2012

Integrating tasks, technology, and the Common Core Standards in the Algebra II classroom

Beth Perkins McInnis
Louisiana State University and Agricultural and Mechanical College

INTEGRATING TASKS, TECHNOLOGY, AND THE COMMON CORE STANDARDS IN THE ALGEBRA II CLASSROOM

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Natural Sciences

in

The Interdepartmental Program in Natural Sciences

by
Beth McInnis
B.S., University of New Orleans, 1990
M. Ed., University of New Orleans, 1999
August 2012

ACKNOWLEDGMENTS

I would like to thank my family and friends for their prayers, patience, support, and encouragement through this process.

I would like to thank and acknowledge Dr. Frank Neubrander and Dr. James Madden for their commitment to improve math education in Louisiana through LaMSTI. I would also like to thank Dr. Robert Perlis and Dr. Ameziane Harhad for serving on my committee.

I would like to thank The Dunham School and my students who supported me every step of the way with this endeavor.

Finally, I would like to express my gratitude and to congratulate my fellow 2010 LaMSTI colleagues. May the relationships formed over the last three summers continue to build strong friendships as we begin this next chapter in our lives.

TABLE OF CONTENTS

ACKNOWLEDGMENTS……………………………………………………………..........ii

ABSTRACT…………………………………………………………………………….....v

INTRODUCTION……………………………………………………………………...1

CHAPTER 1. BACKGROUND INFORMATION……………………………………….5

1.1 Curriculum Alignment to the CCSSM……………………………………………...5

1.2 The 21st Century Classroom………………………………………………………...9

1.2.1 Task Based Activities…………………………………………………………11

1.2.2 The Student Centered Classroom……………………………………………..12

1.2.3 Formative Assessment………………………………………………………...15

1.3 Technology at The Dunham School………………………………................16

1.3.1 MathXL………………………………………………………………………16

1.3.2 One-to-One Environment……..……………………………………………...20

CHAPTER 2. SAMPLE ALEGBRA II TASKS………………………………………...24

2.1 Interpreting Distance -Time Graphs...……………………………………………...24

2.1.1 Introduction……………………………………………………………………..24

2.1.2 Student Centered Group Task Day 1…………………………………………..28

2.1.3 Student Centered Group Task Day 2…………………………………………..30

2.1.4 Student Centered Group Task Day 3…………………………………………..33

2.1.5 Student Centered Group Task Day 4…………………………………………..33

2.1.6 Reflection…………………………..34

2.2 Maximizing Volume: The Open Crate Problem……………………………………..45

2.2.1 Introduction……………………………………………………………………..45

2.2.2 Student Centered Group Task Day 1…………………………………………...48

2.2.3 Student Centered Group Task Day 2…………………………………………...52

2.2.4 Reflection……………………………………………………………………………….53

2.3 Car Task………………………………………………………………………………..59

2.3.1 Introduction…………………………………………………………………......59

2.3.2 Individual Student Task……………………………………………………….61

2.3.3 Reflection……………………………………………………………………….64

CHAPTER 3. CONCLUDING THOUGHTS………………………………………………...67

REFERENCES………………………………………………………………………………..71

APPENDIX

A: STANDARDS FOR MATHEMATICAL PRACTICE……………………….............73

B: SUMMER MATH INFORMATION………………………………………………..76

C: APPLICATIONS INVOLVING QUADRATICS…………………………………..78

D: MAXIMIZING VOLUME TASK……………………………………………………..80

E: RUBRIC FOR MAXIMIZING VOLUME TASK…………………………………..83

F: STUDENT REPSONSES TO MAXIMIZING VOLUME TASK…………………..84

G: MODIFIED RUBRIC FOR ASSESSING TASKS………………………………….85

H: SAMPLE EXPERT TASK FROM MAP WEBSITE……………………………….86

I: SAMPLE APPRENTICE TASK FROM MAP WEBSITE…………………………87

J: CAR TASK………………………………………………………………………………..89

K: STANDARDS ADDRESSED IN EACH TASK…………………………………...91

VITA……………………………………………………………………………………….…..94

ABSTRACT

The Common Core State Standards for Mathematics were released in June of 2010. The standards were developed by a team of over 75 teachers and specialist in response to improve math education in the United States through more focused, coherent, rigorous standards to help our students be competitive in the 21st century. As of June 2012, 45 states had adopted the new standards which better coordinate what students at individual grade levels should know and be able to do to in order to be college and career ready by grade twelve. With new computer based assessments being developed and set to be given to students as early as 2014 to assess understanding of these standards, it is necessary for teachers to begin implementing instructional shifts in the classroom that prepare students for both the Standards for Mathematical Content and the Standards for Mathematical Practice. The concept of dual intensity emphasizes that both procedural and conceptual skills are of equal importance in the classroom and changes should be made to provide opportunities for students to experience both in an atmosphere that is rigorous and intense. Providing students opportunities to demonstrate The Standards for Mathematical Practice will involve the most change in the classroom. It will involve a change in the classroom environment involving the roles of both the teacher and the student. This thesis discusses how the use of tasks and technology were used in the Algebra II classroom to implement the Common Core Standards and describes student misconceptions and lesson revisions for future use that include connections to calculus. The process of formative assessment was used to provide information to both the teacher and the student intended to improve teaching and learning in the classroom. Information gained from the formative assessment reinforced the need to provide more opportunities for students to connect the Standards for Mathematical Content and the Standards for Mathematical Practice using tasks and technology.

INTRODUCTION

The release of the Common Core State Standards for Mathematics (CCSSM) in June of 2010 has prompted much discussion and debate among districts and classroom teachers in the United States regarding the benefits and challenges of implementing national standards. As of June, 2012 over 45 states had adopted these new standards. The CCSSM were developed by the National Governors Association Center for Best Practices and the Council of Chief State School Officers in response to the need for a more focused and coherent set of national standards. According to Kober & Rentmer (2011), the development of the national standards "grew out of concern that the array of different standards in every state is not adequately preparing students in our highly mobile society with the skills needed to compete globally" (p. 2). According to the website http://www.corestandards.org the mission statement for the initiative is as follows:

> The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

This effort extends on the work started by the National Council of Teachers of Mathematics (NCTM) in 1989 with the release of the *Curriculum and Evaluation Standards for School Mathematics*. The original goals of the NCTM initiative were to "delineate a vision of school mathematics sufficient to prepare students for the 21st century and, in doing so, to detail what mathematics these students should know and be able to use" (Crosswhite, Dossey, & Frye, 1989, p. 515). In 1999, NCTM released *Principles and Standards for School Mathematics*, which added Process Standards to Content Standards for school mathematics and clarified and elaborated the 1989 Standards. In 2006, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* extended that work and described the most

significant concepts and skills at each grade level. *Focus in High School Mathematics: Reasoning and Sense Making* released in 2009 detailed NCTM's vision of the critical roles of reasoning, communication, connections, and mathematical problem solving in the teaching and learning of all content in secondary mathematics. The work done by the CCSSM is an attempt to build upon and expand the work started by NCTM and cannot be looked at as a totally distinct and different initiative. Instead, the work done by NCTM should be used to supplement and provide additional background for the work done by the CCSSM. In February 2011, NCTM released *Making It Happen A Guide to Interpreting and Implementing Common Core State Standards for Mathematics*. The work done by NCTM with standards-based math reform can be used as a tool for educators to aid in the implementation of the Common Core State Standards. The adoption of the CCSSM is not a totally new approach to teaching math, but more emphasis is added to mathematical practices and opportunities for students to demonstrate their ability to secure mathematical knowledge by completing problems that are interwoven with new concepts and application of prior knowledge. In addition, the CCSSM it is an attempt to better coordinate what students at individual grade levels should know and be able to do in order to be successful in the technological world that we live in today.

According to the website http://www.parcconline.org/about-parcc the Partnership for the Assessment of Readiness for College and Careers (PARCC) is working to develop a set of computer based assessments for grades 3-12 that will assess what is in the CCSSM and what skills students need to be college and career ready by the end of high school. This is similar to work being done by the Smarter Balanced Assessment Consortium (SBAC) that is also working on creating computer-based assessments linked to the CCSSM and college and career readiness (http://www.smarterbalanced.org/about/). The SBAC assessments are similar to the ones being

created by PARCC, but they will include an adaptive technology component, which will individualize student questions based on student responses. For example, the difficulty level of a question will be adjusted throughout the assessment based on student answers. These new assessments are set to be implemented in the 2014 – 2015 school year and will include two summative mandatory computer-based assessments that are designed to provide feedback to determine if students are on track for college and career readiness and to provide data regarding student performance toward the standards. In addition, there will be optional non-summative computer-based assessments that will provide information to teachers that can be used to determine interventions needed or assistance for future lesson design and implementation by tailoring professional development for teachers to meet the needs found based on the assessment results.

In order to prepare students for these assessments and to aid in the ability for students to be college and career ready, the next phase after adopting the standards is the successful implementation of the standards in the classroom. This thesis will describe the process used at an independent, private school in Baton Rouge regarding the implementation of the Common Core Standards specifically in the Algebra II classroom and how the classroom environment was changed to implement these standards. Although private schools will not be mandated to implement the PARCC tests, it is evident that this movement will directly impact private school students, as the educational goal for our students is acceptance and success in college. As stakeholders begin the process of collaborating with higher education institutions to determine college and career readiness indicators, it is our duty to make sure that our students are prepared for these expectations and given opportunities to demonstrate these skills. The main goal of this thesis is to increase awareness of the standards and how they may differ from what is currently

being done in the classroom and to discuss changes that were implemented in the classroom to promote the Standards for Mathematical Practice. Implementation of the standards is more than just rearranging what content is taught at certain grade levels using the standard teacher directed method of instruction where the emphasis is on procedural knowledge or the vertical alignment of the curriculum. Although content is important, the new emphasis will be on creating an environment that balances both procedural and conceptual development for students. This thesis will describe the process used to locate and refine rich problems that provide opportunities for students to demonstrate application of a wide range of content standards and to communicate solutions using a variety of representations and technology available in the classroom at our school. One of my key objectives was to be able to provide students with opportunities that would require them to make connections between mathematical topics and to build upon prior learning of mathematical concepts.

The process that I used in the 2011-2012 school year involved four phases and is an ongoing reflective practice of review and revision with the goal of improving teaching and learning in the classroom so that students are college and career ready when they graduate. The first phase was a collaborative effort among the math teachers to align current curriculum to the new standards. The second phase involved locating and revising currently available resources to create opportunities for students to demonstrate understanding of both the Standards for Mathematical Content and the Standards for Mathematical Practice. The third phase involved incorporating the use of available technology to support practice, collaboration, communication, and multiple representations of solutions in the classroom. The fourth phase was the process of reflecting on how to improve the implementation of the CCSSM in the Algebra II classroom and making revisions and suggestions to be implemented in the upcoming school year.

CHAPTER 1. BACKGROUND INFORMATION

1.1 Curriculum Alignment to the CCSSM

The Common Core State Standards are comprised of Standards of Mathematical Practice and Standards of Mathematical Content. Together these standards comprise the conceptual and procedural knowledge that students need to develop in order to be college and career ready by the end of grade twelve. PARCC and SBAC assessments are currently being designed and piloted with a full implementation scheduled in the 2014-2015 school year to assess student progress towards these standards. In October of 2010, the Center on Education Policy surveyed states regarding progress and challenges in implementing the Common Core State Standards. Responses were received from 42 states and listed below is a summary of some of the key findings (Kober & Rentmer, 2011).

- Most of the responding states that have adopted the new standards plan to make related changes in assessment, curriculum, teacher policies, and other areas, but timelines vary. Many states anticipate that it will take until 2013 or later to fully implement complex changes.
- Most states are expecting, rather than requiring, districts to undertake such activities as developing new curriculum materials and instructional practices, providing professional development to teachers and principals, and designing and implementing teacher induction programs and evaluations related to the standards.
- Developing curriculum materials tied to the standards and implanting new assessments aligned with the standards are major implementation challenges.
- Many states are requiring school districts to implement the common core state standards, but most are not requiring districts to initiate new programs or practices to support or complement implementation.

Although the role of the teacher is an important factor in the effective implementation of innovations in the classroom, the findings stated above indicate that in many states teachers will have only limited support and that the implementation of the standards may result in the teacher having to search for high quality instructional materials to support implementation and assessment of the standards. Also, since students will be expected to perform on new

assessments as early as 2014, it is equally important for students to be given ample opportunities to prepare for these new types of assessments. The new PARCC and SBAC assessments that are being developed will be used to provide information regarding college and career readiness for students and to provide data related to the success of implementation of the standards in the classroom. Since I teach at a private school, we do not have the benefit of support or training from the district or state level, so I find myself in a position similar to what many teachers at public schools may face in regards to support and implementation.

According to the website www.dunhamschool.org the first two lines of the school mission statement are as follows:

> The Dunham School seeks to provide students with the opportunity for college preparatory education set in the framework of Christian instruction and example. To this end, the school offers boys and girls rigorous academic instruction, challenging athletic and recreational activities, and creative expression in the fine arts.

This statement is engrained in the faculty and administrators at our school. As a result, there is a desire by the teachers to provide students with the most rigorous academic program of study and a commitment by faculty to professional growth and development. Although not required, many of our teachers participate in professional development opportunities over the summer as there is a commitment to life long learning among the faculty. As an example, this summer we will have over half of our core fifth – twelfth grade teachers either attending Advanced Placement or Pre AP institutes, working on a summer research program at Notre Dame, attending science workshops sponsored by universities, participating in College Readiness Programs, or completing graduate degrees. In addition to the faculty commitment to learning, beginning in the summer after sixth grade students and their parents meet with the college counselor and principal to map out a six-year course plan of study that is most challenging to the individual student based on test scores, classroom data, interest, and abilities. Students in the middle school are able

to accelerate into honors level high school credit courses to extend opportunities for more exposure to advanced placement courses in high school or dual enrollment credit opportunities. All of this goes back to the mission of the school to provide opportunities for students for college preparatory education.

Over the last several years we have taken on an analysis and review of our school curriculum. The math department has researched a variety of standards to implement a program of study for our students which include NCTM , College Board, standards from other states, as well as the Louisiana Grade Level Expectations (GLEs). Our school is in a unique position since we serve students in grades pre-kindergarten through twelfth grade. Teachers are able to collaborate and discuss the vertical alignment of our math curriculum. One tool that we use to accomplish this collaboration is the use of the Rubicon Curriculum Mapping program. Curriculum Mapping is a procedure for collecting data about the actual curriculum being taught in the classroom using a web-based program and the school calendar to organize information in an electronic format (Jacobs, 1997). Teachers are also able to upload documents such as projects and assessments given in the course. All teachers have computer access to see what curriculum is being taught at each grade level, what resources or assessments are being used, and at what time in the school year the content is being taught. During the initial use of the program the teachers simply used the program as a diary map. It gave a picture as to what was actually being taught in the classroom. Once the diary maps were completed, teachers began the process of reviewing maps for gaps and overlaps in the curriculum. Changes were submitted to administrators for review to determine if a change was warranted. GLEs were used as a minimum expectation, but the curriculum also infused some of the other sources listed previously. This refinement process continued for the next several years. The release of the Common Core Standards connects all of

these pieces of information from the different stakeholders in math standards to provide a pathway for students to be college and career ready. I was first exposed to the Common Core State Standards during my graduate program of study in the summer of 2010. Preliminary work was done on becoming familiar with the standards both on a personal level and a result of a graduate course taken that summer. Since I do not teach at a public school, we are not required to implement the Louisiana Comprehensive Curriculum or the state Grade Level Expectations. Our students are not mandated to be part of the high stakes testing or the state accountability system, but we do use other tests such as the PSAT, SAT, PLAN, EXPLORE, and ACT to evaluate student performance. We also have a common desire to see that our students are college and career ready. Even though our students may not be required to take the PARCC or SBAC assessments currently being developed, being knowledgeable about these assessments and the role that they may play in college readiness is something that we want to make sure we are addressing in our program of study at the school. As a result, a decision was made by administrators and math teachers to implement the CCSSM at our school.

During the 2010-2011 school year all math teachers began the process of aligning what was in our current curriculum maps to the CCSSM. For grades K-8, there are content specific standards for what should be taught at each grade level. For the high school courses, there are suggested model course pathways. The first part of the process involved matching the standards to what is currently being taught in the classroom. The second phase involved reviewing the suggested pathway and comparing the suggestions to what was currently being taught. The third part involved looking for gaps and overlaps in the maps. At this point, the decision was made not to adopt the model course pathways in their entirety, and only minor adjustments were made to the curriculum for the upcoming year. As we transition into full adoption of these standards, it is

expected that shifts in the curriculum will occur. It was noted that there was an emphasis on breadth of coverage as opposed to depth of coverage in our current curriculum, so in the future some course content may be removed or combined with other topics at other grade levels. After creating the units and aligning the standards, it became evident that our classrooms were dominant on the procedural aspect of the content standards. It was determined that in order to implement the practice standards of the CCSSM (see Appendix A), learning experiences that provide opportunities for students to experience situations that would connect the Standards for Mathematical Practice with the Standards for Mathematical Content needed to be added to the learning environment. One new feature of the mapping program allows CCSSM to be aligned to specific units and linked to assessments. The reports that can be run will enable us to analyze the extent to which standards are assessed and how those standards are assessed. We hope to utilize this feature in the future as we continue to refine our curriculum.

1.2 The 21st Century Classroom

Both procedural and conceptual knowledge are related to the process of teaching math for understanding (Eisenhart et al., 1993). Teaching and learning mathematics for understanding requires a change to the traditional classroom roles of teacher and student. Opportunities for students to demonstrate an understanding of math concepts involve the selection of problems that develop students' ability to reason. In the article, *What is Mathematics For,* Underwood Dudley (2010) refers to the idea behind teaching mathematics as one that is not to provide students with real world math problems that they may encounter outside of the math classroom, but to teach reasoning. He states, "Reasoning needs to be learned, and mathematics is the best way to learn it" (p. 612). Many teachers believe that they are implementing new reform standards, but are actually still rooted in the traditional methods of teaching (Hiebert et al., 2005). Results from the

TIMMS 1995 video study validate this claim. Video comparisons comparing an 8th grade Japanese classroom and an 8th grade United States classroom show vast differences in the experiences of both the teacher and the students. In the Japanese classroom the teacher would pose a problem, students would reflect on the problem and present ideas, the teacher would summarize information and then the students would spend time practicing problems. In contrast, in the United States classroom the teacher would instruct and solve problems and then the students would practice problems. The majority of the practice problems in United States classrooms were devoted to practicing procedures, whereas in the Japanese classroom the time was split evenly between practicing problems and collaborating on procedures and analyzing solutions to new and different problems in multiple ways (Wiggins & McTighe 2005, Hiebert et al., 2005).

A new concept of dual intensity has appeared in some state documents related to instructional shifts needed for implementation of the Common Core Standards in the classroom (http://engageny.org/resource/common-core-shifts/). The idea is that students are both practicing and understanding mathematics in the classroom with more than a balance between the two but both occurring with intensity. The use of MathXL in the classroom has enabled me to create opportunities for students to practice problems and has provided more time in the classroom to create opportunities for students to extend those concepts to problems that demonstrate student reasoning and conceptual understanding of the content. Finding the equilibrium point between procedural and conceptual knowledge will involve a myriad of instructional practices and assessment forms to provide optimal opportunities for intensity of content and practice skills. One of the instructional shifts that I used involved the use of tasks to assess student understanding of procedural and conceptual knowledge.

