome of my experiment. Nevertheless, these results were very encouraging. I hope that the supplements I created would be of some benefit to improving the Comprehensive Curriculum's effectiveness in Unit 4: Measurement and Geometry.

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APPENDIX A: NCTM FOCUS SKILLS

In grades 5-8, the mathematics curriculum should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can--

- identify, describe, compare, and classify geometric figures;
- visualize and represent geometric figures with special attention to developing spatial sense;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- develop an appreciation of geometry as a means of describing the physical world.

Focus

Geometry is grasping space . . . that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it. (Freudenthal 1973, p. 403).

The study of geometry helps students represent and make sense of the world. Geometric models provide a perspective from which students can analyze and solve problems, and geometric interpretations can help make an abstract (symbolic) representation more easily understood. Many ideas about number and measurement arise from attempts to quantify real-world objects that can be viewed geometrically. For example, the use of area models provides an interpretation for much of the arithmetic of decimals, fractions, ratios, proportions, and percents.

Students discover relationships and develop spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures. Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments. The informal exploration of geometry can be exciting and mathematically productive for middle school students. At this level, geometry should focus on investigating and using geometric ideas and relationships rather than on memorizing definitions and formulas.

The study of geometry in grades 5-8 links the informal explorations begun in grades K-4 to the more formalized processes studied in grades 9-12. The expanding logical capabilities of students in grades 5-8 allow them to draw inferences and make logical deductions from geometric problem situations. This does not imply that the study of geometry in grades 5-8 should be a formalized endeavor; rather, it should simply provide increased opportunities for students to engage in more systematic explorations.

APPENDIX B: 2009-2010 EBRPSS CURRICULUM PACING GUIDE– GRADE 8

Table B.1 – The Curriculum Pacing Guide

UNIT#	UNIT NAME	APPROXIMATE TIME
1	Rational Numbers, Measures, and Models	5 weeks Aug 7 – Sept 11
2	Rates, Ratios, and Proportions	5 weeks Sept 14 – Oct 16
3	Angles and Line Relationships	5 weeks Oct 19 – Nov 20
4	Measurement and Geometry	2 weeks Nov 30 – Dec 11
Exams		Dec 14 – Dec 18
4 CONT.	Measurement and Geometry	3 weeks Jan 5 – Jan 22
5	Algebra and Integers	4 weeks Jan 25 – Feb 19
6	Growth and Patterns	3 weeks Feb 22 – March 12
7	What are the Data?	3 weeks March 15 – April 9
Leap Testing		April 12 – April 16
8	Examining Chances	4 weeks April 19 – May 14
Exams		May 17 – May 20

Unit 4: Measurement and Geometry	**<u>Time Frame:</u>**(approx.) **4 weeks**

<u>Unit Description:</u>

In this unit, basic 2- and 3-dimensional shapes, their surface areas, and their volumes are explored. Nets will be used to help students visualized three dimensional solids and understand surface areas. Conversions of volume within the same system and comparisons of relative sizes of units of volume across systems are made. Density is connected to algebraic relationships. Analyses of rates of change of sides, areas, and volumes of similar figures are also revisited. Such analyses are also applied to the lengths of sides, areas, and volumes of similar figures due to changes in one or more of the dimensions. Single and multiple event probabilities are explored.

<u>Guiding Questions/Student Understandings:</u>

1. Can students describe the nature of surface area, volume, and capacity as measures of size?
2. Can students apply and interpret the results of surface area and volume considerations applied to prisms, cylinders, pyramids, and cones?
3. Can students make appropriate estimates of volume and capacity and use these in applications?
4. Can students discuss the rate of change of derived measures (density)?
5. Can students find single and multiple event probabilities?
6. Can students draw and use planar nets to construct polyhedra, noting the relationships of sides, edges, and vertices?

<u>GLE(s):</u>

17. Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M)
18. Apply rate of change in real-life problems, including density, velocity, and international monetary conversions (M-1-M) (N-8-M) (M-6-M)
19. Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating (M-2-M) (G-1-M)
20. Identify and select appropriate units for measuring volume (M-3-M)
21. Compare and estimate measurements of volume and capacity within and between the U.S. and metric systems (M-4-M) (G-1-M)

22. Convert units of volume/capacity within systems for U.S. and metric units (M-5-M)

27. Construct polyhedra using 2-dimensional patterns (nets) (G-4-M)

(Continued on next page)

GLE(s): cont.

