3-dimensional flow measurements in square coaxial jets

Thomas Lagarde
Louisiana State University and Agricultural and Mechanical College, tlagar1@lsu.edu
3-DIMENSIONAL FLOW MEASUREMENTS IN SQUARE COAXIAL JETS

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Thomas Lagarde
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Dedication

To my family and my close friends

Also to high quality and shiny shoes
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Abstract

Results from un-forced experiments conducted in the three-dimensional flow emanating from coaxial contoured nozzles with square cross section and parallel sides are presented and discussed. Two experiments were conducted with the inner to outer jet nominal velocity ratio equal to \( \lambda = 0.5 \) and \( \lambda = 1.5 \). The Reynolds number of the outer jet based on the hydraulic diameter of the outer nozzle exit was in both cases 16,000.

A multi-position hot-wire anemometry method was used for measurements in the three-dimensional turbulent flow using a dual film sensor (X-probe). A methodology was developed and implemented to derive the mean-velocity components and Reynolds stresses corrected for velocity gradient effects.

Axis switching, a phenomenon readily obtained with non-axisymmetric section nozzles, was observed as it evolves in the near-field using the three-dimensional flow maps obtained through this method. The overall entrainment was calculated from the measurements and compared to the case of equivalent circular coaxial nozzles under the same experimental conditions, where axis switching does not occur. The entrainment rate of the square geometry was found to clearly exceed the one of the circular geometry in the nozzle exit region. This brings evidence of overall mixing enhancement accomplished with the square nozzle.

In addition to velocity spatial distribution, this study presents a spectral analysis that reveals the distribution of discrete modal amplitudes in three dimensions.
Chapter 1  Introduction

The object of the present work is to study the characteristics of the three-dimensional flow emanating from coaxial contoured nozzles with a square cross section. Results will be compared to results previously obtained for coaxial contoured nozzles with circular cross section. One objective is to conclude on the quantitative influence of the nozzle geometry to promote mixing enhancement.

Different studies have previously investigated passive forcing as a way to modify the flow characteristics by changing the geometry of the nozzle. This chapter addresses a brief literature survey of up-to-date studies conducted on nozzles of different cross section geometries. Results on axis switching, a phenomenon encountered in jets induced by non axisymmetric nozzles are then presented.

This study is the continuation of the studies of Bonnafous (2001) and Piffaut (2003) on square coaxial nozzles. The chapter ends with a brief review of the different techniques used to measure mean quantities of a three-dimensional turbulent flow using constant temperature anemometry.

1.1 Free Turbulent Axisymmetric Jets

A distinction is usually made between the near field region of the jet close to the nozzle exit, also called the developing region where the potential core is still intact, and the far field region called the fully developed region starting about five or more exit-diameters from the nozzle exit depending on initial conditions.

The near field region is strongly dependent on the nozzle exit conditions. The flow in the potential core behaves as inviscid and the velocity of the jet remains of the order of 99% of the centerline exit velocity as pointed out by Wygnanski and Fielder
Michalke (1971) and Ho and Huerre (1984) found similarity profiles and proposed a hyperbolic tangent profile for the non-interacting shear layers.

In the far field region, the mean velocity and shear stress are known to have similarity profiles with almost a Gaussian distribution: $\bar{U}/U_c = f_1(r/l)$ and $\bar{u}'v'/U_c^2 = f_2(r/l)$, where $r$ is the radial distance from the center line, $\bar{U}_c (\propto x^{-1})$ is the mean velocity at the center line and $l$ ($\propto x$) is the half-width of the jet.

![Figure 1.1: Schematic of a jet](image)

Turbulent jet flows are characterized by the presence of high fluctuating level vorticity. It results in the formation of eddies of different frequencies and length scales. Turbulent eddies are mostly produced and spread in the interface regions where the gradients of the velocity are high, for example in the shear layers. Their largest scale is of the order of the momentum thickness $\theta$ (small eddies). Large eddies are also present in the jet and are of the scale of the jet diameter $D$ in the mid field.

The shedding frequency of eddies is characterized by the Strouhal number $St$. Numerical works on the Orr-Sommerfeld stability problem verified by experimental
studies have shown that the Strouhal number in shear layers $St_{\theta} = \frac{f \cdot \theta}{\Delta U}$ (where $\Delta U$ is the shear layer velocity jump) lies in the range of 0.012 to 0.020. Similarly, the Strouhal number $St_D = \frac{f \cdot D}{U_c}$ ($D$ is the diameter of the jet and $U_c$ the mean velocity at the center line) characterizing eddies of larger scale (also called jet column mode or preferred mode of the jet) was found to be in the range of 0.2 to 0.5.

In their evolution downstream of the exit, large eddies loose energy, resulting in their break down into smaller scale rotating structures, causing transition to finer scale turbulence. The scale of the smallest dissipative eddies is characterized by the Kolmogorov micro scale $\eta = \left( \frac{\nu}{\varepsilon} \right)^{1/4}$, where $\nu$ is the dynamic viscosity and $\varepsilon$ is the dissipation.

1.2 Passive Forcing

Several studies have shown that the properties of the jet flow depend strongly on initial conditions and the geometry of the nozzle exit plane. In particular, the axis-switching phenomenon that has been identified to substantially promote mixing, can be observed in non-circular nozzle flows but is absent in circular flows. This important feature has motivated several studies to investigate the characteristics of the flow induced by different geometries of non-circular nozzles, in particular for elliptical, triangular, square and rectangular geometries.

Studies on elliptical nozzles by Ho and Gutmark (1987), and Hussain and Husain (1989) have shown that the mixing rate is considerably higher (as much as 8:1) than in circular jets. Mixing enhancement was observed by Schadow et al. (1988) and Koshigoe et al. (1988) in equilateral and isosceles triangular jets. Square jets have been investigated
by Sforza et al. (1966), duPlessis et al. (1974) and Trentacoste and Sforza (1976). Quinn and Militzer (1988) have found a similar spreading rate for the square jets as for the circular jet at the same distance from the exit of the nozzle and for identical hydraulic diameter. Simulations by Grinstein et al. (1995) and Grinstein and De Vore (1996) on square jets have led to similar conclusions. The rectangular nozzle with an aspect ratio of 2 or 3 was compared to square nozzles by Grinstein (2001). Jet spreading was found to be significantly larger for the square geometry.

1.3 Axis Switching

The difference of jet spreading and mixing enhancement between non-circular and circular nozzles is explained by the deformation of the shear layer vortex ring, which is responsible for the axis-switching phenomenon.

Axis switching refers to the evolution of the axial cross section of the jet along the jet axis, which can have a similar boundary profile further downstream of the exit to that of the exit of the nozzle, with the difference that it appears rotated around the jet axis by an angle characteristic to the initial shape (Grinstein, 2001). As pointed out by Tsuchiya et al. (1985) and Ho and Gutmark (1987), this apparent rotation does not correspond to a helical turning of the jet column around the jet axis.

Depending on the initial conditions, one, several or no axis switching have been observed experimentally.

Grinstein and De Vore explain the axis-switching phenomenon by the occurrence of a self-induction mechanism of a coherent vortex tube with high curvature described by Batchelor (1967). In the case of non-circular exit geometries, a substantial self-induced velocity develops in the regions of high curvature in the direction of the binormal vector.
to the plane containing the local vortex ring segment. Higher local vortex-line curvature and vortex strength result in a higher local induced velocity.

![Diagram of vortex tube with induced velocity](image)

**Figure 1.2:** Self-induced velocity on curved vortex ring (a), deformation of vortex rings (from Grinstein and De Vore) (b)

For instance in the case of square nozzles, the self-induced velocity is important at the corners (high curvature zones) whereas it is null at the straight sides (no curvature). According to the previous explanation, as we move farther from the exit along the jet axis, the vortex ring deforms first at the corners which start moving upwards away from the nozzle. The vortex ring deforms and the curvature at the sides increases, which creates locally an induced velocity bringing the side to move outwards. During its evolution downstream the exit, the initial square vortex ring contained in the exit plane evolves to a non-planar shape and eventually reaches again a planar square shape identical to the initial one but rotated by 45 degrees around the jet axis.
Grinstein (2001) suggested that the axis switching is influenced by a strong interaction between streamwise and azimuthal vorticity: the stretching of the streamwise vorticity results in the formation of braid vortices (of hairpin vortices) that interact with the vortex ring and can provoke transition to turbulence (Gutmark and Grinstein, 1999). Moreover, streamwise vorticity can delay or accelerate the axis switching depending on the initial conditions (Zaman, 1996).

Several variables were found to influence the occurrence of axis switching, the number of switch-overs, and the distance from the exit of the first crossover (Koshigoe et al. 1988, Grinstein et al. 1995) among which are (a) the aspect ratio of the exit of the nozzle in the case of rectangular and elliptic nozzles, (b) Reynolds and Mach numbers of the jet, (c) the initial conditions of the jet (azimuthal distribution of the momentum thickness, size of the momentum thickness and initial turbulence level).

Mach and Reynolds number have been proved to have almost no influence on axis switching according to the work of Husain and Hussain (1983), and Grinstein et al. (1995).

The distance from the exit of the first crossover for a rectangular jet was experimentally found to vary linearly with the aspect ratio of the nozzle (Sforza et al., 1966, Sfeir, 1979, Krothapalli et al., 1981, Tsuchiya 1985). This can be partially explained by the variation of the distance for the width along the minor axis to overtake the width along the major axis. Similar conclusions were reached by Grinstein (1995), who also found a linear relationship between the axis rotation period and the aspect ratio for rectangular and elliptic jets.

The initial azimuthal distribution of momentum thickness has a strong influence
on the axis switching according to Grinstein et al. (1995). If the initial shear layer grows at the same rate on the major and minor axis or if the initial momentum thickness on the major axis (corner for the square nozzle) is greater than at the minor axis (flat side for square nozzle) then no cross over would be observed. For the same reason axis switching is more prominent for contoured nozzles than for orifices and faster for orifices than for channel of pipe nozzles (Krothapalli et al., 1981, Tsuchiya et al., 1985, Husain and Hussain, 1989 and Grinstein and Gutmark, 1995).

1.4 Square Coaxial Jets

Single and coaxial circular jets along with square single nozzle have been extensively studied. Yet jets with non-axisymmetric coaxial geometries were less completely investigated.

