2013

Instability analysis and suppression of instability in low-density gas jets

Sukanta Bhattacharjee
Louisiana State University and Agricultural and Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_theses
Part of the Mechanical Engineering Commons

Recommended Citation
https://digitalcommons.lsu.edu/gradschool_theses/906

This Thesis is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Master's Theses by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.
INSTABILITY ANALYSIS AND SUPPRESSION OF INSTABILITY IN LOW-DENSITY GAS JETS

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Masters of Science in Mechanical Engineering

in

The Department of Mechanical Engineering

by

Sukanta Bhattacharjee
B.S., Bangladesh University of Engineering and Technology, 2008
August 2013
This thesis is dedicated to my parents, Manik Chandra Bhattacharjee and Supriti Rani Chakraborty, who are with me along these years and assist in terms of guidance, encouragement, advice, monetary support, mental support, and time.
ACKNOWLEDGEMENTS

I would like to thank my advisor Dr. Sumanta Acharya who provides me support and advices in my research work during these years.

I thank all of my friends and colleagues I worked with, Baine, James, Krishnendu, Pranay, Sheng, Srinibas, Susheel and especially Prasad. These people helped me a lot when I was facing difficulties in my works. I am very grateful to my friends whom I got wonderful moments in their company and for helping me in many regard at my research and academic courses.

Finally, my acknowledgement is due to my parents back in home, who were always behind me with their prayer, love and blessings, encouraged me and provided tremendous support throughout my life. Without their help, I would not have been here. I am very grateful to all of my family members who always support me in this endeavor.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ......................................................................................... iii

ABSTRACT ............................................................................................................... vi

CHAPTER 1: INTRODUCTION ............................................................................... 1
  1.1. Background ............................................................................................... 1
  1.2. Low-Density Jet ....................................................................................... 3
  1.3. Literature Survey ..................................................................................... 4
    1.3.1 Experimental studies ......................................................................... 4
    1.3.2. Numerical Studies ........................................................................ 15

CHAPTER 2: DEFINITIONS AND NOTATIONS .................................................. 18
  2.1. Notations .................................................................................................. 18

CHAPTER 3: SIMULATIONS ............................................................................. 20
  3.1. Geometry .................................................................................................. 20
  3.2. Mesh .......................................................................................................... 22
  3.3. Governing Equations ............................................................................... 24
  3.4. Boundary conditions ............................................................................... 27
  3.5. Data Processing ....................................................................................... 30
    3.5.1. Q-criterion ...................................................................................... 31
    3.5.2. Determination of the visualization method .................................... 31
  3.6. Configuration studied ............................................................................. 32
  3.7. Turbulent energy dissipation and Kolmogorov length scale .................... 33
  3.8. Grid independence and validation ......................................................... 35
    3.8.1. Grid independence study ................................................................ 35
    3.8.2. Flow field validation ...................................................................... 39

CHAPTER 4: RESULTS AND DISCUSSION ..................................................... 41
  4.1. Basic Jet .................................................................................................... 41
    4.1.1 Evolution of mean velocity ................................................................ 45
    4.1.2. Velocity fluctuations ......................................................................... 47
    4.1.3. Evolution of the species .................................................................. 48
    4.1.4. Species fluctuation .......................................................................... 50
    4.1.5. Flow structures .................................................................................. 52
    4.1.6. Generation of the instability ............................................................ 54
    4.1.7. Iso-surface of vortex structures ...................................................... 57
    4.1.8. Generation of the instability ............................................................. 64
    4.1.9. Varicose mode generation ............................................................... 64
    4.1.10. Iso-surface of density ................................................................... 69
    4.1.11. Vortex merging & hairpin vortex ................................................... 72
    4.1.12. Spectral analysis & universal scaling .......................................... 74
  4.2. Controlled jet ............................................................................................. 80
4.2.1. Vortex structures ......................................................................................... 80
4.2.2. Suppression of instability ........................................................................... 82
4.3. Conclusion ......................................................................................................... 88

REFERENCES ......................................................................................................... 90
APPENDIX A ........................................................................................................ 94
APPENDIX B ........................................................................................................ 98
APPENDIX C ........................................................................................................ 103
APPENDIX D ........................................................................................................ 108
VITA ..................................................................................................................... 112
ABSTRACT

A numerical study is conducted to understand the global instability of very low-density jets (as encountered in thermal plasmas). The simulations have been carried out for different parameters of density ratios, $S = \rho_j / \rho_\infty$ ranging from 0.5 to 0.03, different Reynolds numbers ranging from 500 to 4000 and different momentum thickness obtained by different extension tube length ranging from 3 to 6 diameter long. The flow parameters and vortex structures has been visualized to understand the details of the evolution of the flow field. The axisymmetric shear layer rolls up in the near field of the jet forming vortex rings. The rings merged, formed secondary mode of instability in the near field before they interacts with each other to form turbulence. Spectra results show that a global instability exists in the range of density ratios investigated. An envelope of the absolutely globally unstable region has been defined in a 3D space of Reynolds number, density ratio and momentum thickness space. A strategy for reducing the instability by altering the density profile of the surrounding gas stream is explored. Specifically, a ramp density profile or a step shape with an outward offset was explored, and it was observed that there was a reduction in the instability amplitude with the modified density profiles. Such a lowering in the instability fluctuations can be beneficial in stabilizing the thermal plasma behavior.
CHAPTER 1: INTRODUCTION

1.1. Background

Thermal spray process are used extensively for applying corrosion & wear preventive coating and heat resistant coatings. Among other techniques, thermal plasma torches are the most common method for propelling and melting ceramic coating and finally making a uniform coating on a metal surface.

![Figure 1-1: Picture of a plasma torch](http://www.westinghouse-plasma.com/wpc_plasma_torches/)

Figure 1-2 represents a schematic of a typical thermal plasma torch. An electric arc is formed between the conical cathode and the water-cooled anode, which heats and accelerates the plasma gas leaving the nozzle. This high temperature, high velocity and very light gas stream melts and accelerates injected powdered ceramic material, which impinges on a substrate to create a dense coating. Although the plasma spray process has been in use for nearly four decades, a more complete understanding of the low-density plasma jet fluid mechanics is needed.
In a DC plasma spraying, the fluid dynamical behavior of the plasma jet leads to the development of turbulence associated with strong entrainment of ambient gas into the plasma jet as sketched in Figure 1-3. Beginning with the engulfment of the ambient air into the plasma jet and the breaking down of eddies the entrainment finally leads to a short jet potential core and thoroughly mixed turbulent flow afterwards.

![Schematic of a thermal plasma jet](image1)

Figure 1-2: Schematic of a thermal plasma jet

![Main regions of a transitional plasma torch](image2)

Figure 1-3: Main regions of a transitional plasma torch
The fluid dynamics behind the entrainment and instabilities involves the unique property of the plasma jet. Typically reaching near 1-10 percent of the density of the ambient air, a plasma jet is well within the regime of low-density jet. So the jet dynamics of low-density jet is the driving mechanism behind all of observed behavior of plasma spray jet.

1.2. Low-Density Jet

Low-density jets are very commonly observed phenomenon in the world of fluid dynamics. A low-density gas jet is characterized by the injection of a low-density fluid into a high-density ambient surrounding. The flow structure of a low-density gas jet can be divided into three regions: Laminar (near field), transition and turbulent (far-field) regimes. Often when these jets are studied they posses characteristics such as rapid spreading, periodic oscillations and the nature of which, depends on the ratio of density between the primary jet and the ambient fluid into which it exits. When pure-frequency oscillations emerge of an intrinsic nature, the flow can be shown to be globally unstable; research has shown that global instability in low density axisymmetric jets can occur when the jet density is approximately less than 70% that of the surrounding fluid.

For naturally evolving free jets at relatively low Reynolds number, a thin shear layer is formed at the nozzle exit. The thin layer then rolled up and formed Kelvin-Helmholtz vortex street. By diffusion and coalescing these vortices grow in size and subsequent tearing due to nonlinear interactions resulting in the appearance of complex fine scale structures in the downstream direction. Stream-wise rib/hairpin vortices develop in the braid region between the primary structures of the shear layer and these vortices are responsible for the creation of small-scale motions outside the potential core. However, the presence of small-scale disturbances at the nozzle exit leads to early transition from K-H ring vortices to complex structures having multiple
length scales. The scale distribution of the flow structures depends on both Reynolds number, density ratio, and stream-wise location.

Practical applications of low-density flows are omnipresent. The density ratios $\rho_j/\rho_a$ generated, for example, in combusting systems and plasma processing are generally much less than unity and therefore exhibit global instabilities. In atmospheric thermal plasma spraying applications, the density ratios are on the order of 0.01, which is well within the regime of conditions known to support unstable global instability. In plasma spraying it is desirable to have an uninterrupted potential core of very hot gas for materials processing. However, the formation of global instabilities facilitate the mixing between the hot plasma gas and cold surrounding air, reducing the efficiency of the plasma deposition process and creating undesirable spray coatings. This very low density jet produced organized vortex structures which were partially responsible for the rapid entrainment of external air. The formation of these organized structures could be disrupted by introducing turbulence, but the rapid entrainment process was not significantly affected. It is precisely this process that has motivated the current work which aims at better understanding the creation of the instability and controlling the global instabilities present in very low-density jet environments. In particular, in an effort to mimic the conditions of an atmospheric spray process, very low density jets were studied, where the jet density was approximately from 1% to 50% of the density of the surrounding fluid.

1.3. Literature Survey

1.3.1 Experimental studies

Round jets with homogeneous shear layers have been studied extensively in the past. A fluid exhausted into a gas of equal density (isothermal flow) develops a shear layer at the interface between the jet gas and the surrounding gas. Near the jet exit region of a round jet has a
direct influence on the flow development in the far field of the jet. Several authors (Crow 1972, Moore 1977, Ffowcs-Williams 1987) have shown that linear stability analysis can accurately models the large-scale structure of the near field of the jet exit. Batchelor and Gill (Batchelor 1962) researched on the stability of steady unbounded axisymmetric homogenous jet flows of the wake-jet type with nearly parallel streamlines. For a small disturbance, the instantaneous pressure and velocity equations to a unidirectional jet were derived and the results were obtained in form of a fourier series with respect to time in cylindrical coordinates. Based on these assumptions, the critical Reynolds number of the jet was obtained.

Mollo-Christensen (1967) measured space-time correlations between different frequency bands with bandpass filters tuned to different frequencies in a turbulent jet. The power spectral density of velocity fluctuations in the near-injector region shear layer was measured and it was found that the spectrum of forced instability oscillations showed sharp distinct peaks at the forcing frequency as well as at sub-harmonic frequencies the near-injector region of the jet responded to the pressure field associated with pressure fluctuations downstream of the jet. Thus, it was suggested that knowing the phase speeds of the disturbance waves, the maxima and decay parameters for the different frequency components and the power spectral density of pressure fluctuations at the jet, the far-field power spectrum could be computed. Crow and Champagne (1971) found that the near-injector large-scale structure of jet turbulence was well modeled by linear stability theory. More recent developments in the understanding of turbulent shear flows have shown that coherent structures control the dynamics of the flow. In the case of free shear flows, such as mixing layers and jets, it has also been shown that external forcing techniques can be used to control the global development of these flows. Gaster et al. (1985) applied the inviscid stability theory to model the large-scale vortex structures that occurred in a forced mixing layer.
The comparison between experimental data and computational models showed that the agreement in both amplitude and phase velocity of the disturbances across the various sections of the flow was excellent on a purely local basis. Armstrong et al. (1977) and Stromberg et al. (1980) found that the large-scale structure of turbulence in a circular jet was dominated by the axisymmetric and the first order azimuthal components. The results of linear stability calculations were successfully applied to these two components to describe the near-field structures of a turbulent jet.

