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Fraction proficiency in gifted middle school students

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FRACTION PROFICIENCY IN GIFTED MIDDLE SCHOOL STUDENTS

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TABLE OF CONTENTS

APPENDIX

LIST OF FIGURES

ABSTRACT

This thesis presents a program to develop fraction proficiency in gifted students who have skipped grades. The program was piloted with 31 gifted and advanced students in grades 6 and 7, and qualitative data on their performance is presented. Also presented here is a test of fraction proficiency that was developed to determine the level that students achieved at the end of the pilot program. The results of this test are presented and analyzed. The net contribution of this work is to present a starting point for a set of catch-up lessons on operations with fractions and to present a starting point for an assessment for testing fraction proficiency.

INTRODUCTION

Fraction proficiency is crucial for success in mathematics. Students who are proficient in fractions have a better chance of being successful in high school and beyond. Fractions are difficult to understand, however, and this creates a challenge in mathematics education. Fraction concepts appear in the upper elementary and middle school grades, giving children a phobia about math early on. The difficulties with fractions have been studied tirelessly by researchers for many years.

As a teacher of gifted and advanced students in the East Baton Rouge Parish School System, I am responsible for middle-school students who have skipped grade levels of mathematics. The district allows students to be placed in accelerated math classes, even though they may not have had every prerequisite. This creates a challenge in that these students must learn most of the information needed to operate appropriately with fractions during their middle school careers. Weakness in fractions would be crippling for these students when they reach high school level math courses. Since most of these students study high school level mathematics courses in middle school, acquiring fraction proficiency rapidly is critical in their success.

Students need to develop a conceptual understanding as well as procedural understanding when learning about fractions. The National Mathematics Advisory Panel (NMP, 2008) recommends developing conceptual understanding by using the number line to teach fraction concepts such as equivalence, ordering, and operations. The Louisiana Comprehensive Curriculum does not use methods recommended by the NMP consistently throughout the process of learning fractions, and must be supplemented to support the growth of conceptual understanding.

In this study, I examined the fraction proficiency of my Gifted and Advanced students. After a careful exploration of the Louisiana Grade Level Expectations, I concluded that the fraction concepts missed by my students were most closely related to proficiency with operations. In response, I developed six catch-up lessons to give them the knowledge necessary for proficiency. It appeared that the more-basic fraction concepts, such as equivalence, had been adequately addressed in their elementary math studies. My lessons focused on making the definitions related to fractions clear and on adding, subtracting, multiplying, and dividing fractions. I included methods that have been shown to develop conceptual understanding in fraction operations. During the lessons, I kept tabs on what the students understood through conversation. After the lessons were taught, students answered a number of questions involving all concepts of fractions. These questions allowed me to study the proficiency my students held in fractions.

The lessons I taught were a pilot run of the catch-up program I created. Despite extensive research, I found no significant guidance in the literature on the issue of designing catch-up programs on fractions; therfore I had to create this program with little assistance from previous examples. The lesson plans, along with worksheets are located in the appendix of this paper. Future users will want to make revisions, according to intended uses. The results reported in this paper should be useful in guiding changes. The program was successful in certain areas, but others may need to be added to. For example, the results of my test showed that students were still had some misconceptions regarding multiplication of fractions.

The test I created is also a possible model for the design of assessments. It can be found in the appendix. Based on my results, the test should be revamped so that the questions better distinguish between performance levels. Some of the questions on the test were apparently too easy, and did not give any information that differentiated between students or help me identify areas of difficulty in the content addressed. These questions should be revised to produce better results. Questions could also be added to cover concepts that I did not address. The concepts of subtraction and division are such examples. This test was based on the Common Core State Standards for Mathematics, but other learning objectives for fractions could also be considered.

The work reported here is subject to the following limitations. The first limitation is that no pre-test was administered. Therefore no estimate can be made of the amount of learning that took place as a direct consequence of the lessons I delivered. Future work with these lessons should include a carefully planned pretest. Another limitation was that there was no control group. Without a control group, it is impossible to compare these lessons to lessons traditionally taught.

CHAPTER 1. A SELECTIVE REVIEW OF FRACTION PROFICIENCY

1.1. *Adding it Up*

1.1.1. Mathematical Proficiency

The book *Adding it Up* suggests that mathematical proficiency should be viewed as consisting of five components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. (Kilpatrick, 2001) These strands are intertwined and interdependent in the development of mathematical proficiency. The authors believe that having these mental abilities allows students to connect concepts and use their understandings in future problem solving. Developing these interrelated strands promotes retention and fluency, which is more powerful than memorization.

Conceptual understanding is defined as “comprehension of mathematical concepts, operations, and relations.” (Kilpatrick, 2001) It is more than knowing isolated facts and methods. It includes the ability to represent a mathematical situation in more than one way and to connect that understanding to the development of other mathematical procedures. Students with conceptual understanding are better able to remember procedures and make fewer critical errors in solving problems.

Procedural fluency is defined as “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.” (Kilpatrick, 2001) With practice accuracy and efficiency can be improved which helps maintain fluency. Students need to be able to work efficiently and accurately with mathematical procedures to free up mental space to deal with new ideas. Students who are fluent in skills and procedures do not waste energy on routine, and can devote their attention to conceptual understanding of the topic.

Strategic competence is defined as “the ability to formulate, represent, and solve mathematical problems.” (Kilpatrick, 2001) Strategic competence relates to problem solving. Students with strategic competence can form mental representations, detect relationships to previously known concepts, and create a model solution for a problem. These students are able to flexibly choose a method for solving problems depending on the demands of the problem.

Adaptive reasoning is defined as “the capacity for logical thought, reflection, explanation, and justification.” (Kilpatrick, 2001) Students who possess adaptive reasoning can justify, or give sufficient evidence, why their answers are correct. Adaptive reasoning allows students to decide logically whether a solution method is appropriate for the problem they are trying to solve.

Productive disposition is defined as “the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and in one’s own efficacy.” (Kilpatrick, 2001) Students need to believe that they are capable of figuring out problems and that these problems are not arbitrary. Students with a productive disposition are more likely to be confident in their abilities and knowledge. Students are more likely to develop other strands of proficiency if they have productive disposition.

1.1.2. Proficiency with Rational Numbers

The rational number system is more complex than the whole number system. Students in middle school encountering rational numbers have greater difficulties learning how to operate with them. One major reason for this is that the rational numbers are represented in more than one way. The same quantity can be written as a decimal, a fraction, and a percent. For example 2/4 = 1/2 = 0.5 = 50%. Another major reason for this is that rational numbers are used in more than one way. Rational numbers can be used to represent part-to-whole relationships, part-to-part relationships, other kinds of ratios, and can be the result of arithmetic operations. Students are less likely to encounter and relate rational numbers to out-of-school experiences. This makes it difficult for teachers to teach the concepts associated with rational numbers and it makes learning about them difficult for students.

When starting to learn about fractions, students use informal knowledge of partitioning and sharing. They are first introduced to sharing in lower elementary and then to partitioning. These ideas, however, do not provide a solid foundation for rational arithmetic. Many students can figure out their own procedures for operations with whole numbers, but rational number operations need to be more teacher-guided. The teacher needs to provide relevant experiences to build understanding and to help students appreciate the procedures for the operations. Of course, teachers themselves need to be aware of the conceptual meaning of the procedures.

Certain main ideas about rational numbers are often underdeveloped. Students may lack a clear and meaningful understanding of the symbols and may fail to make connections between equivalent quantities represented using different symbols. For example, students may fail to recognize that 2/5=0.2=10/25. Another main idea students struggle with is the idea that rational numbers are also numbers. Proficient students should be able to identify the location of rational numbers on the number line. The notion of fractions being numbers is important when operations are applied. Students need to think of a fraction as a number and understand the symbols related to rational numbers before they begin learning about operations of fractions.

Students need to understand the procedures for operations with fractions conceptually Memorization of the procedures is not enough to be proficient. Research shows that students who were instructed in the procedures, but were not encouraged to understand the reasons why the procedures work, were less likely to remember the procedures later. (Behr et al., 1983)

Students need lessons to help develop conceptual understanding from the start, building on "students' intuitive understanding of fractions" and using objects that help students understand the operations. (Kilpatrick, 2001) Students who can use the procedures correctly, but do not understand why the procedures work are not proficient.

1.2. Common Core State Standards vs. Louisiana Grade Level Expectations

1.2.1. Common Core State Standards for Mathematics

The Common Core State Standards for Mathematics (CCSS) suggests students should begin working with fractions in 3rd grade and should become proficient in fractions in the 6th grade. The bulk of the material on fractions is prescribed for grades 4 and 5. Each grade level focuses on a new concept about fractions, building on the concepts learned from the previous grade level. At each level, objectives are stated and then broken down into subparts for a more precise description of what should be learned.

Fractions are first viewed as parts of wholes in grade 3. Very soon after, fractions are described as numbers on the number line. The fact that fractions are numbers is given a lot of attention. Students are also taught that equivalent fractions represent the same quantity and are located at the same position on the number line.

In grade 4, students explain why two fractions are equivalent to one another, compare fractions by determining which of two fractions is larger, and order fractions from least to greatest. Students learn how to add and subtract mixed numbers with like denominators. They also learn how to multiply fractions by whole numbers using equations and visual fraction models.

In grade 5, equivalent fractions are used as strategies for adding and subtracting fractions with unlike denominators. Procedures used to understand multiplication with whole numbers are used to help understand multiplication of fractions. Division of fractions by whole numbers is taught using previous understandings of division. In grade 6 procedures used to understand division with whole numbers are used to help students understand division of fractions.

1.2.2. Louisiana Grade Level Expectations

The Louisiana Grade Level Expectations (GLEs) in Mathematics group the learning objectives into 6 categories that apply to all grades from kindergarten to grade 12. In each category, a list of objectives is given. Separate objectives are not explained in detail, but the Louisiana Comprehensive Curriculum includes activities that clarify the intent and aid in planning instruction. Many of the same objectives appear at multiple grade levels, but paired with different activities in the Comprehensive Curriculum.

The Louisiana GLEs list expectations for fractions in the category “Number and Number Relations.” According to the GLEs, students should begin working with fractions in Grade 2 and should complete their study of fractions in Grade 7.

Simple fractions are introduced in Grade 2 and are illustrated using regions and sets. In Grade 3, fractions with denominators up to ten are modeled with regions and sets. In Grade 4, students develop the ability to model fractions with denominators up to 12 using regions and sets and to compare and order such fractions. They also learn to use the first two decimal places. In Grade 5, addition and subtraction of fractions with like denominators is introduced. The number line is used to compare fractions. Equivalent fractions are recognized and computed. In Grade 6, equivalent fractions are computed, fractions are compared using symbols and the number line, and fractions are added and subtracted “in real-life situations”. (Louisiana Department of Education, 2010) Multiplication and division of fractions is addressed in Grade 7.

There are major differences between the CCSS and the GLEs in terms of when and how fraction skills and concepts are taught. The CCSS represent fractions on a number line in Grade 3. The GLEs do not place fractions on a number line until grade 5, and they focus more on using partitions to represent fractions. The GLEs do not define a fraction as a number, as the CCSS do. Arithmetic operations with fractions are taught earlier in the CCSS and given far more emphasis. The GLEs lack any explicit expectations for developing proficiency in arithmetic with fractions. The GLEs do not relate back to previously learned objectives as the CCSS objectives do. The CCSS list objectives divided into subparts, giving precise explanations of what should be learned. For instance in Grade 5 of the CCSS the following objectives and subparts are listed under the broad topic of fraction arithmetic:

> **Use equivalent fractions as a strategy to add and subtract fractions.**
>
> 5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
>
> 5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* (National Governors Association, 2010)

In the Louisiana Comprehensive Curriculum the GLEs list the objectives to be mastered, but do not give details. For instance, in Grade 6, the entire set of expectations related to fractions is as follows:

4. Recognize and compute equivalent representations of fractions and decimals (i.e., halves, thirds, fourths, fifths, eighths, tenths, hundredths).
5. Decide which representation (i.e., fraction or decimal) of a positive number is appropriate in a real-life situation.

6. Compare positive fractions, decimals, and positive and negative integers using symbols (i.e., <, =, >) and number lines.
7. Read and write numerals and words for decimals through ten-thousandths.
8. Demonstrate the meaning of positive and negative numbers and their opposites in real-life situations.
9. Add and subtract fractions and decimals in real-life situations.
10. Use and explain estimation strategies to predict computational results with positive fractions and decimals. (Louisiana Department of Education, 2010)

1.3. Teaching Fractions

Fractions are an integral part of the middle school math curriculum and are used throughout students' careers. This makes fractions a major concept that needs to be mastered. Some research (Peck and Jencks, 1981), (Neimi, 1996), and (Cramer, Behr, Post, and Lesh, 2002) shows that lessons devoted to developing conceptual understanding of fractions enhances learning about fractions.

Peck and Jencks (1981) interviewed hundreds of students to determine their conceptual understandings of fractions. The authors hoped to determine whether students were applying the rules used for operations with fractions mechanically, or if they actually had a conceptual understanding. Students used sketches to justify their answers. Concentrating on the results from 20 sixth-graders, the authors found that these children were capable of learning conceptually. The students knew that to illustrate a fraction, the whole—often depicted as a pie—had to be divided into the number of pieces specified by the denominator. Many children did not indicate that the pieces needed to be of equal size, and even the students who could define and draw pictures of fractions were unable to use this understanding to explain or justify operations with fractions. The students may have been able to come up with an answer but were unable to connect this to a conceptual explanation. Most of the students who could answer the questions correctly were only able to use rules that were taught to them. They were not able to explain the answer conceptually or to justify why the solution was correct. The authors conclude that students "are going through the motions of operations with fractions but they have not been exposed to the kinds of experience that could provide them with necessary understandings." (Peck, 1981) Curriculums need to be reorganized to have a conceptual focus on fractions.