In the case of axisymmetric coaxial jets, Ko and Au (1985) observed the existence of shedding vortex in the inner mixing regio, developing alternately from the inner and outer lips of the inner nozzle rotating inward and outward respectively. For the inner-to-outer jet nominal velocity ratio $\lambda$ less than 0.5, the inner vortices called coflowing-wake-vortices develop on the outer lip of the inner nozzle and rotate inward in direction of the center of the jet. The strength of the coflowing-wake-vortices decreases as $\lambda$ increases, unlike the alternate shedding vortices. Both type of vortices coexist for $\lambda$ in the range of 0.5 to 0.8. For values of $\lambda$ in the range of 0.8 to 1.0, the alternate shedding vortical structures prevail and move outwards the jet.

Flow visualizations were conducted on coaxial non-contoured nozzles of different shapes (square, triangular, lobbed and circular) by Bitting et al. (1997). The experiments were not conclusive since they observed no axis switching and found a modest mixing
improvement with respect to the circular case for similar conditions. Bonnafous (2001) and then Piffaut (2003) studied the case of square coaxial jets induced by appropriately designed contoured nozzles, at a co-flow jet Reynolds number of $Re=16,000$ and inner-to-outer jet nominal velocity ratios of $\lambda = 0, 0.5, 1.5$. Axis switching was for the first time shown to occur in square coaxial jets in the near field region, resulting in considerable jet spreading increases. The spreading rates (directly related to the mixing rate) of the inner and outer mixing regions of the square nozzles are clearly higher than the ones observed under similar conditions for the circular geometry with the same hydraulic diameters. These results are based on measurements conducted on a center plane parallel to the square nozzle sides and on a diagonal plane of the square nozzle. A comparison between those results and the circular case give good hope that the entrainment rate will be more significant for the square geometry.

The present will present the three-dimensional map of the flow induced by square coaxial nozzles by conducting flow measurements on the same center and diagonal planes along with intermediate planes, for the same experimental setting and with similar flow conditions.

For the previous studies by Bonnafous (2001) and Piffaut (2003) the center and diagonal planes were chosen because the mean flow has no cross-plane component. In general the flow is three dimensional with two dominant components depending on the chosen plane. In order to fully evaluate the mixing advantages and the 3-D evolution of the flow, three dimensional measurements are necessary.
1.5 Measurement of the Mean-Flow and the Reynolds Stresses Field in a Three-Dimensional Flow

Results obtained by Bonnafous (2001) and then Piffaut (2003) were derived from measurements conducted using constant temperature anemometry, and using an X-wire probe. The data processing method used is based on several reductive assumptions, and in particular on the symmetry-based hypothesis that the flow induced by the nozzle in the diagonal and side planes has two non-zero mean components (i.e. the mean azimuthal component of the velocity field is negligible compared to the radial and streamwise components). At the exit of the nozzle the side and diagonal planes are mean symmetry planes. Yet the assumption is not in general valid in any other planes of measurement at the exit plane and in other planes downstream (particularly because of the axis-switching phenomenon).

Therefore, another method is needed to obtain accurate results in three-dimensional regions of the flow field.

The objective of this part is to review some of the available methods used for the measurements of the mean-velocity components and Reynolds stresses using hot-wire anemometry and a two-wire probe (in particular an X-probe). It is needed to emphasize that those methods provide with reasonably high accuracy estimates of average quantities such as mean flow components and mean Reynolds stresses at any location in the flow. Yet they are not appropriate to derive instantaneous quantities that would be required for a frequency space analysis.

Even though some techniques provide approximations of flow quantities of order two (Müller 1982b), they usually fail in providing higher order quantities such as triple products. In this case, it is again preferable to use triple-sensor probes.
Direct instantaneous measurements of three-dimensional flow quantities are not possible using a two-wire sensor but it is still possible to estimate with satisfactory accuracy mean flow quantities using a multi-position X-probe technique. It consists of conducting measurements with the probe set in different orientations with respect to the flow for each location in space. Results are obtained indirectly by combining the data from the different orientations.

Literature proposes different data processing techniques that mainly differ on the basis of the assumptions used. Bradshaw and Terrel (1969), Johnston (1970), Mojola (1974), Cutler and Bradshaw (1991) are working under the assumption that (a) velocity fluctuations are a small fraction of the mean velocity and that (b) the probe is precisely aligned with the mean flow direction which is required to be known at each spatial position. The quantities $\overline{U}, \overline{u'^2}, \overline{v'^2}, \overline{w'^2}, \overline{u'v'}, \overline{u'w'}$ are derived from measurements with probe orientated in position $0^\circ$ (reference orientation) and $90^\circ$, and measurements with probe in orientation $45^\circ$ and $135^\circ$ (or $-45^\circ$) provide $\overline{v'w'}$ (Figure 1.2).

In practice, it is not always possible to align the probe axis with the main flow direction and these techniques may then fail to work accurately. Elsenaar and Boelsma (1974), Müller and Krause (1979), Müller (1982b) proposed techniques that allow not to align precisely the probe axis with the mean flow direction. Among the latter Müller uses the less reductive assumptions and is thus the most accurate.

In those techniques, it is assumed that the two wires of the X-probe are measuring simultaneous velocities at the same spatial location. However, in reality they are built a small distance ($\delta = 0.94$ mm in our case) away from each other to avoid prong interferences (Figure 1.3).
As pointed out by Bell and Metha (1989), a velocity gradient in the direction perpendicular to the plane of measurement can result in errors in the estimation of the velocity components. Cutler and Bradshaw (1991) and Hirota et al (1988) propose a linear correction on the effective cooling velocities. Similarly, Bell and Metha (1989) derived a linear relationship involving the instantaneous velocity components. The mean velocities and Reynolds stresses are corrected using a linear relationship.

None of the previous research proposes a measuring and data processing technique that gives accurate results in turbulent three-dimensional flows and includes a correction for the gradient of velocity.

Since the flow induced by a non-axisymmetric nozzle is clearly three-dimensional and the boundary layers are regions of high velocity gradient, it was necessary to develop and implement a methodology to account for velocity gradient effects and 3D-turbulence.
Figure 1.4: Geometry of the X-probe

1.6 Objectives

The first objective consists in implementing and developing a procedure to obtain, from measurements using hot wire anemometry with a dual film X-probe, the mean components and Reynolds stresses of a three-dimensional turbulent flow, characterized by regions of strong spatial velocity gradient. This procedure is explained in Chapter 3 and the validation is presented in Chapter 4.

The second part completes the study of Bonnafous (2001) and Piffaut (2003) who studied the flow emanating from square coaxial jets in two vertical planes (side and corner planes) parallel to the jets axis for an inner to outer jet nominal velocity ratios of $\lambda =0.5$ and 1.5 respectively. The present study provides the flow field for similar velocity ratios in horizontal planes obtained using the three-dimensional technique. The low and high velocity ratio cases are presented in Chapters 5 and 6 respectively.
Chapter 2 Experimental Apparatus and Procedures

Experiments were carried out in the Flow Control Laboratory of the LSU Center for Turbine Innovation and Energy Research (TIER).

2.1 Square and Circular Nozzles

This part briefly describes the circular and square coaxial nozzles on which the experiments were carried out. Circular and square coaxial nozzles are described in greater details by Choy (2001) and Choy and al. (1999) respectively.

The near-field flow characteristics depend strongly on initial conditions at the exit, such as the momentum thickness of the boundary layers, turbulence level and velocity ratio. Those initial conditions depend on the flow upstream the exit in the nozzles, which is ensured to be identical between different cross sections if the hydraulic diameters are matched. Consequently, circular and square nozzles are designed to have identical hydraulic diameters throughout, from the settling chamber to the exit, the hydraulic diameter being defined as equal to four times the area to perimeter ratio: \( D_h = 4 \frac{A}{P} \). Additionally, matching of the hydraulic diameters results in matching of the area ratio, ensuring that for any cross section of the circular and square coaxial nozzles, the area ratios match. Dimensions are shown in Figure 2.1.

The hydraulic diameters of the inner and the outer nozzles are \( Dh_i = 0.6 \text{ in} = 15.24 \text{ mm} \) and \( Dh_o = 0.8 \text{ in} = 20.32 \text{ mm} \) respectively.

The nozzles were manufactured using numerically controlled machining tools. Contours of the inner and outer nozzles are defined by fifth-order polynomial contours with zero first order derivatives at both ends. The design prevents any flow separation on the inner jet surface throughout the nozzle until the exit. The contraction ratio for the
inner stream is equal to 25:1 and for the outer stream equal to 13.6:1. The outer-to-inner area ratio at the exit is 4.89.

The lip that separates the inner from the outer jets has been designed thin so that the mixing of the two jets happens closer to the exit plane.
The body of the nozzles is supported by a stand mounted on a 45 seconds accurate rotary table. Four speakers mounted on the body, can be used to acoustically force the outer flow. The inner flow can be excited using an extra speaker mounted out of the body. The speakers are presented in Figure 2.4 and the outer speakers are visible in the picture of Figure 2.2 (b). Since active forcing was not studied in the present study, the acoustic forcing system is not described with more details.

Centers of the inner and outer square nozzle coincide with a 25μm (0.001 in) precision equal to the machining tolerance. The vertical axes of the coaxial jets and of the rotary table do not coincide perfectly, standing at the nozzle exit at 50μm (0.002 in) away from each other.

2.2 Air Supply

A schematic of the jet-testing facility is presented in Figure 2.4.

A 2.0-Mpa-rated (~300 psi) main storage tank is supplied in dried air by a compressor at a pressure of 0.9 Mpa (~140 psi) approximately. Alternately, air-drying
can also be achieved by a pressure-swing desiccant type compressed air dryer located downstream the main storage tank. The air is stored in a secondary 1.0-Mpa-rated (~150 psi) buffer tank that helps maintain bulk flowrate stability. From there, the flow divides into two metering runs. Each line is equipped with a Rosemount orifice flow meter. High precision pressure regulator valves keep a constant 550kPa (~80 psi) upstream pressure. Flowrates in both branches feeding the inner and outer flows are adjusted using flow control valves downstream the line.

The air goes then through a distribution chamber, passes through eight 3/8” Teflon tubes for the outer jet and four 1’ rubber pipes for the inner jet, and goes through four wire mesh screens (coarse to fine) to get a more uniform flow and to reduce fluctuations.

2.3 Operating Conditions

Choy (2001) studied the axis-symmetric coaxial jets with a Reynolds number
$\text{Re}_{Dho}$ equal to 15,300. Results were obtained for different values of $\lambda$: 0.5, 1.0, 1.5 where $\lambda$ is defined as the ratio of the inner to outer jet nominal velocities. Bonnafous (2001) studied the square coaxial jet with the same Reynolds number and a 0.5 velocity ratio. Piffaut (2003) extended the study to the 1.5 velocity ratio.

