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Teaching Strategies for Proof Based Geometry

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TEACHING STRATEGIES FOR PROOF BASED GEOMETRY

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ABSTRACT

This study aims is to discover the best methods for geometry students to master proof writing. Students who are taught how to write proofs in a traditional setting find proofs to be very difficult - struggling throughout the school year writing proofs on their own. Studies have been conducted regarding the use of dynamic geometry software in proof writing. To further study the effects of proof writing using dynamic geometry software, forty-eight freshmen students enrolled in an honors geometry course at a high performing suburban high school in Louisiana were given several proofs to complete, along with self-reflection surveys. During phase one of this research, twenty-four students were allowed to use Geometer's Sketchpad (GSP) while writing their proofs, while the other twenty-four students were using only paper and pencil to explore the figure involved in the proof. During phase two of testing, the control and experimental groups swapped places to uphold the equality standards of the course. Student self-reflection surveys show that some students enjoy writing proofs when using GSP, while others are indifferent. Along with the student surveys, the present study is an analysis of student work from those who had access to GSP to improve proof writing skills.

CHAPTER 1: INTRODUCTION

Proof writing is considered very important in the high school geometry. However, proofs involve perseverance and abstract reasoning, which may be one of the reasons students struggle so much with proof writing. Students must recall a great deal of previously learned geometry theorems and postulates in order to complete proofs, along with deductively reason to successfully write the proof. Proofs are unlike most tasks in any other math course. The math problems that students are exposed to up to this level involve a very short number of steps. These problems usually contain computation and procedural skills. However, proof writing requires a great deal of persistence to complete a sequential list of arguments and justifications in order to reach a desired goal. Students who are exposed to proof writing for the first time often do not know what exactly is expected of them. It is up to the teacher to carefully navigate through the process of teaching students to confidently and correctly write proofs.

The Common Core State Standards implements eight mathematical practices for students to master throughout his/her school career. They are as follows:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Students should develop these math practices through the math courses taken at all levels. Seven of the eight mathematic practices directly involve problem solving with reasoning and proof, along with communication, representation, and connections. There has been a recent drive for students to build meaningful connections between several concepts learned in a course. Proof writing helps students to do this very thing.

This research explores the idea of using dynamic geometry software as a tool to teach proof writing effectively. Geometer's Sketchpad (GSP) is a dynamic software that allows students to manipulate figures to observe how angles and segments change as a result. Throughout this study, students were asked to work on several proof writing exercises. One group was allowed to explore the figure involved in GSP during the proof writing process, while the other group relied on their own illustrations of the figure. Upon completion of the proof writing exercise, the students answered a self-reflection survey regarding their confidence on specific areas of the proof writing task and the reason for their confidence.

This research addresses four specific objectives. First, what is the best practice in teaching proof writing and deductive reasoning? Secondly, is dynamic geometry software such as Geometer's Sketchpad (GSP) useful in the best methods for teaching proof writing? Thirdly, do students find proof writing thought-provoking and exciting while using GSP? Finally, is there any correlation between proof writing competency and proficiency in answering geometry multiple choice questions? Student performance on ACT geometry based questions was tracked throughout the school year. The multiple choice questions were geometry-based from the math portion of a previously administered ACT test. The purpose of this study is provide appropriate tasks and tools that can be utilized for students to master the skill of proof writing.

Chapter two outlines literature that connects proof writing to the use of technology, the current educational standards involving proof writing, and why proof writing is such a necessary skill for students to master. Chapter three addresses the setting of this research. Chapter four highlights the tasks, self-reflection surveys, and GSP exploration activities that were used in the research. Lastly, the research results are found in chapter five.

CHAPTER 2: LITERATURE REVIEW

2.1: Purpose of Proof Writing

Students are being asked to think critically and use logical sequencing skills more and more in today's schools. As the math curriculum changes to fit the need for a more diverse student and future member of the workforce, proof writing is at the forefront of this transformation. Members of the workforce must have the ability to understand and analyze problems that arise in any situation. Proof writing allows students to carefully study and practice these skills. When a student is able to argue a truth using a logical system of axioms, then he/she is most likely able to argue another truth within a different logical system of axioms. Students who master the skill of proof writing are able to compete globally in the future workforce.

2.2: Common Core State Standards

The third mathematical practice implemented by the Common Core State Standards requires students to "construct viable arguments and critique the reasoning of others." This practice involves making conjectures and being able to build a logical progression of statements in order to test the truth values of those statements. Proof writing falls under this practice. The fifth mathematical practice is "use appropriate tools strategically." The tools that high school students are expected to use effectively at the appropriate time include "pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software" (CCSS). This research directly addresses the need for tools such as dynamic geometry software, along with the use of a straight edge and protractor when using only pencil and paper to construct figures. The Common Core State Standards (CCSS) emphasize that dynamic geometry environments allow students to explore and model to investigate occurrences within plane geometry. The types of proofs students are expected to learn in their geometry course are theorems about lines, angles, triangles, and

parallelograms. Although the CCSS gives a list of *types* of proofs students should learn, it is not known which proof students will be assessed on at the end of the year. This research explores the idea of proof writing exercises that force the student to analyze what information is given through the eyes of GSP, giving the student the ability to manipulate the figure. This initial step of investigating the figure allows the student to problem solve and devise a game plan for the preceding proof writing exercise. When students jump straight into writing a proof before seeing what lies ahead to form a clear action plan, those students will most likely be unsuccessful with the assignment. Students might be inclined to make false assertions that are not based on any actual evidence.

2.3: Previous Research

There have been studies done across the world that support the idea that GSP improves geometry performance scores. However, there have also been several studies that support no significant difference when GSP is utilized. One such study overseen by Kamariah Abu Bakar involves Malaysian secondary school students from two different schools who traveled to a University to participate in a study involving GSP. The study only lasted for six hours for one day with 90 students. The control group received traditional teaching at the University, while the treatment group received a GSP introduction so they are familiar with the program's features. Then, they worked on several activities using GSP to explore geometry concepts. They were both given the same post test. The results found indicated that the post-test means were close (Bakar, 2008). This research shows that the time spent learning GSP and using it to write proofs is significant.

A second study in Bursa, Turkey involving forty-two seventh grade students using GSP was conducted by Kesan and Caliskan. There was a control and experimental group with

twenty-one students each. The treatment group was given worksheets and activities created by the researcher that would supplement GSP exploration on the geometric concepts including lines, angles, and triangles. These students discovered geometric relationships by drawing the figures and dragging vertices to change the features of the figures to make conjectures like a mathematician. The control group was in traditional styled classrooms. Both groups were given a geometry achievement test as a pre and post test. The Mann Whitney U test was used to analyze the data, which indicated that there was a significant difference in the experimental and control group performance scores. This study also considered the retention level to determine which method is more effective long term, which yielded higher scores in the experimental group (Kesan & Caliskan, 2013). It is clear from the literature that there is still some question in how help GSP can be while writing proofs.

A non-empirical study conducted by John Olive, tracks several activities that can easily be done in GSP that would otherwise be very difficult to draw tedious figures on paper. Olive noted that a triangle on a paper merely represents a static triangle, while a triangle constructed on GSP represents a prototype for all possible triangles. Prototypes can effortlessly be explored by students, which in turn allows those students to make generalized conjectures about geometric relationships (Olive, 1991). Giving the ability to manipulate a prototype of a figure allows the students to interact with the figure to be better acquainted with the necessary steps of the proof writing process.

Likewise, Zhonghong Jiang conducted research at Florida International University involving secondary school mathematics pre-service teachers. The driving force of this study comes from the notion that knowledge is not passively received from the instructor, but rather actively constructed by the student. Over the course of ten weeks, the control group was

provided with opportunities to work on carefully selected tasks by the researcher, answer thought provoking questions, all while exploring freely through the use of GSP (Jiang, 2001).

McGivney and DeFranco write about the importance of a teacher-student dialogue that guides students in such a way that allows the student to grow in knowledge. There is an art to questioning that provokes the student to critically think and analyze without revealing too much information. This kind of questioning allows the student to learn by discovering solutions themselves rather than being told what to do step by step. There is a fine line between holding the student's hand along the way and carefully guiding them towards the solution from a distance. Questioning in such a way can be found frustrating in a culture where students are used to receiving assistance at the first sign of struggle. The ability for students to logically reason through problems can be achieved through this questioning process. The Third Committee on Geometry, composed of twenty-six prominent teachers in the field of mathematics completed a questionnaire regarding the teaching of geometry. These teachers discussed the teaching in a traditional geometry course.

> "There is almost unanimous agreement that demonstrative geometry can be so taught that it will develop the power to reason logically more readily than other school subjects, and that the degree of transfer of this logical training to situations outside geometry is a fair measure of the efficacy of the instruction. However great the partisan bias in this expression of opinion, the question 'Do teachers of geometry ordinarily teach in such a way as to secure transfer of those methods, attitudes, and appreciations which are commonly said to be most easily transferable?' elicits an almost unanimous but sorrowful 'No.'"(McGivney & DeFranco)

Another study done by Yang and Lin regarding reading comprehension of proof writing mentions that several approaches are taken when teaching students proof writing. These include listening, speaking, writing, and doing. "Activities of doing proofs, like conjecturing and

proving, are designed to have students manipulate physical models of geometric figures, engage in visualization, and observe relationships between or within the attributes of figures" (Yang and Lin, 2007). However, this method of doing so by visualization does have its challenges. Students could confuse conjecturing for verifying during the proving process. Manipulating a figure and seeing that an angle in a triangle measures ninety degrees during several cases when working with the figure is still leaving room for error. The students must recognize that verifying several cases where the angle measures ninety degrees does not always imply that the angle always measures ninety degrees in every possible case. Students must also understand that visualization is a tool for forming a hypothesis, rather than serving as a proof itself.

Hargrave researched the best method to provide critical feedback to students writing proofs. "The feedback tools will be a proof writing checklist that defines exactly what goes into a geometric proof, and a consultation format in which students receive feedback on how best to adapt their writing" (Hargrave, 2013). Tools used in the classroom to help students successfully write proofs comes in many forms. While Hargrave used a checklist and consolation format, McAllister researched how mathematical writing exercises could improve proof writing. In her research, McAllister found that her students used accurate and appropriate mathematical vocabulary when completing their proof-writing assignments (McAllister, 2013). There has been extensive research dealing with teaching students how to write proofs. It is clear that students struggle when first introduced to proof writing and would benefit from additional help outside the traditional lecture style teaching.

The present study focused on the idea that students would use GSP to visualize the figure as it is being manipulated by the student, which would only jump start the proof writing process. The demonstration of all possible figures obtained by repeated motion is not a substitute for a

proof itself since a proof must be justified by accepted mathematical postulates and theorems. By writing a mathematical proof of statements, students learn that it stands the best of times, in contrast to data-driven conclusions.

CHAPTER 3: NATURE OF THE STUDY

3.1: Population and Setting

This study was conducted in a high performing suburban high school outside of Baton Rouge, Louisiana with approximately 1,500 students. The demographics of the student population is nearly 50% African American and 50% Caucasian with 38% receiving free or reducing lunch.

This study involves forty-eight freshmen enrolled in an honors geometry course. Two sections of the course were taught by the same teacher, all completing a full course of algebra honors during their eighth grade year. Furthermore, all students were assessed using the End of Course (EOC) test at the completion of taking the geometry course with a score of *Excellent* or *Good* (A or B), none earning a *Fair* or *Needs Improvement*.

3.2: Rationale

Proof writing has always been deemed very important in the geometry curriculum. Most recently, the Common Core Curriculum has made a significant push towards students learning to reason and problem solve that involves students linking several different concepts together to arrive at the solution or answer. The eight mathematical practices outlined by the common core address the importance for students to make connections throughout their math career, which includes reasoning and proof-writing.

> "The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections." (Common Core State Standards Initiative)

Most geometry courses cover proof writing as a major component of the coursework although it still causes students to be frustrated when they are not able to perform on assessments in proof writing. As with other learned math concepts, unending practice seems to be the answer for several classrooms. Even varying the nature of the proofs that are discussed and completed as practice, students still struggle when exposed to a new proof exercise for the first time. Research shows that students, when asked to complete a proof on their own, find that the biggest obstacle to overcome is knowing where to start. Several people view proofs as a specific genre of mathematics (Pimm and Wagner, 2003). Proof writing requires a great deal of mathematical expertise. Students must be able to use prior knowledge, understand what it is they are setting out to prove, and make connections between these two through the process of writing proofs. Once students are able to confidently make connections between what they have already learned and understand the purpose of theorems and postulates, then students will be able to complete proof writing exercises with ease.

Students should enjoy writing proofs. If students felt comfortable when writing proofs, then they would feel less frustrated when doing these exercises. This research set out to help students make connections between prior geometry knowledge and their ability to understand how to use that to justify each statement required of a geometry proof. In addition, this research documents the students' delight in using GSP as they complete proof writing exercises. GSP allows students that animate figures with simple clicking and dragging motions. This enables students to make connections between angles and segments that have been altered on the figure. The manipulation of the figures also brings light to how angles and side lengths may or may not remain the same when other angles are changed in the figure. For example, an inscribed angle on a circle will always measure ninety degrees if the diameter forms one side of the triangle,

regardless of where the angle is located on the circle. Students looking at this figure on a paper without the ability to animate or easily manipulate the angle's location while keeping everything else constant might find it difficult to conclude that the angle is always ninety degrees. The first object in Figure 3.1 shows what students would initially draw using paper and pencil. The second two objects in Figure 3.1 show an example of what a student could observe while using GSP to drag the inscribed angle along the circle. The student using GSP is more likely to see for him/herself that the angle remains ninety degrees.

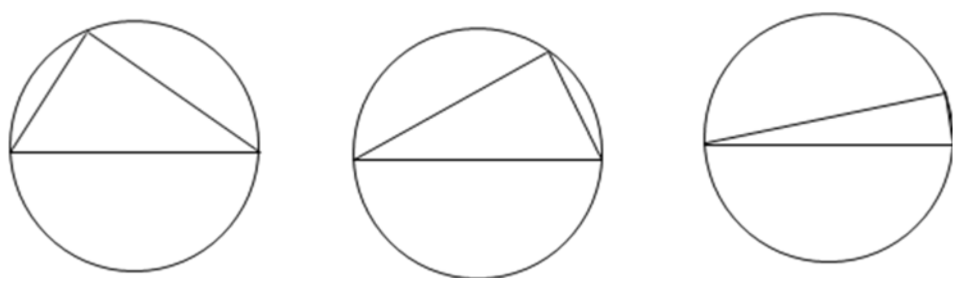


Figure 3.1 Inscribed Angle on a Circle

If students find difficulty in understanding the hypothesis they are setting out to prove, they will also find difficulty in writing a proof on that very same hypothesis. When using GSP, students are able to create the circle and inscribed angle that intercepts an arc measuring half the circle fairly easily. Those students that are using the software to see how the angle continues to measure ninety degrees as it is dragged along the circle are more likely to understand what it is they are trying to proof. In addition to understand their goal, students generally enjoy using the software as it provides an outlet for their creativity and geometry at the same time. There are several GSP activity workbooks, but very few set the stage for improved proof writing as an

effect of these activities. This research outlines several GSP activities that align directly to proof writing exercises.

3.3: Research Design

This research was designed in such a way that there is one group of students able to use GSP to complete several proof writing exercises, while the other group is allowed to use paper and pencil to explore the figures involved in the proof writing process. There were two phases of testing and data collection. In phase one of this research, the students using GSP were given time to become familiar with the software. They worked through three GSP activities several weeks before the first proof writing exercise was administered. This ensured that students who were using the software would be familiar with the features that would be helpful during the proof writing process. They were given instructions on how to create geometric figures such as segments, angles, circles, etc. and taught how to measure angles and segments.

For the first exploration using sketchpad, the students were instructed to create an angle bisector and a perpendicular bisector. Upon creating the complete figure, they were asked to observe properties of both the angle bisector and perpendicular bisector. Regarding the angle bisector properties, the students were asked to make a conjecture about any point on the angle bisector. Regarding the perpendicular bisector, the students were asked to make a conjecture about any point on the perpendicular bisector. Most students were able to determine that any point on the perpendicular bisector is equidistant to the two endpoints of the segment being bisected.

The second exploration using sketchpad involved properties of parallel lines and a transversal. The students were instructed to create a pair of parallel lines and transversal using the sketchpad construction tools. Once that was complete, they measured the angles between the

parallel lines and the transversal. Afterwards, they documented their observations of the mathematical relationships between corresponding angles, alternate interior angles, alternate exterior angles, and same-side interior angles. The students were able to notice that corresponding angles, alternate interior angles, and alternate exterior angles were congruent, while the same-side interior angles were supplementary fairly easily with the help of sketchpad.

The third exploration the students completed involved the triangle inequality theorem. Students were allowed to use sketchpad to explore the side length requirements for a triangle to be created. They observed the sum of the two smaller sides must be greater than the third side of the triangle in order for a triangle to be created. They also observed that the smallest angle is always across from the shortest side and the largest angle is always across from the longest side.

Upon completing the exploration activities to orient the students with the software of GSP, they were then given three proof writing exercises to complete. The group using GSP and the other group not using GSP were both given the same three proofs to ensure the validity of test results. Phase two of the testing took place approximately two months later. In phase two, the group using GSP was not able to use the software, and the group not using GSP in phase one were now allowed to use it in phase two. Each group received the same three proof writing exercises again, but different from the three proofs administered in phase one. All proofs from both phases were scored using a rubric that can be found in Appendix B. The actual scoring was completed by the teacher of the course.

Although this research intended to keep everything constant between the two groups in addition to phase one and phase two, it is important to note some limitations found in this research. First, phase one took place earlier in the school year, while phase two took place at the end of the school year. Throughout this school year, students worked on proof writing exercises

within the constraints of the research and design and within the curriculum used for a typical honors high school geometry course. It is possible for the scores to be skewed in phase two simply due to the time factor. Students were able to have more practice with writing proofs once phase two started, while phase one was in the middle of the school year. The second possible cause for skewed data in phase two is the lack of time available for the students to become familiar with the features and tools of GSP. In phase one, the students using GSP were given three opportunities to explore GSP through the activities provided by the teacher prior to the proof writing exercises. Phase two of the research did not allow for this to take place; however, the students were given an abbreviated version of GSP orientation.