Early studies of jet instability were reviewed by Michalke (1984). These studies were carried out to understand the transition from laminar flow to turbulence and to describe the evolution of the large-scale coherent structures in the near-injector field. The analysis was based on the locally parallel flow assumption and the neglect of fluid viscosity, heat conduction and dissipation, and gravitational effects. Small disturbances in the form of normal modes were introduced in the equations of motion and the equations were linearised. The disturbance equations could then be reduced to one equation governing the amplitude of the pressure disturbance. The equation was solved for specified axial velocity and temperature profiles, and the unstable wave number were documented as a function of frequency. The critical Reynolds number denoting the transition from laminar to turbulent flow was found. Also, the streak line patterns obtained suing the results of the linear stability analysis were in good agreement with the first-stage of the vortex rolling-up process that was observed experimentally in the near field of the jet exit. Cohen and Wygnanski (1987) concluded that the linear stability analysis was able to correctly predict the local distribution of amplitudes and phases in an axisymmetric jet that was excited by external means. The analysis was also capable of predicting the entire spectral
distribution of velocity perturbations in an unexcited jet over a short distance in the stream wise direction.

Monkewitz and Sohn (1988) re-examined the linear inviscid stability analysis of compressible heated axisymmetric jet with particular attention to the impulse response of the flow. Monkewitz and Sohn assumed a velocity profile

\[ u = 1 - A + \frac{2A}{1 + (er^2ln2 - 1)^N} \]

Where \( A \) was the co-flow velocity ratio given as

\[ A = \frac{u_c - U_c}{u_c + U_c} \]

With \( U_c \) being the centerline velocity and \( U_c \) the ambient gas uniform velocity, \( N \) was a jet parameter that controlled the jet mixing layer thus indicated the axial location of the jet and thickness, \( r \) was the radial direction from the jet exit. They used the concept of the turbulent Prandtl number proposed by Reichardt (Schlichting, 1979) to specify a relationship between the density profile and the velocity profile as

\[ \frac{1}{\rho} - 1 = \left( \frac{u + 2A - 1}{2A} \right)^P \]

Where \( S \) was the ratio of the mass density at the jet centerline to that of the ambient gas and \( P \) was the Prandtl number. Two different responses were identified based on the result. In one case, the flow was absolutely unstable, when a locally generated small disturbance grew exponentially at the site of the disturbance and eventually affected the entire flow region. In the other case termed convective instability, the disturbance was convected downstream leaving the mean flow undisturbed. Monkewitz and Sohn (1988) documented the boundaries between the absolute and convective instability assuming the locally parallel flow, infinite Froude number,
and zero Eckert number. It was shown that heated (low-density) jets injected into ambient gas of high density developed and absolute instability and became self-excited when the jet density was less than 0.72 times the ambient gas density. Monkewitz and Sohn also studied the effect of the azimuthal wave number on the absolute stability of the hot jet. The critical density ratio (demarcating absolute and convective stability) was much lower, about 0.35, for the first azimuthal wave number mode than that for the axisymmetric wave number mode. This indicated the dominance of the axisymmetric mode during jet instability for high density ratios.

Yu and Monkewitz (1990) carried out a similar analysis for two-dimensional inertial jets and wakes with non-uniform density and axial velocity profiles. Again, gravitational effects were neglected and the flow was assumed to be locally parallel. The assumed a velocity profile of

$$u = 1 - A + \frac{2A}{1 + \sinh^{2N}(y \sinh^{-1}1)}$$

and for the temperature they used a profile, similar to the velocity profile, of

$$T = 1 + \frac{(s^{-1} - 1)}{1 + \sinh^{2N}(y \sinh^{-1}1 - 1)}$$

Where, $y$ was the direction normal to the flow axis and other symbols are as specified above the Sohn and Monkewitz discussion. An ideal gas was assumed to produce the variable density by heating. It was found that the absolute frequencies and wave numbers of small disturbances in the near field of the jet exit region scaled with the jet/wake width and not on the thickness of the individual mixing layer, suggesting that the absolute instability was brought about by the interaction of the two mixing layers.

Jendoubi and Strykowski (1994) considered the stability of axisymmetric jets with external co-flow and counter flow using linear stability analysis. The boundaries between absolute and convective instability were distinguished for various parameters: jet-to-ambient
velocity ratio, density ratio, jet Mach number and the shear layer thickness. Jendoubi and Strykowski assumed a hyperbolic tangent velocity profile of the form

\[ U(r) = 1 + A \tanh \left( \frac{D}{\theta} \left( \frac{1}{r} - r \right) \right) \]

where \( D \) was a characteristic diameter of the jet, \( \theta \) the momentum thickness, \( A \) the co-flow velocity ratio similar to that of Monekowitz and Sohn (1988) and \( r \), the radial distance from the jet axis. The temperature distribution was assumed to satisfy the Busemann-Crocco relation (Schlichting, 1979) and thus the density profile was given as

\[ \rho(r) = \frac{S}{1 + \left( \frac{1}{2A} \right)(U-1-A) + \left( \frac{M^2}{1+A} \right)^2 \left( A^2 - 1 + 2(U - U^2) \right) \]

with \( S \) being the density ratio, \( M \) the centerline Mach number and \( \gamma \) the specific heat ratio. Two distinct axisymmetric instability modes were identified: the first became absolutely unstable in the presence of co-flow whereas the other mode required counter flow to become absolutely unstable. The onset of global self-excitation identified in laboratory jets agreed reasonably well with the predictions.

Kyle and Sreenivasan (1993) preformed experiments at low Richardson numbers to study the instability and breakdown of axisymmetric helium/air mixtures emerging into ambient air. Kyle et al. made high-speed motion pictures of the jet by illuminating the flow with a continuous laser sheet. They made velocity measurement of the flow using a laser-Doppler velocimeter in the forward scatter-mode. The velocity profile of the jet approximated the Blasius profile. A hot wire anemometer operated on the constant temperature mode was also used to obtain velocity measurements in air jets (constant density within the shear layer). Kyle and Sreenivasan observed an intense oscillating instability (the oscillating mode) of the jet when the ratio of the jet exit density to the ambient fluid density was less than 0.6. From the high-speed films, they
found that the overall structure of the oscillating mode repeated itself with “extreme” regularity which often extended to the fine-scale structure of the jet.

Raynal et al. (1996) carried out experiments with variable-density plane jets issuing into ambient air. A range of density ratios (0.14 to 1) and a range of Reynolds numbers based on the slot width and the jet fluid viscosity (250 to 3000) were studied. It was found that when the jet to ambient fluid density ratio was less than 0.7, the jet exhibited self-excited oscillations. The frequency and amplitude of these oscillations scaled with the nozzle width and not the thickness of the shear layer, indicating that the oscillatory behavior was a whole-jet phenomenon. Theoretical results based on the analysis of Yu and Monkewitz (1990) indicated the presence of absolute instability for density ratios less than 0.94 for plane jets, which was different from the value of 0.7 observed in experiments. Raynal et al. (1996) attempted to explain this discrepancy by taking into account the differences in the location of the inflection points in the density and velocity profiles for the cases of heated jets injected into air and the isothermal helium-air jets injected into air. It was found that the difference in the profiles alone could not entirely account for the discrepancy, and that the differences in the location of the inflexion points of the velocity and density profiles was important. The growth rate of the absolute instability was maximum when the inflection points in the density and velocity profiles coincided with one another. Also, a critical value for the difference the difference in the location of the inflexion points of the velocity and density profiles was noted, above which the absolute instability became convectively unstable.

Richards et al (1996) used Mie scattering to visualize helium-air injected into air. Jets at varying density ratios and Reynolds number were studied. Intense mixing and vortex interactions characterized the self-excited helium jets at a density ratio of 0.14. using an aspirating probe,
concentration measurements at several radial locations were obtained that provided evidence of side-jets which produce vigorous mixing.

Note that gravitational effects have been neglected in all of the analysis listed above. The neglect of gravitational effects has been justified by the authors because of the small magnitude of the Richardson number based on the jet velocity and jet diameter. However, within the shear layer where the density and velocity change from jet to ambient values, the gravitational effects may not be negligible. Literature on the effects of gravity and/or buoyancy on the stability characteristics of low-density gas jets injected in higher density ambient gases is limited.

Early studies of buoyant planar and axisymmetric jets are reviewed by Gebhart et al. (1984), Hamins et al. (1992) used a shadowgraph technique to observe the near-field behavior of a non-reacting buoyant helium plume discharged from a round tube into air. Hamins et al. compared the pulsation in non-reacting helium plumes to the pulsation in diffusion flames. Pulsation in the non-reacting helium were not observed until a minimum flow rate that was much greater than the flow rate required to initiate pulsation in the flames. Like methane flames, the location of the formation of the vortex structures depended on the helium exit velocity. The Strouhal number-Froude number relationship for the non-reacting helium plume was $St \propto \left( \frac{1}{Fr} \right)^{0.38}$, unlike that for the flames which was $St \propto \left( \frac{1}{Fr} \right)^{0.57}$. The Strouhal number was defined as $St = fD/V$; where, $f$ is the pulsation frequency, $D$ is the jet diameter and $V$ is the exit velocity. While the Froude number was defined as $Fr = V^2/(gD)$; where $g$ is the acceleration due to gravity. Hamins et al. (1992) concluded that difference in the Strouhal number Froude number relations was due to the difference in the density profiles in the shear layers of the non-reacting and reacting flows.
Subbarao and Cantwell (1992) performed experiments on a co-flowing buoyant jet to study the scaling properties and effects of Richardson number and Reynolds number, independently, on the natural frequency of the jet. A helium jet, with a parabolic exit velocity profile, was injected into co-flowing air at half the helium jet velocity. The presence of co-flow eliminated the random meandering usually present in buoyant plumes with no co-flow. The jet exhibited periodic oscillations; even the fine-scale structure of these oscillations was highly regular and repeatable at high Richardson numbers. The Strouhal number defined as

\[ \text{St} = \frac{fD}{U_j} \]

was plotted as a function of the square root of the Richardson number and was separated into three regimes. Note, for the graph, Subbarao and Cantwell (1992) defined the Richardson number as

\[ Ri = \frac{gD}{U_j^2} \left( \frac{1 - \frac{\rho_j}{\rho_\infty}}{\frac{\rho_j}{\rho_\infty}} \right) \]

At low Richardson numbers, the flow Strouhal number scaled with an inertial timescale, \( DU_j \), while at high Richardson number the Stronghal number scaled with buoyancy timescale, \( [\rho_j D^3 g (\rho_\infty \rho_j)]^{0.5} \). Between the Richardson numbers of 0.7 and 1, a transition regime occurred.

Subbarao and Cantwell (1992) defined a “buoyancy Strouhal number” as

\[ \text{St}_\theta = \frac{fD}{U_j^2} - K_1 \]

where, \( K_j \) is an empirical constant. Subbarao and Cantwell (1992) suggested that at high Richardson numbers the “buoyancy Strouhal number” was constant at a value.

Cetegen and coworkers studied buoyant plumes experimentally Cetegen and Kasper (1996) performed experiments on the oscillatory behavior of axisymmetric buoyant plumes of
helium and helium-air mixtures. The effects of varying nozzle diameters, source velocities and plume densities were investigated. Cetegen and Kasper (1996) observed that the plume boundaries exhibited periodic oscillations that were repeatable and evolved into toroidal vortices similar to those found in pool fires. When Strouhal number was defined as $St = fD/U_j$ and Richardson number defined as

$$Ri_j = \frac{(\rho_x - \rho_j)gD}{\rho_x U_j^2}$$

Cetegen and Kasper (1996) determined that three regimes could be identified for the Strouhal number, $St$, of buoyant jets: for $Ri_j < 100$, $St = 0.8 \, Ri_j^{0.38}$; for $Ri_j > 500$, $St = 2.1 \, Ri_j^{0.28}$; and for $100 < Ri_j < 500$ a transition regime occurred. Cetegen and Kasper (1996) explained that the effect of turbulent mixing on local plume density and a resulting change of the convection speed of the vortices were the reason for the different observed flow regimes. Cetegen (1997-1) investigated the effect of sinusoidal forcing on an axisymmetric buoyant plume. Helium and helium-air mixtures were again used. A loud speaker was used upstream of the plumes to force them. The plumes responded readily to the imposed oscillations with the toroidal vortices forming at the frequency of forcing. Mushroom-shaped small-scale vortex pairs observed in the early part of the forced plumes that were not observed in unforced plumes. As the plumes evolved downstream, the frequency spectra became more broadband and contained frequencies other than the imposed one and its harmonics.