Table 2.1: Flow conditions

<table>
<thead>
<tr>
<th></th>
<th>Circular jets (Wai-Ho Choy)</th>
<th>Square jets (side)</th>
<th>Square jets (diagonal)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outer jet</td>
<td>Inner jet</td>
<td>Outer jet</td>
</tr>
<tr>
<td>Flowrate $Q$ (SCFM)</td>
<td>21.75</td>
<td>7.46</td>
<td>25.33</td>
</tr>
<tr>
<td>Bulk velocity $\overline{U}$ (m/s)</td>
<td>11.51</td>
<td>16.90</td>
<td>11.51</td>
</tr>
<tr>
<td>Max velocity $U_{\text{max}}$ (m/s)</td>
<td>13.55</td>
<td>21.91</td>
<td>14.45</td>
</tr>
<tr>
<td>Centerline velocity $U_{\text{CL}}$ (m/s)</td>
<td>13.25</td>
<td>21.35</td>
<td>13.92</td>
</tr>
<tr>
<td>$U_{\overline{c}} = (U_{\text{CL,o}} + U_{\text{CL,i}})/2$</td>
<td>17.30</td>
<td>17.59</td>
<td>17.89</td>
</tr>
<tr>
<td>$\text{Re}_{Dho}$</td>
<td>15,360</td>
<td>16,620</td>
<td>15,250</td>
</tr>
<tr>
<td>$\text{Re}_{Dho,CL}$</td>
<td>17,690</td>
<td>21,370</td>
<td>18,440</td>
</tr>
<tr>
<td>$\overline{\lambda}$</td>
<td>1.507</td>
<td>1.526</td>
<td>1.526</td>
</tr>
<tr>
<td>$\lambda_{\text{CL}}$</td>
<td>1.611</td>
<td>1.527</td>
<td>1.493</td>
</tr>
</tbody>
</table>

where $\text{Re}_{Dho} = \frac{U_{\text{o}} D_{ho}}{v}$, $\text{Re}_{Dho,CL} = \frac{U_{\text{o,CL}} D_{ho}}{v}$ both based on the hydraulic diameter of the co-flow, and $\overline{\lambda} = \frac{U_{\overline{c}}}{U_{\text{o}}}$, $\lambda_{\text{CL}} = \frac{U_{\text{i,CL}}}{U_{\text{o,CL}}}$.

The present study is complementing the previous results of the square coaxial jets for the cases of $\lambda = 0.5$ and 1.5 and identical Reynolds number. So far, the side and the diagonal planes had been investigated. In this study, experiments are conducted on 8 planes: 110°, 120° (side plane), 130°, 140°, 150°, 160°, 165° (diagonal plane), 170° (with
three extra planes 145°, 153°, 156° for case \( \lambda = 0.5 \) for the first height only), at five different heights with respect to the exit plane.

The ambient temperature during experiments varies from 21°C to 27°C. The fluid used is air at 80 psi with a viscosity in the range of \( 1.81 \times 10^{-5} \) to \( 1.85 \times 10^{-5} \) kg/m/s. The flow conditions used are similar to the ones presented in Table 2.1.

2.4 Constant Temperature Anemometry

The constant temperature anemometry technique is used. Measurements are done using TSI 1241-10 dual-film X-probes and TSI IFA 300 acquisition system. A thermocouple mounted between the buffer tank and the pressure regulators before the separation in two lines, provides the IFA software with the air stream temperature standing for the ambient temperature.

At each location in the flow, 65,536 measurements are acquired with a sampling rate of 50,000 Hz. Data are filtered by a 20 kHz de-aliasing low-pass filter.

2.5 Calibration of the X-Probe

The dual-film X-probe was calibrated on a TSI calibrator nozzle set for a instantaneous velocity range of 0.0 m/s to 50 m/s. The calibration procedure was adapted...
for measurements in three-dimensional flows. For this reason, both sensitivities within and perpendicular to the plane of measurement of the probe were determined. More traditional calibration procedures for probes of the same type do not determine the sensitivity of the sensor perpendicular to the plane of measurement (the plane of measurement is parallel to both sensor films), given that during experiments the component of the velocity in this direction is assumed to be null.

Therefore the calibration procedure adopted is composed of three distinct steps. Firstly, for both films of the probe the anemometer equation relating the Wheatstone bridge output voltage to the corresponding effective velocity is determined. The second and third steps calculate for both probe films the yaw \( k \) and the pitch \( h \) coefficients as defined in the Jörgensen’s cooling law:

\[
V_{\text{eff}}^2 = U_{N1}^2 + k^2 U_T^2 + h^2 U_{N2}^2
\]

where \( V_{\text{eff}} \) is the effective velocity, \( U_{N1}, U_T \) and \( U_{N2} \) are the components of the flow velocity as defined in Figure 2.5.

The yaw coefficient \( k \) stands for the film sensitivity in the direction tangential to the film axis, the pitch \( h \) coefficient is related to its sensitivity in the direction perpendicular to the plane of measurement.

Anemometer equations and yaw coefficients were calculated using the calibration procedure available in the TSI IFA 300 software. The anemometer equation is expressed as a fourth-order polynomial equation as introduced by George et al. (1981, 1989a). Different results (Bruun et al., 1988, Bruun and Tropea, 1980, 1985, Swaminathan et al., 1986a) confirm that even though the polynomial relation is less accurate than a King’s-type equation, it remains acceptable when a fourth-order polynomial is used.
The studies conducted by Irwin (1971), Vagt (1979), Adrian et al. (1984) and Lekakis (1988) have shown that the yaw coefficient is a function of the flow velocity and can be considered as independent from the yaw angle if the latter remains in the range of 0 to 70°. The yaw coefficient is computed consistently with those results from data acquired at the three different velocities approximately equal to 7.4, 15.5 and 25.4 m/s. For each one, the yaw coefficient is calculated from acquisitions conducted at eleven different yaw angles: -30°, -24°, -18°, -12°, -6°, 0°, 6°, 12°, 18°, 24°, 30° with respect to the main flow direction.

![Figure 2.6: Velocity vector decomposed in the frame of the film](image)

The pitch coefficient $h$ was determined following the method described by Müller (1982b): the flow in the calibrator nozzle is set to $U_0$ (successively $U_0=25$ m/s and $U_0=35$ m/s), the probe axis is aligned with the flow direction so that the component normal to the plane of measurement vanish, and the corresponding mean effective $V_{eff}(U_0)$ is recorded. The probe is then inclined at $\sigma=-6^\circ$ in pitch, the velocity in the calibrator nozzle is set to $U_0/\cos(-6^\circ)$, and $V_{eff}(U_0/\cos(-6^\circ))$ is recorded. This step is repeated for a $\sigma=+6^\circ$ pitch angle. The pitch coefficient $h$ is then calculated using the formula (see Müller, 1982):
Müller (1982b) found that \( h \) did not show any significant variation with the velocity. An average value of 1.25 and 1.35 for film 1 and 2 respectively was found by averaging the values of \( h \) found at the two different velocities \( U_0 = 25 \text{ m/s} \) and \( U_0 = 35 \text{ m/s} \) and for \( \sigma = +6^\circ \) and \( \sigma = -6^\circ \). Müller found an average value of \( h = 1.2 \) consistently with our results.

### 2.6 Probe Support and Traverse System

The X-probe is connected to the lower end of the probe support (looking downward). The latter is aligned with the vertical and is connected to the traverse system through the R56-572 Edmund Industrial Optics micro manual rotary stage allowing rotating precisely the X-probe around its vertical axis for measurements with different orientation.

A two-dimensional traverse system controlled by the IFA system allowed to move the probe with two degrees of freedom with a 0.05 mm precision in positioning. Data were recorded after a 0.6 second delay after each move.

For each case, the probe goes through five different heights depending on the value of \( \lambda \).

### 2.7 Measurement Technique

After each move of the rotary stage or the X-probe, misalignment and adequate adjustments were done using the GAETNER Traveling Telescope Cathetometer M912. Ideally, measurements should be taken at the same time in the whole flow, but in practice, acquisition is done point after point during an experiment that can last 48 hours. This introduces errors due to any possible fluctuations of the flow conditions.
As discussed earlier, errors due to temperature drift is corrected but no correction is applied for flowrate fluctuation. The results were accepted when the velocities $U_{oCL}$

Figure 2.7: Schematic of the probe and probe support
and $U_{iCL}$ at the exit plane differ from the target velocities by less than $\pm 5\%$, otherwise the experiment was repeated.

The X-probe is used in two different orientations, orientation $0^\circ$ when the plane of measurement coincides with the traverse plane, and orientation $90^\circ$ when the probe is rotated around its axis by a $90^\circ$ angle. By convention, the X-probe is plugged and rotated as described in Figure 2.6, to be consistent with the formula described in the data process section. Measurements are taken in vertical planes that contain the nozzles vertical axis $x$.

It should be emphasized that the instantaneous velocity vector has to remain in the quadrant of validity defined on Figure 2.7. Measurement of velocities out of this domain results in invalid readings and are discarded.

2.8 Data Processing with IFA 300 Software

After one experiment, two files with data are stored for each measuring location correspond, containing voltages in binary format and the average ambient temperature. TSI IFA 300 software processes the data using the calibration file of the X-probe and provides ASCII time history of the output bridge voltages.
Chapter 3  Data Reduction Methods

3.1 Data Reduction Method for 3-D Low Turbulence Flows with Velocity Gradient

3.1.1 Introduction

The methodology employed to process the measurements conducted in the flow emanating from coaxial coutoured nozzles with square section is described in the present section. Since the flow is three-dimensional with low-turbulence (less than 30%) and characterized by regions of high velocity gradient, this method was develop to be accurate for three-dimensional flow measurements with turbulent level less than 30%, and great care was taken to include a correction for bias introduced by velocity gradients. Additionally, corrections are added to take into consideration the air stream temperature and ambient static pressure difference between calibration and experiment.

This home-developed method computes mean velocities and mean Reynolds stresses from data acquired from two different orientations of a dual-film X-probe for each measurement location in the jet stream. The probe is first positioned so that the two principal mean velocity components remain in the plane of the sensors, i.e. the plane of measurement in parallel to the vertical and radial axis. It is then rotated around its axis by a 90° angle (Figure 3.2).

The present method differs from the various others proposed in the literature by fewer number of assumptions used to simplify the problem. In our case, the azimuthal mean velocity component $\bar{W}$ was assumed to be null for simplicity as in previous cases but fluctuations $w'$ in the same direction were not neglected.