CHAPTER 4: THE RESEARCH PROCESS

4.1: Phase One

During phase one, one group was presented with proof writing exercises that were more difficult than the daily proof writing exercises. These proofs can be found in Appendix B. Each proof was supplemented with helpful hints to guide the students towards the proof. All proofs included a common theme of necessary auxiliary lines. The first proof called for the construction of an auxiliary line segment to be the radius of a circle. Students were given Figure 4.1 and instructed to prove that angle ABD is always ninety degrees. Here, C denotes the center of the circle. Students constructed auxiliary segment BC in order to divide the triangle into two smaller triangles. Next, students labelled angles accordingly and were able to prove that angle ABD is always ninety degrees using algebraic properties.

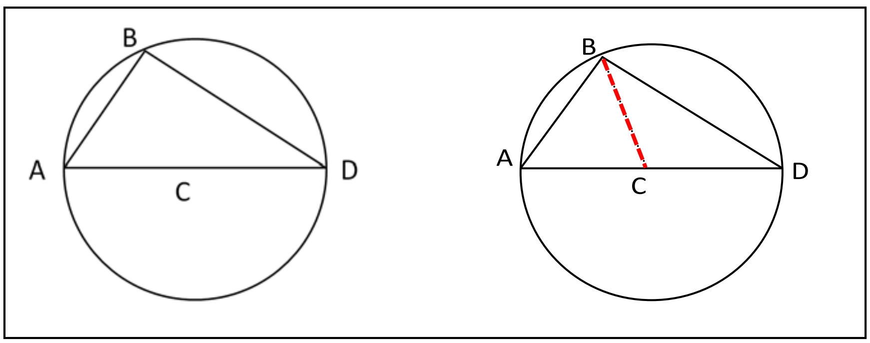


Figure 4.1: Figure from Proof #1 with and without auxiliary line drawn

The second proof needed an auxiliary line to be drawn to extend past the set of parallel lines, creating a transversal. Students were shown the picture on the left in Figure 4.2 and told to prove $x + y = z$. Students had the option of creating an auxiliary line that extends segment MP or segment LP. The picture on the right in Figure 4.2 shows one of these examples. Once this auxiliary line is drawn on their paper or constructed on the screen of GSP for those using the

dynamic software, students were then able to prove the conjecture that x + y = z. Some chose to use the exterior angle theorem at this point; however, most chose to use alternate interior angles and triangle sum theorem to prove the conjecture.

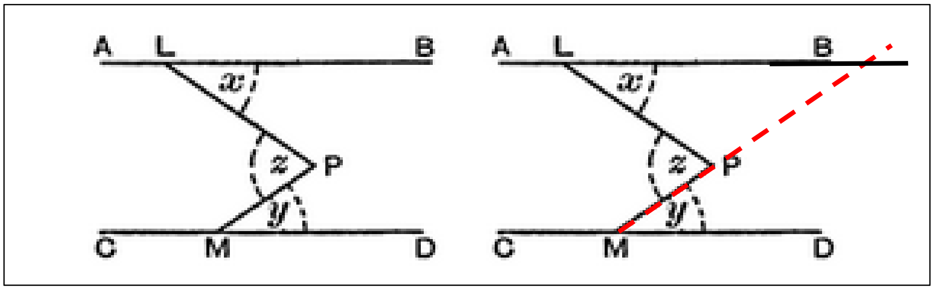


Figure 4.2: Figure from Proof #2 with and without auxiliary line drawn

The third proof was very similar to the second proof as it also involved transversals and parallel lines; however, it needed an auxiliary line to be constructed parallel to the other two lines and passing through the middle point. The figure given to the students is found on the left in Figure 4.3. Students were instructed to draw a parallel auxiliary line that passes through the vertex of angle C. The picture on the right in Figure 4.3 shows how students drew this line.

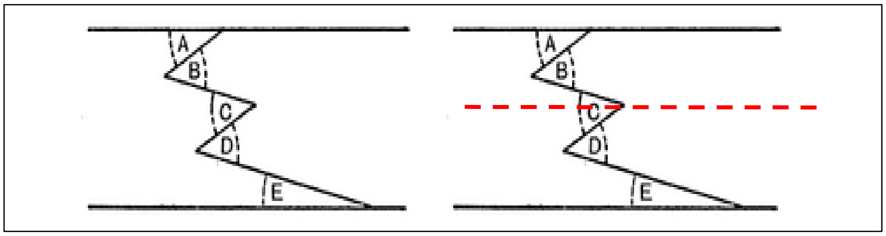


Figure 4.3: Figure from Proof #3 with and without auxiliary line drawn

Once the students were looking at this picture, it became very clear how it was related to proof #2. The purpose of constructing auxiliary lines become more clear to the students as they worked through these exercises, the control and experimental groups alike.

The students from both groups were originally paired with one partner to work on the proof writing exercise. As the students started understanding the proof and what additional constructions it called for, the pairs of students were allowed to converse with others on their thoughts. The group not allowed to use GSP was allowed to use the miniature dry erase boards to explore the figure's properties and communicate their arguments with the others in the group. Working in groups was familiar to these students because they had worked in groups several times in this geometry class; although, this proof writing group dynamic fostered a more enthusiastic approach. The group using GSP really enjoyed using the dragging feature to see all possible figures in regards to their proof, while the other group enjoyed using the miniature dry erase boards to communicate their thoughts about the proof to others in the group.

4.2: Phase Two

Phase two occurred two months after the conclusion of phase one. The groups were exchanged so the group not using GSP was now able to use it as the proof writing exercises took place, while those students able to use GSP were instructed to write their proofs without the use of GSP. Both groups were given proof writing exercises involving the need of auxiliary segments as well. These proofs may also be found in Appendix A. The first proof presented in this phase called for two auxiliary line segments to be drawn in such a way that they are the radii of a circle. They were asked to prove that OM is perpendicular to AB given that M is the midpoint of AB. Figure 4.4 shows the original figure presented to the students and what the figure looks like with the two radii drawn. Once the auxiliary segments were drawn, different

approaches were taken to prove that OM is perpendicular to AB. Some chose to use isosceles triangle properties to ultimately show that the two smaller triangles are congruent using the side-angle-side triangle congruence postulate. Others using the side-side-side triangle postulate to prove the two smaller triangles are congruent based on the reflexive property of segments and definition of radii of a circle.

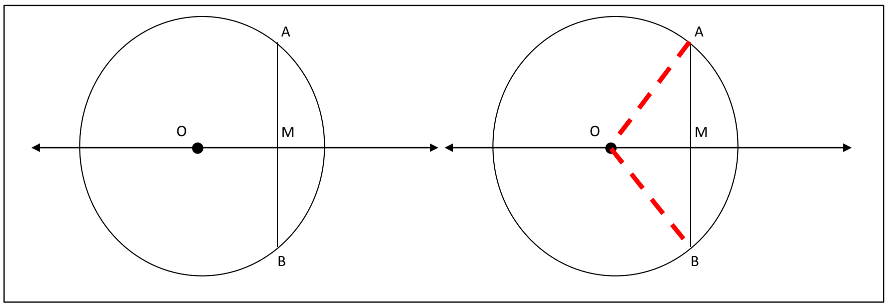


Figure 4.4: Figure from Proof #4 with and without auxiliary line drawn

In the second proof of this phase, the auxiliary segment was in the form of a perpendicular bisector. The students were given the first picture of Figure 4.5 and asked to prove that BD is the perpendicular bisector or AC. The construction of the auxiliary segment BD was much more obvious in this proof than with the other proofs because it was included in the hypothesis to be proven. Students then set out to prove that BD is the perpendicular bisector of AC by using congruent triangles.

The third proof required the drawing of an auxiliary segment parallel to a line segment in the figure in order to complete the proof. See Figure 4.6 for figure shown to students and figure with the parallel auxiliary segment constructed. Students were asked to prove $\frac{CL}{LF} = \frac{CK}{KD}$. In order to do this, similar triangle properties were explored within this figure. Students showed that

ΔCLK is similar to ΔKMD. Once the triangle similarity is proven and that segment LF is congruent to segment KM, then the ratio can be proven.

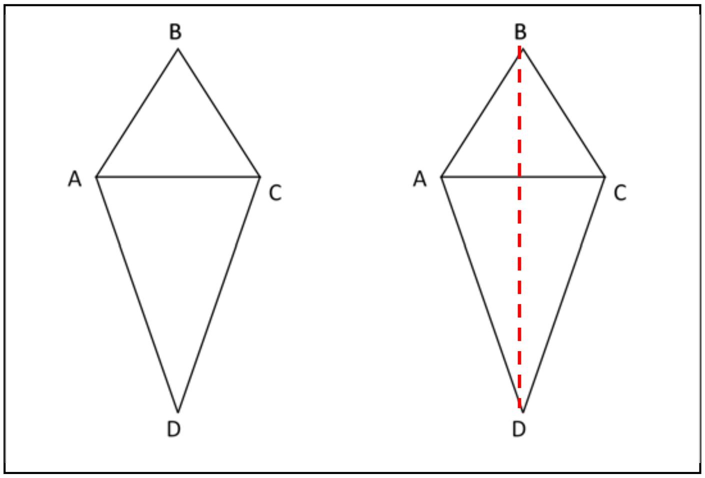


Figure 4.5: Figure from Proof #5 with and without auxiliary line drawn

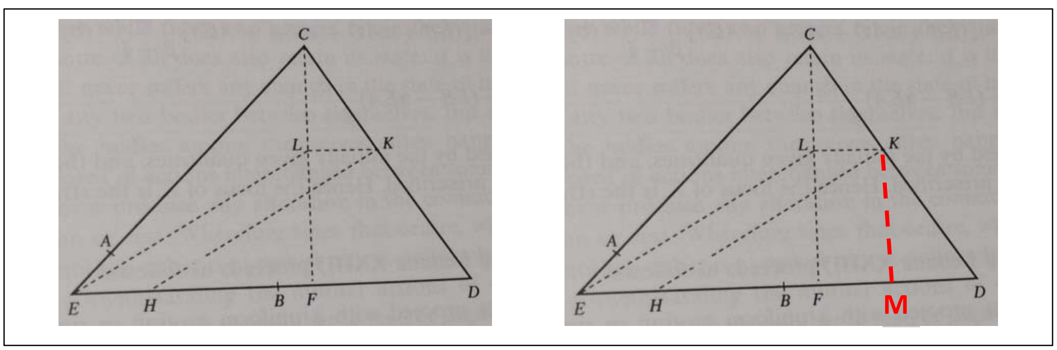


Figure 4.6: Figure from Proof #6 with and without auxiliary line drawn

Once again, the notion of auxiliary segments used in proofs became quite familiar to the students upon completing phase two of this research.

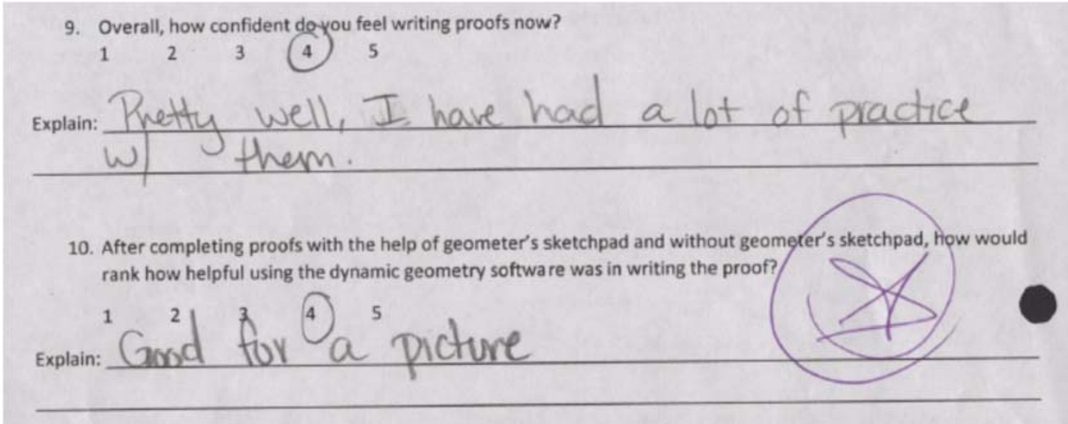
Similar to phase one, the students were paired up in phase two to get started on the proof writing exercises. Once each pair had a grasp of the figure being argued in the proofs, pairs were allowed to converse with other groups. The conversation fostered during this process was

thought provoking. The students were arguing like mathematicians; they would ask each other, "Why is that true?" or "How do you know it will always be that measurement?" Some of the students were observed telling others that they must include every step in the proof and there can be no statement that is skipped over.

4.3: Student Surveys

After each phase of this research, the students were given post proof reflections which consisted of questions about the completed proofs. These questions were Likert scale questions focusing on the student's understanding on what was given, what was being asked to prove, the theorems and postulates used in the proof, the proof writing process as a whole, along with several other questions. The post proof reflection can be found in Appendix C. Some of the students really enjoyed using GSP, while others found it a difficult to use. Figures 4.7 – 4.11 show student responses to the questions: "Overall, how confident do you feel writing proofs now?" and "After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would you rank how helpful using the dynamic geometry software was in writing the proof?"

Some students really enjoyed using GSP, while others found it difficult to use the tools it has to offer. Additional time to become familiar with the software could be an adjustment to this study. When asked how *helpful* the dynamic geometry software was when writing the proof, 45% of the students ranked three or higher on a one to five scale (1-lowest confidence, 5-highest confidence) and 48% of the students ranked three or higher when asked how much they *enjoyed* using the software.


9. Overall, how confident do you feel writing proofs now?

1 2 3 4 5

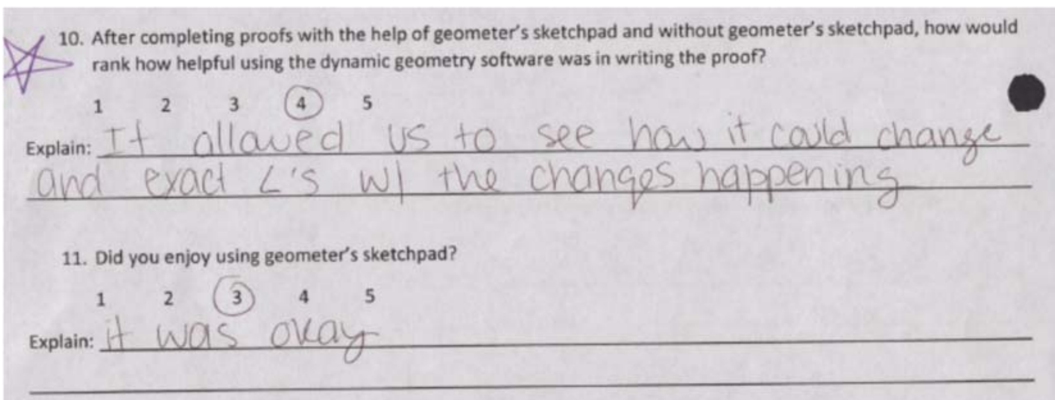
Explain: Pretty well, I have had a lot of practice w/ them.

10. After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would rank how helpful using the dynamic geometry software was in writing the proof?

1 2 3 4 5

Explain: Good for a picture

Figure 4.7: Student Response to Post Proof Reflection


10. After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would rank how helpful using the dynamic geometry software was in writing the proof?

1 2 3 4 5

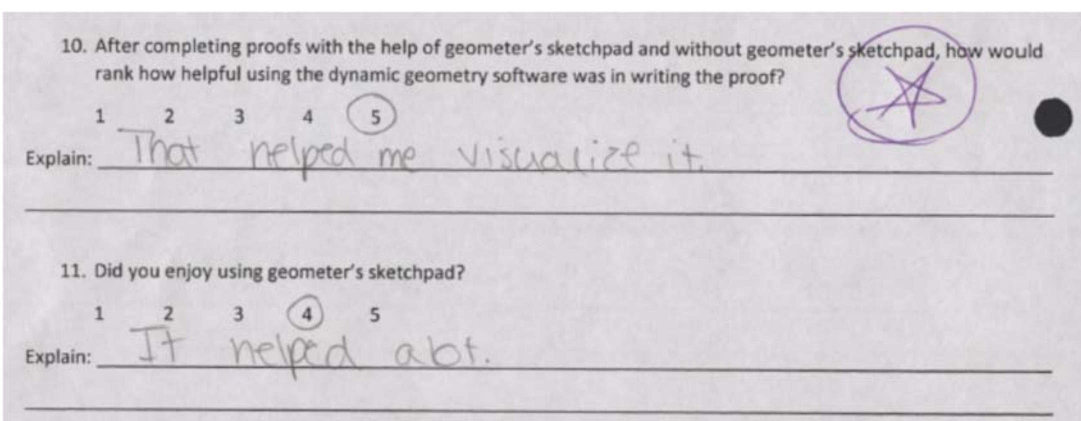
Explain: It allowed us to see how it could change and exact ∠'s w/ the changes happening

11. Did you enjoy using geometer's sketchpad?

1 2 3 4 5

Explain: it was okay

Figure 4.8: Student Response to Post Proof Reflection


10. After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would rank how helpful using the dynamic geometry software was in writing the proof?

1 2 3 4 5

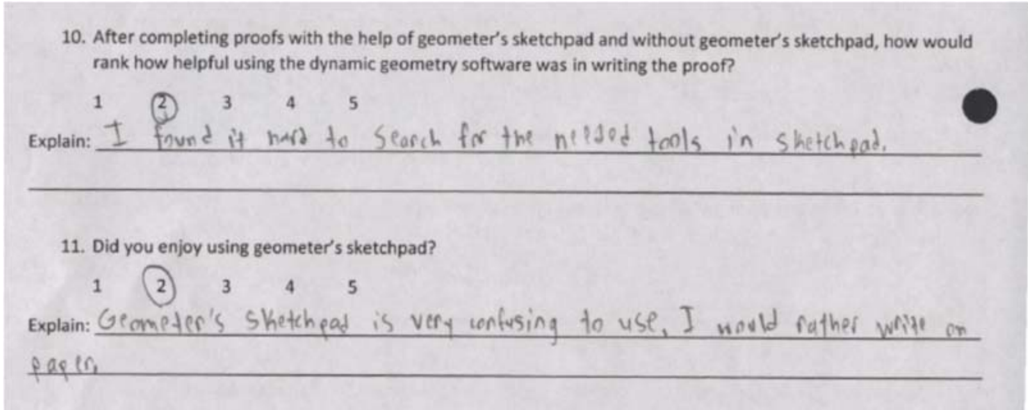
Explain: That helped me visualize it.