1.2.1 Task Based Activities

"Mathematical understanding and procedural skill are equally important and both are assessable using math tasks of sufficient richness" (CCSSM, 2010, p. 4). What is a mathematical task? The Mathematics Assessment Project (MAP) defines a task as a problem that provides opportunities for "students to use their mathematics in routine or non-routine situations to design, plan, estimate, evaluate and recommend, review and critique, investigate, represent information, explain, define concepts, and show their skills in routine technical exercises" (http://map.mathshell.org/materials/index.php). Expectations that begin with "understand" are good opportunities to connect practices and content (CCSSM, 2010). What is the meaning of understanding? Bloom (1956) defines understanding as the ability to use skills and facts wisely and appropriately through effective application, analysis, synthesis, and evaluation. Douglas Newton (2000) discusses how understanding cannot be transmitted to students, but can be supported by engaging the learner in experiences that allow the learner to construct knowledge. Understanding involves more that just being able to get a correct answer, but rather a student needs to be able to explain why a particular skill or approach is or is not appropriate for a situation (Wiggins & McTighe, 2005). Tasks may take on different formats, be of different lengths, and serve different assessment purposes. The tasks used in this thesis were designed to have students make connections between procedural and conceptual processes, to support student application of mathematical knowledge, and to provide opportunities for students to demonstrate secured mathematical knowledge. Secured mathematical knowledge in this context refers to the student's ability to use prior mathematical knowledge from previous math courses or current course lessons to complete a problem. The initial task was done in a group setting and reinforced the concept of multiple representations. It also utilized technology and contained an extension

that connected several mathematical topics studied in earlier courses with current content. The topics addressed in the first task included: distance versus time graphs, equations of lines, the concept of domain, piecewise linear functions (which will be referred to as piecewise functions), function notation, and the correct use of mathematical symbols. The second task was also done in a group setting, but the format was intended to guide students through the modeling process. Students were to determine which type of representation to use when solving the problem. Students were given access to graphing calculators and Geometer's Sketchpad software. The last task was an individual task that again required students to make connections and use prior knowledge to determine how to solve the problem. Technology was also available to students for the last task.

Curriculum should be organized around tasks that engage students in reflection and communication (Hiebert et al., 1997). The authors Danielson and Marquez (1998) suggest a framework for teachers to use when selecting tasks for use in the classroom. Tasks should be engaging, authentic, and aligned to instructional goals, which may require making adjustments to the task. It may also be necessary to enhance the relevance of the task to reflect the group of students. Directions should be clear and concise and align to a grading rubric if the task is used for summative assessment. The authors also recommend starting small, suggesting four – six tasks per year in the first year. Teachers should also reflect on tasks once they are given in class and revise accordingly. A recommendation was made to pilot tasks in the classroom for a year.

1.2.2 Student Centered Classroom

When implementing tasks in the math curriculum there are certain features that are essential for classrooms to successfully foster an environment that promotes mathematical understanding (Hiebert et al., 1997). The role of the teacher will change from the sole

disseminator of information to a facilitator in the learning process. Students will be exposed to various ways of solving problems and the teacher will need to allow time for learning to be constructed and be available to clarify and summarize methods for solving problems presented by students. Sullo (1999) discusses the impact of a positive environment on learning. He states, "In a positive environment we want to learn more, share more, make new connections, and continue the process of discovery" (p. 94). Tasks that engage students in reflection and communication will require a collaborative, safe community in the classroom. Students will need to understand the role of math tools in solving problems and be open to various methods and approaches for solving problems.

According to Glasgow (1997), in the teacher centered classroom it is the role of the teacher to distribute and interpret information for the students via lectures, readings, demonstrations, and selected activities. The teacher is the primary person doing the work and the students take a more passive role in the learning experience. In contrast, in the student centered classroom it is the student who takes responsibility for the learning and becomes an active participant in the process. Advantages of the student centered classroom involve students "learning to learn" so that they can problem solve outside of the classroom and the students learn to evaluate their own thinking and that of their peers to evaluate strengths and weaknesses in a solution (Glasgow, 1997).

A Dunham distinctive is the use of the Harkness method of instruction in Humanities courses. In 1930, Edward Harkness suggested to the principal of Philips Exeter Academy the idea of a method of instruction where students sit around a table and discuss course content with the teacher (http://www.exeter.edu/admissions/109_1220_11688.aspx). As a result the Harkness method was established. In this method teachers are trained to guide students in the discovery of

information through dialogue. Students are responsible for reading and preparing for discussions. The teacher is responsible for tracking student progress and making sure that the topics stay focused, but the students drive the discussion. The Harkness director at The Dunham School describes the Harkness method as follows:

> Learning is a collaborative effort in a Harkness classroom where each student has a vested interest in the discussions. Students learn to annotate the text and bring to "the table" their observations, questions and prejudices from the reading, equipped to support their comments with textual evidence. They also learn to become listeners, as one goal of Harkness Method is to enrich perspective, not merely to share a sole opinion. Harkness is student-centered; the method builds confidence because each student's opinions are valued. The teacher directs the discussion of the day's readings, occasionally changing the direction of the discussion and suggesting other areas of importance. Students speak as they see fit rather than waiting to be called on by the teacher . Since the discussion at the table depends on active participation, the expectation is to come into class prepared to do the "work of discussion." The teacher does not give students the answers because there is no value in that. There is great value, however, in learning to think critically. This of course, takes tremendous effort as students sit and struggle with the text at hand. The correct answer is not as important as the means by which they arrived at the answer. Thinking critically will affect the way they approach a new problem tomorrow, or ten years from now (S.Towry, personal communication, June 5, 2012).

As a result, students at The Dunham School have multiple opportunities to collaborate with each other. There is an established set of norms for responding to remarks of others and for the appropriate use of comments. Exposure to the method of Harkness teaching in Humanities classes made the transition to student centered learning tasks in the math classroom easier to implement when it came to providing an environment where the students felt safe sharing ideas. However, the students had a difficult time applying critical thinking in determining how to solve a problem by selecting an appropriate method, representing solutions to problems using multiple representations, and communicating the process of how the answer was determined. Students wanted to write an answer, but not explain how they arrived at the answer or the meaning of the answer. It became evident that guidelines for how to deconstruct and interpret solutions to mathematical problems would need to be developed.

1.2.3 Formative Assessment

Assessment is a key piece to the teaching and learning environment and its central role is to get information to improve teaching and learning (Klum, 1994). Assessment may be conducted in the classroom for a multitude of reasons. Danielson and Marquez (1998) suggest that assessments may be used for instructional decision-making, feedback to students, and communication with parents. Tasks can be used for formative or summative assessment. Summative assessment is used to give quantitative feedback to students whereas formative assessment is used to provide qualitative feedback to the teacher and the student. Individual feedback may be given to students in the form of questions to help guide them in evaluating, comparing, and improving solutions to tasks or teachers may use the information gained from a formative assessment to assess understanding or misconceptions which is helpful when planning future lessons. Klum (1994) suggests that tasks that are used for assessment should have the following basic criteria:

- A task should give all students the chance to demonstrate some knowledge, skill, and understanding.
- A task should be rich enough to challenge students to reason and think and go beyond what they expect they can do.
- A task should allow the application of a wide range of solution approaches and strategies.

The method of formative assessment was used to evaluate the tasks selected for this paper. I evaluated student work and determined a list of class questions for the students to reflect on as I created additional opportunities for students to refine responses and discuss various approaches to solving problems. Students were given the opportunity to use the interactive whiteboard and the document camera to explain how they arrived at their solutions or how they refined their solution based on the class reflection questions. Although my original intent was to provide individual feedback on student work, I found this part of the process difficult. It is easy to grade

procedural problems for correct answers, but it was more challenging to provide feedback in the form of questions to improve student understanding. I also used the questions that I chose to help guide me in the development of future lessons, warm-ups, and practice problems in MathXL. The web resource http://map.mathshell.org/materials/index.php provides a list of pre-made questions associated with each formative assessment lesson that are correlated to common student misconceptions. For my first task, I used this resource to select my set of guiding questions that were presented to the students.

1.3 Technology at The Dunham School

1.3.1 MathXL

In 2008 a conversation with a former student revealed to me information about a program that was being used on the campus of Louisiana State University (LSU) utilizing MathXL. Although my former student felt comfortable with her high school preparation for the content of what was being taught in College Algebra and College Trigonometry, the method of delivery on the computer via MathXL was an adjustment. MathXL is a web-based program that can be personalized by the teacher to provide homework, quizzes, and tests on mathematical content. The program has an abundance of help features available to students including access to an online textbook, explanations to similar problems, videos, and animations. Students receive immediate feedback and are given the opportunity to keep practicing problems until mastery is obtained. My student explained to me how assignments, quizzes, and tests were administered on the computer at the college level and how classes were structured with a lecture/lab component. In an attempt to better prepare my students for what was happening in the college environment, I began to investigate what was being done in the freshmen level courses at LSU in regards to

math. I connected with Phoebe Rouse who in 2004 began overseeing LSU's College Algebra redesign.

During the summer of 2008 I was able to take part in a College Readiness program also designed to enable high school math teachers to deliver college content to qualified students using MathXL. The idea was that the students would be able to not only earn high school credit in advanced math or precalculus, but qualified students would also have the opportunity to earn up to six hours of college credit through the program. The program was called AMCAT, Advanced Math with College Algebra and Trigonometry. Although I was not going to teach that course at my school, I suggested to Ms. Rouse that I would be interested in incorporating MathXL into my Algebra II courses with the direct intent to help prepare potential students for the subsequent course that would enable them to earn college credit. As a result of my participation in the program, I became part of a team of four other teachers who created a course of study in MathXL to supplement instruction in Algebra II.

During the 2008-2009 school year, I was allowed to pilot MathXL with my students in Algebra II. Although my students did not have access to a computer lab, I was fortunate that all of my students had access to a computer with Internet access at home. During the pilot year of use of MathXL, students only completed their homework in the program. Quizzes and tests were given in the traditional paper/pencil format. The students were motivated by the immediate feedback and worked problems multiple times to achieve 100% on assignments. This was a vast difference to the attitude that many students took previously regarding homework assignments. Students were eager to admit that in the past they would sometimes write anything down for an answer to a homework problem and rarely checked the answer in the back of the book. If an answer was checked in the back of the book and was incorrect, they would choose to wait until

the next day to ask the teacher how to work the problem. As a result, a large amount of class time was spent answering questions and reteaching. The student was often an observer in the process. The addition of MathXL to my math classroom transformed the learning environment. The immediate feedback and help features motivated students to keep working until the correct answer was attained. Students were eager to explain to other students how to work a problem after "getting the concept" through practice. As a result, time in the classroom was spent to "cover" more material. After the first year of use, there was an increase in the number of students who earned an 'A', 'B', or 'C' in the course as documented by course grades, more content was covered than in previous years, and students were more actively engaged in the procedural part of "doing" math. However, there was also a downside to the use of the program. Many students would try to work the problems in their head or on scratch paper and then would enter the answer in the computer. In some instances there was no record of the process taken to get to the correct answer. Students also became heavily dependent on the view an example button within the program. Although the student was able to get a correct answer to a problem by looking at how a similar problem was solved, it became evident on the paper/pencil assessments that there was not a mastery of understanding of the concept. Students were trying to just memorize a procedure for a particular type of problem. When an assessment involved a variety of problem types some students were not able to determine appropriate methods for solving a type of problem.

As a result of these observations, changes were made to how the program was used in the classroom in subsequent years. In the spring of 2010, I was invited to San Diego to attend the MathXL for School Development Summit for high school teachers. The summit was an opportunity for teachers to share and discuss strengths and weaknesses of using MathXL in the high school classroom and how the program was being implemented in the classroom. A group

of about fifteen teachers were assembled to provide feedback and suggestions to each other and to Pearson representatives. Although we had encountered some problems with the implementation of MathXL, solutions were discussed and ideas presented for future enhancements. One idea that we implemented at our school was a composition notebook requirement for students to document the process of how a solution is attained. Notebook checks are done once or twice a quarter and feedback is provided to students in the form of comments and suggestions for improvement. A minimal (10 – 20 point) grade is attached to the feedback to assist in holding students accountable for the notebook. Additional features that were forthcoming as a result of feedback from teachers was the ability to copy problems that could be added to assignments in which the students would not have access to the help features such as the view an example button. This feature allows teachers the ability to mix problems within the assignment in such a way where some problems within the assignment have the help feature and others do not. Additionally, it is possible to create an entire assignment where students do not have access to help features or the teacher can control which help features are available to students. The later method is applied to all problems in an assignment. This feature is generally used at our school to create spiral review problems to determine if students internalize procedures over a given period of time since students do not have access to the help features. Finally, another enhancement provided the ability to create personalized assignments for students based upon their performance on a prerequisite assignment. Help features are not available when the students complete the assessment, but a personalized assignment is created for each individual student based on what content was mastered. On the assignment students are given credit for topics mastered and only need to work on the topics not mastered or additional problems assigned to all students and required by the teacher. MathXL also has an item analysis

feature that enables the teacher to see the median time spent on each problem assigned. This provides an additional opportunity for teachers to adjust instruction to provide additional examples on problems that students are taking a long time to solve. It also provides information on individual student performance, which the teacher can use to conference with the student or for additional data during a parent conference.

As a result of these new enhancements and a desire to instill the importance of retention of mathematical concepts, our school now has a mandatory summer math requirement that utilizes MathXL. The department chair reviewed curriculum and created a series of content specific math problems that students need to complete during the summer to help reinforce the procedural knowledge needed for the upcoming course. Students are given a pretest and certain requirements are set before a student is allowed to practice problems missed and retake the assessment. Students are required to earn a 90% on the practice problem assignment before they are allowed to retake an assessment. A minimal score of 70% is needed on the assessment before a student is allowed to move to the next concept. The letter and information sheet that was sent home to parents and students regarding summer math requirements for Algebra II is in Appendix B.

1.3.2 One-to-One Environment

In the 2009-2010 school year, The Dunham School became a one-to-one MacBook school. All students in grades three through twelve were issued their own MacBook to use during the school year. As a result of going to a one-to-one environment, each classroom was equipped with an interactive whiteboard. All high school students have access to Geometer’s Sketchpad software, MathXL, spreadsheet programs such as Numbers and Excel, presentation software such as Keynote and PowerPoint, as well as TI-84 graphing calculators. In addition, my

classroom has a document camera, a printer, access to TI-Smartview software, and five CBR motion detectors that can connect to the computer or a graphing calculator. Teachers are required to attend a three-day summer technology institute and a minimum of two professional development sessions throughout the year on incorporating technology into the curriculum. The school hired a director of technology and one additional technology support person.

With the one-to-one environment and after the successful pilot of MathXL in the Algebra II classroom, it was determined that we would continue the use of MathXL in the Algebra II classroom and offer qualified students in precalculus the opportunity to earn college credit through a dual enrollment program. In addition to another class that used MathXL, the way that MathXL was used in the classroom began to change. Students were required to have a composition notebook to show their work for all problems in MathXL. Students were also encouraged to use pens or highlighters to make notes about how to solve certain problems, to record why certain problems were missed, to keep track of problems where the help features were used, and to record questions that needed clarification. Students in Algebra II continued to have their homework assignments in MathXL, but some quizzes were done in MathXL and practice tests were available for the students but not required. A hybrid approach of technology and paper/pencil assessments was still the medium in the algebra classroom, but all tests and quizzes in the precalculus classroom were given on the computer and created by the University. During our first year of implementation of the dual enrollment program several of the students struggled with the fact that all of the quizzes and tests were completed on the computer and that partial credit was not awarded. A determination was made that the precalculus teacher would grade work for partial credit for the high school grade. This meant that it was possible for students to earn different grades for dual enrollment and the high school course grade. The

precalculus teacher was also allowed to include other grades such as notebook or binder checks in the gradebook for the high school grade. At the end of our second year of implementation, the decision was made to include more required computer based assessments in Algebra II to help better prepare the students for what would be expected in dual enrollment. A summary of the data on our student success rate in dual enrollment over the last three years is in Table 1.

Table 1- Summary of Dual Enrollment Data from 2010-2012

3 year summary	College Algebra	College Trigonometry
Number of students enrolled	80	86
Number of students earning credit	78	74
Number of students who dropped the course	2	12
Percent of students who earned credit	97.5%	86.05%

Students had to earn a 70% or better in order to earn credit for the course, which was denoted with a *P* on their college transcript. In the spring of 2012 instead of having students earn a *P* on their transcript, a decision was made for the actual grade earned by the student to appear on the transcript. The original *P* policy was to enable students to earn credit without starting college with a grade that could negatively impact their college grade point average. The reason for the change in 2012 was due to the fact that some colleges wanted an actual grade rather than a *P* to allow the credit to transfer. Students did have the option to drop the course if warranted. As a result, in the spring of 2012 fourteen students earned an ‘A’ in College Trigonometry, six earned a ‘B’, one earned a ‘C’, and four students dropped the course.

After the second year of implementation of MathXL at our school it was decided that MathXL would be used as a supplement in all math courses from algebra – calculus. One teacher from each grade level attended a summer College Readiness Program at LSU to get training on

how to supplement math instruction using MathXL as a resource. In addition, required technology meetings were held to inform parents about MathXL and how the program would be used in the classroom. MathXL was used as a resource in the classroom to assist students in the development of procedural knowledge and it provided teachers with an opportunity to "cover" more information, as more time was made available in the classroom as a result of using MathXL. However, after the curriculum review and knowledge of what was needed to implement the CCSSM Practice Standards, it was determined that instead of "covering" more content with the time saved by the technology enhancements or instead of using the technology to just present information, the role of technology in the classroom could be used to assign activities that would support critical thinking, communication, collaboration and problem solving. The three tasks selected for this thesis involved using technology as a tool to solve problems and make connections between multiple representations of situations. The first two tasks were student centered group tasks and the last one was an individual task.

CHAPTER 2. SAMPLE ALGEBRA II TASKS

2.1 Interpreting Distance -Time Graphs

2.1.1 Introduction

Interpreting distance-time graphs is a formative assessment lesson from the Mathematics Assessment Project (MAP) website (http://map.mathshell.org/). The website contains a variety of resources that are designed to assist teachers in the implementation of the CCSSM. Some of the resources include classroom challenges, summative assessment items, PowerPoint resources, as well as professional development videos. MAP is a collaboration between the University of California, Berkeley and the Shell Center team at the University of Nottingham, with support from the Bill & Melinda Gates Foundation.

After using MathXL to have the students review the concept of slope and writing equations of lines, this lesson was used to assess student understanding of these concepts and to extend understanding of the concepts to include multiple representations, piecewise defined functions, domain of a function, and interpreting functions that arise in applications in terms of context. The original lesson and some of the resources that I used to design the task for my class can be found on the website for the Mathematics Assessment Project given above. However, I did revise and extend the lesson for my classroom to include the use of technology, the connection to piecewise defined functions, and an extension into some concepts found in calculus that will be used next year. The lesson in its original format was a classroom challenge intended to be a conceptual development lesson for middle school students. Since my group of students had not been exposed to this type of task, I felt it was appropriate to start with the basic premise and extend to meet the needs of the Algebra II content. In addition, this task provided an

opportunity for students to be able to demonstrate application of prior knowledge learned in previous math courses.