Cetegen (1997-2) used digital particle image velocimetry to measure the velocity field of a naturally pulsating plume of helium-air mixture in the presence of co-flowing air. The density ratio for the plume was 0.25, source velocity was 0.05 m/s and Richardson number of 283. The Richardson number was defined as $= \frac{gd(\rho_x - \rho_j)}{\rho_x U_j^2}$, where $g$ is the acceleration due to gravity, $d$ is
the nozzle diameter \( (d = 10 \text{ cm}) \), \( V \) is the source velocity, \( \rho_j \) is the plume density and \( \rho_\infty \) is the ambient air density. The oscillation frequency of the plume was between 3 and 4.5 Hz. Velocity measurement were performed in a phase resolved manner so as to characterize different phase of the plume. The velocity field around the toroidal vortex at 1.5 cm from the nozzle exit was characterized. Cetegen (1997-2) observed that the formation of the toroidal vortex structure occurs at about an axial location of 0.5d. The vortex then convects downstream and significantly affects the flow field in its vicinity.

Pasumarthi (2000) conducted experiments to investigate the flow structure of a pulsating helium jet injected into air using quantitative rainbow schlieren deflectometry. Pasumarthi (2000) observed the flickering of the helium jet that revealed periodic global oscillations in the flow field. Like Subbarao and Cantwell (1992), Pasumarthi (2000) observed that the Richardson number had a more significant effect on the flow structure than the jet exit Reynolds number. His data revealed a linear relationship between the Strouhal number \( \left( \frac{fD}{U_j} \right) \) and Richardson number, defined as \( \frac{gD}{U_j} \left( \frac{\rho_\infty - \rho_j}{\rho_j} \right) \), for the range of Richardson numbers, 0.5 to 6.5, investigated.

More recently, Pier & Huerre (2001) embarked on a comprehensive examination of the nonlinear global modes and the conditions under which disturbance will lead to global flow resonance (see also the review by Chomaz, 2005). By studying the nonlinear dispersion relation of a slowly diverging wake, Pier & Huerre concluded that the real wake frequency is selected by the first streamwise station of non-negative absolute growth, a conjecture supported by their numerical simulations and the earlier results of Monkewitz & Nguyen (1987). More recent findings of Lesshafft et al. (2005) explore this connection in the context of low density axisymmetric jets, by conducting both DNS simulations as well as spatio-temporal stability
calculations on the computed base flow. The frequency predictions of the linear theory were substantially lower than those found in the DNS simulations, a point noted by the authors as possibly being associated with the high Reynolds number of the simulation.

One of the most important discovery in the field of low-density gas was found by Hallberg & Strykowski (2006) in their study to find an universal scaling law which governs the global modes. They showed that that the onset of global mode depends on three independent parameters of Reynolds number, the density ratio and the momentum thickness. They changed the momentum thickness in their experimental study by changing the entry length of the jet inlet. The non-dimensional number they construct are
\[
\text{Re} \left( \frac{D}{\theta_0} \right)^{1/2} (1+S^{1/2}) \quad \& \quad fD^2/\nu
\]
A domain constructed by these variables can set all of the previous works in a linear relation of these parameters which can predict any combination between these three independent parameters.

In 2007 Nichols et al. in their study of self sustained oscillations in variable density round jets shows the vorticity dynamics of low density jet and supported it with the instability study and DNS study. One other breakthrough is the finding of Lesshafft & Marquet (2010) of an optimal velocity and density profiles for the onset of absolute instability in jets in their theoretical study.

1.3.2. Numerical Studies

Recent advances in computational techniques have seen the advent of computational fluid dynamics (CFD) to gain insight into the detailed flow structure of jets. The CFD analysis has been used for basic studies of the fluid dynamics, for engineering design of complex flow
configurations, and for predicting the interactions of chemistry with fluid flow for plasma, flame and propulsion applications. The CFD analysis can also assist in the interpretation of experimental data and probe regimes that are inaccessible or too costly to study experimentally.

Mell et al. (1996) achieved good agreement between computed and measured flow oscillation frequencies. The computational study by Soteriou et al. (2002) provided flow structure details of planar low-density gas jet. The nature of the instability and relative roles of buoyancy and diffusion in triggering/sustaining the instability were also explored. Using intuitive arguments, they proposed that diffusion was important in initiating the flow instability. Diffusion however assumed a secondary role after the instability was manifested.

Foysi et al. (2010) showed a comprehensive and detailed nature of the low-density jet, both for planer and for round jets, in their LES study. They extensively studied the growth rate of the low-density jet from different aspect and found a good agreement with the findings of Hallberg et al. (2006).

In spite of the vast literature on jet flows, the detailed flow structure in the near field of low-density buoyant jets has not been documented in past studies. Experimental studies have described the flow using either velocity or concentration measurements, which are inadequate to elucidate the strong coupling between velocity and concentration fields in low-density gas jets. In this regard, the analysis based on computational fluid dynamics (CFD) is useful because it provides flow structure details including simultaneous visualization of vector-scalar fields throughout the region of interest. However, past computational studies have considered either the global features of the oscillating jet (Mell et al., 1996) or the planar jets (Soterious et al. 2002).
Detailed CFD analysis is therefore needed to not only characterize the flow structure of oscillating jets, but also to explain the mechanism of instability in circular jets.

The objective of the present computational study is to examine the flow structure of low-density buoyant gas jets issued from a circular tube into quiescent ambient, which is among the most practical jet configuration. Numerical simulations were performed to identify and explain interactions among velocity and concentration fields over a range of jet Reynolds numbers. Computations were done using an unsteady model and no external perturbations were introduced to ensure that the flow oscillations, if present, were self-excited. The jet flow is characterized by velocity vectors and profiles of jet half radius, axial velocity and helium concentration etc. both instantaneous and mean structure is presented in case of the oscillating flow. Results are presented to highlight unique features of steady and oscillating low-density gas jets. Besides providing details of the flow structures, the mechanism of instability is explored.
CHAPTER 2: DEFINITIONS AND NOTATIONS

Before going into more details about the model setup and procedure, the variables used in this thesis must be introduced as well as the definitions of the main parameters of our experiments.

2.1. Notations

The following are the few notations which is used extensively thorough out the entire document.

\[ D = \text{Diameter of the central jet} \]
\[ f = \text{Frequency} \]
\[ L = \text{Jet extension tube length} \]
\[ \nu = \text{Viscosity of the gas} \]
\[ r = \text{Radial distance from the center of the jet} \]
\[ \text{Re} = \text{Reynolds number} \]
\[ \rho_j = \text{Jet density} \]
\[ \rho_\infty = \text{Ambient density} \]
\[ S = \text{Density ratio (} \rho_j / \rho_\infty \text{)} \]
\[ \text{SF}_6 = \text{Sulfur Hexa-fluoride} \]
\( \theta \) = Momentum thickness

\( U \) = Velocity of jet at the entry of the tube

\( U_0 \) = Velocity at the center of the jet

\( u(x, y, z, t) \) = Axial Component of Velocity [m/s]

\( z/D \) = Axial location in terms of jet diameter from exit

**Basic jet:** Basic jet means a jet without any control technique to modify the jet spreading, entrainment and other flow properties.

**Controlled jet:** Controlled jet means a jet with some kind of control technique to alter the flow properties to facilitate or suppress the jet spread and entrainment.

**Low-density jet:** A free jet with lower density than the ambient fluid. It has a density ratio lower than unity.

**Homogeneous jet:** A free jet with the same density as the ambient fluid. The density ratio is unity.
CHAPTER 3: SIMULATIONS

3.1. Geometry

The geometric domain used for the numerical simulation is a rough model of a typical very low-density jet experimental set-up. A typical very low-density jet experimental set-up consists of a high-pressure high-density gas chamber, low-density gas nozzle, after nozzle extension tube, the open to atmosphere chamber where the gases mix with other peripheral parts such as the porous medium, high density and low density gas pipes. The open atmosphere gas chamber is generally open in the top side (figure 3-1). The high-density gas enters into the open chamber from bottom through a porous medium in very low velocity to ensure a quiescent flow. After that through a contoured contraction nozzle and extension tube, helium gas exhausts into the dense SF6 gas that is already into the open chamber and goes in upward direction. The extension tube before the jet exit, the sulfur hexa-fluoride chamber size, the length of the knife-edge adjacent to jet exit and the domain extent in axial, radial and azimuthal direction are a direct function of the jet exit diameter.

Figure 3-1: A typical very low-density jet experimental set-up
For simulation, a domain directly related to the flow is considered, hence, discarding the high-pressure chamber, porous medium through which SF6 enters into the chamber and helium contraction nozzle. The rectangular box structure of the chamber is also altered in the simulation to a cylindrical chamber. The box is wide enough to consider as an open environment. Since the cylindrical chamber ensures uniform grid spacing, with proper boundary condition it will serve the same purpose that the box does.

The jet exit has a diameter, D of 5 millimeter, situated at the center of the bottom and perpendicular with the horizontal plane. The cylindrical open chamber is extended 12 D in radial direction, 15 D in the axial direction. The role of the extension tube in experiment is to modify the momentum thickness. In the simulations extension tube of 3D, 6D and 12D, feeding the jet, are used to alter the jet momentum thickness. The jet exit is located exactly 0.06D downstream from the beginning of the open chamber.

Figure 3-2: The geometric domain
3.2. Mesh

The meshing of the geometry has been performed with the commercially available software GAMBIT. The computational mesh is structured and consists of approximately 2.8 million hexahedral elements. Fine elements are used near the jet wall, jet exit, the region of shear flow and coarse elements are used in place around the chamber peripheral boundary and top boundary.

The meshing of the jet and quiescent flow is a very delicate part of the operation. Since it involves a large density variation due to the species difference, without proper tailoring the elements near the shear layer it is very susceptible to divergence. To reduce the number of skewed elements in the cylindrical section of the jet, an ‘O-grid’ block has been employed. It arranges the grid lines into an ‘O’ shape where a block corner lies on the continuous curved surface. Figure 3-3 shows the mesh inside the helium pipe. A high mesh density has been used around the periphery and low mesh density is used around the center.

![Figure 3-3: Computational mesh inside the helium jet](image)

It can be observed that in the central square section of the jet is composed of rectangular elements exclusively and the peripheral section is composed of slightly skewed elements. 900 square elements of size 0.0167D (edge length) result from the uniform meshing of the inner...
square. The corner of the inner square and the circular section are linked to each other by radial edges containing 15 elements per edge. Therefore, the jet cross-section is composed of 2700 elements. The depth of the elements in the jet extension tube has been set up to a variable length of 0.018D in the both end to 0.1D in the middle. The maximum cell squish is 0.9, which is well within the acceptable limit, is achieved inside the helium jet extension tube.

![Figure 3-4: Computational mesh at a cross section after the jet exit and longitudinal section in the middle of the domain](image)

Figure 3-4 represents the mesh of the cross-section after the jet exit. The mapped mesh resulting from the meshing of the open chamber from the jet outer radius to the chamber inner radius. 120 elements in the azimuthal direction and 100 elements in the radial direction made a total of 12000 elements in one plane of the cross section.

To resolve the flow properly near the area of boundary layer, the shear layer and the near field of the jet exit, which is our region of high interest, maximum mesh density of fine resolution is applied. As we are not much interested to the flow regions near to the boundary and
the far downstream from the jet exit, the mesh is more spread there. The mesh size increases linearly upwards at a rate of 1.05 until 10D then has a constant size afterwards up to the top end.