11. Did you enjoy using geometer's sketchpad?

1 2 3 4 5

Explain: It helped alot.

Figure 4.9: Student Response to Post Proof Reflection



10. After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would rank how helpful using the dynamic geometry software was in writing the proof?

1 2 3 4 5

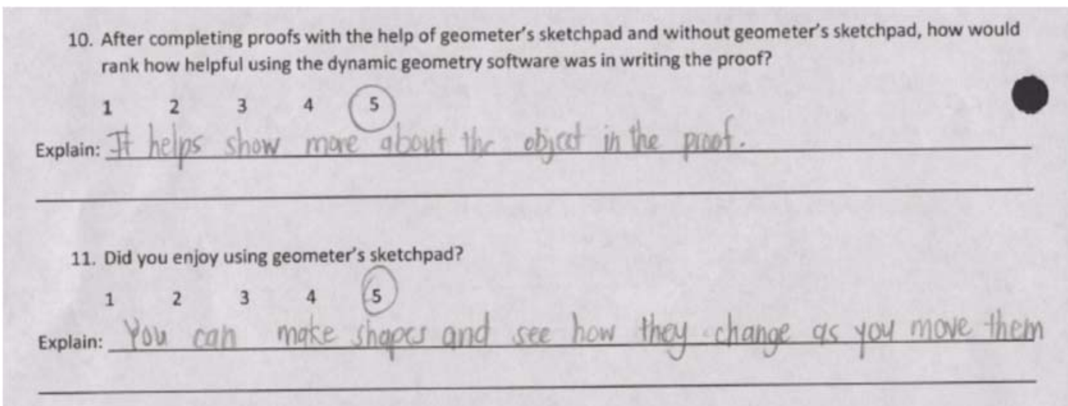
Explain: I found it hard to Search for the needed tools in Sketchpad.

11. Did you enjoy using geometer's sketchpad?

1 2 3 4 5

Explain: Geometer's Sketchpad is very confusing to use, I would rather write on paper.

Figure 4.10: Student Response to Post Proof Reflection



10. After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would rank how helpful using the dynamic geometry software was in writing the proof?

1 2 3 4 5

Explain: It helps show more about the object in the proof.

11. Did you enjoy using geometer's sketchpad?

1 2 3 4 5

Explain: You can make shapes and see how they change as you move them

Figure 4.11: Student Response to Post Proof Reflection

In phase one of testing, students using GSP to help in the proof writing process reported a high confidence. Both groups of students were asked to rate on a scale of one to five, "How confident were you in writing the 2-column proof statements?" The group using GSP averaged an answer of 3.5 and the group without GSP averaged an answer of 3.0. Students were also asked to rate on a scale of one to five if they were able to complete the entire proof with confidence. The group using GSP provide an average rate of 3.3 and the group not using GSP had an average answer of 2.7.

4.4: Performance Based Assessment

Teachers are encouraged to use performance based tasks as often as possible to help students think critically and demonstrate what they have learned. Upon completion of phase one and two of this research, a performance based assessment was administered. The performance based task involved proof writing exercises with a court room presentation style. The students were place into teams of three, each playing a vital role on a “legal team.” Each group was told if they were on the defending team or the prosecuting team. Both teams received the same proof writing exercise to plan for their court case. See Appendix D for proof writing exercise. The defense team presented their proof to the court, in a question answer format using “witnesses” from their team to explain how they came to the conclusion from the given statements of their particular proof. The prosecution team had the opportunity to ask the defense team about the statements and reasons they presented in their proof, in an effort to point out mistakes of the proof. If the prosecution team was able to point out mistakes in the defense’s proof, then they were able to successfully charge them with “proof writing fraud.” The students not presenting in the court at that moment were acting as the jury. The jury completed grading rubrics as the courtroom demonstrations took place so they were able to keep up with the validity of the statements and reasons in the presented proof. Those students not presenting their case were also able to easily stay engaged as the presentations took place.

After a full course of geometry and much practice with writing proofs, the students were able to enjoy presenting proofs in the courtroom setting. Students who originally struggled with writing proofs and not enjoying the proof process were able to excel in the courtroom presentations. Observations of team discussion as they prepared for the courtroom show that the students felt much more confident with writing proofs. Student dialogue included questions such

as, “Let’s see what we have given about the figure?” and “What theorems did we learn that can help us with this proof?” As students questioned their witnesses and debated in the courtroom, they used precise and accurate mathematical terminology, which had been quite difficult in the beginning of the course with most students.

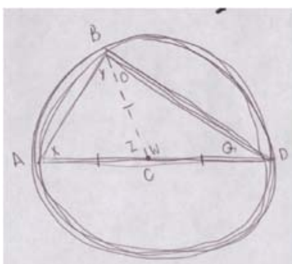
CHAPTER 5: FINDINGS

This research was done to determine if GSP is beneficial to geometry students learning to write proofs. This chapter will analyze the data collected regarding the proofs administered throughout phase one and phase two of this study. Additionally, pre- and post-test scores were collected to compare ability to proof write and perform on ACT type multiple choice questions. This chapter will also show and describe student work that was documented throughout this research.

5.1: Student Work from Phase One

Student work was collected throughout this research project. Student responses that are correct and incorrect will be explored in this section. The first student responses discussed are from proof #1. As mentioned before, the proof writing exercises for this research all require the construction of auxiliary lines. The first proof required the construction of an additional radius of the circle, forming two isosceles triangles in the circle. Figures 5.1 and 5.2 show student responses from the group not using GSP in phase one. Both of these proofs lack necessary steps for a complete proof. In particular, the last line in the two column proof of Figure 5.1 shows that the student has misconceptions about what a linear pair is. If this student was able to animate the figure in GSP, he/she would see that the angles Z and W are not always equal to each other. Figure 5.2 shows work from a student that did not include a picture at all with proof. Student responses from those students using GSP during the proof writing process can be found in Figures 5.3 and 5.4. The student providing the response in Figure 5.3 is much more knowledgeable about the proof than those students providing responses in Figures 5.1 and 5.2. The student who wrote the proof in Figure 5.3 utilized GSP during the proof writing exercise. This student was able to drag point B along the circle to observe how the inscribed angle seems

to always measure ninety degrees. This act of figure manipulation allows the student to explore the figure prior to starting the proof writing process. Another student in the group using GSP takes a different approach as seen in Figure 5.4. This student chooses to use two different variables rather than expressing each angle measurement in terms of one variable. Figure 5.4 shows that the student noticed two isosceles triangles inside of the circle. He/she was able to use GSP to see as point B is moved along the circle, the two triangles formed are still isosceles regardless of the location of point B.



TWO COLUMN PROOF

Given: A circle with center C and diameter AD, where points A, B, & D lie on the circle.

Prove: ∡ABD is always a right angle.

Statements:	Reasons:
C is center of circle	Given
AD is the diameter	Given
A B D lie on the circle	Given
Segments AC BC DC are congruent	Def. of radius
Angle Z is congruent to angle W	Def. Linear Pair

PARAGRAPH EXPLANATION

How will you prove the conjecture that angle B is always a right angle if AD is the diamater of the circle and point B lies on the circle with center C?

Type your resonse below:

It is given that the circles center is C and its diameter is AD and points A B D lie on the circle. Prove that ABD is always a right angle. First you draw a line connecting B C which creates a radius of the circle. When you do that you create two triangles within the bigger one, they are both isosceles. Then you notice that lines AC, BC, and DC are all congruent. Then you label all of the angles

Figure 5.1: Student response to proof #1

TWO COLUMN PROOF

Given: a circle with center C and AC

Prove: angle ABD is always a right angle where points A,B, and D lie on the circle with AD being the diameter

Statements:	Reasons:
• Draw an auxiliary line from point B to C	• Midpoint
• Angles AC,DC, and BC are all congruent	• Definition of isosceles triangle

PARAGRAPH EXPLANATION

How will you prove the conjecture that angle B is always a right angle if AD is the diamater of the circle and point B lies on the circle with center C?

Type your resonse below:

It is given that the circles center is C and its diameter is AD and points A B D lie on the circle. Prove that ABD is always a right angle. First you draw a line connecting B C which creates a radius of the circle. When you do that you create two triangles within the bigger one, they are both isosceles. Then you notice that lines AC, BC, and DC are all congruent. Then you label all of the angles

Figure 5.2: Student response to proof #1

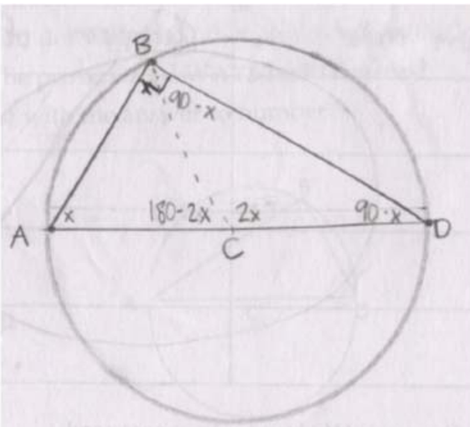
The second proof writing exercise the students worked on involved one pair of parallel lines and angle measurements. The auxiliary line needed for this proof is used to extend one of the line segments in order to form a transversal. Once this transversal is created, the students could then take two approaches to complete the proof. The student response found in Figure 5.5 is from a student in the group using GSP. He/she was able to draw the needed auxiliary line using the construction tools that the software provides. As the student manipulates the figure in GSP, he/she is also able to see what happens to the figure as angle Z is changed. As angle Z changes, the angles that are formed with the transversal are also changed. A student with the ability to witness the figure change as certain angles and lines are changed might be better equipped to start the proof writing process. As seen in Figure 5.6, students not able to use GSP

PARAGRAPH EXPLANATION

How will you prove the conjecture that angle B is always a right angle if AD is the diamater of the circle and point B lies on the circle with center C?

Type your resonse below:

First construct an auxillary line from point B to point C. This shows a radius on the circle that is equal to $\overline{AC}$ and $\overline{CD}$. We know this because C is the center of the circle and all radii are equal. Now we can tell that the two triangles formed by the auxillary line are isosceles b/c in an isosceles triangle, two sides have to be equal - which they are. Two bases of each isosceles triangle will be equal so $m\angle ABC = m\angle A$ and $m\angle D = m\angle CBD$. We can label $\angle ABC$ and $\angle A$ as x since they are equal. Then, using the triangle sum theorem, we can figure out that the $m\angle ACB = 180 - 2x$. Knowing that $\angle ACB$ is a linear pair with $\angle BCD$, we can figure out that $m\angle BCD = 2x$. To figure out the other sides measures, we use the triangle sum theorem again and see that $m\angle D$ and $m\angle CBD = 90 - x$ since they are equal. Now to prove that $\angle B$ is a right angle, we can add the two measures that make up $\angle B \rightarrow x + 90 - x$, and always get 90° which is the measurement of a right triangle!



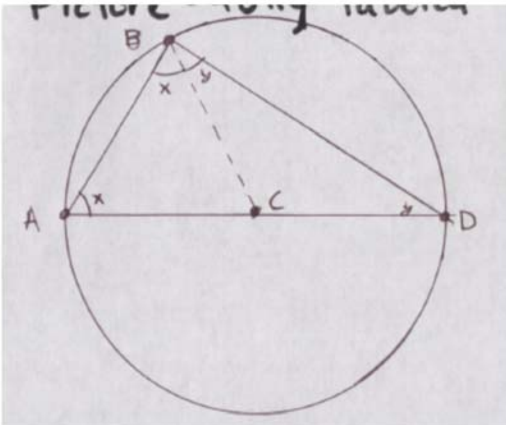
TWO COLUMN PROOF

Given: Center = C Radius = $\overline{AC}$

Prove: $\angle ABD$ is a right angle

Statements:	Reasons:
Center = C Radius = $\overline{AC}$	given
$\overline{AC}, \overline{BC}, \overline{AD}$ are congruent	definition of radius of a circle
Triangle BAC + Triangle BCD are isosceles	definition of isosceles triangle
$m\angle ABC = m\angle A$	bases of isoscelec triangles
$m\angle ACB = 180 - 2x$	triangle sum theorem
$m\angle BCD = 2x$	definition of linear pair
$m\angle D = m\angle CBD$	bases of isosceles triangles
$m\angle D = 90 - x$ $m\angle CBD = 90 - x$	triangle sum theorem
$m\angle ABC + m\angle CBD = \angle B$	angle addition postulate
$x + 90 - x = 90°$	substitution property of equality
$m\angle B = 90°$	transitive property of equality
$m\angle B$ is a right angle	definition of a right angle

Figure 5.3: Student response to proof #1



PARAGRAPH EXPLANATION

How will you prove the conjecture that angle B is always a right angle if AD is the diameter of the circle and point B lies on the circle with center C?

Type your response below:

In order to prove that <B is always a right angle, an auxiliary line must be drawn from Point B to Point C. The triangle is now divided into two isosceles triangles by segment BC. <BAC and <ABC are congruent, because they are part of an isosceles triangle. <CBD and <CDB are also congruent because they are part of an isosceles triangle. In terms of x, $x + (x + y) + y = 180$. ($(x + y)$ is <B) When you combine like terms (substitution) you get $2x + 2y = 180$. Using the Division Property of Equality, you get $x + y = 90$.

TWO COLUMN PROOF

a. **Given:**
   i. a circle with center C and diameter AD,
   ii. points A, B, & D lie on the circle,

b. **Prove:**
   i. ∡ABD is always a right angle

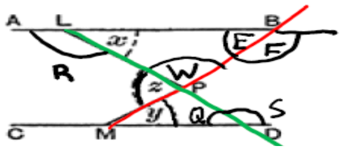
Statements:	Reasons:
<BAC ≅ <ABC	Definition of Isosceles Triangle
<CBD ≅ <CDB	Definition of Isosceles Triangle
<BAC = x	Substitution
<ABC = x	Substitution
<CBD = y	Substitution
<CDB = y	Substitution
$x + (x + y) + y = 180$	Triangle Sum Theorem
$x + x + y + y = 180$	Triangle Sum Theorem
$2x + 2y = 180$	Combine Like Terms
$\frac{2x + 2y = 180}{2}$	Division Property of Equality
$x + y = 90$	Simplification

Figure 5.4: Student response to proof #1

would not be permitted the freedom to manipulate the figure using dynamic geometry software. Although the student response in Figure 5.6 shows the extension of a segment to create a transversal, his/her thought process is unclear. The proof is incomplete and does not have a discernable path from the given information to the final conclusion. It is possible that this student would have been able to complete the proof writing process if able to utilize the figure manipulation tools in GSP.

Given: AB and CD are two parallel straight lines. Point L lies somewhere on AB and point M lies somewhere on CD.

Prove: x + y = z



Statements: **Reasons:**

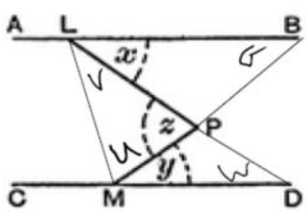
AB and CD are two parallel lines	Given
<Z + <W = 180 degrees	Definition of linear pair
<E = <Y	Definition of alternate interior angles
<X = <Q	Definition of Alternate interior angles
<X + <W + <E =180 degrees	Triangle sum theorem
<X + <W + <Y= 180 degrees	Substitution postulate
<X+<W+<Y=<Z+<W	Substitution postulate
<X+<Y=<Z	Subtraction property of quality

In order to prove if x + y = z, then we must first better label our diagram. I added two auxiliary lines in order to better understand the angles. Angles R, W, E, F, Q, and S were added along with these. First, we recognized that lines AB and CD are parallel lines as that was our given. Next we found that angles z and w both equal 180 degrees. This will be important later on in the proof. We knew angles e and y were equal due to the alternate interior angle definition. Angles x, y, and e equal 180 degrees which is due to the triangle sum theorem. We then use the substitution postulate to substitute angle e for y (because they equal), which means angles x, w, and y equal 180 degrees. Since we also knew angle z and w equal 180 degrees, we substituted 180 degrees for angles z and w, giving us the equation <x + <w + <y = <z + <w. We then use the subtraction property of equality and subtract angle w from the equation, thus giving us our answer, <x + <y = <z.

Figure 5.5: Student response for proof #2

Given: AB and CD are two parallel straight lines. Point L lies somewhere on AB and point M lies somewhere on CD.

Prove: x + y = z



Statements: **Reasons:**

AB and CD are two parallel straight lines. Point L lies somewhere on AB and point M lies somewhere on CD.	Given
<Y is congruent to <1	Alternate Interior Angles Theorem
<1 + <X = <2	Exterior Angles Theorem
<X + <Y = <Z	Substitution

Your given is always first and then when you look at the model angle Y is congruent to angle 1 which makes them alternate interior angles. Angle 1 plus angle X is equal to angle 2 which are then exterior angles. This means that angle X plus angle Y is equal to angle Z which completes the proof and proves the statement.

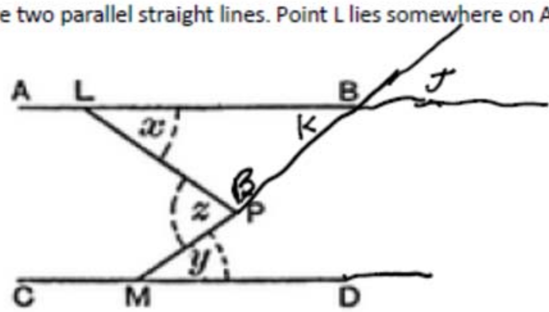
Figure 5.6: Student response for proof #2

Some of the students took the approach to prove the hypothesis x + y = z using the exterior angle theorem. It should be noted that six students in the group using GSP and six students in the group not using GSP all used the exterior angle theorem to complete their proof. Figure 5.7 shows one example of this. Since there were the same amount of students in each group using the exterior angles theorem, it is hard to say if GSP had a role in the particular thought process involving the exterior angle theorem. It is possible that some students were able to recall the exterior angles theorem regardless of using GSP.

