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Fault observability in distributed power system

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FAULT OBSERVABILITY IN DISTRIBUTED POWER SYSTEM

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
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in

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Science

by

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ABSTRACT

Fault observability as well as fault location algorithms in distributed power system are studied in this thesis. The importance of finding the fault location in a distribution system with the purpose of increasing reliability and decreasing the maintenance time and cost is discussed. Then, different existing fault location algorithms and approaches in the literature are introduced and compared.

Subsequently, a new strategy to achieve fault observability of power systems while aiming minimum required number of Phasor Measurement Units (PMUs) in the network is proposed. The method exploits the nodal voltage and mesh current analyses where the impedance and admittance matrices of the network and its dual circuit are developed and utilized for fault location. The criterion of determining the number and the places of PMUs is that the fault location and impedance can be obtained in a unique manner without multi estimation. In addition, the method considers faults along the lines as opposed to the faults only on system buses available in the literature. The proposed approach provides an economical solution to decrease measurement costs for large power networks, distributed generation networks, and micro grids. Simulation results for IEEE 7-bus, 14-bus, and 30-bus systems verify the effectiveness of the proposed approach.

1. INTRODUCTION

1.1 Importance of Fault Observability in Distribution System

The energy chain contains producing the energy, its transmission, and delivering the energy to the customer where the distribution feeders are the last links of this chain. The importance of the latter part is no less than the former parts in the electrical power system if not higher. Thus, many commercial experts believe that the distribution system should be taken into consideration more seriously as all the efforts to generate electrical power is with the purpose of delivering it to the customer [1].

One of the challenging and important issues for the customer is the reliability of the provided electrical energy. At the same time electric utilities wish to reduce the revenue loss caused by outage. For this purpose, the distribution system has to be highly reliable and efficient under not only normal condition but also emergency conditions. Customer Average Interruption Duration Index (CAIDI) caused by weather condition, human error, or equipment aging in the system is one of the factors that affect the reliability index [2]. Under the situation that distribution system consisting of a number of radial feeders are always subject to the various types of fault caused by storms, lightning, snow, freezing, rain, insulation breakdown, and short circuits caused by birds and other external objects, desired reliability cannot be achieved very easily. In order to improve the reliability, utility should be able to detect and recognize the fault location and type immediately after fault occurs. The faster the fault location is identified or at least estimated with reasonable accuracy, the more accelerated the maintenance time to restore normal energy supply.

In traditional methods, customers' calls are the base of outage troubleshooting. That is,

usually the utility starts to identify faults when they are informed by consumers about a fallen electric pole, broken cable, or when they receive complains about the cut in power supply [3]. In order to specify the exact location of the fault there has to be a precise overlap between the geographic location of the caller and the connectivity of the distribution network. In addition, if the fault occurs during the night-time, the utility might not receive any calls, which poses a problem for the operator in locating the fault. Also, barriers such as practical difficulties to install the measurement devices at each distribution system bus or problems such as communication failures limit the possibility of measuring currents in the lines and voltages at the distribution transformers.

In some rural distribution systems, basic approaches such as visual inspection, switching procedure or using fault indicators, are employed to locate a fault. Although these approaches are time consuming, lack of advance monitoring equipment such as SCADA system in rural distribution systems results in these methods being still in practice. From practical aspect it is really important that the exact location of fault can be assessed only with limited available measurements and with lowest human interaction around the substation. Therefore, an automatic fault-locator is an essential and unavoidable component of a distribution system.

Fault locating in power systems has been a major subject for power and protection engineers in recent years for the reason of system reliability. Power engineers devote a lot of time to develop different fault locating algorithms in order to overcome this challenge in power system. However, it should be mentioned that in distribution systems, due to large variations of fault impedance, fault location problem has more complications than in transmission and generation systems [4]. Furthermore, it is not economically viable to equip distribution networks with advanced high-cost protection equipment.

In recent years, the presence of distribution generation (DG) and utilizing renewable energy sources in distribution networks have been attracted much attention. DGs can use clean and environmentally compatible energy resources to generate electricity with lower operational capacity in comparison with large power plants. Therefore, they are considered as small generation units. Due to small generation capacity of DGs, the energy produced by them is not transferred through the power transmission lines. Instead, DGs generally support the distribution systems loads [5], [6]. Connection of DGs to distribution systems has not only changed the simple and conventional radial configuration of the distribution network but also makes their operation, control and protection more complicated. Consequently, identifying the exact location of occurred faults will be more important and complicated in distribution systems including DGs compared with transmission systems [7].

Usually, in high voltage transmission lines, faults have low impedance. Therefore, it is not so complicated a task to find the fault location that can be accomplished simply by distance relays. In contrast, distribution systems encounter various and relatively large impedance (low current) faults which occur in residential urban and rural regions. Consequently, the available classic fault location methods will not suit to identify the fault location in distribution systems due to the variety and wide range of fault impedances in such systems [4].

Outage time reduction in distribution systems can be attained through introducing condition-based maintenance in asset management, and developing fault location algorithms. That is, system reliability can be improved by utilizing an accurate fault location program which leads to reduce the average time for the field crew to find and isolate the faults more efficiently. In other words, the accuracy of fault location algorithms may have a huge impact on system reliability by reducing the duration of unexpected outages (outage caused by faults) and

narrowing the search for faults within a relatively small area [8].

In brief, the following benefits can be achieved by having a fast and efficient fault location scheme:

- Fault location algorithm assists to accelerate the restoration process
- By specifying the faulted node location there is the chance to reduce the affected area through implementing sectionalize switching operations
- By locating permanent faults it is possible to plan for preventive maintenance tasks and avoid future faults [2].

Once a fault has been located in the system, a maintenance engineer is able to figure out the fault causes by checking the location of the fault. Moreover, a prevention plan can be scheduled to avoid the same fault problem in the future. Thus, the reliability of the system can be improved.

1.2 Barriers of Fault Location in Distribution Systems

Although, in distribution system the basic approach to calculate the fault location using voltage and current measurement is still the same as in the transmission system case, fault locating in the distribution system is not an easy task because of its high complexity and difficulty caused by the non-homogeneity of lines, fault resistances, load uncertainty, and phase imbalance [9],[10].

The major bottlenecks in fault location, particular to distribution systems are:

- Limited number of measurement devices along the distribution feeder
- Lack of advanced protection data such as circuit breaker (CB) and relay statuses, as well as measurements of multiple line segment, dissimilar to transmission system
- Presence of DG and changing the number of generators in operation throughout the day

that makes the fault measurement and location depend on time [3].

1.3 Thesis Subject

The subject of this thesis is to propose a novel algorithm to identify the fault location and impedance based on optimal Phasor Measurement Unit (PMU) placement in the distribution power system.

1.4 Thesis Objective and Approach

In this work the concept of fault observability is applied via the condition that the location and impedance of faults are detected uniquely. In other words, if with limited number of measurements, fault location and impedance can be uniquely determined in the entire network, the chosen measurement nodes are adequate for full fault observability. That is, the uniqueness of the fault location and impedance makes an appropriate solver to find the fault location and impedance from the measured data. In this work the minimum number of measurement units accompanied by their best locations are targeted in the distribution network. The PMU is able to measure the current phasor as well as the voltage phasor; an added capability that could help to decrease the number of PMUs for fault observability. Thus, here voltage and current measurements are utilized to enhance fault observability and reduce the number of PMUs. In this method the power system is first used to find the impedance matrix of the system. Then, by employing the concept of duality, the dual circuit of the network is obtained followed by its impedance matrix. By using the impedance matrix as a powerful tool for fault analysis, the voltage changes in the dual network represent current changes in the actual network. When a fault occurs in the system, these matrices will change according to the place and impedance of the fault. Therefore, the impedance, which is a function of the fault location and fault impedance is obtained. Using the Impedance functions of the actual network and its dual, the effect of each

fault on the voltage and current phasors recorded by PMUs at the measurement points can be calculated. The goal is the placement of the PMUs in certain nodes that provide a unique set of voltages and currents for any given fault in the network. In this thesis three-phase symmetrical faults are analyzed; however, the proposed method can be easily utilized for other types of faults.

2. VARIOUS FAULT LOCATION METHODS

2.1 Introduction

Over The past fifty years the number of customers who benefit from the electrical energy has increased significantly. Consequently, in order to meet this demand electric power systems have grown rapidly by largely increasing the power lines as well as employing new energy sources such as renewable energy and distribution generation. The problem arises when these lines experience faults in their operation which happens with high probability due to the large length of lines. In most cases, the major effects of electrical faults appear in the form of mechanical damage, which must be repaired before returning the line to service. For this reason the presence of a fault location algorithm can accelerate the restoration procedure when the location of the fault is either known or can be estimated with reasonable accuracy.

Temporary faults caused by trees penetration or insulators degradations known as high resistance faults are difficult to detect as they do not result in breaker operations and mainly cause minor damage that is not easily visible in inspection. Thus, a fault locator that is able to estimate both sustained and transient faults is necessary [11].

The research that has been done so far mostly focuses on finding the locations of transmission line faults. The reason is mainly the time required to physically check the lines in transmission system that is much more than in the distribution systems. However, nowadays the location of faults on distribution systems has taken more attention because recent deregulation in utility industry has forced power distribution companies to supply reliable and quality power at minimum cost to customers.

Distribution system contributes towards most number of disturbances experienced by

customers and major revenue loss to utilities on account of lost business opportunities and repair cost. In spite of major technological advances and widespread use of Intelligent Electronic Devices for measurement and monitoring, accurate fault location in distribution lines is still a challenge and considerable interesting to electric power utility engineers and researchers for over twenty years. Fault location accuracy in distribution lines varies vastly because of nonhomogeneous nature of distribution network as opposed to transmission lines.

The history of fault location methods starts from visual inspection in rural area to using voltages and currents measured at one or both terminals of a line which contains of methods that are based on traveling waves, methods based on high frequency components of currents and voltages and, methods that use the fundamental frequency voltages and currents measured at the terminals of a line [11].

