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The Production of Eta-Mesons in Pion-Nucleon Collisions.

Thomas Alan Moss
Louisiana State University and Agricultural & Mechanical College

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requirements for the degree of
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in

The Department of Physics and Astronomy

by

Thomas Alan Moss
B.S., Louisiana Polytechnic Institute, 1955
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May, 1964
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ABSTRACT

The $\pi + N \rightarrow \eta + N$ reaction in the neighborhood of threshold is studied from a field-theoretical viewpoint. The effects of the second resonant-state of the pion-nucleon system are taken into account and a parameter study of the reaction is performed in which the variable parameters are the coupling constants involved in the reaction and the resonance width of the second resonant-state. The angular distributions and excitation function for the reaction are calculated for various relative magnitudes of the coupling constants and two values of the width. A comparison between these calculated data and experimental angular distributions and excitation functions in the neighborhood threshold is suggested at such time as experimental data in this energy range become available. It is pointed out that this comparison may give an indication of the importance of the effect of the second-resonant state on the $\eta - N$ interaction thus suggesting the possibility of strong d-wave $\eta - N$ interaction at low energy.
CHAPTER I
INTRODUCTION

As is well known, the $\eta$-meson is a member of the pseudoscalar meson octet ($\pi, \eta, K, \bar{K}$) and is therefore one of the most fundamental particles. The investigation of the $\eta - N$ interaction on the basis of field theoretic techniques is intended to contribute to the knowledge of elementary particle interactions. One of the most elementary processes involving the $\eta - N$ interaction is the reaction $\pi + N \rightarrow \eta + N$. In this paper we study this reaction.

We now consider which elementary processes of the type $\pi + N \rightarrow \eta + N$ are experimentally available. Two such processes are

\begin{align*}
\pi^+ + n &\rightarrow \eta + p \quad \text{(1a)} \\
\pi^- + p &\rightarrow \eta + n. \quad \text{(1b)}
\end{align*}

Notice that for two particle final states with initial states of $\pi^+ + p$ and $\pi^- + n$, $\eta$-mesons will not be produced due to conservation of charge. Reaction (1b) might seem to be the most preferable due to the availability of proton targets but it has the serious drawback that two neutrals are produced in the final state making detection difficult. Reaction (1a), on the other hand, presents a problem in obtaining neutron targets. This problem may be overcome by using deuteron targets, thus

\begin{equation}
\pi^+ + D \rightarrow \eta + p + p. \quad \text{(2)}
\end{equation}
The proton may be assumed to play the role of a spectator in this reaction since as pointed out before, $\pi^+ + p$ will not produce $\eta$-mesons (that is to say $\pi^+ + D \rightarrow \eta + n + p$ does not conserve charge). In reaction (2) only one neutral is produced making determination of angular distributions, etc., possible. This reaction has been investigated experimentally by Pevsner, et al. at an incident pion momentum of around 1200 MeV/c.

In our study we shall be concerned with the reaction in the neighborhood of threshold. In this energy range the second resonant-state of the pion-nucleon system $N^{***}(I = \frac{1}{2}, M = 1512$ MeV) should have its most important effects on the reaction while the effects of the third resonant-state $N^{***}(I = \frac{1}{2}, M = 1688$ MeV) should be negligible. If the $N^{***}$ plays an important role in this reaction then its effects will show up clearly in this energy region. Since the $N^{***}$ is the $d_3^n$ resonant-state in the pion-nucleon system, the angular distribution of the produced $\eta$'s will not be isotropic in the center-of-mass system despite the closeness of threshold. In fact calculations show that the angular distribution of the produced $\eta$'s in the reaction $\pi + N \rightarrow N^{***} \rightarrow \eta + N$ just at resonance has a form like $(1 + 3 \cos^2 \theta)$, while the angular distribution from the second-order perturbation terms is almost isotropic. Thus a comparison between experimental angular distributions and calculated ones in the neighborhood of resonance will indicate whether or not the effects of the $N^{***}$ are important. If the effects of the $N^{***}$ are important then the $\eta - N$ interaction in the neighborhood of threshold is predominately a d-wave interaction rather than an s- or p-wave interaction.
At this time no experimental data concerning angular distributions or excitation functions in the neighborhood of threshold are available. The work done by Pevsner, et al., was at energies considerably above threshold where the cross-section is larger, making measurements considerably easier. Then since experimental data are not available in the neighborhood of threshold, a parameter study of the \( \pi + N \rightarrow \eta + N \) reaction is done. In Chapter II we calculate the angular distributions and excitation functions for this reaction assuming various relative weights of the perturbation processes and the effects of the \( N^{\ast \ast} \). We proceed in the calculation of the angular distributions and excitation functions in the following way. The \( N^{\ast \ast} \) is treated approximately in terms of a Rarita-Schwinger particle with spin 3/2. We calculate the matrix elements for the Feynman diagrams corresponding to the perturbation expansion and for the diagram of the process \( \pi + N \rightarrow N^{\ast \ast} \rightarrow \eta + N \). The matrix elements have as parameters the coupling constants for each interaction and in the latter case, the width of the \( N^{\ast \ast} \). We combine these matrix elements and calculate the angular distributions and excitation functions in the neighborhood of threshold for various ratios of the coupling constants involved in one process to the coupling constants involved in the other. The relative weight or importance of each of these processes is governed by the relative magnitude of the coupling constants. Thus angular distributions and excitation functions are calculated for different relative importances of each process.