The third proof exercise directly follows from the second proof writing activity. Similar to the second proof, the third proof involves relationships between parallel lines and angles. The figure in the third proof had several segments in the form of a zig-zag pattern between the two parallel lines. The auxiliary line needed for this figure is an additional parallel line constructed in a way that the auxiliary line is between the two given parallel lines and intersects the vertex of angle C. Figure 5.8 shows student work from a student who used GSP while writing the proof. The proof found in Figure 5.8 is complete showing a clear and concise path from the hypothesis to the conclusion of the given proof. Comparing Figure 5.8 with Figure 5.9, the student work in Figure 5.9 shows signs of mistakes regarding the angle bisector reason. It is important to note that the student work found in Figure 5.9 is from a student who was only using paper and pencil to complete the proof. The auxiliary parallel line constructed does not necessarily cut angle C in half. Students using GSP to manipulate the figure would easily see that this is true. As seen in Appendix A, students are able to drag angle C along the auxiliary line in such a way that it is not being bisected by the auxiliary line. The student work found in Figure 5.8 supports the idea that students using GSP are able to clearly understand that angle C is cut into two parts that are not necessarily equal parts.

Given: AB and CD are two parallel straight lines. Point L lies somewhere on AB and point K lies somewhere on CD.

Prove: x + y = z



Statements: **Reasons:**

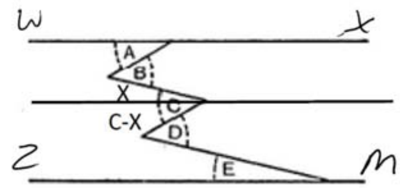
Statements	Reasons
Angle y= angle J	Corresponding angles theorem
Angle j= angle k	Vertical angles theorem
Angle x + k= z	Exterior Angles theorem
Angle x + y= z	Substitution Property of Equality

First I extended the lines of the transversals, then labeled the angles created by extending the lines through the parallel lines. Next, I said that angle y is equal to angle j because of the corresponding angles theorem. Then I said angle j is equal to angle k because of the vertical angles theorem. Then angle x + k = z because of the exterior angles theorem. Lastly, angle x + y = z because of the substitution property substituting the k for y.

Figure 5.7: Student response for proof #2

Given: Figure similar to 1st proof with parallel lines

Prove: a + c + e = b + d



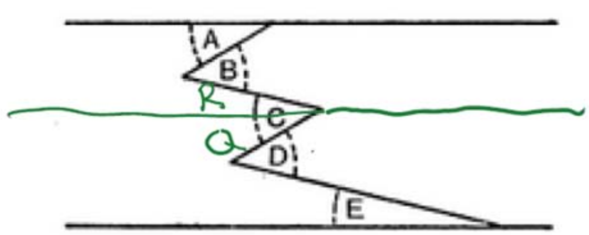
Statements:	Reasons:
Segment WX is parallel to segment ZM	Given
A + X = B	Previous Proof
C-X + E = D	Previous Proof
X + C-X = C	Angle Addition Postulate
A + X + C-X + E = B + D	Adding Equivalent Expressions together
A + C + E = B + D	Subtraction

Segment WX is parallel to segment ZM. You add a line that goes through point C and is parallel to segment WX and segment ZM. Angle C is divided into two angles: X and C-X. A + X = B according to the previous proof. C-X + E = D according to the previous proof. X + C-X = C according to the angle addition postulate. Then, you add A + X + C-X + E = B + D because you are adding equivalent expressions together. Lastly, you come to the conclusion that A + C + E = B + D because of subtraction.

Figure 5.8: Student response for proof #3

Given: Figure similar to 1st proof with parallel lines

Prove: a + c + e = b + d



Statements:	Reasons:
R=Q and R+Q=C	Definition of angle bisector
A+R=B	Previous proof
Q+E=D	Previous Proof
A+R+Q+E=B+D	Transitive property of equality
A+C+E=B+D	Substitution

Construct a parallel line that bisects the angle C. Name the new angles R and Q. From the previous proof we know that if you add angle A and R it will equal angle B. If you add Q and E it will equal D as well. Since A and R equal B and Q and E equal D you can say A plus R plus Q plus E equals B plus D because of the transitive property of equality. You can substitute R and Q for C. The final answer is A plus C plus E equals B plus D.

Figure 5.9: Student response for proof #3

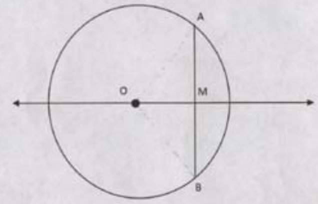
5.2: Student Work from Phase Two

Phase two occurred approximately two months after the completion of phase one. The groups were exchanged allowing for the group without GSP to explore through the dynamic software while proof writing and taking away that ability from the group that was able to use it in phase one. As in phase one, there were three proofs administered to both groups during phase two. It is also worth mentioning that phase two is taking place towards the end of the course. Although the groups were switched moving from phase one to phase two, both groups are practicing proof writing on a regular basis throughout the geometry course to uphold the education requirements enforced by the common core state standards. The group not using GSP in phase two seems to have excelled in proof writing more than the group not using GSP in phase one. This could be attributed to the timeline of the course and the fact that all students practiced traditional proof writing throughout the course.

The first proof given during phase two, proof #4, can be found in Figure 5.10 and 5.11. The student response found in Figure 5.10 is a sample from the group not using GSP and the student response found in Figure 5.11 is a sample from the group able to construct and manipulate the figure while completing the proof. The sample in Figure 5.10 shows only a few mistakes. The second line of the two column proof states that OA is congruent to OB based on the definition of midpoint. This student was not able to use GSP and therefore not able to see how both OA and OB are radii of the circle and will always be the same length regardless of the circle size. The student whose response is in Figure 5.11 includes a very clearly labeled figure. It is important to note that this student explored the figure in GSP in order to observe how the segments and angles relate to one another as the circle is changing sizes on the computer screen.

Given: In a circle O, M is the midpoint a chord AB.

Prove: OM is perpendicular to AB (***Hint***: Draw auxiliary segments OA and OB)



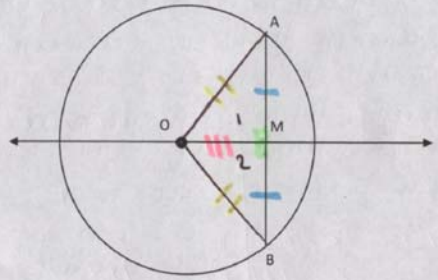
Statements:	**Reasons:**
M is the midpoint of chord AB	Given
AM is congruent to MB	Def. of midpt
$OA \cong OB$	Def. of midpt
OM is a bisector	Def. of bisector
$OM \cong OM$	Reflexive Property
$\triangle OAM \cong \triangle OBM$	SSS
$\angle 1 + \angle 2 = 180$	Def of Linear Pair
$\angle 1 \cong \angle 2$	CPCTC
$\angle 1 = 90°; \angle 2 = 90°$	Substitution
OM is perpend	Def. of perpendicular bisector

Given that in circle O, M is the midpoint of AB, it will be proven that OM is perpendicular to AB. AM is congruent to BM, because of the definition of midpoint. Also, segments OA and OB are congruent b/c of midpoint. OM is a bisector, b/c of the definition of bisectors. Then, segment OM is congruent to MO, b/c of the reflexive property. Therefore, triangle OAM is congruent to OBM, b/c of the Side-Side-Side postulate. So, angles 1+2 = 180, b/c of the definition of a linear pair. Angle 1 is congruent to Angle 2, because Corresponding parts of congruent triangles are congruent. Substitute and it is discovered that angles 1 and 2 are 90°. Therefore, OM is perpendicular to AB.

Figure 5.10: Student response for proof #4

Given: In a circle O, M is the midpoint of a chord AB.

Prove: OM is perpendicular to AB (*Hint*: Draw auxiliary segments OA and OB)



Statements:	**Reasons:**
M is the midpoint of chord AB	given
$\overline{AM} \cong \overline{BM}$	definition of midpoint
$\overline{OA} \cong \overline{OB}$	radii of same circle are congruent
$\overline{OM} \cong \overline{OM}$	reflexive property
$\triangle OAM \cong \triangle OBM$	side side side
$\angle 1 \cong \angle 2$	CPCTC
$m\angle 1 + m\angle 2 = 180°$	def. of linear pairs
$m\angle 1 + m\angle 1 = 180$	substitution property of equality
$2(m\angle 1) = 180$	Combining like terms
$m\angle 1 = 90$	division property of equality
$\overline{OM}$ is perpendicular to $\overline{AB}$	def. of a perpendicular line

This is where you will type your ...

In this proof, it is given that point M is the midpoint of chord AB. In order to prove that OM is perpendicular to chord AB, auxillary lines $\overline{AO}$ and $\overline{BO}$ must be drawn. These auxillarly lines are congruent because radii of the same circle are congruent. To add, line segment OM is congruent to itself because of the reflexive property. Line segment AM is also congruent to line segment BM because of the definition of a midpoint. Therefore, triangle OAM is congruent to triangle OBM because they have three congruent sides. Because the two triangles are congruent, angle one is congruent to angle two because of CPCTC. Then, the angles add up to 180° because they are a linear pair. Next, an equation can be written substituting angle two with angle one, and then combining like terms. Finally, by dividing both sides by two, angle one equals 90°. Beause of the definition of perpendicular lines, OM is perpendicular to AB.

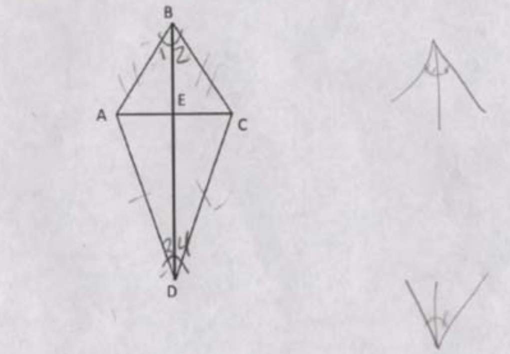
Figure 5.11: Student response for proof #4

The second proof administered in phase two, proof #5 can be found in Figures 5.12 and 5.13. Figure 5.12 shows work from a student not using GSP and Figure 5.13 shows work from a student using GSP during the proof writing process. Notice the comparison between the two figures in regard to the labelling alone. More specifically, the writing response in Figure 5.13 is missing a few steps in the two column proof that are necessary to state the two smaller triangles are congruent in the middle of the proof.

The third and final proof administered in phase two is proof #6. This proof writing exercise is based on Newton's theory of traveling objects. There was a story to tell which offered background knowledge on where the figure came from. Setting the scene for students in an interesting and thought provoking manner invites the student to become instantly engaged in the proof writing discussion, which then translates to the proof writing process itself. Even with a background story, it is unclear to the reader what the student is thinking in the sample response in Figure 5.14. This is a sample from a student not using GSP. The sample work in Figure 5.15a and 5.15b is an example of a student writing a proof using GSP. As Figure 5.14 shows a path with missing steps leading the hypothesis to the conclusion of the prove line, Figure 5.15a and 5.15b shows a path that is quite clear. The construction of the auxiliary line is mentioned in this proof but not mentioned once in the proof from Figure 5.14, although it is drawn on the figure.

Given: Polygon ABCD in which BD bisects <ABC and <ADC.

Prove: BD is the perpendicular bisector of AC. (Hint: What does a perpendicular bisector do?)



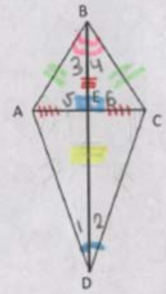
Statements:	Reasons:
BD bisects ∠ABC and ∠ADC	Given
∠1 ≅ ∠2	Def. of bisector
∠3 ≅ ∠4	Def. of bisector
$\overline{BD} \cong \overline{BD}$	Reflexive Property
△ABD ≅ CBD	ASA
$\overline{AB} \cong \overline{BC}$	CPCTC
△ABE ≅ △CBE	SAS
$\overline{AD} \cong \overline{CD}$	CPCTC
△ADE ≅ △CDE	SAS
BD is perpendicular bisector of AC	Properties of a Kite

We are given that BD bisects angle ABC and angle ADC. Since the angles are bisected, we know that ∠1 is congruent to ∠2. Not only that, but we also know that ∠3 is congruent to ∠4. BD is congruent to itself because of the reflexive property. Because of the ASA property, we know that △ABD is congruent to △CBD. From that congruent statement, we know that $\overline{AB}$ is congruent to $\overline{CB}$ because of CPCTC. We also know that △ABE is congruent to △CBE because of SAS. Another conclusion we can draw from the first triangle congruence statement is that $\overline{AD}$ is congruent to $\overline{CD}$. Now we know that △ADE is congruent to △CDE because of SAS. Since we have proved most of the properties of a kite, we can conclude that BD is the perpendicular bisector of $\overline{AC}$.

Figure 5.12: Student response for proof #5

Given: Polygon ABCD in which BD bisects <ABC and <ADC.

Prove: BD is the perpendicular bisector of AC. (Hint: What does a perpendicular bisector do?)

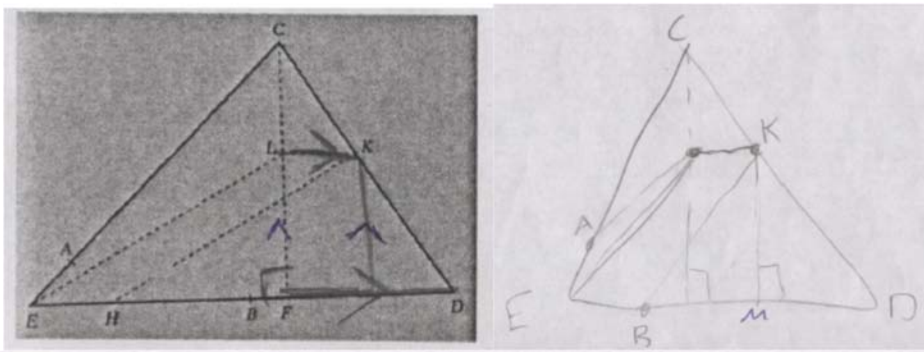


Statements:	**Reasons:**
$\overline{BD}$ bisects ∠ABC and ∠ADC	Given
∠1≅∠2 + ∠3≅∠4	def. of a bisector
$\overline{BD}$ ≅ $\overline{BD}$	reflexive property
△BCD ≅ △BAD	Angle-side-Angle
$\overline{AB}$ ≅ $\overline{CB}$	CPCTC
$\overline{BE}$ ≅ $\overline{BE}$	reflexive property
△ABE ≅ △CBE	side-Angle-side
∠5 ≅ ∠6	CPCTC
m∠5 + m∠6 = 180	def. of linear pair
m∠5 + m∠5 = 180	substitution
2(m∠5) = 180	Combining like terms
m∠5 = 90	division P.O.E
$\overline{AE}$ ≅ $\overline{CE}$	CPCTC
$\overline{BD}$ is perpendicular bisector of $\overline{AC}$	def. of perpendicular bisector

This is where you will type your paragraph proof

In this proof, it is given that segment BD bisects the polygon at angles ABC and ADC. From the definition of a bisector, the angles labeled 1 and 2 are congruent and angle 3 is congruent to angle 4. Because of the reflexive property, segment BD is congruent to itself. Triangle BCD is congruent to triangle BAD because of angle-side-angle. Then, AB≅CB because of CPCTC, and BE≅BE because of the reflexive property. Next, triangle ABE is congruent to triangle CBE because of side-angle-side. Now, it is clear that angle 5 is congruent to angle 6 because of CPCTC. Angle 5 and angle 6's measures added together equals 180 degrees. In order to determine the measure of angles 5 and 6, the substitution property of equality must be used. Then, the terms need to be combined in order to get the equation 2(m∠5)=180. In order to find that angle 5 equals 90 degrees, both sides of the equation needs to be divided by two. In addition, because of CPCTC segment AE is congruent to CE. Lastly, segment BD is a perpendicular bisector of segment AC because of the definition of a perpendicular bisector.