![Total Computational Mesh](image)

**Figure 3-5: Total computational mesh**

The quality of the meshing has been tested by using GAMBIT mesh checker. For the code the maximum allowable aspect ratio in the bulk flow is lower than 5 and around 10 in the boundary layer. The maximum aspect ratio for this mesh is well within this limit.

### 3.3. Governing Equations

A commercially available CFD code, FLUENT, is used in the investigation. Incompressible Navier-Stokes equations are solved with binary species on a generalized, curvilinear coordinate system using finite volume based discretization. The spatial and temporal discretization used are second order accurate and use a weighted scheme. A second order, implicit time stepping scheme is used for the temporal differencing. In order to accurately
capture the evolution of the flow features, the physical time step was chosen for each grid so that the CFL number was restricted to be about 0.8 in the most refined portion of each grid. The physical time step is also chosen in a way so that the maximum frequency expected to be captured must be $1/30^{th}$ of the time step so that the code can accurately capture it.

The conservation equations are expressed as follows:

$$\frac{\partial \rho}{\partial t} + \mathbf{V} \cdot (\rho \mathbf{u}) = 0$$

Equation 3-1: Continuity equation

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \mathbf{V} \cdot (\rho \mathbf{u} \mathbf{u}) = -\mathbf{V} P + \mathbf{V} \cdot (\mathbf{\bar{\tau}}) + \rho \mathbf{g}$$

Equation 3-2: Momentum equation

Where, $\mathbf{\bar{\tau}}$ is the stress tensor and $\rho \mathbf{g}$ is the gravitational force. The stress tensor is given by

$$\mathbf{\bar{\tau}} = \mu \left[ (\mathbf{\bar{u}}\mathbf{u}) + \mathbf{V} \mathbf{u} \mathbf{u}^T \right] - \frac{2}{3} \mathbf{V} \mathbf{u} \mathbf{u}$$

Equation 3-3: Stress tensor equation

Where $\mu$ is the molecular viscosity, $I$ is the unit tensor, and the second term on the right hand side is the effect of volume dilatation.

$$\frac{\partial (\rho Y_i)}{\partial t} + \mathbf{V} \cdot (\rho \mathbf{u} Y_i) = -\mathbf{V} \mathbf{J}_i$$

Equation 3-4: Species transport equation

Where, $Y_i$ is the local mass fraction of each species. $\mathbf{J}_i$ is the diffusion flux of species $i$, which arises due to gradients of concentration. The code uses dilute approximation/Fick’s law to model mass diffusion due to concentration gradient, under which the diffusion flux can be written as,
\[ \dot{J}_i = -\rho D_{i,m} \nabla Y_i - D_{T,i} \frac{\nabla T}{T} \]

Equation 3-5: Mass diffusion equation

Here \( D_{i,m} \) is the mass diffusion coefficient for species \( i \) in the mixture, and \( D_{T,i} \) is the thermal diffusion coefficient. For calculating the mass diffusivity/mass diffusion coefficient of helium into SF6 the following formulation is used.

\[
D = \frac{0.00186 \cdot T^{3/2} \sqrt{\frac{1}{M_1} + \frac{1}{M_2}}}{P \cdot \sigma_{12}^2 \Omega}
\]

Equation 3-6: Chapman-Enskog equation

This is the equation of Chapman-Enskog for no-polar low-pressure binary mixture, where \( T, M, P \) indicates temperature (K), Molar mass (g/mol), Pressure (atm.) respectively. \( \Omega \) is temperature-dependent collision integral (usually of order 1; dimensionless). \( D \) is diffusion coefficient (which is expressed in cm\(^2\)/s).

Now:

- \( \sigma_{12} = (\sigma_1 + \sigma_2)/2 \)
- \( \Omega = (44.54T^* + 4.909 \cdot 1.911T^* \cdot 1.575)^{0.1} \); Where \( T^* = kT/\varepsilon_{12} \)
- \( kT/\varepsilon_{12} = T/(\varepsilon_1/k \cdot \varepsilon_2/k)^{0.5} \); Where \( \varepsilon_i/T = 0.75 \) critical temperature.
- The critical temperature of helium and SF6 are 5.19 K and 327.7K.
- Form these values, \( \Omega = 0.745 \) (which is of order 1, as predicted)
- \( \sigma = 2.44(Tc/Pc)^{1/3} \)
  - For helium \( \sigma = 3.2287 \) and for SF6 \( \sigma = 5.0213 \). So the average value of \( \sigma = 4.125 \)
- \( Mab = 0.5068^2 \)
- From all of these values, mass diffusivity of Helium into SF6 is = 0.21422 cm\(^2\)/s
Every case is initialized after the initial transient conditions have washed away and the fully developed operating condition is reached, which is approximately 20 domain flow-through times. Time averaging is then begun and the time dependent quantities has been collected. Averaging is done by taking enough time steps so that an acceptable statistical convergence is ensured. Depending on the flow conditions this averaging requires usually around 12800 time steps, which is around 640D/velocity seconds. FFT data was then taken for 620D/velocity seconds again. Louisiana State University cluster Pandora as well as Pelican are used to calculate this simulations.

3.4. Boundary conditions

Figure 3-2 shows some of the boundary conditions applied during the simulations. Adiabatic, no-slip wall boundary conditions are applied in the helium extension tube/pipe wall, jet exit. At the inlet of the helium extension tube a velocity inlet boundary is applied with no swirl and no turbulence. The constant uniform velocity is determined from the operating Reynolds number for the specific case. The species concentration is always set to 100% helium. The extension tube makes the proper boundary layer and momentum thickness at the jet exit.

Most of the cases closely follow the experiment of Hallberg and Strykowski (2006). As like the experimentation, real values of the gas properties were used to create the same flow properties. For 0.03 and 0.14 density ratio Helium-SF6 and Helium-Nitrogen gas is used to match the experiment. For higher density ratio mixture of nitrogen and helium has been used.

A knife-edge had been created, as mentioned in case of the experiment, for initiation of the shear layer. Like Hallberg et al. extension tubes are used to create the same momentum
thickness, which is an important parameter in onset of global instability, and are tested to check if they are generating the momentum thickness as the experiments or not.

\[ \delta^{**} = \sqrt{\int \left( \frac{u}{U_{e}} \left( 1 - \frac{u}{U_{e}} \right) \right) dy} \]

From the boundary layer generated inside the extension tube of helium inlet the momentum thickness is calculated using the above equation and match has been found with the experimental value. Moreover the square root dependence on Re corroborates that the boundary layers exiting the tubes are laminar over the range of conditions reported. Experimental investigations have shown that the turbulent intensity for such setups are below 0.2%.

Hotwire anemometry was the primary quantitative diagnostic used in the experiment to characterize the jet response. One-second data sets were captured at a sampling rate of 5kHz for the majority of situations, but up to 50kHz when the primary frequencies were over 2.5kHz. In simulation a time step size of 0.00001 seconds were used for all of the cases which gives a frequency output up to 50kHz, exactly similar with the experimental probe. That also satisfies the requirement of the Kolmogorov time scale shown earlier. For the experiment the background turbulence was broadband with no distinct frequencies dominating the spectrum suggesting no facility instability or resonance was present. Same conditions has been applied to the simulation conducted for the current study.

In the experiments of Hallberg et al. SF6 was introduced into a glass box (1ft³) open to the atmosphere through 80 in² of 316 stainless porous media of 1 micron pore size (Media grade 0.1). The driving pressure of the SF6 was approximately 4psi to maintain a steady flow of SF6.
From this manufacturers specification for 0.1 media grade the velocity was calculated for the simulation, which is 0.01 m/s. This velocity is used to specify a velocity inlet boundary condition for SF6.

![Graph showing pressure drop vs. media grade and acfm/ft²](image)

Figure 3-6: Gas flow for porous media

Other boundary and initial conditions of no-slip for pipe wall, steep top hat density profile for helium jet exit and pressure outlet with atmospheric pressure at the outlet of the domain is used to match with the experimental boundary and initial conditions.

The flow get out from the domain through the cylindrical side boundary and the top boundary. Pressure outlet boundary conditions require the specification of a static pressure at the outlet boundary, which is 101325 Pascal in this case. The value of the specified static pressure is used only while the flow is subsonic. Should the flow become locally supersonic, the specified
pressure is no longer used; pressure is extrapolated from the flow in the interior. All other flow quantities are extrapolated from the interior. For the back flow 100% SF6 enters into the domain.

For pressure velocity coupling Pressure-Implicit with Splitting of Operators (PISO) scheme is used. It is part of the SIMPLE family of algorithms, based on the higher degree of the approximate relation between the corrections for pressure and velocity. One of the limitations of the SIMPLE and SIMPLEC algorithms is that new velocities and corresponding fluxes do not satisfy the momentum balance after the pressure-correction equation is solved. As a result, the calculation must be repeated until the balance is satisfied. To improve the efficiency of this calculation, the PISO algorithm performs two additional corrections: neighbor correction and skewness correction. Along with these parameters a suitable under relaxation factor is used for different equations.

3.5. Data Processing

The numerical data have been processed with the commercial software Tecplot 360. The large scale structures resulting from the interaction of a jet with the dense environment can be visualized by three different methods. Because the core of a large-scale structures is associated with strong vorticity and therefore local pressure minima. Jeong and Hussain (Jeong 1994) have established three definitions which capture the pressure minima; they are based on the Laplacian pressure, Q-criterion or $\lambda_2$-criterion. In this research only Q-criterion has been used to visualize the large scale vortex structures.
3.5.1. Q-criterion

For incompressible flows, the Laplacian pressure is directly related to the second invariant of the velocity gradient tensor \( Q \), which is defined in equation 3-7. It depends on the strain rate tensor \( S \) and rotation rate tensor \( \Omega \).

\[
Q = -\frac{1}{2} u_{i,j}u_{j,i} = -\frac{1}{2} (S_{ij}s_{ji} + \Omega_{ij}\Omega_{ji})
\]

Equation 3-7: Definition of the second invariant of the velocity gradient tensor \( Q \)

After the following calculation the above equation can be reduced to a simpler form for calculating in Tecplot 360.

\[
Q = -\frac{1}{2} (S_{ij}s_{ji} + \Omega_{ij}\Omega_{ji})
\]

\[
= \frac{1}{2} (||\Omega||^2 - ||S||^2)
\]

\[
= \frac{1}{2} (tr(\Omega\Omega^T) - tr(SS^T))
\]

\[
S = \left(\frac{1}{2}\right) [\nabla \vec{u} + (\nabla \vec{u})^T] \quad \Omega = \left(\frac{1}{2}\right) [\nabla \vec{u} - (\nabla \vec{u})^T]
\]

After several manipulation it finally reduces to:

\[
Q = -\left(\frac{\partial v}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial v}{\partial z}\right) - \left(\frac{1}{2}\right) \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]
\]

Equation 3-8: Simplified form of the Q-criterion

3.5.2. Determination of the visualization method

From their work, Jeong and Hussain stated, in the most cases almost same results obtained for the identification of the vortex cores by using the Q-criterion or \( \lambda_2 \)-criterion. Slight differences can be observed only for vortices with strong core dynamics; the vortex core boundary based on both definitions varies a little. It has to be notified that Jeong and Hussain did
not use iso-surface of the Laplacian pressure to visualize the coherent structures. Therefore no comparison has been preformed with it. However, since the Laplacian pressure is proportional to $Q$ for incompressible flows, it is quite legitimate to assume that the comparison between the Laplacian pressure and $Q$-criterion leads to the same conclusion. Furthermore, it is impossible to automate the calculation of $\lambda_2$ due to software limitations, the vertical structures will be visualized with iso-surface of the $Q$-criterion.