Although there is a big archive of methods for identifying or estimation of fault locations in radial transmission lines, some features of distribution system such as non-homogeneity of lines, presence of 'laterals', load taps, and DGs cause a significant error when these methods are applied for distribution lines [12],[13]. The available fault location methods in literature can be categorize in five major groups which each of them has their own advantages and disadvantages [14], [15]. This chapter reviews selected fault location techniques proposed for distribution systems.

2.2 Classification of Fault Location Methods

Fault location methods use different measurements and techniques as follows:

- Impendence method
- Traveling wave method
- Signal injection method

- The zero sequence component based method
- Composite location method

2.2.1 Impedance Method

When a fault occurs in the system by assuming that the line is uniform and based on measured voltage and current at the measurement point the impedance of fault loop can be calculated. Since the calculated impedance is proportional to the line length, distance between measured point of device and fault location can be reached. This principle is used in relays such as distance relays.

Double-end impedance method is utilized to increase the accuracy of fault location. The basis of the analysis in this method is impedance. Also, there is no need for the data synchronization. In addition, this method can cover the effect of line parameters [14]. In brief, the advantages and disadvantages of this method include:

- Characterized by simple calculation,
- Easy to program,
- Easily realized with a mass of existing devices and less hardware investment, and
- Only realizes fault location without branch in line.

2.2.2 Travelling Wave Method

Travelling wave method is defined as a fault location method in which the phenomenon of traveling wave is employed. Reflected traveling wave difference, between healthy phase and faulted phase of traveling wave can be used for fault location. In literature there are different approaches based on this method to calculate the distance; namely, A, B, C, D, E and F. Among

these methods only C-Traveling method is utilized in distribution networks with multi-terminal, usually called single-ended injection traveling wave method. This method is based on sending pulse to healthy and faulted phases and records the reflected waveforms. By comparing waveforms of healthy and faulted phase and wave characteristic at fault point the fault area can be estimated. In brief, the advantages and disadvantages of this method include:

- High ranging precision,
- Matching application prospect in power system,
- Lack of solution for the problems of white noise in actual line and waveform distortion,
- Impaired return signal of traveling wave due to more complex structures and more branches as well as high grounding resistance in low voltage distribution network, and
- Requirement of a high technology on data processing and time synchronization.

2.2.3 Signal Injection Method

In this method particular frequency of current is injected through bus PT to ground circuit. When a fault occurs, the injected signal flows into the earth along the faulted line and grounded point. Then, a signal detector is employed to detect the faulted line while injected signal flows through it. Finally, the location of the fault can be obtained by signal tracking along the line using portable detector. Injection method is the short term of “injecting signal tracing method.”

It has been proved that to reduce the influence of capacitance to ground and high grounding resistance on the accuracy of this method, the frequency of the injected signal should be reduced to decrease the circuit distributary of ground reactance and increase the circuit reactance of faulted phase [14]. In brief, the advantages and disadvantages of this method include:

- Effective in locating single-phase to ground fault in distribution network,
- Not effective with overlong lines and oversize grounding resistance,
- Difficult fault location when complex structures and too many branches exist.

2.2.4 Zero Sequence Component Based Method

In this method by considering the appropriate reference direction where the direction of line head end to load terminal is the positive direction, the phase of zero-sequence current in the faulted line (before fault point) lags behind zero-sequence voltage around 90° while the phase of zero-sequence current in healthy line (healthy branch, after fault point) leads zero-sequence voltage around 90° . If the phase angle of zero-sequence voltage is considered as the reference phase angle, then zero-sequence current phase in faulted line is opposite to it in healthy line. Moreover, the maximum amplitude of zero-sequence current along the fault path happens in the fault point.

This method has been proposed for single-phase to ground fault and the recorded current waveform is analyzed after determination of the fault region by inquiring the current transformers (CTs) installed nearby the sectionalized switches. In brief, the advantages and disadvantages of this method include:

- Has high accuracy and good prospect for application,
- Needs to install a certain number of zero-sequence CT.

2.2.5 Compositive Location Method

Complexity of recent distribution networks has increased the necessity of using an approach which is able to locate the fault precisely. Therefore, the research on a comprehensive location method is one of power engineers concerns recently. The basic idea of this method is

employing two different methods in principle for fault location which can effectively compensate the shortage of using any each method alone, and benefiting from advantages of both to improve the accuracy of fault location.

2.3 Impedance Based Fault Location Method

Impedance based fault location method is more suitable and widely used than the other methods since it relies on minimum data and does not require costly hardware. Also, it uses simple algorithm and no communication is required [14]. Therefore, the proposed method in this thesis is based on impedance approach.

Impedance-based methods require the following steps:

- Measure the voltage and current phasors,
- Extract the fundamental components, determine the phasors and fault type, and
- Apply impedance algorithm.

3. PROPOSED ALGORITHM

3.1 Introduction

Faults and device failures affect the power quality in power systems and cause losses for both electric utilities and customers. In distributed generation, safety and secure operation of the system highly rely on the level of power system operating condition monitoring. In traditional approaches the measurements provided by the remote terminal units (RTU) at the substations are sent to the control center. This data include real/reactive power in different lines as well as bus voltages and branch currents. The unmeasured states of the system are then estimated by state estimators that reside in the control center. Recently, PMUs are proposed and used at substations in power systems to record operational data. PMUs employ global positioning system (GPS) signals to provide synchronized measurements of real time phasors of voltages and currents and provide a significant improvement in monitoring and control of power systems over the traditional instruments through accurate and synchronized measurement of the system states. PMUs are utilized in various applications [16]-[25] such as static analysis, recording dynamics of the power network and post-contingency analysis, identifying and calibrating the dynamic models of power system, transient stability prediction and control, voltage and frequency stability monitoring and control, analyzing and damping the low frequency oscillations, global feedback control, parameter measurement of transmission line, fault location, etc.

The current and potential applications of PMUs in power system enhance normal observability and fault observability. Normal observability in power system is defined as knowing the voltage phasors of all the buses [26]. However, the voltage phasors at many buses can be calculated by a limited number of measured voltages in the system along with the

parameters of the network and the load information of other buses. Fault observability, on the other hand, is defined such that a system becomes fault observable when the voltage at two ends of each line and the current at any end of the line are determinable [26]. In general, a system, which is observable in the normal condition, may not be observable for the fault condition. In this thesis the application of PMUs in fault location is investigated and a method is proposed to effectively reduce the number of required PMUs while retaining the accuracy of fault location. Thus, fault observability is the main focus in this work.

Different algorithms are proposed to minimize the number of PMUs required for fault observability. However, available methods still require a lot of PMUs in the system to measure and/or estimate all the voltages, which cause both high initial cost and communication problems. Unlike normal observability [27]-[32], the optimal or minimum number of PMUs for fault observability in the power system has not been studied as extensively. Moreover, in the past studies, it is usually assumed that faults occur on network buses. The work of [33] and [34] have described one-bus-spaced strategy for PMU placement as well as fault location by measuring the fault current and voltage phasors at the two ends of the faulted line. The results show that the number of required PMUs is about one third of buses. In [26] the authors minimized the number of PMUs by utilizing available flow measurements; the drawback is that not all conventional devices employ high resolution and synchronized data collection. Reference [35] proposed a method to minimize the PMUs allocation based on avoiding same measurements for different faults, taking into account faults on buses only, while multi estimation caused by line faults is not addressed. In the algorithm presented in [36] the objective function for minimization is based on voltage change while considering faults on the network busses only.

By contrast, in this work fault observability is defined and applied such that it ensures the

location and impedance of faults are detected uniquely. In other words, if with limited number of measurements, fault location and impedance can be uniquely determined in the entire network, the chosen measurement nodes are adequate for full fault observability. That is, the uniqueness of the fault location and impedance help an appropriate solver find the fault location and impedance from the measured data. The PMU is able to measure the current phasor as well as the voltage phasor; an added capability that could help decrease the number of PMUs for fault observability. Thus, in this work voltage and current measurements are utilized to enhance fault observability and reduce the number of PMUs. In this method the power system is first used to find the impedance matrix of the system. Then, by employing the concept of duality, the dual circuit of the network is obtained followed by its impedance matrix. By using the impedance matrix as a powerful tool for fault analysis, the voltage changes in the dual circuit represent current changes in the actual network. When a fault occurs in the system, these matrices change according to the place and impedance of the fault. Therefore, impedance matrices elements, which are functions of the fault location and impedance, are obtained. These functions of the network and its dual can be utilized to calculate the effects of each fault on the voltage and current phasors recorded by PMUs at measurement points. The goal is the placement of the PMUs in certain nodes such that a unique fault is identified in the network for the given set of measured voltages and currents. In this work three-phase symmetrical faults are analyzed; however, the proposed method can be easily utilized for other types of faults.

The rest of the chapter is organized as follows. In Section 3.2, the principles of the proposed algorithm on the basis of the multi-estimation problem in fault location are described. Then, the functions of voltage and line current changes due to fault are developed. Next, in Sections 3.3, simulation results obtained from applying the proposed algorithm on IEEE 7-bus,

14-bus and 30 bus systems and comparison with previous works are provided. Finally, concluding remarks are in Sections 3.4.

3.2 Methodology

PMU placement at each bus helps measure the voltage phasor at that bus and the current phasors in all the branches connected to that bus. When a fault occurs in the system, according to the location and impedance of the fault all voltages and currents of the network change including at PMU nodes (busses.) But the problem arises when more than one fault (with possibly different impedances and locations) cause the same change in the voltages and currents at PMU busses. This problem is called multi estimation. The focus of this thesis is to find the proper locations for PMUs that reduce multi estimation when faults occur, and to use measured voltage and current changes at the PMU locations to uniquely identify a fault as opposed to the available methods that only rely on voltage measurement/estimation. Introducing current measurement can reduce the number of required PMUs.