32. Model and explain the relationship between the dimensions of a rectangular prism and its volume (i.e., how scale change in linear dimension(s) affects volume) (G-5-M)

33. Graph solutions to real-life problems on the coordinate plane (G-6-M)

39. Analyze and make predictions from discovered data patterns (D-2-M)

45. Calculate, illustrate, and apply single- and multiple-event probabilities, including mutually exclusive, independent events and non-mutually exclusive, dependent events (D-5-M)

48. Illustrate patterns of change in dimension(s) and corresponding changes in volumes of rectangular solids (P-3-M)

Prerequisite Skills/Knowledge:

Students must have some prior knowledge to be successful in this unit. Specific skills or knowledge is listed below. Teachers need to ensure these concepts are mastered before going into the curriculum activities or review these concepts as needed. A review during the warm-up may be sufficient. Please see intervention strategies for more specific ideas.

- Students should be able to measure using rulers (cm and inches)
- Students should be able to compute perimeter/circumference and area for 2-dimensional shapes (i.e. quadrilaterals, triangles, and circles).
- Students should be able to apply the Pythagorean Theorem.
- Students should be familiar with common customary units and metric units.

Vocabulary:

Polyhedrons	Nets	Volume	Surface area
Solid	Prism	Pyramid	Cone
Cylinder	Edge	Vertex	Faces

Materials/ Resources:

Textbook	Compass	Grid/ graph paper	Ruler
Common solids	Cubes (in/cm)	Tape	Paperclips

Scissors	Computer access	Newsprint	Tape measure
Triple beam balance	Math learning log	Markers/ Colored Pencil	Snap cubes
6 containers/ boxes (rectangular prisms)		Beans/ rice/ popcorn kernel	

Intervention Strategies:

These intervention strategies may be used at any point during the unit when students need support for mastery of a particular concept or skill.

- To help students with nets and visualizing surface area, allow students to outline the faces of solids on paper (i.e., to draw a pyramid, students can first outline the base. Then, without otherwise moving the pyramid, students can tilt the pyramid onto one of its faces, outline that face, and then tilt the pyramid back to its base. Repeat this process for each side.)
- Review Pythagorean Theorem before starting on Activity 9. This can be done through a warm-up.
- Allow students many physical models (i.e. cereal box) that students can touch, take apart, and explore to help them with understanding solids.
- To help students organize the different shapes and solids they are learning in this unit, teacher may want to start a “formula card” (can use an index card). This formula card should have the name of the shape/solid, illustrations with labels, and formulas for area and volume. The card should be a summary (do not include examples) and it should be ongoing. Students will add to it as they learn the concepts and teachers should allow students practice on using the card.
- The fictional book “Flatland” can be incorporated to help students understand the relationships between different dimensions (i.e. cubic units, square units, linear units, etc.). This book also works well as an enrichment and allow for cross-disciplinary studies.
- In prior courses (6^{th} & 7^{th} grade Math), students should have practiced converting customary and metric units within the same system for area. Mastery of this concept will make conversions for volume (this unit) easier to grasp. If necessary, teachers may want to review the units in both system, linear conversions (i.e. inches to feet) and then go into square conversions (i.e. cubic inches to cubic feet) before moving on to cubic conversions.
- A day or two (at most) can be spent on reviewing finding areas of 2-dimensional shapes. Students must know this skill in order to find the surface area.
- Teachers may provide some students guided notes with answers
- Optional activities may be used for enrichment or remediation
- McDougal Littell textbook has accompanying workbooks that give three levels of practice and ideas for differentiated instructions.

UNIT 4 ACTIVITIES:

Unit 4 Activity 1: Volume and Surface Area (GLE: 17)

Give student pairs a given set of 16 one-inch cubes or centimeter cubes, and ask them to build all possible rectangular solids. Have students count the number of cubes to determine the volume of the solids and count the number of exposed faces to calculate the surface area of each solid built, recording information on the Volume and Surface Area handout. Have students make sketches of the solids and label the dimensions of the rectangular prisms built.