### 3.6. Configuration studied

According to the studies of Hallberg et al. (2006), the instability of the jet completely depends upon some defined non dimensional numbers. The most relevant non dimensional numbers related to this case are

- **Density ratio, $S$**
  - Range: 0.03, 0.14, 0.25, 0.5
- **Reynolds number, $Re$**
  - Range: 500, 1000, 1500, 2000, 3000, 4000
- **Strouhal number, $St$**
  - Range: 0.025 – 0.1
- **Diameter/Momentum thickness, $D/\theta_o$**
  - Range: 0.12 – 0.28
- **Roshko number, $fD^2/\nu$**
  - Range: 26 – 1500

Density ratio, Reynolds number and Diameter/Momentum thickness are the driving parameters which dictates the flow field. Density ratio and Reynolds numbers are directly altered by changing the density of the jet fluid and velocity. By changing the extension tube length momentum thickness has been changed while the diameter of the jet remained fixed. Three extension tube are considered that are 3D, 6D and 12 D long.
3.7. Turbulent energy dissipation and Kolmogorov length scale

The turbulent dissipation $\varepsilon$ depends on the local strain-rate of the flow field. For calculating the strain-rate, the following formulation has been used.

$$S = \left(\frac{1}{2}\right) [\nabla \vec{u} + (\nabla \vec{u})^T]$$

Now the rate at which kinetic energy is dissipated in a fluid is

$$\varepsilon = 2. \nu \left(\text{tr}[S S^T]^{1/2}\right)^2$$

$$\varepsilon = 2\nu \left[ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 \right]$$

Equation 3-9: Expression for turbulent energy dissipation

Using this equation, kinetic energy dissipation has been calculated for the flow. Values are given at specific location in where the power spectrums are shown. From this energy dissipation term Kolmogorov length scale, $\eta$ can be calculated as

$$\eta = \left(\frac{\nu^{3}}{\varepsilon}\right)^{1/4} \quad \text{and} \quad t_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$$

Equation 3-10: Kolmogorov length and time scale

Table 3-1: Energy dissipation, Kolmogorov length and time scale

<table>
<thead>
<tr>
<th>Location (x, y, z)</th>
<th>(0, 0, 0)</th>
<th>(0, 0, 1)</th>
<th>(0, 0, 2)</th>
<th>(0, 0, 3)</th>
<th>(0, 0, 4)</th>
<th>(0, 0, 5)</th>
<th>(0, 0, 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$ (m$^2$/s$^3$)</td>
<td>1.5619</td>
<td>1.4268</td>
<td>14.6087</td>
<td>2067.3011</td>
<td>360.8725</td>
<td>101.9015</td>
<td>26.9155</td>
</tr>
<tr>
<td>$\eta$ (m)</td>
<td>0.0010413</td>
<td>0.0010651</td>
<td>0.00059545</td>
<td>0.00017264</td>
<td>0.0001018</td>
<td>0.0000998</td>
<td>0.0001169</td>
</tr>
<tr>
<td>$t_\eta$ (s)</td>
<td>0.008854</td>
<td>0.009264</td>
<td>0.002895</td>
<td>0.0002434</td>
<td>0.0003062</td>
<td>0.0004608</td>
<td>0.0007981</td>
</tr>
</tbody>
</table>
Figure 3-7: Comparison of the Kolmogorov length scale and the grid size

Figure 3-8: Comparison of the Kolmogorov time scale and the simulation time step

In the Figure of Kolmogorov length scale and the grid size it is clear that the grid is adequate to capture the large-scale instabilities. While after 4D downstream of the jet exit the grid size grows slightly bigger than the Kolmogorov scale which is obvious for a stretched grid. It should be noted that the grid size is always in the order of the Kolmogorov scale after 4D location. Moreover, the grid is finer in the shear layer region than that of the jet centerline to ensure that all of the fluctuations are captured.
The time step size is also smaller than the Kolmogorov time scale, which is shown the second figure. This time step can capture frequency up to 50,000 Hz that is way beyond our frequency of interest as well as the main flow instability frequencies.

3.8. Grid independence and validation

3.8.1. Grid independence study

In order to find the correct grid resolution level, grid independent study has been performed. A systematic approach is followed to determine four grid refinement levels for comparison in used. Initially a very refined mesh is created, which in this case in 4.9 million cells. After that a scale factor of 1.2 is chosen to provide and even spacing between the coarsened grid levels. Coarsening the grid in each dimension by the assigned factor makes a coarse grid of factor of $(1.2)^3 = 1.73$, resulting a medium and coarse grid of 2.8 million and 1.6 million cells, respectively.

Table 3-2: Summary of different grids

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of grid points in axial direction, nz</th>
<th>Number of grid points in azimuthal direction, nθ</th>
<th>Number of grid points in radial direction, nr</th>
</tr>
</thead>
<tbody>
<tr>
<td>920K</td>
<td>210</td>
<td>80</td>
<td>64</td>
</tr>
<tr>
<td>1.6M</td>
<td>252</td>
<td>96</td>
<td>76</td>
</tr>
<tr>
<td>2.8M</td>
<td>302</td>
<td>116</td>
<td>90</td>
</tr>
</tbody>
</table>
To get a clearer idea about the trend a very coarse grid of 920 thousand cells is run. Once the time-averaged solutions are obtained on the three grid refinement levels, both the flow field and the species concentration field have been sampled at various points. A determination of whether or not convergence has been reached can be made based on these observations.

The following are the grid independent study carried out inside of the helium extension tube (before the jet exit) to show that the boundary layer and development of the axial velocity is independent of the grid resolution. Since momentum thickness is an important parameter in low-density jet study, special care has been given to the proper development of the boundary layer.

Figure 3-9 and figure 3-10 show the results for the development of the axial velocity in the jet centerline and at the radial direction just before the jet exit for $Re = 100, L/D=3$. They indicate that adequate convergence of the boundary layer and the centerline mean axial velocity have been reached at the 2.8 million cell level.

For these same grids grid independence study also been carried out at the jet area at different location. Figure 3-11 (a), figure 3-11 (b), figure 3-11 (c) and figure 3-11 (d) show the time averaged axial velocity in centerline at $z/D = 1.5, 3, 6$ and $z/D = 4, r/D = 1$ locations downstream of the jet exit. Similarly with the previous result results shows that the 2.8 million grid resolution is fine enough to get a correct result.
Figure 3-9: Boundary layer comparison before the jet exit, for extension tube L/D = 3

Figure 3-10: Comparison of development of axial velocity at centerline (extension tube L/D = 3)
Figure 3-11: Grid independent study result for velocity at different points in the jet

Figure 3-12 represents the stream wise species concentration along the jet centerline from the jet exit. The 4.9 million and 2.8 million grid sets produce almost same kind of result in the time averaged species concentration graph. The other two coarse grids 1.6 million and 920 thousand respectively produce a shorter helium jet core. Similarly to the previous velocity results the 2.8 million shows the most efficient and optimum grid to work with.
3.8.2. Flow field validation

The flow field validation is done with respect to the hot wire anemometry data created in the experimental study of Hallberg et al. (2006). Flow field validation is done for the Reynolds number of 1000 with an extension tube of length of 6D. The time averaged total pressure shows good agreement with the experimental result. Since no other experimental flow field data is available for this current set up, it is not possible to compare the stream-wise jet spread and species concentration data.

Figure 3-13: Validation of the centerline total pressure data
Along with the flow field data the FFT of the axial velocity at the near field of the jet shows excellent agreement with the experimental FFT. The dominant frequency and the sub-harmonics from CFD calculation and experimental data all matches very well.
CHAPTER 4: RESULTS AND DISCUSSION

4.1. Basic Jet

As observed by Hamins et al. (1992), a minimum jet exit velocity is required to initiate periodic flow oscillations. Previous studies have attributed unsteady flow oscillations to relative role of diffusion and buoyant convection. Motivated by these ideas, computations have been performed at several jet exit velocities/Reynolds numbers to demarcate the steady and unsteady flow regimes. No external perturbations are introduced and computations are performed for sufficiently long time such that the flow reach either steady state condition or unstable condition. Since the flow field is dependent upon Reynolds number, density ratio and momentum thickness/extension tube length, the Reynolds number is changed with other parameters fixed at S and L/D respectively. Figure 4-1 plots the non-dimensional flow frequency versus the jet Reynolds number. The flow was steady for Re=500 and the change from steady to unsteady state occurred approximately at Re=1000 or U=25 m/s.

Figure 4-1: Variation of non-dimensional frequency with jet Reynolds number
The computed velocity required to initiate the oscillations matches well with the measured Reynolds number of approximately 1000 reported by Hallberg et al. (2006). Such agreement further substantiate the effectiveness of the present model. The scaled oscillation frequency (Roshko number) increased almost linearly from \( \text{Re} = 1000 \) to \( \text{Re} = 4000 \) which is in complete congruence with the universal scaling law. The operating conditions for these test cases are summarized in table 4.1.

Table 4-1: Summary of test conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>S</th>
<th>Re</th>
<th>L/D</th>
<th>Global Absolute Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.03</td>
<td>500</td>
<td>3</td>
<td>Stable</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
<td>500</td>
<td>6</td>
<td>Stable</td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td>500</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td>1000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>6</td>
<td>0.03</td>
<td>1000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>7</td>
<td>0.03</td>
<td>1000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td>2000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>9</td>
<td>0.03</td>
<td>2000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
<td>2000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>11</td>
<td>0.03</td>
<td>3000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>12</td>
<td>0.03</td>
<td>3000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>13</td>
<td>0.03</td>
<td>3000</td>
<td>12</td>
<td>Unstable</td>
</tr>
</tbody>
</table>
(Table 4-1 Continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>S</th>
<th>Re</th>
<th>L/D</th>
<th>Global Absolute Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.03</td>
<td>4000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>15</td>
<td>0.03</td>
<td>4000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>16</td>
<td>0.03</td>
<td>4000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>17</td>
<td>0.14</td>
<td>1000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>18</td>
<td>0.14</td>
<td>1000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>19</td>
<td>0.14</td>
<td>1000</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>20</td>
<td>0.14</td>
<td>2000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>21</td>
<td>0.14</td>
<td>2000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>22</td>
<td>0.14</td>
<td>2000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>23</td>
<td>0.14</td>
<td>3000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>24</td>
<td>0.14</td>
<td>3000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>25</td>
<td>0.14</td>
<td>3000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>26</td>
<td>0.14</td>
<td>4000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>27</td>
<td>0.14</td>
<td>4000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>28</td>
<td>0.14</td>
<td>4000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>29</td>
<td>0.25</td>
<td>1000</td>
<td>3</td>
<td>Stable</td>
</tr>
<tr>
<td>30</td>
<td>0.25</td>
<td>1000</td>
<td>6</td>
<td>Stable</td>
</tr>
<tr>
<td>31</td>
<td>0.25</td>
<td>1000</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>32</td>
<td>0.25</td>
<td>2000</td>
<td>3</td>
<td>Unstable</td>
</tr>
</tbody>
</table>
(Table 4-1 Continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>S</th>
<th>Re</th>
<th>L/D</th>
<th>Global Absolute Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0.25</td>
<td>2000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>34</td>
<td>0.25</td>
<td>2000</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>35</td>
<td>0.25</td>
<td>3000</td>
<td>3</td>
<td>Stable</td>
</tr>
<tr>
<td>36</td>
<td>0.25</td>
<td>3000</td>
<td>6</td>
<td>Stable</td>
</tr>
<tr>
<td>37</td>
<td>0.25</td>
<td>3000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>38</td>
<td>0.25</td>
<td>4000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>39</td>
<td>0.25</td>
<td>4000</td>
<td>6</td>
<td>Unstable</td>
</tr>
<tr>
<td>40</td>
<td>0.25</td>
<td>4000</td>
<td>12</td>
<td>Unstable</td>
</tr>
<tr>
<td>41</td>
<td>0.5</td>
<td>1000</td>
<td>3</td>
<td>Stable</td>
</tr>
<tr>
<td>42</td>
<td>0.5</td>
<td>1000</td>
<td>6</td>
<td>Stable</td>
</tr>
<tr>
<td>43</td>
<td>0.5</td>
<td>1000</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>44</td>
<td>0.5</td>
<td>2000</td>
<td>3</td>
<td>Stable</td>
</tr>
<tr>
<td>45</td>
<td>0.5</td>
<td>2000</td>
<td>6</td>
<td>Stable</td>
</tr>
<tr>
<td>46</td>
<td>0.5</td>
<td>2000</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>47</td>
<td>0.5</td>
<td>3000</td>
<td>3</td>
<td>Stable</td>
</tr>
<tr>
<td>48</td>
<td>0.5</td>
<td>3000</td>
<td>6</td>
<td>Stable</td>
</tr>
<tr>
<td>49</td>
<td>0.5</td>
<td>3000</td>
<td>12</td>
<td>Stable</td>
</tr>
<tr>
<td>50</td>
<td>0.5</td>
<td>4000</td>
<td>3</td>
<td>Unstable</td>
</tr>
<tr>
<td>51</td>
<td>0.5</td>
<td>4000</td>
<td>6</td>
<td>Stable</td>
</tr>
</tbody>
</table>
(Table 4-1 Continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>S</th>
<th>Re</th>
<th>L/D</th>
<th>Global Absolute Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>0.5</td>
<td>4000</td>
<td>12</td>
<td>Stable</td>
</tr>
</tbody>
</table>

4.1.1 Evolution of mean velocity

Figure 4-2 and 4-3 represents a test of the similarity of centerline stream-wise velocity and cross-stream variations of stream-wise velocity. In Figure 4-2 the total pressure in stream-wise direction is shown for different density ratio, non-dimensionalized by the jet exit total pressure. With the increase of the density ratio, the jet potential core length also increase, though with increasing Reynolds number the effect on potential core length is not so explicit.