Having the impedance matrix of the system, namely Z , makes it easy to calculate the voltage change at each bus caused by the fault [37]. However, the impedance matrix of the system does not provide any useful information regarding current changes. Note that, the conventional admittance matrix can be utilized to obtain nodal voltages based on nodal equations using lines and load admittances as well as the current sources feeding the network. This work proposes a novel method to calculate the effect of the faults on the line currents. Here, the concept of “mesh impedance,” namely Z' , matrix is developed and the mesh equations of the network are obtained. That is $V_s = Z'I_{mesh}$ where V_s is the vector of voltage sources feeding the network and I_{mesh} is the vector of mesh currents. Then, the network mesh

currents are obtained from the inverse of Z' , namely “mesh admittance matrix” Y' , using line and load impedances as well as the voltage sources feeding the network.

3.2.1 Problem Statement

Suppose that pair $m_{hP} = (\Delta V_{hP}, \Delta I_{hP})$ represents the measured changes in the voltage and current at PMU bus h due to the fault P where P , defined as $P = (j, k, D, R_f)$ contains the location and impedance of the fault where k and j are sending and receiving end buses of the faulted line, respectively, D is percent distance from the sending end bus, and R_f is the fault impedance. Define $\Omega = \{m_{hP} | 1 \leq h, j, k \leq N, 0 \leq D \leq 1, R_f \in \mathbb{R}^+\}$ as a set of all possible pairs m_{hP} , with N being the total number of network busses. Also, define $M_l \subseteq \Omega$, with $1 \leq l$ an integer, a subset of Ω that can uniquely identify all faults (location and impedance) in the network. That is, for any two different faults P_1 and P_2 where $m_{hP_1} \& m_{hP_2} \in M_l$ one has $m_{hP_1} \neq m_{hP_2}$ for some h . A trivial solution for most networks occurs when $M_l = \Omega$. In other words, placing PMUs on all the network busses uniquely identifies all faults. Once subset M_l , when exists, is identified, fault location and impedance can be found using M_l and available methods with no possibility of multi estimation. Thus, the target is to find the smallest subset M_l (that might not be unique.)

3.2.2 Change in the Measured Voltage

The power system fault analysis benefits from the impedance matrix based on nodal analysis. Although impedance matrix can be obtained by inverting the admittance matrix, in the case of large-scale and/or distributed power systems with large number of nodes, it imposes a significant computational burden and error, and thus, direct development of impedance matrix

[37] is preferred. As a result, by having the impedance matrix before the fault, the impedance matrix after the fault can be calculated avoiding the matrix inversion.

Figure 1 shows single-line diagram of the part of system before and after fault occurs at point p on the line connecting buses j and k . When a fault occurs on the line between buses j and k , the point of fault on the line can be assigned a new bus number. Subsequently, Z from pre-fault condition can be modified to accommodate the new bus.

The line has series impedance z_b while the percent distance of the fault to the sending-end is D . The following steps are required to modify the existing impedance matrix Z [37] to reflect the new condition:

- Remove the line between bus j and bus k by adding line impedance $-z_b$ between those two buses which can be formulate as follows [37]:

$$z_{hi(1)} = z_{hi(0)} - \frac{(z_{hj(0)} - z_{hk(0)})(z_{ji(0)} - z_{ki(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} \quad (1)$$

where $Z_{hi(1)}$ is the element of row h and column i of modified impedance matrix Z while index (1) shows the level of modification which is the first step and $Z_{hi(0)}$ is the same element but before modification.

- Add $D \times z_b$ from a new bus p to an existing bus j which leads to adding a new row and column to Z as follows:[37]

$$z_{pi(2)} = z_{ji(1)} \quad (2)$$

$$z_{hp(2)} = z_{hj(1)} \quad (3)$$

$$z_{pp(2)} = z_{jj(1)} + Dz_b \quad (4)$$

- Add $(1-D) \times z_b$ between the new bus p and existing bus k which new elements calculated

as:[37]

$$z_{hi(3)} = z_{hi(2)} - \frac{(z_{hp(2)} - z_{hk(2)})(z_{pi(2)} - z_{ki(2)})}{z_{pp(2)} + z_{kk(2)} - 2z_{pk(2)} + (1-D)z_b} \quad (5)$$

where $z_{hi(3)}$ and $z_{hi(2)}$ are the element of row h and column i of modified impedance matrix Z at third and second step respectively.

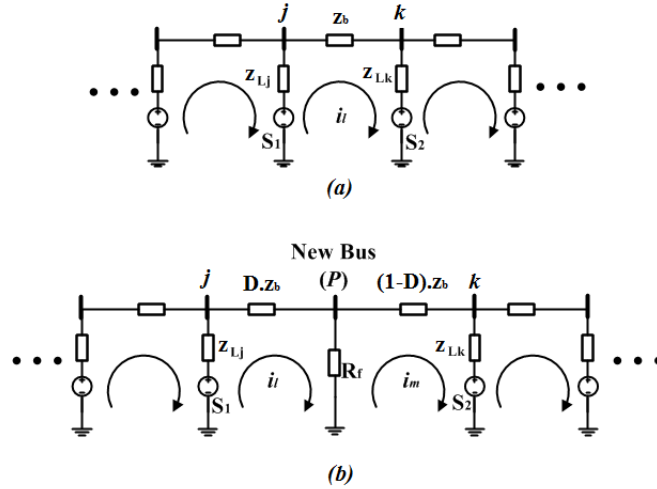


Figure 3.1 Power Network a) before the Fault b) after the Fault on Point p

. According to the standard fault analysis [37], by knowing the elements of the modified impedance matrix, the change in the bus h voltage due to the fault current at bus p can be obtained as:

$$\Delta V_{hp} = z_{hp(new)} \times I_f \quad (6)$$

where $z_{hp(new)}$ is the element in row h and column p of modified matrix Z ($z_{hp(3)}$),

$$I_f = \frac{V_{pref(p)}}{z_{pp(new)} + R_f}, \quad R_f \text{ is fault resistance, } V_{pref(p)} \text{ is the pre-fault voltage at point } p \text{ and } z_{pp(new)} \text{ is}$$

the diagonal entry in row p and column p of modified matrix Z ($z_{pp(3)}$), which is equal to

Thevenin impedance at bus p [37].

It is reasonable to assume that the voltage drop is linear along the line. Also, for

simplicity, the capacitive effects of the power lines can be ignored especially for small- and medium-size distributed generation networks. The pre-fault voltage at point p between bus j and k can be calculated as follows:

$$V_{pref(p)} = V_k + (1 - D) \times (V_j - V_k) \quad (7)$$

By combining equations (1), (2) and modified $z_{pp(new)}$ and $z_{hp(new)}$, the voltage change measured at bus h due to the fault located at point p is obtained. The fault is specified by knowing the faulted line receiving and sending end buses j and k , the distance from bus j (D), and fault impedance R_f . The relationship for voltage change based on fault location and fault impedance can be given as

$$\Delta V_{hp} = \frac{\alpha_1 D^2 + \alpha_2 D + \alpha_3}{\beta_1 D^2 + \beta_2 D + \beta_3 + \beta_4 R_f} \quad (8)$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \beta_4$ are coefficients which are calculated for each line and are functions of elements of pre-fault impedance matrix and two end voltages of that line (the calculations can be found in Appendix A.)

3.2.3 Line Current Change Formulation Using Network Dual Circuit

In this section a method based on mesh current analysis is proposed for line current measurement. Here, the main problem is that there is no systematic approach for modifying mesh-admittance matrix Y' , unlike with its counterpart Z . The proposed solution is to find the dual circuit of the network where the nodal equations in the dual circuit are the same as mesh equations in the original circuit. The crucial task is to find the changes in the dual circuit after occurrence of a fault and to apply those changes to the nodal-impedance matrix \hat{Z} of the dual circuit.

Adding a new bus, which is connected to the ground by the fault impedance, in the line between buses j and k provides the system configuration of the original network after the fault as shown in Fig. 1. Comparing the dual circuit before and after the fault helps find the required steps to modify the nodal-impedance matrix of the dual circuit.

Figure 2 shows a part of the network of the dual circuit before and after the fault occurs at point p , which is the dual of the circuit shown in Fig. 1. The faulted line has series impedance z_b while the percent distance of the fault to sending-end is assumed as D . The following steps are required to reflect the new condition in the dual circuit where the dual values are represented by a hat sign “^”:

- Omit impedance $1/\hat{y}_b$ where $\hat{y}_b = z_b$ between bus j' and ground (adding the impedance $-1/\hat{y}_b$ between bus j' and ground,)

$$\hat{z}_{hi(1)} = \hat{z}_{hi(0)} - \frac{(\hat{z}_{hj'(0)}\hat{z}_{ji(0)})}{\hat{z}_{jj'(0)} - \frac{1}{\hat{y}_b}} \quad (9)$$

- Remove the impedance $1/\hat{y}_{Lk}$ where $\hat{y}_{Lk} = z_{Lk}$ between bus j' and k' (adding the impedance $-1/\hat{y}_{Lk}$ between bus j' and k')

$$\hat{z}_{hi(2)} = \hat{z}_{hi(1)} - \frac{(\hat{z}_{hk'(1)} - \hat{z}_{hj'(1)})(\hat{z}_{k'i(1)} - \hat{z}_{ji(1)})}{\hat{z}_{jj'(1)} + \hat{z}_{kk'(1)} - 2\hat{z}_{jk'(0)} - \frac{1}{\hat{y}_{Lk}}} \quad (10)$$

- Add a new bus p' connected to bus j' through conductance G_f ,

$$\hat{z}_{p'i(3)} = \hat{z}_{ji(2)} \quad (11)$$

$$\hat{z}_{hp'(3)} = \hat{z}_{hj'(2)} \quad (12)$$

$$\hat{z}_{p'p'(3)} = \hat{z}_{jj'(2)} + \frac{1}{G_f} \quad (13)$$

The modified impedance matrix \hat{Z} of the dual circuit is the same as the modified mesh admittance matrix $Y' = (Z')^{-1}$ of the original circuit; i.e., $Y' = \hat{Z}$.