Figure 5.13: Student response for proof #5

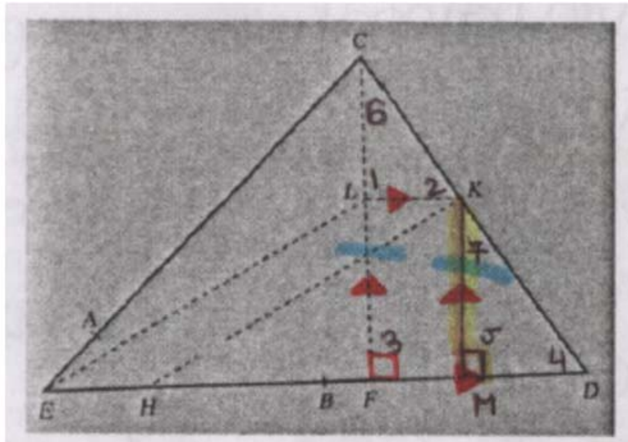


Statements: | **Reasons:**

Statements	Reasons
$LK \parallel FD$, $CF \perp$ to ED	Given
$\angle 1 \cong \angle 7$; $\angle 7 \cong \angle 4$	Def. of Corresponding ∡'s
$\angle 3 \cong \angle 6$	Def of corresponding ∡'s
$\triangle KLM \sim \triangle KMD$	AA postulate
$\frac{CL}{LF} = \frac{CK}{KD}$	Def. of similar figures

Given that LK is parallel to FD, and that CF is perpendicular to ED, it will be proven that $\frac{CL}{LF}$ is equal $\frac{CK}{KD}$. First, angle 1 ~~and~~ is congruent to angle 7 and 6. angle 7 is congruent to angle 4, because of the defintion of corresponding angles. Also, angle 3 is congruent to angle 6, also because of the defintion of corresponding angles. Therefore, triangle KLM is similar to triangle KMD, because of the Angle-Angle Postulade. Therefore $\frac{CL}{LF} = \frac{CK}{KD}$

Figure 5.14: Student response for proof #6



Given: LK is parallel to FD, CF is perpendicular to ED

$\frac{CK}{KD} = \frac{CL}{LF}$ | Substitution

Prove: $\frac{CL}{LF} = \frac{CK}{KD}$

Hint: Draw auxilary perpendicular line connecting point K to FD. Prove similar triangles in order to prove equivalent ratios

Statements:	**Reasons:**
LK is parallel to FD	Given
CF is perpendicular to ED	Given
auxiliary line KM is parallel to CF	Construction of a parallel line
$m\angle 3 = 90°$	def. of perpendicular line
$m\angle J = 90°$	def. of perpendicular line
$m\angle 3 = m\angle 1$	Corresponding angles
$m\angle J = m\angle 1$	Substitution
$\angle 2 \cong \angle 4$	Corresponding angles
$\triangle CLK \sim \triangle KMD$	Angle - Angle
$LF \cong KM$	def. of a rectangle
$\frac{CK}{KD} = \frac{CL}{KM}$	Corresponding sides of similar triangles are proportional

Figure 5.15a: Student response for proof #6

In this proof, it is given that segment LK is parallel to segment FD, and it is also given that segment CF is perpendicular to segment ED. In order to prove that segments CL and LF are proportional to segments CK and KD, auxillary line KM must be constructed. Segment KM must also be proportional segment CF. Because of the definiton of a perpendicular line, angles 3 and 5 equal 90 degrees. Because angle 3 and angle 1 are corresponding angles, their measures are equal. Next, substitute angle 3 with angle 1. Angle 2 is congruent to angle 4 because of corresponding angles. Therefore, triangle CLK and triangle KMD are similar because of angle angle. In addition, segment LF and segment KM are congruent because two opposite sides of a rectangle are congruent. This makes segments CK and segment KD ~~congru~~ are proportional to segments CL and KM. ~~3~~ Finally, because of substitution segments CK and KD are proportional to segments CL and LF.

Figure 5.15b: Student response for proof #6

5.3: Data Analysis

The students were given a pre-test to ensure that both groups started at the same academic level. The pre-test was in the form of sixteen multiple choice geometry based questions released from a previously administered ACT test. Figure 5.16 shows the data analysis on the pre-test scores from a random sample of twenty students taken from both groups of twenty four students. The data supports that both groups of students are starting at the same base line since the p-value is 0.327673176 ($P \geq 0.05$).

ACT Pre Test Analysis		
t-Test: Two-Sample Assuming Unequal Variances		
	With GSP	*Without GSP*
Mean	6.15	6.85
Variance	6.134210526	3.818421053
Observations	20	20
Hypothesized Mean Difference	0	
df	36	
t Stat	-0.992302473	
P(T<=t) one-tail	0.163836588	
t Critical one-tail	1.688297714	
P(T<=t) two-tail	0.327673176	
t Critical two-tail	2.028094001	

Figure 5.16

The same test was administered as a post test at the end of phase one to both groups. The mean of both groups are similar. Since the p-value is large($P \geq 0.05$), there is no significant difference between the two means. The two groups contain a certain equality of the mean values, showing no direct correlation from working with GSP to performing on the ACT multiple choice geometry based questions.

ACT Post Test Analysis		
t-Test: Two-Sample Assuming Unequal Variances		
	With GSP	*Without GSP*
Mean	9.1	9.05
Variance	4.515789474	4.471052632
Observations	20	20
Hypothesized Mean Difference	0	
df	38	
t Stat	0.074590144	
P(T<=t) one-tail	0.470466052	
t Critical one-tail	1.68595446	
P(T<=t) two-tail	0.940932104	
t Critical two-tail	2.024394164	

Figure 5.17

ACT Post-Pre Difference Analysis		
t-Test: Two-Sample Assuming Unequal Variances		
	With GSP	*Without GSP*
Mean	2.95	1.8
Variance	8.260526316	10.58947368
Observations	20	20
Hypothesized Mean Difference	0	
df	37	
t Stat	1.184560266	
P(T<=t) one-tail	0.121872434	
t Critical one-tail	1.68709362	
P(T<=t) two-tail	0.243744867	
t Critical two-tail	2.026192463	

Figure 5.18

The ACT pre and post test was further analyzed by running a two-sample t-Test assuming unequal variances on a random sample of twenty students from each group of twenty four students. Figure 5.18 shows that the mean of the group using GSP was calculated at 2.95, while the mean of the group not using GSP was 1.8. In this data analysis, the p-value 0.243547515 is also found to be large, supporting that there is no significant difference between the two means. This further supports that there was no substantial change in how students perform on the ACT multiple choice geometry based questions with the use of GSP during proof writing exercises.

Data was collected on the proof writing exercises as well. The rubric used to score proofs was divided into three categories: Statement of Problem, Logical Argument, and Knowledge of Definitions, Postulates, and Theorems. The *Statement of Problem* category had five possible points to earn, the *Logical Argument* category had ten possible points to earn, and the *Knowledge of Definitions, Postulates, and Theorems* had five possible points to earn. This rubric was used to score every proof writing exercise within this research. While there was not a substantial difference in the scores between the students using GSP and those not using GSP in the categories of *Statement of Problem* and *Knowledge of Definitions, Postulates, and Theorem,* there was a significant difference in the category of *Logical Argument*. There were some student absences during phase one of this research, so data was carefully analyzed regarding proof scores for phase two of testing and data collection. Since phase two consisted of three proof writing exercises, students could earn a total of thirty points in the category for *Logical Argument*. As seen in Figure 5.19 and Figure 5.20, the scores for the students using GSP are distributed differently than those students not using GSP. Notice the students using GSP while writing the proofs scored either a twenty-eight or thirty, while the students not using GSP earned scores ranging from twenty-two to thirty. The different distribution could be attributed to the freedom

to manipulate the figure in the proof, giving the students the foundational understanding as they set out to begin their proof. As students observe how a figure is altered as one piece of the figure is changed, they may start to build a logical sequence of ideas regarding why the changes are occurring.

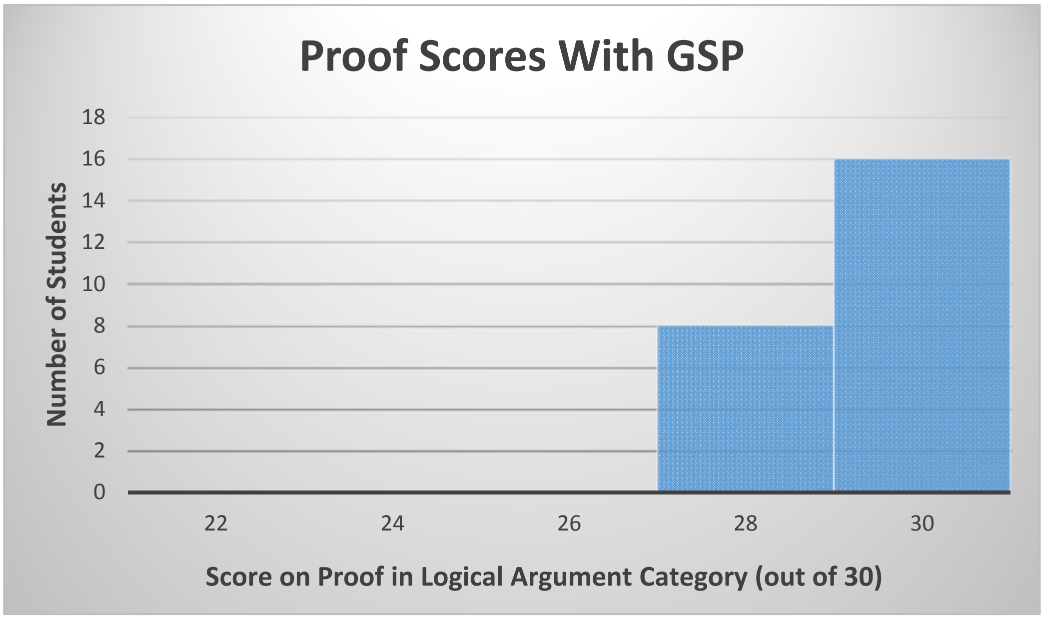


Figure 5.19

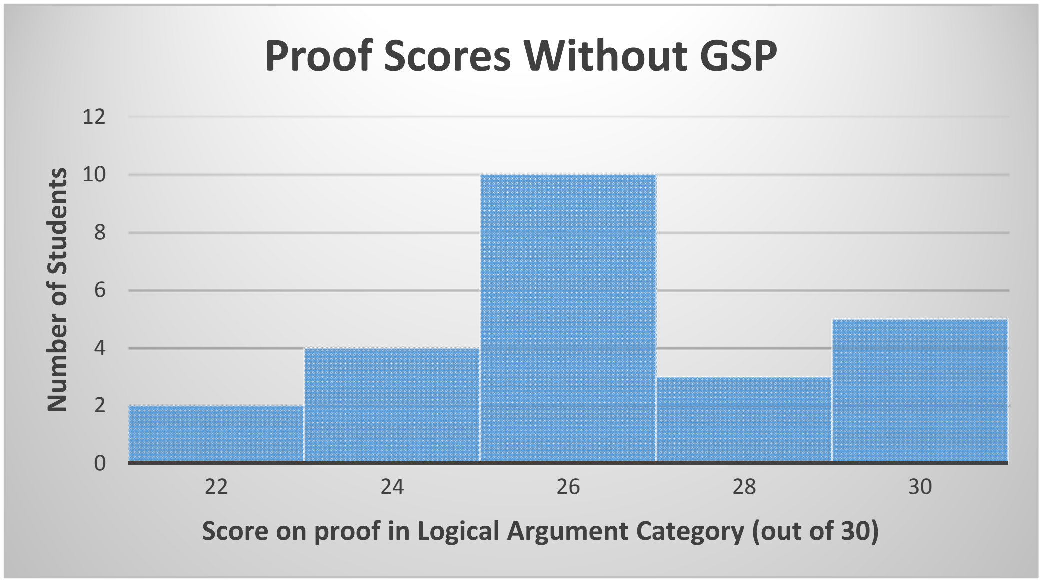


Figure 5.20

5.4: Conclusions

It is extremely important for students to understand the proof writing process. The skills that are fine-tuned through the proof writing exercises can directly translate to the skills needed to compete in the global workforce of today and the future. While this research could not conclude that the use of GSP positively affects students' performance on geometry based ACT questions, the use of GSP is helpful when writing proofs, more specifically improvement of logical proficiency. The student work discussed in this research and the data analysis on the proof scores supports this idea. The amount of time allotted for students to be well acquainted with the dynamic geometry software GSP is critical. As seen in the student responses on the opinion surveys, some students thought it was difficult to use. If students are not able to effectively operate the tools needed to animate and manipulate the figures in question, then the use of GSP is unlikely to be beneficial, especially with the proof writing process. This study had limitations regarding the GSP orientation time allowed in phase two. It would be interesting to see how students who are completely comfortable with the software excel in proof writing compared to those students not using the software.

This study explored methods that could easily be used in a classroom setting that engaged students in the proof writing process. Although some students did not find it helpful as they wrote proofs, the majority of the students in this study found GSP to be fun and interactive. If students become more involved in the mathematics, then they are more likely to persevere through the extensive proof writing process. Previous research studies and opinion surveys have shown that proof writing is by far the most undesirable practice of a high school geometry course. When students start to understand how to think logically as they write proofs, they will feel more comfortable with the proof writing process. The students' ease of proof writing is

noticeable throughout the courtroom presentations at the end of this research. Students were able respectfully argue with one another in the form of prosecution or defense team in a courtroom setting debating the validity of a written proof. It is noteworthy to observe students transition from being discouraged when presented with a proof writing exercise into eager presenters of the classroom. This transition can be credited with the use of GSP throughout this study and the use of the six compiled proof writing exercises found in this research. If teachers are able to facilitate an environment where students buy in to the process of sequencing statements and reasons in such a way that they are completing a proof, then student will no longer dread writing proofs, but rather find joy in the process.

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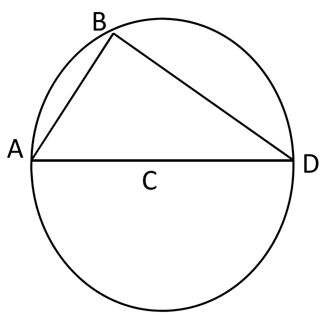
APPENDIX A: PROOF WRITING EXERCISES

PROOF #1

Complete the following proof. Show your reasoning throughout the process. The format of the proof should be both paragraph and 2 column form. First, write a paragraph to explain how you will show your hypothesis to be true. Second, write a 2-column proof to line out the details of the sequence of your reasoning.

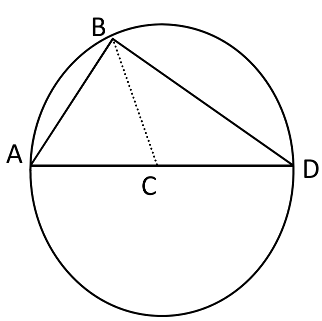
GIVEN: a circle with center C and diameter AD, where points A, B, & D lie on the circle

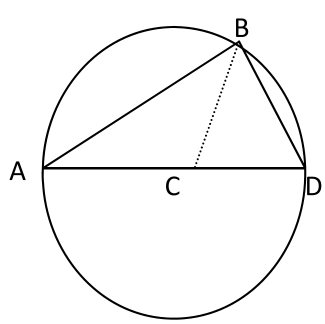
PROVE: ∡ABD is always a right angle

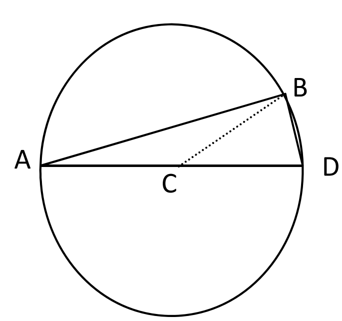


Proof Guidance: ***What are your observations about angle ABD? How can you prove your conjecture?***

1) Construct an auxiliary line segment BC.
2) Observe what relationship BC has with AC and DC.
3) Observe what characteristics the triangles in the circle have.
4) Calling ∡CAB as x, determine all other angles within the triangles in terms of x.
5) How can ∡ABD be proven to ALWAYS be 90 degrees?
6) Set up your proof with the answer to number 5.







PARAGRAPH EXPLANATION

How will you prove the conjecture that angle B is always a right angle if AD is the diameter of the circle and point B lies on the circle with center C?

Type your response below:

TWO COLUMN PROOF

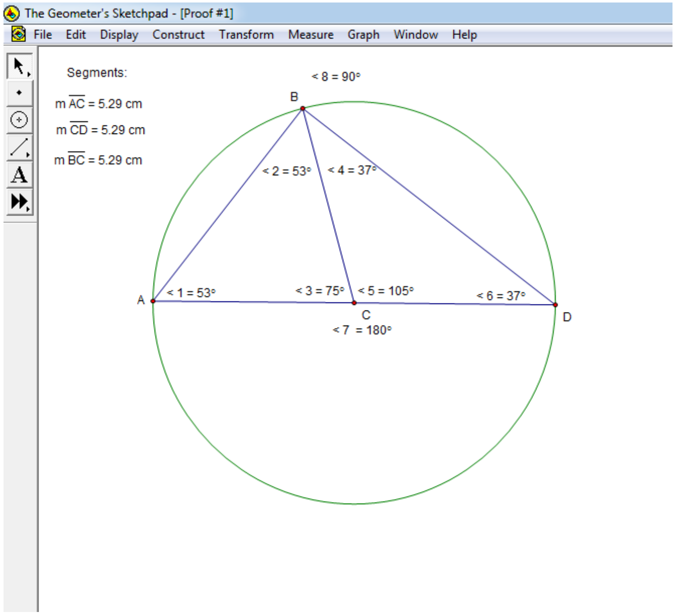
Given:

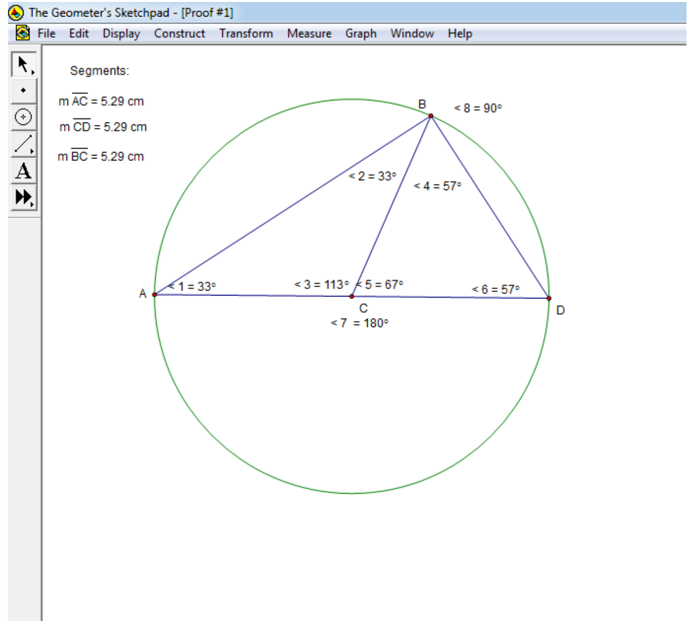
Prove:

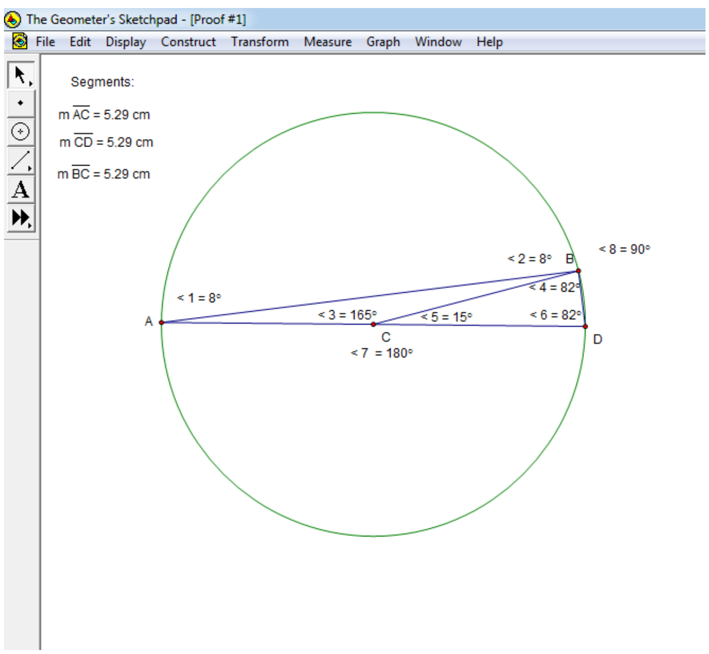
Statements:	Reasons:

(You may add/subtract rows as necessary)

PROOF #1: GEOMETER'S SKETCHPAD SCREENSHOTS





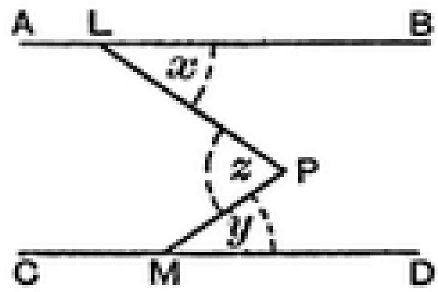


PROOF #2

Complete the following proof. Show your reasoning throughout the process. The format of the proof should be both paragraph and 2 column form. First, write a paragraph to explain how you will show your hypothesis to be true. Second, write a 2-column proof to line out the details of the sequence of your reasoning.