The figure 4-3 shows that the slight backflow observed on the edge of the jet vanishes in the far field. It is seen that the velocity profile features a classical transition from an initial top hat shape to a fully developed Gaussian like shape.

![Figure 4-2: Time averaged stream-wise variations of axial velocity at centerline](image)
Figure 4-3: Time averaged cross-stream variations of axial velocity at different stream-wise locations, normalized by (a) exit centerline velocity and (b) local centerline velocity

Figure 4-4 presents the stream-wise evolution of the jet half width at several axial location. The jet half width (or radius) is an important parameter to quantify radial growth of the jet in the stream-wise direction. The jet half width is defined as the radial location where the axial velocity or helium concentration is half of that at the jet center (r/D=0.0). In case of the oscillating flow, the time averaged profiles were used to determine the jet half width.

Figure 4-4: Time averaged stream-wise variation of the jet half-width
The total length of the calculation is 0.31 second or 12800 time steps. The variation are compared to previous DNS results from the literature (Sarkar et al. 2002). The comparison is good; some discrepancies are observed, however and in particular the velocity decay is faster. The graph reveals several unique features of low-density gas jets. First the jet half width very slightly decreases after the jet exit until 3D downstream and then it starts increasing almost linearly.

4.1.2. Velocity fluctuations

![Graph showing downstream evolution of the fluctuation of the axial velocity](image)

Figure 4-5: Downstream evolution of the fluctuation of the axial velocity

The downstream evolution of the longitudinal/axial centerline fluctuation intensity is presented in Figure 4-5. The fluctuating quantities develop slowly towards a self-preserving behavior. Compared to the study by Capone et al. (2012), which is for s=1, Re=3200, the stream-wise r.m.s value increases later for low density jet, but achieves a very high level of fluctuations as in the reference study. The cross-stream variations of stream-wise turbulence intensity are shown in Figure 4-6. These sections at different z/D location of 1, 2, 3, 4, 5 and 7 are chosen because it belongs to the region where self-similarity in the fluctuating quantities applies and is still far from the outflow boundary.
4.1.3. Evolution of the species

The evolution of species in stream wise direction and cross-stream direction is shown in the Figure 4-7 and Figure 4-8 for the basic jet configuration. The stream-wise helium mass concentration start to decreasing at around 4D downstream of the jet exit and abruptly reduces unlike the axial velocity. This indicates the entrainment of heavier environmental SF6 into the helium jet core. Around 10 D downstream along the jet centerline the helium mass concentration reduces to around 0.1. As mass transportation can be analogous to heat transfer it can be inferred that the temperature profile would follow same trend as like the mass concentration profile along the centerline. Such bad heat loss actually reduces the efficiency of the thermal plasma jet in spray process of applying thermal barrier coating on gas turbine blade surface.
The cross stream helium mass concentration for different centerline location shows the evolution of helium concentration profile. It should be noted that unlike the helium mole concentration, the helium mass concentration does not show the jet density profile, as mass concentration is not proportional with the density. The mass concentration profile gradually reduces to a Gaussian profile from a top hat shape. The concentration field near the jet exit
shows that the helium has reached beyond \( r/D = 0.6 \) and pure helium is present up to \( r/d < 0.5 \). Until 3D downstream from the jet exit the center of the jet has purely helium and then it starts reducing. The entrainment of the ambient gas and mixing into the helium core gets more stronger after \( z/D = 3 \) when the toroidal vortex breaks into parts and interacts with each other.

### 4.1.4. Species fluctuation

For basic jet configuration the downstream evolution of the r.m.s of helium mass concentration is shown in Figure 4-9. The r.m.s mass concentration of helium amplitude is small from \( z/D = 0 \) to \( z/D = 2.5 \), a region coinciding with the potential core. After that the amplitude reaches 0.3 at a location \( z/D = 4 \), in a linear fashion, then starts reducing again. It should be noted that this curve is not normalized with the local helium mass concentration.

Figure 4-10 depicts the time averaged r.m.s helium mass concentration along the cross-stream line for a specific downstream direction. The graph reveals that the mass concentrations fluctuates the highest at the shear layer region and it reaches upto 0.35 in \( z/D = 3 \) location. This is because of the vortex structures convection from the jet exit to the downstream direction. In the jet centerline the concentration is rather steady as the vortex structures were initially present at the jet shear layer. Later at more downstream location, the centerline fluctuation starts to increase as the vortex structures breaks up and interacts with each other. This inward shifting of the mass concentration fluctuation from the wake region \( 0.4 < r/D < 0.6 \) contaminate the most of the flow at \( z/D = 4 \).
Figure 4-9: Streamwise evolution of the fluctuation of the helium mass concentration

Figure 4-10: Time averaged cross-stream fluctuation of helium mass concentration at different stream-wise locations
4.1.5. Flow structures

Advanced flow visualization technique has been employed to visualize different flow properties by using commercially available software Tecplot 360 and Fluent.

Figure 4-11: A crude comparison of the jet structure from CFD calculations with the experimental schlieren image from Hallberg et al. (2006)

In Figure 4-12 the instantaneous density field is shown for a jet with density ratio of 0.03, Reynolds number of 1000 and with extension tube length of 6D. After the jet exit about 1.5D downstream a vortex street of Kelvin Helmholtz instability is clearly visible indicating the entrainment of ambient gas into the jet. The vortex structures are generated around the round jet makes a toroidal vortex structure. Depending on the flow condition the nature of the jet spread and the time averaged half width of the jet changes. For certain flow conditions, a clear side jet nearly 90 degree with the main jet can be created. The general trend is with the increase of the density ratio the jet potential core increases. The nature of the jet spread does not very strongly depend on the Reynolds number, though the potential core length is proportionally related with the Reynolds number after the threshold of the onset Reynolds number.
The vorticity contour for the same flow condition further corroborates the generation of vortex. Later we will show the 2D velocity vectors along with 3D vortex structure iso-surface to explain the vortex generation mechanism for low density jets. It should be noticed that there are some vorticity present along the wall of the helium tube. That is generated from the boundary layer inside the helium tube.

The corresponding instantaneous velocity field shows from Figure 4-14 shows how the potential core diminishes after a very short distance from the jet exit. The instantaneous velocity in some downstream position can reach much more higher than the jet exit centerline velocity.

Figure 4-12: Instantaneous density contour in the mid plane, S=0.03, Re=1000 & L/D=6

Figure 4-13: Instantaneous vorticity contour in the mid plane of a jet with S=0.03, Re=1000 & L/D=6
4.1.6. Generation of the instability

Figure 4-15 (a) to (j) shows a sequence of velocity vectors plots superimposed with helium mole percentage contours during the oscillation cycle for the low-density jet in presence of gravitational field. From these snapshots, a careful observation can show that the buoyancy accelerates the jet core, which contracts to conserve the mass. This results in the entrainment of the ambient fluid to produce a toroidal vortex. Later the jet shows characteristic of larger recirculation region accompanied with greater contraction of the jet core, evident from the inward indentations in the concentration field near z/D=1. Subsequently, the vortex grows in size, convects downstream and contaminates a greater portion of the jet core. The vortex gains strength by transfer of the momentum from the jet core. This feature is evident in Figure (g) and (f) wherein the velocity vectors downstream of the vortex core are larger than those upstream before interacting with the jet core.
Figure 4-14: Instantaneous velocity vectors superimposed on species mole fraction contour each after 0.0002 sec
As the vortex propagates downstream, the jet expands near the tube exit to initiate another vortex at later stages to repeat oscillation cycle. These results illustrate self-excited periodicity in the jet at a frequency of about 600 Hz. The iso-surface of the vortex structures can explain further about the dominant frequency. It also shows how the vortex structures interact with each other to create turbulence.
4.1.7. Iso-surface of vortex structures

If the flow is nowhere absolutely unstable, i.e. convectively unstable or stable everywhere, all local modes and consequently all global modes are time-damped, provided the system contains no additional global pressure feedback. Therefore such a flow behaves as an ‘amplifier’ that spatially amplifies selected external disturbances and ultimately returns to the undisturbed base state if all external excitation is turned off. On the other hand, if a system, starting from the undisturbed state, only needs a small push to develop time-growing oscillations, it is self-excited and termed an ‘oscillator’. As suggested by Chomaz et al. (1988), in the absence of global pressure feedback, such behavior is only possible if the flow contains a region of absolute instability with a sufficient streamwise extent. Following the initial exponential growth of the disturbance, the system in many cases settles into a limit cycle. This final nonlinear state generally represents a global response which is intrinsic to the system and frequently independent of the nature of the initial impulse or even the nature of any additional, continuous, low-level forcing. Loosely stated, self-excitation of a system generally results in the largest possible amplitude of its ‘preferred’ mode, often associated with optimal momentum, heat and mass transport, in our case across the jet shear layer. The term ‘preferred’ mode is thereby not accidental but reflects the connection to the ‘preferred’ or ‘jet column’ mode in the homogeneous jet, established by Monkewitz, Huerre & Chomaz (1987).

In Figure 4-15 a full mechanism of the vortex generation is explained. The whole mechanism can be explained by the help of the finding from experimental study of A. J. Yule (1978). The Figure 4-16 explains the physics behind the vortex mechanism. After the jet exit, the natural instability of the initial laminar shear layer produces a street of vortex-ring-like vorticity concentrations. As these vortex rings move downstream they gradually coalesce with
neighboring rings, so that the scale and separation of the vortex rings increase with distance from the nozzle.

In Figure 4-15 the instantaneous iso-surface of Q-criteria is shown for a jet with density ratio of 0.03, Reynolds number of 1000 and with extension tube length of 6D. After the jet exit towards downstream a vortex street of Kelvin Helmholtz instability is clearly visible indicating the entrainment of ambient gas into the jet. The vortex structures are generated around the axisymmetric jet makes a toroidal vortex structure. Depending on the flow condition, the nature of the jet spread and the time averaged half width of the jet changes which intern changes the convective velocity of these vortices. For certain flow conditions a clear side jet nearly 90 degree with the main jet can be created. The general trend is with the increase of the density ratio the jet potential core increases. The nature of the jet spread does not very strongly depend on the Reynolds number, though the potential core length is proportionally related with the Reynolds number after the threshold of the onset Reynolds number.