In the dual circuit, one can calculate the nodal voltages from the current sources feeding the nodes as $\hat{V} = \hat{Z} \cdot \hat{I}_s$ where \hat{I}_s is the vector of current sources feeding the dual network.

Since the nodal voltages in the dual circuit are the same as mesh currents in the original circuit, one can conclude that

$$I_{mesh} = Y' \cdot V_s \quad (17)$$

where $I_{mesh} = [I_1 \ I_2 \ \dots \ I_W]^T$ is the vector of mesh currents, $V_s = [V_{s1} \ V_{s2} \ \dots \ V_{sW}]^T$ is the vector of mesh voltage sources, $Y' = \hat{Z}$, and W is the number of meshes in the original circuit.

As depicted in Fig. 1, when a fault happens a new loop is added to the original network. The faulted node can be modeled by a voltage source which connects that node to the ground and its value is equal to the fault impedance multiplied by the fault current. Since this branch is common between two loops the voltage source for both loops are affected as

$$\begin{bmatrix} I_1 \\ \vdots \\ I_h \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11}' & \dots & Y_{1m}' & \dots & Y_{1h}' & \dots & Y_{1l}' & \dots & Y_{1n}' \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Y_{h1}' & \dots & Y_{hm}' & \dots & Y_{hh}' & \dots & Y_{hl}' & \dots & Y_{hn}' \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Y_{n1}' & \dots & Y_{nm}' & \dots & Y_{nh}' & \dots & Y_{nl}' & \dots & Y_{nn}' \end{bmatrix} \times \begin{bmatrix} V_1 \\ \vdots \\ V_m - \Delta V_f \\ \vdots \\ V_h \\ \vdots \\ V_l + \Delta V_f \\ \vdots \\ V_n \end{bmatrix}. \quad (18)$$

According to (18), the change in the corresponding mesh current at bus h (measured by PMU at bus h) due to the fault at point p between loops l and m is calculated as

$$\Delta I_{hp} = (Y'_{hl(new)} - Y'_{hm(new)}) \times \Delta V_f \quad (19)$$

where $\Delta V_f = I_f \times R_f = \frac{V_{pref(p)}}{z_{pp(new)} + R_f} R_f$, R_f is the fault resistance, $V_{pref(p)}$ is pre-fault voltage at point p and $z_{pp(new)}$ is the diagonal entry in row p and column p of modified matrix Z which is equal to Thevenin impedance at bus p [37], $Y'_{hl(new)}$ and $Y'_{hm(new)}$ are developed mesh admittance matrix Y' elements and equal to $\hat{z}_{hj'(6)}$ and $\hat{z}_{hp'(6)}$ respectively. Considering linear voltage change along the power lines, one can calculate the pre-fault voltage at point p between bus j and k using (7). Next, by using equation (19), the current change measured at bus h due to the fault at point p is obtained.

3.2.4 PMU Allocation Optimization Algorithm

In this section, the proposed algorithm to find the best place by minimum number of PMUs to achieve fault observability is described. Given two faults at locations p_1 and p_2 with impedances R_{f1} and R_{f2} , it is desirable to have the conditions

$$\begin{aligned} \Delta V_{hp_1} &\neq \Delta V_{hp_2} \\ \Delta I_{hp_1} &\neq \Delta I_{hp_2} \end{aligned} \quad (20)$$

as a result of proper placement of PMU where ΔV_{hp_1} , ΔV_{hp_2} , ΔI_{hp_1} , and ΔI_{hp_2} are defined in (6) and (19). The proposed placement algorithm includes the following general steps:

- 1) By using (6) and (19), develop the voltage and current change functions for the specific measurement (PMU) point and fault (location and impedance)
- 2) Create “sectors” comprising any two lines of the network; for instance, lines 1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4, etc.
- 3) For each possible measurement point (PMU buses) check if conditions (20) are

violated in any “sector” comprised of any two power lines; that is, two faults on two different lines cause equal voltage and current phasor changes at the selected measurement point (PMU bus)

- 4) Sort the measurement points according to the number of the undesired “sectors”; that is, “sectors” that violate (20). The measurement point (bus) that has the minimum undesired “sectors” is chosen as the first place for the PMU placement. This approach can be continued by adding more PMUs under the condition that the undesired sectors for any new PMU has minimum overlap with those of the previous ones until the intersection of all undesired sectors of all utilized PMUs becomes an empty set where the required level of observability is reached.

In step 3, using the calculated voltage change functions in step 1, the function

$$f = \Delta V_{hp_1} - \Delta V_{hp_2} \quad (21)$$

is defined where h is the measurement point and p_1 and p_2 are two different fault locations. If (21) becomes zero it implies that there are two different faults (locations and impedances) that provide the same voltage change on a selected measurement point (h); that is, a multi estimation has occurred. Consequently, the goal is to find all two lines (“sectors”) per each measurement point (PMU bus) for which there is potential multi estimation. For this purpose the equation $f = 0$ is solved for all “sectors”. Since each two lines create a “sector” and the number of lines is considered as L there would be $C(L,2) = L \times (L-1)/2$ sectors. The next criterion is the equality in the measured current change; that is,

$$g = \Delta I_{hp_1} - \Delta I_{hp_2} = 0 \quad (22)$$

If the roots of $f = 0$ satisfy (22) in a sector, the sector is considered as undesired and

causes multi estimation. The novelty of this algorithm is to use the current criterion as well as voltage to minimize the number of PMUs and solving the multi estimation problem by applying the obtained current and voltage change functions for the entire network and for a large range of fault impedance. This algorithm specifies the best measurement points to identify the faults uniquely on the basis of the number of undesired sectors for each measurement point (PMU bus.) Finding the roots of $f = 0$ leads to solving the nonlinear equations with 4 unknowns D_1, D_2, R_{f1} and R_{f2} . These equations can be solved by minimizing the f^2 through Levenberg-Marquardt algorithm. Once the solutions of $f = 0$ is found, they are checked in (22).

The procedure of the entire algorithm is shown in Fig. 3.

3.3 Simulation Results

The IEEE 7-bus, 14-bus and 30-bus benchmarks are considered as the case studies as shown in Figs. 4, 5 and 6.

The proposed algorithm is applied to all networks to find the optimum number of PMUs besides the best place for them. The results are shown separately in Table I. As illustrated in Table 3.1, for 7-bus system after solving equation (8), 30 undesirable sectors are found where each measurement point on buses contains at least one undesired sector. However, when the current criterion is taken into the account those 30 sectors reduce to 2 undesired sectors. Nevertheless, there are measurement points (PMU buses) in the network that do not experience any undesired sectors, and thus, can be considered as the best places for PMU. As a result, by using this approach for 7-bus system only one PMU is needed for fault observability on bus 1 or 2 or 4 or 5 or 7.

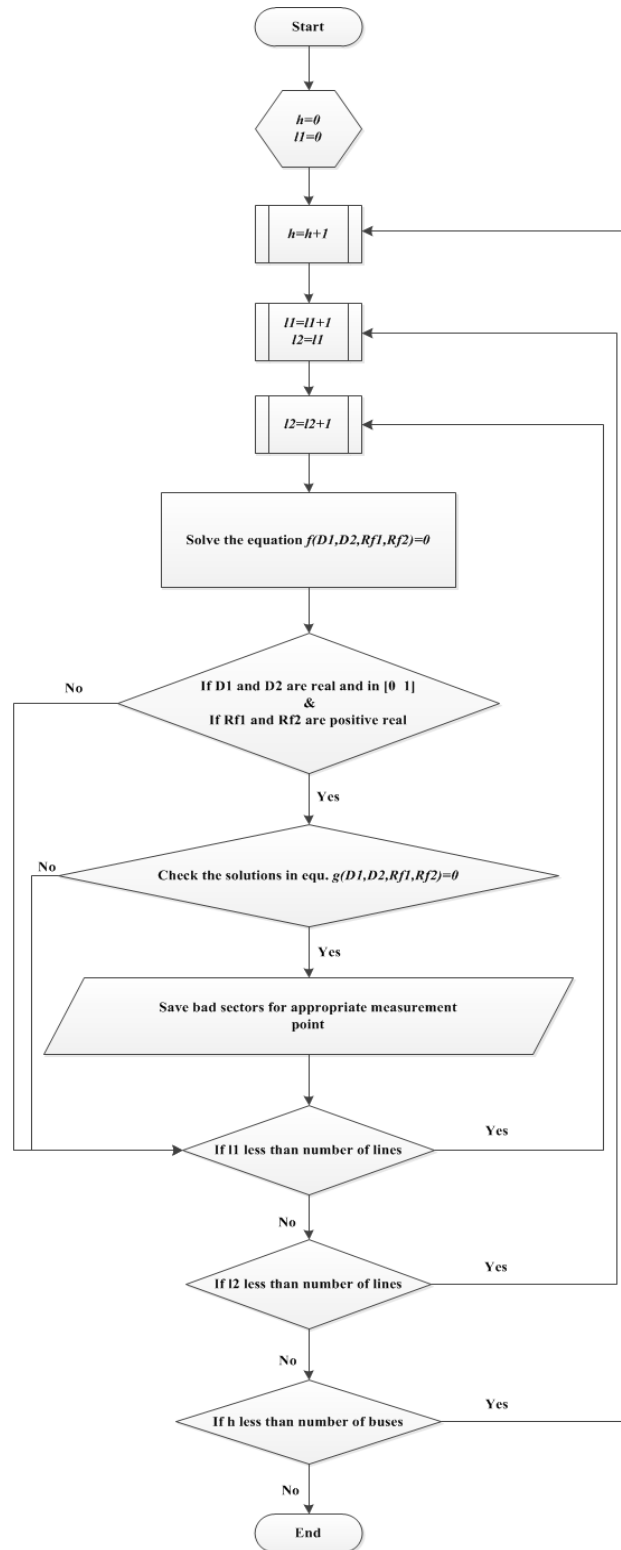


Figure 3.3 Algorithm of Finding the Best PMU Placement

Next, the proposed approach is applied to the IEEE 14-bus system. Again, the proposed algorithm reduces the undesired sectors from 115 to 40 when current change criterion is utilized along with the voltage change. Also, the minimum number of PMUs is determined to be one and appropriate locations are on bus 1 or 2 or 4 or 5, where no multi estimations occur (no undesired sectors.)