One of my essential questions for the course is to have students communicate mathematically using multiple representations and to see connections between numerical, graphical, analytical and verbal models. This lesson captured all of these skills. The lesson was covered over a four-day period. Prior to the first day of the lesson, I connected a motion detector to a computer and projected the image of a distance-time graph. I asked the students to study the graph and then asked for a volunteer to try to produce the graph by walking in front of the motion detector. I did not give too many instructions, but wanted to assess student understanding of how the volunteer should walk to match the graph. After the first attempt, we had a large group discussion regarding how the motion detector gathered data and then represented the data graphically. We compared the numerical data with the graphical data and discussed how the volunteer could improve the strategy to match the graph. I did not want to spend a lot of time with the motion detector, but I wanted the students to see the connection between the numerical and graphical data and how the volunteer chose to walk in front of the detector. After the brief introduction to distance-time graphs, I gave the students The Journey to the Bus Stop task to complete individually (Figure 1).

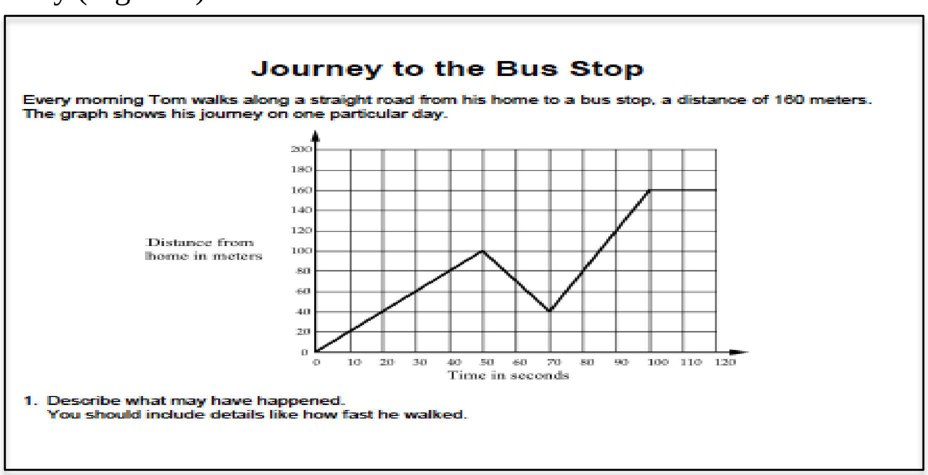


Figure 1- Journey to The Bus Stop Task

I told the students to do their best to answer each question as thoroughly as possible and that I would use the responses to determine how to structure the remaining lessons. One of the features on the MAP website is to provide teachers with suggested questions and prompts based on student misconceptions as a tool for providing feedback to students. Although I did not assign a letter grade to the task, I used this list to come up with class questions and additional activities to reinforce concepts based on student responses. Figure 2 includes some samples of actual student responses.

<table>
<tr><td colspan="2">Student Responses
Question 1
Describe what may have happened. You should include details like how fast he walked.</td></tr>
<tr><td>Well we can’t assume that Tom walked in a straight line just because the graph is made of straight lines. Tom could have been walking up or down a hill and that could be what changed the direction of the slope. Also, Tom probably wasn’t going as fast as the graph says. 160 m in 120 seconds is insanely fast. I can barely swim 50 m in 33 seconds.</td><td>1) Tom leaves his house walking fast.
2) At 100 meters he realizes that he dropped his glasses.
3) He gets his glasses then runs to catch the bus.
4) At 160 meters he waits for the bus.</td></tr>
<tr><td>He begins walking towards the bus but at 50 seconds he notices that he dropped his backpack behind him so he walks back to get it. Now he is late so he runs to catch the bus and gets on the bus at 100 seconds.</td><td>He speeds up for about a third of the way then starts to slow down for a little bit before speeding up again for about the same time that he slowed down. After that he stays at a steady speed the rest of the way.</td></tr>
<tr><td>1- He started walking away (forward)
2- He started walking back (backwards)
3- He walked quickly away (forwards)
4- He stopped walking altogether</td><td>1) He started off walking very quickly because he probably had a lot of energy. 2) He started to slow a little because he was getting tired. He was sort of taking a break but still walking. 3) He rested enough and got his energy back so he started picking up speed again. 4) He finishes at a strong fast speed that doesn’t change.</td></tr>
<tr><td colspan="2">1) Tom began walking at a pace of 20m/10sec, a simple stroll with no rush.
2) When he reaches 100 meters, he remembers that he lost his glasses at the 40 meter mark and goes back (faster pace).
3) After picking up his glasses, he gets chased by a dog and speeds up until he reaches the bus stop.
4) When he reaches 160 m, he stops and spends 20 pointless seconds smelling flowers.</td></tr>
</table>

Figure 2- Sample Student Responses to Question 1

Many students either did not answer question two on the task or gave an answer without fully explaining or justifying their conclusion. If the students had been allowed more time to respond to the questions, the responses may have been different for question two. Figure 3 includes samples of actual student responses.

Student Responses Question 2 Are all sections of the graph realistic? Fully explain your answer.	
Yes, because it shows the distance he traveled in the amount of seconds it took.	Yes, because it shows realistically a journey of 160 meters over 120 seconds. Speeding up when necessary and slowing down as well.
No, when the direction changes it's almost instant with no time to turn around.	No, because I don't think Tom would walk backwards on his way to the bus stop.
No, it is impossible to walk 100 meters in 50 seconds. Tom would have to run when he walked and when he ran he would need to sprint.	Yes, I think so, though I am not sure someone can walk so perfectly.
To some extent. It depends on what happened during the downward slope of the graph. It could definitely be realistic.	I think the measurement of time should be in minutes as to seconds. With minutes we'll be able to tell a lot easier the time it took Tom to walk to the bus stop.
All sections of the graph are pretty realistic besides the last major positive slope. He would have had to have been really fast to go that far that fast.	Tom's slowest distance is going 100 meters in 50 seconds. Tom's fastest time was going 120 meters in 30 seconds. Both of these times are realistic, especially if he does run the 120 meters in 30 seconds.

Figure 3- Sample Student Responses to Question 2

The activities that I decided to do on the subsequent days were structured to help students with some of the misconceptions found based on their answers to the task prompt. I noticed that some students interpreted the graph as a picture; some students recognized that Tom was walking at varying speeds throughout the journey but only a few students actually linked the slope to his speed. Some students thought that the section of the graph when the segment had a negative slope meant that Tom slowed down or that he was walking backwards. I was not clear if backwards referred to walking back towards home or if the student actually though that Tom was

walking backwards. Some students thought that the horizontal segment of the journey indicated that Tom was walking at a constant rate. As a result of these responses, I compiled the following reflection questions to help guide future lessons and to highlight student misconceptions.

- If a student walked at a steady speed up a hill and at a different steady speed down the hill, directly away from home, what would the distance time graph look like?
- How can you tell if Tom is traveling away from or towards home?
- How far has Tom traveled altogether?
- Can you provide more information about how fast Tom is traveling over different sections of his journey?
- What is Tom's fastest speed and is it realistic? Why or Why not?

2.1.2 Student Centered Group Task Day 1

The next day for a warm-up, I projected a distance-time graph and asked the students to determine which story matched the graph (Figure 4).

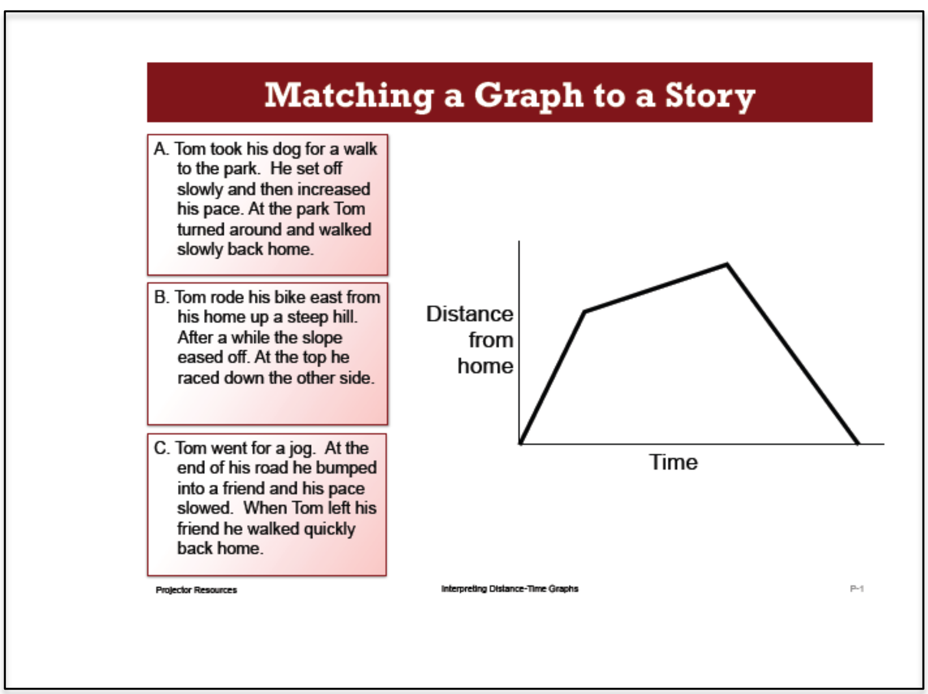


Figure 4 – Matching a Graph to a Story

Out of eighteen students the responses were evenly split between the three choices. I selected three volunteers to mark on the graph and explain each part of the journey to match the verbal description selected. As a whole group we discussed the responses. After a decision was made regarding the correct response, I asked the students to use their mini white boards to sketch a graph of the following: If a student walked at a steady speed up a hill and at a different steady speed down the hill, directly away from home, what would the distance-time graph look like? Students held up responses and this helped me to see if students understood that a distance-time graph represented how far away something was from a fixed location after a given period of time instead of interpreting the graph as a picture of the story.

A second activity that was done to help students make connections between verbal models and graphs involved placing the students into groups of two or three students. Each group was given an envelope that contained ten verbal descriptions and ten graphs which were part of the resources from the MAP website. Students were asked to match the graph to the verbal description while I circulated questioning students regarding choices. Students were given a worksheet to record their answers and each group was encouraged to write on each graph. After about fifteen minutes, one person from each group went and visited with another group to check findings. Upon returning to their group the students discussed results with their partners and had the ability to change their answers as needed. The remaining fifteen minutes of class involved whole group instruction about making up data to match a graph using another resource from the MAP website (Figure 5). Students were to analyze the graph and select distance values that would be appropriate for the situation to match the story being told by the graph. The lesson concluded by reminding students of the connections between verbal, graphical and numerical representations of data.

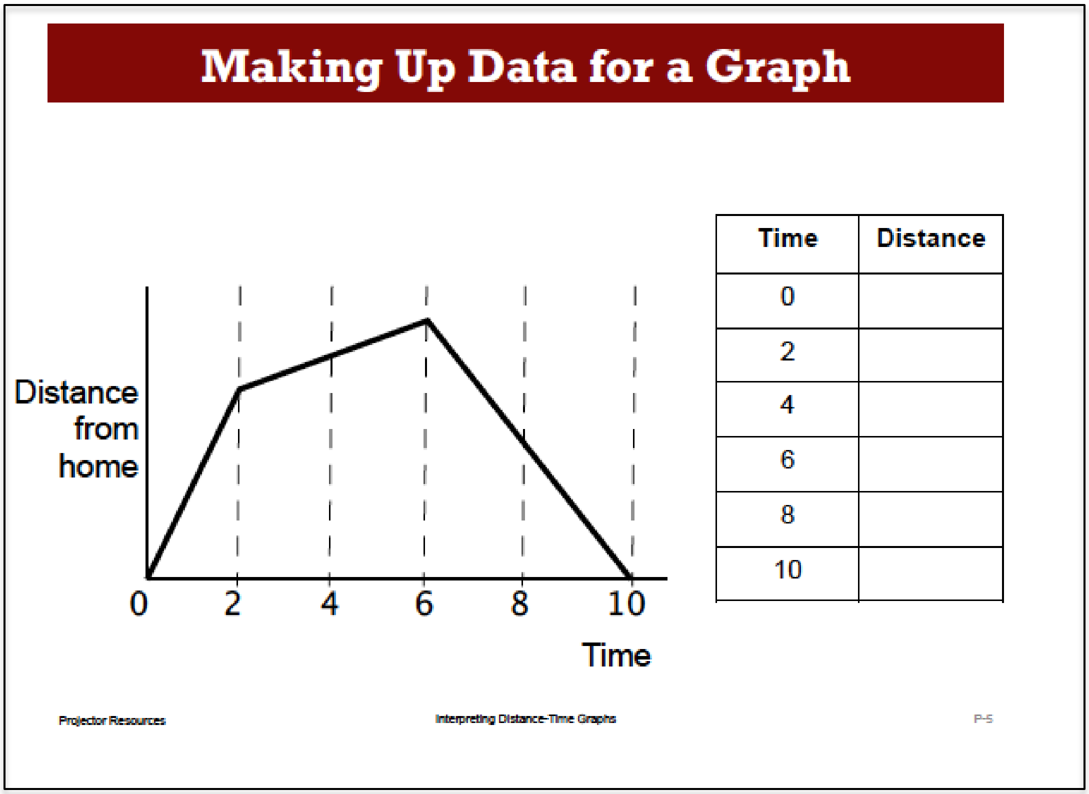


Figure 5- Making up Data for a Graph

2.1.3 Student Centered Group Task Day 2

During day two of the lesson students were given back their envelopes that contained the ten verbal descriptions, ten graphs, and this time ten tables of data had been added. The groups were instructed to reassemble the pairs from yesterday and to match the correct table of data to each pair. Students were instructed to alternate matching tables and graphs and to discuss reasons for matching data sets to graphs. Again I circulated and questioned groups. After about fifteen minutes, one member from each group (the person that had not moved the first day) went and visited another group to discuss matches. Upon returning to their partner the students were also given the opportunity to discuss results and make changes to their answers as needed. Students recorded their answers on a worksheet, which I collected to check for understanding. Students were issued a grade based on their group participation and correctness of responses between verbal, graphical and numerical matches.

As a whole group students were again asked to study a distance-time graph projected on the interactive board in front of the class and to write down a verbal description on a mini whiteboard to describe the graph. After sharing some of the responses, I then asked if we could create a mathematical model to represent the situation. The idea was to introduce the concept of a piecewise defined function and to reinforce the concept of writing equations of lines. We made the assumption that home was represented by the point (0,0). As a whole class, we divided the graph into three pieces and then wrote the equation of each piece of the function using the most efficient manner (Figure 6). For example, for the first piece of the function the students identified the y-intercept and then used data points from the graph to compute the slope. The second part of the function was represented with a horizontal line and the third part of the function involved students identifying two points on the segment and then using the point-slope formula to write the equation of the line that contained the segment. Once the equations for each piece of the function were found, I introduced the concept of a piecewise defined function and the correct notation for piecewise functions. We discussed how the domain was restricted for each piece of the function and how this was represented in the notation. Finally, students were asked to evaluate the function for different values of x using both the graph and the piecewise defined function. It was my intent to provide students with a concrete example of piecewise defined functions and the notation that is associated with such functions. The notation used for piecewise functions often confuses students. I used this concrete situation to ask students to find the value of the function for various input values using function notation and to explain the meaning in the context of the situation. I also asked students to explain how to use the graph of the piecewise function to answer the same questions. It was my desire to have students see the connection between the notation, the graph, and the piecewise defined function.

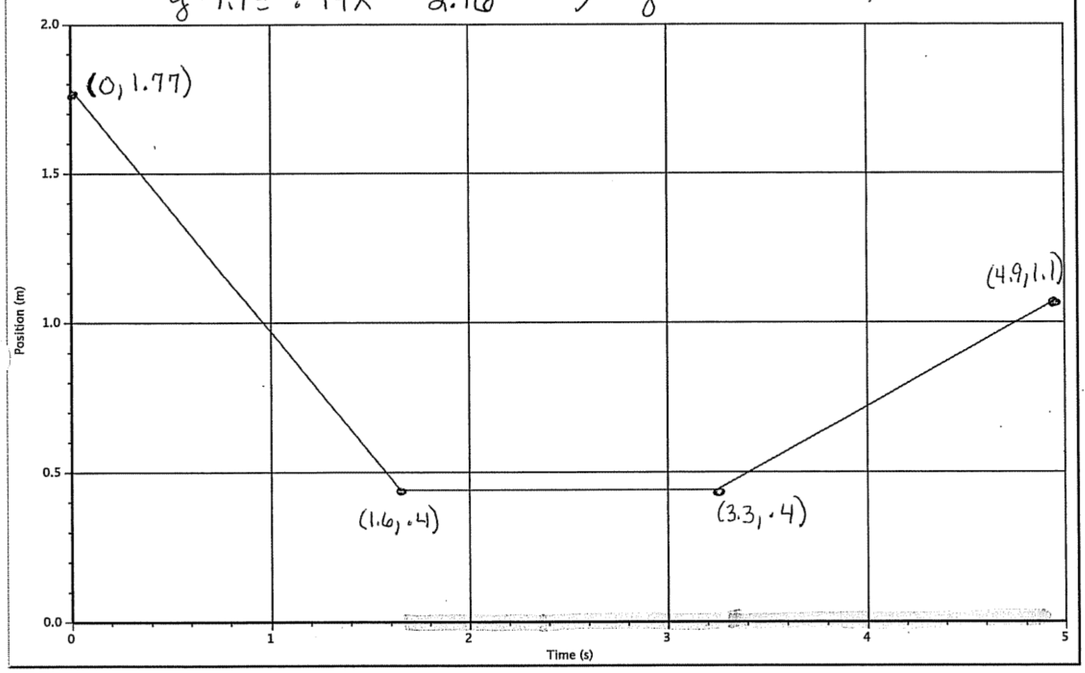
① $m = \frac{1.77 - .4}{0 - 1.6} = \frac{1.37}{-1.6} \approx -.86 \text{ m/s}$ y int = 1.77

② $m = 0$ horizontal line

③ $m = \frac{1.1 - .4}{4.9 - 3.3} = \frac{.7}{1.6} \approx .44 \text{ m/s}$ use point-slope formula

$y - 1.1 = .44(x - 4.9)$

$y - 1.1 = .44x - 2.16 \longrightarrow y = .44x - 1.06$



$$f(x) = \begin{cases} -.86x + 1.77 & 0 \le x < 1.6 \\ .4 & 1.6 \le x \le 3.3 \\ .44x - 1.06 & 3.3 < x \le 4.9 \end{cases}$$

Beth Mcinnis Friday, October 14, 2011 7:39:31 AM CT

Figure 6- Piecewise Defined Linear Function Work Sample

2.1.4 Student Centered Group Task Day 3

On the third day of the lesson students were divided into groups of three for the next lesson activity. Since our school is a one-to-one laptop school, students have the Loggerlite software installed on their individual computers. The motion detector can be attached to the laptop with a USB connector and data from the detector is collected and displayed with the Loggerlite software. Each group used one laptop, one motion detector, and was given a verbal description of a situation that they had to recreate with the motion detector. Once the students had produced a graph to model the situation they were instructed to type the verbal description on their graph, to print their graph, and to write a piecewise function to model their situation for homework. Although the data was collected as a group, each individual member was responsible for writing their own piecewise function. Students were allowed to collaborate, but it was ultimately each individual's choice as to what points would be used to write the equations of the line segments. Students turned in their work the next day and I graded the assignment out of ten points. I did not have a rubric that I gave to students ahead of time that was used, but after the activity I saw a need to develop a rubric based on the following criteria: collaboration within the group, following instructions, interpretation of the verbal/graphical model, representation of the graph using the algebraic model of a piecewise function, correct use of mathematical notation.