We anticipate in the near future that experimental angular distributions and the excitation function in the neighborhood of
threshold will become available for the $\pi + N \rightarrow \eta + N$ reaction. Then
by comparing the calculated angular distributions with the measured
ones, the relative magnitudes of the coupling constants may be
estimated thereby indicating the importance of the $N^{***}$ effects. If the
$N^{***}$ plays an important role in the reaction then comparison of the
calculated and experimental excitation functions might give an
indication of the width of the $N^{***}$ resonant-state.
CHAPTER II
CALCULATION OF THE MATRIX ELEMENTS

We now consider the Feynman diagrams which must be included in the calculation of the matrix elements for the reaction \( \pi + N \rightarrow \eta + N \).

First, we must take into account the perturbation diagrams shown in Fig. 1. Next we consider the effect of the \( N^{\ast\ast} \) resonant-state. In this study, the incident pion energy (total pion energy) is restricted to the region 460-580 MeV. The excitation energy of the \( N^{\ast\ast} \) resonant-state corresponds to a pion energy of 470.9 MeV in the center-of-mass system, thus in the energy range considered, it may be expected that the \( N^{\ast\ast} \) has its most prominent effect on \( \eta \)-meson production. The diagrams involving \( N^{\ast\ast} \) are shown in Fig. 2. Of course, only pion-nucleon resonant-states with isotopic spin \( \frac{1}{2} \) must be considered in this calculation since the isotopic spin of the final state \((\eta + N)\) is \( \frac{1}{2} \). Although there are other resonant-states with \( I = \frac{1}{2} \) they have no large effect on the reaction in this energy region. \(^3\) There are many resonant-states in the meson family which have been well established. However, there is no need to take any of these into account as will be shown below. Let us consider the diagrams involving the exchange of a \( \pi, \omega, \rho, f, \) or \( \phi \) (cf. Fig. 3).

1) The \( \pi \)-meson exchange: The \( \pi \)-meson is a \( 1(0^{-}) \) meson, where the notation \( I(J^{PG}) \) means isotopic spin = \( I \), spin = \( J \), parity = \( P \), G-parity = \( G \). The \( \eta \)-meson is a \( 0(0^{+}) \) meson. Thus there is no
Fig. 1. The Perturbation Diagrams.
Fig. 2. The diagrams for the case in which the effect of N is taken into account.
Fig. 3. The Meson Exchange Diagram.
\(\pi - \pi - \eta\) interaction due to the conservation of angular momentum and parity.

2) The \(\omega\)-meson exchange: The \(\omega\)-meson is a \(0(1^-)\) meson, thus \(\eta\)-mesons cannot be produced through one \(\omega\)-meson exchange in the reaction \(\pi + N \rightarrow \eta + N\) due to conservation of isotopic spin.

3) The \(\rho\)-meson exchange: The \(\rho\)-meson is a \(1(1^+)\) meson, thus \(\eta\)-mesons cannot be produced through one \(\rho\)-meson exchange in the reaction \(\pi + N \rightarrow \eta + N\) due to conservation of \(G\)-parity.

4) The \(f\)-meson exchange: The \(f\)-meson is a \(0(2^+)\) meson, thus \(\eta\)-mesons are not produced in the \(\pi + N \rightarrow \eta + N\) reaction for the same reason as in 3).

5) The \(\phi\)-meson exchange: The \(\phi\)-meson is a \(0(1^-)\) meson, thus \(\eta\)-mesons cannot be produced in the \(\pi + N \rightarrow \eta + N\) reaction for the same reason as in 2).

We then see, as stated above, that none of these one-meson exchange diagrams represent an allowable mode for the reaction.