Finally, the IEEE 30-bus system is tested. It can be seen the undesired sectors are reduced from 405 to 162 when current change criterion is utilized along with the voltage change. Also, the minimum number of PMUs is determined to be 2 and appropriate locations are on buses 10 and 25.

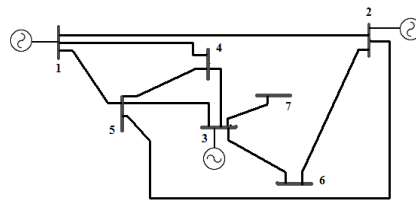


Fig. 3.4

Figure 3.4 7 Buses IEEE Benchmark

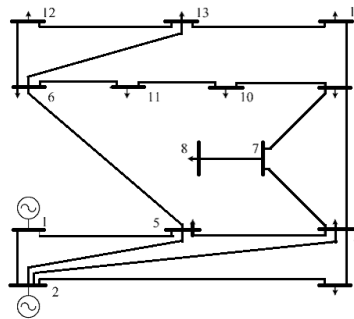


Figure 3.5 14 Buses IEEE Benchmark

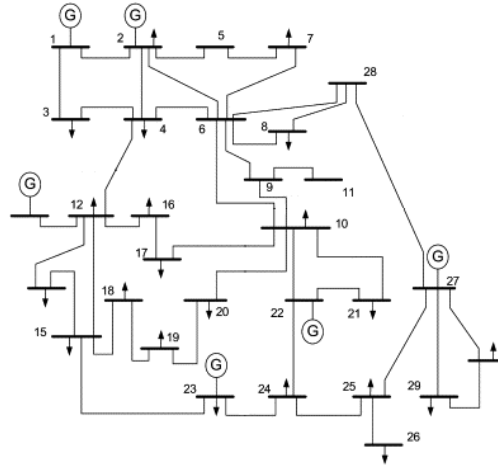


Figure 3.6 30 Buses IEEE Benchmark

The results are compared with available approaches in literature [26] and [38] and are shown in Table 3.2 where the effectiveness of the proposed algorithm is shown by reducing the number of PMUs from 5 to 1 for IEEE 7-bus, 8 to 1 for IEEE 14-bus and 17 to 2 for IEEE 30-bus networks.

The proposed algorithm is also capable of considering a desired range for voltage and current measurement error. In this simulation this error is assumed to be as $1e-5$ which is about 1mV error in 100V. By assuming higher measurement error, more undesired sectors are likely to appear leading to more number of required PMUs. For instance, if the error increases to $1e-4$, in case of IEEE 7-bus, it leads to 7 undesired sector instead of 2 but still one PMU on bus B1 or B2 or B7 could solve the multi estimation problem. However for IEEE 14-bus the consequence of the same simulation is two required PMUs instead of one and 56 undesired sectors instead of 40. Although the suggested number of PMUs satisfies the theoretical requirement, more number may be desirable to provide required robustness.

Table 3.1 Results of Applying Proposed Approach

Test System	Based on Voltage change only	Based on both Voltage and current change		
	# undesired sectors	# undesired sectors	# PMUs	Location
IEEE 7 Bus	30	2	1	B1 or B2 or B4 or B5 or B7
IEEE 14 bus	115	40	1	B1 or B2 or B4 or B5
IEEE 30 bus	405	162	2	B10 and B25

Table 3.2 Comparison between Proposed and Prior Approaches

Test System	Number of PMUs	
	Prior Approach	Proposed Approach
IEEE 7 Bus	5	1
IEEE 14 bus	8	1
IEEE 30 bus	17	2

3.4 Summary

In this chapter, a new method for fault analysis is developed based on mesh equations. This method allows calculating the line current change when a fault occurs in the system. Then, the voltage and current changes are formulated for the entire system in the form of functions of fault location and impedance. Subsequently, the network is tested for all possible PMU locations and faults to reduce multi estimation. It is shown that by utilizing the voltage change criterion along with the current change criterion at the PMU buses, the likelihood of multi estimation is greatly lowered. In addition, the required number of the PMUs for fault observability is significantly reduced compared to the past work.

4. CONCLUSION

The fault location in the distribution system has been a controversial issue because of the complexity of such a system caused by:

1. System imbalance due to existence of unbalanced loads,
2. Different types of load include resistive, inductive or dynamic loads,
3. Presence of laterals along the main feeder,
4. Presence of the load taps along the main feeder and laterals,
5. Heterogeneity of feeders caused by different cable size and length as well as presence of overhead and underground lines, and
6. Presence of the renewable energy and DGs.

As a result, recently there have been a lot of research and effort to propose an efficient fault location algorithm in order to improve the reliability of the distribution system by reducing duration of unexpected outages (outage caused by faults) and confining the search for faults within a relatively small area. Although there are different approaches to solve the multi estimation problem for fault location in the literature, just a few of them focus on optimal PMU placement to achieve fault observability in the given distribution system.

In this thesis a novel approach is proposed to find the minimum number and best places of Phasor Measurement Units in the distribution system based on voltage and current measurement. In order to develop the approach the concepts of mesh-impedance matrix and mesh-admittance matrix in the network are introduced to calculate the current changes at the points of measurements as well as voltage drops caused by occurred fault. Finally, the minimum required number of PMUS and the best place of them are specified so that measured voltages and currents provide a unique location for the happened fault.

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APPENDIX A: FORMULA DEVELOPMENT

$$\Delta V_{hp} = z_{hp(new)} \times I_f \quad (1)$$

$$I_f = \frac{V_{pref(p)}}{z_{pp(new)} + R_f} \quad (2)$$

$$z_{hp(new)} = z_{hp(3)} = z_{hp(2)} - \frac{(z_{hp(2)} - z_{hk(2)})(z_{pp(2)} - z_{kp(2)})}{z_{pp(2)} + z_{kk(2)} - 2z_{pk(2)} + (1-D)z_b} \quad (3)$$

$$z_{pp(new)} = z_{pp(3)} = z_{pp(2)} - \frac{(z_{pp(2)} - z_{pk(2)})(z_{pp(2)} - z_{kp(2)})}{z_{pp(2)} + z_{kk(2)} - 2z_{pk(2)} + (1-D)z_b} \quad (4)$$

$$V_{pref(p)} = V_j + (1-D) \times (V_j - V_k) \quad (5)$$

Substituting (2)-(5) in (1), one can have:

$$\Delta V_{hp} = z_{hp(new)} \times \frac{V_{pref(p)}}{z_{pp(new)} + R_f} = \frac{z_{hp(2)} - \frac{(z_{hp(2)} - z_{hk(2)})(z_{pp(2)} - z_{kp(2)})}{z_{pp(2)} + z_{kk(2)} - 2z_{pk(2)} + (1-D)z_b} \times (V_j + (1-D) \times (V_j - V_k))}{z_{pp(2)} - \frac{(z_{pp(2)} - z_{pk(2)})(z_{pp(2)} - z_{kp(2)})}{z_{pp(2)} + z_{kk(2)} - 2z_{pk(2)} + (1-D)z_b} + R_f} \quad (6)$$

where

$$z_{hk(2)} = z_{hk(1)} = z_{hk(0)} - \frac{(z_{hj(0)} - z_{hk(0)})(z_{jk(0)} - z_{kk(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} \quad (7)$$

$$z_{kk(2)} = z_{kk(1)} = z_{kk(0)} - \frac{(z_{kj(0)} - z_{kk(0)})(z_{jk(0)} - z_{kk(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} \quad (8)$$

$$z_{kp(2)} = z_{kj(1)} = z_{kj(0)} - \frac{(z_{kj(0)} - z_{kk(0)})(z_{jj(0)} - z_{kj(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} \quad (9)$$

$$z_{pk(2)} = z_{jk(1)} = z_{jk(0)} - \frac{(z_{jj(0)} - z_{jk(0)})(z_{jk(0)} - z_{kk(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} \quad (10)$$

$$z_{hp(2)} = z_{hj(1)} = z_{hj(0)} - \frac{(z_{hj(0)} - z_{hk(0)})(z_{jj(0)} - z_{kj(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} \quad (11)$$

$$z_{pp(2)} = z_{jj(1)} + Dz_b = z_{jj(0)} - \frac{(z_{jj(0)} - z_{jk(0)})(z_{jj(0)} - z_{kj(0)})}{z_{jj(0)} + z_{kk(0)} - 2z_{jk(0)} - z_b} + Dz_b \quad (12)$$

Equation (6) can be rewritten by substituting (7)-(12) in it as follows:

$$\Delta V_{hp} = \frac{\alpha_1 D^2 + \alpha_2 D + \alpha_3}{\beta_1 D^2 + \beta_2 D + \beta_3 + \beta_4 R_f} \quad (13)$$

where

$$\alpha_1 = (z_{hk(2)} - z_{hp(2)}) \times z_b \times (V_k - V_j) \quad (14)$$

$$\alpha_2 = (z_{kk(2)} z_{hp(2)} - z_{kp(2)} z_{hp(2)} - z_{hk(2)} z_{kp(2)} + z_{hk(2)} z_{jj(2)} + z_{hp(2)} z_b) \times (V_k - V_j) + (z_{hk(2)} - z_{hp(2)}) \times z_b \times V_j \quad (15)$$

$$\alpha_3 = (z_{kk(2)} z_{hp(2)} - z_{kp(2)} z_{hp(2)} - z_{hk(2)} z_{kp(2)} + z_{hk(2)} z_{jj(2)} + z_{hp(2)} z_b) \times V_j \quad (16)$$

$$\beta_1 = z_b \quad (17)$$

$$\beta_2 = (z_{kk(2)} - z_{kp(2)} + z_b) \times z_b \quad (18)$$

$$\beta_3 = (z_{kk(2)} z_{jj(2)} - z_{pk(2)}^2 + z_{jj(2)} z_b) \quad (19)$$

$$\beta_4 = (z_{kk(2)} + z_{jj(2)} - 2z_{kp(2)} + z_b) \quad (20)$$

Note that $z_{hk(2)}$, $z_{kk(2)}$, $z_{kp(2)}$, $z_{pk(2)}$, $z_{hp(2)}$, and $z_{pp(2)}$ can be calculated from (7), (8), (9), (10), (11), and (12) respectively.