2.1.5 Student Centered Group Task Day 4

Individual work from the previous day was collected from each student on day four. We had a whole class discussion about what was learned from the activity on the previous day. The last activity that was done with this lesson was done as a whole class. The motion detector was attached to my computer and projected for the entire class. A student was selected to walk a path in front of the motion detector and using their mini whiteboards students were asked to create

their own verbal description to match the graph. A few of these were shared during class. As a large group we wrote a piecewise function to model the situation and discussed the meaning of the y-intercept and how it translated to the situation. I questioned students about how the graph could be used to give directions to someone regarding how to match the graph. Questions arose such as should the person walk at a constant rate of speed the entire time or does the graph indicate that a change in rate is represented. Another question that was discussed was the direction that the person should walk and how this information could be determined from the graph. Students were also asked to evaluate and interpret the meaning of the piecewise function over different values in the domain. Since the data was collected in meters/sec, students were asked to compute the rate at which the person was walking in miles/hour to see if this seemed realistic. As a class we then went back to the Journey to the Bus Stop task to see how we could improve our responses to the questions and I went back to revisit some of the earlier questions selected based on student misconceptions. Students used their mini whiteboards to respond to the following questions:

- How can you use the graph to tell if Tom is traveling away from or towards his home?
- Mary thinks that Tom traveled a distance of 160 meters, but Paul says that she is not correct. Who is correct and why? How far did Tom travel altogether?
- What is Tom's fastest speed? Convert your answer to miles per minute to see if this seems realistic.
- What do you think may have been happening on the interval [100, 120]?

2.1.6 Reflection

The use of tasks in the classroom changes the environment and roles of the teacher and the student. Although the lesson did take up many days in the classroom, it provided an

opportunity for students to see connections to math concepts instead of just viewing math problems as individual separate items. Although students had the procedural knowledge of how to find the slope and y intercept of a line, write the equation of a line, determine the domain of a function, and evaluate functions as evidenced by their success in completing the corresponding MathXL assignments and paper/pencil assessments, when all of these concepts were put into one problem and the students were asked to explain and interpret the meaning this was challenging to most. It became clearly evident that many of my students were merely memorizing procedures, but had a difficult time applying the procedure in the context of a real problem. The addition of MathXL in the classroom alone was not preparing my students to meet some of the key points in the CCSSM that include, "opportunities for students to practice applying mathematical ways of thinking to real world issues and challenges; or helping students develop a depth of understanding and ability to apply math to novel situations" (www.commoncorestandards.org). Although we have seen the benefits of MathXL on student performance in math and student attitudes toward math, it became clearly evident that more needed to be done to provide opportunities for students to demonstrate their understanding of math through problems that involve the Standards for Mathematical Practice. MathXL, however, is a key component that enables me to have time in my classroom to provide such opportunities for Mathematical Practice and a tool that aids in producing an environment of dual intensity between conceptual and procedural knowledge.

The addition of problems in the classroom that allow students to apply mathematics changes the environment of the math classroom. Instead of the teacher being the primary person disseminating information, the time spent in small group collaboration or whole group discussion is the primary mode of learning in the classroom. I had to share the role of teacher in my

classroom with my students. It was important to have an atmosphere where the students did not depend on me as the sole provider of information, but could trust their classmates. This trust could only be established in an environment where they felt safe presenting their ideas. Students had to listen to comments of their classmates and be able to respond in an appropriate manner. For example, the day that the students had to determine which of the three verbal models represented the graphical model students in one class were evenly split with six students choosing the first choice, six choosing the second choice, and six choosing the third choice. One student from each scenario was asked to come to the front of the room and defend their choice. The other students were given the opportunity to question the responses. Since only one response was correct, I had to clearly make sure that everyone understood how we could learn together from each other through possible misconceptions. It was evident that in order for collaborative learning to be positive, students must be in an environment where the ideas of others are heard and respected. Since I am in a Christian school that focuses on the method of Harkness instruction in the humanities classes, students appeared to be comfortable with this method of collaboration. However, this experience reinforced the importance of a safe environment and guidelines for how to respond to comments of others that are different from your own when implementing this method in a class.

A final lesson that I learned from this activity was that although I was very comfortable grading for correct answers in procedural problems, it was difficult to assess students on their ability in regards to mathematical practice standards. I am currently trying to learn more about rubrics and how to use rubrics to assess student performance. The use of formative assessment and providing written questions was time consuming. Originally, I wanted to write individual

responses on student work, but due to time constraints I went with a list of questions for the entire class to think about based on some of the more prevalent misconceptions.

After sharing about this lesson with my colleagues at other grade levels, it became evident that this activity could be presented at many different levels. At the sixth grade level students could be exposed to the idea of how a distance-time graph related to a verbal model. Students at this age could make the connection between the coordinate plane, ordered pairs, and producing a graph from a table of values related to a situation that they could model and test. In seventh grade the concept of slope is presented from a procedural aspect. Students are shown how to compute the slope when given two ordered pairs using the formula $\frac{y_2-y_1}{x_2-x_1}$. This activity could be used to support the concept of slope as a rate of change. Students would be able to compute the slope and relate it to the rate at which a person may have walked in front of the motion detector. Students would need to pay attention to the scale on the graph as well as the units. Students at this level could also convert from meters per second to miles per hour or another unit rate. At the Algebra 1 level, the students could expand on the foundation started in grades six and seven to include writing the equation of a line to model a situation as well as the introduction of function notation. Students could be asked to answer questions related to the graph, the table and the equation. It was also determined that the initial part of the activity, when the students match the verbal, graphical, and tabular representations, could be done at the middle school level. At the Algebra II level, I could present students with the verbal model and ask them to use technology to produce the graph and then use piecewise functions to model the graph. After reflecting on this activity, enhancements will be made for next year to help prepare students for concepts they will see in calculus.

Since standard F-IF 6 in the CCSSM states that students should calculate and interpret the average rate of change of a function, one new extension that will be added next year will be the concept of average velocity. Since Tom did not walk at a constant rate for the entire journey students would be asked to compute Tom's average velocity over the course of his journey using their mini whiteboards. Responses would be shared as a class. I anticipate that the students would want to find the average of his rates on the four parts of the journey. However, this would provide an opportunity to discuss the concept of a weighted average since the time intervals are not the same for each part of his journey. Students would be able to see from either the distance-time graph (Figure 7) or the velocity-time graph (Figure 8) why a weighted average would be needed. Since velocity is equal to distance/ time, the slope of the line segment on each interval represents Tom's velocity for each part of the journey. These calculations are in Table 2.

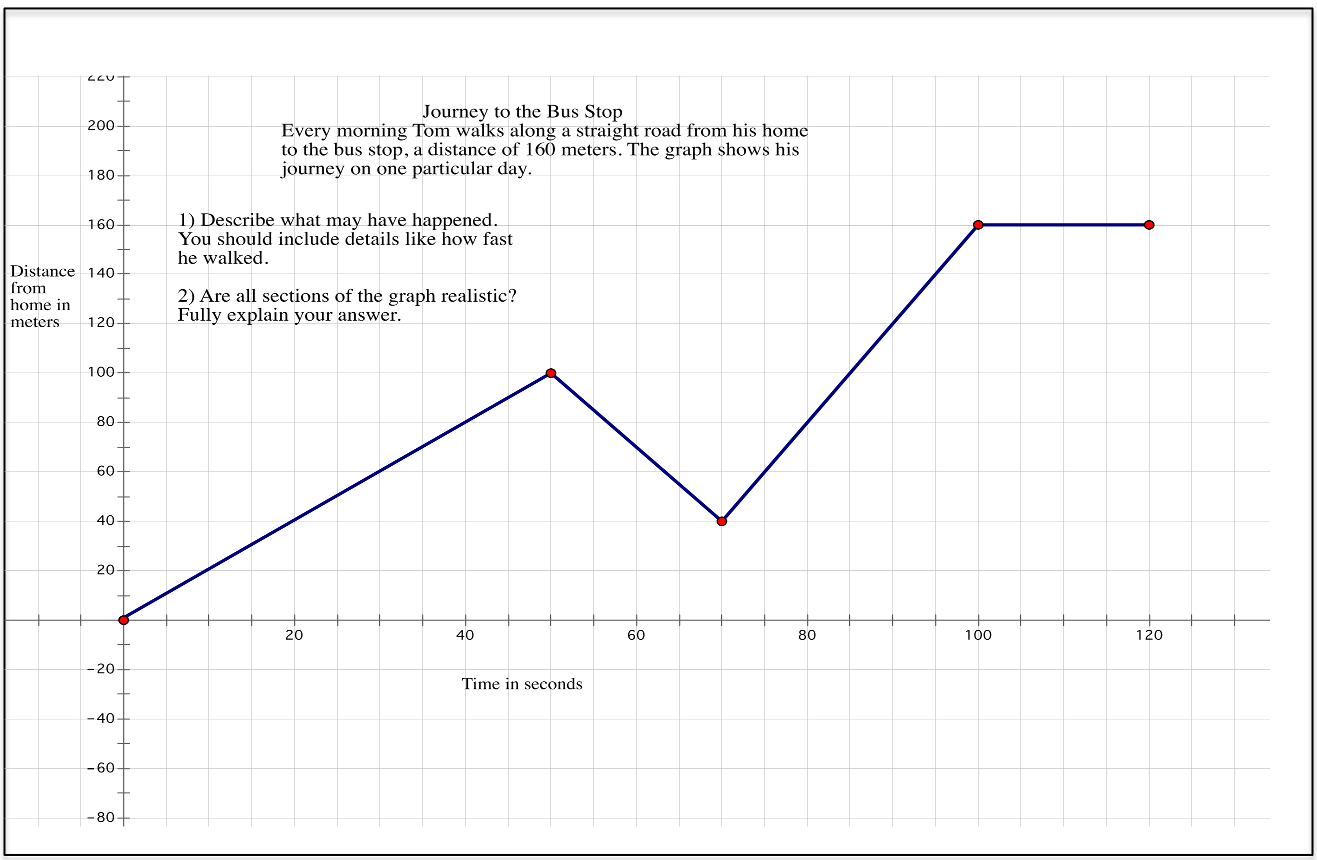


Figure 7- Distance-Time Graph for Tom's Journey

Table 2- Velocity Calculations for Tom's Journey

Interval	Velocity = distance/time
[0,50]	$\frac{100-0}{50-0}=\frac{100}{50}=2\mathrm{m/s}$
[50,70]	$\frac{40-100}{70-50}=\frac{-60}{20}=-3\mathrm{m/s}$
[70,100]	$\frac{160-40}{100-70}=\frac{120}{30}=4m/s$
[100,120]	$\frac{160-160}{120-100}=\frac{0}{20}=0m/s$

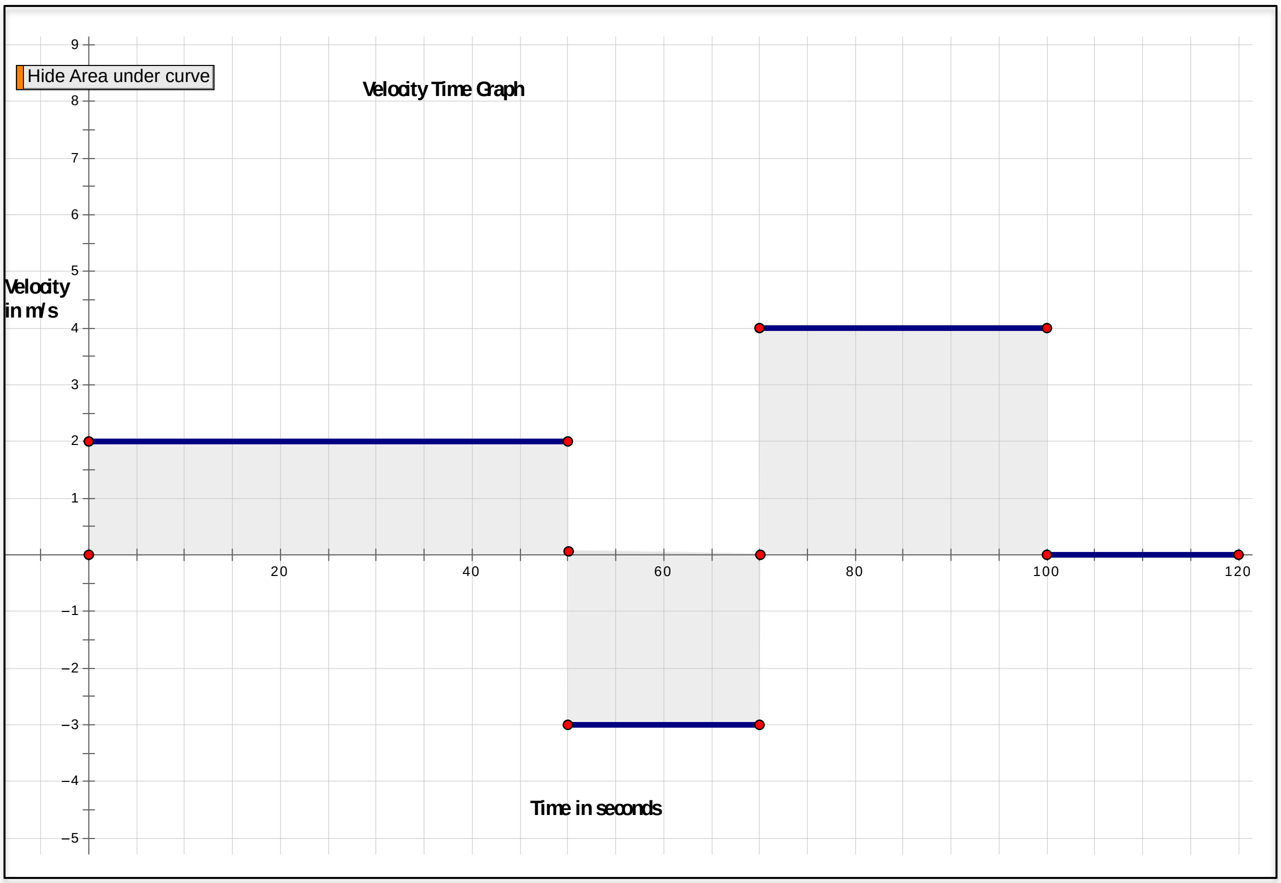


Figure 8 – Velocity-Time Graph for Tom's Journey

The average velocity on [0,120] can be computed by calculating the weighted average of the velocity for each part of the journey.

$$\text{Weighted Average} = \frac{w_1 v_1 + w_2 v_2 + w_3 v_3 + w_4 v_4}{w_1 + w_2 + w_3 + w_4}$$

$$= \frac{50s\left(\frac{2m}{s}\right)+20s\left(\frac{-3m}{s}\right)+30s\left(\frac{4m}{s}\right)+20s\left(\frac{0m}{s}\right)}{50s+20s+30s+20s}$$

$$= \frac{4}{3}\text{m/s}$$

After the average velocity is computed, I will make the claim that the average velocity for any path that Tom chooses for his journey from home ending at the bus stop at point (120,160) will be $\frac{4}{3}$m/s. I will challenge students to use sketchpad to create their own journey for Tom to take from home to the bus stop that involves at least three changes in Tom's velocity along his journey. Students will then calculate an average velocity for Tom's new journey by calculating the weighted average. When the students compare answers they should see that the average velocity from home to the bus stop is the same regardless of the path of Tom's journey. A sample of two other possible paths for Tom journey is in Figure 9. When the students question why this is the case, I will mention to them two topics that will be studied in calculus, the fundamental theorem of calculus and the mean value theorem. I would not expect to explain these concepts to the students, however it would provide an opportunity to plant a seed for topics that will be studied in future math courses. I will explain to them that concepts they will learn in calculus will show that the slope of the secant line through the starting and ending points of the journey will represent the average velocity. This will provide a foundation for students to understand why no matter what path Tom is given from home to the bus stop, the average velocity will always be $\frac{4}{3}$m/s since the secant line drawn from home (0,0) to the point at (120,160) will always be the same. Table 3 shows the average velocity calculations for the two additional paths.