A method can be developed in a similar way for cases where $\bar{W}$ cannot be neglected, by simply deriving the equations with $W = \bar{W} + w$. The steps of the processing
method deriving the mean velocities $\overline{U}$ and $\overline{V}$ along with the Reynolds stresses $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$, $\overline{u'v'}$, and $\overline{u'w'}$ from the bridge output voltages are now presented.

![Figure 3.1: Schematic of the top view of the square coaxial nozzle showing the probe-holder frame (X_p, Y_p, Z_p) for one particular path](image)

### 3.1.2 Temperature and Pressure Corrections

The correction for the effect of temperature change between the calibration and the data acquisition is applied on the Wheatstone bridge output voltages $E_w$ using the formula: $E_w^c = E_w \left( \frac{T_w - T_c}{T_w - T_e} \right)^{1/2}$, where $E_w^c$ is the corrected voltage, $T_w$ the sensor operating temperature, $T_c$ and $T_e$ are respectively the temperatures during calibration and experiment.

Effective velocities are then obtained from the corrected voltages using the $4^{th}$ order polynomials determined during the calibration:
\[ V_{\text{eff},i} = a_i + b_i \cdot E_{\text{wi}} + c_i \cdot E_{\text{wi}}^2 + d_i \cdot E_{\text{wi}}^3 + e_i \cdot E_{\text{wi}}^4, \ i=1 \text{ or } 2 \text{ for wire 1 or 2}. \]

Changes in atmospheric pressure between calibration and experiment are taken into account using the formula derived from King’s law: \[ V_{\text{eff}}^c = V_{\text{eff}} \cdot \frac{P_c}{P_e} \]
where \( V_{\text{eff}}^c, V_{\text{eff}} \) are the corrected and uncorrected effective velocities respectively, \( P_c \) and \( P_e \) are the calibration and experimental atmospheric pressures respectively.

### 3.1.3 Velocity Gradient Correction

Because a small distance separates the two wires of the X-probe \( \delta \approx 0.94 \text{ mm} \), the effective velocities seen by the wires do not correspond to the same spatial point. This can consequently bias the results if the flow contains regions of high velocity gradients.

The main mathematical difficulty is to solve simultaneously for the gradient-corrected velocity field components. The trick used to simplify the solution of the system consists in decoupling the velocity gradient correction problem, and avoiding solving more complicated sets of equations.

The equations are derived using notations described in Figure 3.2. Assuming that the velocities vary linearly over the distance \( \frac{\delta}{2} \), we can express \( U_{n1,i}(M) \) (the effective velocity as wire \( i \) would see it at point \( M \)) as a function of \( U_{n1,i} \) (the effective velocity actually seen by wire \( i \) at a distance \( \frac{\delta}{2} \) from \( M \)):

\[
\begin{align*}
U_{n1,i} &= U_{n1,i}(M) + \frac{\delta}{2} \cdot \frac{\partial U_{n1,i}(M)}{\partial z} + O(\delta^2) \quad \text{for probe orientation } 0^\circ, \ i=1(-) \text{ and } 2(+) \\
U_{n1,i} &= U_{n1,i}(M) + \frac{\delta}{2} \cdot \frac{\partial U_{n1,i}(M)}{\partial y} + O(\delta^2) \quad \text{for probe orientation } 90^\circ
\end{align*}
\]

for wire 1 and 2 respectively.
Proceeding the same way for $U_{n, i}$ and $U_{i, i}$, using the definition of the instantaneous effective velocity $V_{eff, i}^2 = U_{n, i}^2 + k_i^2 U_{i, i}^2 + h_i^2 U_{n, i}^2$ (Jørgensen’s cooling law), we obtain the gradient correction formula:

$$
\begin{align*}
V_{eff, i}^2 &= V_{eff, i}^2(M) \pm \frac{\delta}{2} \frac{\partial V_{eff, i}^2(M)}{\partial z} + O(\delta^2) \quad \text{for probe orientation } 0^0 \\
V_{eff, i}^2 &= V_{eff, i}^2(M) \pm \frac{\delta}{2} \frac{\partial V_{eff, i}^2(M)}{\partial y} + O(\delta^2) \quad \text{for probe orientation } 90^0
\end{align*}
$$

for wire 1 and 2 respectively.

After separating mean and fluctuating parts, manipulating and dropping terms of order $\delta^2$ and higher:

$$
\begin{align*}
\frac{V_{eff, i}^2}{V_{eff, i}^2} &= V_{eff, i}^2(M) \pm \frac{\delta}{2} \frac{\partial V_{eff, i}^2(M)}{\partial z} + O(\delta^2) \quad \text{for probe orientation } 0^0, \text{ and} \\
\frac{V_{eff, i}^2}{V_{eff, i}^2} &= V_{eff, i}^2(M) \pm \frac{\delta}{2} \frac{\partial V_{eff, i}^2(M)}{\partial y} + O(\delta^2)
\end{align*}
$$
\[
\begin{align*}
V_{eff_i}^2 &= V_{eff_i}^2(M) \pm \frac{\delta}{2} \frac{\partial V_{eff_i}^2(M)}{\partial y} + O(\delta^2) \\
V_{eff_i}^2 &= V_{eff_i}^2(M) \pm \frac{\delta}{2} \frac{\partial V_{eff_i}^2(M)}{\partial y} + O(\delta^2)
\end{align*}
\]

for probe orientation 90°, \(i = 1(-)or 2(+)\) for wire 1 and 2 respectively.

We should emphasize that it is not possible to derive a similar correction for the cross terms \(v_{eff_1,v_{eff_2}}\). Consequently, the final results are calculated from values of \(v_{eff_1,v_{eff_2}}\) uncorrected for the gradient effects. Therefore the velocity own-correlations are less credible in the high shear regions than the remaining velocity statistics.

**3.1.4 Linearized Hot-Wire Response Equations**

The linearized hot-wire response equations are derived consistently with the work of Müller (1982b) but less reductive assumptions are made.

It was chosen not to use the ‘cosine law’ that describes the cooling law for an ideal infinite hot-wire. This law is based on the assumption that the effective velocity is only a function of the velocity normal to the wire in the wire axis. In practice, the effects of the hot-wire response in a three-dimensional flow depends also on the tangential components because of the effects of convection along the wire-sensor. The cooling law proposed by Jörgensen (1971) includes those effects and are thus used as earlier with the following notations:

\[V_{eff}^2 = U_{n_1}^2 + k^2 U_{n_1}^2 + h^2 U_{n_2}^2,\] where \(V_{eff}\) is the effective velocity, \(k\) and \(h\) are respectively the yaw and pitch coefficients, \(U_{n_1}\) and \(U_{n_2}\) are respectively the instantaneous velocity normal to the wire in the plane of measurement of the probe and perpendicular to it, \(U_i\) is the instantaneous velocity tangential to the wire (see Figure 2.5).
Figure 3.3: Schematics and expressions of the normal and tangential velocity components as functions of the velocity components $U$, $V$, $W$ in the probe-holder frame $(X_p, Y_p, Z_p)$ for wire 1 for the probe in orientation $0^\circ$ (a), and $90^\circ$ (b)

\[
U_{n1} = -U \cos\left(\frac{\pi}{4} + t_0\right) + V \cos\left(\frac{\pi}{4} - t_0\right) \\
U_{t1} = -U \sin\left(\frac{\pi}{4} + t_0\right) - V \sin\left(\frac{\pi}{4} - t_0\right)
\]

\[
U_{n2} = -U \cos\left(\frac{\pi}{4} - t_0\right) - V \sin\left(\frac{\pi}{4} - t_0\right) \\
U_{t2} = -U \sin\left(\frac{\pi}{4} - t_0\right) + V \cos\left(\frac{\pi}{4} - t_0\right)
\]

Figure 3.4: Schematics and expressions of the normal and tangential velocity components as functions of the velocity components $U$, $V$, $W$ in the probe-holder frame $(X_p, Y_p, Z_p)$ for wire 2 for the probe in orientation $0^\circ$ (a), and $90^\circ$ (b)

\[
U_{n3} = -U \cos\left(\frac{\pi}{4} - t_1\right) - W \sin\left(\frac{\pi}{4} - t_1\right) \\
U_{t3} = -U \sin\left(\frac{\pi}{4} - t_1\right) + W \cos\left(\frac{\pi}{4} - t_1\right)
\]

\[
U_{n4} = -U \cos\left(\frac{\pi}{4} + t_1\right) + W \sin\left(\frac{\pi}{4} + t_1\right) \\
U_{t4} = -U \sin\left(\frac{\pi}{4} + t_1\right) - W \cos\left(\frac{\pi}{4} + t_1\right)
\]
By geometrical considerations for each wire in both orientations, the velocity components in the probe frame \( (X_p, Y_p, Z_p) \) can be expressed using the velocity components in the reference frame \( (X, Y, Z) \) along with \( t_0 \) and \( t_1 \), the angles between the two frames around respectively \( Z \) and \( Y \), that are standing for misalignment during the experiment between the probe axis \( X_p \) and the vertical axis \( X \) in the frame of reference (see Figure 3.3).

The deviatory angles \( t_0 \) and \( t_1 \) are estimated by applying the following trigonometric relations at a point in the flow where the velocities are known. The point used here is the center of the nozzles in the exit plane, where it is a good assumption to say that the mean velocity is aligned with the vertical axis and the secondary mean velocities are null.

In general,

\[
\begin{aligned}
U &= U^* \cos(t_0) + V^* \sin(t_0) \\
V &= -U^* \sin(t_0) + V^* \cos(t_0) \\
W &= -U^* \sin(t_1) + W^* \cos(t_1)
\end{aligned}
\]

Since \( V = W = 0 \) at the center point, then

\[
\begin{aligned}
t_0 &= \tan^{-1}(\frac{V^*}{U^*}) \\
t_1 &= \tan^{-1}(\frac{W^*}{U^*})
\end{aligned}
\]

The starred and non-starred quantities are expressed in the probe frame \( (X_p, Y_p, Z_p) \) and \( (X, Y, Z) \) respectively.