Have students repeat the exercise using a different number of cubes and record the information in the chart.

Ask students to study their findings and list their observations. Make sure observations include the relationship of the surface area and the shape of the rectangular solid (*i.e. the closer to the shape of a cube, the smaller the surface area).*

Have students respond to the prompt in their math *learning log* (view literacy strategy descriptions).

> Measurements of rectangular solids can be linear, square and cubic units. These units refer to . . .'

Unit 4 Activity 1: Volume and Surface Area

Name ____________________________ Date ________________ Period ______

Table D.1 – Exploring Volume and Surface Area

# cubes used for model	Length of rectangular prism built (linear units)	Width of rectangular prism built (linear units)	Height of rectangular prism built (linear units)	Volume of rectangular prism built (cubic units)	Surface Area of rectangular prism built (square units)
16					

Unit 4 Activity 2: Rectangular prisms (GLEs: 17, 21)

Have the students refer to the Volume and Surface Area handout used in Activity 1. Review observations made with shapes formed in Activity 1. Have each student construct a net for a one-centimeter cube from centimeter grid paper, fold the net to form a cube, and tape the cube to hold its shape. Ask students to make and record a prediction as to how many of these one centimeter cubes it will take to make a cube with dimensions of 2 cm x 2 cm x 2 cm. Have them work in groups of four to make enough centimeter cubes to form the 2 cm x 2 cm x 2 cm cube. Discuss the concept that the number of centimeter cubes that it takes to make a 2 cm x 2 cm x 2 cm cube is the volume of the new cube and is recorded as cm^3.

Distribute the LEAP Reference Sheet and have groups of four use their centimeter cubes and make the connection between the formula for volume as stated on the LEAP Reference Sheet and the 2 cm x 2 cm x 2 cm cube that they formed. Discuss the dimensions of the cube, the concept that three edges meet at a vertex of a cube and that there are 8 vertices. Each edge is a dimension and there are12 edges in the cube.

Have students construct a net 3 cm x 3 cm x 3 cm. Have students predict how many of the centimeter cubes would fit inside of a 3 cm x 3 cm x 3 cm cube. Have students fold their net into a cube and again relate the volume to the formula on the LEAP Reference Sheet. Ask, **"**How many cubic centimeter blocks would fit into a 4 x 4 x 4 cube?" Have students state the rule to use for finding volume of a rectangular prism, and make sure the students are relating the number of cubic units needed to form the cube to the volume and are recording asnwers in cubic units as they multiply length, width, and height.

As an extension, have students determine the surface areas of the 2 cm, 3 cm, and 4 cm cubes to help with understanding the difference between surface area and volume.

Use the *SQPL* strategy (view literacy strategy descriptions) to challenge the students to further explore volume measurements. Put the following statement on the board or overhead for students to read: "It would take more than 10,000 one inch cubes to fill a cube that is 8 ft on each side." Have students work with a partner and brainstorm 2-3 questions that would have to be answered to prove or disprove the statement. As a whole class have each pair of students present one of their questions and write this question on chart paper or the board. Give the class time to read each of the questions presented. Give pairs of students time to select the ideas that they would use to prove or disprove the statement. *It is important that the students understand that when changing units of volume, all three dimensions have to be changed.*

Using the *professor-know-it-all* strategy (view literacy strategy descriptions), have different pairs of students explain their proof and answer questions from the class. Using this strategy, the teacher randomly selects pairs of students, not volunteers.

Next, have the students construct and cut out a net for a rectangular prism with dimensions of 1 cm x 2 cm x 2 cm. Ask students to determine the number of cubic centimeter cubes that will fit inside of the rectangular prism. Ask the students what this is called (volume). Have students fold their net to form a rectangular prism. Ask someone to show the class the three dimensions of the rectangular prism. Challenge students to find the surface area of the prism.

Have students mark the edges, vertices and faces with colored pencils or markers. Instruct students to work in their groups to find a rule for determining the number of centimeter cubes that will fit into a rectangular prism (volume). Discuss conjectures that students make. Use the LEAP Reference Sheet, and have the students explain how the formulas for volume and surface area of a rectangular prism relate to the models they have built in Activities 1 and 2 of this unit. Challenge groups to test their conjectures.