Niemi (1996) suggests that students are not getting a deep understanding of fractions due to the methods teachers use to teach fractions. Most curriculums treat fractions strictly as part-to-whole relationships. For example, the fraction 3/4 is associated with a pie cut into 4 pieces with 3 pieces being shaded. Students count the shaded pieces and place this result over the count of the whole. This forces students to view fractions as two separate whole numbers referring to two separate counts. Students are not seeing the most important meaning, namely that fractions are numbers; they represent single quantities. Niemi tested the conceptual understanding of fractions after two different types of lessons. 540 fifth-grade students were divided into two groups, each receiving one of the two types of lessons. One group of students was taught using lessons "designed to teach principles derived from relatively novel measurement applications of fractions." (Niemi, 1996) The other group of students was taught using traditional lessons found in textbooks. To test the groups, an essay question was posed, asking the students to explain everything a fifth grader should know about fractions. The results of this study showed that students receiving the lessons based on novel measurement applications scored significantly higher than those students receiving traditional instruction. The students scoring higher were able to present and explain more principles of fractions than other students.

Hung-Hsi Wu (2005) suggests that there are three main topics in which middle school math curriculums revolve around. These three concepts are rational numbers, introduction to algebra, and basic geometry. Rational numbers are important due to the fact that "what students learn here about this topic would have to serve them until at least the first two years of college." (Wu, 2005) Rational numbers are difficult to teach at this level because students do not yet possess the conceptual sophistication needed to work knowingly with rational numbers. Learning rational numbers is also important because it is the first "excursion into abstraction." (Wu, 2005) To learn fraction concepts, students need a clear and accurate definition of the term, they need to be able to regard fractions as numbers, and also need to be fluent in fraction arithmetic.

The National Mathematics Advisory Panel (NMP), (2008) agrees that conceptual and procedural knowledge is crucial in the learning of fractions. Algebra I teachers surveyed by the NMP concluded that students are poorly prepared in "rational numbers and operations involving fractions and decimals." (NMP, 2008) When the learning of fractions is difficult, it becomes an obstacle for progress in mathematics in the future. The NMP recommends that the curriculum on rational numbers should focus on methods that have been effective, such as using the number line to show representations as well as ordering fractions. The curriculum on rational numbers needs to devote enough time to ensure conceptual and procedural knowledge of fractions and proportional reasoning.

Cramer, Behr, Post, and Lesh (2009) also confirm that the rational number curriculum needs to focus on developing conceptual understanding, and they see this as possibly more important than procedural understanding. They created a curriculum based on this idea, and recruited 66 teachers to participate in a study devoted to the effectiveness of the lessons. Thirty-three teachers used the curriculum produced by the authors, known as the Rational Number Project (RNP), and thirty-three used the textbook. This experiment was performed on fourth and fifth grade students. (Cramer, 2002) Students who were taught using the RNP curriculum outscored students taught using the textbook in all areas. The RNP students were also able to verbalize their thinking about fractions. The authors suggest that students need to master initial fraction ideas, such as developing concepts, ordering, and equivalence ideas, before they begin learning about operations. The lessons were divided into two parts based on this hypothesis. During the study the authors worked with the children and interviewed the teachers to discover the misunderstandings concerning fractions that the children had. They found that students had difficulty with the idea that a fraction represents a single entity. Ordering fractions, based on what students previously learned about whole numbers and fraction equivalence, also presented problems. Based on these observations, the authors have produced lessons devoted to students' conceptual understanding of fractions.

Literature covering catch-up programs involving fractions is limited. I searched *Google*, *Google Scholar* and the Louisiana State University online database**.** These searches did not turn up any material on fraction catch-up programs. The research on fractions mostly contains programs of extensive coverage of fractions. This is not helpful to this project because of the limited time I had to get my students caught up.

CHAPTER 2. CATCH-UP IN FRACTIONS FOR GIFTED STUDENTS

2.1. Gifted Students in the East Baton Rouge Parish School System

The East Baton Rouge Parish School System provides Gifted classes for students with exceptional abilities. Students qualify by scoring at a certain percentile on a high security IQ exam that is administered by Pupil Appraisal Services, a division of the East Baton Rouge Parish Exceptional Student Services. The East Baton Rouge Parish School System also provides Advanced classes to students selected based on LEAP/iLEAP test scores and GPA. (LEAP/iLEAP is the statewide high-stakes standardized test.) To be considered for the Advanced classes, students must score at the Mastery level on two sections of the LEAP/iLEAP and must maintain a 2.5 GPA. Both the Gifted and Advanced programs allow for advanced math courses above the grade level of the student. In both programs students become eligible to take high school credit courses as middle school students, positioning them for yet higher level math courses in high school and college.

The fact that students move into math courses above their grade level means that they may miss instruction in some topics. In some cases, students skip two to three courses to be placed in a course that is deemed appropriate and therefore miss out on fractions. Although students have seen fractions in elementary school, they generally have not mastered the concepts on the level needed for the course they are enrolled in. As the National Math Panel has said, proficiency with fractions is essential to be successful in Algebra and other high school math courses. (NMP, 2008) Many Pre-Algebra and Algebra concepts involve fractions. Fractions are constantly used in solving equations ,hence, students need to be comfortable working with fractions so that this does not overshadow the procedures they need to learn to solve equations.

Fractions play another important role. Fractions are the first abstract concept students are introduced to. Learning to work with fractions lays a foundation for studying other abstract concepts that appear in high school math courses. Being successful with the abstract topic of fractions gives students the ability and confidence to work with other abstract concepts that are more difficult. Unfortunately, many students do not learn about fractions in this light. The mathematician Han-Hsi Wu stated that "most American curricula of grades 5-7 downplay the inherent abstraction of the subject of fractions and teach it exclusively by the use of metaphors, analogies, and manipulatives." (Wu, 2008)

I worked with 6^{th} grade Gifted, 7^{th} grade Gifted and 7^{th} grade Advanced students enrolled in an Introduction to Algebra class. This class is usually taught to 8^{th} grade students. Fraction concepts usually taught in 6^{th} and 7^{th} grade are a prerequisite for Introduction to Algebra. All of the students I worked with have skipped at least one grade level of mathematics. The 6^{th} grade students skipped traditional 6^{th} grade and traditional 7^{th} grade math. The 7^{th} grade students took a course in the previous year that combined 6^{th} and 7^{th} grade math courses. The combination course does not give the students the time needed to master all the fraction concepts normally taught in the two courses.

2.2. Conceptual Make-up of Catch-up Lessons

After reviewing the Louisiana GLEs, I observed that my students who skipped Grade 6 and Grade 7 math missed out on fraction operations. The lessons I created were based on this fact. My students should have been exposed to all other fraction concepts needed to be proficient in elementary school.

The lessons in my catch-up program focused on two main objectives. The first objective was to give students a better understanding of fractions. Based on informal observations students could illustrate a fraction, but had difficulty defining the term mathematically. For example, students could draw out 3/4 using a pie, but could not give a definition such as a fraction is a part to whole relationship or a fraction is a comparison of two distinct objects, or most importantly a fraction is a number. They could use partitions to represent fractions, but did not think of fractions as numbers. Though most of my students were able to simplify fractions and order simple fractions from least to greatest, they lacked a deeper understanding of what they were doing. The second objective was to show students the procedures for fraction arithmetic and to explain how and why these procedures relate to the parts of arithmetic they already understand. I also wanted to give them practice, because students need to be able to comfortably compute with fractions in order to be successful in Algebra 1 and all math courses that follow.

Because fractions can be used in multiple ways they may be a difficult topic for middle school students to fully understand. For example, fractions have multiple interpretations. The first lesson defined the term fraction in three ways and presented examples of situations representing each of the definitions. The symbols used in fraction notation are another difficult concept to understand. Some students relate the numerator and denominator to two separate numbers instead of thinking of a fraction as one quantity. The parts of a fraction were labeled and discussed and the meaning and significance of the parts were illustrated.

Many curriculums involving fractions teach only the procedures. These curriculums do not explain what happens conceptually when fractions are being added. Adding fractions on the number line may help. The number line allows students to see the necessity of common denominators. Students can also make a connection between adding whole numbers and adding fractions when working with the number line. When students are taught to add and subtract whole numbers on the number line they are using a unit of 1. They see that whole numbers use a common unit, or common denominator, just like rational numbers use a common denominator. This allows them to make the connection that when adding and subtracting any number a common unit must be used. Multiplication and division of fractions also needs a conceptual basis. In Singapore, students learning to multiply fractions use sets and fraction bars to visualize what is happening. Grouping the elements of a set is used to show multiplication of whole numbers and fractions. This allows students to make the connection between the term “of” and the multiplication operation. Singapore students also use fraction bars to demonstrate fractions multiplied by fractions. This visualization allows the students to create procedures for multiplying fractions. The division of two fractions is also represented by fraction bars. Visualization of the process allows students to create procedures for dividing fractions. Students can see that the dividend is being multiplied by the reciprocal of the divisor.

In order to teach concepts to students, teachers need to be aware of what students do and do not know about the concept. The only way to determine student knowledge is to give the students some kind of assessment. Each lesson began with a starter problem for the students to work on which always focused on the lesson being presented that day. These questions allowed me to assess students for a better understanding of their knowledge about the lesson. The lessons could then be modified according to what I gathered from the class discussion of the starter problems.

2.3. Implementation of Lessons

The following lessons were taught following Unit 4 of the East Baton Rouge Parish Comprehensive Curriculum for Grade 8. The lessons and the assessment that followed took place during the weeks of January 10-21, 2011. There were a total of six lessons, delivered over 8 days. The assessment period lasted two days. The assessment was given during the last week of school. Unit 4 of the Comprehensive Curriculum focused on rates, ratios, and proportions. I felt that this was the correct time to give the lessons since the students had just finished working with ratios, which are fractions. During Unit 4, students studied simplifying ratios. I taught students to simplify ratios using prime factorization. These lessons were taught to a Gifted 6th grade and Gifted 7th grade combination class and an Advanced 7th grade class.

Lesson 1, 2, 3, and 4 were taught on consecutive days. After lesson 4, a day was spent working on word problems that involved adding and subtracting fractions. Lesson 5 and 6 were also taught consecutively, each lesson getting its own day. After these two lessons, students spent a day working on word problems involving division and multiplication with fractions. The two days that were spent working on word problems allowed me time to advise students who were still having difficulties with the concepts, and this gave all the students more practice and connected the concepts to the real-world.

2.3.1. Lesson 1

Lesson 1 focused on defining the term fraction and defining the parts of a fraction. To do an informal assessment of the knowledge my students had of the definition of a fraction, I placed the number 3/4 on the board. I then asked the students to write down a description of the picture on the board. The students took about two minutes to write down their thoughts. I then asked students to share their thoughts with a partner. Partners were assigned at the beginning of the school year and were used often during class discussions. Partners discussed their results for about two minutes. I then asked the students to share with the class. Many of the students said that 3/4 was 0.75 or 75%. Students also compared 3/4 to 3 out of four objects. For example three out of four slices of pizza. I gathered that students could relate fractions to decimals and percents and had an understanding of fractions being part of a whole.

After this discussion we talked about the fact that a fraction can be interpreted in more than one way. I presented three definitions to the class. I defined a fraction as a part-to-whole relationship, as quotient, and as a comparison of two distinct objects. After each definition, I presented an example to illustrate it. I allowed the students to come up with their own examples in order to give a quick informal assessment of their understanding. Students gave me examples such as 4 red marbles to 11 total marbles and 4 cookies out of 5 total cookies for part to whole relationships. Examples of quotients were 2 divided by 5 and 1 divided by 2. Examples of ratios were 3 blue marbles to 5 red marbles and 6 trucks to 7 cars.

Next, we discussed the parts to a fraction. I asked the class to list the parts of a fraction and explain what each part did. Most students knew that there were a numerator and a denominator, but some were confused as to where each was located in a fraction. None of the students viewed the fraction bar as being part of the fraction. When I asked what they thought the bar was, many of them said that they had never really thought about it until I brought it up.

After the discussion, I wrote a fraction on the board and labeled the three parts. I then gave a description of what each part represented. The numerator is how many objects are being considered. The fraction bar separates the numerator and denominator and is used as a grouping symbol. This symbol is also a symbol meaning to divide. The denominator names the objects being counted. For example in the fraction 3/4, 1/4 is the object being counted, or the unit, and 3 represents how many times 1/4 is being considered. We also had a discussion about whether or not students thought that fractions were numbers. Some students responded with the following: no, because you cannot count using fractions, yes because you can have a fraction on the number line, and yes because we learned about the real numbers. To clear up this matter we reviewed the real number system and used the number lines to illustrate that fractions are numbers. We discussed the fact that, just like whole numbers, fractions represent a quantity.

At the end of the lesson, we reviewed how to simplify fractions. This topic had been taught previously during the year. In Unit 4 students studied simplifying ratios using prime factorization. Because a ratio is one interpretation of a fraction, fractions were also simplified in this manner. We had a class discussion on the procedures that were taught previously and we discussed how to tell if a fraction is in simplest form. The students were able to explain to me how to simplify a ratio and they were also able to tell me that it was in simplest form if the two numbers were relatively prime.

After this lesson students were aware that different definitions of fractions existed. Students now had an understanding of what purpose the fraction bar holds and could give several examples of situations represented by fractions. I realized that students had not explored the parts of a fraction as much as they should have and that many students were confused about what a fraction represented.

2.3.2. Lesson 2

Lesson two focused on adding fractions with like denominators. To gain a better understanding of my students' knowledge, I wrote the following scenario on the board:

> Sara walked 3/7 of a mile on Monday and 2/7 of a mile on Tuesday. How far did she walk altogether? Draw a picture to illustrate your findings.

Students were given three minutes to work on this problem on their own. Students then discussed their findings with group members. After five minutes with their groups we had a class discussion. I asked two students to copy their illustrations on the board. One student drew a number line with the numbers 1-10. She started at 3 and moved two more spaces to arrive at 5.