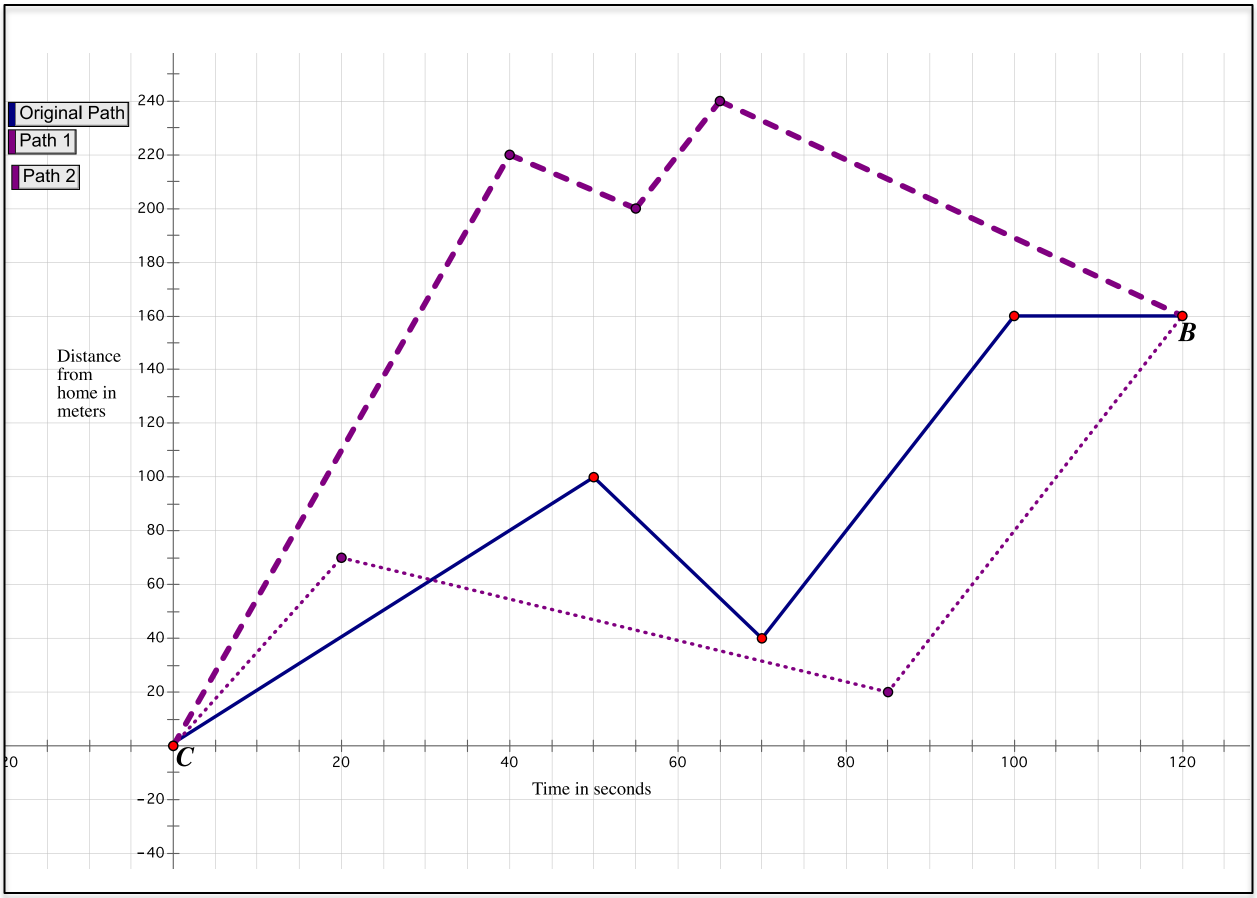


Figure 9 – Additional Paths for Tom's Journey

Table 3- Velocity Calculations for Additional Paths

Interval Path 1	Velocity = distance/time	Interval Path 2	Velocity = distance/time
[0,40]	$\frac{220-0}{40-0}=\frac{220}{40}=\frac{11}{2}\text{m/s}$	[0,20]	$\frac{70-0}{20-0}=\frac{70}{20}=\frac{7}{2}\text{m/s}$
[40,55]	$\frac{200-220}{55-40}=\frac{-20}{15}=-\frac{4}{3}\text{m/s}$	[20,85]	$\frac{20-70}{85-20}=\frac{-50}{65}=-\frac{10}{13}\text{m/s}$
[55,65]	$\frac{240-200}{65-55}=\frac{40}{10}=4m/s$	[85,120]	$\frac{160-120}{120-85}=\frac{140}{35}=4m/s$
[65,120]	$\frac{160-240}{120-65}=\frac{-80}{55}=-\frac{16}{11}m/s$		

$$\text{Weighted Average Path 1} = \frac{40s\left(\frac{11m}{2s}\right)+15s\left(\frac{-4m}{3s}\right)+10s\left(\frac{4m}{s}\right)+55s\left(\frac{-16m}{11s}\right)}{40s+15s+10s+55s} = \frac{4}{3}\,\mathrm{m/s}$$

$$\text{Weighted Average Path 2} = \frac{20s\left(\frac{7m}{2s}\right)+65s\left(\frac{-10m}{13s}\right)+35s\left(\frac{4m}{s}\right)}{20s+65s+35s} = \frac{4}{3}\,\mathrm{m/s}$$

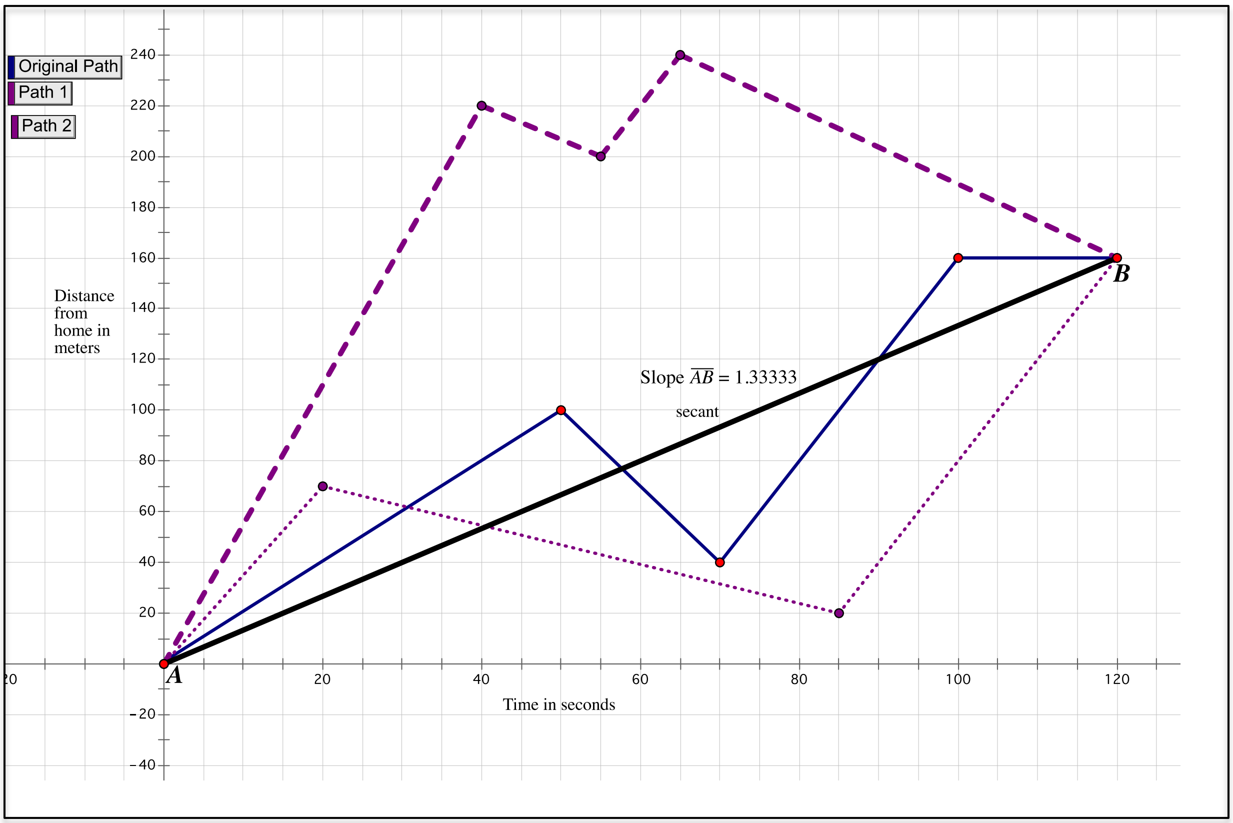


Figure 10- Average Velocity Represented by the Slope of the Secant Line

Figure 10 graphically represents the calculus connection that the slope of the secant line on the interval [0,120] represents the average velocity.

$$Average\ Velocity = \frac{1}{b-a}\int_a^b V(t)dt$$

$$= \frac{d(b)-d(a)}{b-a}$$

= slope of secant line

Another extension would involve more work converting between distance-time graphs and velocity-time graphs. In the past I have never presented students with velocity-time graphs in my course. After studying distance-time graphs, I would have a task that would ask students to create a distance-time graph from a velocity-time graph (Figure 11).

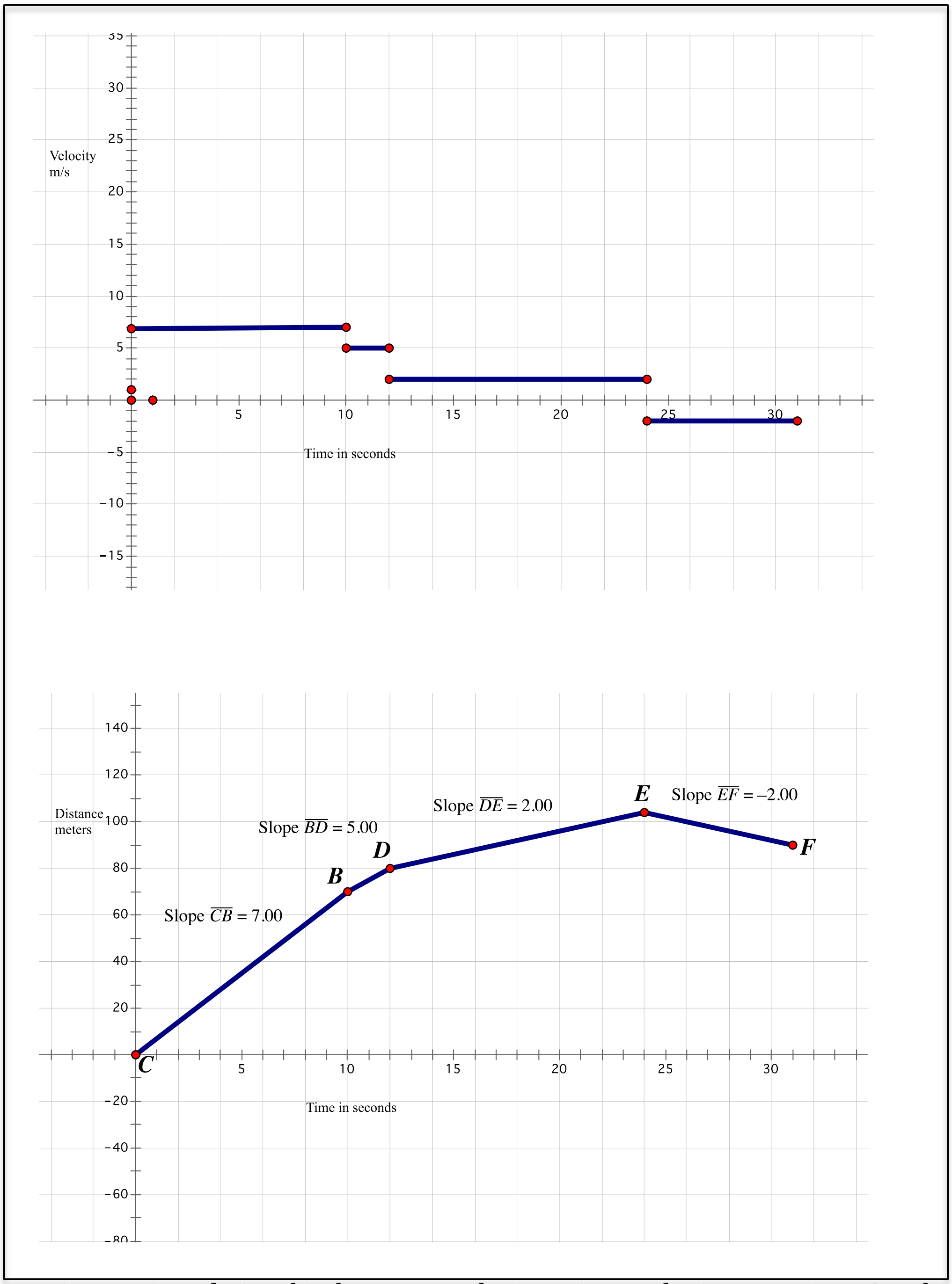


Figure 11- Relationship between Velocity-Time and Distance-Time Graph

I would also present students with a velocity-time graph and ask the students to calculate the area under the curve and explain what the meaning represented. By adding up all of the areas, the total distance traveled could be found. This part of the activity would also reinforce the concept of dimensional analysis that is an important concept in science. Since the height of the rectangle is represented by the absolute value of the velocity and the length of the rectangle is represented by time, the students should be able to see that the product of velocity and time is equal to a distance. It is relatively easy to compute the total distance traveled by simply looking at a distance-time graph, but this concept is a little more challenging with a velocity-time graph. Presenting students with the information in the form of a velocity-time graph and asking this question about the total distance traveled would require the students to think about the context of the problem and the information given by the graph. It would also help prepare students for the idea of finding the area under a curve, which is the foundation for integration.

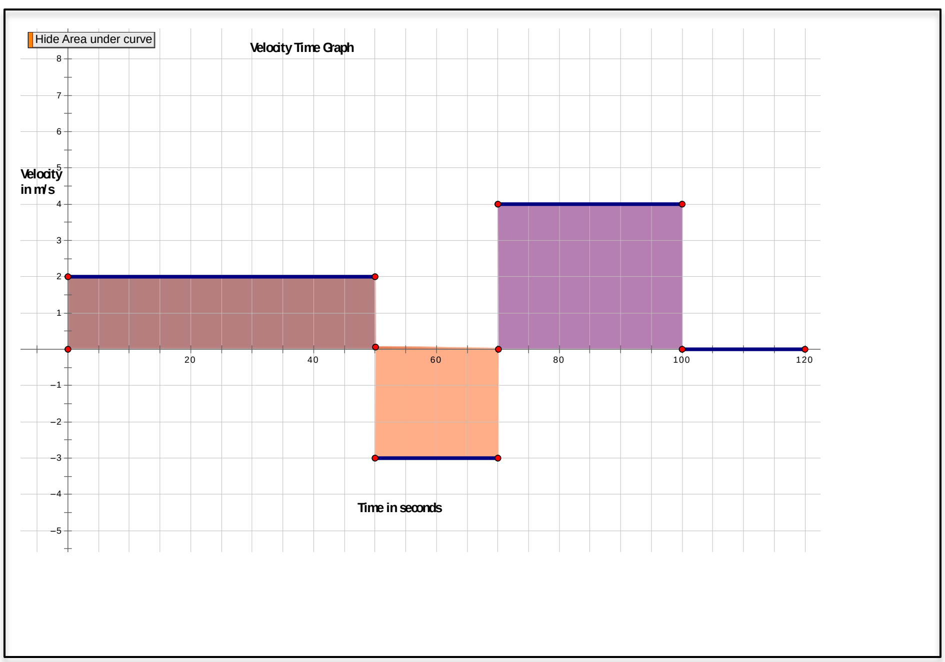


Figure 12 – Area Under the Velocity Graph

The total distance traveled by Tom may be found from the velocity-time graph by calculating the area of each rectangle in Figure 12 and then adding the areas.

Sum of areas = (2m/s)(50s) + (|-3| m/s)(20s) + (4m/s)(30s)

= 100 m + 60 m + 120 m

= 280 m

Students would be given opportunities to create piecewise linear function distance-time graphs and then challenge a partner to create a velocity-time graph to represent the situation. The next group would then have to draw the distance-time graph represented by the velocity-time graph. At this point, all of the piecewise functions would represent linear functions, but the objective would be to get students comfortable with the concept of both distance-time graphs and velocity-time graphs and the information that can be determined from each type.

2.2 Maximizing Volume: The Open Crate Problem

2.2.1 Introduction

The maximizing volume activity was designed to have students make real-world connections using the graph of a polynomial function to solve a design problem that involved starting with a given amount of material. This is a common problem that is found in many textbooks that I modified for this task. It represents one type of optimization problem that refers to finding the minimum or maximum value associated with a function. Although I recognize that the idea of optimization is much broader than this context, it was a place to start to introduce the concept of optimization using mathematical modeling and technology. By exposing students to the idea of optimization before they enter calculus, it is my hope that the students would have a better understanding of the concept. The task was intended to guide students through the process of mathematical modeling. It was student centered rather than teacher directed although the handout deconstructed the problem to help the students in the process of how to model with mathematics. The students had previously worked with quadratic functions to determine

minimum and maximum values when given quadratic equations (see Appendix C), however, the students did not have much experience where they were not given the mathematical equation and they had to identify the maximum or minimum value to solve the problem. The problems referenced above were mainly procedural questions where the students had to calculate the maximum or minimum value from a given equation to answer a question. The forthcoming canoe problem was presented to demonstrate the process of how mathematics is used to find a solution to a problem by breaking the problem into steps. It involved reading and understanding the problem, defining variables, organizing data, looking for patterns in the data, translating from a pattern to a function rule, and then evaluating and interpreting the rule to best answer the question being asked. It was my intent to demonstrate for the students how a situation could be modeled with mathematics in order to find a solution. As a class, two days prior to the maximizing volume activity, we solved a modified problem from http://www.purplemath.com/modules/quadprob3.htm involving modeling with quadratics where we had to find the optimal price to charge to maximize profit when given certain conditions.

> You run a canoe-rental business around the lakes at LSU. You currently charge $12 per canoe and average 36 rentals a day. An industry journal says that, for every fifty-cent increase in rental price, the average business can expect to lose two rentals a day. Use this information to attempt to maximize your income. What should you charge?

We discussed how this problem was different from the others because the mathematical function had to be determined from the information given. The expectation was not to just go right to the equation, but to look for a pattern in the process using concrete numbers that would help to create an equation to represent the situation. I modeled the process on the interactive whiteboard

of how to use a table and data to determine a pattern that would help to develop a mathematical model for the revenue in terms of x price hikes (Table 4).

Table 4- Table Used to Organize Data for the Canoe Problem

price hikes	price per rental	number of rentals	revenue
none	$12.00	36	$12.00*36 = $432.00
1	$12.00 + 1(.50)	36 – 1(2)	$12.50*34= $425.00
2	$12.00 + 2(.50)	36 – 2(2)	$13.00*32= $416.00
3	$12.00 + 3(.50)	36 – 3(2)	$13.50*30 = $405.00
4	$12.00 + 4(.50)	36 – 4(2)	$14.00*28 = $392.00
x	$12.00 + x(.50)	36 – x(2)	(12 +. 5x)(36-2x)

Since the equation was quadratic, we discussed how the vertex could be used to determine the number of price hikes needed to maximize the profit. We discussed how the x -coordinate of the vertex determined the number of price hikes needed to maximize profit under the given conditions. We first rewrote the expression (12+. 5x)(36-2x) in standard form using the distributive property. Next we wrote the function to model the situation where R(x) represented the total revenue after x price hikes, $R(x) = -x^2 - 6x + 432$. Students were given the option of either using $\frac{-b}{2a}$and $R(\frac{-b}{2a})$ to determine the x and y- coordinates of the vertex or completing the square to rewrite the equation in vertex form. After finding the vertex, (-3,441), algebraically we used technology and key information about the graph (vertex and intercepts) to determine a suitable window for viewing the graph on the TI-84 calculator and we verified the vertex graphically and numerically using the graph and table (Figure 13).

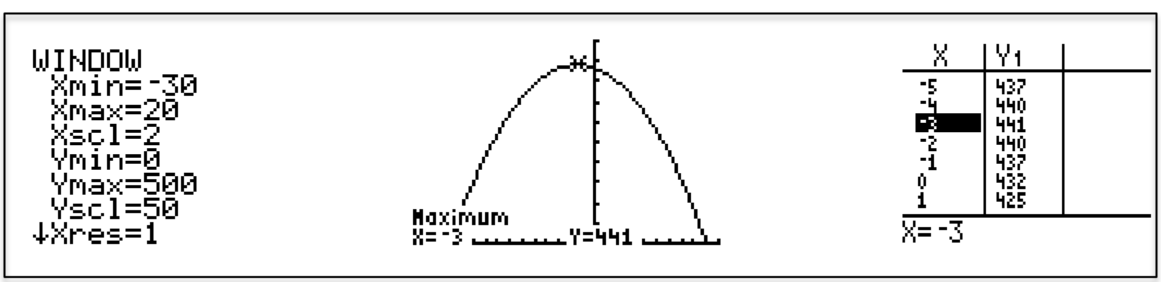


Figure 13- Graphing Calculator Screen Shots

The fact that the x-coordinate was -3 prompted some discussion. It was determined that since the x-value was negative, this indicated that instead of raising the price to maximize the profit, if the information in the journal is correct, then the price charged should be decreased with three price reductions. This would mean that in order to earn a maximum profit of $441 under the given conditions, the price charged per rental should be $10.50. The day prior to the maximizing volume activity students used sketchpad to explore polynomial functions of degree three and higher. The terms relative minimum, relative maximum, and end behaviors were introduced. The concepts of zeros, domain, and range were reviewed.