GIVEN: AB and CD are two parallel straight lines. Point L lies somewhere on AB and point K lies somewhere on CD.

PROVE: x + y = z



Proof Guidance:

1) Construct auxiliary lines if necessary
2) Observe relationships between the angles and the lines
3) Label other angles as needed to reference in the proof
4) Observe relationships between x, y, and z
5) Set up equations to represent the above relationships
6) You may refer to previously proven theorems in this proof

PARAGRAPH EXPLANATION

How will you prove the conjecture **x + y = z**?

Type your response below:

TWO COLUMN PROOF

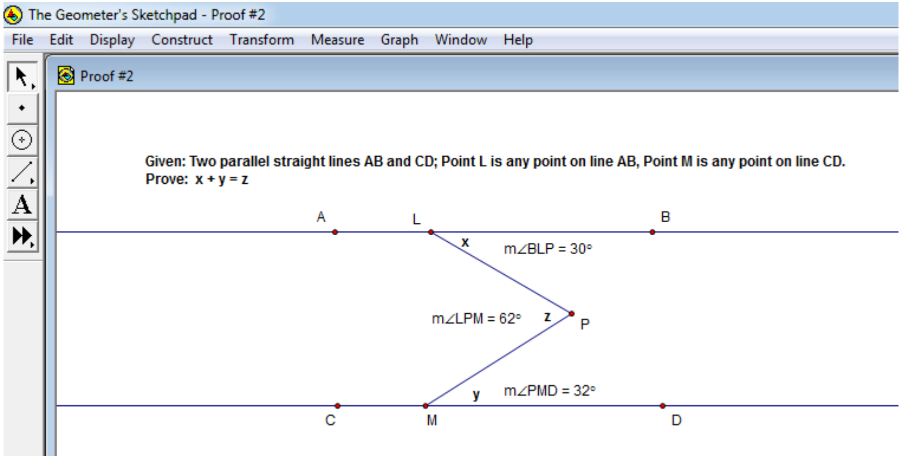
Given:

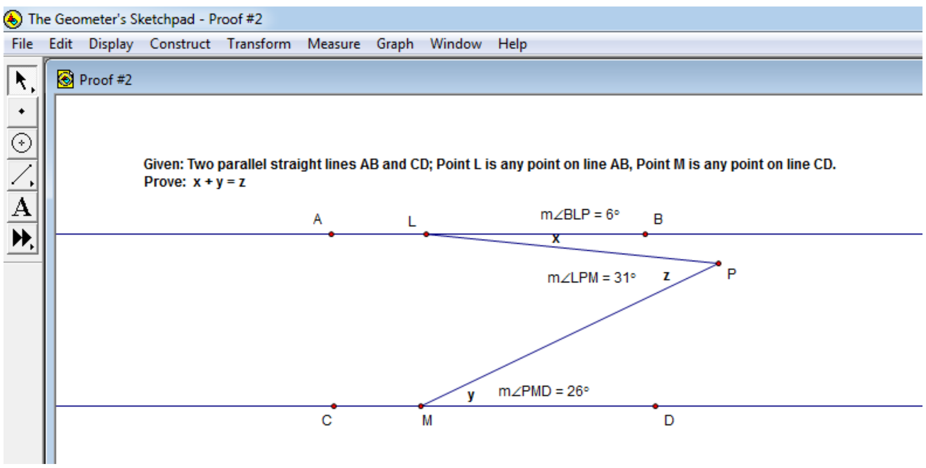
Prove:

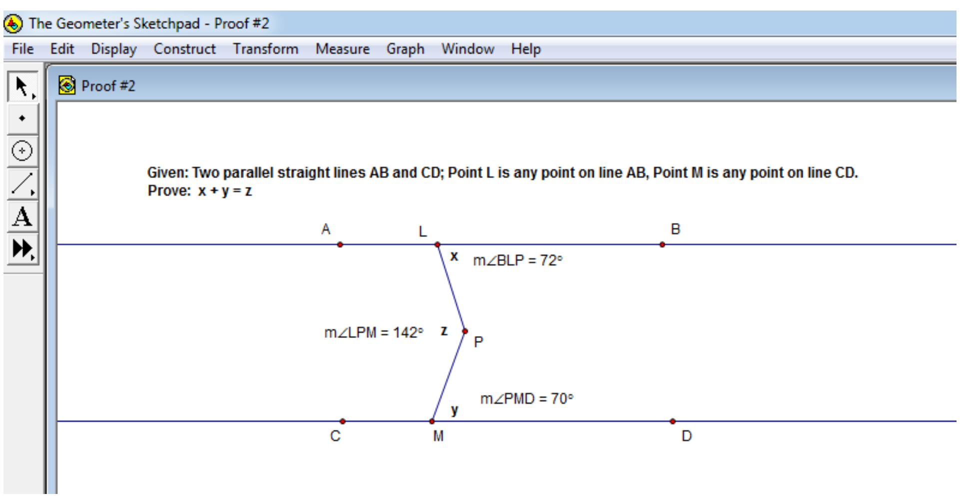
Statements:	Reasons:

(You may add/subtract rows as necessary)

PROOF #2: GEOMETER'S SKETCHPAD SCREENSHOTS





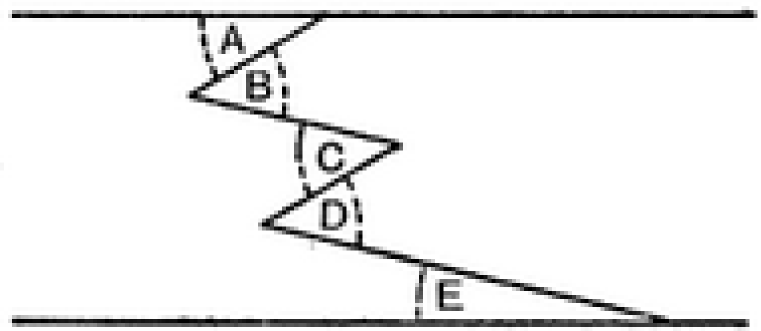


PROOF #3:

Complete the following proof. Show your reasoning throughout the process. The format of the proof should be both paragraph and 2 column form. First, write a paragraph to explain how you will show your hypothesis to be true. Second, write a 2-column proof to line out the details of the sequence of your reasoning.

GIVEN: Figure similar to 1st proof with parallel lines

PROVE: a + c + e = b + d



Guide outline Notes:

1) Construct auxiliary lines if necessary
2) Observe relationships between the angles and the lines
3) Label (and Re-Label) other angles as needed to reference in the proof
4) Observe relationships between a, b, c, d, and e
5) Set up equations to represent the above relationships
6) You may refer to previously proven theorems in this proof

PARAGRAPH EXPLANATION

How will you prove the conjecture **$a + c + e = b + d$?**

Type your response below:

TWO COLUMN PROOF

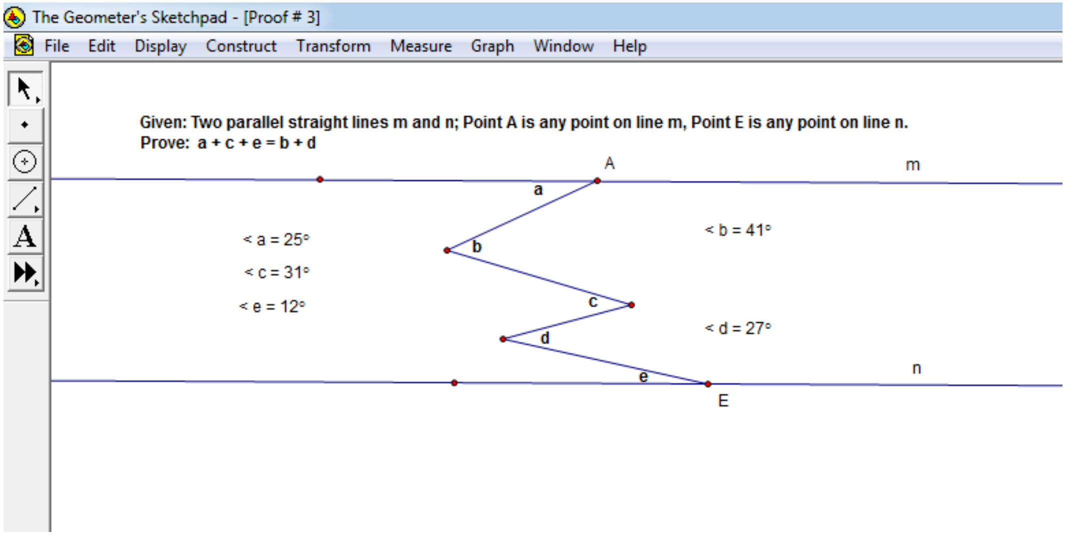
Given:

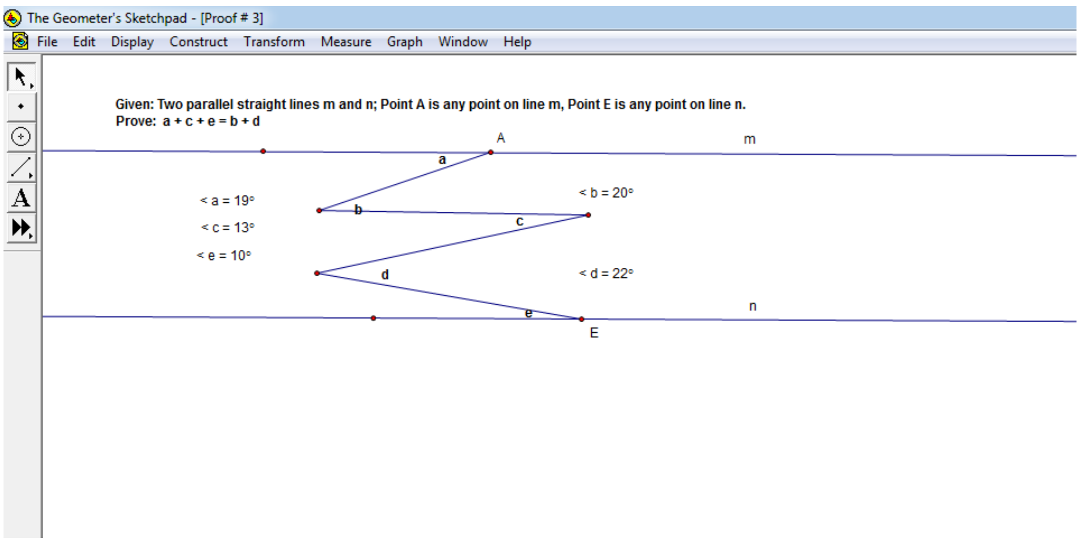
Prove:

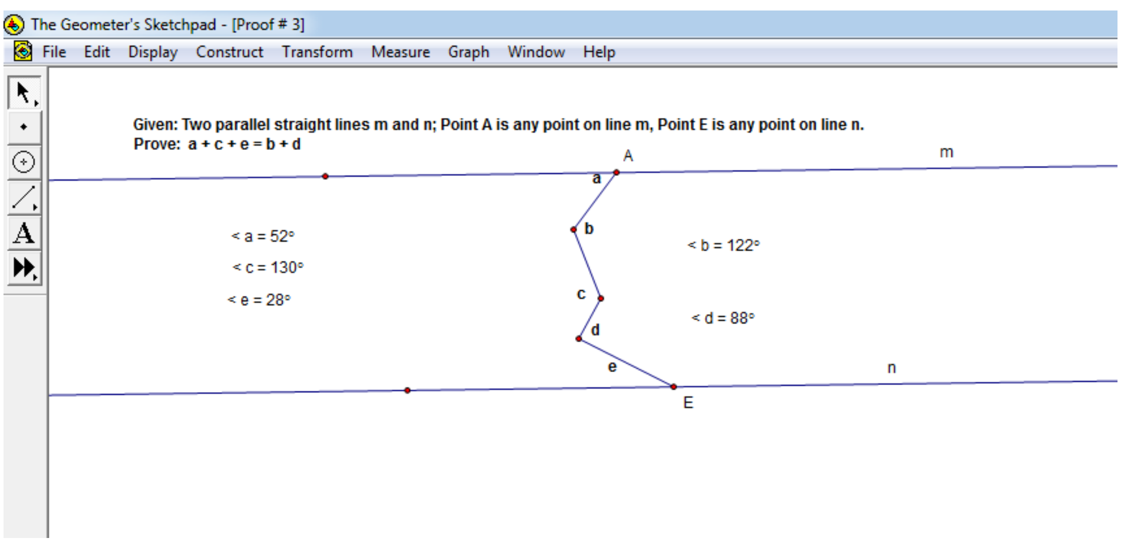
Statements:	Reasons:

(You may add/subtract rows as necessary)

PROOF #3: GEOMETER'S SKETCHPAD SCREENSHOTS





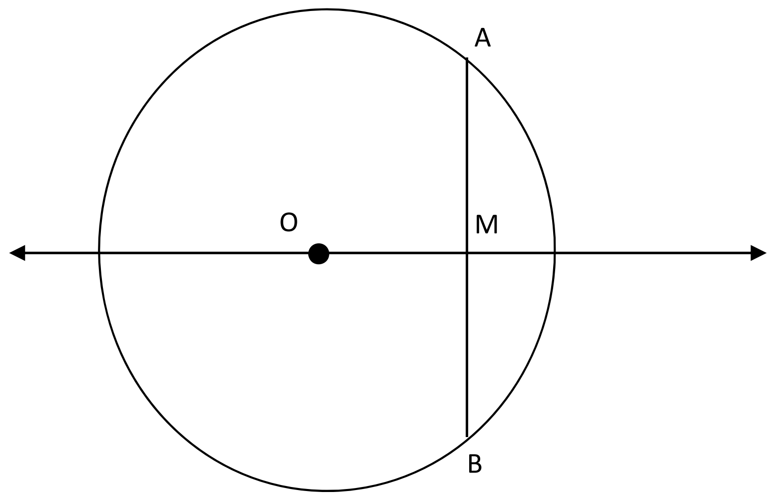


PROOF #4

Complete the following proof. Show your reasoning throughout the process. The format of the proof should be both paragraph and 2 column form. First, write a paragraph to explain how you will show your hypothesis to be true. Second, write a 2-column proof to line out the details of the sequence of your reasoning.

GIVEN: In a circle O, M is the midpoint a chord AB.

PROVE: OM is perpendicular to AB



Proof guidance:

1) Draw auxiliary segments OA and OB.
2) What does it mean to prove lines are perpendicular?

PARAGRAPH EXPLANATION

How will you prove the conjecture OM is perpendicular to AB?