Then she concluded that Sara walked 5/7 of a mile. Another student drew a rectangle divided into seven parts. He shaded in three to represent 3/7 and then shaded in two more to represent 2/7. He counted the shaded regions and concluded that Sara walked 5/7 of a mile. I gathered that students were able to interpret the problem as addition. I also noticed that students were able to make the connection that, since both fractions had a denominator of 7, the numerators could be added together without manipulating the denominators.

I asked the students to look at the two illustrations on the board. We had a discussion on whether or not the students viewed the solutions as correct. All of the students agreed that the illustrations made since, therefore both were right. We then discussed the illustrations one at a time. I had them pay close attention to the numbers that were placed on the number line. I told them that 3 and 3/7 was not the same number. One of the students argued that since both fractions had the same denominator then it did not matter. I told the students that the numbers 2 and 2/7 were not the same and all numbers need to be correctly labeled when working with the number line. Fractions should not be viewed as whole numbers due to the fact that they are not whole numbers.

From the discussion, I had to change the way I had originally planned to teach. In my notes I assumed that number lines would not appear in our discussion. To clarify using number lines, I drew a number line with only 0, 1/2, and 1. I asked the students if this number line could be used to solve the problem. The students concluded that this number line would not be a good choice, but suggested that a number line with 1/7-7/7 could be used. I told them that this was correct. Just like when we added whole numbers on the number line we could add fractions, as long as there was a common unit. We then used number lines to add other examples of fractions.

Students commented that this procedure took a long time, but helped them to understand why common denominators were necessary.

Once the students were comfortable using the number line to add fractions, we worked on creating a procedure that could be used, in place of the number line, to add fractions with like denominators. The students mentioned that in all of the examples we added up the numerators, but the denominators did not change. For the procedure we concluded that if fractions have like denominators, to add them add the numerators, place the sum over the common denominator, and simplify the result if necessary.

After this lesson I realized that my students already had a good understanding of adding fractions using pictures. The students learned that they could solve word problems involving fractions by illustrating the solution and they also learned the procedure used for adding with like denominators and practiced using it. Students developed a conceptual understanding of why fractions being added need a common denominator by using the number line as a tool.

2.3.3. Lesson 3

Lesson 3 focused on adding fractions with unlike denominators. To get a better understanding of the students' previous knowledge the following problem was placed on the board:

> Peter ate 2/3 of a cake. His friend David ate 1/6 of the same cake. How much of the cake has been eaten? Illustrate your findings.

Students worked on the problem individually for three minutes then shared their results with their groups. The results were discussed in groups for about 5 minutes.

Two students copied their illustrations on the board. One student drew a circle and divided it into 3 parts. She shaded 2 of the three parts. Then she drew another circle with six parts and shaded in one of the parts. She drew a third circle with 6 parts and shaded in 5 of the parts. She explained that her first circle represented 2/3 and the second circle represented 1/6. The third circle represented the sum of 2/3 and 1/6. I asked the student why she partitioned the third circle into six pieces and she explained that 1/3 is the same as 2/6 and she needed to add 1/6 to that quantity so six partitions made the most sense. The second student drew two rectangles of equal size and placed one directly above the other. She divided the first rectangle into three sections and the second rectangle into six pieces. She shaded 2 sections of the first rectangle and said that the shaded region represented 2/3. She shaded 1 section of the second rectangle and said that this shaded region represented 1/6. She then mentioned that two sections of the second rectangle made up one section of the first rectangle. In order to combine the two quantities she drew a third rectangle of equal size and divided it into six sections. She said that four sections represented 2/3 and one section represented 1/6, so added together you get 5/6. I gathered that the students were able to create a common unit, and that they used this common unit to aid them in their solutions. Most students had a strong conceptual understanding for adding fractions, but were unaware of the procedure used in symbolic computations.

After both presentations we discussed them as a class. The class concluded that both students gave a correct answer and that both students went through the same process but used different shapes to illustrate their process. I then posed the question "Can you use a number line to illustrate this?" The general response was yes, but no one gave an explanation for their choice. I placed a number line with the numbers 0, 1, and 2 and asked the class if this number line could be used. One student said no because the number line did not have any fractions, but our problem did. Another student said you could use it but you would have to divide it into smaller sections. Then I placed a number line with 0, 1/3, 2/3, and 1 and asked if I could use this one. The class concluded that you could use this number line to represent 2/3, but you could not add 1/6 to it. We then discussed how we could change the number line so that it could be used. The students concluded that a number line with 0, 1/6,…, 6/6 would allow this to happen. The fractions 2/3 and 4/6 are in the same position. After this, I then gave them the problem 2/3 + 2/5 and asked the students to explain how to use a number line to add them. The students worked individually for five minutes. The class concluded that the number line would have to be divided by fifteenths to find the sum of the problem. We then made the connection between the least common denominator of the fractions and the amount of partitions the number line needed to be. The students concluded that the denominators of both fractions must be the same to be able to add the fractions.

From this conversation, I asked the students why, when adding whole numbers, we do not have to keep using different units like we do in fractions. After many incorrect answers, one student suggested that it had to do with the fact that all whole numbers had a denominator of one. We discussed this as a class and the students agreed that this was a good answer to the question. I then told the students that when adding two quantities a common unit must be used. The common unit for all whole numbers is one. The common unit for fractions will have to be found using common denominators.

Next, we discussed how the procedure for adding with unlike denominators should occur. Students concluded that a common unit must be established before any additions can be performed. So the first step would be to make both fractions have the same denominator. We discussed how this should be accomplished. Some students commented that the numerators would stay the same, but the denominators would change. I then drew an illustration of the fraction 4/5 on the board and an illustration of the fraction 4/10 on the board showing the students that just changing the denominator changes the quantity of the fraction. We need to find a way to keep the same quantity but represent the quantity in a different way. We then discussed how to fix this. We concluded that equivalent fractions denote the same quantities, so equivalent fractions need to be used here. To make both fractions have the same denominators, equivalent fractions with the common denominators must be produced. Once the fractions were rewritten with common denominators, the rules for adding fractions with like denominators can be applied.

The procedure was demonstrated by me three times and then students volunteered to work out examples on the board. After this, students worked in pairs to complete a worksheet with sample problems for adding fractions with unlike denominators.

This lesson allowed the students to make the connection between their illustrations and the procedures used for adding fractions with unlike denominators. It appeared that previously the students were able to create common denominators without knowing what they were doing. The students now had a rationale for the procedures used for adding fractions.

2.3.4. Lesson 4

Lesson 4 focused on subtracting fractions with like and unlike denominators. To get a better understanding of students' knowledge the following problem was placed on the board:

Michael had 5/8 of a cake. He ate 3/8 of the cake. What fraction of the cake was left? Illustrate your solution.

Students worked on this problem for three minutes individually and then shared their results in small groups. After five minutes, there was a class discussion of the results.

Two students placed their results on the board. The first student drew a circle partitioned into eight pieces. He shaded five of the pieces and explained that the shaded region represented

5/8 of a cake. He then double shaded three of the already shaded regions and explained that these pieces represented the pieces he ate. He then concluded that there were 2/8 of the cake left because 2/8 of the shaded region had not been double shaded. The second student illustrated the problem using a number line. He drew a number line from 0 to 1 divided into eighths. He started at 5/8 then moved to the left 3 times. He drew a circle around 2/8 and concluded that 2/8 of the cake would be left. He also added that 2/8 was not simplified and that the actual answer is 1/4. I gathered, from these solutions, that students were beginning to connect fractions to the number line and that they could correctly interpret a word problem involving subtraction.

Next, the class compared the two solutions on the board. One student commented that the first solution was solved correctly, but was not simplified. Another student commented that the double shading was confusing because she could not tell if the answer was the single shaded region or the double shaded region, without looking at the answer given. More of the students used number lines to illustrate this problem due to the fact that we used number lines in the previous two lessons to show addition.

After this, we worked three more examples of subtraction with like denominators using the number line. The students concluded that the procedure for subtracting fractions with like denominators was closely related to the procedure for adding fractions with like denominators. They concluded that the procedure for subtracting fractions with like denominators was to first subtract the numerators and then place the difference over the like denominator. I added that the answer must always be simplified.

We then moved on to subtracting fractions with unlike denominators. I placed the problem 3/4 – 1/2 on the board and asked if we could use the procedure we used for subtracting the previous set of problems. The students agreed that the above procedure would not work because the denominators of the two fractions were not the same. We discussed how the procedure would have to be changed in order to solve the problem. One student said that we needed to change the denominators so that they would be the same. I asked him to elaborate. He said we could use 8 as the common denominator because both denominators divided 8. If we do this, we have 3/8 – 1/8 and the answer is 2/8 or 1/4. Another student disagreed with this solution. She said that the fractions having the denominator of 8 were not equivalent fractions to the fractions in the original problem. She said that the fractions should be 6/8 – 4/8 giving and answer of 2/8 or 1/4. This led to a discussion about whose procedure was correct, since the result of both were the same. I allowed the students to discuss this in their groups for about 5 minutes. One group concluded that it was just a coincidence that the same answer was produced. They commented that they tried both methods out, using illustrations to help them, with two other problems and concluded that the second method was correct. The other groups agreed, with one of the groups adding that the denominator of 4 is the least common denominator for the original problem.

After this, we wrote a procedure for subtracting fractions with unlike denominators. The first step would be to find a common denominator and then rewrite equivalent fractions having this denominator. Second, subtract the numerators and place the difference over the common denominator. Third, simplify the solution if necessary. Students then worked on practice problems with a partner.

After completing the lesson I concluded that students were able to develop procedures easier now than when we first started learning how to compute with fractions. Students understood what a common denominator was and how and why it was used when adding or subtracting fractions.

2.3.5 Lesson 5

Lesson 5 focused on multiplying fractions. To get a better understanding of students' knowledge I placed the following question on the board:

> Noah had $20. He went to a book store and spent 2/5 of his money on a book. How much did the book cost? Draw a picture to illustrate this.

The students were given three minutes to work on the problem individually, and then shared their results with their groups. I asked two students to copy their solutions on the board.

The first student drew 20 rectangles, divided the rectangles into groups of 5, circled two of the groups, and wrote the number 8 on the board. He explained that the rectangles represented the $20. He then said that he divided the money into five groups because he needed to know how much money represented one fifth. He explained that he circled two of the groups because he needed two fifths. He then counted the rectangles in the groups and concluded that the book cost $8. I asked the student what operation he performed and he told me that he performed division because the answer was smaller than $20. The second student drew one rectangle partitioned into twenty sections. He then drew a line after every fourth partition until he had five groups. He shaded the rectangles in two of the groups. He explained that he needed to figure out how much one fifth was before doubling that to get two fifths. He also concluded that the answer was $8. I asked him what operation he performed and he also said it was division.

I gathered that the students could work out the problem using pictures to guide them, but they were not translating their understanding into the appropriate operations. The logic behind their solutions was solid, but they lacked the knowledge needed to realize the problem was referring to multiplication and not division. I realized that students were not fully aware of the meaning of the multiplication operation.

I then led a discussion about reading word problems and using key words to help determine the operation to perform. We went over the problem carefully and the class decided that the key word was "of." The students remembered that "of" meant to multiply and therefore concluded that the operation must mean to multiply.

Then we looked at why multiplication is the correct operation. We looked at a word problem involving whole numbers:

> A case of water contains 24 bottles. If Jane bought 7 cases, how many bottles of water did she buy?

To illustrate this word problem, 7 groups <u>of</u> 24 were drawn on the board. We discussed the fact that "7 <u>of</u> 24" has the same connection to the operation of multiplication as "2/5 of 20." Anytime you are finding an amount of something else, multiplication is used.

Next, we discussed the procedure for multiplication. I used the example 20 * 2/5 to aid in thinking about the procedure. We looked at what occurred to get the solution of 8. I explained that 20 was divided by 5 and then multiplied by 2. This procedure works when one of the factors is a whole number. After this, we looked at the problem 3/4 * 2/3. First, we looked at how to illustrate this. I drew a rectangle divided into three equal parts and shaded two of them. I explained that I illustrated 2/3 first because a multiplication problem is read 3/4 of 2/3. I then divided the shaded region into fourths and double shaded three of the sections. I explained that this double shaded region represented 3/4. Once my shading was complete, I asked the students to tell me how much of the entire picture the double shaded region represented and they answered 1/2.

We looked at three more examples similar to the above example. After doing this, I explained the procedure for multiplying fractions. Students were told to multiply numerator by numerator and denominator by denominator. The solution is always simplified when necessary. We then had a discussion about why common denominators were not used. I explained that multiplication does not call for a common unit; therefore a common denominator was not necessary. The students worked out a problem using common denominators, and then a problem not using common denominators. They concluded that they could be used, but were not necessary and in some cases made the problem more difficult. For example it is easier to multiply 1/8 * 1/3 than it is to multiply 3/24 * 8/24.

After this lesson students could use a procedure to multiply fractions and were aware of the fact that common denominators were not necessary in multiplying fractions. I realized that students had a difficult time understanding why whole numbers multiplied by some fractions produced a number smaller than the whole number. Students still seemed to be weak in this area after the discussions that took place.

2.3.6 Lesson 6

Lesson 6 focused on division of whole numbers by fractions and division of fractions by fractions. To get a better understanding of students' knowledge the following problem was written on the board:

> How many pieces of string, each 1/5 m long, can be cut from a string 3 m long? Illustrate your solution.

Students were given three minutes to work individually and then spent five minutes discussing their answers as a group.

Two students placed their solutions on the board. The first student drew a line and divided it into three sections. She then divided each section into five sections, producing 15 sections. She explained that the line represented the 3 m of string. The first subdivision represents each individual meter and then the second subdivision divides each individual meter into fifths. She then counted her sections and concluded that 15 pieces of string 1/5 m long could be produced. I asked her what operation she performed and she replied that she would have originally chosen multiplication, but since there was confusion in multiplication being thought of as division, this is probably division. The second student drew three separate rectangles and explained that each represented 1 m of the string. Then he divided the rectangles into five pieces and said that each piece represented 1/5 m. He counted the 1/5 m pieces and concluded that fifteen 1/5 m pieces could be made. I also asked him what operation he performed and he answered that he was doing division. When a question asks how many, then you should know that division is taking place. From these solutions, I gathered that the students were making strategic decisions based on previously learned topics.