Finally $f = \Delta V_{hp_1} - \Delta V_{hp_2}$ can be calculated from:

$$f = \frac{\alpha_1 D^2 + \alpha_2 D + \alpha_3}{\beta_1 D^2 + \beta_2 D + \beta_3 + \beta_4 R_f} - \frac{\alpha'_1 D^2 + \alpha'_2 D + \alpha'_3}{\beta'_1 D^2 + \beta'_2 D + \beta'_3 + \beta'_4 R_f} \quad (21)$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \beta_4$ are coefficients which are calculated for fault occurs at point p_1 and $\alpha'_1, \alpha'_2, \alpha'_3, \beta'_1, \beta'_2, \beta'_3, \beta'_4$ are coefficients which are calculated for fault occurs at point p_2

APPENDIX B: MATLAB CODE FOR OPTIMAL PMU PLACEMENT

% PROGRAM TO MAKE THE DESIRED BUS AND LINE MATRICES ACCORDING TO BUS AND LINE INFORMATION OF IEEE 30-BUS

```
bus=[
1 1.06 0 0 0 0.000 0.000 0 0 1
2 1.043 0 0.4 0 0.217 0.127 0 0 2
3 1 0 0 0 0.024 0.012 0 0 3
4 1 0 0 0 0.076 0.016 0 0 3
5 1.01 0 0 0 0.942 0.190 0 0 2
6 1 0 0 0 0.000 0.000 0 0 3
7 1 0 0 0 0.228 0.109 0 0 3
8 1.01 0 0 0 0.300 0.300 0 0 2
9 1 0 0 0 0.000 0.000 0 0 3
10 1 0 0 0 0.058 0.020 0 0.19 3
11 1.082 0 0 0 0.000 0.000 0 0 2
12 1 0 0 0 0.112 0.075 0 0 3
13 1.071 0 0 0 0.000 0.000 0 0 2
14 1 0 0 0 0.062 0.016 0 0 3
15 1 0 0 0 0.082 0.025 0 0 3
16 1 0 0 0 0.035 0.018 0 0 3
17 1 0 0 0 0.090 0.058 0 0 3
18 1 0 0 0 0.032 0.009 0 0 3
19 1 0 0 0 0.095 0.034 0 0 3
20 1 0 0 0 0.022 0.007 0 0 3
21 1 0 0 0 0.175 0.112 0 0 3
22 1 0 0 0 0.000 0.000 0 0 3
23 1 0 0 0 0.032 0.016 0 0 3
24 1 0 0 0 0.087 0.067 0 0.043 3
25 1 0 0 0 0.000 0.000 0 0 3
26 1 0 0 0 0.035 0.023 0 0 3
27 1 0 0 0 0.000 0.000 0 0 3
28 1 0 0 0 0.000 0.000 0 0 3
29 1 0 0 0 0.024 0.009 0 0 3
30 1 0 0 0 0.106 0.019 0 0 3
];
```

```
line=[
%From To R X B
1 2 0.019200 0.057500 0.026400 1.00 0.00
1 3 0.045200 0.185200 0.020400 1.00 0.00
2 4 0.013200 0.037900 0.004200 1.00 0.00
2 5 0.047200 0.198300 0.020900 1.00 0.00
2 6 0.058100 0.176300 0.018700 1.00 0.00
4 6 0.011900 0.041400 0.004500 1.00 0.00
5 7 0.046000 0.116000 0.010200 1.00 0.00
6 7 0.026700 0.082000 0.008500 1.00 0.00
6 8 0.012000 0.042000 0.004500 1.00 0.00
6 9 0.000000 0.208000 0.004500 0.978 0.00
```

```

6 10 0.000000 0.556000 0.004500 0.969 0.00
9 11 0.000000 0.208000 0.004500 1.00 0.00
9 10 0.000000 0.110000 0.004500 1.00 0.00
4 12 0.000000 0.256000 0.004500 0.932 0.00
12 13 0.000000 0.140000 0.004500 1.00 0.00
12 14 0.123100 0.255900 0.004500 1.00 0.00
12 15 0.066200 0.130400 0.008500 1.00 0.00
12 16 0.094500 0.198700 0.018700 1.00 0.00
14 15 0.221000 0.199700 0.010200 1.00 0.00
16 17 0.082400 0.192300 0.010200 1.00 0.00
15 18 0.107000 0.218500 0.010200 1.00 0.00
18 19 0.063900 0.129200 0.010200 1.00 0.00
19 20 0.034000 0.068000 0.010200 1.00 0.00
10 20 0.093600 0.209000 0.010200 1.00 0.00
10 17 0.032400 0.084500 0.010200 1.00 0.00
10 21 0.034800 0.074900 0.010200 1.00 0.00
10 22 0.072700 0.149900 0.020900 1.00 0.00
21 22 0.011600 0.023600 0.020900 1.00 0.00
15 23 0.100000 0.202000 0.020900 1.00 0.00
22 24 0.115000 0.179000 0.021400 1.00 0.00
23 24 0.132000 0.270000 0.021400 1.00 0.00
24 25 0.188500 0.329200 0.021400 1.00 0.00
25 26 0.254400 0.380000 0.021400 1.00 0.00
25 27 0.109300 0.208700 0.021400 1.00 0.00
27 28 0.000000 0.396000 0.021400 0.968 0.00
27 29 0.219800 0.415300 0.021400 1.00 0.00
27 30 0.320200 0.602700 0.021400 1.00 0.00
29 30 0.239900 0.453300 0.021400 1.00 0.00
8 28 0.063600 0.200000 0.021400 1.00 0.00
6 28 0.016900 0.059900 0.006500 1.00 0.00
];

```

```

[br,bc]=size(bus);
for i=1:br
    if bus(i,10)==3;
        bus(i,6)=(bus(i,6)-bus(i,4));
        bus(i,4)=0;
        bus(i,7)=(bus(i,7)-bus(i,5));
        bus(i,5)=0;
    else
        bus(i,4)=(bus(i,4)-bus(i,6));
        bus(i,6)=0;
        bus(i,5)=(bus(i,5)-bus(i,7));
        bus(i,7)=0;
    end
end

```

```

bus2=bus;
line2=line;

```

```

bus2=[bus2,bus(:,1)];
line2=[line2,line(:,1:2)];

```

%this part place the slack Gens in the first row

```
[lr,lc]=size(line);  
for i=1:br  
    if bus2(i,10)==1  
        Bus_temp=bus2(1,:);  
        bus2(1,:)=bus2(i,:);  
        bus2(i,:)=Bus_temp;  
        bus2(1,1)=1;  
        bus2(i,1)=i;  
  
        for j2=1:lr  
            if line2(j2,1)==1  
                line2(j2,1)=i;  
            elseif line2(j2,1)==i  
                line2(j2,1)=1;  
            end  
  
            if line2(j2,2)==1  
                line2(j2,2)=i;  
            elseif line2(j2,2)==i  
                line2(j2,2)=1;  
            end  
  
        end  
    end  
end
```

%this part place the Gens in the first rows after slack

```
j1=2;  
for i=1:br  
    if bus2(i,10)==2  
        Bus_temp=bus2(j1,:);  
        bus2(j1,:)=bus2(i,:);  
        bus2(i,:)=Bus_temp;  
        bus2(j1,1)=j1;  
        bus2(i,1)=i;  
  
        for j2=1:lr  
            if line2(j2,1)==j1  
                line2(j2,1)=i;  
            elseif line2(j2,1)==i  
                line2(j2,1)=j1;  
            end  
  
            if line2(j2,2)==j1  
                line2(j2,2)=i;  
            elseif line2(j2,2)==i
```

```

        line2(j2,2)=j1;
    end
end
j1=j1+1;
end
end

bus=bus2(:,1:bc);
line=line2(:,1:lc);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

b=size(line);
b=b(1,1);
clear b
clear bus1

bus2;
line2;

busINIT = bus;