Unit 4 Activity 3: What's the Probability? (GLE: 45)

Use *brainstorming* (view literacy strategy descriptions) and have the students recall all they know about probability. Write their ideas on a chart or the board. Brainstorming is used in this lesson to pre-assess what the students recall from 7th grade and earlier grades about probability. Model the use of a graphic organizer (view literacy strategy descriptions) to organize the ideas that students have brainstormed. A circle map is a good one to use to gather this information and pre-assess the students' knowledge of probability. An example of one possible graphic organizer is shown below.

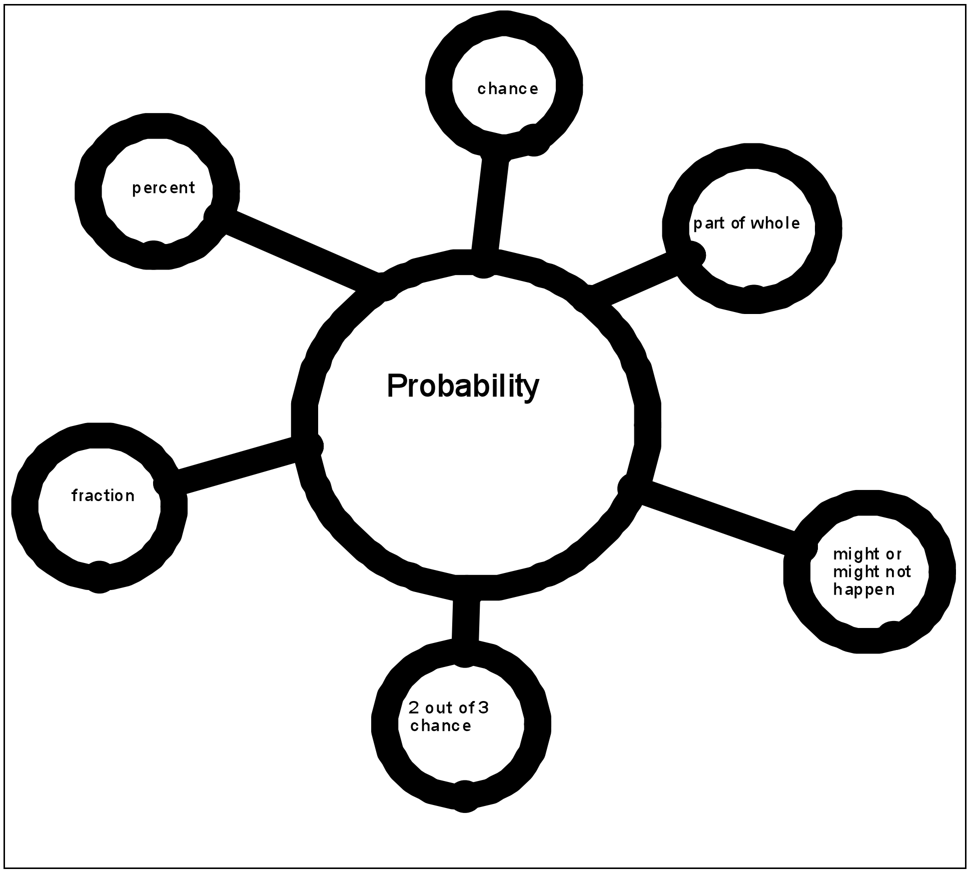

Figure D.1 – Probability Circle Map

Distribute the What's the Probability handout and give the students time to work independently on the probability situations given. Discuss results briefly prior to the next situation.

Provide the students with this website to view a basketball court and its dimensions (http://www.betterbasketball.com/basketball-court-dimensions/basketballcourt2.html) or use the printout from this site and make copies for each student. Give them the following situation:

> Jason's basketball coach told the starters that they must score at least 10 points during the next four games. Jason practiced shooting only two point shots at the gym. When he left, he realized that he had dropped his house key and his practice schedule. He knew that the objects could be anywhere on the court, but he thought that the practice schedule would most likely be on the half of the court where he was practicing. Find the probability that the practice schedule will be found on this end of the court. Find the probability that his key is inside the free-throw area of his end of the court. What is the probability that both the key and practice schedule are inside of the free-throw area?