Next, we looked at three more examples of dividing whole numbers by fractions and fractions by whole numbers. We discussed the similarities in each of the problems, and the students concluded that dividing by a fraction was the same as multiplying by the reciprocal of the fraction. I asked them if this also was the case for the example of dividing a fraction by a whole number. The students agreed that the fraction was being multiplied by the reciprocal of the whole number. I explained that this was also true for dividing a fraction by a fraction. The procedure is to multiply the dividend by the reciprocal of the divisor.

I worked three examples of fractions divided by fractions on the board and then students volunteered to work more examples on the board. After about 10 students explained their work, students completed a worksheet containing practice problems of dividing fractions by fractions.

After this lesson most students were able to divide fractions by fractions quickly and correctly. As I walked around, while the students worked on problems, I realized that this was the most difficult concept for the students to work with. In the previous lessons, students were able to correctly work most of the problems when starting the worksheet. In this lesson, students were still struggling to correctly perform the procedure half way through the worksheet.

2.4. Assessment

After teaching the lessons and having the students work on practice problems, each student was given 12 questions to analyze and answer. In developing these questions I referred to the Common Core Standards for Mathematics. In the Common Core Standards objectives for understanding fractions are explained in Grades 3-6. Each of these goals is explained in detail on a general basis. I took these generalities and turned them into problems appropriate for middle school students.

Questions 1 and 2 dealt with illustrating and explaining fractions. Students were asked to depict the fractions 1/3 and 3/8 and to explain what each depiction meant. These questions were testing the idea that students understood fractions as part-to-whole relationships. These questions were also designed to find out if students would only stick to the part-to-whole relationship or if the students would associate the fractions with any of the other definitions presented to them in Lesson 1.

Questions 3 and 4 dealt with ordering fractions on a number line. Question 3 listed common fractions, 1/2, 1/3, 1/4, 2/3, 3/4, 3/2, to order and question 4 listed unusual fractions, 5/2, 7/3, 3/8, 7/8, 13/8, to order. These questions tested students' abilities to correctly order fractions from least to greatest. Common fractions as well as unusual fractions were used to see if this caused any discrepancy in students' abilities to compare and order fractions.

Questions 5 and 6 dealt with equivalence of fractions. Question 5 compared 1/2 and 2/4 and question 6 compared 4/6 and 6/9. Question 5 showed a multiple of another fraction and question 6 compares two fractions that are simplified to the same fraction. This tested students' understanding of what is meant by equivalent in more than one way.

Question 7 and 8 dealt with comparing fractions. Both questions asked the students to tell which fraction was bigger. Question 7 compared 1/2 and 1/8 and question 8 compared 4/7 and 3/5. Question 7 tested students' ability to compare fractions with the same numerator but different denominators. Question 8 tested students' ability to compare fractions with different numerators and different denominators.

Question 9 and 10 dealt with adding fractions. In Question 9 students were given an example problem and asked to tell if the work was done correctly. In question 10, the students were asked to explain how to add the two fractions to a classmate. These questions tested the knowledge gained from Lesson 3. It tested students' ability to correctly add fractions using common denominators.

Question 11 dealt with multiplying fractions. The students were given an example problem and asked to explain if the student was right or wrong. The example showed common denominators being used. This tested students' understanding of when common denominators were necessary. Question 12 dealt with comparing the factors to the products of rational number multiplication. This question asked to compare the size of the product to the size of the rational factors once multiplication is applied.

CHAPTER 3. FINDINGS

From the discussions of the lessons, I noted areas where the students were more involved than usual and areas that seemed to create some confusion. I also reviewed the results found from the test I piloted and noted questions that yielded little significance to learning which students were still struggling with the concept in the question. I also noted questions that produced results that I could use to restructure the catch up program in the future.

3.1. Reflections on Lessons

The lessons I taught contained some good points, but there were also points that could be improved. Reflecting on the lessons, there are some aspects I would change and there are some that I would expand and build on. Overall, the lessons were successful in discovering the knowledge my students' previously held as well as in discovering the knowledge the students' gained during the lessons.

Students reacted well to the first lesson, which highlighted multiple interpretations of fractions. The discussions that followed these interpretations were enthusiastic and the students were 100% engaged. The students also reacted well to being able to give examples of their own. Students began to understand that fractions were not as simple as whole numbers. They also began to understand that a part-to-whole relationship was not always the best way to explain a fraction. Identifying a fraction as a number lying on the number line allowed the students to see that fractions represented one single quantity and not two distinct whole numbers bundled together in a mysterious way.

Students enjoyed aiding in the process of developing the procedures for fraction operations. The need to be precise was emphasized, and by the time we reached the lesson on multiplication, students were correcting me for not being precise enough. Students like the idea of being included in the teaching process—not just the learning process. Developing the procedures for themselves gave students a sense of accomplishment. Students were better able to remember the procedures throughout the year.

The lessons on adding and subtracting fractions were also successful. The students truly grasped the reasons why common units are needed to add and subtract numbers by using the number line. The number line gave the students a good visual for the role of common denominators; however, students began to realize this method was time consuming. Student liked the idea that a procedure could be developed that saved time. Students were able to answer questions relating to common units and were able to make the connection between adding whole numbers and adding fractions. They realized that whole numbers also have a common unit, and are special fractions with denominator 1.

Allowing students to spend an entire class period working on word problems, involving adding and subtracting fractions, was also successful. Students were able to identify which questions dealt with addition and which ones dealt with subtraction. The word problems helped the students connect these concepts with the real-world, giving them a purpose for learning the material.

The multiplication lesson was not as effective as I had hoped. When teaching this lesson in the future, I will try to find a new method for explaining what occurs when two fractions are being multiplied that makes the concepts more apparent. An example of a new method would be using grid models. In grid models, students label two sides of a rectangle with the units associated with two different fractions, and then divide the rectangle into smaller area units. Grid models connect multiplication to area, which is a concept they are already familiar with, and to the distributive law.

I would also spend more time reinforcing the idea that common denominators are not necessary when multiplying. Looking at the results of the assessment, I realized that this idea was not addressed in enough detail for the students to fully absorb. Having the students work the problems out in both ways, allowing students to work examples using both methods, common denominators and just multiplying before using common denominators, may help to bring this home.

Although students saw what was occurring when whole numbers were divided by fractions and could draw conclusions from this, we did not explore what happens when fractions are divided by fractions. Students need to see both concepts. This is a difficult concept to show conceptually and more investigation is needed to create a method for inviting students to explore this concept.

I did not include mixed numbers in the lessons; however, some problems involving mixed numbers were included in the worksheets that students completed. Mixed numbers should be added to the lessons to give the students time to work with them in a setting where the teacher can give assistance. I had to stop the students during the time allotted for them to work on their worksheets to do examples involving mixed numbers. We discussed whether or not fractions needed to be changed into improper fractions before you could perform operations. I also showed them the borrowing method when subtracting mixed numbers.

3.2. Data Analysis

The lessons I taught were specifically on operations with fractions. However, since students need to be proficient in all aspects of fractions, I tested them on more than just fraction operations. The questions on my written test were designed to discover whether or not my students could define a fraction, compare fractions, order fractions, and understand addition and multiplication of fractions.

Students answered the questions in a variety of ways. Question 1 asked the students to illustrate the fraction 1/3 and describe the illustration, and Question 2 asked students to illustrate 3/8 and describe the illustration. The answers varied in style and description. Some students used circles, some used rectangles or fraction bars. One student used a number line in his drawing, but the illustration was inappropriate because it showed no fractions. In the cases of circles and fraction bars, when drawing 1/3, students shaded in one section and left the other two not shaded. When describing the pictures, students either stated that 1/3 of the object was left or 1/3 of the object had been taken away. In most of these cases the object drawn was a piece of food, such as a pie, a cake, or a chocolate bar. Only a small number of students made the effort to note explicitly that the object had been divided into equal pieces, although most of the pictures were drawn to reflect this idea.

Question 3 asked students to order common fractions on the number line. (see Appendix D) Students had the most difficulty with placing the fraction 3/4 in the correct spot. 21 out of 31 students placed this correctly; see Figure 1. Placing 2/3 and 3/2 were also difficult for students. 24 out of 31 students placed these correctly. Some of the mistakes were placing 2/3 after 1 or after 3/4, and placing 3/2 before 1 and before 1/2. All of the students placed 1/2 correctly; one student omitted the number 1.

A statistical test rejects the null hypothesis that the factions in question 3 were equally likely to be placed correctly, with a significance level of $p < .01$. In other words, the differences in height of the columns in Figure 1 cannot be attributed to chance alone.

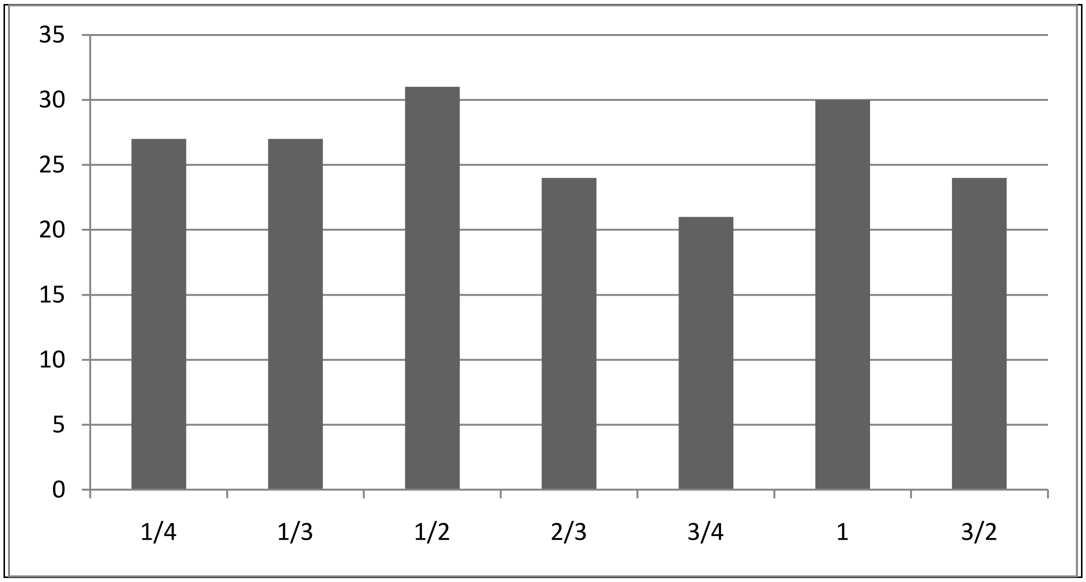

Figure 1. Number of students out of 31 who accurately placed fractions from question 3 on the number line.

Question 4 asked students to order unusual fractions on the number line. (see Appendix D) Students had the most difficulty placing 3/8 and 13/8 on the number line. 20 out of 31 placed them correctly; see Figure 2. A common mistake seen when placing 3/8 was students placed it between 0 and 1/2, but placed it closer to zero. The most common mistake seen when placing 13/8 was that students placed it after 2, and not between 3/2 and 2. 21 out of 31 students placed 7/3 in the correct place. Mistakes seen when placing 7/3 were placing it after 5/2, placing it before 2, and giving it equal value to 2. 25 out of 31 students correctly placed 5/2. The most common mistake was placing it significantly closer to 2 than to 3. Mistakes made when placing 2 on the number line were probably due to lack of attention to detail. The number was not out of order, but was too far away from 1 when considering the distance between 0 and 1.

A statistical test rejects the null hypothesis that the factions in question 4 were equally likely to be placed correctly, with $p < .0011$. In other words, the differences in height of the columns in Figure 2 cannot be attributed to chance alone

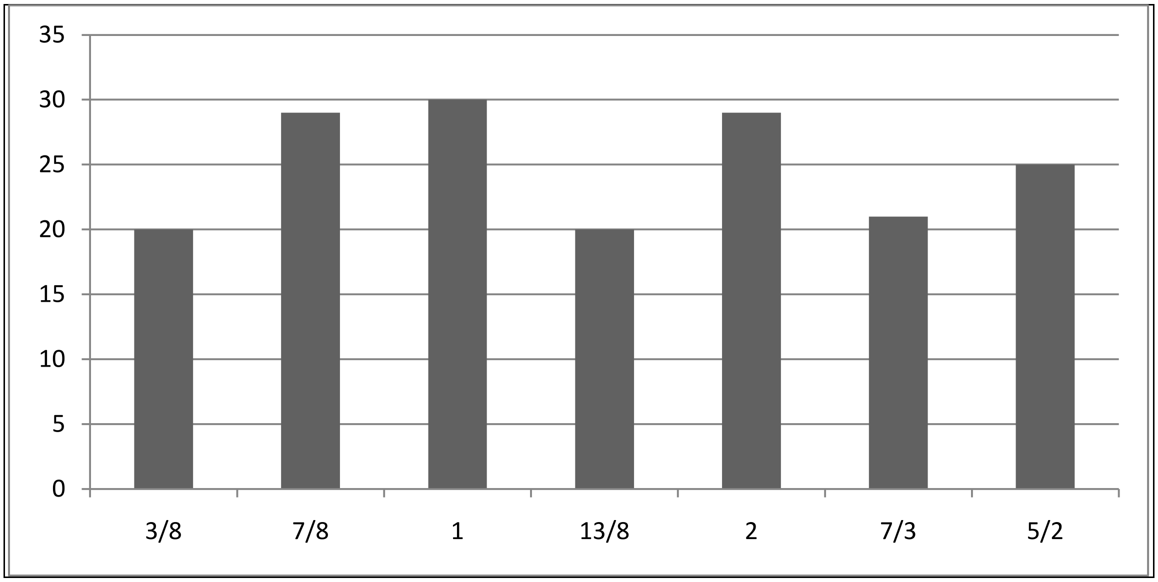

Figure 2. Number of students out of 31 who accurately placed fractions from question 4 on the number line.