2.2.2 Student Centered Group Task Day 1

Students were introduced to Maximizing Volume: The Crate Problem on the third day of the lesson. The lesson started with a procedural warm-up problem to remind students about volume (Figure 14). A week prior to the lesson the students were assigned a MathXL assignment that reviewed various types of geometry problems. Some of the problems addressed volume and surface area of both prisms and cylinders. All of the problems were procedural in nature.

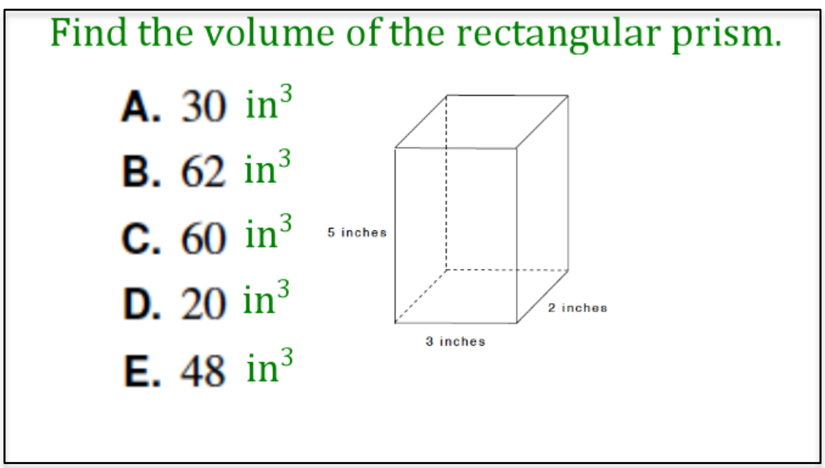


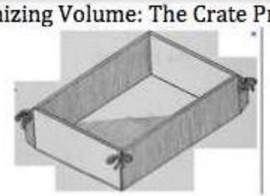
Figure 14 - Procedural Warm-Up

The maximizing volume activity was designed to guide students through the modeling process by first constructing a concrete model and then progressing to using a table to look for a pattern

before attempting to create a mathematical model to represent the situation. It was my hope that the students would use the activities and information that was presented from the previous days lessons to assist them in the process. After the students completed the warm-up we discussed the difference between volume and surface area. We talked about the appropriate use of units and how the units were connected to the rules of exponents. We also discussed the use of the formula V= Bh to calculate volume for prisms and cylinders where the capital B represented the area of the base. We also discussed what type of base was present in cylinders and different types of prisms. I then asked the students how we could create a 3-dimensional open top box from a sheet of paper. The idea that surfaced included folding the paper, but it was not suggested to cut the paper. I took a tissue box apart and cut out the top to show the students that the original material was just a flat piece of cardboard. They were able to see that the corners had been cut and the material folded to create the box. However, rectangles had been cut from the corners and the students were able to see that this created an open top box, but the side heights were not uniform causing two of the sides to be shorter than the other two sides. This was not seen in the finished design since the box had a top in the original design. This did provide an opportunity to discuss what we should do if we wanted to create an open top crate where all of the sides were a uniform height. I also used the opportunity to ask students about other factors that would need to be considered in the design process. Comments included that it would be important to know what was going to be placed in the crates since this may impact the design regarding the size and weight of the objects. We discussed how engineers would have a variety of variables to consider, but that for our activity we wanted to know what size square should be cut from our original material to produce a maximum volume for the crate. We discussed the use of a scale model in the design process. Students were then given a copy of the task Maximizing Volume: The Crate

Problem (see Appendix D). I reviewed the overall objective with the students by projecting a copy of the handout on the whiteboard in front of the class, but did not give too many instructions other than what was on the handout. The students were also given a copy of the rubric that I designed for the activity (see Appendix E). The objective was to put the students in a position that would incorporate the Standards for Mathematical Practice with the Standards for Mathematical Content. The overview of the activity that was presented to the students appears in Figure 15.

Maximizing Volume: The Crate Problem



Overview: You are part of a design team charged with assembling a crate to hold a vendor's product using a specified amount of material. You will explore the changes of the shape and volume of a scale model of the crate with respect to the size of the square that is cut from each corner. You will find a function that relates the volume of the box to the size of the square cut from each corner. Finally, you will use technology to approximate the maximum volume and the size of the square that will allow you to maximize the volume. Each person should submit a copy of the completed worksheet, but only one model and one set of graph paper sketches is due per group.

Figure 15 - Overview of Maximizing Volume Task

The activity was intended to guide the students through the modeling process by first having the students create a concrete model using an 8 inch x 12 inch piece of construction paper. Students were instructed to select the size of a square that should be cut from each corner to maximize the volume and then to compute the volume of their model using the tools available. On a side table in the room I had scissors, construction paper, tape, rulers, graph paper, plain white paper, and markers. Students also had access to sketchpad and graphing calculators. The second phase of the activity guided students to use graph paper models to draw diagrams to complete a table

comparing height, length, width, and volume of various box types when squares of a certain length were removed from the original material and to look for patterns. The third phase of the activity was intended to have students look for a pattern in the table or from the graph paper sketches to create an equation to model the volume of the open top box in terms of x, where x represented the length of the side of the square cut from each corner. The fourth objective was to have students use their mathematical model to compute the value of the function when x was 2.7 and explain its meaning. I referred to this as compute V(2.7). The fifth question was to see if students could make the connection between the model and the graph using key features such as intercepts and end behaviors. The sixth question was designed to see if students could connect the domain to the real-world context of the problem. Being able to identify the domain in the context of the problem would also help students determine an appropriate viewing window when using the graphing calculators. Students were to demonstrate how to use sketchpad and the graphing calculator to answer the question of what size square should be cut from each corner to maximize the volume. An extension question was added for students to create the general form of the equation that could be used to model the volume of an open top box regardless of the original size of the material if a square of length x is cut from each corner of the material. My original plan had the students complete this activity in one 45-minute session. However, the warm-up and the explanation took about 15 minutes, which meant that the students only had 30 minutes on day one. Another problem that I encountered that I was not expecting was that the students spent a large amount of time discussing and trying to decide what size square should be cut from the material in order to maximize the volume. I expected the students to just guess a size and complete question one in about 5 minutes. This was not the case. On the original rubric I only had question one worth one point because I did not expect the amount of discussion and

collaboration that would go into the creation of the model. Many of the students wanted to be sure of the size square that should be cut before attempting to assemble the model. Although unexpected, it did provide an opportunity for students to explore and discuss possibilities before creating a model. On the other hand, it presented a problem in regards to how I was going to manage the activity when most students were still working on question two when the class ended. I made the decision to have the students place their materials on a side table and told them that we would continue the next day.

2.2.3 Student Centered Group Task Day 2

The second day of the task began with asking one of the students from the class to summarize the idea behind the task that was presented the day before. I also realized that I had a couple of groups that still had not successfully assembled a model. I asked the students to take out their copies of the task and the rubric. Before beginning I asked the students to read over the rubric to get a clear understanding of what was expected and to pay attention to how they were being assessed. In addition, in order to save time I instructed the students to use either a graphing calculator or sketchpad to represent the solution to the problem instead of having to do both. I told them that the rubric would be amended. In addition, I pointed out that the extension problem was not being assessed on the rubric. It was intended as an exercise for early finishers, but I did not want the students to be overwhelmed by the task. Students were instructed to resume the activity where they left off and I originally intended to give the students an additional 20 minutes to complete the task. However, it became clear that some of the groups were not able to take the data from the table to create a mathematical model to represent the volume of the box when a square of side length x was removed from each corner. Although eventually twenty-three out of thirty-five students were able to come up with the correct function to model the situation about

1/3 of the students were not able to come up with the correct model. Even with the twenty-three students who were able to come up with the correct function, it took more time than I expected and guidance on my part. In each class only one group was able to complete the task unassisted and get to the extension activity. A summary in Appendix F gives a breakdown of student responses to certain sections of the task. Another issue that was surprising to me was the ability of the students to correctly evaluate the function created for volume of the box in terms of *x* at 2.7, but not be able to explain its meaning. Even if the student had the wrong function, I looked to see if they understood the process of how to evaluate a function at a certain value of x. This again reinforced the idea that procedural knowledge did not assure conceptual knowledge, and encouraged me to continue presenting problems to students that would connect the two ideas.

2.2.4 Reflection

Despite the emphasis this year on making connections between multiple representations, I learned that more opportunities are needed for students to be able to make those connections on their own. This was the first time that students were expected to create a higher degree mathematical model on their own by looking for patterns. Although I had modeled this in previous lessons, I learned that the students need more independent exposure to such problems. I originally anticipated that this activity could be completed in one class period. However, students were given approximately thirty minutes on day one and forty minutes on day two. I did not expect students to take as long as they did to create the concrete model from the construction paper. Although time consuming, it did generate discussion between students about measurement techniques and what size square should be cut from each corner to maximize the volume of the scale model. At least two student groups wanted to alter the original size of the material

(construction paper) to start with a square piece of material. The other groups were reluctant to select a square size to cut for fear of not selecting the “correct” size square. In the future if students decide to create a concrete model of the situation, I will remove the question select the size of the square that you think will produce the largest volume for the box. Students will just be instructed to cut congruent squares from each corner of the construction paper and I will adjust the questions and layout of the activity.

My second obstacle for this lesson dealt with how I should assess the task. Although I created a rubric prior to the lesson, after the second day of the lesson it became apparent that I needed to adjust how I assessed the students. I decided to devote the seventy minutes to the lesson, but did not allow students to continue to work past that time period. I told the students that I was going to assess what they were able to complete and adjust scoring. Next year, instead of the rubric that I attempted to use the first time with the task, I am going to try a rubric that would provide more individual feedback to students in regards to areas that they should work on in order to improve their solutions. It would be more of a formative assessment approach where students could evaluate their individual performance on tasks. The rubric will involve assessing a student in regards to their ability to compute and represent solutions mathematically, make connections between patterns and relationships, and to communicate their solutions (see Appendix G).

What I found most helpful to me in the redesign of this activity was the analysis of the student responses to the various questions. Overall only eight students were able to use the correct mathematical model to predict what size square should be cut from each corner to maximize the volume. Five students did not get past question two. Twenty- five students were able to compute the value of the function at 2.7 even if the wrong model was used, but only nine

were able to explain the meaning correctly. Although twenty-three students were able to create a mathematical equation to model the situation, only nine students were able to predict what the graph of the polynomial function would look like. Fifteen students did not produce any graph on their own and seven students thought that a parabola would model the situation. When the students were allowed to use technology to view the graph, only three students correctly identified the domain for this situation (Figure 16).

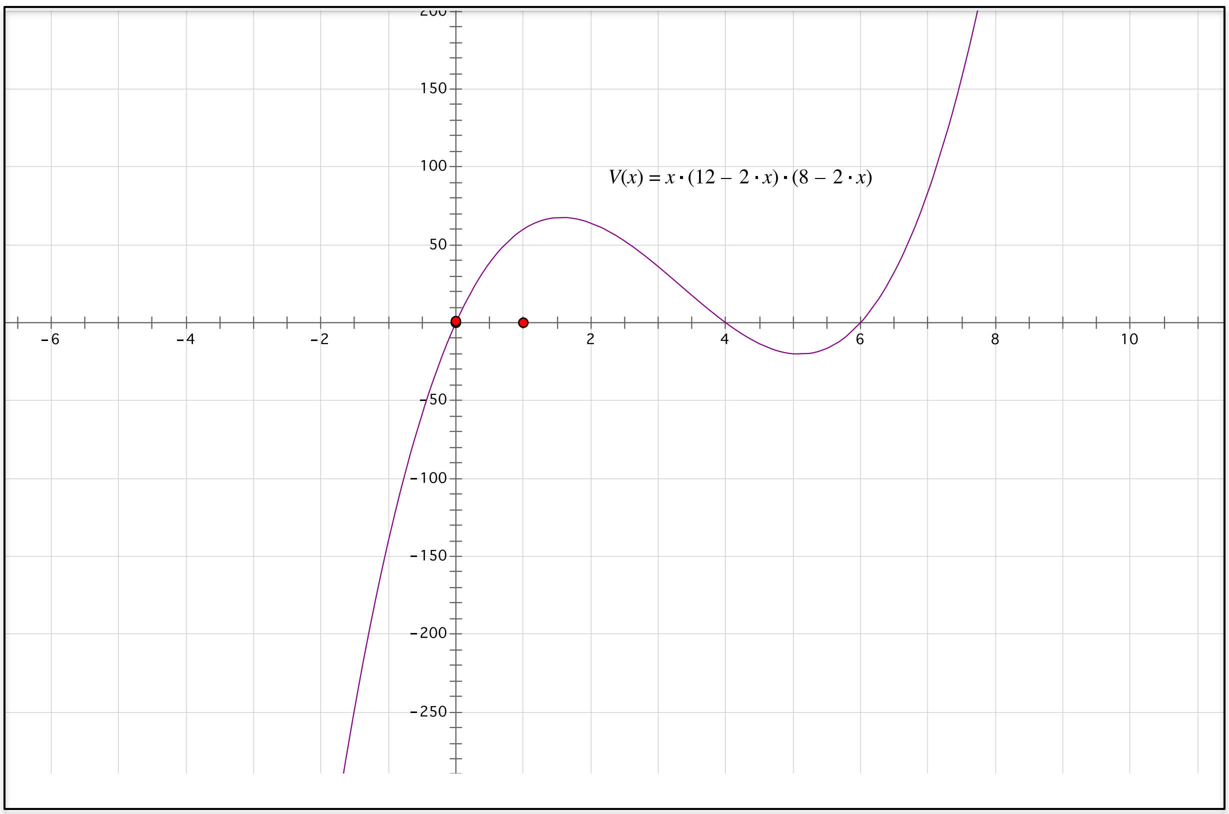


Figure 16- Graph of Polynomial Function V(x) = x(12-2x)(8-2x)

Since the size of the material used to create the box was limited, the size of the square that could be cut from each corner was restricted to the interval (0,4). Six students thought the domain of the function was $(-\infty,\infty)$. Although this is correct for the polynomial function V(x) = x(12-2x)(8-2x) in general, the context of the problem must be considered. Eight students answered that the domain was $(0, \infty)$. This indicated to me that the students grasped the idea that the x value must

be positive since it represented the length of a side of a square, but the students did not recognize that any positive x value would not work since the size of the original material would limit the domain. Two students indicated that the domain was (0,4) U (6, ∞) . This indicated to me that the students made the connection that only x values that would produce a positive volume made sense in the context of the problem, but they did not connect the fact that there was a finite piece of material with which they were given to work that put an additional constraint on the problem.

This activity will be redesigned in a number of ways. First, I noticed that I gave too much structure on to how to solve the problem. In the future I will have two versions of this activity. One where the students are just presented with the task and there is a table of materials that they may use to help them with the task (Figure 17).

You are part of a design team that is charged with using an 8 inch by 12 inch sheet of cardboard to create a scale model of an open-top box by removing congruent squares from each corner and folding up the sides to create the box. Determine the height of the box that will give a maximum volume. Explain how you arrived at your answer.

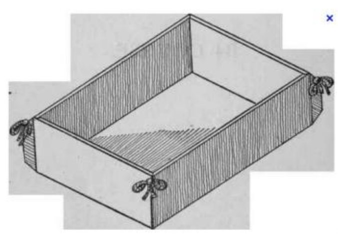


Figure 17 – Redesigned Overview of Maximizing Volume Task

If students struggle with this option, then I will have sections available that will offer more guidance in the process. I will also redesign how some of the questions are worded on the guided parts of the activity. For example, if a group needs or wants to create a concrete model I will

pose question one in a different manner. I will guide the students by adjusting the first question as follows:

- Side length of square removed from each corner ________
- Height of scale model ________
- Length of scale model ________
- Width of scale model ________
- Volume of your model ________
- What relationship do you see between the length of the side of the square removed and the height of your model?
- What relationship do you see between the length of the side of the square removed and the length and the width of your model?

Students would be required to show me this part of the activity before moving onto question two. This would ensure that I was able to spend time with each group and it would also help me to interact with groups that need additional assistance. If this guidance did not get the student on the right track to answer the problem, then I would provide additional suggestions to help them with the process. Thirty-five students participated in this activity and there were five students who did not complete through question two even though they were given two days. This made me realize that the more vocal groups commanded my attention and that I need to make sure that I monitor the progress of all students. The groups that did not finish were the quieter groups who were also very concerned with the design of their concrete model. In the future I will continue to use this task, but I will also provide students with more opportunities to work with creating mathematical models from a verbal model or a table of data before assigning this task.

When I first piloted this task I did not make the calculus connection as it relates to the concept of optimization. However, in the future I will add some extensions to help make more connections to calculus concepts that will be seen by some students in future course. Although this problem involved optimization, the constraint on the problem, the size of the material the student was given to work with, was not expressed with an equation. Only the volume was represented with the function $V(x) = x(12-2x)(8-2x)$ and the students used a graphical or numeric approach to solve the problem. Although I would not expect students to solve the problem using calculus, it does provide an opportunity to discuss how knowing calculus would allow for an analytical solution to the problem. By finding the first derivative of $V(x)$ and setting the first derivative equal to zero, the x values where the slope of the tangent line is equal to zero would determine the critical points (minimum and/or maximum values). In order to determine if the point was a minimum or a maximum the second derivative of $V(x)$ could be found to determine concavity. In this example, since the dimensions of the paper used to make the model were 8 inches by 12 inches, the maximum size square that could be cut from each corner in theory would have to be less than 4 inches. The analytical solutions found using calculus and the quadratic formula were 1.6 inches and 5.1 inches when rounded to the nearest tenth of an inch. Since 5.1 inches does not make sense in the context of the original size of the material that the students had to work with, only the 1.6 inches approximates a possible solution. In order to determine if 1.6 inches would determine a maximum or a minimum volume, the second derivative test could be applied to affirm that the graph was concave down when the x-value was 1.6. This would indicate that a maximum volume would occur at the point when $x= 1.6$. Next year I will include additional task problems that involve optimization, but in these examples the constraint will be modeled by an equation. One extension is from the Mathematics Assessment

Project website and is presented in two forms. The first task is denoted as an expert task where students are given limited direction in the problem and it requires more of the mathematical practice skills (see Appendix H). Another version of the task is listed as an apprentice task on the website because more guidance is given to students to help lead them solve a similar type of problem, but it does not require the use of as many of the mathematical practice skills (see Appendix I). The resources include a rubric that I will use to assess the tasks and the entire task can be found on the MAP website at the link http://map.mathshell.org/materials/tasks.php?taskid=252&subpage=apprentice.