Since probability has not been extensively covered at this point (it was covered in the 7th grade curriculum), have students work through a similar situation prior to assigning this activity with groups of students. Draw any geometric figure, inscribe a second figure inside (shade one part), and work with the students to determine the probability that an object falling randomly will land in the shaded area. It is a good time to use figures that the students will subdivide to compute the area. Have students prepare a poster to show how they figured the probability. Use the *professor know-it all* strategy (view literacy strategy descriptions) to discuss results as a class. Randomly select groups to share and justify their thinking.

Unit 4 Activity 5: Cylinders (GLEs: 17, 21) *optional*

SPAWN writing (view literacy strategy descriptions) is an informal writing with students responding to a given prompt. Begin by explaining to the students that they will reflect on what they know about volume and surface area using the 'P" or Problem Solving of *SPAWN writing*. Write the following prompt on the board:

> We have been studying volume and surface area of rectangular prisms and cubes. What measurements will be necessary when finding the volume of a cylinder?

Ask students to record their thoughts in their math *learning logs* (view literacy strategy descriptions). This is informal SPAWN writing and should not be taken as a grade. It is important that the students know the importance of communicating mathematically (give points for completion if necessary). Use the students' SPAWN writing responses for whole class discussion.

Have students create a cylinder from standard $8\frac{1}{2}$ inch x 11 inch paper. Engage the class in a discussion about how to make a cylinder with a circumference of $8\frac{1}{2}$ inches without cutting or tearing the paper. Have students roll paper to form a cylinder shape. Ask what the height of this cylinder is. (*The height of the cylinder will be 11 inches*). Ask the students to predict whether the cylinder with a circumference of $8\frac{1}{2}$ inches or the cylinder with a circumference of 11 inches made from a full sheet of this paper would have the larger volume.

Show students the circles formed when the paper is rolled. Ask them again what the circumferences of the circles are and how they know. (The circumference is formed by the side of the sheet of paper which is 8½ inches long.) Ask students if they remember the formula for the circumference of the circle. Then ask how they can find the diameter if the circumference is known. This is a good time to use the LEAP Reference Sheet and have them substitute values into the circumference formula to find the diameter of the cylinder using 3.14 for *pi.*

Unit 4 Activity 6: Pyramids and Cones

Name ____________________________ Date _____________ Period _______

Fill in the chart below using at least 4 different measurements for area of base and heights of pyramids and rectangular prisms. Assume that the area of the base and the height is the same for each set of figures.

Table D.2 – Volume of Pyramids and Prisms

Area of square base	Height	Volume of pyramid	Volume of Prism
4 in^2	3 in		

Unit 4 Activity 7: Comparing Cones (GLE: 17)

Distribute the Comparing Cones handout and the Model for Cone handout. Have students cut out the circle leaving the points so that they can use them for the activity.

Have students cut along the radius and form a cone by moving point L so that it lies on top of point A. Instruct students to measure the diameter, circumference, and height of the cone and to record the measures in the Comparing Cones handout. Have students calculate the volume of the cone. Next, have students form a second cone by sliding point L so that it lies on top of point B. Each time a new sized cone is formed, have students record the diameter, circumference and height. After the students have completed forming cones by moving point L to at least 5 different locations, have them find the volume of each of the cones and develop a conjecture about how the change in circumference affects the volume in their math *learning log* (view literacy strategy descriptions).

Unit 4 Activity 8: Common Containers (GLEs: 17, <u>19</u>, 20, <u>21</u>, 22)

Provide student pairs with several common containers (rectangular solids and cylinders) found in the grocery or hardware store (with the labels removed or volume information covered) and a copy of the Common Containers handout. Prior to class, put letters A, B, C, D, E, F on the containers for each group. If the labels have information relative to volume in cubic units, save the labels for later use and mark the labels with the same letter as the container. Have students estimate the volume of each container, record this in the correct column on the Common Containers handout, and arrange the containers in order from smallest to largest volume.

Once the students have estimated the volumes, provide measuring instruments and have students determine the volume of each container using U.S. units and record the volume on the Common Containers handout. Remind the students that the formulas needed are found on their LEAP Reference Sheet. It would be a good idea to have students first write the formula, show substitution of values so that use of correct values can be determined if an error is found, and then give the answer. Make sure students give the correct unit on their answers.