Substituting those expressions in Jörgensen’s equation and decomposing velocities in mean components and fluctuating part denoted by lower case letters, assuming \( W \) null, the cooling velocities becomes:

\[
V_{eff}^2 = F_i^2(\overline{U_i}(\overline{U_i} + f_i(\overline{U_i}, u_i) + g_i(u_i, u_i), i=1 \text{ and } 2 \text{ for wire 1 and 2 with probe in orientation } 0^\circ, i=3 \text{ and } 4 \text{ for wire 1 and 2 with probe in orientation } 90^\circ).
\]
where
\[ \begin{align*}
F_1^2 &= A_1 \vec{U}^2 + B_1 \vec{V}^2 + C_1 \vec{U} \vec{V} \\
F_2^2 &= A_2 \vec{U}^2 + B_2 \vec{V}^2 + C_2 \vec{U} \vec{V} \\
F_3^2 &= A_3 \vec{U}^2 + B_3 \vec{V}^2 \\
F_4^2 &= A_4 \vec{U}^2 + B_4 \vec{V}^2 \\
\end{align*} \]
\[\begin{align*}
f_1 &= D_1 u \vec{U} + E_1 v \vec{V} + F_1 u \vec{V} + G_1 v \vec{U} \\
f_2 &= D_2 u \vec{U} + E_2 v \vec{V} + F_2 u \vec{V} + G_2 v \vec{U} \\
f_3 &= D_3 u \vec{U} + E_3 v \vec{V} + F_3 v \vec{U} \\
f_4 &= D_4 u \vec{U} + E_4 v \vec{V} + F_4 v \vec{U} \\
\end{align*}\,

and
\[ \begin{align*}
g_1 &= H_1 u^2 + I_1 v^2 + J_1 w^2 + K_1 u v \\
g_2 &= H_2 u^2 + I_2 v^2 + J_2 w^2 + K_2 u v \\
g_3 &= H_3 u^2 + I_3 v^2 + J_3 w^2 + K_3 u w \\
g_4 &= H_4 u^2 + I_4 v^2 + J_4 w^2 + K_4 u w \\
\end{align*}\]

\[ \begin{align*}
A_1 &= \cos^2 \left( \frac{\pi}{4} + t_0 \right) + k_1^2 \sin^2 \left( \frac{\pi}{4} + t_0 \right) \\
B_1 &= \cos^2 \left( \frac{\pi}{4} - t_0 \right) + k_1^2 \sin^2 \left( \frac{\pi}{4} + t_0 \right) \\
C_1 &= 2 \cos \left( \frac{\pi}{4} - t_0 \right) \cos \left( \frac{\pi}{4} + t_0 \right) (k_1^2 - 1) \\
D_1 &= 2 A_1 \\
E_1 &= 2 B_1 \\
F_1 &= G_1 = C_1 \\
H_1 &= A_1 \\
I_1 &= B_1 \\
J_1 &= h_1^2 \\
K_1 &= C_1 \\
\end{align*}\] ,

\[ \begin{align*}
A_2 &= \cos^2 \left( \frac{\pi}{4} - t_0 \right) + k_2^2 \sin^2 \left( \frac{\pi}{4} - t_0 \right) \\
B_2 &= \cos^2 \left( \frac{\pi}{4} + t_0 \right) + k_2^2 \sin^2 \left( \frac{\pi}{4} + t_0 \right) \\
C_2 &= 2 \cos \left( \frac{\pi}{4} - t_0 \right) \cos \left( \frac{\pi}{4} + t_0 \right) (1 - k_2^2) \\
D_2 &= 2 A_2 \\
E_2 &= 2 B_2 \\
F_2 &= G_2 = C_2 \\
H_2 &= A_2 \\
I_2 &= B_2 \\
J_2 &= h_2^2 \\
K_2 &= C_2 \\
\end{align*}\]

\[ \begin{align*}
A_3 &= \cos^2 \left( \frac{\pi}{4} - t_1 \right) + k_1^2 \sin^2 \left( \frac{\pi}{4} - t_1 \right) \\
B_3 &= h_1^2 \\
C_3 &= 0 \\
D_3 &= 2 A_3 \\
E_3 &= 2 B_3 \\
\end{align*}\]

\[ \begin{align*}
F_3 &= 2 \cos \left( \frac{\pi}{4} - t_1 \right) \cos \left( \frac{\pi}{4} + t_1 \right) (1 - k_1^2) \\
G_3 &= 0 \\
H_3 &= A_3 \\
I_3 &= B_3 \\
J_3 &= \cos^2 \left( \frac{\pi}{4} + t_1 \right) + k_1^2 \sin^2 \left( \frac{\pi}{4} + t_1 \right) \\
K_3 &= F_3 \\
\end{align*}\], and
\[
\begin{align*}
A_4 &= \cos^2\left(\frac{\pi}{4} + t_1\right) + k_2^2 \cdot \sin^2\left(\frac{\pi}{4} + t_1\right), \quad F_4 = 2 \cdot \cos\left(\frac{\pi}{4} - t_1\right) \cdot \cos\left(\frac{\pi}{4} + t_1\right) (k_2^2 - 1) \\
B_4 &= h_2^2, \quad G_4 = 0 \\
C_4 &= 0, \quad H_4 = A_4 \\
D_4 &= 2A_4, \quad I_4 = B_4 \\
E_4 &= 2B_4, \quad J_4 = \cos^2\left(\frac{\pi}{4} - t_1\right) + k_2^2 \cdot \sin^2\left(\frac{\pi}{4} - t_1\right) \\
K_4 &= F_4
\end{align*}
\]

where \(k_1, k_2\) the yaw and \(h_1, h_2\) the pitch coefficients for wire 1 and 2 are considered as calibration constants.

By separating the mean and fluctuating parts of the cooling velocities, and using binomial expansion when needed, we get the following open system of 14 equations at different orders.

\[
\begin{align*}
\bar{V}_{eff_i}^2 &= F_i^2 + g_i, \\
\bar{V}_{eff_i} &= F_i + \frac{g_i}{2F_i} - \frac{f_i^2}{8F_i^3} + O\left(\frac{u_ju_ku_l}{U_mU_n}\right), \\
\bar{V}_{eff_i}^2 &= \left(V_{eff_i} - \bar{V}_{eff_i}\right)^2 = \frac{f_i^2 + 2f_i g_i}{4F_i^2} - \frac{f_i^3}{8F_i^4} + O\left(\frac{u_ju_ku_m}{U_mU_o}\right), \\
\left(V_{eff_i} \pm V_{eff_{(i+1)}}\right)^2 &= \frac{1}{4} \left[ \frac{f_i \pm f_{i+1}}{F_i F_{i+1}} \right]^2 + \left[ \frac{f_i \pm f_{i+1}}{F_i F_{i+1}} \right] \left[ \frac{g_i \pm g_{i+1}}{F_i F_{i+1}} \right] + \frac{1}{8} \left[ \frac{f_i \pm f_{i+1}}{F_i F_{i+1}} \right] \left[ \frac{f_i^2 \pm f_{i+1}^2}{F_i^3 F_{i+1}^3} \right] + O\left(\frac{u_ju_ku_ju_m}{U_mU_o}\right) \\
&\quad \text{for } i = 1 \text{ and } 3.
\end{align*}
\]

We should emphasize again that all but the cross correlation terms, \(V_{eff_3}, V_{eff_4}\) of the effective velocities are corrected for the effect of velocity gradient.
The numerical resolution of the system appeared to be a difficult task, and the routine employed did not easily converge to the solution with the precision required even by providing initial values close to the solution (less than 5% of relative difference). Thus, the system was solved in a more indirect way as explained next.

3.1.5 System Solution

For measurements in flows with low turbulence intensity (about 10% to be conservative), terms of order higher than two are neglected so that only mean flow components and Reynolds stresses are retained as unknowns in the equations, allowing closing the system of equations. The system to order 2 contains the 8 unknowns

\[ \overline{U}, \overline{V}, \overline{u^2}, \overline{v^2}, \overline{w^2}, \overline{uv}, \overline{uw}, \overline{vw} \]

\[
\begin{align*}
\text{set#1} & \\
\begin{cases}
F_1^2 + g_1 \overline{V_{\text{eff}}^1} = 0 \\
F_2^2 + g_2 \overline{V_{\text{eff}}^2} = 0 \\
F_3^2 + g_3 \overline{V_{\text{eff}}^3} = 0 \\
F_4^2 + g_4 \overline{V_{\text{eff}}^4} = 0 \\
\end{cases} \\
\begin{cases}
f_1^2 - 4.\overline{F_1^2} \cdot \overline{V_{\text{eff}}^1} = 0 \\
f_2^2 - 4.\overline{F_2^2} \cdot \overline{V_{\text{eff}}^2} = 0 \\
f_3^2 - 4.\overline{F_3^2} \cdot \overline{V_{\text{eff}}^3} = 0 \\
f_4^2 - 4.\overline{F_4^2} \cdot \overline{V_{\text{eff}}^4} = 0 \\
\end{cases} \\
\begin{cases}
8.\overline{F_1^4} + 4.g_1.\overline{F_1^2}^2 - f_1^2 - 8.\overline{F_1^3} \cdot \overline{V_{\text{eff}}^1} = 0 \\
8.\overline{F_2^4} + 4.g_2.\overline{F_2^2}^2 - f_2^2 - 8.\overline{F_2^3} \cdot \overline{V_{\text{eff}}^2} = 0 \\
8.\overline{F_3^4} + 4.g_3.\overline{F_3^2}^2 - f_3^2 - 8.\overline{F_3^3} \cdot \overline{V_{\text{eff}}^3} = 0 \\
8.\overline{F_4^4} + 4.g_4.\overline{F_4^2}^2 - f_4^2 - 8.\overline{F_4^3} \cdot \overline{V_{\text{eff}}^4} = 0 \\
\end{cases} \\
\begin{cases}
4.\overline{F_1.\overline{F_2} \cdot \overline{V_{\text{eff}}^1} \cdot \overline{V_{\text{eff}}^2}} = f_1 \cdot f_2 \\
4.\overline{F_3.\overline{F_4} \cdot \overline{V_{\text{eff}}^3} \cdot \overline{V_{\text{eff}}^4}} = f_3 \cdot f_4 \\
\end{cases}
\]

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In theory, equations from sets #1 and 3 are linearly independent but in reality they behave as redundant. Set #1 is preferred to set #3 since its equations are not approximated. Other multi-position techniques (Müller, 1982b) usually derive the quantity \( \overline{v'w'} \) from extra measurements obtained with the probe oriented at 45° and 135°. Since the values found are usually biased, we decided not to solve for this unknown. This results in the major advantage of taking measurements with probe in only two different orientations, which reduces considerably the acquisition time. The quantity \( \overline{v.w} \) is discarded by setting \( \overline{vw} = \frac{1}{2} (\overline{uw} + \overline{vw}) \) which is a reasonable assumption since the Reynolds cross stresses are of the same order of magnitude. Results using this assumption were compared to the case where \( \overline{vw} \) was set to zero. Mean and Reynolds stresses apart from \( \overline{uw} \) were influenced by less than 1% whereas \( \overline{uw} \) was affected by more than 50% in the shear layer regions. Thus, results concerning the latter must be taken with great care.