From the above discussion we see that only the diagrams shown in Figs. (1) and (2) must be considered within the meaning of the lowest order coupling constants in order to calculate the matrix elements for the reaction \(\pi + N \rightarrow \eta + N\). We begin by calculating the matrix elements corresponding to the perturbation diagrams. Assuming pseudoscalar coupling between the pseudoscalar fields and the spin \(\frac{1}{2}\) field, the interaction Hamiltonians for the \(\eta - N\) and \(\pi - N\) interactions are given by

\[
H_{\eta N} = g_\eta \, \bar{\eta} \gamma_5 \eta \gamma_\alpha \chi_
\]

\[
H_{\pi N} = g_\pi \, \bar{\pi} \gamma_5 \tau_\alpha \gamma_\alpha \phi_\alpha \quad (\alpha = 1, 2, 3)
\]
where $g_\pi$ = coupling constant for the pion-nucleon interaction.

$g_\eta$ = coupling constant for the $\eta$-nucleon interaction.

$\gamma$ = field operator associated with the nucleon field.

$\phi$ = field operator associated with the pion field.

$\zeta$ = field operator associated with the $\eta$-meson field.

Using the usual rules for Feynman diagrams, the matrix elements corresponding to the diagrams shown in Figs. (1a) and (1b) are given by $R_a$ and $R_b$ respectively where

$$R_a = \frac{(-1)g_\pi g_\eta}{2^2(2\pi)^2} \delta^4(I + P - F - q) \bar{\gamma}(F) \frac{i\gamma_P}{(1 + P)^2 + m^2} \gamma(1) \phi(P) \zeta^*(q)$$ (4a)

$$R_b = \frac{(-1)g_\pi g_\eta}{2^2(2\pi)^2} \delta^4(I + P - F - q) \bar{\gamma}(F) \frac{(-i\gamma_P)}{(1 - q)^2 + m^2} \gamma(1) \phi(P) \zeta^*(q)$$ (4b)

$I$ = four momentum of the incident nucleon.

$F$ = four momentum of the outgoing nucleon.

$P$ = four momentum of the incident pion.

$q$ = four momentum of the outgoing $\eta$-meson.

$m$ = the mass of the nucleon.

These matrix elements must later be combined with that associated with the $N^{**}$ resonance contribution to calculate the differential cross-section for $\eta$-meson production.

Next we consider the effects of the $N^{**}$. Since the $\eta$-meson is a $0(0^{-+})$ meson, and the $N^{**}$ is the $\Delta$ resonant-state in the pion-nucleon system, the interaction Hamiltonians for the $\pi - N - N^{**}$ and $\eta - N - N^{**}$ systems are given by
\[ H_{\pi NN^{*\ast}} = \frac{G_{\pi}}{\mu} \gamma_\lambda \gamma_\nu \frac{\partial \phi}{\partial x_\lambda} + \text{comp. conj.} \quad (5a) \]

\[ H_{\eta NN^{*\ast}} = \frac{G_{\eta}}{m_\eta} \gamma_\lambda \gamma_\nu \frac{\partial \phi}{\partial x_\lambda} + \text{comp. conj.} \quad (5b) \]

where \( \mu \) = mass of the pion.

\( m_\eta \) = mass of the \( \eta \)-meson.

\( G_{\pi} \) = coupling constant for the \( \pi-N-N^{*\ast} \) interaction.

\( G_{\eta} \) = coupling constant for the \( \eta-N-N^{*\ast} \) interaction.

\( \gamma_\lambda \) = field operator associated with the \( N^{*\ast} \) field.

The \( N^{*\ast} \) is considered to be described approximately in terms of a Rarita-Schwinger particle with spin \( 3/2 \). Then the propagation function of the \( N^{*\ast} \) is given by

\[
S(1 + P)_{\mu \nu} = \frac{1}{(1 + P)^2 + \mu^2} \left\{ \delta_{\mu \nu} (i \gamma (1 + P) - M) \right. \\
+ \frac{2}{3} (1 + P)_{\mu} (i \gamma (1 + P)) (1 + P)_{\nu} - \frac{4}{3} M (1 + P)_{\mu} (1 + P)_{\nu} \\
- \left. \frac{i}{3 M} [(1 + P)_{\mu} (i \gamma (1 + P)) \gamma_{\nu} + \gamma_{\mu} (i \gamma (1 + P)) (1 + P)_{\nu}] \right\} \quad (6)
\]

where \( M \) is the mass of the \( N^{*\ast} \), and \( (1 + P) \) its four momentum.