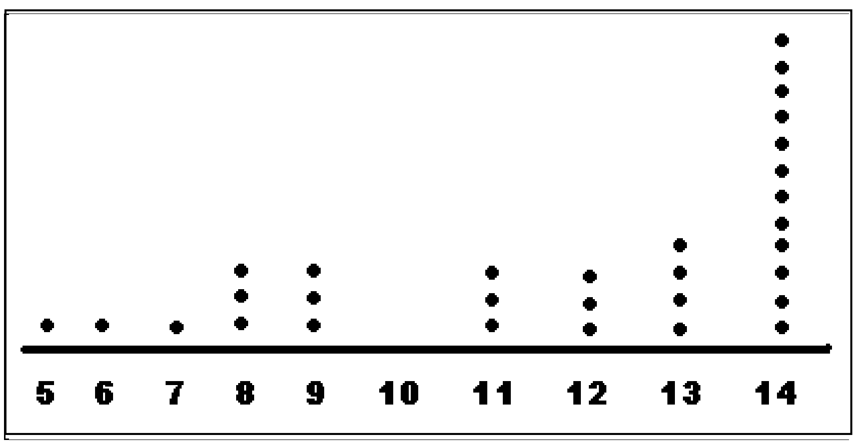

Figure 3. Distribution of the total scores of 31 students on the fraction-placing questions. The maximum possible score was 14.

Figure 3 shows how many students placed the given number of fractions correctly. The results show that most of the students placed 11-14 fractions correctly, which I interpret as indicative of an understanding of where fractions are located on the number line. The three students who scored at the lower end, placing 5-7 fractions correctly, are special cases. Two of these students were weaker in all respects than the rest of the class, consistently receiving lower grades in all math assignments. The other student tended to exhibit good understanding of mathematical concepts in other assignments. After reviewing her answer I concluded that she misinterpreted the directions. She correctly ordered the fractions, but did not place them on the number line.

Questions 5 and 6 compared equivalent fractions. Question 5 compared 1/2 = 2/4 and question 6 compared 4/6 = 6/9. The results show (Figure 4 and Figure 5) that students could easily tell that the fractions in question 5 were equivalent, but had some difficulty with the fractions in question 6. The one student who did not get full credit for question 5 answered yes to the question, but did not give an explanation. The most common explanation for answering question 6 incorrectly, with no, was that 6/9 could not be simplified into 4/6. Students answering it correctly gave explanations stating that both fractions simplify into 2/3 or when given a common denominator, the two fractions have the same numerator. These explanations show that these students have a deep understanding of what is meant by fractions being equivalent. Comparing these two questions reveals that my students are able to identify equivalence when fractions are multiples of each other, but have more difficulty when the fractions are not. Students who have a deep understanding of equivalence should be able to identify equivalence when dealing with either situation.

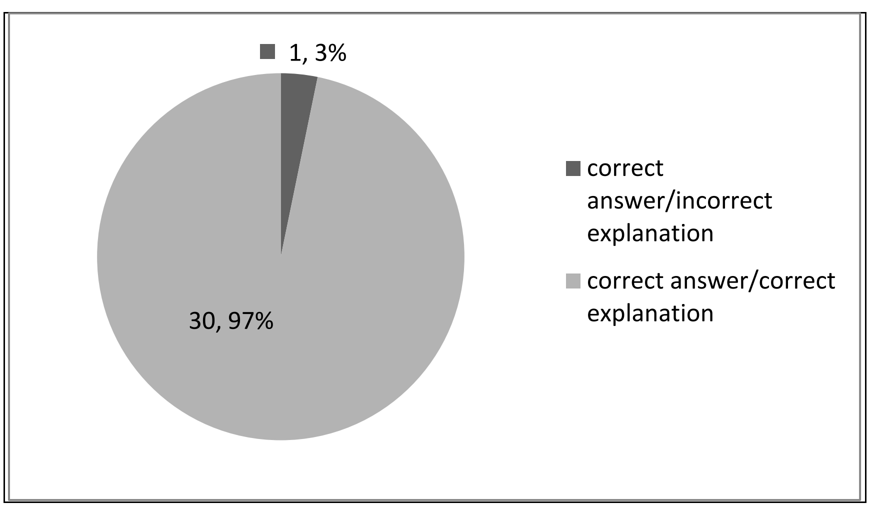

Figure 4. Results of question 5.

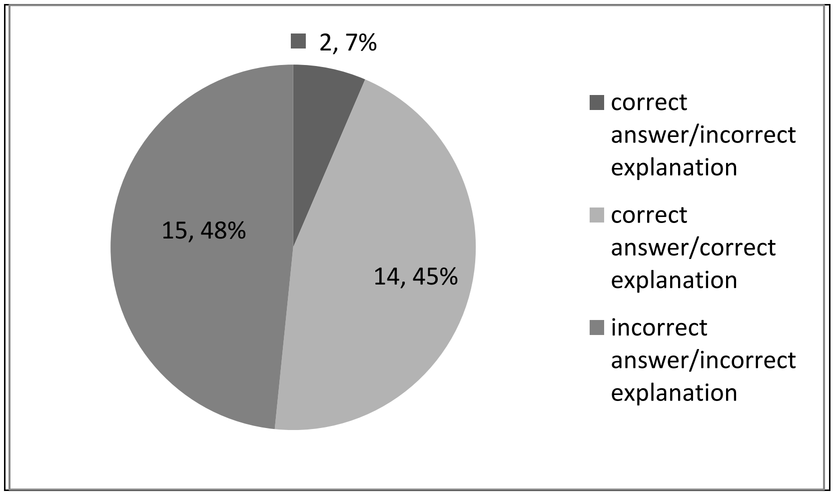

Figure 5. Results of question 6.

Question 7 and 8 compared sizes of fractions. Question 7 asked students to determine if 1/2 or 1/8 was bigger and explain their reasoning. Question 8 asked students to determine if 4/7 or 3/5 was bigger and explain their reasoning. The most common method used for explaining question 7 was that when given common denominators, 1/2= 4/8 making it larger than 1/8. Another explanation was that if a whole was cut into halves the pieces would be bigger than the pieces cut into eighths. These students were able to use proportional reasoning to help answer the question. Students who did not receive full credit usually gave a response that was not informative such as 1/2 is bigger than 1/8. The results (see Figure 6 and Figure 7) show that students are better able to compare simple fractions. When given more difficult fractions like 4/7 and 3/5 students needed to be able to use a method such as finding a common denominator to determine which was bigger. Some of the incorrect explanations of question 7 was that 3/5 was bigger than 4/7 because a whole cut into fifths has bigger pieces than a whole cut into sevenths. This logic can only be used when fractions have a common numerator. This was not the case in this example.

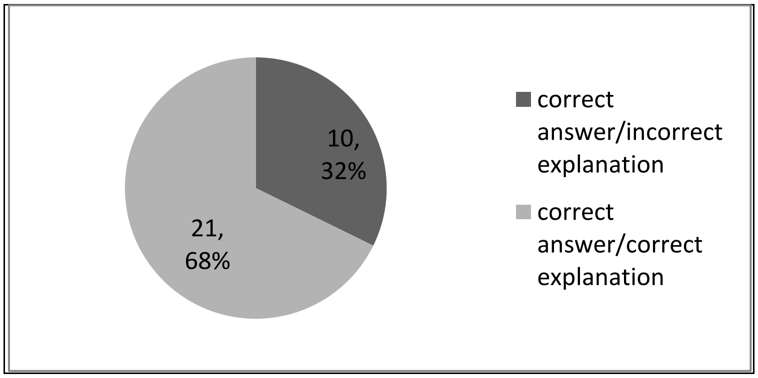

Figure 6. Results of question 7.

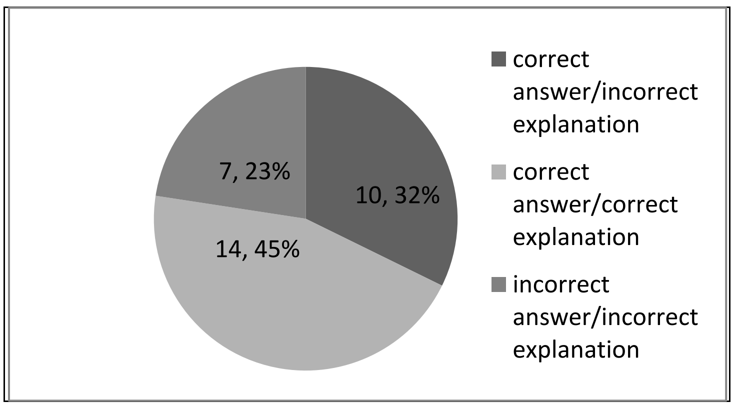

Figure 7. Results of question 8.

Questions 9-12 covered operations with fractions. Questions 9 and 10 were on adding fractions and questions 11 and 12 were on multiplying fractions. Question 9 and 10 received better results than questions 11 and 12. (see Figure 8- Figure 11) Question 9 had students analyze a sample answer of an addition problem where the student gave an incorrect answer. The results (Figure 8) show that most of the students, 84%, were able to correctly identify the problem to the sample solution. Question 10 asked students to explain how to add fractions. The results (Figure 9) show that 90% of the students could correctly explain the procedures. Question 11 had students analyze a sample solution of a multiplication problem. Students were supposed to be able to explain that common denominators were not necessary to solve the problem. The results (Figure 10) show that only 52% of the students could do this. The results conclude that multiplication with fractions is still a concept that students are struggling with. Question 12 asked students to compare the results of a multiplication problem to the factors of the problem. The results (Figure 11) show that only 13%, or four students, were able to give an appropriate answer to the problem. After reviewing the answers these four students gave, I concluded that these students truly understood what happens when multiplying fractions. These students knew how to manipulate the factors to produce the correct quantity as an answer.

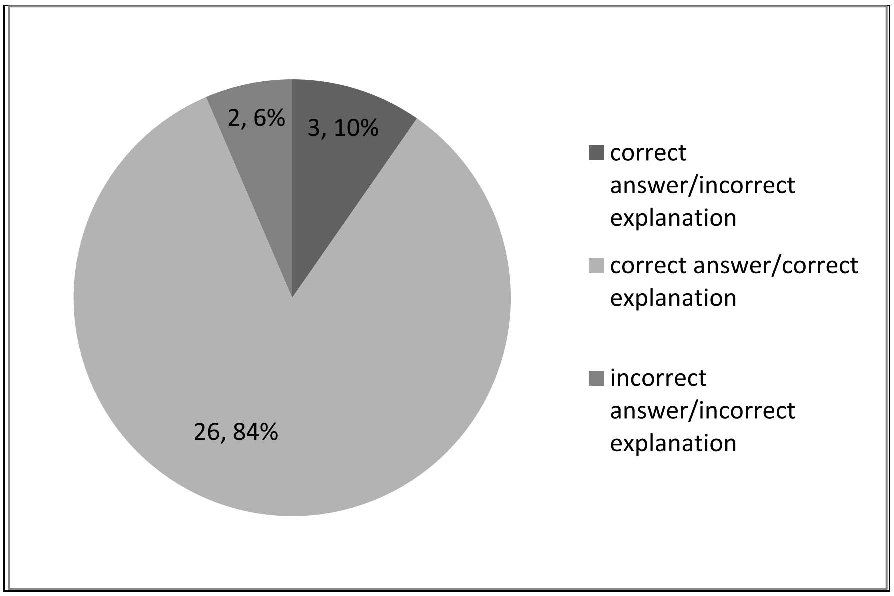

Figure 8. Results of question 9.

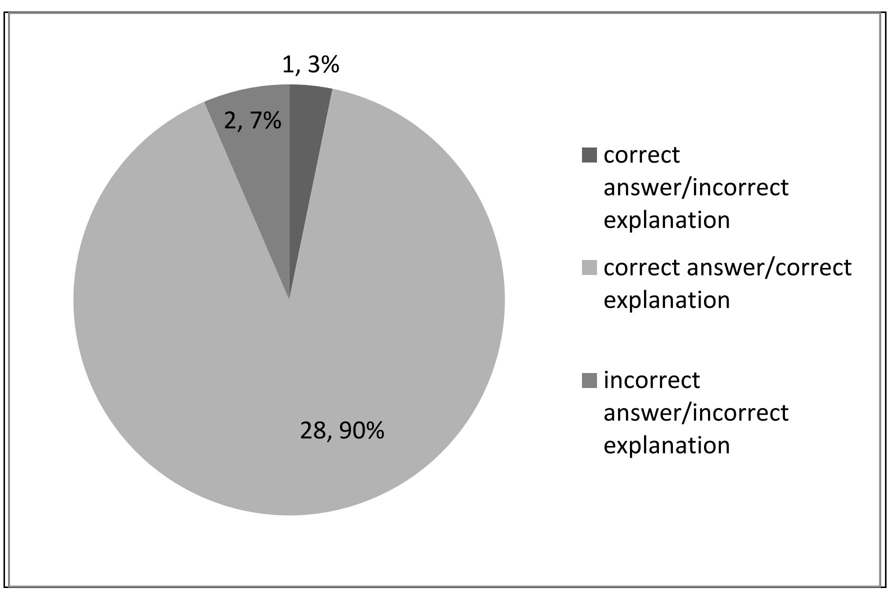

Figure 9. Results of question 10.