Hence, the problem is reduced to the solution of a system of 10 equations (after set #3 was discarded) and 7 unknowns. Since the procedure employed by the TSI IFA 300 post-processor computes the velocity field using measurements corresponding to the probe in position 0°, equations involving effective velocities obtained from position 0° were preferred to the ones from position 90°.

The final system reduces to:

\[
\begin{align*}
\text{(eq1)} & \quad \overline{F_1^2 + g_1} = \overline{V_{\text{eff}1}}^2 \\
\text{(eq2)} & \quad \overline{F_2^2 + g_2} = \overline{V_{\text{eff}2}}^2 \\
\text{(eq3)} & \quad \overline{f_1^2 - 4F_1\overline{v_{\text{eff}1}}^2} = 0 \\
\text{(eq4)} & \quad \overline{f_2^2 - 4F_2\overline{v_{\text{eff}2}}^2} = 0 \\
\text{(eq5)} & \quad 4F_2F_2\overline{v_{\text{eff}1}}\overline{v_{\text{eff}2}} = f_1f_2 \\
\text{(eq6)} & \quad \overline{f_3^2 - 4F_3\overline{v_{\text{eff}3}}^2} = 0 \\
\text{(eq7)} & \quad \overline{f_4^2 - 4F_4\overline{v_{\text{eff}4}}^2} = 0
\end{align*}
\]
To simplify the solution, the system was divided into two coupled sub-systems, sub-system 1 being constituted by the first five equations and sub-system 2 by equations 6 and 7. Sub-system 1 was chosen for the solution of $\overline{U}, \overline{V}, \overline{u^2}, \overline{v^2}, \overline{uv}$ assuming temporary values for $\overline{w^2}$ and $\overline{uw}$, sub-system 2 was used to solve for $\overline{w^2}, \overline{uw}$ using temporary values for the other unknowns. This choice is motivated by the fact that equations 1 and 2 only contain through $\overline{g_1}$ and $\overline{g_2}$ the unknown $\overline{w^2}$ ($\overline{uw}$ is not present), which is a second order term in both equations. This procedure is in agreement with Müller (1982b) who derived analytical solutions for $\overline{u^2}, \overline{v^2}, \overline{uv}$ and $\overline{w^2}, \overline{uw}$ from equations 3, 4, 5 and 6, 7 respectively, with the mean of the additional simplifying assumptions $\overline{V} \ll \overline{U}$, $h_1 = h_2 = 1.0$, $k_1 = k_2 = 0.0$ and $t_0 = t_1 = 0^\circ$ which are not employed in our case.

Solutions are finally obtained performing iterations until the relative difference between solutions from two consecutive iterations become less that 1\% for the mean components and 5\% for the Reynolds stresses. The resolution algorithm is presented in Figure 3.5.

The Reynolds stresses $\overline{u^2}, \overline{v^2}, \overline{uv}$ are calculated analytically from temporary values for the others unknowns using equations 3, 4 and 5:

\[
\begin{align*}
\overline{uv} &= \frac{num}{denum} \\
\text{where} &\quad num = [(M_2.P_1 - M_1.P_2).((N_1.M_3 - N_3.M_1) - (M_3.P_1 - M_1.P_3).((N_1.M_2 - N_2.M_1))] \\
&\quad denum = [(M_2.O_1 - M_1.O_2).((N_1.M_3 - N_3.M_1) - (M_3.O_1 - M_1.O_3).((N_1.M_2 - N_2.M_1))] \\
\overline{v^2} &= [(M_2.P_1 - M_1.P_2) - (O_1.M_2 - O_2.M_1).\overline{uv}] / (N_1.M_2 - N_2.M_1) \\
\overline{u^2} &= (P_1 - N_1.\overline{v^2} - O_1.\overline{uv}) / M_1
\end{align*}
\]
\[
M_1 = [D_1\overline{U} + F_1\overline{V}]^2
\]

with
\[
M_2 = [D_2\overline{U} + F_2\overline{V}]^2
\]
\[
M_3 = D_1D_2\overline{U}^2 + (D_1F_2 + D_2F_1)\overline{U}\overline{V} + F_1F_2\overline{V}^2
\]
\[
N_1 = [G_1\overline{U} + E_1\overline{V}]^2
\]
\[
N_2 = [G_2\overline{U} + E_2\overline{V}]^2
\]
\[
N_3 = E_1E_2\overline{U}^2 + (E_1G_2 + E_2G_1)\overline{U}\overline{V} + G_1G_2\overline{V}^2
\]
\[
O_1 = 2[D_1G_1\overline{U}^2 + (D_1E_1 + F_1G_1)\overline{U}\overline{V} + E_1F_1\overline{V}^2]
\]
\[
O_2 = 2[D_2G_2\overline{U}^2 + (D_2E_2 + F_2G_2)\overline{U}\overline{V} + E_2F_2\overline{V}^2]
\]
\[
O_3 = [(D_1G_2 + D_2G_1)\overline{U}^2 + (D_1E_2 + D_2E_1 + F_1G_2 + F_2G_1)\overline{U}\overline{V} + (E_1F_2 + E_2F_1)\overline{V}^2]
\]
\[
P_1 = \overline{F_1}^2 \cdot \frac{\nu_{\text{eff} 1}}{2}
\]
\[
P_2 = \overline{F_2}^2 \cdot \frac{\nu_{\text{eff} 2}}{2}
\]
\[
P_3 = 4\overline{F_1F_2} \cdot \frac{\nu_{\text{eff} 1} \cdot \nu_{\text{eff} 2}}{2}
\]

Likewise, \( \overline{w^2}, \overline{uw} \) are calculated from analytical relations derived from equations 6 and 7:
\[
\overline{w^2} = (N_5P_4 - N_4P_3)/(M_4N_5 - M_5N_4)
\]
\[
\overline{uw} = [P_4 - M_4\overline{w^2}] / N_4
\]

with
\[
M_4 = F_3^2 \overline{U}^2
\]
\[
M_5 = F_4^2 \overline{U}^2
\]
\[
N_4 = 2D_3F_3\overline{U}^2 + E_3F_3\overline{U}\overline{V}
\]
\[
N_5 = 2D_4F_4\overline{U}^2 + E_4F_4\overline{U}\overline{V}
\]

\[
P_4 = -D_3^2 \cdot \overline{U}^2 - E_3^2 \cdot \overline{V}^2 - (2D_3E_3 + E_3F_3)\overline{U}\overline{V} + 4F_3^2 \cdot \nu_{\text{eff} 3}^2
\]
\[
P_5 = -D_4^2 \cdot \overline{U}^2 - E_4^2 \cdot \overline{V}^2 - (2D_4E_4 + E_4F_4)\overline{U}\overline{V} + 4F_4^2 \cdot \nu_{\text{eff} 4}^2
\]
3.2 Data Reduction for Fourier Transform Calculation

3.2.1 Instantaneous Velocity Components Calculation

The previous method provides solution averaged in time but do not provide the instantaneous field for a three-dimensional flow. It is thus necessary to utilize a simpler but less accurate method in order to derive results in the frequency space.

The alternate method is based on several assumptions which do not influence the results in frequency space when viewed in a relative sense: (i) the instantaneous velocity vector remains in the plane of the two sensors. In other words, the flow is perfectly two-dimensional and the velocity component perpendicular to the plane of measurement is null. Consequently, the term containing the pitch coefficient in Jörgensen’s equation vanishes, (ii) the velocity stays within the planar region defined by the two wires and described in Figure 2.7.

The instantaneous effective velocities are processed the same way as in the former method from the output bridge voltages. The velocity components to be used for spectral analysis are found using the Jörgensen’s equation, which becomes under the previous assumptions:

\[
\begin{align*}
V_{\text{eff}1}^2 &= U_{n1}^2 + k_1^2 U_{t1}^2 \\
V_{\text{eff}2}^2 &= U_{n2}^2 + k_2^2 U_{t2}^2
\end{align*}
\]

where \( U_{n1} \) and \( U_{t1} \) (or \( U_{n2} \) and \( U_{t2} \)) are the normal and tangential components of the instantaneous velocity contained in the cone of validity with respect to wire 1 (or wire 2).

Since the angle between the two wires is equal to 45°, we have \( U_{n1} = U_{t2} \) and \( U_{t1} = U_{n2} \).
Initial values: $\overline{u^2} = \overline{v^2} = \overline{w^2} = \overline{uv} = \overline{uw} = \overline{vw} = 0$

Solve numerically for $\overline{U}$ and $\overline{V}$ using equations 1 and 2

Solve analytically for $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$ using equations 3, 4 and 5

Convergence criterion: 1% and 5% in relative errors for mean components and Reynolds stresses respectively

Solve analytically for $\overline{w^2}$ and $\overline{uw}$ using equations 6 and 7

Convergence criterion: 5% in relative errors for Reynolds stresses from two successive iterations

Final values of $\overline{U}, \overline{V}, \overline{u^2}, \overline{v^2}, \overline{w^2}, \overline{uv}, \overline{uw}$ corrected for temperature and velocity gradient with the assumptions that:

$\overline{W} = 0, \overline{vw} = 1/2.(\overline{uv} + \overline{uw})$

$terms \ in \ o(\overline{u^3}) \ are \ neglectable$

Figure 3.5: Algorithm followed during the 3-D data reduction process
The two instantaneous components of the velocity $U$ and $V$ are then obtained using the following relations:

\[
\begin{align*}
U_{i1} &= \frac{V_{eff1}^2 - k_1^2 \cdot V_{eff2}^2}{1 - k_1^2 \cdot k_2^2} \\
U_{n1} &= \frac{V_{eff2}^2 - k_2^2 \cdot V_{eff1}^2}{1 - k_1^2 \cdot k_2^2}
\end{align*}
\] and

\[
\begin{align*}
U &= \frac{U_{i1} + U_{n1}}{\sqrt{2}} \\
V &= \frac{U_{i1} - U_{n1}}{\sqrt{2}}
\end{align*}
\]

The formulas involve instantaneous quantities. The mean components, Reynolds tensor and fluctuating terms of higher orders are easily derived by performing arithmetic averages over the acquisition time. The results are biased by velocity gradients but are still useful for spectral information.