Since the matrix elements corresponding to Fig. 2b are negligible compared to those of Fig. 2a, we take into account only the latter. Making use of Eqs. (5) and (6) the matrix elements corresponding to Fig. 2a are given by...
\[ R_{11} = \frac{1G \pi G_{\eta}}{2^3 (2\pi)^3} \frac{\delta^4 (1 + P - F - g)(AR_3 + BR_0)}{(1 + P)^3 + M^3} \]  

(7)

where \( A = [(IP) - (FP) - \mu^2](M - m) \)

\[ + \frac{2m}{3M^3}[2(IP) - (1F) - (FP) - m^2 - \mu^2](\mu^2 - (IP)) \]

\[ + \frac{4}{3M}[2(IP) - (1F) - (FP) - m^2 - \mu^2][(1P) - \mu^2] \]  

(8)

\[ - \frac{1}{3M}[2(IP) - (1F) - (FP) - m^2 - \mu^2](2(IP) - \mu^2) \]

\[ + 2m/3M^4[2(IP) - (1F) - (FP) - m^2 - \mu^2](\mu^2 - (IP)) \]

\[ + \frac{4}{3M}[2(IP) - (1F) - (FP) - m^2 - \mu^2][(1P) - \mu^2] \]

\[ - \frac{1}{3M}[2(IP) - (1F) - (FP) - m^2 - \mu^2](2(IP) - \mu^2) \]

\[ + \frac{1}{3M}[\mu^2 - 2(IP)][(1P) - \mu^2] - \frac{1}{M^3} \left[ \frac{2}{3}(1 + P)^3 + M^3 \right][2(IP) - \mu^2](M - m). \]

\( B = m[(IP) - (FP) - \mu^2] + \frac{m^2}{3M}[(IP) - \mu^2] \)

\[ + m[2(IP) - (1F) - (FP) - m^2 - \mu^2][1/3M(2/IP)((IP) - \mu^2) + 1] \]

(9)

\[ - 1/M^2(2/3(2/IP) - m^2 - \mu^2) + M^2) \]

\[ - m/M^2[2/3(2/IP) - m^2 - \mu^2] + M^2][2m(m - M) - (1P)] \]

\[ R_3 = \overline{\gamma}(F) \gamma(1) \]  

(10)

\[ R_0 = \overline{\gamma}(F) \frac{1+iP}{m} \gamma(1). \]  

(11)

and the notation \((IP)\) means \(\sum_{i=1}^{L} i_P_i\). Note that all the matrix
elements can be expressed in terms of $R_1$ and $R_2$. We now have obtained the matrix elements $R_1(= R_a + R_b)$ and $R_{||}$ which must be combined in order to calculate the cross-section for $\eta$-meson production.

The matrix element, $R$, for the reaction $\pi^+ N \rightarrow \eta + N$ in the neighborhood of threshold can be expressed as follows

$$R = R_1 + R_{||}$$  \hspace{1cm} (12)

where

$$R_1 = \frac{-iG_{\pi\eta} \delta^4(1 + P - F - q)}{2^2(2\pi)^2} \left[ \frac{R_2}{(1 + P)^2 + m^2} - \frac{R_2}{(1 - q)^2 + m^2} \right]$$ \hspace{1cm} (13)

$$R_{||} = \frac{iG_{\pi\eta}}{2^2(2\pi)^2 \mu m_\eta} \left[ \frac{A R_1 + B R_2}{(1 + P)^2 + M^2} \right]$$ \hspace{1cm} (14)

In the expression for $R_{||}$ (Eq. (14)) we observe that at just resonance energy the denominator $(1 + P)^2 + M^2$ becomes zero. In order to correctly estimate $R_{||}$ in the neighborhood of resonance, the width $\Gamma$ of the $N^{**}$ must be taken into account, by modifying the denominator. The denominator of Eq. (14) can be expressed as

$$(1 + P)^2 + M^2 = E_R^2 - E^2$$ \hspace{1cm} (15)

where $E_R$ = $M$, the total energy at resonance and $E = \sqrt{p^2 + m^2} + \sqrt{p^2 + \mu^2}$, the total energy of the colliding system. Comparing the right hand side of Eq. (15) with the denominator of the Breit-Wigner one-level formula, we see that the width may be introduced phenomenologically in the following way
\[ E_R^2 - E^2 = (E_R + E)(E_R - E) - (E_R + E)(E_R - E - i\Gamma/2) \]  

(16)

The introduction of the width in this manner is more satisfactory for our purpose since a theoretical estimate of the width using the damping theory would involve many unknown coupling constants. The introduction of Eq. (16) into Eq. (14) gives

\[ R_{11} = \frac{iG_{\pi}G_{\eta}}{2a(2\pi)^3 \mu \eta} \left[ \frac{AR_1 + BR_0}{(E_R + E)(E_R - E - i\Gamma/2)} \right]. \]  

(17)