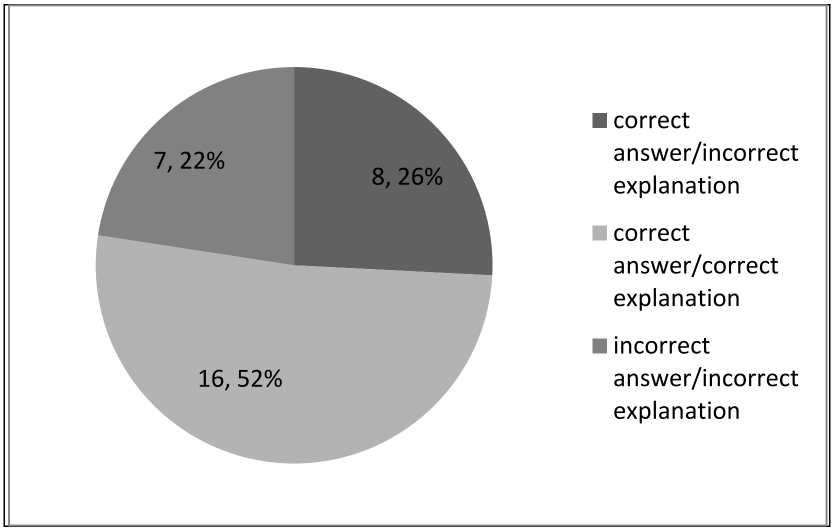

Figure 10. Results of question 11.

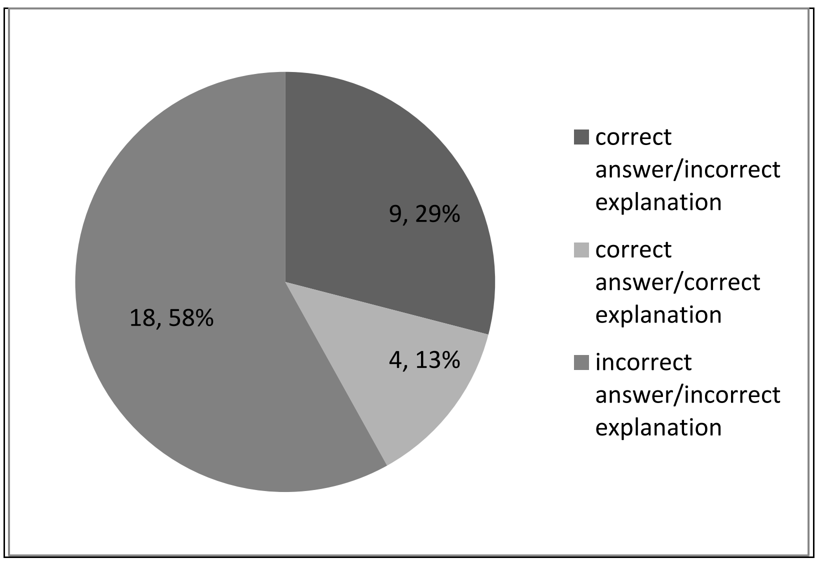

Figure 11. Results for question 12.

3.3. Reflections on Assessment

There were some interesting relationships between how the fractions were represented in questions 1 and 2 and the overall performance of the students on questions 3-12. Students who scored at the lower end often described the fraction they depicted as being an example of food. For example, one student who described the fraction 1/3 as one piece of cake, which had been eaten from a whole cake, was not able to identify equivalence in 4/6 and 6/9 or correctly explain why common denominators were unnecessary in question 10. I also noticed that students who scored at the lower levels on the assessment were typically the students who struggled in many concepts throughout the school year.

The results of my assessment were difficult to categorize. After carefully grading all of the questions, I decided to compare the results of questions related to one another. I feel that there are other ways in which I could have reviewed the results in order to get an even better understanding of what level of proficiency each student was at.

Some of the questions underestimated the abilities of my students, giving insufficient data on which of the students were weaker in the area addressed. These questions could be rewritten so as to categorize students better. Questions 5 and 7 are good examples of this error. These questions produced nearly a 100% success rate. The questions could contain more difficult fractions instead of the common fractions that were presented.

After reviewing the answers of my students, I noticed that Question 8 could have been more informative if I would have used 3/5 and 5/7 instead of 3/5 and 4/7. From question 7 students concluded that 1/2 was larger than 1/8 because a whole cut into halves had bigger portions than a whole cut into eighths. These students also used this proportional reasoning to answer question 8, but this reasoning cannot be applied due to the fact that the fractions in question 8 did not have common numerators. Students using this logic answered the question correctly by choosing 3/5, but did not make the connection about the numerators. Having used 5/7 would have made 5/7 bigger contradicting the proportional logic used.

Question 9 and question 12 was well structured, allowing the students' responses to clearly express their understanding. Question 9 asked the students to determine whether or not an addition problem was done correctly and explain how to perform it correctly. Reviewing these results allowed me to determine which students were not having a clear understanding of the necessity of common denominators. The results to question 12 significantly showed students who had developed a conceptual understanding of multiplying with rational numbers.

Question 11 allowed me to see how many of the students understood that common denominators were not necessary for multiplying. The way the question was presented seemed to be confusing to some students based on the methods they chose to answer the question. After reviewing the solutions some of the students wrote, I concluded that many of them were unsure about exactly what the question was asking.

CONCLUSION

Fraction proficiency is vital to success in mathematics in middle school and high school. Some children miss out on developing this essential skill set, and for them it is desirable to have a special program. However, there is not much literature on catch-up programs devoted to fractions. I have created and piloted such a program. Though limited in ways identified in the introduction, this is a start. Some specific areas for future work and modification have been identified in my discussions. (see p. 34-36)

Testing fraction proficiency is difficult because it is a complex, interconnected set of skills and understandings. I have also piloted an assessment that is intended to serve this purpose. The reflections, data analysis, and observations I gathered from an administration of this test to 31 students have shown that some questions are good indicators of proficiency, because they separate students into distinct ability groups. (see p. 36-46) Other questions failed to distinguish between students because almost all answered in the same way. Like the lessons, the test had specific limitations. For example, it might be improved by including questions about subtracting and dividing fractions. These were not included, but are a vital factor in fraction proficiency.

The question that motivated this work was, “How can I assure that my gifted students are proficient in fractions?” I answered this by developing a set of lessons to help my students catch up with what they need to know and by developing an assessment to test their level of proficiency. Although I have not been able to gauge the success of these items with a high level of precision, there is evidence of positive outcomes. Perhaps more important, the work presented here opens the door for other teachers interested in this same question. The lessons and the assessment provide a good resource and a great starting point for other teachers to build upon.

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APPENDIX A. LESSON PLANS FOR CATCH-UP PROGRAM

PROGRESSION OF LESSONS

The following lessons should be given over a two week or longer span. Any time less than this would be inefficient in the learning process. The lessons should be taught early in the school year and concepts should be revisited frequently throughout the school year in all areas of mathematics study.

Lesson plans, along with outlines for the teacher to follow, are included. Worksheets for each lesson are also included. The activities are specifically working on word problems involving operations with fractions.

Day 1-Lesson 1 (see Lesson Plans and Teacher Outline)

Day 2-Lesson 2 (see Lesson Plans and Teacher Outline)

Day 3-Lesson 3 (see Lesson Plans and Teacher Outline)

Day 4-Lesson 4 (see Lesson Plans and Teacher Outline)

Day 5-Activity 1

ACTIVITY 1
Have students work in groups on the first 10 problems. Assign each group two the problems to present at the board. Discuss solutions to the problems as a class. Have students complete the problems individually. Monitor the students and give assistance when needed. Worksheets 5 and 6 are necessary for this activity.

Day 6-Lesson 5 (see Lesson Plans and Teacher Outline)

Day 7-Lesson 6 (see Lesson Plans and Teacher Outline)

Day 8-Actiivty 2

ACTIVITY 2 (Day 8)
Have students work in groups on the first 10 problems. Assign each group two the problems to present at the board. Discuss solutions to the problems as a class. Have students complete the problems individually. Monitor the students and give assistance when needed. Worksheet 9 and 10 are necessary for this activity.

<table>
<tr><td colspan="3">Lesson 1 What is a fraction? (Day 1)</td></tr>
<tr><td colspan="3">Objectives:
1. To define a fraction as a number
2. To establish relationships between part-to-whole relationships, quotients, and ratios to fractions</td></tr>
<tr><td>Starter problem:
List ideas that come to mind when you see 3/4.
students should work independently (3 min)
Students should discuss answers with a partner (3 min)</td><td colspan="2">Discussion questions:
What is the definition of a fraction?
Would you say that a fraction is a number?</td></tr>
<tr><td>Instruction:</td><td>Materials:</td><td>Assessment/Discussion questions:</td></tr>
<tr><td>Part 1:
Interpretations of fractions

1. Part-Whole: Part whole indicates that a whole has been divided into equal parts. Ex. 1/8 indicates that something has been divided into 8 equal pieces and 1 is being considered.
2. Quotient: Fractions may also be considered a quotient, the result of a division problem. Ex. 1/8 can be interpreted as 1 divided by 8.
3. Ratio: A fraction can also be interpreted as a ratio, which is a comparison of two distinct objects. Ex. There is one boy for every 8 girls.

Complete Part 1 of Worksheet 1.

Part 2:
Parts of a fraction
1. Numerator- number of parts being considered. The top number in a fraction.
2. Denominator- names the objects being counted
3. Fraction bar- grouping symbol, symbol meaning to divide

Discuss the fact that a fraction is a number located on the number line
Complete Part 2 of Worksheet 2.

Part 3:
Review how to simplify fractions: Remind students that fractions are in simplest form if the numerator and denominator are relatively prime.</td><td>Worksheet 1</td><td>Can you give an example of a part-whole relationship?

Can you give an example of a quotient relationship?

Can you give an example of a ratio relationship?

Allow students to share examples with the class.

Give examples of fractions and have students identify the parts and explain their meanings.

Have students place examples of fractions on the number line.

Give examples of how to simplify fractions using prime factorization, showing what is meant by relatively prime.</td></tr>
</table>

<table>
<tr><td colspan="3">Lesson 2 Adding Fractions with Like Denominators (Day 2)</td></tr>
<tr><td colspan="3">Objectives:
1. To use a number line to add fractions with like denominators
2. To develop a procedure for adding fractions with like denominators</td></tr>
<tr><td>Starter problem:
Sara walked 3/7 of a mile on Monday and 2/7 of a mile on Tuesday. How far did she walk altogether? Draw a picture to illustrate your findings.
students should work independently
Students should discuss answers with a group
Have students present findings at board for group discussion</td><td colspan="2">Discussion questions:
Why can you add the numerators and get the correct answer?

Can you use a number line to show this?</td></tr>
<tr><td>Instruction:</td><td>Materials:</td><td>Assessment/Discussion questions:</td></tr>
<tr><td>Part 1:
Use a number line to show how fractions with common denominators are added.

Compare this method to the method used when adding whole numbers on the number line.

Part 2:
Have students aid in developing a procedure for adding fractions with common denominators.

Step 1: Identify whether or not the denominators are common.
Step 2: Add the numerators (denominators should be left alone.
Step 3: Place the sum of the numerators over the common denominator.
Step 4: Simplify the fraction if necessary.

Be sure students understand that the denominator creates a unit, and a common unit must be present to add numbers.

Complete Worksheet 2. Students may work with partners in class. Worksheet should be completed as homework to be reviewed in class.</td><td>Worksheet 2</td><td>Give multiple examples of how to use the number line.

Allow students to work out some examples at the board.

Have a class discussion of this method.

Give several examples using the procedure to add fractions with common fractions.

Does this method work for all fractions with common denominators?

Why does it work?</td></tr>
</table>

<table>
<tr><td colspan="3">Lesson 3 Adding fractions with unlike denominators (Day 3)</td></tr>
<tr><td colspan="3">Objectives:
1. To use the number line to understand the need for common denominators
2. To develop a procedure for adding fractions with unlike denominators</td></tr>
<tr><td>Starter problem:
Peter ate 2/3 of a cake. His friend David ate 1/6 of the same cake. How much of the cake has been eaten? Illustrate your findings.
students should work independently
Students should discuss answers with a group
Have students present findings at board for group discussion</td><td colspan="2">Discussion questions:
What must happen to be able to add these fractions?

How does this differ from adding fractions with common denominators?</td></tr>
<tr><td>Instruction:</td><td>Materials:</td><td>Assessment/Discussion questions:</td></tr>
<tr><td>Part 1:
Use the number line to show that creating common denominators yields the unit necessary for adding the fractions.

Compare adding fractions with like and unlike denominators.

Part 2:
Have students aid in developing a procedure for adding fractions with unlike denominators.
Step 1: Identify whether or not the fractions have unlike denominators.
Step 2: Create common denominators using the method for finding the least common multiple.
Step 3: Rewrite equivalent fractions having the common denominator found in Step 2.
Step 4: Add the numerators of the equivalent fractions produced.
Step 5: Place the sum over the common denominator.
Step 6: Simplify the fraction if necessary.

Complete Worksheet 3. Students may work with a partner. The worksheet should be completed for homework to be reviewed in class.</td><td>Worksheet 3</td><td>Allow students to explore with the number line, creating common units, or common denominators.

Have students explain why common units are necessary for adding.

Do the numerators of the fractions change as you create common denominators?

What are these fractions called?

Why are the denominators not added as well as the numerators?

Give examples of adding fractions with unlike denominators using the procedure produced.

Allow students to work problems at the board. Have them explain the process.</td></tr>
</table>

<table>
<tr><td colspan="3">Lesson 4 Subtracting fractions (Day 4)</td></tr>
<tr><td colspan="3">Objectives:
4. To use the number line to subtract fractions
5. To develop procedures for subtracting fractions with like and unlike denominators.</td></tr>
<tr><td>Starter problem:
Michael had 5/8 of a cake. He ate 3/8 of the cake. What fraction of the cake was left? Illustrate your solution.
(students should work independently
Students should discuss answers with a group
Have students present findings at board for group discussion</td><td colspan="2">Discussion questions:
How does this example differ from addition?

Could you use the number line in this example to show your findings?</td></tr>
<tr><td>Instruction:</td><td>Materials:</td><td>Assessment/Discussion questions:</td></tr>
<tr><td>Part 1:
Use the number line to show the result of a subtraction problem.

Review moving left to subtract whole numbers and connect this to subtracting fractions using the number line.