Type your response below:

TWO COLUMN PROOF

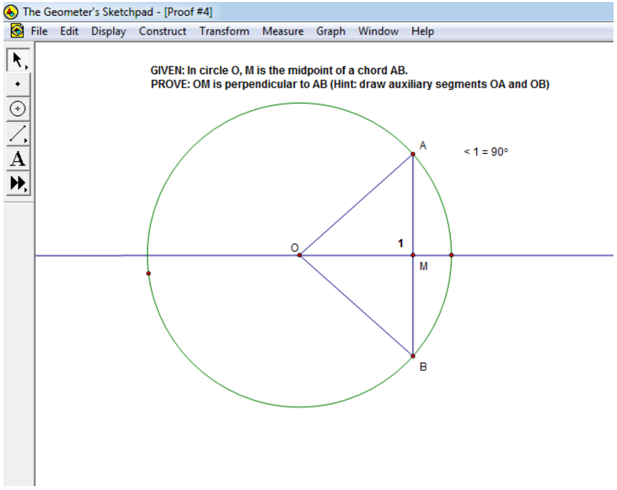
Given:

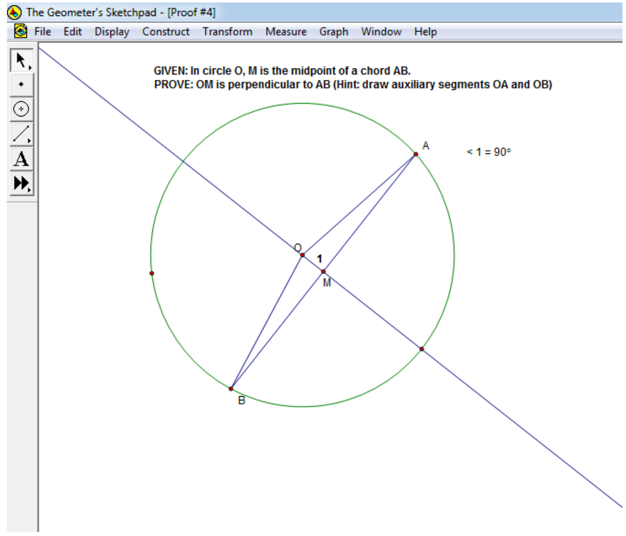
Prove:

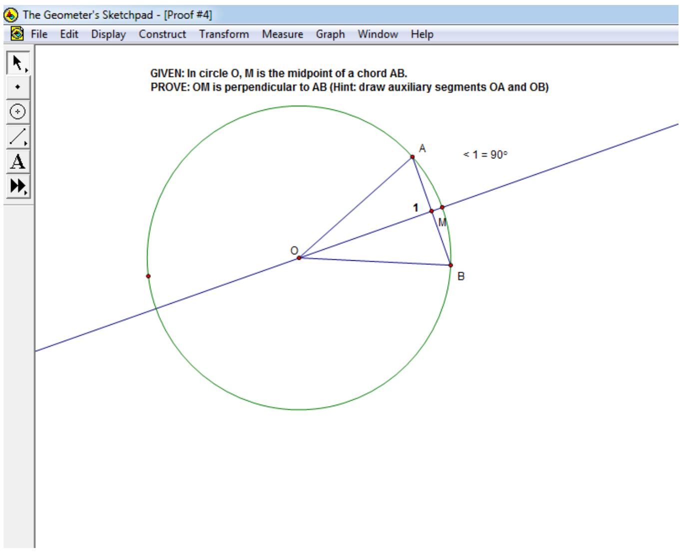
Statements:	Reasons:

(You may add/subtract rows as necessary)

PROOF #4: GEOMETER'S SKETCHPAD SCREENSHOTS





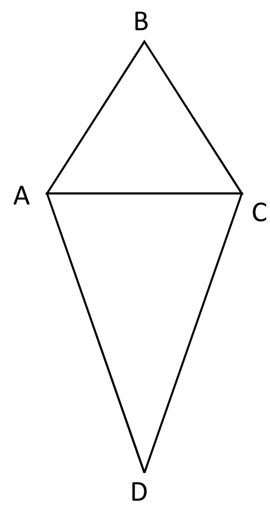


PROOF #5

Complete the following proof. Show your reasoning throughout the process. The format of the proof should be both paragraph and 2 column form. First, write a paragraph to explain how you will show your hypothesis to be true. Second, write a 2-column proof to line out the details of the sequence of your reasoning.

Given: Polygon ABCD in which BD bisects <ABC and <ADC.

Prove: BD is the perpendicular bisector of AC.



Proof Guidance: What does a perpendicular bisector do?

1) Draw auxiliary segment AC.
2) How can you prove lines are perpendicular?
3) How can you prove a segment is bisected?
4) Can congruent triangle play a role in this proof?

PARAGRAPH EXPLANATION

How will you prove the conjecture BD is the perpendicular bisector of AC?

Type your response below:

TWO COLUMN PROOF

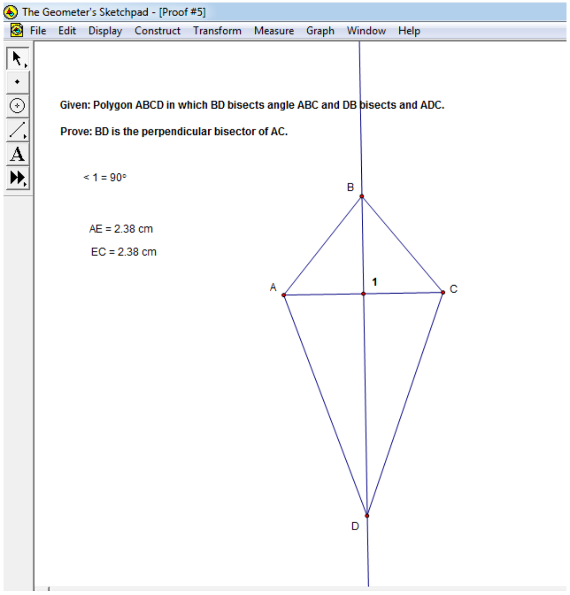
Given:

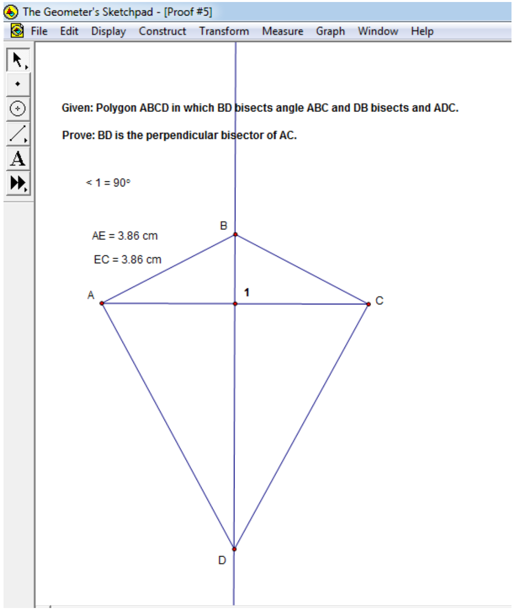
Prove:

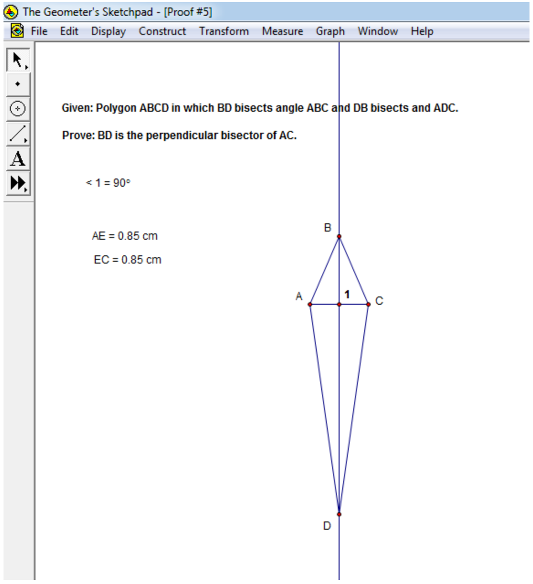
Statements:	Reasons:

(You may add/subtract rows as necessary)

PROOF #5: GEOMETER'S SKETCHPAD SCREENSHOTS



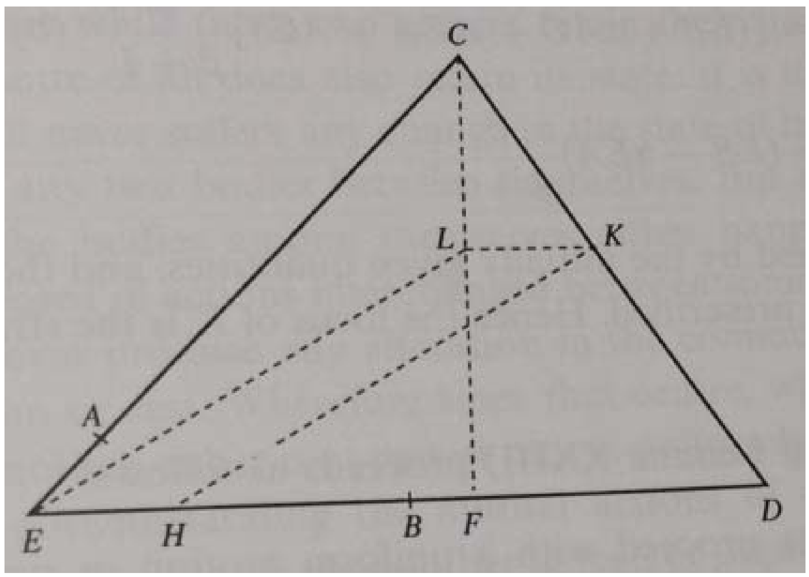




PROOF #6

Complete the following proof. Show your reasoning throughout the process. The format of the proof should be both paragraph and 2 column form. First, write a paragraph to explain how you will show your hypothesis to be true. Second, write a 2-column proof to line out the details of the sequence of your reasoning.

Background information: *<u>Newton's Theory of traveling objects</u>* → two objects are traveling at a constant rate (so the ratios of the distance traveled are constant). If two given lines, as AC, BD, terminating in given points A, B, are in a given ratio one to the other, and the line CD, by which the undetermined points C, D are joined is cut in K in a given ratio: I say, that the points K will be placed in a given line.



Given: LK is parallel to FD, CF is perpendicular to ED

Prove: $\frac{CL}{LF} = \frac{CK}{KD}$

PARAGRAPH EXPLANATION

How will you prove the conjecture $\frac{CL}{LF} = \frac{CK}{KD}$?

Type your response below:

TWO COLUMN PROOF

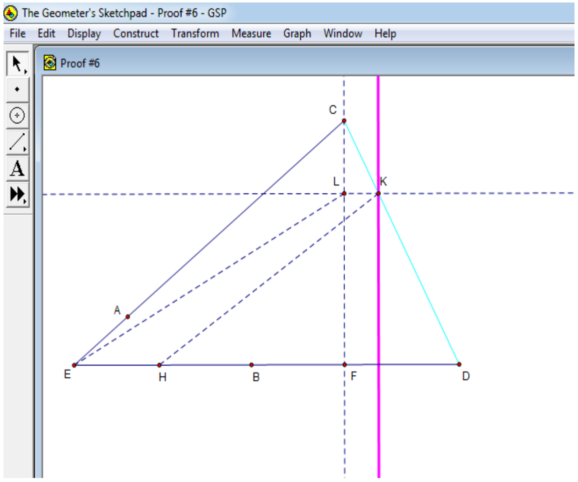
Given:

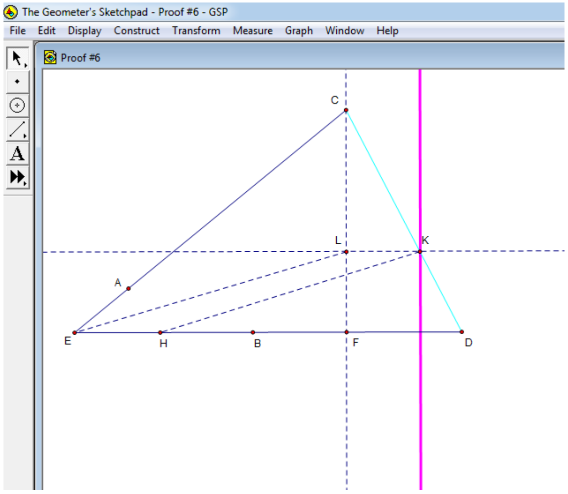
Prove:

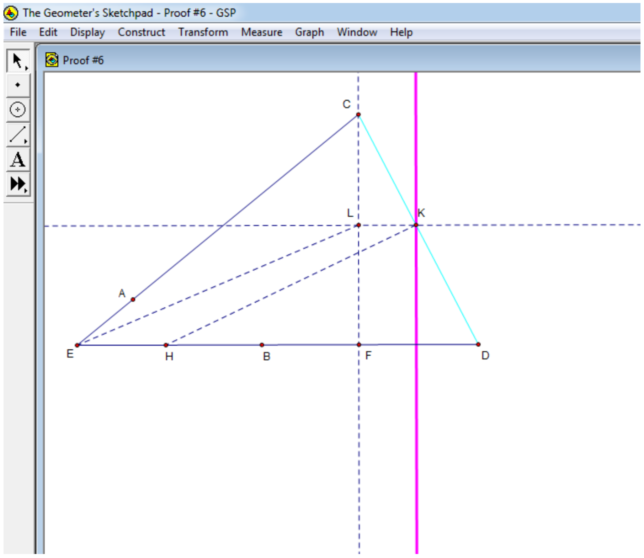
Statements:	Reasons:

(You may add/subtract rows as necessary)

PROOF #6: GEOMETER'S SKETCHPAD SCREENSHOTS







APPENDIX B: PROOF WRITING GRADING RUBRIC

Description	Unacceptable (F)	Struggling (D)	Improving (C)	Achieving (B)	Excelling (A)	
Statement of Problem	**Points: 1** Given/Prove statements and picture entirely absent.	**Points: 2** Picture is entirely missing.	**Points: 3** Incorrect or absent given or prove statement AND absent or incomplete picture.	**Points: 4** Incorrect or absent given or prove statement OR absent or incomplete picture.	**Points: 5** Given statement and statement to be proved present above proof. Labeled picture beside.	**Your proof will be graded using this rubric.** **Your proof will be completed on white computer paper.**
Logical Argument (Double Points)	**Points: 2** No interpretable attempt made at building argument.	**Points: 4** No discernable path from given statement to be proved.	**Points: 6** Many missing or unnecessary steps from given statement to statement to be proved.	**Points: 8** Discernable path from given statement to statement to be proved, a few missing or unnecessary steps.	**Points: 10** Clear and concise argument from given statement to be proved.	
Knowledge of Definitions, Postulates, and Theorems	**Points: 1** No evidence of knowledge of definitions, postulates, and theorems.	**Points: 2** Misuse of definitions, postulates, and theorems. Makes up or mixes up definitions, postulates, and theorems.	**Points: 3** Regular misuse of definitions, postulates, and theorems.	**Points: 4** A few missuses or incorrect definitions, postulates, and theorems.	**Points: 5** Appropriate usage of definitions, postulates, and theorems.	
					Project Grade	/20

APPENDIX C: POST PROOF REFLECTION

1. Did you feel successful in completing the last 3 proofs? Explain.

2. Did you feel the partner/group discussions leading up to the proof writing exercise were beneficial? How so?

3. How confident were you in what was given in the proof? (1 = low confidence, 5 = high confidence)
   1 2 3 4 5

Explain:

____

____

4. How confident were you in what was asked to be proved?
   1 2 3 4 5

Explain:

____

____

5. How confident were you in understanding the theorems needed to prove your hypothesis?
   1 2 3 4 5

Explain:

____

____

6. How confident were writing in the paragraph to set up the proof?
   1 2 3 4 5

Explain:

____

____

7. How confident were you in writing the 2 column proof statements?

   1 2 3 4 5

Explain:

____

____

8. How confident were in justifying your statements with reasons?

   1 2 3 4 5

Explain:

____

____

9. Overall, how confident do you feel writing proofs now?

   1 2 3 4 5

Explain:

____

____

10. After completing proofs with the help of geometer's sketchpad and without geometer's sketchpad, how would rank how helpful using the dynamic geometry software was in writing the proof?

   1 2 3 4 5

Explain:

____

____

11. Did you enjoy using geometer's sketchpad?

   1 2 3 4 5

Explain:

____

____

12. What do you think helped your understand writing proofs the most?

APPENDIX D: COURTROOM PROOF WRITING EXERCISES

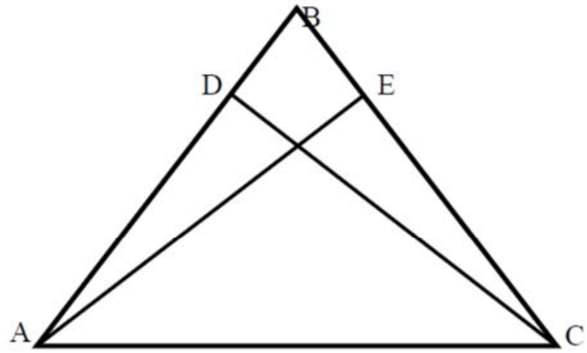
Proof Case #1

Given: $\overline{CD}$ is the altitude to $\overline{AB}$

$\overline{AE}$ is the altitude to $\overline{BC}$

$\overline{CD} \cong \overline{AE}$

Prove: ΔABC is isosceles

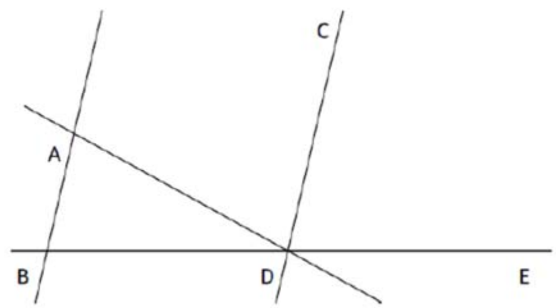


Proof Case # 2

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$\overleftrightarrow{DC}$ bisects $\angle ADE$