In the calculation of the cross-section we will require the quantity \( |R|^2 \). Then from Eq. (12)

\[ |R|^2 = |R_1|^2 + |R_{11}|^2 + R_1 R_1^* + R_{11} R_{11}^* \]  

(18)

where \( R_1 \) is given by Eq. (13) and \( R_{11} \) by Eq. (17). Next we must average \( |R|^2 \) over the initial spin states and sum \( |R|^2 \) over the final spin states. Indicating by \( \frac{1}{2} \Sigma |R|^2 \) the quantity \( |R|^2 \) summed and averaged we find

\[ \frac{1}{2} \Sigma |R|^2 = \frac{1}{2} \Sigma |R_1|^2 + \frac{1}{2} \Sigma |R_{11}|^2 + \frac{1}{2} \Sigma (R_1 R_{11}^* + R_{11} R_1^*) \]  

(19)

\[ \frac{1}{2} \Sigma |R_1|^2 = \frac{(g_{\pi}^2/4\pi)(g_{\eta}^2/4\pi) m^2}{4(2\pi)^3} \left[ \frac{1}{(1 + P)^2 + m^2} \right] \]  

\[ - \frac{\Sigma (R_1 R_{11}^* + R_{11} R_1^*)}{(1 + P)^2 + m^2} \]  

\[ + \frac{\Sigma |R_0|^2}{(1 - q)^2 + m^2} \]  

(20)
\[ \frac{1}{2} \Sigma | R_{11} |^2 = \frac{G^2 \eta^2}{4\pi \mu^2 m^2} \left( A^2 \left( \frac{1}{2} \Sigma | R_{11} |^2 \right) + AB \left( \frac{1}{2} \Sigma \left( R_{01} + R_{02} \right) \right) + B^2 \left( \frac{1}{2} \Sigma | R_{01} |^2 \right) \right) \]

\[ \frac{1}{2} \Sigma (R_{11}^* R_{11} + R_{11} R_{11}^*) = \frac{-2m\left[ G \eta^2 \eta \right]}{4\pi \mu^2 m} \left( \frac{E_R - E}{D_1} \right) \left[ \frac{1}{D_1} - \frac{1}{D_0} \right] \]

\[ \cdot \left[ A \left( \frac{1}{2} \Sigma R_{01}^* R_1 \right) + B \left( \frac{1}{2} \Sigma | R_{01} |^2 \right) \right] \]

where \( D_1 = (E_R + E) \left[ (E_R - E)^2 + \Gamma^2 / 4 \right] \)

\[ D_0 = (1 + p)^2 + m^2 \]

\[ D_0 = (1 - q)^2 + m^2 \]

The quantities \( \frac{1}{2} \Sigma R_{11}^* R_{11} \) in Eqs. (20), (21), and (22) are given by

\[ \frac{1}{2} \Sigma | R_{11} |^2 = 2(m^2 - (1F)) \]

\[ \frac{1}{2} \Sigma R_{11}^* R_{11} = \frac{1}{2} \Sigma R_{01}^* R_{01} = 2((1P) + (FP)) \]

\[ \frac{1}{2} \Sigma | R_{01} |^2 = 2m^2 \left[ 2(1P)(FP) + \mu^2 (m^2 + (1F)) \right]. \]

We can now express the differential cross-section for the production of \( \eta \)-mesons by the processes discussed above using the
relation

\[ \frac{d\sigma}{d\Omega} = \frac{4\pi^2 |\mathbf{q}| (1+3m)}{|\mathbf{p}| \left( \sqrt{p^2 + m^2} + \sqrt{p^2 + \mu^2} \right)^2} \]  \hspace{1cm} (29)

where \( \mathbf{p} \) = three momentum of the incident pion.

\( \mathbf{q} \) = three momentum of the out-going \( \eta \)-meson.

The total cross-section is, of course, given by

\[ \sigma = \int \frac{d\sigma}{d\Omega} d\Omega. \]  \hspace{1cm} (30)

A study of the angular distribution, \( \frac{d\sigma}{d\Omega} \), and the excitation function, \( \sigma(E_\eta) \), gives the information necessary in studying \( \eta \)-meson production in pion-nucleon collisions.
CHAPTER III
RESULTS AND CONCLUSIONS