### 3.2.2 Fourier Transform Calculation

A fast Fourier transformation is applied on the instantaneous velocity field obtained. At each location in space, a Fourier transform is computed using 23 blocks of 10,000 measurements with a sampling rate of 50,000 Hz and a resolution of 5 Hz, from a sample of 32,768 points acquired during 0.65536 seconds at 50,000 Hz.
Chapter 4  Validation of the 3-D Data Reduction Method

In the present chapter the effects of the velocity gradient, angle deviation, three-dimensional flow and low turbulence assumption are quantified. In each case, results before and after correction are compared. Then, a comparison of the results obtained using the TSI IFA post-processor and the 3D dimensional method is presented. One should note that the quantities presented are mean quantities.

4.1 Effect of the Velocity Gradient Correction

Figure 4.1: Position of the X-probe wires with respect to the scan mesh for the orientations 0° and 90°
The measured effective velocities are biased by the effect of the velocity gradient. As mentioned earlier, the two films constituting the dual film X-probe are measuring at two different locations separated by a distance of $\delta \approx 0.94$ mm.

The measured effective velocities are corrected at each location in the flow for measurement with the probe orientated at $0^\circ$ and $90^\circ$. Corrections formulae are based on a first order interpolation:

$$\overline{V_{\text{eff}1}}^2(P)^C = \overline{V_{\text{eff}1}}^2(P) + (\overline{V_{\text{eff}1}}^2(r^{(n+1)}, \theta^{(n)}) - \overline{V_{\text{eff}1}}^2(r^{(n)}, \theta^{(n)})) \delta l(\delta, r^{(n+1)} - r^{(n)})$$

$$\overline{V_{\text{eff}1}}^2(P)^C = \overline{V_{\text{eff}1}}^2(P) + (\overline{V_{\text{eff}1}}^2(r^{(n+1)}, \theta^{(n)}) - \overline{V_{\text{eff}1}}^2(r^{(n)}, \theta^{(n)})) \delta l(\delta, r^{(n+1)} - r^{(n)})$$

for probe orientation $0^\circ$ (azimuthal correction), and

$$\overline{V_{\text{eff}1}}^2(P)^C = \overline{V_{\text{eff}1}}^2(P) + (\overline{V_{\text{eff}1}}^2(r^{(n)}, \theta^{(n)}) - \overline{V_{\text{eff}1}}^2(r^{(n)}, \theta^{(n-1)})) \delta l(\delta, r^{(n)} - \theta^{(n-1)})$$

$$\overline{V_{\text{eff}1}}^2(P)^C = \overline{V_{\text{eff}1}}^2(P) + (\overline{V_{\text{eff}1}}^2(r^{(n)}, \theta^{(n)}) - \overline{V_{\text{eff}1}}^2(r^{(n)}, \theta^{(n-1)})) \delta l(\delta, r^{(n)} - \theta^{(n-1)})$$

for probe orientation $90^\circ$ (radial correction), where $\overline{V_{\text{eff}1}}^2(P)^C, \overline{V_{\text{eff}1}}^2(P)$ are the corrected and uncorrected mean squared effective velocities respectively at point P for wire 1, $\overline{V_{\text{eff}1}}^2(P), \overline{V_{\text{eff}1}}^2(P)$ are the mean of the corrected and uncorrected fluctuating effective velocities squared respectively at point P for wire 1, $r$ and $\theta$ are defined in Figure 4.1. Similar formulas are derived for the wire 2.

First order corrections were preferred to second order ones, which introduce non-physical corrections in regions of high velocity gradients combined with velocities near the resolution limits.

The spatial resolution of the grid in the radial direction is in the range of 0.1 to 1 mm, depending on the region, and of the order of 5.0 mm in the azimuthal direction. The
radial correction is thus more accurate. A better accuracy could be reached by conducting measurements for additional values of $\theta$.

Corrected and uncorrected effective velocities are presented in Figures 4.2 and 4.3. $V_{\text{eff}^2}$ is the mean of the effective velocity squared seen by wire #1 at the first level in the side plane. The radial correction results in a translation of the profiles in the direction of increasing radius for wire 1 and in the opposite direction for wire 2, making

Figure 4.2: Comparison of the effective velocity profiles before and after velocity gradient correction for probe orientation 0°, $\theta=130°$. $y/D_{ho}$ is the radius from the center.
coincident the shear layer regions seen by wire 1 and 2. The relative error in the inner shear layer region at the location \( x/D_{ho} = 0.98 \) and \( y/D_{ho} = 0.4 \) (radial and azimuthal velocity gradient is about 2.5 and 1 m/s/mm respectively) is of the order of 10\% and 20\% for all the quantities related to orientation \( 0^\circ \) and \( 90^\circ \) respectively, whereas it is almost null (less than 1\%) at the center of the inner jet where the velocity gradient is almost null.

Figure 4.3: Comparison of the effective velocity profiles before and after velocity gradient correction for probe orientation \( 90^\circ \), \( \theta = 130^\circ \), \( y/D_{ho} \) is the radius
Figure 4.4 (a): Comparison of the velocity profiles before and after velocity gradient correction, $\theta = 130^\circ$, $y/D_{ho}$ is the radius from the center.
Figure 4.4 (b): Comparison of the velocity profiles before and after velocity gradient correction, $\theta = 130^\circ$, $y/D_{ho}$ is the radius from the center

The influence of this correction on the velocity calculus can be evaluated in Figure 4.4 presenting the velocity profiles obtained from the corrected and uncorrected effective velocities, using the processing method described in Chapter 3. At the same location, the relative error is less that 1% for $\bar{U}$ and is of 3% for $\bar{u}^2$ and $\bar{v}^2$. More significant relative error are observed for $\bar{V}$ (up to 75%), and $\bar{w}^2$, $\bar{uv}$ and $\bar{uw}$ (30% approximately).

4.2 Effect of the Angle Correction

As explained in section 3.1.4, misalignments between the direction of the probe $X_p$ and the vertical axis of the reference frame bias the calculation of the velocity, due to the bending of the probe holder or misalignments during the experiment. The correction is equivalent to a geometrical rotation of the velocity components using angles computed
at the center of the nozzle in the exit plane. It is needed to emphasize that the values of the angles $t_2$ and $t_3$ defined in Figure 4.5 cannot be determined this way. Since they were found by direct measurement on the experimental setting to be less than 1°, they were assumed to be zero.

Results obtained with and without angle correction are presented in Figure 4.6. In both cases, the velocity gradient correction is not applied. Consistently with the formula, the corrected mean component $\bar{V}$ vanishes at the center.

Figure 4.5: Definition of the deviation angle of the probe out of the plane of measurement for both orientations
4.3 Effects of the 3D-Flow Assumption

The effects of the 3D-flow assumption can be determined by comparing the results with the ones obtained using the same method but with the difference that $w^2$, $uw$ and $vw$ are set to zero. Results are presented in Figure 4.7. The relative error is less that 1% for all the components. The benefit of the 3D assumption is that it provides values for $w^2$ and $uw$ that obviously cannot be considered null.

4.4 Effects of the Low Turbulence Level Assumption

The TSI IFA 300 post-processor do not use the assumption that the turbulent level is low. Nevertheless, it assumes that the flow is two-dimensional. Consequently, the
Figure 4.7 (a): Comparison of the velocity profiles before and after 3-D correction, $\theta = 130^\circ$, $y/D_{ho}$ is the radius from the center
Figure 4.7 (b): Comparison of the velocity profiles before and after 3-D correction, \( \theta = 130^\circ \), \( y / D_h \) is the radius from the center

Figure 4.8 (a): Comparison of the velocity profiles with and without low turbulence level assumption, \( \theta = 130^\circ \), \( y / D_h \) is the radius from the center
effects of the low turbulence level assumption can be obtained by comparison between
the results from the post-processor and the results obtained with the 3D data reduction
method with the additional assumption that $\overline{w^2}$, $\overline{uw}$ and $\overline{vw}$ are equal to zero. Results
are presented in Figure 4.9 and are not corrected in angle.

The correction results in no significant change for the mean components (less than
1%) and results in relative errors of less than 10% for the Reynolds stresses in the outer
shear layer where the turbulence level $T_u = \sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}} / U_0$ is about 0.3.

4.5 Comparison with the TSI IFA Processor

We may recall that unlike the 3D data reduction method, the TSI IFA post-
processor does not correct for velocity gradient and that the flow is supposed to be purely
two-dimensional. Velocity profiles obtained from both ways are plotted in Figure 4.9.
The results from the TSI IFA post-processor are also corrected for the angle deviation.
Figure 4.9 (a): Comparison of the effective velocity profiles corrected for angle deviation with the TSI IFA processor and with the 3D reduction method ($y/D_{ho}$: radius)
Figure 4.9 (b): Comparison of the effective velocity profiles corrected for angle deviation with the TSI IFA processor and with the 3D reduction method ($y/D_{ho}$: radius)

Significant error is found for $\bar{V}$ (about 70% is the shear layer region). The correction is less important for the Reynolds stresses and not significant for $\bar{U}$. The computed $\bar{w}^2$, $\bar{uw}$ are found to be not zero.
Chapter 5 Passive Forcing on Coaxial Jets for Case $\lambda=0.5$

Bonnafous (2001) studied this low velocity ratio case and presented results for a large number of span wise locations for the side (corresponding to 120°) and the diagonal (165°) planes, providing information in vertical planes. Results are based on the assumption of a two-dimensional flow on these planes. This chapter is completing them by investigating the flow in horizontal planes for 5 span wise locations.

5.1 Time Space Results

5.1.1 Mean Streamwise Velocity

Time results obtained in this study are all derived using the 3-dimensional flow method described in chapters 3 and 4. The conditions are similar to the ones of Bonnafous (2001) experiments, the jet Reynolds number being about 16,000.