Prove: ΔABD is isosceles



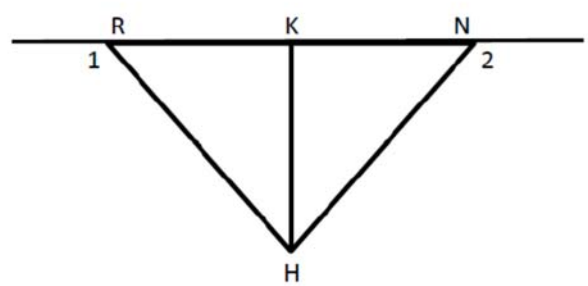
Proof Case #3

Given: $\angle 1 \cong \angle 2$

$\overline{HK}$ bisects $\angle RHN$

$\overline{HR} \cong \overline{HN}$

Prove: $\overline{HK} \perp \overline{RN}$



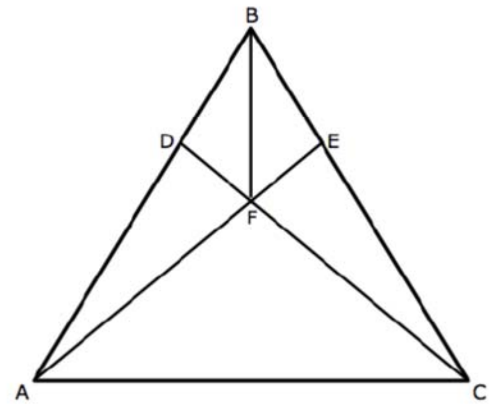
Proof Case #4

Given: $\angle FAC \cong \angle FCA$

$\overline{FD} \perp \overline{AB}$

$\overline{FE} \perp \overline{BC}$

Prove: $\overline{BF}$ bisects $\angle DBE$

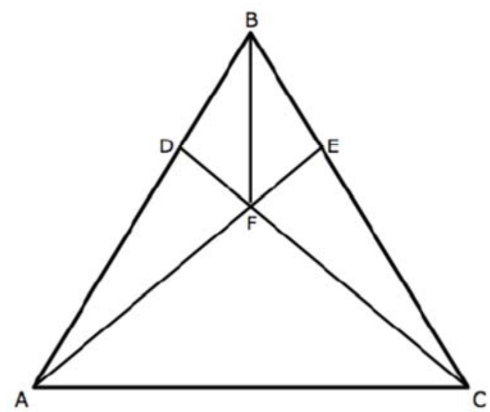


Proof Case #5

Given: $\overline{BD} \cong \overline{BE}$

$\overline{FD} \cong \overline{FE}$

Prove: ΔAFC is isosceles

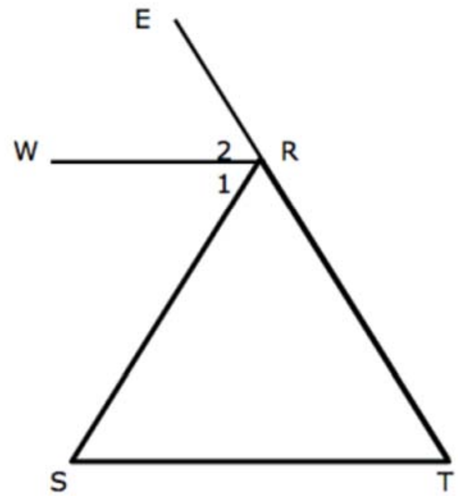


Proof Case #6

Given: $\overline{WR} \parallel \overline{ST}$

$\overline{WR}$ bisects $\angle SRE$

Prove: ΔSRT is isosceles



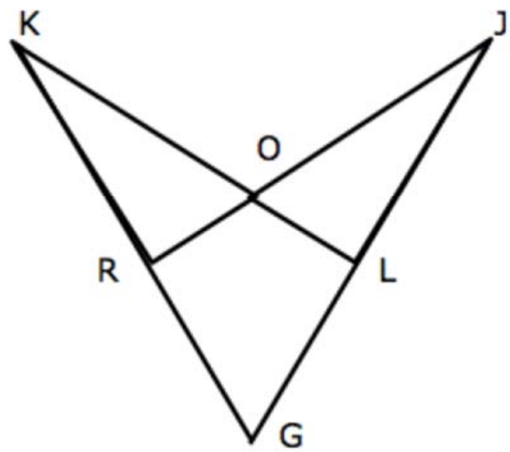
Proof Case #7

Given: $\overline{JR} \perp \overline{KG}$

$\overline{KL} \perp \overline{JG}$

$\overline{KL} \cong \overline{JR}$

Prove: $\Delta KLG \cong \Delta JRG$



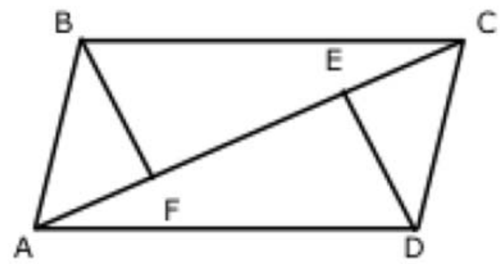
Proof Case #8

Given: $\overline{BF} \perp \overline{AC}$

$\overline{DE} \perp \overline{AC}$

$\overline{AB} \cong \overline{DC}$

$\overline{AE} \cong \overline{CF}$



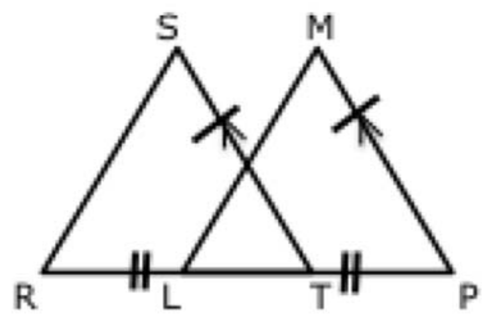
Prove: $\Delta AFB \cong \Delta CED$

Proof Case #9

Given: $\overline{ST} \cong \overline{MP}$

$\overline{ST} \parallel \overline{MP}$

$\overline{RT} \cong \overline{LP}$



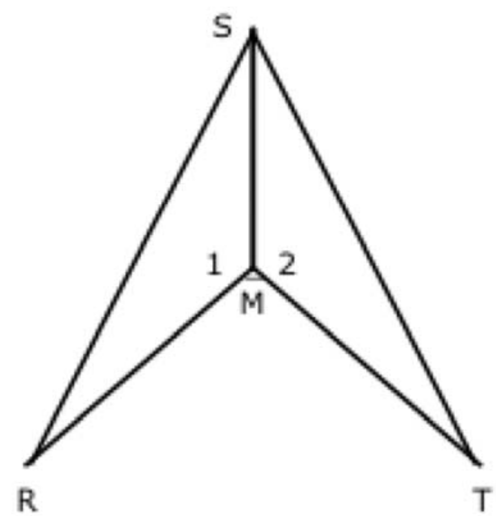
Prove: $\overline{RS} \parallel \overline{LM}$

Proof Case #10

Given: $\angle 1 \cong \angle 2$

$\overline{RM} \cong \overline{TM}$

Prove: $\overline{SM}$ bisects $\angle RST$



APPENDIX E: DATA ANALYSIS ON ACT PRE- AND POST-TEST

ACT Pre-Post Test Analysis							
With GSP				*Without GSP*			
Random Sample	**Post - Pre Difference**	**Pre test score**	**Post test score**	**Random Sample**	**Post - Pre Difference**	**Pre test score**	**Post test score**
101	-3	10	7	201	2	8	10
107	2	4	6	217	1	3	4
118	5	3	8	211	5	6	11
117	4	4	8	204	0	7	7
110	3	8	11	215	6	5	11
124	5	6	11	202	-1	8	7
121	3	7	10	225	6	5	11
108	5	8	13	222	0	9	9
122	4	5	9	218	3	6	9
102	0	7	7	221	4	5	9
119	-2	9	7	203	4	8	12
111	6	4	10	206	-1	9	8
105	4	7	11	219	5	7	12
103	5	7	12	214	4	3	7
112	1	6	7	212	1	8	9
104	0	6	6	224	3	7	10
120	2	8	10	207	2	10	12
123	5	7	12	205	-1	9	8
115	1	8	9	223	-8	8	8
109	9	-1	8	209	1	6	7
Average	**2.95**	**6.15**	**9.1**	**Average**	**1.8**	**6.85**	**9.05**

APPENDIX F: DATA ANALYSIS ON PROOF SCORES

With GSP - PHASE ONE												
	Proof # 1				Proof #2				Proof #3			
Student Code	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score
101	5	10	5	20	5	10	5	20	5	8	5	18
102	3	4	2	9								
103	4	8	4	16	5	10	5	20	5	8	5	18
104	4	10	4	18	4	8	5	17	5	8	5	18
105	5	8	5	18	5	10	5	20	5	10	5	20
106					3	8	3	14	4	8	4	16
107	4	8	4	16	3	6	4	13	4	8	4	16
108	3	8	5	16	5	10	5	20	5	10	5	20
109	4	8	4	16	4	6	4	14	5	8	5	18
110	4	6	3	13	5	8	5	18	5	10	5	20
111	4	6	4	14	4	8	4	16	5	8	5	18
112	3	6	4	13	5	8	4	17	5	8	5	18
113	5	8	4	17	4	6	4	14	5	8	5	18
114	5	6	4	15	5	8	4	17	5	10	5	20
115	3	6	3	12								
116	5	8	4	17	5	10	5	20	5	8	5	18
117					4	10	5	19	5	8	5	18
118	5	8	4	17	5	10	4	19	5	8	5	18
119	4	6	3	13								
120	4	10	4	18	5	10	5	20	5	10	5	20
121	5	10	5	20	4	8	5	17	5	10	5	20
122	5	10	4	19	5	10	5	20	5	10	5	20
123	5	10	4	19	4	8	4	16	5	8	5	18
124	4	8	4	16								
Average	**4.2**	**7.8**	**4.0**	**16.0**	**4.5**	**8.6**	**4.5**	**17.6**	**4.9**	**8.7**	**4.9**	**18.5**

Without GSP - PHASE ONE												
Student Code	Proof #1				Proof #2				Proof #3			
	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score
201	5	8	4	17	4	10	4	18	4	10	4	18
202	3	8	4	15	3	10	5	18	4	10	4	18
203	3	8	4	15	4	6	4	14	4	8	4	16
204	5	6	4	15	5	10	5	20	5	10	5	20
205	3	2	2	7								
206												
207	3	4	2	9	5	10	5	20	5	10	5	20
208	2	6	4	12	5	10	5	20	3	8	4	15
209	4	4	4	12	4	10	5	19	4	10	4	18
210	3	4	4	11								
211					4	10	5	19	4	8	4	16
212	4	8	4	16	4	10	5	19	4	8	4	16
213	4	4	3	11	4	8	4	16	4	8	4	16
214	5	6	5	16	5	8	5	18	4	8	5	17
215	5	8	4	17	4	10	5	19	4	8	4	16
216	3	8	3	14	4	10	5	19	4	10	4	18
217	2	4	2	8	5	10	5	20	4	10	5	19
218												
219	5	10	5	20	4	10	5	19	4	8	4	16
220	3	6	4	13								
221	3	6	4	13	5	10	5	20	5	10	5	20
222	3	6	3	12	4	10	5	19	5	8	5	18
224	4	2	4	10	3	8	4	15	4	8	4	16
225	3	10	5	18								
Average	**3.6**	**6.1**	**3.7**	**13.4**	**4.2**	**9.4**	**4.8**	**18.4**	**4.2**	**8.9**	**4.3**	**17.4**

Without GSP - PHASE TWO												
Student Code	Proof # 4				Proof #5				Proof #6			
	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score
101	5	10	5	**20**	5	8	5	**18**	5	8	5	**18**
102	4	8	4	**16**	4	8	5	**17**	5	10	5	**20**
103	5	10	5	**20**	5	8	4	**17**	5	8	5	**18**
104	5	8	5	**18**	4	8	4	**16**	5	10	5	**20**
105	5	8	4	**17**	5	10	4	**19**	5	10	4	**19**
106	4	8	4	**16**	4	8	4	**16**	4	8	4	**16**
107	5	10	5	**20**	5	10	4	**19**	5	10	4	**19**
108	5	10	5	**20**	5	10	5	**20**	5	8	5	**18**
109	5	10	5	**20**	5	10	5	**20**	5	6	4	**15**
110	5	8	4	**17**	5	6	4	**15**	5	8	5	**18**
111	5	8	4	**17**	5	8	4	**17**	5	8	4	**17**
112	4	10	5	**19**	3	8	4	**15**	5	8	5	**18**
113	5	10	5	**20**	5	10	4	**19**	5	10	4	**19**
114	5	10	5	**20**	5	8	4	**17**	5	8	4	**17**
115	5	8	5	**18**	5	8	4	**17**	4	6	4	**14**
116	5	10	5	**20**	4	10	4	**18**	4	10	4	**18**
117	5	10	5	**20**	5	10	5	**20**	5	10	4	**19**
118	5	10	5	**20**	5	8	4	**17**	5	10	5	**20**
119	5	10	5	**20**	5	8	4	**17**	4	8	4	**16**
120	5	10	4	**19**	5	8	4	**17**	5	8	4	**17**
121	4	10	4	**18**	5	8	4	**17**	4	8	4	**16**
122	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
123	5	8	3	**16**	5	8	4	**17**	5	8	5	**18**
124	5	10	5	**20**	5	8	4	**17**	5	6	5	**16**
Average	**4.8**	**9.3**	**4.6**	**18.8**	**4.8**	**8.6**	**4.3**	**17.6**	**4.8**	**8.5**	**4.5**	**17.8**

With GPS - PHASE TWO												
	Proof #4				Proof #5				Proof #6			
Student Code	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score	Statement of Problem	Logical Argument	Knowledge of Definitions, Postulates, and Theorems	Total Score
201	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
202	5	10	5	**20**	4	8	5	**17**	5	10	5	**20**
203	5	10	5	**20**	5	10	5	**20**	5	10	4	**19**
204	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
205	5	10	5	**20**	5	10	5	**20**	5	8	5	**18**
206	5	10	4	**19**	5	10	5	**20**	5	10	4	**19**
207	5	10	5	**20**	5	10	5	**20**	5	8	4	**17**
208	5	10	5	**20**	5	10	5	**20**	5	8	5	**18**
209	4	10	5	**19**	4	8	5	**17**	4	10	4	**18**
210	5	10	5	**20**	5	10	5	**20**	4	10	5	**19**
211	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
212	5	10	5	**20**	5	8	4	**17**	5	10	5	**20**
213	4	10	5	**19**	5	10	5	**20**	5	10	5	**20**
214	4	10	5	**19**	5	10	5	**20**	5	10	5	**20**
215	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
216	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
217	5	10	4	**19**	5	10	5	**20**	5	10	5	**20**
218	4	10	5	**19**	4	10	5	**19**	4	10	5	**19**
219	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
220	5	10	5	**20**	4	10	5	**19**	4	8	4	**16**
221	5	10	5	**20**	5	10	4	**19**	5	10	5	**20**
222	5	10	5	**20**	5	10	5	**20**	5	10	5	**20**
224	5	10	5	**20**	5	8	5	**18**	5	10	5	**20**
225	4	10	5	**19**	5	10	5	**20**	5	10	5	**20**
Average	**4.8**	**10.0**	**4.9**	**19.7**	**4.8**	**9.7**	**4.9**	**19.4**	**4.8**	**9.7**	**4.8**	**19.3**

Without GSP – Phase Two	
Logical Argument Total Score	**Number of students**
22	2
24	4
26	10
28	3
30	5

With GSP – Phase Two	
Logical Argument Total Score	**Number of Students**
22	0
24	0
26	0
28	8
30	16

APPENDIX G: IRB APPROVAL

Application for Exemption from Institutional Oversight

LSU
Institutional Review Board
Dr. Robert Mathews, Chair
130 David Boyd Hall
Baton Rouge, LA 70803
P: 225.578.8692
F: 225.578.5983
irb@lsu.edu
lsu.edu/irb

Unless qualified as meeting the specific criteria for exemption from Institutional Review Board (IRB) oversight, ALL LSU research/ projects using living humans as subjects, or samples, or data obtained from humans, directly or indirectly, with or without their consent, must be approved or exempted in advance by the LSU IRB. This Form helps the PI determine if a project may be exempted, and is used to request an exemption.

– Applicant, Please fill out the application in its entirety and include the completed application as well as parts A-F, listed below, when submitting to the IRB. Once the application is completed, please the completed application to the IRB Office or to a member of the Human Subjects Screening Committee. Members of this committee can be found at http://sites01.lsu.edu/wp/ored/human-subjects-screening-committee-members/

– A Complete Application Includes All of the Following:
- **(A)** A copy of this completed form and a copy of parts B thru F.
- **(B)** A brief project description (adequate to evaluate risks to subjects and to explain your responses to Parts 1&2)
- **(C)** Copies of all instruments to be used.
  - *If this proposal is part of a grant proposal, include a copy of the proposal and all recruitment material.
- **(D)** The consent form that you will use in the study (see part 3 for more information.)
- **(E)** Certificate of Completion of Human Subjects Protection Training for all personnel involved in the project, including students who are involved with testing or handling data, unless already on file with the IRB. Training link: (http://phrp.nihtraining.com/users/login.php)
- **(F)** IRB Security of Data Agreement: (https://sites01.lsu.edu/wp/ored/files/2013/07/Security-of-Data-Agreement.pdf)

1) Principal Investigator: Kristina Chaves **Rank:** graduate student

Dept: Natural Sciences **Ph:** 337-280-7951 **E-mail:** kristina.chaves@zacharyschools.org

2) Co Investigator(s): please include department, rank, phone and e-mail for each
*If student, please identify and name supervising professor in this space

Dr. Padmanabhan Sundar (Mathematics Department)
Rank: Professor
Phone: 225-578-1611 Email: sundar@math.lsu.edu

IRB# E8397 LSU Proposal #
Complete Application
Human Subjects Training
IRB Security of Data Agreement

3) Project Title: Does Exploration Through Geometer's Sketchpad Better Develop Proof Writing Skills Compared to Teacher Instruction?

STUDY EXEMPTED BY:
Dr. Robert C. Mathews, Chairman
Institutional Review Board
Louisiana State University
130 David Boyd Hall
225-578-8692 / www.lsu.edu/irb
Exemption Expires: 9/12/2016

4) Proposal? (yes or no) NO **If Yes, LSU Proposal Number**

Also, if YES, either
- **This application completely matches the scope of work in the grant**

OR
- **More IRB Applications will be filed later**

5) Subject pool (e.g. Psychology students) 55 Geometry 9th grade Zachary High School students

*Circle any **"vulnerable populations" to be used**: (children <18; the mentally impaired, pregnant women, the ages, other). Projects with incarcerated persons cannot be exempted.

6) PI Signature **Date** (no per signatures)

**** I certify my responses are accurate and complete.** If the project scope or design is later changes, I will resubmit for review. I will obtain written approval from the Authorized Representative of all non-LSU institutions in which the study is conducted. I also understand that it is my responsibility to maintain copies of all consent forms at LSU for three years after completion of the study. If I leave LSU before that time the consent forms should be preserved in the Departmental Office.

Screening Committee Action: Exempted ✓ Not Exempted ____ Category/Paragraph 1

Signed Consent Waived?: Yes / No

Reviewer Mathews **Signature** [signature] **Date** 9/13/13

VITA

Kristina Chaves was born to Joseph and Lynn Chaves in November of 1985 in Lafayette, Louisiana. She attended elementary and middle school at Our Lady of Fatima Catholic School in Lafayette, then St. Thomas Moore Catholic High also in Lafayette. Upon her graduation in 2004 from St. Thomas Moore Catholic High, she attended Louisiana State University in Baton Rouge, Louisiana. Kristina graduated in 2009 with a Bachelor of Arts and Sciences. She became certified to teach 7-12 grade level of mathematics through the Geaux Teach program at Louisiana State University. She is currently entering her sixth year of teaching algebra I honors and geometry honors at the high school level. She is also a candidate for the Master of Natural Sciences Degree at Louisiana State University.