Part 2:
Have students aid in developing a procedure for subtracting fractions with common denominators.
Step 1: Identify whether or not the denominators are common.
Step 2: Subtract the numerators (denominators should be left alone.)
Step 3: Place the difference of the numerators over the common denominator.
Step 4: Simplify the fraction if necessary.

Part 3:
Have students create a procedure for subtracting fractions with unlike denominators.
Step 1: Identify whether or not the fractions have unlike denominators.
Step 2: Create common denominators using the method for finding the least common multiple.
Step 3: Rewrite equivalent fractions having the common denominator found in Step 2.
Step 4: subtract the numerators of the equivalent fractions produced.
Step 5: Place the difference over the common denominator.
Step 6: Simplify the fraction if necessary.</td><td>Worksheet 4</td><td>How does using the number line change from addition?

Why are common denominators also important when subtracting fractions?

How do the procedures for adding and subtracting fractions compare to one another?</td></tr>
</table>

<table>
<tr><td colspan="3">Lesson 5 Multiplying fractions (Day 6)</td></tr>
<tr><td colspan="3">Objectives:
1. To use fraction bars and grouping to show what occurs when fractions are multiplied
2. To develop a procedure for multiplying fractions</td></tr>
<tr><td>Starter problem:
Noah had $20. He went to a book store and spent 2/5 of his money on a book. How much did the book cost? Draw a picture to illustrate this.
students should work independently
Students should discuss answers with a group
Have students present findings at board for group discussion</td><td colspan="2">Discussion questions:
What operation is being performed?

Why does multiplication of a fraction and a whole number produce a quantity smaller than the whole number?</td></tr>
<tr><td>Instruction:</td><td>Materials:</td><td>Assessment/Discussion questions:</td></tr>
<tr><td>Part 1:
Use grouping to show multiplication of whole numbers times fractions.

Part 2:
Use fraction bars to show multiplying fractions by fractions.

Review: “of” means to multiply

Part 3:
Have the students aid in producing a procedure for multiplying fractions.

Step 1: Multiply the numerators.
Step 2: Multiply the denominators.
Step 3: Place the product of Step 1 over the product of Step 2.
Step 4: Simplify if necessary.

Complete worksheet 6.</td><td>Worksheet 7</td><td>Give multiple examples of multiplying whole numbers by fractions.

What is occurring in all of these examples?

Give multiple of examples of multiplying fractions by fractions.

What is occurring in all of these examples?

Do you feel that common denominators are necessary for multiplying fractions?</td></tr>
</table>

Lesson 6 Dividing fractions **(Day 7)**		
Objectives: 1. To use fraction bars to show what happens when dividing by a fraction 2. To develop a procedure to divide fractions		
Starter problem: How many pieces of string, each 1/5 m long, can be cut from a string 3 m long? Illustrate your solution. students should work independently Students should discuss answers with a group Have students present findings at board for group discussion	Discussion questions: What operation is occurring? How does this example differ from the example given for multiplication?	
Instruction:	Materials:	Assessment/Discussion questions:
Part 1: Use fraction bars to show division of whole numbers by fractions. Use fraction bars to show students that when dividing by a fraction you are multiplying by the reciprocal. Review what a reciprocal is. Part 2: Have students aid in developing a procedure for dividing fractions. Step 1: Find the reciprocal of the divisor (second fraction). Step 2: Rewrite the problem so you are multiplying the first fraction by the reciprocal of the second fraction. Step 3: Multiply the numerators. Step 4: Multiply the denominators. Step 5: Place the product of Step 3 over the product of Step 5. Step 6: Simplify if necessary. Complete Worksheet 7.	Worksheet 8	Why does dividing a whole number by a fraction produce a number greater than the whole number? Give several examples of dividing whole numbers by fractions. What do you see happening in each of these examples? What do you call a fraction after it is “flipped?” Give several examples of dividing fractions by fractions. Allow students to try some at the board.

APPENDIX B. DETAILED INSTRUCTIONS FOR LESSONS

Lesson 1 (Part 1)
Fractions (Overview)

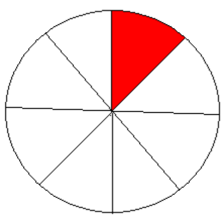
Starter Problem: Place the fraction ¾ on the board and pose the question: What does this mean to you?

Have the students take 3 minutes to write down an answer, and then have the students separate into groups to discuss their findings. Give the students 5 minutes to share their ideas and then have a class discussion about the group discussions.

Lecture Notes

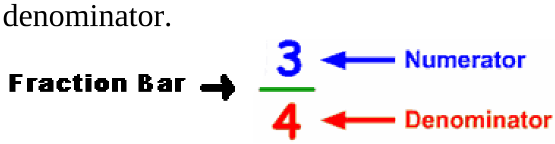
There are three interpretations of a fraction

1. Part-Whole: Part whole indicates that a whole has been divided into equal parts.
   Ex. 1/8 indicates that something has been divided into 8 equal pieces and 1 is being considered.



2. Quotient: Fractions may also be considered a quotient.
   Ex. 1/8 can be interpreted as 1 divided by 8.
3. Ratio: A fraction can also be interpreted as a ratio, which is a comparison of two distinct objects.
   Ex. There is one boy for every 8 girls.

Each piece to a fraction has a name. The number on the top is called the numerator, the line separating the two numbers is called the fraction bar, and the number on the bottom is called the denominator.



Lesson 1 (Part 2)
Simplifying Fractions

Prerequisites- Finding GCF of numbers
Fractions are in simplest form when the GCF of the numerator and the denominator is 1. (When the two numbers are relatively prime)

Lesson 2
Adding fractions with like denominators

Starter problem: Sara walked 3/7 of a mile on Monday and 2/7 of a mile on Tuesday. How far did she walk altogether? Draw a picture to illustrate your findings.

Have a couple of students present their findings up at the board. Discuss their findings as a group.

Lecture Notes

Show students how this problem is solved using a number line:

- Place a number line on the board with only whole numbers-Ask the students if it is possible to use this number line to illustrate the problem. How can we fix this?
- Place a number line with 7ths on the board and show them how to use it to add the
- fractions.

Ex 1. Use the number line to add 4/5 and 3/5.

Ex 2. Use the number line to add 2/3 and 5/3

Show the students that you can add these fractions because you have a common unit of measure. A common denominator

Discuss what is meant by "like" or "common" denominators: Denominators are "like denominators" or "common denominators" when they have equal value.

The fractions 2/5 and 4/5 have like denominators. The denominator of 5 is present in both fractions.

The fractions 2/3 and 8/9 do not have like denominators. The denominators are not of equal value. $3 \neq 9$

To add a fraction with common denominators, add the numerators and place the result over the common denominator. Always simplify your sum.

Lesson 3
Adding Fractions using unlike denominators

Starter Problem: Peter ate 2/3 of a cake. His friend David ate 1/6 of the same cake. How much of the cake has been eaten? Illustrate your findings.

Have 2 or 3 students place their findings on the board.

Lecture Notes:

Part 1.

- Place a number line on the board with only whole numbers and ask the question- Can we use this number line to solve the problem? No!
- Place a number line divided into 3rds and ask the question- Can I use this number line to solve the problem? No!
- The number line needs to be divided into a common unit. How can we do this?
  The number line will need to be broken into 6ths to have a common unit.

Ex. 1 Use a number line to find a common unit of measure to add 2/3 + 2/5

Ex. 2 Use a number line to add 7/10 plus 5/6

Part 2.
When adding fractions with different denominators, we must find a common denominator. (We looked at the reason why in Part 1)

To find a common denominator, you must find a common multiple
Ex. 1 Add 7/9 + 5/6 by finding a common denominator
Multiples of 9 -> 9, 18, 27, 36
Multiples of 6-> 6, 12, 18, 24
18 is the first common multiple, so we will use 18 as the common denominator.
Once a common denominator is found, you must rewrite your fractions to have this denominator.
7/9 is equivalent to what fraction with the denominator of 18? Since 9x2 is 18 we must multiply 7 by 2 as well. Our new fraction will be 14/18.
5/6 is equivalent to 15/18.
14/18 + 15/18 = 29/18
Always simplify your sum.

Lesson 4
Subtracting Fractions with like and unlike denominators

Starter Problem: Michael had 5/8 of a cake. He ate 3/8 of the cake. What fraction of the cake was left? Illustrate your solution.

Lecture Notes:

Review the definition of subtraction: Subtraction is an operation in which the difference of two amounts is calculated. When subtracting using a number line, you move to the left.

- Use a number line to illustrate 5/7 – 3/7
- Use a number line to illustrate ¾ - ½
- Have a student illustrate the example 9/10 – 2/3

To subtract fractions with common denominators, subtract the numerators and place the result over the common denominator.

To subtract fractions with different denominators, a common denominator must be found. Once a common denominator is found, rewrite the fractions and follow the directions for subtracting fractions with common denominators.

Always simplify your difference.

Lesson 5
Multiplying Fractions

Starter problem: Noah had $20. He went to a book store and spent 2/5 of his money on a book. How much did the book cost? Draw a picture to illustrate this.

Have a couple of students place their findings on the board. Discuss.

Lecture Notes:

Let's use illustrations to show multiplication:

First we will use grouping to find 1/3 of 12

- Draw 12 circles on your paper in a horizontal row
- To illustrate 3rds what should we do? Divide the 12 circles into 3 groups
- Identify 1 of the 3 groups-How many circles are in the group?
- So 1/3 of 12 is 4

How can we illustrate ¾ of 2/3?

- Draw a 3 unit box and shade 2/3 of it
- Brake the 2/3 into 4ths and shade ¾ of this
- Now compare the "double shaded area" to the three units from the beginning

When you are finding a value of another value, you are performing multiplication:

- ¾ 0f 2/5 means ¾ times 2/5
- 7/8 of 2 means 7/8 times 2

Do we need to find a common denominator to multiply fractions? Why or why not?

- It is not necessary to find a common denominator to multiply because multiplication does not depend on a specific unit of measure like we saw with addition
- You are finding a part of something, not finding how many of something.

When multiplying fractions, multiply the numerators, and then multiply the denominators.

Always simplify your product.

Lesson 6
Dividing Fractions

Starter Problem: How many pieces of string, each 1/5 m long, can be cut from a string 3 m long? Illustrate your solution.

Have students break into groups and discuss their answers.
Allow the groups to present their findings.

How does this problem differ from the previous starter problem we discussed?

Lecture Notes:

Let's find 3 divided by ½

- Draw three circles on your paper
- There are 2 halves in one whole. How many halves are in three wholes?
- 3 ÷ ½ = 6

Let's divide 2 by 1/3

- You should ask yourself: How many thirds are there in 2 wholes?
- Draw two wholes, divide, and then count the pieces
- Can you see the rule for dividing with fractions?

Try 2 ÷ 1/5 3 ÷ 1/7 9 ÷ 1/9

When dividing you are simply multiplying by the reciprocal of the divisor.

Let's divide 3 by ¾

- How many ¾'s can be made from 3 wholes?
- Draw 3 circles on your paper and cut them into fourths. How many groups of ¾ can you create?
- Dividing by ¾ is the same as multiplying by 4/3 (the reciprocal)

Let's divide 2/3 by 3/5

- Previously we saw that we could obtain our answer by multiplying by the reciprocal
- 2/3 ÷ 3/5 = 2/3 ▪ 5/3
- Once you set up the multiplication, multiply numerator by numerator then denominator by denominator
- Your answer is 10/9

Try 3 ÷ 2/3 5/8 ÷ 2/5 4 ÷ 6/7

APPENDIX C. WORKSHEETS FOR LESSONS

WORKSHEET 1

Fractions/Simplifying Fractions   **Name:**_____________________________

Part 1: Relationships

1. List two examples of a fraction representing a part-whole relationship and explain.

A.

B.

2. List two examples of a fraction representing a quotient and explain.

A.

B.

3. List two examples of a fraction representing a ratio and explain.

A.

B.

4. List the following fractions from least to greatest using a number line.

5/6, 3/5, 2/3, 7/8, 4/3

Part 2: Simplifying Fractions

Simplify the following fractions using prime factorization.

1. $\frac{54}{66}$ =

2. $\frac{24}{44}$ =

3. $\frac{12}{28}$ =

4. $\frac{35}{42}$ =

5. $\frac{24}{33}$ =

6. $\frac{12}{21}$ =

7. $\frac{15}{30}$ =

8. $\frac{4}{22}$ =

9. $\frac{24}{66}$ =

10. $\frac{10}{12}$ =

Resource: http://www.dadsworksheet.com

WORKSHEET 2

Adding Fractions with common denominators **Name: _______________________**

Use the procedure for adding fractions. Illistrate your solutions.

1.	$\frac{2}{3}$	+	$\frac{1}{3}$	=	
2.	$\frac{1}{2}$	+	$\frac{1}{2}$	=	
3.	$\frac{1}{3}$	+	$\frac{1}{3}$	=	
4.	$\frac{1}{3}$	+	$\frac{2}{3}$	=	
5.	$\frac{1}{5}$	+	$\frac{1}{5}$	=	
6.	$\frac{3}{5}$	+	$\frac{2}{5}$	=	
7.	$\frac{2}{3}$	+	$\frac{1}{3}$	=	
8.	$\frac{2}{4}$	+	$\frac{1}{4}$	=	
9.	$\frac{2}{5}$	+	$\frac{1}{5}$	=	
10.	$\frac{1}{2}$	+	$\frac{1}{2}$	=	

Resource: http://www.dadsworksheet.com

11. $\frac{14}{58}$ + $\frac{3}{58}$ =

12. $\frac{7}{42}$ + $\frac{12}{42}$ =

13. $\frac{4}{10}$ + $\frac{2}{10}$ =

14. $\frac{61}{79}$ + $\frac{17}{79}$ =

15. $\frac{11}{13}$ + $\frac{2}{13}$ =

16. $\frac{22}{77}$ + $\frac{11}{77}$ =

17. $\frac{20}{86}$ + $\frac{39}{86}$ =

18. $\frac{11}{13}$ + $\frac{2}{13}$ =

19. $\frac{17}{24}$ + $\frac{4}{24}$ =

20. $\frac{59}{84}$ + $\frac{24}{84}$ =

Resource: http://www.dadsworksheet.com

WORKSHEET 3

Adding Fractions with Unlike Denominators **Name: ________________________**

Add the following fractions using the procedure for adding fractions.