Figure 4-15: Sequence of vortex generation and conversion into hairpin structure (each frame after 0.0001 seconds)
(Figure 4-15 continued)
(Figure 4-15 continued)
The figures presented above are each 0.0001 seconds apart and arranged in a column wise manner. The periodicity appears after around each eight figures indicating a frequency of around 12000 Hz. Afterwards this is changed into 600 Hz by following some mechanism explained later. This frequency data is also supported by the FFT which will be presented in later sections.
However, although point velocity measurement in this transitional flow exhibit periodic correlations and peaked spectra, there is considerable random variation in the movements and strength of the coalescing vortices. These can be further seen in the spectra section later in this chapter. Furthermore, the vortex rings lose their phase agreement across the jet as they move downstream. The gradual increase of $w$ fluctuations with distance from the nozzle is caused by the gradual, almost linear, growth of orderly wave deformations of the cores of the vortex rings, and this growth also results in a decrease in the level of circumferential cross-correlations.

![Physical structure of transitional jet](image)

Figure 4-16: Physical structure of transitional jet

[Large scale structures in mixing layer of round jets. A.J. Yule, JFM (1978), Vol 89]
4.1.8. Generation of the instability

Three types of jet instability govern the formation as well as interaction of large scale coherent structures in a jet. The first is called the shear layer instability and is associated with the initial shear layer that forms pass the nozzle exit. It is also called a Kelvin-Helmholtz instability, and it is responsible for the formation of vortex ring in the flow. With the second type, the jet column instability arises from the instability of the potential core. The nature of the large structure is significantly depends on a third instability, known as the azimuthal instability. Depending on the modes, this instability can either be called helical or axisymmetric instability.

However, although point velocity measurement in this transitional flow exhibit periodic correlations and peaked spectra, there is considerable random variation in the movements and strength of the coalescing vortices. These can be further seen in the spectra section later in this chapter. Furthermore, the vortex rings lose their phase agreement across the jet as they move downstream. The gradual increase of $w$ fluctuations with distance from the nozzle is caused by the gradual, almost linear, growth of orderly wave deformations of the cores of the vortex rings, and this growth also results in a decrease in the level of circumferential cross-correlations.

4.1.9. Varicose mode generation

Iso-surface of Q-criteria of the instantaneous flow field shown in Figure 4-17 reveals that the spatial evolution and the emergence of the individual instability modes in the stream wise direction. In the area near to nozzle exit, jets can be considered to have an axisymmetric shear layer. The primary growth of the K-H instability in that region is similar to instability in the plane shear layer reported by Bernal and Roshko (1986).
Figure 4-17: Iso-surface of vertical structures, dominance of axisymmetric mode is apparent before transition to turbulence

As can be seen from Figure 4-18 the varicose mode generation and growth can be explained from the baroclinic torque term in the vorticity transport equation. When the jet fluid convected downstream it is subjected to continuous entrainment of the denser fluid from the ambient to the jet core.

Figure 4-18: Vortical structure (Varicose mode) growth mechanism
As a result the jet density in the shear layer region gradually increases. From the schematic in the right of Figure 4-18 explains that. It should be noticed that the density gradually increases from point B1 to A2. So when a vortex convects a little further (A), by and random perturbation in any point in azimuthal direction, it faces higher baroclinic torque due to higher density gradient between the upper and lower part of the vortex and the left behind part (B) has lower torque. This effect stretch the different part of vortex differently in downstream and the varicose mode generated.

The rollup of the shear layer into primary vortical structures appears as a strain of round vortex rings. The plane of the vorticity is orthogonal to the stream-wise direction. The parallelism of the vortex rings in the near field reveals he existence of varicose instability mode. Secondary instability develop in the shear layer after the vortex rings are convected downstream by the main flow. Consequent to the secondary instability, the primary structures, namely the vortex rings, breakdown and become corrugated resulting the formation of ribs that look similar to hairpin structures.

Figure 4-19: Brown-Roshko Structures
Secondary structures are generated because of the instability and the interaction among the primary structures. Far downstream, secondary structures are generated and interaction among various structures (primary and secondary; primary and primary; secondary and secondary) causes the flow to undergo a transition before becoming fully turbulent. Location of the inception of the transition depends on the Reynolds number, density ratio and momentum thickness.

As in the plane mixing layer, the primary flow in a circular jet becomes azimuthally unstable in space as well as in time. The secondary instability appears as stream-wise vortex structures as shown in Figure 4-19 & Figure 4-20 and interactions with the primary structure, the vortex ring. The secondary structures are similar to the Brown-Roshko (1974) structures found in mixing layer. After this interaction in the development of the jet, potential core region ends and the centerline velocity starts to decay. Consequently, the velocity difference between the ambient fluid and the high-speed core of the jet decreases and attenuates the shear that supports the vortex.
rings of the jet. The spatial development of the jet shear layer is then seen to be quite rapid with downstream distance. The near field shows regular coherent structures while the far field reveals the interaction of these coherent structures as they travel downstream and form complex patterns.

From one up to at least three regions of vortex coalescence can be observed in the transition region, depending on the jet Reynolds number and nozzle boundary layer thickness. Vortex ring coalescence cannot be observed downstream of the end of the transition region. The coalescing of the vortices is a mechanism by which the jet structure tends to ‘forget’ the conditions at the jet nozzle. This is evident in Figure 4-21 which shows that the wide range range of Strouhal numbers near the nozzle tends to a Reynolds number independent distribution with increasing z/D value.

The last coalescence of vortices, prior to turbulent flow, involves vortex rings which have core deformations larger than a critical size. These rings entangle, thus producing enhanced vorticity stretching and small scales of motion. The remains of these entangled vortices are often visible in the turbulent region up to 5D downstream of transition and they are thus larger eddies in the turbulent region.

Figure 4-21: Strouhal number calculated from the potential core axial velocity spectra peaks in basic low density jet, Re=1000 & 4000, S=0.03, L/D=6
The iso-surface vortex structures of other Reynolds number flows are shown in the appendix. The higher Reynolds number jet vortex stretches less than the lower Reynolds number vortex hence the spread is also lower.

4.1.10. Iso-surface of density

Figure 4-22 shows the iso-surface of the density gradient for different time steps in both 2D and corresponding 3D manner. The 2D contour is taken at the same time instance of the 3D iso surface. A careful observation between these two by imagining a planer section along the Y axis shown in the right top corner of the 3D iso-surfaces reveals that the density gradient starts to grow immediately after the jet exit. After that as the jet inner part starts to move forward at a faster pace the protruded bulges of density gradient/species gradient remained still in a specific z/D location while growing in the radial direction and eventually diminishes as the helium fully mixes with the ambient. It should be noted that these density gradient images can be considered as ‘computational schlieren’ as they are analogous to the experimental schlieren.

In the next Figure of 4-23 the relationship of the primary and secondary instabilities is shown, as a low density jet transitions to turbulence. The jet fluid enters into the domain from the bottom as a laminar flow, then it quickly undergos an axisymmetric primary instability which is self-sustained for sufficient lower values of S. Further downstream, side jets form, which are visible as coherent axially alligned structures immediatley after the primary instability at the bottom of the figure.
Figure 4-22: Time series density gradient 2D contour along with correspondent 3D iso-surface
Figure 4-23: Primary and secondary instability from the density iso-surface

Figure 4-24: Clearly visible separate side jets nearly 90° with the main jet visible at S=0.325, Re=1500 (figures are taken at 0.004 seconds apart)

The small side jets in the basic jet in Figure 4-24 makes a star-shaped cross section as shown. Finally, after about 1D downstream in the jet, it turned to a fully turbulent flow. In previous vortex structures visualized by plotting the iso-surface of Q criteria indicates that the primary instability is caused by the train of Kelvin Helmholtz rings. As these vortex rings convect downstream, they develop this secondary instability which manifests itself as azimuthal oscillations on each ring. More figures of the density iso-surface are given in the appendix for
different flow conditions. From there it can be easily seen that the jet entrainment happens faster in the inner core than in the outer side, as explained before by the velocity vortex superimposed on density contours.

### 4.1.11. Vortex merging & hairpin vortex

The Figure 4-25 shows the Q iso-surface colored with helicity on a density gradient 2D background. The extreme color on the left and right side of a single vortex tube confirms the formation of hairpin structures. It also shows the merging of vortex tubes at around 1D from the jet exit. The vortex tubes always grow inside the maximum density gradient/species gradient line and after 3D downstream from the jet exit intertwined with each other and forms a complex vortex structure.

![Time series of vortex structure merging superimposed with helicity contour in a density gradient background](image)

Figure 4-25: Time series of vortex structure merging superimposed with helicity contour in a density gradient background
Browand and Laufer (1975) conducted extensive flow visualization experiments of jet in water for the Reynolds number range of 5000-15000. The study showed that the initial development of the jet is controlled by the coalescence or pairing of discrete vortex structures. These structures occur in both axisymmetric and helical shapes. The coalescence of an axisymmetric vortex produces a structure with a passage frequency which is one-half of the original value. Another experiment by Stromberg et al. (1980) also observed the one-half and two-third sub-harmonics of the fundamental frequency. Here in this case $\frac{1}{2}$ frequency ratio confirms the presence of axisymmetric mode. The one-half criteria for pairing of varicose mode reported by Browand and Laufer and the pairing process shown in Figure 4-21 of the present simulation appear to be consistent as the visual observation of the vertical structures in Figure 4-25 suggests the formation of varicose structure.

At moderate Reynolds number the generation of coherent vortical structures or eddies in the shear layer is inherent property of Kelvin-Helmholtz instability and they play an important role in the transition of an initially laminar jet to turbulent. The free shear layer originates from the separation of the boundary layer from the surface of a trailing edge and is followed by exponential growth. Due to non-linear growth downstream, vortices roll up and this phenomenon occurs at regular intervals, referred to as the vortex shedding frequency. Becker et al. determined that the distance from a nozzle exit to the location of vortex roll-up length, is a function of the jet diameter and Reynolds number.

In an attempt to quantify the vortex merging for low-density gas jet, a plot has been made to have an idea about the change in vortex merging length with the change of Reynolds number. The trend can be seen from the following figure, where with the increase of Reynolds number the vortex pairing length decreases gradually while all other parameters are kept constant.
4.1.12. Spectral analysis & universal scaling

Energy spectrum in wave number domain has been shown here in Figure 4-27 to better understand the energy dissipation. At higher wave number the energy cascaded to smaller structures by following -5/3 rule. Further downstream kinetic energy cascaded to smaller structure by -5/3 rule until the Kolmogorov length scale indicating turbulence. After that scale
direct entropy cascade at higher rate. Towards downstream as the flow grows into more turbulent the energy is more spread into different size of structures, while at upstream energy is localized into specific frequencies.

Figure 4-28 shows the power spectral density of the jet at different downstream positions from the jet exit. A dominant peak with harmonic content is detected in the spectrum. It is also evident that the eddies are being convected and getting stronger in downstream position at the same frequency. The dominant frequency of the jet is about 580 Hz for $S=0.03$ with $L/D=6$ at $Re=1000$.

![Figure 4-28: Power spectral density for different downstream position from jet exit](image)

From Figure 4-29 it can be observed that energy is transferred into the 580 Hz frequency from the other frequencies as the flow advances downstream. For sufficiently low-density ratio, numerical simulations have shown that the oscillations seem to saturate to global self-excited mode. The plot is scaled with each graphs highest value to show a difference in amplitude in a same scale. It shows how the instability gets stronger globally by taking energy from the other harmonics.
Figure 4-29: Normalized FFT of velocity magnitude at jet exit, 1D downstream & 1.5D downstream

Figure 4-30: Comparison of FFT at different downstream position at the same Reynolds number along the jet centerline

Here in Figure 4-30 shows the FFT for same density ratio of 0.14 and Re=1000 with an extension tube length of L/D=6 in downstream positions as z/D=1, 1.5, 2, 4, 6, 8. The dominant
frequency is around 810 Hz for all the downstream position on the centerline. As the flow grows downstream the power gradually increases in other sub-harmonics and decreases in the dominant frequency. The non-dimensionalized primary frequency or the Roshko number is 184 and the second sub-harmonic frequency is 369. The Strouhal number based on the jet initial velocity is 0.18 for the main frequency and 0.36 for the sub-harmonic.