In the above discussion it is seen that the angular distribution, \( \frac{d\sigma}{d\Omega} \), and the excitation function, \( \sigma(E_n) \), can be obtained explicitly if the values of \( G_\pi G_\eta \), \( g_\pi g_\eta \), \( M \), and \( \Gamma \) are known. The width has been determined experimentally in at least two experiments. The earlier experiment\(^7\) indicated the width of the \( N^{**} \) to be about 60 MeV while the more recent experiment\(^8\) indicated the width to be about 130 MeV. Although the latter may be more reliable than the former we have done all calculations of the angular distributions and excitation functions for both \( \Gamma = 60 \) MeV and \( \Gamma = 120 \) MeV in order to study both cases. The mass of the \( N^{**} \) is taken to be 1512 MeV as indicated by experiment.\(^9\)

The values of the quantities \( G_\pi G_\eta \) and \( g_\pi g_\eta \), however, are not known to any degree of accuracy. It is therefore necessary to consider these quantities as parameters and perform a parameter study of \( \frac{d\sigma}{d\Omega} \) and \( \sigma(E_n) \) to see how these functions behave as the parameters are varied. The functions

\[
\frac{d\sigma}{d\Omega} \quad \text{and} \quad \frac{\sigma}{(G_\pi^2/4\pi)(G_\eta^2/4\pi)}
\]

or

\[
\frac{d\sigma}{d\Omega} \quad \text{and} \quad \frac{\sigma}{(g_\pi^2/4\pi)(g_\eta^2/4\pi)}
\]
are calculated depending on which is more suitable.

The results of these calculations are shown in Figs. 4 through 19. Figs. 4 through 11 show the angular distributions just at the $N^{\ast\ast}$ resonance ($E_\pi = 471$ MeV) for ratios of $G_{\pi\eta}/g_{\pi\eta} = 10$ to $G_{\pi\eta}/g_{\pi\eta} = 1/10$ with $G_{\pi\eta} = 0$, $g_{\pi\eta} = 0$, and $G_{\pi\eta}/g_{\pi\eta} = -\sqrt{10}$ included. Figs. 12 through 19 show the excitation functions in the energy range $E_\pi = 460$ MeV to $E_\pi = 540$ MeV for the same $G_{\pi\eta}$ and $g_{\pi\eta}$ values. These results can be divided into the following five cases for the purposes of discussion:

1) $g_{\pi\eta} = 0$: In this case the second-order perturbation processes do not contribute and the angular distribution is determined by the effects of the $N^{\ast\ast}$ resonant-state. From Fig. 10 the form of the angular distribution is seen to be $(1 + 3 \cos^2 \theta)$. The excitation function at resonance is rather flat and then increases sharply with increasing energy as seen in Fig. 18.

2) $G_{\pi\eta} = 0$: In this case the $N^{\ast\ast}$ resonant-state has no effect on the angular distributions or the excitation function of the produced $\eta$'s. The angular distribution is very slightly peaked backward as seen in Fig. 9 and the excitation function shown in Fig. 17 rises just above threshold (at the $N^{\ast\ast}$ resonance energy) and then levels off with increasing energy.

3) $G_{\pi\eta}/g_{\pi\eta} > 1$: In Figs. 4 and 5 the angular distributions have minimum values at $\cos \theta = 0$ showing the effect of the $N^{\ast\ast}$ on the reaction. The excitation functions shown in Figs. 12 and 13 also have steep slopes at energies above resonance characteristic of the $N^{\ast\ast}$ effects.
Fig. 4. The Angular Distribution of the Produced $\eta$-Mesons at Resonance for $G_{\pi\eta} / G_{\pi\eta} = 10$. 
Fig. 5. The Angular Distribution of the Produced $\eta$-Mesons at Resonance for $G_\pi^2 G_\eta^2 / g_\pi g_\eta = \sqrt{T_0}$. 
Fig. 6. The Angular Distribution of the Produced η-Mesons at Resonance for $G_{\pi^+\eta}/g_{\pi^0\eta} = 1$. 
Fig. 7. The Angular Distribution of the Produced $\eta$-Mesons at Resonance for $G_{\pi\eta}/g_{\pi\eta} = 1/\sqrt{10}$. 
Fig. 8. The Angular Distribution of the Produced $\eta$-Mesons at Resonance for $G_{\pi\eta}/g_{\pi\eta} = 1/10$. 

\[ \frac{d\Phi}{d\cos \theta} \text{ (mb/steradian)} \]
Fig. 9. The Angular Distribution of the Produced $\pi$-Mesons at Resonance for $G_{\pi\eta\eta} = 0$. 
Fig. 10. The Angular Distribution of the Produced $\eta$-Mesons at Resonance for $g_{\pi\eta} = 0$. 
Fig. 11. The Angular Distribution of the Produced $\eta$-Mesons at Resonance for $G_{\pi\eta}/G_{\pi\pi} = -\sqrt{10}$. 