```

% PROGRAM IN MATALB TO CALCULATE $f = \Delta V_{hp_1} - \Delta V_{hp_2}$ FOR EACH SECTION

```
function F=myfun30(x)
global h l1 l2;
BusLine30;
nn=size(bus,1);
mm=size(line,1);
[Y,nSW,nPV,nPQ,SB] = ybus(bus,line);
Y_bus=full(Y);
Z0=Y_bus\eye(nn);
[bus_sol,line_flow]=loadflow(bus,line,1e-11,100,0,2,1,'n',1);
p=nn+1;
Final_R=[];

k=line(l1,1);
j=line(l1,2);
Z2=zeros(nn+1,nn+1);
Vk=bus_sol(k,2)*exp(sqrt(-1)*bus_sol(k,3)*pi/180);
Vj=bus_sol(j,2)*exp(sqrt(-1)*bus_sol(j,3)*pi/180);
Zjk=-1/Y_bus(j,k);
Z2(j,j)=Z0(j,j)-(Z0(j,j)-Z0(j,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(j,p)=Z0(j,k)-(Z0(j,j)-Z0(j,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(p,j)=Z2(j,p);
Z2(h,p)=Z0(h,k)-(Z0(h,j)-Z0(h,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(h,j)=Z0(h,j)-(Z0(h,j)-Z0(h,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(k,k)=Z0(k,k)-(Z0(k,j)-Z0(k,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
m=Z2(j,j)*Z2(h,p)-Z2(j,p)*Z2(h,p)-Z2(h,j)*Z2(j,p)+Z2(h,j)*Z2(k,k)+Z2(h,p)*Zjk;
n=(Z2(h,j)-Z2(h,p))*Zjk;
M=-m*Vk;
N=-(m*Vj-m*Vk+n*Vk);
Q=-n*(Vj-Vk);
Mp=Z2(k,k)*Z2(j,j)-Z2(p,j)^2+Z2(k,k)*Zjk;
Xp=Z2(k,k)+Z2(j,j)-2*Z2(p,j)+Zjk;
Np=(Z2(j,j)-Z2(j,p)+Zjk)*Zjk;
Qp=Zjk;

k=line(l2,1);
j=line(l2,2);
Z2=zeros(nn+1,nn+1);
Vk=bus_sol(k,2)*exp(sqrt(-1)*bus_sol(k,3)*pi/180);
Vj=bus_sol(j,2)*exp(sqrt(-1)*bus_sol(j,3)*pi/180);
Zjk=-1/Y_bus(j,k);
Z2(j,j)=Z0(j,j)-(Z0(j,j)-Z0(j,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(j,p)=Z0(j,k)-(Z0(j,j)-Z0(j,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(p,j)=Z2(j,p);
Z2(h,p)=Z0(h,k)-(Z0(h,j)-Z0(h,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(h,j)=Z0(h,j)-(Z0(h,j)-Z0(h,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(k,k)=Z0(k,k)-(Z0(k,j)-Z0(k,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
s=Z2(j,j)*Z2(h,p)-Z2(j,p)*Z2(h,p)-Z2(h,j)*Z2(j,p)+Z2(h,j)*Z2(k,k)+Z2(h,p)*Zjk;
```



```

t=(Z2(h,j)-Z2(h,p))*Zjk;
S=-s*Vk;
T=-(s*Vj-s*Vk+t*Vk);
R=-t*(Vj-Vk);
Sp=Z2(k,k)*Z2(j,j)-Z2(p,j)^2+Z2(k,k)*Zjk;
Tp=(Z2(j,j)-Z2(j,p)+Zjk)*Zjk;
Yp=Z2(k,k)+Z2(j,j)-2*Z2(p,j)+Zjk;
Rp=Zjk;

F=((M+N*x(1)+Q*(x(1).^2))./(Mp+Np*x(1)+Qp*(x(1).^2)+Xp*x(3))-
(S+T*x(2)+R*(x(2).^2))./(Sp+Tp*x(2)+Rp*(x(2).^2)+Yp*x(4)));

end

```

% PROGRAM IN MATALB TO SOLVE THE EQUATION $f = \Delta V_{hp_1} - \Delta V_{hp_2}$ AND CHECK ITS ROOT IN
THE EQUATION $g = \Delta I_{hp_1} - \Delta I_{hp_2} = 0$

```
clear all
clc
Result_F=[];
Result=[];
Result_t=[];
lb=[];
ub=[];
global h l1 l2
options = optimset('Algorithm','levenberg-marquardt');
tic
    for h=1:30
        for l1=1:40
            for l2=l1+1:40
                [h l1 l2]
                for I=0:.5:.5
                    for J=0:0.5:.5
                        for K=0:.5:.5
                            for M=0:.5:.5
                                x0=[rand*.5+I; rand*.5+J; rand*.5+K; rand*.5+M];
                                [x,resnorm,residual] = lsqnonlin(@myfun30,x0,lb,ub,options);
                                realx=real(x);
                                imagx=imag(x);
                                if abs(imagx(1:2,1))<=.001
                                    if abs(imagx(3:4,1))<=.01
                                        if realx>=0
                                            if residual<=1e-4
                                                Result=[Result,[x;h;l1;l2]];
                                                D1=x(1);
                                                D2=x(2);
                                                R1=x(3);
                                                R2=x(4);
                                                Best_PMU_Place_Voltage_F_30
                                                Best_PMU_Place_Current_F_30
                                                if abs(E*Vf*R1/(Zpp+R1)-Ep*Vfp*R2/(Zppp+R2))<1e-4
                                                    Result_F=[Result_F,[D1;D2;R1;R2;h;l1;l2]];
                                                end
                                            end
                                        end
                                    end
                                end
                            end
                        end
                    end
                end
            end
        end
    end
end
```

```
end
    t=toc
Result
Result_F

save Result_sol_30_VC4 Result_F Result
```

% PROGRAM IN MATALAB TO CALCULATE THE PARAMETERS OF $g = \Delta I_{hp_1} - \Delta I_{hp_2}$ FOR EACH SECTION

```

BusLine30
nn=size(bus,1);
mm=size(line,1);
[Y,nSW,nPV,nPQ,SB] = ybus(bus,line);
Y_bus=full(Y);
Z0=Y_bus\eye(nn);
[bus_sol,line_flow]=loadflow(bus,line,1e-11,100,0,2,1,'n',1);
p=nn+1;
Final_R=[];

k=line(l1,1);
j=line(l1,2);
Z2=zeros(nn+1,nn+1);
Vk=bus_sol(k,2)*exp(sqrt(-1)*bus_sol(k,3)*pi/180);
Vj=bus_sol(j,2)*exp(sqrt(-1)*bus_sol(j,3)*pi/180);
Zjk=-1/Y_bus(j,k);
Z2(j,j)=Z0(j,j)-(Z0(j,j)-Z0(j,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(j,p)=Z0(j,k)-(Z0(j,j)-Z0(j,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(p,j)=Z2(j,p);
Z2(h,p)=Z0(h,k)-(Z0(h,j)-Z0(h,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(h,j)=Z0(h,j)-(Z0(h,j)-Z0(h,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Z2(k,k)=Z0(k,k)-(Z0(k,j)-Z0(k,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);
Zpp=Z2(k,k)+D1*Zjk-(Z2(p,j)-Z2(k,k)-D1*Zjk)*(Z2(j,p)-Z0(k,k)-D1*Zjk)/(Z2(j,j)+Z0(k,k)-
2*Z0(j,k)+Zjk);
m=Z2(j,j)*Z2(h,p)-Z2(j,p)*Z2(h,p)-Z2(h,j)*Z2(j,p)+Z2(h,j)*Z2(k,k)+Z2(h,p)*Zjk;
n=(Z2(h,j)-Z2(h,p))*Zjk;
M=-m*Vk;
N=-(m*Vj-m*Vk+n*Vk);
Q=-n*(Vj-Vk);
Mp=Z2(k,k)*Z2(j,j)-Z2(p,j)^2+Z2(k,k)*Zjk;
Xp=Z2(k,k)+Z2(j,j)-2*Z2(p,j)+Zjk;
Np=(Z2(j,j)-Z2(j,p)+Zjk)*Zjk;
Qp=Zjk;
Vf=(1-D1)*Vk+D1*Vj;

A=(M+N*D1+Q*(D1.^2))./(Mp+Np*D1+Qp*(D1.^2));
C=(m+n*D1)./(Mp+Np*D1+Qp*(D1.^2));

k=line(l2,1);
j=line(l2,2);
Z2=zeros(nn+1,nn+1);
Vk=bus_sol(k,2)*exp(sqrt(-1)*bus_sol(k,3)*pi/180);
Vj=bus_sol(j,2)*exp(sqrt(-1)*bus_sol(j,3)*pi/180);
Zjk=-1/Y_bus(j,k);
Z2(j,j)=Z0(j,j)-(Z0(j,j)-Z0(j,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);

```

$Z2(j,p)=Z0(j,k)-(Z0(j,j)-Z0(j,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);$
 $Z2(p,j)=Z2(j,p);$
 $Z2(h,p)=Z0(h,k)-(Z0(h,j)-Z0(h,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);$
 $Z2(h,j)=Z0(h,j)-(Z0(h,j)-Z0(h,k))*(Z0(j,j)-Z0(k,j))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);$
 $Z2(k,k)=Z0(k,k)-(Z0(k,j)-Z0(k,k))*(Z0(j,k)-Z0(k,k))/(Z0(j,j)+Z0(k,k)-2*Z0(j,k)-Zjk);$
 $s=Z2(j,j)*Z2(h,p)-Z2(j,p)*Z2(h,p)-Z2(h,j)*Z2(j,p)+Z2(h,j)*Z2(k,k)+Z2(h,p)*Zjk;$
 $t=(Z2(h,j)-Z2(h,p))*Zjk;$
 $S=-s*Vk;$
 $T=-(s*Vj-s*Vk+t*Vk);$
 $R=-t*(Vj-Vk);$
 $Sp=Z2(k,k)*Z2(j,j)-Z2(p,j)^2+Z2(k,k)*Zjk;$
 $Tp=(Z2(j,j)-Z2(j,p)+Zjk)*Zjk;$
 $Yp=Z2(k,k)+Z2(j,j)-2*Z2(p,j)+Zjk;$
 $Rp=Zjk;$
 $Vfp=(1-D2)*Vk+D2*Vj;$