Finally, have students repeat the process using a metric measurement tool.

Lead a discussion comparing measurements within and between systems. Once the volumes of the containers have been determined, have students convert their answers to another unit in the same system (i.e., convert from cubic inches to cubic feet and vice versa—include conversions with metric units, also). If there were labels which had volume written in cubic units, then allow students to compare their results with the information provided on the labels.

Repeat the activity with larger containers. For example, show a picture of a silo to the students. Explain to them that silos have been used for many years to store grain. Provide the dimensions of a silo, and have students determine its volume. Buildings can also be used as examples. The Superdome is in a cylindrical shape with a domed roof. Find the actual dimensions of the Superdome at www.superdome.com. Go to the *About Us* section and scroll down to the table of facts. Assuming the roof is flat, have students approximate the volume of the Superdome.

A basketball gym is typically a rectangular solid. Have students determine the volume of their school's gym. In each case, have students express their answers in both U.S. and metric units. Discuss selecting appropriate units for measuring volume and capacity.

Unit 4 Activity 8: Common Containers

Name ______________________________ Date __________ Period ___

Table D.3 – Comparing Customary and Metric Units

Container	Estimated Volume	Volume in US Customary Measure (write formula, show substitutions, and provide answer)	Volume in metric measure (write formula, show substitutions, and provide answer)
A			
B			
C			
D			
E			
F			
G			

Unit 4 Activity 9: The Net (GLE: 27)

Using a shoe box from home and one other rectangular prism box, have students discuss the number and location of faces, vertices and edges. Have measurements of the boxes used for modeling written on the boxes and the board. Lead a discussion about how measures are involved when finding surface area.

Provide students with the Rectangular Prism handout and have them fold and tape it together to form a rectangular prism. If time is a factor, have students cut out and tape together these nets at home the day before this activity begins. Ask student to determine the number of faces, edges, and vertices. Have students find the area of one face of the prism. Have students determine which other faces of the box would have the same area. Have the students work in pairs to determine the surface area of the rectangular prism, and then discuss method(s) used. Have the students list methods used to find surface area on the board so that comparisons of methods can be made. Make comparisons of these methods and the formula used on the LEAP Reference Sheet.

Next, provide students with the Triangular Prism handout and have students construct the prism by appropriately folding and taping it together. Determine faces, edges, and vertices. Have students discuss shapes that make up each face of the triangular prism. Determine a method of finding the area of each face. Provide rulers for measuring lengths so that the groups can find the areas. The Triangular Prism handout is an equilateral triangular prism. Make sure the students realize that this is not a right triangle and that they have to find the height of the equilateral triangles. These triangles are located at either end of the center rectangle region of the net and the students will discover that they can fold it in congruent parts to find the height of the triangles. This is also a good time for a discussion about congruency.

A Right Triangular Prism handout has been included. Have students identify faces, edges, and vertices. Have students use the Pythagorean Theorem to determine the area of the right triangular ends of the prism as they find the surface area of the right-triangular prism. Have students share methods by putting different methods on the board for discussion.

Extend this activity by having the students bring a box from home which is cut so that the six faces are clearly distinguishable. Students should find the surface area of the box that they brought from home but keep the results secret. Have students place their ‘nets’ on a table in the room. Label these with letters and have the students rank the ‘nets’ from largest to smallest surface area without measuring. Discuss results.