Strong oscillating mode together with the strong vortex pairing events phenomena points towards the existence of the global instability in this simulation. Evidence for is shown in figure plotting the power spectrum of the stream wise velocity and various stream wise locations. The power spectrum clearly shows the fundamental frequency and its sub-harmonics at various stream wise positions shifted for better visibility.

Figure 4-31: Comparison of FFT for different Reynolds number at same position
For density ratio, $S=0.03$ and extension tube length $L/D=6$ the FFT is shown for different Reynolds numbers at a fixed point at $z/D=1.5$ and $r/D=0$. As the Reynolds number increases the dominating frequency shifts to higher frequency as predicted by Hallberg et al. (2006).

Here the FFT is shown for density ratio, $S=0.03$, 0.14 & 0.25 at $Re=2000$ and extension tube length $L/D=6$ at $z/D = 1.5$. The dominating frequencies are around 1850 Hz, 2200 Hz and 2500 Hz respectively. Nondimensionalized Roshko numbers are 375, 450 and 525 respectively.

Figure 4-32: Comparison of FFT for different density ratio ($S$) at same Reynolds number

In the Figure 4-33 the scaled frequency is plotted against the density ratio. Frequency seems to increase with the increase of density ratio. However, as it is suggested by Hallberg et al. the relation is not linear.
The jet follows the universal scaling proposed by Strykowski et al. This indicates clearly that the jet is globally unstable. The steep density difference in jet fluid and ambient fluid is the driving source for creating global modes of oscillations. This property of the low density jet is then used to design a control strategy to suppress the instability. Deviation from the scaling can be explained by the choice of spectral peak, which with harmonics present is not clear and is simply made by choosing the highest peak.
Results show that the frequency of the unstable phenomena depends on $S$, $D/\theta_0$ and $Re$. As nature is bound by this operating space, Hallberg et al. suggests an universal law for low. He assumes that the global instability is responsible for a periodic traveling wave, the frequency was non-dimensionalized via inertial or viscous time scales. From previous studies it was clear that Strouhal number is unable to fully capture the physics in the present experiment. So the frequency was normalized by the viscous time scale $D^2/\nu$; in so doing the Reynolds number was retained in the frequency dependence. It was conjectured that the convective velocity of the traveling wave would be a function of density ratio. From previous studies it was also evident that the onset of global instability strongly depends on the non-dimensional momentum thickness $D/\theta_0$. The scaling validates the importance of the Reynolds number in addition to $D/\theta_0$ and $S$ in capturing the physics of global oscillations in low-density jets having initially steep density profiles. The current CFD study completely satisfies the scaling criteria with the explanation of the vortex structures and how they interacts, shedding light on the physical mechanism of the global instability of the low-density jet.

4.2. Controlled jet

4.2.1. Vortex structures

A globally absolute unstable low-density gas jet is characterized by the formation of periodic vortex structures and pairing of that and finally interacts with each other to form turbulence. Three types of jet instability govern the formation as well as interaction of large-scale coherent structures in a jet. The first is called the shear layer instability and is associated with the initial shear layer that forms pass the nozzle exit. It is also called Kelvin-Helmholtz instability, and it is responsible for the formation of vortex ring in the flow. With the second type, the jet column instability arises from the instability of the potential core. The nature of the
large structure is significantly depends on a third instability, known as the azimuthal instability. Depending on the modes, this instability can either be called helical or axisymmetric instability.

In the following figure the shape of the jet in the braid region is not round indicating that the azimuthal instabilities exist in the braid region, whereas the ring is smooth. This shows that the instabilities grow in the braid. Then they interact with the primary K-H vortex structures, stretch them in the axial direction, and induce the waviness into them. Careful observation with the axis direction reveals that the Benard Roshko (1962) structure creates a side jet in the positive Y direction (facing to the observer), explained by Brancher et al. (1993). Secondary structures are generated because of the instability and the interaction among the primary structures. Far downstream, secondary structures are generated and interaction among various structures (primary and secondary; primary and primary; secondary and secondary) causes the flow to undergo a transition before becoming fully turbulent. Location of the inception of the transition depends on the Reynolds number, density ratio and momentum thickness.

![Figure 4-35: Primary and secondary instability with the fluid movement in radial direction](image)

Figure 4-35: Primary and secondary instability with the fluid movement in radial direction
This secondary axisymmetric instability can be attributed to the non-uniform mixing in azimuthal direction, which leads to non-uniform convection of the vortices in the streamwise direction. A region in the interface between the main jet fluid and the ambient fluid that has more intense mixing has experienced a lower convective velocity and vice versa. This mixing further facilitates negative baroclinic torque. As a result of this velocity gradient in azimuthal direction and baroclinic torque, stretching and pulling has occurred to the vortex structures. From the vorticity transport equation, it is evident that the convective and buoyant terms are the most important parameters for pulling and stretching of the vortices. Along with that the diffusive term facilitates better growing of vortices inside the jet rather than heavier outer side.

\[
\frac{d\vec{\omega}}{dt} = \frac{\partial \vec{\omega}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{\omega} = (\vec{\omega} \cdot \vec{\nabla})\vec{v} + \frac{1}{\rho^2} \vec{\nabla} \rho \times \vec{\nabla} \rho + \vec{\nabla} \times \left( \frac{\vec{\nabla} \cdot \tau}{\rho} \right)
\]

4.2.2. Suppression of instability

Previous researches have indicated that globally unstable flows can be altered if the global mode is suppressed. By linear stability theory, Raynal et. al (1996) theoretically showed that changing the density profile or offsetting the sudden density change (hat profile) can improve the flow stability and suppress the global instability modes of the jet. So, to achieve that altered density profile, a surrounding environment was created, composed of the mixture of the same gases present in the central jet and ambient. By changing the boundary condition for gas fraction, a ramp like density profile was created just at the exit of the main jet. Results shown in Figure 4-38 indicate a change in the amplitude and strength of dominant frequency after altering the density profile of the surroundings.
The power spectral density of the jet reduced considerably after employing a changed density profile. The density profile was changed by issuing the lighter gas from the periphery of the main jet exit where the density of the gas increasing with increasing radial location. Density resulted from this gradual increase of ambient gas creates a ramp shape profile. The maximum density in this profile is same with the surrounding fluid and the minimum density is same with the main jet.
Figure 4-38: FFT for basic jet with density ratio of 0.03 and Reynolds number of 1000 without any control & FFT after the suppression of global modes by altering the density profile

In Figure 4-38 the FFT for a basic jet without any control and a jet with controlled surrounding density profile is presented. The result shows considerable amount of power reduction in the dominant frequency of the jet. The morphing of the jet from globally unstable mode to a convective unstable mode suggested that the ramp like density profile around the main jet could effectively suppress the instability of a very low-density jet.

Figure 4-39, 4-40 and 4-41 corroborates the results observed in spectral analysis for basic jet and altered jet. The jet cross stream axial velocity profile, normalized by the mean jet velocity, contracted considerable amount after employing the control technique. The result clearly shows a top hat profile of velocity where the momentum transfer is less for controlled jet than basic jet as the spread is higher for the later one. However, as the ramp profile is not tailored for the best outcome from theoretical point of view, so using an optimum density profile could extend the length of the core farther.
Figure 4-39: Time averaged jet velocity profile for basic and controlled jet

Figure 4-40: Cross stream mean velocity at different downstream position for basic and controlled jet
Figure 4-41: Cross sectional time averaged velocity profile for basic and controlled jet

In Figure 4-42, 4-43 and 4-44 the normalized r.m.s of the axial velocity is presented for different axial position. The r.m.s of axial velocity is higher in the shear layer where most of the wakes are generated. In the jet center area and out of the shear layer the velocity fluctuation is considerably lower as also stated by Foysi et al. (2010). After employing the control technique, the maximum point shifted inward direction. In addition of that for the controlled jet the maximum r.m.s value on the jet edge is less than the basic

Figure 4-42: Time averaged axial velocity fluctuation for basic and controlled jet
Figure 4-43: Cross-stream mean axial velocity fluctuation for basic and controlled jet

Figure 4-44: Cross sectional time averaged velocity fluctuation profile for basic and controlled jet
The potential core length and pressure results for basic jet (S=0.03, Re=1000, R/D=6) and controlled jet by ramp density profile. With the ramp-density profile, the potential core is lengthened, reducing mixing between the core jet and the surrounding.

4.3. Conclusion

A numerical investigation has been carried out to understand the characteristics and controlling parameters of very low-density axisymmetric jets using a variety of Reynolds number, momentum thickness/extension tube length and density ratio.

The controlling parameters have been grouped into two non-dimensional entities, proposed by Hallberg et al. (2006), to define the domain of the flow characteristics of the low-density jets and the results have been checked with the experimental results from other researchers. This is a representation commonly followed in literature to identify global instability in similar type of flow field. The results from the present study show that the low-density jets follow the similar trend of global instability reported by earlier experimental studies. A 3D
envelope has been constructed to define the onset of global absolute instability in a 3D space consists of Reynolds number, density ratio and extension tube length.

The basic jet in investigated for jet spread, dominant frequency and the transition of oscillation energy. Results shown that eddies get energy from the higher sub-harmonic frequency to lower dominant frequency, which is proved both by the visual vortex structures and by one-half law of frequency shifting while advancing from the jet exit to further downstream position. After a distance downstream of the exit, the whole field resonates with the dominant frequency. The power spectra confirms turbulent flow after 4 diameter downstream at very-low density jet case which matches with other results.

The flow physics and vortex generation mechanism has been studied from various aspects to explain the underlying reason. Different mode of instability namely Kelvin-Helmholtz vortex street, varicose structures along with stream-wise Bernal-Roshko structures has been confirmed as the driving mechanism behind the low-density jet instability. Vortex generation mechanism has been analyzed. A model of the varicose vortex structure growth has been proposed. And the vortex merging distance from the jet exit has been quantified for different flow parameters.

In this study, ramp-type density profile was chosen as controlling method to suppress the global mode. Power spectra of the basic and controlled velocity magnitudes revealed that a partial suppression of the global mode of instability was achieved. The potential core results further corroborates the positive results after employing the altered surrounding density profile. It can be said that the changed density profile play an important role in the suppression process.
REFERENCES


92


APPENDIX A

Vortex structure for Re=2000, S=0.03

Figure A: Time series vortex structure for Re=2000, S=0.03
APPENDIX B

Vortex structure for Re=3000, S=0.03

Figure B: Time series vortex structure for Re=3000, S=0.03
(Figure B continued)
(Figure B continued)
APPENDIX C

Vortex structure for Re=4000, S=0.03

Figure C: Time series vortex structure for Re=4000, S=0.03
(Figure C continued)
(Figure C continued)
(Figure C continued)
APPENDIX D

Density iso-surface, Re=3000, S=0.25

Figure D: Time series iso-surface of density for Re=3000, S=0.25
(Figure D continued)
(Figure D continued)
(Figure D continued)
VITA

Sukanta Bhattacharjee was born in the district of Mymensingh, Bangladesh in November, 1984. He completed his schooling from Zilla School, Mymensingh and intermediate education from Krishi Bishwabidyalay Intermediate College, Mymensingh. His interest in Engineering and Fluid dynamics lead him to join the Mechanical Engineering Department of Bangladesh University of Engineering and Technology, Dhaka, Bangladesh, where he earned a Bachelor of Science Degree in Mechanical Engineering. He decided to pursue higher education and enrolled in Louisiana State University, Baton Rouge, Louisiana, USA in Spring 2010. He will be graduating in August 2013 with Master's of Science in Mechanical Engineering.