2.3 Car Task

2.3.1 Introduction

The last task that I implemented was an individual task that I gave to the students at the end of the year to help me determine progress that was being made toward connecting procedural and conceptual knowledge on tasks using mathematical content that had been taught throughout the year. I wanted to see if students could take information learned during the year and apply it to a situation that would require them to model with mathematics and discern what type of mathematical model was best suited for the situation. I also wanted to see how students tackled the problem individually and how they communicated their answers. Throughout the year students have worked with functions and function notation in addition to multiple representations of solutions to problems. Linear models were studied in the first semester and the second to last unit covered in the school year involved exponential functions. The introduction to the unit on exponentials functions involved students using basic knowledge of percentages to answer questions about the value of a vehicle after a given number of years if the vehicle depreciated at a certain rate per year (Figure 18). I was surprised to see the large number of students that

answered the problem incorrectly. After explaining the concept of depreciation, the majority of the students thought that the value of the car after five years would be $6,250. The misconception here was that the vehicle would lose a constant amount each year of $3,750. To find the value of the jeep at the end of the fifth year, the students just multiplied $3,750 by 5 and subtracted this result from the original price of the jeep. This misconception gave me an opportunity to discuss the difference between a linear model, which represented a constant rate of change, and an exponential model. In order to help the students see that this situation did not represent a constant rate of change I asked the following questions:

- At the end of the first year what is the value of the car?
- Is the value of the car at the start of the second year the same as when you first purchased the vehicle? Will this impact the amount that the car depreciates in year two?

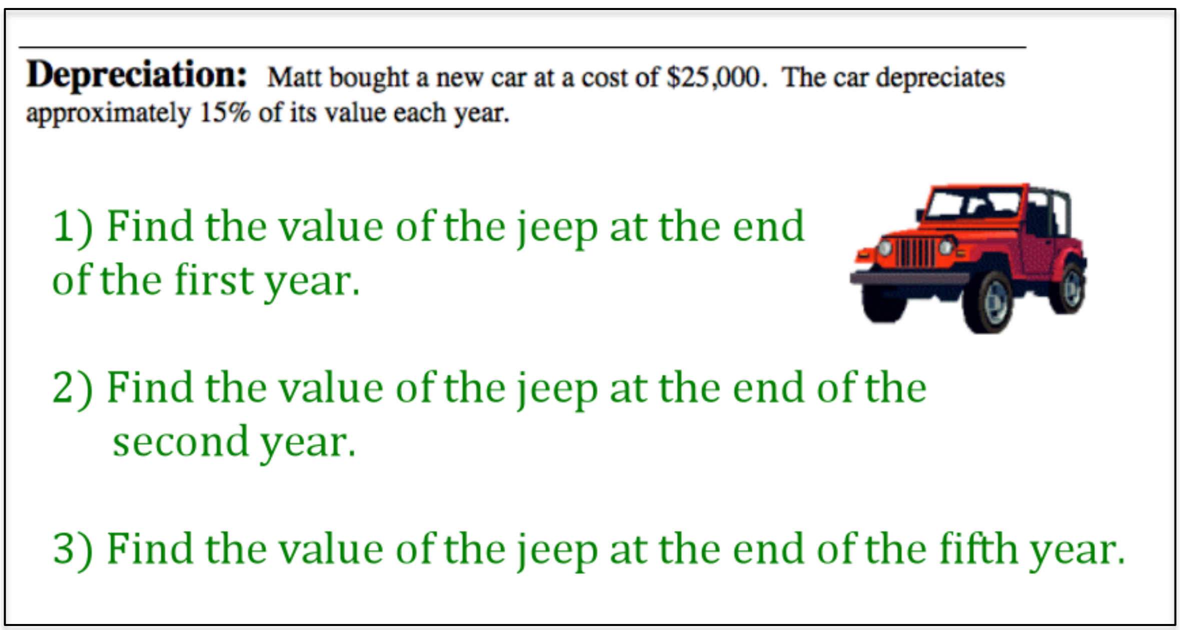


Figure 18 – Introduction to Exponential Unit

These questions enabled students to see that since the value of the car was changing each year, the amount that is was depreciating would change as well. Although the depreciation percentage

was the same each year, the numeric amount would vary each year because the value of the car was changing each year. We plotted some of the data points to discuss what the graph would look like and I used the opportunity to compare linear functions represented by a constant rate of change with the introduction of an exponential function. We discussed applications of growth and decay using the compound interest formula and depreciation models. I also used the opportunity to help the students see the connections between the mathematical model, the table and the graph. The rest of the unit contained procedural information on solving exponential and logarithmic equations. There were a few application problems in MathXL involving compound interest and some problems dealing with growth or decay, but the problems were set up for the students and the students only had to substitute values into the formulas that had already been determined based on the information presented. The problems in MathXL did not require the students to read, interpret, and determine which type of model should be used. The concept of exponential growth/decay was first introduced on April 12.

2.3.2 Individual Student Task

The individual task was given to students near the end of May (see Appendix J). Students were given twenty minutes to complete the task. I used a situation that involved a vehicle since many of my students were at the driving age of sixteen. Before handing out the task, I reminded the students of the various problem-solving strategies that we had studied throughout the year, and we reviewed the different types of functions studied throughout the course. In addition, students were encouraged to use appropriate mathematical tools and were required to communicate how they arrived at their answer. I was not prepared for the questions that the students would have regarding downpayment and financing. It became clear within a few minutes that although we had talked about depreciation, the majority of the students did not

understand the concept of financing. At that point, I stopped and gave a mini lesson on financing, interest, and the idea of credit. With this explanation students were then able to get to work on the task. Although the students were given the opportunity to solve the problem using a method of their choice, I did ask for the students to represent each situation using a function rule. Once the clarification was made about some of the terminology used in the problem, the majority of the students were able to answer the questions on the first page of the task by either using basic math concepts, evaluating the function rule for different input values, or entering the function rule into the calculator and then using the table to answer the questions. None of the students used the graph to answer the question. Multiple representations of the solution are in Figure 19.

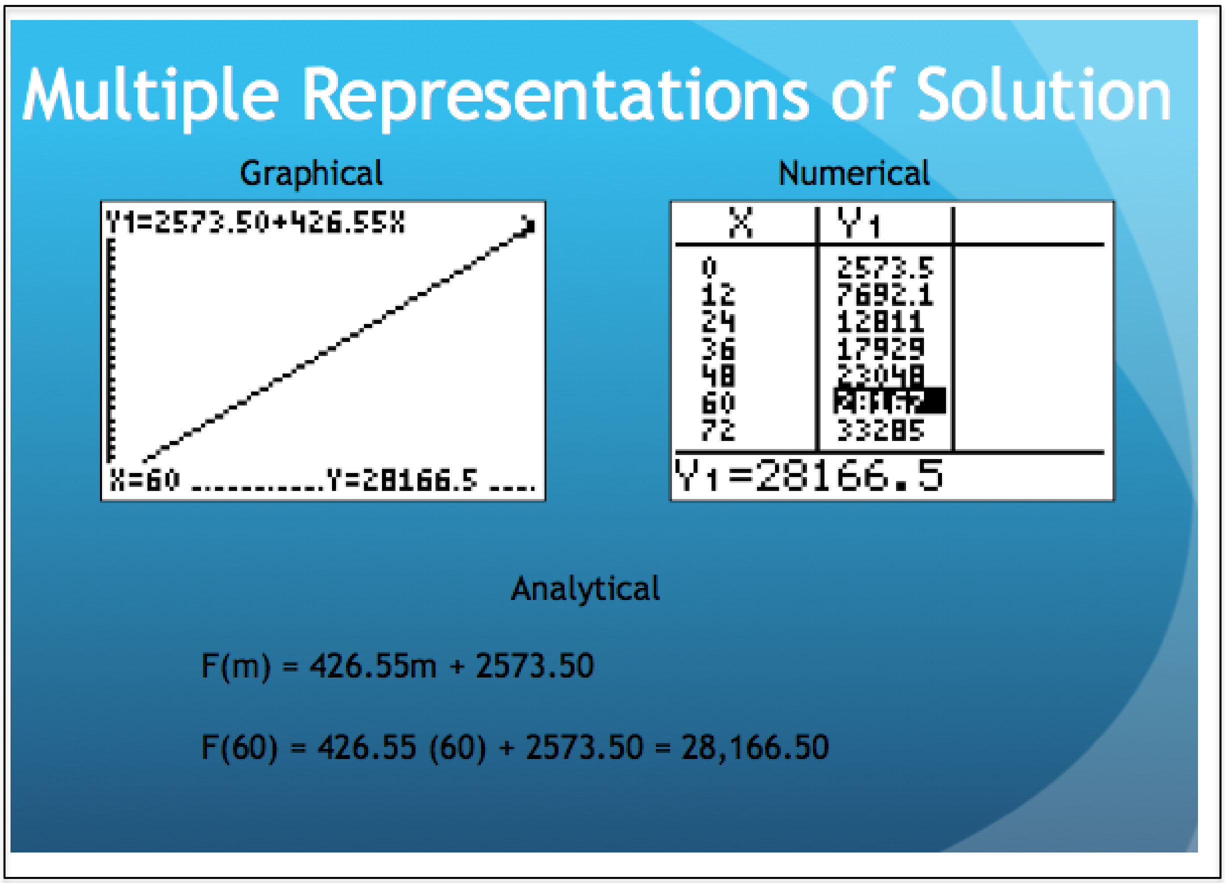


Figure 19 – Multiple Representations of Solution

Five students did not write any function rule to represent the situation. Fifteen students were able to come up with a function rule that was used successfully, however there was

incorrect use of notation. For example, several students wrote the function f(x) = 2573.50 + 426.55m to represent the situation. This indicates a syntax error in the use of the notation since the variables x and m were used. One student used the function f(x) = 2573.50 + 5118.60x, but did not clearly communicate that x stood for number of years. Five students did not include the downpayment in the function rule. Six students wrote an incorrect rule or tried to use the compound interest formula to write the rule. This showed me that there was a lack of conceptual understanding on the use of the compound interest formula and recognizing how to write a mathematical model for the situation. The majority of the students recognized that it was necessary to convert years to months when using the function rule, since most had written the function based on the information related to the monthly payment. For the second page of the task twenty-four students were able to get the correct answer using percentages to complete the table. Five additional students had the correct process for solving the problem using percentages, but made a calculation error that made them get an incorrect answer. Six students gave answers that were incorrect because they did not understand how to use percentages to answer the question. Only eight students were able to model the situation with an exponential function.

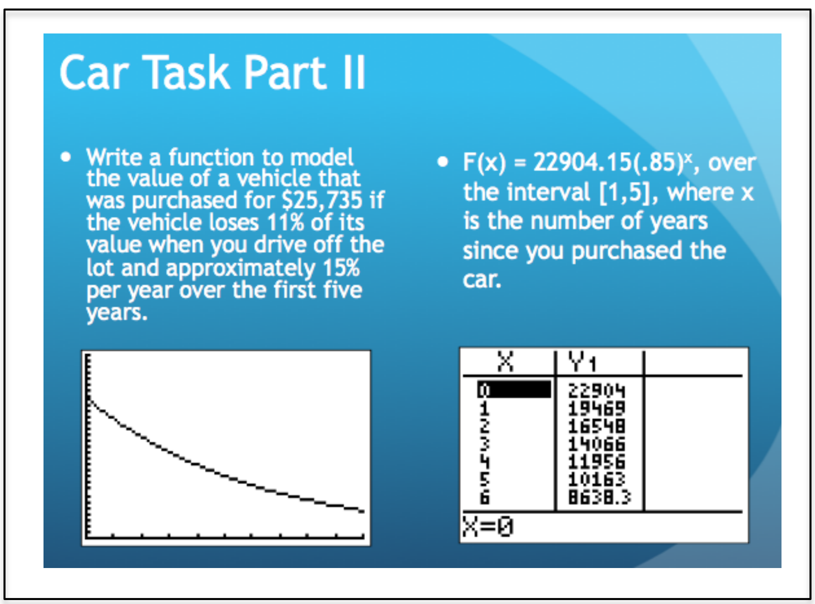


Figure- 20 Exponential Model Representations

Three students recognized that an exponential function should be used to model the situation, but did not account for the almost immediate 11% decrease in value when the car is driven off the lot. Six students thought that the model should be linear, and nineteen students were not able to represent the situation with any type of model. The students had to recognize that the situation was modeled by an exponential decay function on the interval [1,5] and that the initial value of 11% had to be calculated before writing the exponential equation.

2.3.3 Reflection

Since this was the first individual task assigned to students it was difficult for them to approach the task on their own. Many wanted to confer with a partner. As I tried to encourage independence, one student actually made the comment, "You are the teacher and it is your job to to tell us how to do the problem". I tried to encourage the students to persevere through the task, but then I noticed that an issue that many students had was with understanding the context of the problem. This made me aware that when I select problems that involve real-life situations, I need to make sure that the students are familiar with the concepts needed to understand the problem. It did not occur to me that the students would not understand the concept of a downpayment or that financing involved paying interest. Initially, some of the students did not understand how the value that you had paid over five years was more than the purchase price of the vehicle. This was eye opening to me and has made me rethink how I will use this activity in the future. Next year I will begin by introducing students earlier in the year to the concepts related to financing. The formula for calculating a monthly payment, $\frac{P(\frac{r}{12})}{(1-(1+\frac{r}{12})^{-m}}$, will be introduced early in the year to reinforce concepts of evaluating formulas, order of operations, percentages, and negative exponents. I will have the students select a vehicle that they would like to purchase and compute the price with tax, and then calculate a downpayment and monthly payment based on at least two

financing options from different lenders. The students will be asked to create an Excel spreadsheet that a finance manager could use to compute monthly payments for a potential buyer when given certain conditions. I will also use this opportunity to talk about advantages and disadvantages of credit. A second activity will follow later in the year that will involve students comparing options between leasing and purchasing a vehicle. Students will be given two scenarios and they will need to come up with a presentation advising a potential buyer about advantages and disadvantages of each option. The intent is that the students would model the situation with a system of linear equations and then use the results to advise the potential buyer regarding which option is better under which conditions. It is my intent to get students to think about what factors will need to be understood in order to determine which option is better. I will continue to introduce the concept of exponential functions using the example of depreciation to model exponential decay. However, I will search for more application problems that involve applications of exponential growth and decay since the problems in MathXL are mainly procedural and not conceptual. Also, we will collect actual data from the Internet to determine the rate of depreciation of a certain type of vehicle over a 5-year period of time to determine if an exponential model is the best fit for this situation. Finally, I will adjust the current task in the following manner:

> After you graduate from college and get your first job, the first thing that you do is purchase a new vehicle. The total with tax, title, and license is $25, 735. You are required to put down a 10% down payment and you will finance the balance. You are able to get a finance rate of 4% for 5 years. How much will you have paid for your vehicle at the end of your loan term? Clearly explain your answer.
>
> On average a new car loses 11% of its value the moment you leave the lot. In addition,

during the first five years a car will depreciate between 15 and 25% each year. Use the deprecation rate of 15% per year to create a model to compute the value of the vehicle after *y* years and then use your model to determine the value of your car after three years. Show work to support your answer. Should you use your model to predict the value of the car after ten years? Why or Why not?

I will again have two versions of the task available. The revised version will be an expert level task that allows the student to deconstruct the problem on their own to determine a solution and the second version of the task will be partially deconstructed to enable students who struggle with the process to be successful. It is my hope that the more exposure students have to these types of problems the more they will understand that the process is as important as the answer.

CHAPTER 3. CONCLUDING THOUGHTS

The goal of my thesis was to integrate tasks, technology, and the Common Core State Standards into the Algebra II classroom. I did this by selecting and revising tasks that were implemented in the classroom that required students to demonstrate understanding of mathematical topics by connecting the Standards for Mathematical Content with the Standards for Mathematical Practice. A correlation to the standards addressed in each task is located in Appendix K. The tasks were a combination of group tasks that lasted several days to an individual task that was to be completed in twenty minutes. Each of the tasks presented fostered multiple representations of solutions, collaboration, and the use of technology to interpret solutions. The tasks selected were revised to meet the needs of the students, to coordinate with the content in the Algebra II curriculum, and to have students apply mathematical knowledge. The tasks were designed to cover a wide-range of standards that required students to make connections between current content learned and mathematical content taught in previous years.

As I reflected on the tasks piloted this year, I revised tasks based on student responses. I found that the process of analyzing student work using formative assessment allowed me to improve the task for future classes. The reflection process also provided me with information that I could use as I developed future instructional lessons. The information gleaned regarding student misconceptions not only enabled me to revise the tasks, but I was able to extend and improve instructional units for the future that will address these misconceptions. One thing that I noticed is that in some instances my tasks were too structured and gave too much direction for some students. One revision in the future will be to include different versions of the task similar to what is done on the MAP website. An expert level task will require students to use and apply more of the Standards for Mathematical Practice in order to solve the problem. These tasks will

be presented to the students without any guidance regarding how to approach solving the problem, but plenty of space for the student to explain or justify the solution to the problem. I will encourage communicating mathematically using multiple representations and correct use of mathematical vocabulary and notation. I will also create a section of my classroom for students to have access to tools that they may use to guide them in the problem solving process. I will continue to use the document camera in my classroom to have students present various approaches to solving problems. Although I had the technology available this year, I was usually the one who would explain different approaches to solving problems using student work. In the future I will have students communicating and presenting more in the classroom. In order to motivate students to preserve through tasks, if a student is not ready for an expert level of a task, I will provide an apprentice level version of the task that has more guided instruction in order for the student to experience success with the task. I will continue to use the idea of formative assessment to provide feedback to students to help them improve solutions.

The classroom environment was a key factor in the success of integrating tasks into the course. An environment conducive to collaborative learning was necessary in order for students to feel safe discussing and critiquing the work of others. Both the teacher and the student need to recognize that learning how to reason through problem solving changes the role of the teacher and the student in the classroom. As a teacher I am very comfortable and confident in explaining procedural knowledge needed for content in Algebra II. However, it was difficult to stand back and see the students struggle with a problem. I wanted to offer assistance, but if I did this too quickly then I was just reinforcing their dependence on me to tell them how to solve the problem, and if I waited too long to offer help the students would become frustrated. I found it difficult to know what questions to ask to help guide them in the process without just telling them how to

solve the problem. It was also difficult when the student became frustrated to provide motivation for them to preserve through solving the problem.

The role of the student changes as well. The student needs to be actively involved in the learning process. Students are very comfortable with the role of sitting back and watching the teacher do all of the work. The introduction of MathXL in the classroom has helped to change the role of the student in the classroom from passive to active. However, the results of the implementation of these tasks made it clear to me that although students are actively working in Math XL, the program was reinforcing procedural knowledge but not reasoning skills, which are needed to be proficient in the Standards for Mathematical Practice. Since I want my students to be college and career ready as evidenced by new assessments implemented in 2014, it is important for me to provide opportunities for students to demonstrate understanding of both the Standards for Mathematical Content and the Standards for Mathematical Practice. The concept of dual intensity was new to me when I started this process, but I now see that in order for students to be prepared for college and career, both of these standards need to be addressed with a significant amount of rigor and intensity. It will be a challenge to create a rigorous, balanced environment that produces opportunities for students to demonstrate both procedural and conceptual knowledge, but a necessity. MathXL is a tool that will make this process easier as it will provide problems, immediate feedback, and help for students to strengthen procedural knowledge in addition to freeing up class time that can be used to have students interact with rich tasks that will allow students to reason, collaborate, and share mathematical solutions to problems.