1.	$\frac{3}{4}$	+	$\frac{1}{7}$	=	
2.	$\frac{2}{7}$	+	$\frac{3}{5}$	=	
3.	$\frac{1}{8}$	+	$\frac{1}{4}$	=	
4.	$\frac{1}{6}$	+	$\frac{2}{7}$	=	
5.	$\frac{2}{3}$	+	$\frac{2}{6}$	=	
6.	$\frac{2}{4}$	+	$\frac{3}{8}$	=	
7.	$\frac{1}{3}$	+	$\frac{3}{6}$	=	
8.	$\frac{1}{3}$	+	$\frac{1}{5}$	=	
9.	$\frac{2}{4}$	+	$\frac{3}{6}$	=	
10.	$\frac{2}{3}$	+	$\frac{2}{6}$	=	

Resource: http://www.dadsworksheet.com

11. $\frac{7}{18} + \frac{1}{14} =$

12. $\frac{8}{14} + \frac{2}{8} =$

13. $\frac{9}{15} + \frac{2}{16} =$

14. $\frac{2}{12} + \frac{2}{8} =$

15. $\frac{4}{9} + \frac{4}{18} =$

16. $\frac{4}{18} + \frac{1}{8} =$

17. $2\frac{2}{6} + 4\frac{1}{4} =$

18. $8\frac{1}{7} + 9\frac{2}{6} =$

19. $2\frac{2}{4} + 5\frac{1}{6} =$

20. $1\frac{3}{4} + 2\frac{1}{5} =$

21. $8\frac{1}{7} + 9\frac{2}{6} =$

22. $8\frac{3}{6} + 9\frac{3}{7} =$

23. $3\frac{2}{5} + 3\frac{1}{3} =$

24. $6\frac{1}{3} + 8\frac{1}{5} =$

25. $1\frac{1}{3} + 7\frac{2}{7} =$

Resource: http://www.dadsworksheet.com

WORKSHEET 4

Subtracting Fractions **Name:** ______________________________

Part 1: Subtract the fractions and illustrate your solution.

1. $\frac{6}{8} - \frac{1}{8} =$

2. $\frac{8}{16} - \frac{7}{16} =$

3. $\frac{7}{8} - \frac{6}{8} =$

4. $\frac{6}{15} - \frac{5}{15} =$

5. $\frac{5}{14} - \frac{4}{14} =$

Part 2: Subtract the following fractions using the correct procedure.

6. $\frac{2}{4} - \frac{2}{5} =$ ______

7. $\frac{2}{6} - \frac{1}{7} =$ ______

8. $\frac{3}{8} - \frac{1}{3} =$ ______

9. $\frac{3}{7} - \frac{1}{4} =$ ______

10. $\frac{3}{8} - \frac{1}{7} =$ ______

Resource: http://www.dadsworksheet.com

11. $\frac{5}{32} - \frac{2}{20} =$

12. $\frac{19}{24} - \frac{5}{28} =$

13. $\frac{19}{20} - \frac{8}{26} =$

14. $\frac{14}{18} - \frac{15}{36} =$

15. $\frac{12}{36} - \frac{1}{9} =$

16. $8\ \frac{1}{2} - 4\ \frac{3}{7} =$

17. $3\ \frac{7}{8} - 1\ \frac{6}{9} =$

18. $8\ \frac{6}{7} - 2\ \frac{2}{4} =$

19. $9\ \frac{1}{4} - 2\ \frac{6}{1} =$

20. $8\ \frac{2}{3} - 7\ \frac{2}{4} =$

Resource: http://www.dadsworksheet.com

WORKSHEET 5

Fraction Addition and Subtraction Word Problems Name: ______________________

Read each problem carefully. Answer all questions in your groups. Explain your solution. Solutions will be presented at the board.

1. John mowed 2/5 of a lawn. His brother mowed 1/4 of it. What fraction of the lawn did they mow?

2. Sammy took 3/4 hour to travel from home to the zoo. He took – hours to return home. How much longer did he take to return home than to go to the zoo?

3. Mary ate 1/8 of a cake. Peter ate another 1/4 of it.
   (a) What fraction of the cake did they eat altogether?
   (b) How much more of the cake did Peter eat than Mary?

4. Ali went to a bookshop. He spent 3/5 of his money on books and 1/4 of it on a pen.
   (a) What fraction of his money did he spend altogether?
   (b) What fraction of his money did he have left?

5. Thomas spent 3/7 of his money on a book and the rest on a racket. What fraction of his money was spent on the racket?

6. Mr. Johnson bought a can of paint. He used 1/2 of it to paint a table. He used 1/8 of it to paint a book shelf. How much paint did he use altogether?

7. A container has a capacity of 3 liters. It contains – liters of water. How much more water is needed to fill the container?

8. Ann planned to spend – hours to cook a meal. She finished the cooking in — hours. How much earlier did she finish cooking?

9. The total length of two ribbons is – m. If one ribbon is – m long, what is the length of the other ribbon?

10. Beth bought – kg of potatoes and – kg of carrots. How much more potatoes than carrots did she buy?

Resource: Primary Mathematics Textbook

WORKSHEET 6

Fraction Addition and Subtraction Word Problems **Name: ________________________**

Read each problem carefully. Answer all questions individually. Explain your solution.

1. Sally ate 1/8 of a cake and her sister ate 3/8 of it. What fraction of the cake did they eat altogether?

2. Mark spent 4/9 of his pocket money and saved the rest. What fraction of his pocket money did he save?

3. Frannie baked a pie. She ate 1/6 of the pie and gave 3/6 of the pie to her friends. What fraction of the pie did she have left?

4. Mary has 3/4 liter of orange juice. She drinks 1/2 liter of it. How much orange juice does she have left?

5. Meredith bought 2/5 kg of shrimp. Courtney bought 1/10 kg less shrimp than Meredith.
   (a) Find the weight of the shrimp bought by Courtney.
   (b) Find the total weight of the shrimp bought by them.

6. Robert jogged – km. His brother jogged – km. Who jogged a longer distance? How much longer?

7. There were – cakes on the table. After breakfast, there were – cakes left. How many cakes were eaten?

8. A container has a capacity of 3 liters. It contains – liters of water. How much more water is needed to fill the container?

Resource: Primary Mathematics Textbook

WORKSHEET 7

Multiplying Fractions **Name: ____________________________**

Multiply the following using the procedure for multiplying fractions.

1. $\frac{1}{3} \times \frac{4}{5} =$	
2. $\frac{2}{4} \times \frac{4}{8} =$	
3. $\frac{2}{5} \times \frac{1}{3} =$	
4. $\frac{4}{8} \times \frac{8}{9} =$	
5. $\frac{1}{2} \times \frac{4}{5} =$	
6. $\frac{4}{7} \times \frac{3}{10} =$	
7. $\frac{7}{9} \times \frac{2}{3} =$	
8. $\frac{2}{3} \times \frac{2}{5} =$	
9. $\frac{2}{8} \times \frac{7}{9} =$	
10. $\frac{5}{7} \times \frac{2}{9} =$	

Resource: http://www.dadsworksheet.com

11. $2\frac{3}{7} \times 5\frac{4}{5} =$

12. $3\frac{2}{5} \times 2\frac{3}{9} =$

13. $5\frac{6}{8} \times 1\frac{4}{9} =$

14. $5\frac{4}{5} \times 4\frac{5}{7} =$

15. $2\frac{6}{7} \times 5\frac{7}{10} =$

16. $3\frac{6}{9} \times 2\frac{4}{7} =$

17. $1\frac{2}{4} \times 3\frac{5}{10} =$

18. $2\frac{4}{10} \times 3\frac{2}{7} =$

19. $3\frac{4}{9} \times 3\frac{5}{6} =$

20. $4\frac{3}{4} \times 5\frac{6}{10} =$

Resource: http://www.dadsworksheet.com

WORKSHEET 8

Dividing Fractions **Name: ______________________________**

Divide the following using the procedure for dividing fractions.

1. $\frac{5}{6} \div \frac{4}{5} =$	
2. $\frac{3}{9} \div \frac{3}{10} =$	
3. $\frac{5}{6} \div \frac{3}{12} =$	
4. $\frac{4}{5} \div \frac{4}{12} =$	
5. $\frac{7}{11} \div \frac{4}{7} =$	
6. $\frac{3}{7} \div \frac{2}{4} =$	
7. $\frac{2}{6} \div \frac{5}{7} =$	
8. $\frac{3}{4} \div \frac{2}{3} =$	
9. $\frac{3}{6} \div \frac{1}{4} =$	
10. $\frac{2}{7} \div \frac{2}{3} =$	

Resource: http://www.dadsworksheet.com

11. $3\frac{6}{9} \div 3\frac{2}{4} =$

12. $4\frac{3}{8} \div 4\frac{6}{9} =$

13. $2\frac{8}{10} \div 1\frac{6}{9} =$

14. $2\frac{4}{6} \div 1\frac{2}{4} =$

15. $1\frac{4}{5} \div 5\frac{6}{7} =$

16. $1\frac{3}{4} \div 1\frac{2}{5} =$

17. $3\frac{4}{5} \div 1\frac{4}{6} =$

18. $5\frac{8}{9} \div 5\frac{2}{6} =$

19. $1\frac{3}{9} \div 4\frac{8}{10} =$

20. $2\frac{4}{5} \div 5\frac{4}{6} =$

Resource: http://www.dadsworksheet.com

WORKSHEET 9 **Name: ___________________________**

Fraction Multiplication and Division Word Problems

Read each problem carefully. Answer all questions in your groups. Explain your solution. Solutions will be presented at the board.

1. Holly earns $350 a month. She saves $70 each month. What fraction of her earnings does she save?

2. Shawn had a piece of string 1/2 m long. He used 1/3 of it to tie a box. Find the length of the string which was used to tie the box.

3. How many bricks weighing 1/4 lb each will have a total weight of 3 lb?

4. Holly had 2 kg of beef. She used 4/5 of it to make stew. How many kilograms of beef did she have left?

5. Kelly had 3/4 qt of cooking oil. She used 2/5 of it to fry some fish. How much oil did she use?

6. Mr. Anderson gave 2/5 of his money to his wife and spent 1/2 of the remainder. If he had $300 left, how much money did he have at first?

7. Tyrone bought a bag of marbles. 1/4 of the marbles were blue, 1/8 were green and 1/5 of the remainder were yellow. If there were 24 yellow marbles, how many marbles did he buy?

8. Mrs. Foster had 3/5 kg of sugar. She used 1/4 of it to make cookies. How much sugar did she use to make the cookies?

9. Mr. Ricci spent 1/3 of his salary on food and 2/5 of the remainder on transport.
   (a) What fraction of his salary did he left?
   (b) If he had $600 left, find his salary?

10. David had 1280 eggs. He sold 3/5 of them on Saturday and 1/4 of the remainder on Sunday. Find the total number of eggs sole on the two days.

Resource: Primary Mathematics Textbook

WORKSHEET 10 **Name: __________________________**

Fraction Division and Multiplication Word Problems

Read each problem carefully. Answer all questions individually. Explain your solution.

1. Sally ate 1/6 of a cake and gave 1/5 of the remainder to her sister. What fraction of the cake did she give away?

2. Nicole used 6 m of string to tie some packages. She used 2/3 m of string for each package. How many packages did she tie?

3. A flower garden occupies 1/2 of a piece of land. 3/5 of the garden is used for growing orchids. What fraction of the land is used for growing orchids?

4. 2/3 of a wall is painted red. 1/4 of the remaining part is painted gray. What fraction of the wall is painted gray?

5. Kimberly cuts 6 pieces of tape, each 4/5 m long, from a roll of tape 5 m long. How many meters of tape are left in the roll?

6. Mrs. Law bought 1/2 of a cake. She cut it into 4 equal pieces. What fraction of the whole cake is each piece?

7. Danny bought 6 packets of drink. Each packet contained 1/4 liter of drink. Find the total amount of drink in liters.

8. Jim had 360 stamps. He sold 1/3 of them on Monday and 1/4 of the remainder on Tuesday. How many stamps did he sell on Tuesday?

Resource: Primary Mathematics Textbook

APPENDIX D. ASSESSMENT

1. Draw a picture to show what 1/3 means. Explain what your picture represents. Put your work inside the box below.

2. Draw a picture to show what 3/8 means. Explain what your picture represents. Put your work inside the box below.

3. Using the line below as a number line, mark the position of 0 to the left and mark the position of 1 near the middle. Show where the following fractions lie:

1/2, 1/3, 1/4, 2/3, 3/4, 3/2.

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4. Using the line below as a number line, mark the position of 0 to the left and mark the position of 3 to the right. Mark the positions of 1 and 2 in between. Show where the following fractions lie:

5/2, 7/3, 3/8, 7/8, 13/8.

---

5. Are 1/2 and 2/4 the same number? Why or why not?

6. Are 4/6 and 6/9 the same number? Why or why not?

7. Which is bigger, 1/2 or 1/8 ? Explain why.

8. Which is bigger 4/7 or 3/5?

Read each question carefully and explain each part of your answer. You may use pictures and statements to explain.

9. A student named J.J. wrote: 1/2 + 1/4 = 2/6. What error did J.J. make? What would you say to J.J. to help him understand the correct way to add fractions?

10. Show J.J. the way to add 2/5 and 1/6. Explain your work for him.

11.After learning about fraction addition, J.J. tries multiplication. He writes 2/3 * 3/4 = 8/12 * 9/12 = 72/144 = 1/2. Is the answer correct? What is J.J. confused about? Explain to J.J. how to do this problem more simply.

12. J.J. says, "When you multiply two rational numbers, the answer can be bigger than both factors, or it can be smaller than both or is can be between the two." Is he right? Show why.

VITA

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