$\frac{d\Gamma}{d\Omega}(2\pi)$ (mb/steradian) vs $\cos \theta$.
Fig. 12. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$ for $G \pi \eta / g_{\pi \eta \eta} = 10$. 
Fig. 13. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$ for $G_{\pi \eta}/g_{\pi \eta} = \sqrt{10}$. 
Fig. 14. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$ for $G_{\pi \eta}^G / g_{\pi \eta} = \gamma$. 
Fig. 15. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$
for $G_\pi^2 G_\eta^2 / g_{\pi\pi}^2 g_{\pi\eta}^2 = 1/\sqrt{\theta}$. 
Fig. 16. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$
for $G_{\pi \eta}/g_{\pi \eta} = 1/10$. 

$\sigma/(G_{\pi \eta}^2)$ versus $E_\pi$ (MeV) for $\Gamma = 60$ MeV and $\Gamma = 120$ MeV.
Fig. 17. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$
for $G_{\pi \eta} = 0$. 
Fig. 18. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$ for $g_{\pi N}^{\eta} = 0$. 
Fig. 19. The Excitation Function for the Reaction $\pi + N \rightarrow \eta + N$
for $G_{\pi\eta}/g_{\pi\eta} = -\sqrt{\Gamma_0}$. 
4) $0 < \frac{G_{\pi \eta}}{g_{\pi \eta}} \leq 1$: In this case the angular distribution is rather flat as seen in Figs. 6, 7, and 8 showing the dominance of the perturbation effects. The excitation functions shown in Figs. 14, 15, and 16 also indicate the importance of the perturbation effects relative to the effects of the $N^{**}$.

5) $\frac{G_{\pi \eta}}{g_{\pi \eta}} = -\sqrt{10}$: For this ratio the angular distribution displays the minimum characteristic of the $N^{**}$ effects as seen in Fig. 11. The angular distribution at resonance has the same shape as for the case of $\frac{G_{\pi \eta}}{g_{\pi \eta}} = \sqrt{10}$. However, at higher energies the angular distributions are not the same since the interference terms have different signs in the two cases, as seen in Fig. 20. Of course, the excitation functions are also different for these two cases.

Fig. 19 shows the excitation function for $\frac{G_{\pi \eta}}{g_{\pi \eta}} = -\sqrt{10}$. Comparing this curve with Fig. 13 showing the excitation function for $\frac{G_{\pi \eta}}{g_{\pi \eta}} = \sqrt{10}$, we see that the excitation function for $\frac{G_{\pi \eta}}{g_{\pi \eta}} = -\sqrt{10}$ rises more sharply with increasing energy than for the case of $\frac{G_{\pi \eta}}{g_{\pi \eta}} = \sqrt{10}$.

When experimental curves of the angular distributions and the excitation function in the neighborhood of threshold become available for the $\pi + N \rightarrow \eta + N$ reaction, comparison of these experimental data with the calculated data above will yield information concerning the $\eta - N$ interaction. These comparisons may be made in the following way. The calculated angular distributions for the various $\frac{G_{\pi \eta}}{g_{\pi \eta}}$ ratios may be fitted to curves of the form $A_1(E) + B_1(E) \cos \theta + C_1(E) \cos^2 \theta$ where the coefficients $A_1(E)$, $B_1(E)$, and $C_1(E)$ depend on the incident pion energy and the ratio $\frac{G_{\pi \eta}}{g_{\pi \eta}}$. It is then expected that it will
Fig. 20. The Angular Distribution of the Produced $\eta$-Mesons for $E_\pi = 520$ MeV.
be possible to fit the experimental angular distributions in the neighborhood of resonance to curves of the same form with coefficients $A(E)$, $B(E)$, and $C(E)$. Now if the relations

$$A(E)/C(E) = A_1(E)/C_1(E) \tag{31}$$

$$B(E)/C(E) = B_1(E)/C_1(E) \tag{32}$$

hold for each value of $E$, the pion energy, for some set of coefficients $A_1(E)$, $B_1(E)$, and $C_1(E)$ corresponding to some predetermined $G_{\pi\eta}/g_{\pi\eta}$ ratio then the value of $G_{\pi\eta}/g_{\pi\eta}$ is determined. If no set of coefficients $A_1(E)$, $B_1(E)$, and $C_1(E)$ satisfy Eqs. (31) and (32) then the proper $G_{\pi\eta}/g_{\pi\eta}$ ratio may not be included in the calculated data. In this event the ratios $A_1(E_R)/C_1(E_R)$ and $B_1(E_R)/C_1(E_R)$ may be plotted as functions of the $G_{\pi\eta}/g_{\pi\eta}$ ratios where $E_R$ indicates the energy at resonance. Then by comparing these two curves with the ratios $A(E_R)/C(E_R)$ and $B(E_R)/C(E_R)$ it can be determined if there exists any ratio $G_{\pi\eta}/g_{\pi\eta}$ for which Eqs. (31) and (32) hold with $E = E_R$. Then if Eqs. (31) and (32) do hold for the same $G_{\pi\eta}/g_{\pi\eta}$ ratio the angular distributions and excitation function may be calculated for that particular value of $G_{\pi\eta}/g_{\pi\eta}$. From these data it can be determined whether or not Eqs. (31) and (32) hold for several values of the incident pion energy in the neighborhood of resonance. If they do hold then the ratio $G_{\pi\eta}/g_{\pi\eta}$ is determined.