Teaching and learning is an ongoing process that involves commitment from all parties involved. It is my desire to prepare my students for success in the 21st century. I would

encourage everyone who reads this to become knowledgeable about the CCSSM, to be intentional about creating an atmosphere of dual intensity in the classroom, and to begin the process of implementing opportunities for your students to demonstrate proficiency in mathematics through reasoning, problem solving, collaboration, and communication.

REFERENCES

Bloom, B.S. (1956). Taxonomy of Educational objectives: Classification of educational goals.Handbook 1: Cognitive domain. New York: Longman, Green & Co.

Common Core State Standards for Mathematics. Retrieved July 11, 2011, from http://www.corestandards.org/

Crosswhite, F. J., Dossey, J., & Frye, S. (1989). NCTM Standards for School Mathematics Visions for Implementation. *Journal for Research in Mathematics Education*, *20*, 513-522.

Danielson, C. & Marquez, E. (1998). *A Collection of Performance Tasks and Rubrics*. Larchmond, NY: Eye on Education, Incorporated.

Dudley, U. (2010). What is Mathematics For? *Notices of the AMS*, 57(5), 608 – 613

Eisenhart, M., Borko, H., Underdill, R., Brown, C., James, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education*, *24*, 8-40.

Glasgow, N. (1997). *New Curriculum for New Times.* Thousand Oaks, CA: Corwin Press, Inc.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Olivier, A., Human, P. (1997). *Making Sense: Teaching and Learning Mathematics with Understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., . . . Gallimore, R. (2005). Mathematics Teaching in the United States Today (And Tomorrow): Results from the TIMSS 1999 Video Study. *Educational Evaluation and Policy Analysis*, *27*(2), 111-132.

Jacobs, H. (1997). *Mapping the Big Picture*. Alexandria, VA: Association for Supervision and Curriculum Development.

Klum, G. (1994). *Mathematics Assessment: what works in the classroom.* Jossey-Bass, Inc.

Kober, N., Rentmer, D. S. (2011). States' Progress and Challenges in Implementing Common Core State Standards: Center on Education Policy.

Mathematics Assessment Project. Retrieved July 4, 2012, from http://map.mathshell.org/materials/index.php

National Council of Teachers of, M. (2011). *Making It Happen: Common Core Standards*: National Council of Teachers of Mathematics.

Newton, D. (2000). *Teaching for Understanding.* New York, NY: RoutledgeFalmer

Sullo, R. (1999). *The Inspiring Teacher: New Beginnings for the 21st Century. MD:* National Education Association.

Wiggins, G. & Mctighe, J. (2005). *Understanding by Design*. Alexandria, VA: Association for Supervision & Curriculum Development.

APPENDIX A: STANDARDS FOR MATHEMATICAL PRACTICE

Mathematics: Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a

logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with Mathematics**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear

definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 . 8 equals the well remembered 7 . 5 + 7 . 3, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2 . 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

These standards are from the Common Core State Standards
http://www.corestandards.org/the-standards

APPENDIX B: SUMMER MATH INFORMATION

Dear Parents,

The end of the school year is an exciting time, and it foreshadows a well-needed time of rest and rejuvenation! We wish all of our students and families a summer filled with fun experiences and peaceful moments. We also want to help our students maintain the academic edge they have worked so hard this year to obtain.

According to the National Association for Summer Learning, students, on average, lose approximately two months of grade-level math skills during the summer if they do not participate in any educational activities. Additional research shows that teachers often spend four to six weeks re-teaching material that was "lost" during the summer months. This lost time becomes more and more critical as students enter more challenging math classes in the middle and upper school grades.[1]

At Dunham, we are committed to providing your children with the best opportunities for a college preparatory education. As such, I am pleased to announce the addition of a summer math program for upper-middle and high school students, similar to the traditional summer reading program, Upper school students will utilize MathXL for their summer math. The goal of our summer program is:

1) Ensure that students maintain the prerequisite skills that are needed for their upcoming math classes.
2) Students that need extra practice can access the lessons and help they need through this individualized tool.
3) Students who are stronger in math will need less time to complete the activities than students who are not as strong.
4) **Students will have the opportunity to start the school year on a strong positive note, with the work they have completed over the summer counting as their first grade of the new quarter.**

It is anticipated that the average student will spend approximately 1 hour per week working in the program. Directions for accessing the program will be posted on the Dunham website under academics. The contents for the Algebra 2 requirements are listed below:

Algebra 2 Summer Math – 6 units

MathXL Course ID: XL0X---31SR---201Y---0CY2

Course format: 6 quizzes and 6 HW (Practice what your missed assignments) -- Students will begin with quiz 1. After submitting quiz 1, student will work on "Practice what I missed on quiz 1". Students may re-take quizzes after working on "practice what I missed" assignment. The "practice what I missed" assignment is diagnostic. It will automatically assign problems based on what was missed on the quiz.

To move to the next quiz, student must score a minimum of 70% on the quiz and 90% on the practice what I missed HW.

Once the required score has been made on the quiz and HW, student may progress to next quiz and HW.

All mathxl homework must be completed by August 8, 2012.

Unit 1 – no calculator: Basic Math Review. Properties of numbers, Fractions and operations with fractions, perimeter, exponents, order of operations (14 questions)

Unit 2 – no calculator: Variable expressions, solving equations and inequalities (11 questions)

Unit 3 – no calculator: Algebra 1 Review. Equations, Inequalities and problem solving, absolute value equations (11 questions)

Unit 4 – no calculator: Systems of equations, exponents, simplifying expressions (10 questions)

Unit 5 – no calculator: Multiplying polynomials and factoring (10 questions)

Unit 6 –no calculator: Solving quadratic equations, simplifying rational expressions, operations with rational expressions, simplifying radical expressions (12 questions)

APPENDIX C: APPLICATIONS INVOLVING QUADRATICS

Applications with Quadratics using Multiple Representations

1) The height in feet of a bottle rocket is given by the function
$h(t) = 160t - 16t^2$ where t is the time in seconds and h(t) is the height in feet.

   a) How long will it take for the rocket to return to the ground? Represent your solution algebraically.

   b) What is the maximum height of the rocket? Represent your solution algebraically.

   c) How would you interpret the solution to parts *a and b* graphically? Draw a sketch and label the solutions.

   d) How would you use the table on the calculator to find the solution to part *a?*

You may choose to use an algebraic, graphical, or numeric approach to answer each question, but you must show work to support your answer or explain how you arrived at your answer.

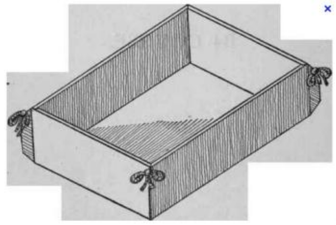
2) The formula $h = -16t^2 + 48t + 160$ gives the height of an object thrown from a building 160 feet high with an initial velocity of 48ft/sec, where t is measured in seconds and the height is in feet.

   a) How long will it take for the object to hit the ground?

   b) How long does it take the object to reach its maximum height and what is the maximum height?

APPENDIX D: MAXIMIZING VOLUME TASK

Maximizing Volume: The Crate Problem



Name:

Date: 3/27/12

Overview: You are part of a design team charged with assembling a crate to hold a vendor's product using a specified amount of material. You will explore the changes of the shape and volume of a scale model of the crate with respect to the size of the square that is cut from each corner. You will find a function that relates the volume of the box to the size of the square cut from each corner. Finally, you will use technology to approximate the maximum volume and the size of the square that will allow you to maximize the volume. Each person should submit a copy of the completed worksheet, but only one model and one set of graph paper sketches is due per group.

Materials: 1 sheet of construction paper with dimensions 8 inches x 12 inches
tape
ruler
scissors
pen
graph paper
graphing calculator
computer (GSP---Sketchpad)

1) Construct a box with an open top from a sheet of construction paper with dimensions 8 inches x 12 inches by cutting congruent squares from each corner and folding up the sides. You will use tape to secure the sides. Calculate the volume of your model. Label the dimensions on your model.

Side length of square selected (select the size of the square that you think will produce the largest volume for the box) ___________________________________

Volume of your model (show work to support your answer) __________________________

2) Using what you learned from question 1, complete the table below for the listed side lengths of the square cut from each corner. Use graph paper to sketch your models, but do not assemble the models. Please label your sketches. Only one set of sketches per group is required.

Original size of material	Length of side of square cut	Height of model	Length of model	Width of model	Volume of model
8 in x 12 in	1 in				
8 in x 12 in	2.5 in				
8 in x 12 in	3 in				

3) Use your model and/or the table above to help you create a function rule to represent the volume of the box if the length of a side of the square you remove is x inches and you are using a sheet of material that is 8 in x 12 in. (Hint: Look for patterns or think about how the height, length, and width are related to the square)

Original size of material	Length of side of square cut	Height of model	Length of model	Width of model	Volume of model
8 in x 12 in	x in				V(x) =

4) Use your equation to calculate V(2.7) and explain its meaning.

5) Predict what the graph of your model will look like. Think about end behaviors and intercepts.

6) What set of x values can be used in the equation given the context of the problem. Explain your answer.

Domain:

Explanation:

Use technology to produce a graph of your mathematical model and to answer questions 7 and 8.

7) To the nearest tenth of an inch, what size square should be cut from each corner to maximize the volume? ______________________________(Be sure to include the appropriate units in your answer.

8) Approximate the maximum volume of the box to the nearest tenth.

____________________________________ (Be sure to include the appropriate units in your answer)

9) Extension: Create a mathematical model to represent the volume of an open top box if the dimensions of the material used are ***a*** inches x ***b*** inches. Express your model of the polynomial function in standard form. Name the degree of the polynomial function and identify the leading coefficient.

V(x) =

APPENDIX E: RUBRIC FOR MAXIMIZING VOLUME TASK

Design Team Members:

	Grading Rubric	Comments
Team Collaboration	3 pts.	
Labeled Model (one per group)	1 pt.	
Worksheet		
Question 1	1 pt.	
Question 2 Table/Calculations	6 pts.	
Question 2 Graph paper sketches (one set per group)	3 pts.	
Question 3 Function Rule V(x) =	2 pts.	
Question 4 V(2.7) and explanation	2 pts.	
Question 5 Graph Prediction	2 pts.	
Question 6 Domain and Explanation	2 pts.	
Question 7 Size of Square	1 pt.	
Question 8 Maximum Volume	1 pt.	
Technology Model Sketchpad	2 pts.	
Technology Model Calculator	2 pts.	
Appropriate use of units	2 pts.	
Total	30 points	

APPENDIX F: STUDENT RESPONSES TO MAXIMIZING VOLUME TASK

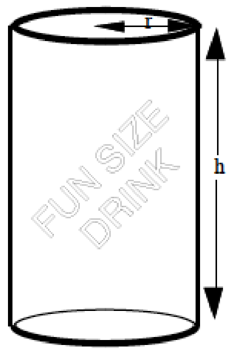
Mathematical Model	
No mathematical model given	6 students
V(x) = 8*12*x	2 students
V(x) = x(12-x)(8-x)	2 students
V(x) = (2x-w)(2x-l)(2x-h)	2 students
V(x) = x(12-2x)(8-2x)	23 students
Graphical Connections	
No graph produced	15 students
Linear graph	2 students
Quadratic graph	7 students
Verbal description	1 student
Identified intercepts only	1 student
Correct polynomial model	9 students
Domain	
None stated	16 students
(-∞,∞)	6 students
(0, ∞)	8 students
(0, 4) U (6, ∞)	2 students
(0,4)	3 students

APPENDIX G: MODIFIED RUBRIC FOR ASSESSING TASKS

	Computation and Execution	Connections	Communication
1	❖ Serious errors in computation led to an incorrect solution. ❖ Representations were seriously flawed. ❖ No evidence was given of how the answer was computed	❖ Unable to recognize patterns and relationships.	❖ Little or no explanation for the work was given. ❖ Mathematical vocabulary was incorrect.
2	❖ Minor computational errors were made. ❖ Representations were mostly correctly, but not accurately labeled. ❖ Evidence for solutions was inconsistent or unclear	❖ Recognized some patterns and relationships.	❖ Explanation was not clearly stated. ❖ Mathematical vocabulary was used imprecisely.
3	❖ Computation was accurate. ❖ Representations were complete and accurate ❖ Work clearly supported the solution.	❖ Recognized important patterns and relationships.	❖ Explanation was easy to follow. ❖ Mathematical vocabulary was used correctly.
4	❖ All aspects of the solution were accurate. ❖ Multiple representations verified the solution. ❖ Multiple ways to compute the answer were shown.	❖ A general rule or formula for solving the related problem was created.	❖ Explanation was clear and concise. ❖ Mathematical vocabulary was used precisely.

APPENDIX H: SAMPLE EXPERT TASK FROM MAP WEBSITE

Bestsize Cans



The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm^3.

In order to keep costs low, the company wants to use the smallest amount of aluminum.

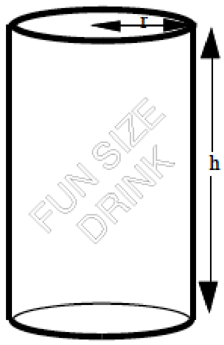
Find the radius and height of a cylindrical can which holds 200 cm3 and uses the smallest amount of aluminum.

Explain your reasons and show all your calculations

This resource is from the website
http://map.mathshell.org/materials/tasks.php?taskid=284&subpage=expert

APPPENDIX I: SAMPLE APPRENTICE TASK FROM MAP WEBSITE

Funsize Cans



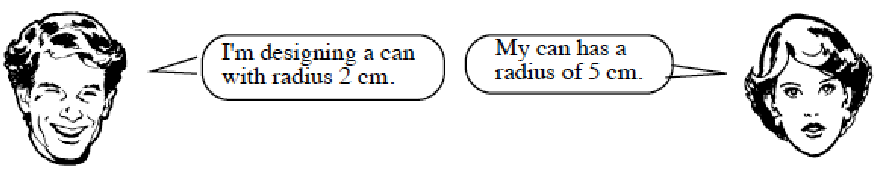
The volume of a cylinder is
$$V = \pi r^2 h$$

The surface area of a cylinder is
$$S = 2\pi r^2 + 2\pi r h$$

The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a `Fun Size' cylindrical can which holds 200 cm^3.

Here are two suggestions for the radius of the cylindrical can.



1. Each of these cans holds 200 cm^3. Find the heights of these two cans.

Are the dimensions of the cans suitable? Explain your answer.

2. Find the surface area of the two cans. Show your work

3. In order to keep costs low, the Fresha Drink Company wants to sell the drink in cylindrical cans that use the smallest amount of aluminum.

   Find the approximate radius and height of a can that holds 200 cm^3 and uses the smallest amount of aluminum. Show clearly how you figured out the size of the can.

   Make your dimensions correct to the nearest 0.5 centimeter.
   (You may find it helpful to use graph paper.)

This resource is from the website
http://map.mathshell.org/materials/tasks.php?taskid=252&subpage=apprentice

APPENDIX J: CAR TASK

After you graduate from college and get your first job, the first thing that you do is purchase a new vehicle. The total with tax, title, and license is $25, 735. You are required to put down a 10% down payment and you will finance the balance. Please answer the following questions. Show your work or explain how you arrived at your answer. You may use an algebraic, numeric, verbal, or graphical representation to express your answers to the questions asked in this task.

1) How much of a down payment do you need?

____

2) How much will you finance?

____

With a 10% down payment the finance manager can get you a rate of 4% for 5 years. He calculates that your monthly payment will be $426.55 per month. Write a function to represent the total amount that you have paid toward your purchase at the end of m months.

3) Function Rule:

____

Find your total cost after 1 year, 3 years, and at the end of your loan term. Justify your answers.

4) a) ____

b) ____

c) ____

On average a new car loses 11% of its value the moment you leave the lot. In addition, during the first five years a car will depreciate by 15 – 25% per year. Use an average of 15% per year to complete the table. Don't forget to adjust for the initial 11% of the value that the car loses almost immediately.

Price of Vehicle with tax, title and fees $25, 735		
	Depreciation amount	Value of vehicle
Off Lot --- 11%	5)	6)
End of 1^{st} year--- 15%	7)	8)
End of 2^{nd} year--- 15%	9)	10)
End of 3^{rd} year--- 15%	11)	12)
End of 4^{th} year--- 15%	13)	14)
End of 5^{th} year--- 15%	15)	16)

17) Write a function to model the value of the vehicle over the first five years. Show work to support your answer. Explain your reasoning for selecting your model.

____

APPENDIX K: STANDARDS ADDRESSED IN EACH TASK

Distance-Time Formative Lesson and Task	
Standards for Mathematical Practice	Standards for Mathematical Content
MP 1 Make sense of problems and persevere in solving them	8.F 2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically, or by a verbal description)
MP 2 Reason abstractly and quantitatively	8. F. 4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph, interpret the rate of change and the initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values
MP 3 Construct viable arguments and critique the reasoning of others	8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g. where the function is increasing or decreasing, linear or nonlinear.) Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
MP 4 Model with mathematics	A- CED. 2 Create equations in two or more variables to represent relationships between quantities; graph equations on the coordinate axes with labels and scales.
	F-IF. 2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.
	F. IF.5 Relate the domain of a function to its graph and where applicable to the quantitative relationship it describes. *
	F. IF. 6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *
	F. BF 1 Write a function that describes a relationship between two quantities. * a. Determine an explicit expression, a recursive process, or steps for calculation from context.

Maximizing Volume Task	
Standards for Mathematical Practice	Standards for Mathematical Content
MP 1 Make sense of problems and persevere in solving them	7.G. 6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and – three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
MP 2 Reason abstractly and quantitatively	A-CED. 2 Create equations in two or more variables to represent relationships between quantities; graph equations on the coordinate axes with labels and scales.
MP 4 Model with mathematics	F-IF 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.
	F. IF. 4 For a function that models the relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship.
MP 5 Use appropriate tools strategically	F. IF.5 Relate the domain of a function to its graph and where applicable to the quantitative relationship it describes. *
	F. IF 7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
	F. BF 1 Write a function that describes a relationship between two quantities. * a. Determine an explicit expression, a recursive process, or steps for calculation from context.
	G-MG. 3 Apply geometric methods to solve design problems.

Car Task	
Standards for Mathematical Practice	Standards for Mathematical Content
MP 1 Make sense of problems and persevere in solving them	6.RP 3.C Find a percent of a quantity as a rate per 100. 6.EE. 9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.
MP 4 Model with mathematics	8. F 4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and the initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph.
MP 5 Use appropriate tools strategically	F-IF 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.
	F. BF 1 Write a function that describes a relationship between two quantities. * a. Determine an explicit expression, a recursive process, or steps for calculation from context.
	F-LE 1 Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.
	F-LE 2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.
	F-LE 5 Interpret the parameters in a linear or exponential function in terms of a context.

VITA

Beth Perkins McInnis was born in Bad Kreuznach, Germany, to Alton S. Perkins and Maureen S. Perkins. She has one younger brother, Scott, and two children, Justin and Meghan. She has been teaching for twenty-two years and has taught at The Dunham School for the last seven years. She is currently teaching Algebra II, Algebra II Honors, and PreAP Precalculus. She received her Bachelor of Science degree in Secondary Math Education from the University of New Orleans in 1990 and a Master of Education in Curriculum and Instruction in 1999. In 2003 she earned National Board Certification in Mathematics/Adolescence and Young Adulthood.