$Ap=(S+T*D2+R*(D2.^2))./(Sp+Tp*D2+Rp*(D2.^2));$
 $Cp=(s+t*D1)./(Sp+Tp*D2+Rp*(D2.^2));$
 $Zppp=Z2(k,k)+D1*Zjk-(Z2(p,j)-Z2(k,k)-D1*Zjk)*(Z2(j,p)-Z0(k,k)-D1*Zjk)/(Z2(j,j)+Z0(k,k)-2*Z0(j,k)+Zjk);$

```

BusLine30
Mesh_find;
nn=size(line,1);

YY_bus=YY;
Z0=YY_bus\eye(nn);

p=nn+1;
Final_R=[];

l=line(l1,1);
m=line(l1,2);
Z2=zeros(nn+1,nn+1);
Zml=-1/YY_bus(m,l);
Yl=Zml;
Yml=LL(m);
RF=0;
Z1(h,l)=Z0(h,l)-(Z0(h,l)*Z0(l,l))/(Z0(l,l)-1/Yl);
Z1(h,m)=Z0(h,l)-(Z0(h,l)*Z0(l,m))/(Z0(l,l)-1/Yl);
Z1(m,l)=Z0(m,l)-(Z0(m,l)*Z0(l,l))/(Z0(l,l)-1/Yl);
Z1(l,l)=Z0(l,l)-(Z0(l,l)*Z0(l,l))/(Z0(l,l)-1/Yl);
Z1(m,m)=Z0(m,l)-(Z0(m,l)*Z0(l,m))/(Z0(l,l)-1/Yl);

Z2(h,l)=Z1(h,l)-(Z1(h,m)-Z1(h,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(h,m)=Z1(h,m)-(Z1(h,m)-Z1(h,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(m,l)=Z1(m,l)-(Z1(m,m)-Z1(m,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(l,l)=Z1(l,l)-(Z1(l,m)-Z1(l,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(m,m)=Z1(m,m)-(Z1(m,m)-Z1(m,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);

Z4(h,l)=Z2(h,l)-(Z2(h,m)-Z2(h,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml);
Z4(h,p)=Z4(h,l);
Z4(p,l)=Z2(l,l)-(Z2(m,l)-Z2(l,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml);
Z4(p,p)=Z2(l,l)-(Z2(m,l)-Z2(l,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml);

Z5(h,l)=Z4(h,l)-(Z4(h,p)*Z4(p,l))/(Z4(p,p)+1/((1-D1)*Yl));
Z5(l,l)=Z4(l,l)-(Z4(l,p)*Z4(p,l))/(Z4(p,p)+1/((1-D1)*Yl));

Z6(h,l)=Z5(h,l)-(Z5(h,l)*Z5(l,l))/(Z5(l,l)+1/(D1*Yl));
%-----%

Z1(h,l)=Z0(h,l)-(Z0(h,l)*Z0(l,l))/(Z0(l,l)-1/Yl);
Z1(h,m)=Z0(h,l)-(Z0(h,l)*Z0(l,m))/(Z0(l,l)-1/Yl);
Z1(m,l)=Z0(m,l)-(Z0(m,l)*Z0(l,l))/(Z0(l,l)-1/Yl);
Z1(l,l)=Z0(l,l)-(Z0(l,l)*Z0(l,l))/(Z0(l,l)-1/Yl);
Z1(m,m)=Z0(m,l)-(Z0(m,l)*Z0(l,m))/(Z0(l,l)-1/Yl);

Z2(h,l)=Z1(h,l)-(Z1(h,m)-Z1(h,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(h,m)=Z1(h,m)-(Z1(h,m)-Z1(h,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(m,l)=Z1(m,l)-(Z1(m,m)-Z1(m,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);
Z2(l,l)=Z1(l,l)-(Z1(l,m)-Z1(l,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);

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$$Z2(m,m)=Z1(m,m)-(Z1(m,m)-Z1(m,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$\begin{aligned} Z4(h,l) &= Z2(h,l)-(Z2(h,m)-Z2(h,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml); \\ Z4(h,p) &= Z2(h,l)-(Z2(h,m)-Z2(h,l))*(Z2(m,l)-Z2(l,l)-RF)/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml); \\ Z4(p,l) &= Z2(l,l)-(Z2(m,l)-Z2(l,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml); \\ Z4(p,p) &= Z2(l,l)-(Z2(m,l)-Z2(l,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml); \end{aligned}$$

$$\begin{aligned} Z5(h,p) &= Z4(h,p)-(Z4(h,p)*Z4(p,p))/(Z4(p,p)+1/((1-D1)*Yl)); \\ Z5(p,l) &= Z4(p,l)-(Z4(p,p)*Z4(p,l))/(Z4(p,p)+1/((1-D1)*Yl)); \end{aligned}$$

$$Z6(h,p)=Z5(h,p)-(Z5(h,l)*Z5(p,l))/(Z5(l,l)+1/(D1*Yl));$$

$$E=(Z6(h,l)-Z6(h,p));$$

$$l=\text{line}(l2,1);$$

$$m=\text{line}(l2,2);$$

$$Z2=\text{zeros}(nn+1,nn+1);$$

$$Zml=-1/YY_bus(m,l);$$

$$Yl=Zml;$$

$$Yml=LL(m);$$

$$Z1(h,l)=Z0(h,l)-(Z0(h,l)*Z0(l,l))/(Z0(l,l)-1/Yl);$$

$$Z1(h,m)=Z0(h,l)-(Z0(h,l)*Z0(l,m))/(Z0(l,l)-1/Yl);$$

$$Z1(m,l)=Z0(m,l)-(Z0(m,l)*Z0(l,l))/(Z0(l,l)-1/Yl);$$

$$Z1(l,l)=Z0(l,l)-(Z0(l,l)*Z0(l,l))/(Z0(l,l)-1/Yl);$$

$$Z1(m,m)=Z0(m,l)-(Z0(m,l)*Z0(l,m))/(Z0(l,l)-1/Yl);$$

$$Z2(h,l)=Z1(h,l)-(Z1(h,m)-Z1(h,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$Z2(h,m)=Z1(h,m)-(Z1(h,m)-Z1(h,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$Z2(m,l)=Z1(m,l)-(Z1(m,m)-Z1(m,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$Z2(l,l)=Z1(l,l)-(Z1(l,m)-Z1(l,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$Z2(m,m)=Z1(m,m)-(Z1(m,m)-Z1(m,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$Z4(h,l)=Z2(h,l)-(Z2(h,m)-Z2(h,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml);$$

$$Z4(h,p)=Z4(h,l);$$

$$Z4(p,l)=Z2(l,l)-(Z2(m,l)-Z2(l,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml);$$

$$Z4(p,p)=Z2(l,l)-(Z2(m,l)-Z2(l,l))*(Z2(m,l)-Z2(l,l))/(Z2(l,l)+RF+Z2(m,m)-2*Z2(m,l)-1/Yml);$$

$$Z5(h,l)=Z4(h,l)-(Z4(h,p)*Z4(p,l))/(Z4(p,p)+1/((1-D2)*Yl));$$

$$Z5(l,l)=Z4(l,l)-(Z4(l,p)*Z4(p,l))/(Z4(p,p)+1/((1-D2)*Yl));$$

$$Z6(h,l)=Z5(h,l)-(Z5(h,l)*Z5(l,l))/(Z5(l,l)+1/(D2*Yl));$$

$$Z1(h,l)=Z0(h,l)-(Z0(h,l)*Z0(l,l))/(Z0(l,l)-1/Yl);$$

$$Z1(h,m)=Z0(h,l)-(Z0(h,l)*Z0(l,m))/(Z0(l,l)-1/Yl);$$

$$Z1(m,l)=Z0(m,l)-(Z0(m,l)*Z0(l,l))/(Z0(l,l)-1/Yl);$$

$$Z1(l,l)=Z0(l,l)-(Z0(l,l)*Z0(l,l))/(Z0(l,l)-1/Yl);$$

$$Z1(m,m)=Z0(m,l)-(Z0(m,l)*Z0(l,m))/(Z0(l,l)-1/Yl);$$

$$Z2(h,l)=Z1(h,l)-(Z1(h,m)-Z1(h,l))*(Z1(m,l)-Z1(l,l))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$Z2(h,m)=Z1(h,m)-(Z1(h,m)-Z1(h,l))*(Z1(m,m)-Z1(l,m))/(Z1(l,l)+Z1(m,m)-2*Z1(m,l)-1/Yml);$$

$$\begin{aligned} Z2(m,l) &= Z1(m,l) - (Z1(m,m) - Z1(m,l)) * (Z1(m,l) - Z1(l,l)) / (Z1(l,l) + Z1(m,m) - 2 * Z1(m,l) - 1/Yml); \\ Z2(l,l) &= Z1(l,l) - (Z1(l,m) - Z1(l,l)) * (Z1(m,l) - Z1(l,l)) / (Z1(l,l) + Z1(m,m) - 2 * Z1(m,l) - 1/Yml); \\ Z2(m,m) &= Z1(m,m) - (Z1(m,m) - Z1(m,l)) * (Z1(m,m) - Z1(l,m)) / (Z1(l,l) + Z1(m,m) - 2 * Z1(m,l) - 1/Yml); \end{aligned}$$

$$\begin{aligned} Z4(h,l) &= Z2(h,l) - (Z2(h,m) - Z2(h,l)) * (Z2(m,l) - Z2(l,l)) / (Z2(l,l) + RF + Z2(m,m) - 2 * Z2(m,l) - 1/Yml); \\ Z4(h,p) &= Z2(h,l) - (Z2(h,m) - Z2(h,l)) * (Z2(m,l) - Z2(l,l) - RF) / (Z2(l,l) + RF + Z2(m,m) - 2 * Z2(m,l) - 1/Yml); \\ Z4(p,l) &= Z2(l,l) - (Z2(m,l) - Z2(l,l)) * (Z2(m,l) - Z2(l,l)) / (Z2(l,l) + RF + Z2(m,m) - 2 * Z2(m,l) - 1/Yml); \\ Z4(p,p) &= Z2(l,l) - (Z2(m,l) - Z2(l,l)) * (Z2(m,l) - Z2(l,l)) / (Z2(l,l) + RF + Z2(m,m) - 2 * Z2(m,l) - 1/Yml); \end{aligned}$$

$$\begin{aligned} Z5(h,p) &= Z4(h,p) - (Z4(h,p) * Z4(p,p)) / (Z4(p,p) + 1 / ((1 - D2) * Yl)); \\ Z5(p,l) &= Z4(p,l) - (Z4(p,p) * Z4(p,l)) / (Z4(p,p) + 1 / ((1 - D2) * Yl)); \end{aligned}$$

$$Z6(h,p) = Z5(h,p) - (Z5(h,l) * Z5(p,l)) / (Z5(l,l) + 1 / (D2 * Yl));$$

$$Ep = (Z6(h,l) - Z6(h,p));$$

VITA

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