If the ratio $G_{\pi\eta}/g_{\pi\eta}$ can be determined by the above methods, this means that the experimental angular distributions can be described in terms of a single parameter $G_{\pi\eta}/g_{\pi\eta}$ independent of energy in the neighborhood of threshold. In other words, the reaction $\pi + N \rightarrow \eta + N$ can be described well by means of the perturbation theory if only the
effects of $\pi^*$ are taken into account. If the ratio $G_{\pi \eta} / g_{\pi \eta}$ cannot be determined, this means that the analysis is incomplete and that some higher order effects should be considered.

Assuming a ratio $G_{\pi \eta} / g_{\pi \eta}$ can be determined by the above procedure, then we shall be able to estimate the values of the products $G_{\pi \eta}$ and $g_{\pi \eta}$ by comparing $\sigma / (4\pi) (\pi \eta)$ (or $\sigma / (4\pi) (\pi \eta)$) with the observed values of the total cross-section. Of course, the width of the $N^{\ast \ast}$ can best be determined from pion-nucleon scattering or photo-meson production experiments. It should be noted, however, that the shape of the calculated excitation functions for the $\pi + N \to \eta + N$ reaction in the cases in which the $N^{\ast \ast}$ plays an important role depend, to some extent, on the value of the width. Thus it might be possible to get an indication of which value of $\Gamma$ (60 MeV or 120 MeV) is better from comparison of calculated and experimental excitation functions.

From the above discussions it can be seen that when experimental angular distributions and the excitation function for the reaction $\pi + N \to \eta + N$ in the neighborhood of threshold become available, by comparing these data with the above calculated data the relative importance of the $N^{\ast \ast}$ resonant-state in this reaction may be estimated and thus the significance of the d-wave contribution to the $\eta - N$ interaction may be assessed. Further, the relative values of the coupling constants involved may be estimated and an indication of which value of the width is better may be obtained.

Additional information may be obtained concerning the $\eta - N$ interaction by comparing the information which may be deduced from the above analysis with information from a similar analysis of the
$\gamma + p \rightarrow \eta + p$ reaction. This process has been studied in another paper and in that study the calculations were parameterized in much the same way as the above calculations. The parameters involved in the calculations for the $\gamma + p \rightarrow \eta + p$ reaction are $G_\gamma \eta$, $g_\eta$, and $\Gamma$, where $G_\eta$, $g_\eta$, and $\Gamma$ are as defined above and $G_\gamma$ is the coupling constant for the $\gamma - N - N^*$ vertex. Since $G_\eta$, $g_\eta$, and $\Gamma$ are the same in both processes, comparison of the results of the analysis of the experimental data in these two reactions should yield additional information. At this time no experimental angular distributions or excitation functions are available for the $\gamma + p \rightarrow \eta + p$ reaction in the neighborhood of threshold. Assuming this information becomes available in the near future an analysis may be made of this data in the same manner as discussed above for the $\pi + N \rightarrow \eta + N$ reaction. And in a similar manner the results of this analysis should yield values of the ratio $G_\eta / g_\eta$ and possibly some indication of the better value of $\Gamma$.

Then combining these results

$$\frac{G_\pi \pi \eta}{G_\gamma \eta} = \text{known constant}$$

or

$$G_\pi / G_\gamma = (\text{known constant}) \cdot (g_\eta / e).$$

Thus the ratio $G_\pi / G_\gamma$ can be found. It will be of physical interest to see if $G_\pi / G_\gamma > 1$ as it should be.
REFERENCES


2. The pion energy in the center-of-mass system at threshold for the $\pi + N \rightarrow \eta + N$ reaction is 454 MeV. The pion energy in the center-of-mass system at resonance for the reaction $\pi + N \rightarrow N^{*\pi} \rightarrow \eta + N$ is 471 MeV.

3. The latest information indicates the possibility of a $P_{11}$ resonant-state in the pion-nucleon system, but its existence is not well established.


VITA

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