Unit 4 Activity 9: The Net

Rectangular Prism

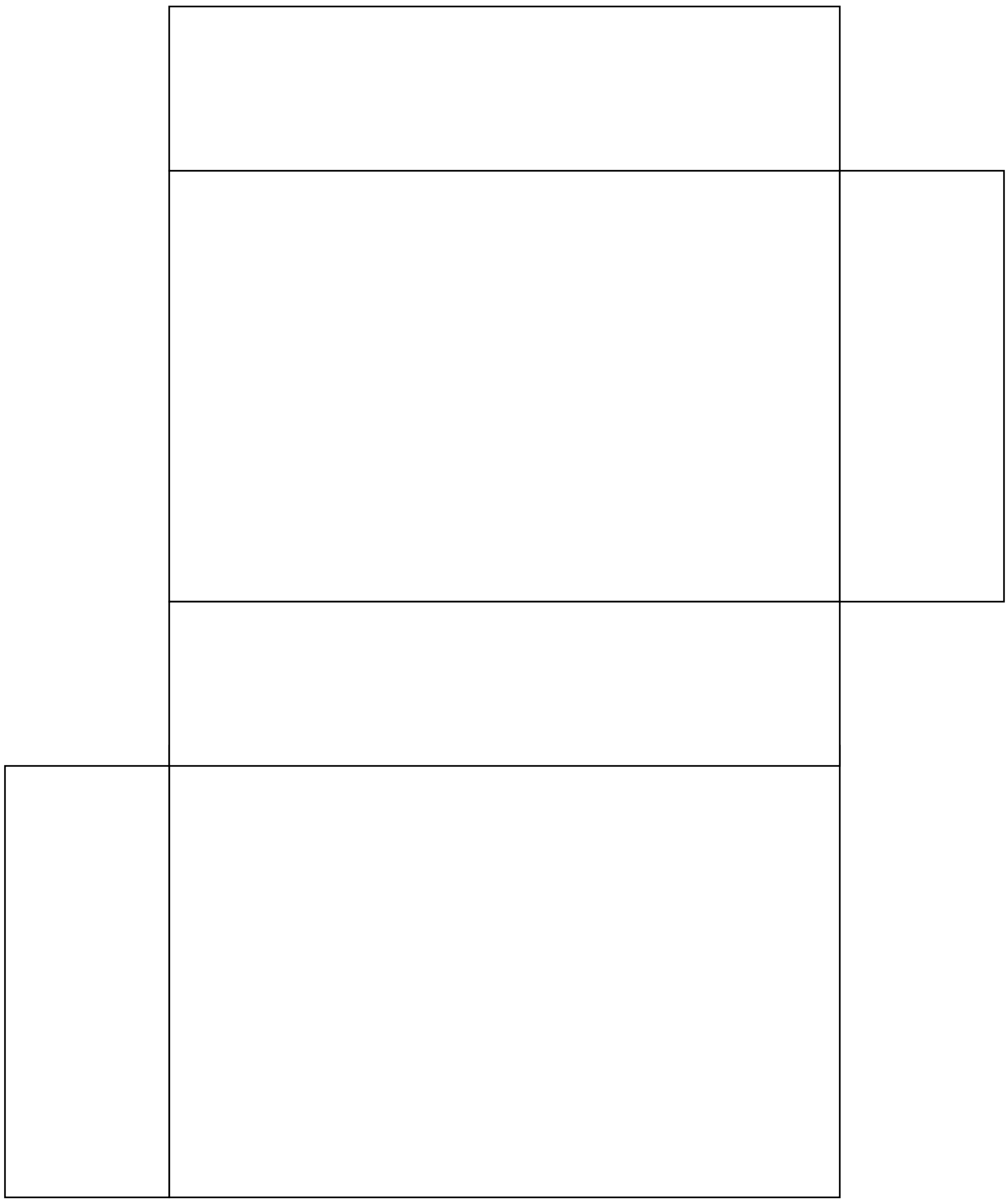

Figure D.2 – A Rectangular Prism Net

Unit 4 Activity 9: The Net

Right Triangular Prism

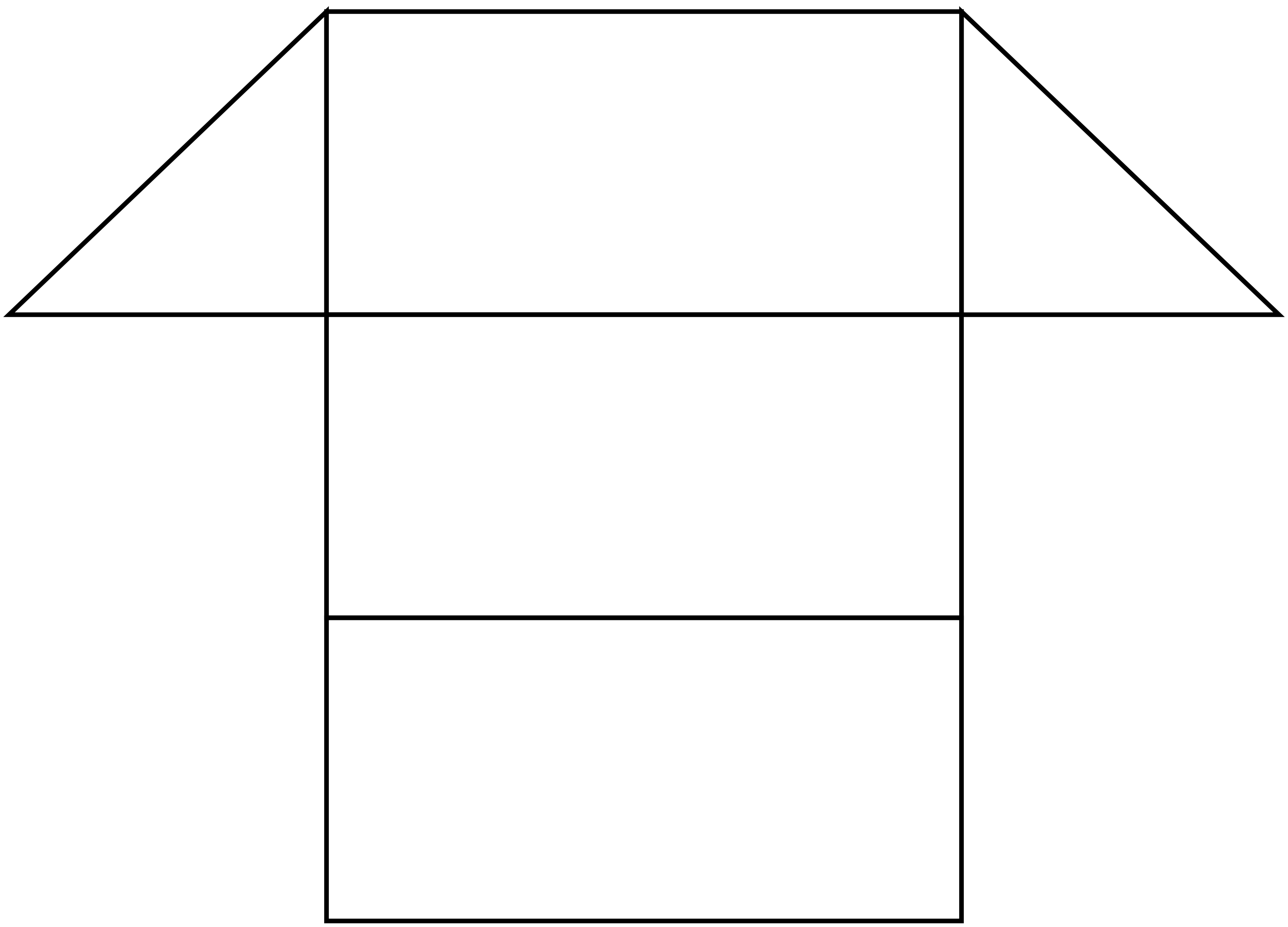

Figure D.3 – A Triangular Prism Net

Unit 4 Activity 10: Changing Areas and Volumes

Name ____________________________ Date _______________ Hour ________

Part 1

SURFACE AREA, VOLUME AND DIMENSIONS

Table D.4 – Scaling a Rectangular Prism

Volume	Dimensions
	Original: 4 units x 3 units x 2 units
	Double width:

Part 2

Table D.5 – Scaling Cubes

Volume	Dimensions
8 cubic units (8 u^3)	Cube:
	Double one side:
	Double two sides:
	Double three sides:
27 cubic units (27 u^3)	Cube:
	Double one side:
	Double two sides:
	Double three sides:

VITA

Verna Marie Richard was born in Baton Rouge, Louisiana. She is a third year Introduction to Algebra teacher at Broadmoor Middle School, who discovered her love for teaching mathematics through the Secondary Teacher Education Preparation thru Science, Technology, Engineering and Mathematics (S.T.E.P thru S.T.E.M) program in 2005. In June of 2007, she received her Bachelor of Science degree in mathematics from Louisiana State University.