Mean streamwise velocity profiles are portrayed in Figure 5.1 for 5 different levels. The scaling factor $U_c=10.6$ m/s is the average of the mean streamwise nominal velocities in the middle of the inner and outer jet at the exit of the nozzle in the plane 120°. The middle shear layer in plane 165° (diagonal plane) shows a velocity deficit as it was observed by Bonnafous in the same conditions and also for the outer single square jet case. This is a local 3-dimensional phenomenon due to a separation region forming on the corner of the inner wall of the outer nozzle. It is located between planes 160° and 170° but not farther as we will see later in Figure 5.7. It disappears at 3.2 hydraulic diameters downstream. The slope of the mean streamwise velocity profile close to the exit in the outer jet is related to the nozzle shape, since it was found opposite by Bitting (1998) with another nozzle design.
Figure 5.1: Axial mean velocity evolution in the side and diagonal plane with respect to the radius $r/D_{ho}$, all at $Re_o=16,000$, $\lambda=0.5$, square coaxial nozzle
The wake caused by the inner lower-velocity jet lip has vanished at a distance of about 0.98 hydraulic diameters. The zones corresponding to the inner and outer jets are still distinct at $x/D_{ho} = 4.43$ (the profile is said to be “wake-like”) but eventually further from the exit, the profile loses this shape to become similar to the one of a single jet (“jet-like”) (not investigated further here).

5.1.2 Shear Layers Evolution

![Figure 5.2: Mean velocity contour plots and evolution of $Y_{01}$, $Y_{05}$ and $Y_{09}$ with respect to $x$ in the side (a) and diagonal (b) planes for the square coaxial jets, $Re_o = 16,000$, $\lambda = 0.5$ (Bonafous, 2001)](image)

Figure 5.2: Mean velocity contour plots and evolution of $Y_{01}$, $Y_{05}$ and $Y_{09}$ with respect to $x$ in the side (a) and diagonal (b) planes for the square coaxial jets, $Re_o = 16,000$, $\lambda = 0.5$ (Bonafous, 2001)
Figure 5.2 presents the contours of the mean streamwise velocity presented by Bonnafous for the square case in the side and diagonal planes. The evolution of the inner and outer shear layers are shown by mean of the traces $Y_{01}$, $Y_{05}$ and $Y_{09}$ defined as the streamwise locations where the mean streamwise velocity equals 10%, 50% and 90% of the shear layer velocity jump respectively. In addition, Figures 5.3 and 5.4 present the streamwise evolution of the shear layer thickness $\delta$ and of its derivative $d\delta/dx$.

The outer shear layer in the side plane of the square nozzle is wider than in the corner plane of the square nozzle and in the plane of the circular nozzle (see Figure 5.3) from the exit until 9 hydraulic diameters.

---

**Figure 5.3:** Streamwise evolution of the outer shear layer thickness $\delta$ (a) and its derivative $d\delta/dx$ (b) in the axisymmetric plane (CC) for the coaxial annular case and in the side (SC) and diagonal (SD) planes for the square coaxial jet $Re_o=16,000$, $\lambda=0.5$ (Bonnafous, 2001)
Figure 5.4: Streamwise evolution of the middle shear layer thickness $\delta$ (a) and its derivative $d\delta/dx$ (b) in the axisymmetric plane (CC) for the coaxial annular case and in the side (SC) and diagonal (SD) planes for the square coaxial jet $Re_\infty=16,000$, $\lambda=0.5$ (Bonnafous, 2001)

5.1.2.1 Outer Shear Layer

It was found from Figure 5.2 that the outer shear layer after a small distance from the exit spreads outwards for all the cases, noticing that it is more pronounced for the side plane of the square coaxial nozzle. This trend is confirmed in Figure 5.5 that shows the evolution of the derivative $dY_{05}/dx$. The negative values for the side plane indicate that close to the exit (before 0.5 hydraulic diameters) the shear layer develops inwards, and starts then moving faster outwards, reaching its fastest growth rate at 1.75 hydraulic diameters. At the corner, the outer shear layer moves inwards before 2 hydraulic diameters, slowing down as it progresses downstream. After 3 hydraulic diameters, it starts to move consistently outwards.

The evolution of the growth rate (Figure 5.3) shows that the size of the shear layer increases from the exit for all cases, showing a faster growth at the side until 5 hydraulic
diameters, the difference being more pronounced until 2 hydraulic diameters.

Given that the mixing is related to the shear layer growth rate, the mixing with the ambient air is likely to be more efficient at the side than at the corner of the square nozzle in the near field region.

5.1.2.2 Inner and Middle Shear Layers

Close to the nozzle exit (less than 0.5 hydraulic diameters), the inner and middle shear layers merge into a single shear layer that we refer to as the middle shear layer (MSL). Unlike for the outer shear layer, the trends of the latter are less pronounced.

![Figure 5.5](image)  
*Figure 5.5: Streamwise evolution of the derivative $dY_{05}/dx$ of the middle and outer shear layers in the side and axisymmetric planes (a) and in the diagonal and axisymmetric planes (b), $Re_o =16,000, \lambda =0.5$ (Bonafous, 2001)*
Figure 5.4 shows that the middle shear layer globally grows as we move downstream. For \( x \leq 1.3D_{ho} \), the shear layer is wider in the diagonal plane. It presents a local maximum at \( x = 0.5D_{ho} \). Unlike in the side plane, the growth rate in the diagonal plane grows at the exit, reaching a maximum at 0.2 hydraulic diameters. In both cases it decreases until 0.5 hydraulic diameters approximately and starts increasing again until \( 2.4D_{ho} \).

Furthermore, the shear layer moves inwards (see Figure 5.5) more significantly for the diagonal plane that for the side plane.

5.1.3 Axis Switching

Bonnafous observed (Figure 5.9) one axis-switching occurrence for the inner and outer jets. The cross over is more pronounced for the outer shear layer. It happens closer to the exit (at \( x = 1.5D_{ho} \)) than for the inner shear layer (at \( x = 3.5D_{ho} \) approximately). At \( x = 1.5D_{ho} \) the outer jet is likely to have lost its square shape, before coming back to it after 3 hydraulic diameters.

This is in full agreement with the results presented in Figure 5.6. The mean streamwise velocity at the exit nozzle at \( x = 0.05D_{ho} \) shows clearly the initial square shape of the inner and outer jets similar to the nozzle geometry. At \( x = 0.98D_{ho} \) the inner and outer jets change and at \( x = 1.48D_{ho} \), which according to Figure 5.6 is the location of the crossover, the outer shear layer looks approximately axisymmetric. Further downstream at \( x = 3.20D_{ho} \) and \( x = 4.43D_{ho} \), the outer shear layer regains a square geometry where the corner is located in the side plane (120°) and the flat side is located in the diagonal plane (165°).
Figure 5.6 (a): Mean streamwise and radial velocity contours in an octant at 5 levels, $Re_o=16,000$, $\lambda=0.5$, $U_c=10.6$ m/s
As indicated by the results from Figure 5.9, the axis switching in the inner jet cannot be fully observed. At the exit, the inner jet has clearly a square shape, which is absent at the other levels. If we look at the mean velocity profiles (Figure 5.1) and since the axis switching is less pronounced it is very unlikely to observe a clear square shape for the inner jet even further downstream.
Figure 5.7: Contours of the dimensionless Reynolds stresses $u'^2$, $v'^2$ and $w'^2$ in an octant at 5 levels, $Re_a = 16,000$, $\lambda = 0.5$, $U_a = 10.6$ m/s
Figure 5.8: Contours of the dimensionless Reynolds stresses $\bar{u}'\bar{v}'$ and $\bar{u}'\bar{w}'$ in an octant at 5 levels, $Re_o = 16,000$, $\lambda = 0.5$, $U_e = 10.6$ m/s
As discussed earlier, the outer shear layer spreads more in the side plane (120°), which is confirmed by the mean radius velocity that shows higher positive values, indicating that the flow grows outwards. One can notice that the maximum peak at the corner spreads at the 2\textsuperscript{nd} and 3\textsuperscript{rd} levels over different planes (from 150° to 110° and further). It remains strong at the 4\textsuperscript{th} level and even at the 5\textsuperscript{th}, narrowing in the azimuthal direction but spreading in the radial one. Similarly the mean radial velocity in the diagonal plane presents a positive peak much weaker that in the side plane (about 3 times weaker in amplitude). It is present from the 2\textsuperscript{nd} level, and spreads slightly preferably in the radius direction as we move downstream, moving towards the center of the nozzle.
5.2 Fourier Space Results

As explained in chapter 3.2, Fourier space results were obtained using a similar data reduction method as Bonnafous (2001), assuming that the flow is two-dimensional. In this section, results are obtained by applying a Fast Fourier transformation on the instantaneous streamwise velocity.

Spectra contours are plotted for frequencies in the range of 0 to 10,000 Hz. The scaling factor \( A u_0 \) is defined as the mean value of the amplitudes over the range of frequencies \([0; 10,000]\) Hz, computed on the nozzle axis at the exit of the nozzle \( (x = 0.05D_{ho}) \) from measurements in the plane \(120^\circ\). The scale is logarithmic.

It is important to note that at the level \(x=1\)mm, for several span wise locations the velocity vector went out of the measurement cone. The missing values have been interpolated from the neighboring points and then smoothed.

### 5.2.1 Streamwise Energy Distribution

The location of both shear layers is marked by the presence of a high fluctuation level. One can first notice that both inner and outer shear layers thicken as we move downstream the exit and that ultimately, the two layers merge into a single one.

The streamwise evolution of the preferred modes in the side and diagonal planes of the square jets is more completely observable in Bonnafous’ research, which presents spectra at appropriate locations that were not investigated in this study.

Near the exit at \(x = 0.05D_{ho}\), the spectrum shows clearly a maximum centered on the frequency \(f_i = 1750\) Hz in the inner and middle shear layers and consequently appearing as twin peaks. It was identified by Bonnafous as the inner (or middle) shear layer mode and was found equal to \(f_i = 1794\) Hz (relative difference of less than 5%.
probably due to differences in initial conditions). In contrast the outer shear layer is characterized by weaker disturbances. One can yet notice the presence of the frequency that was observed more downstream by Bonnafous and identified as the sub-harmonic $f_o/8$ of the outer shear layer mode $f_o=1327$ Hz almost not observable. The associated energy seems to be stronger in the outer shear layer, spreads through the outer jet and vanishes in the inner (or middle) shear layer.

At the next level for $x=0.98D_{ho}$ the turbulence level has increased in the outer shear layer and fluctuations have clearly amplified in the inner shear layer. The natural frequency $f_i$ is still dominating and has originated a frequency cascade: the harmonics are clearly present and decreasing in amplitude with increasing frequency. The side and neighboring planes, unlike close to the diagonal, also present a maximum though weaker, corresponding to the sub-harmonic at the frequency $f_i/2$, originating a secondary cascade. The outer shear layer is characterized by a high fluctuation level corresponding to a continuous range of frequencies spreading from 0 to 1000 Hz approximately. The peak associated to $f_o=1327$ Hz can be still identified as well as $f_o/8$ that spreads through the outer core until the inner shear layer.

Downstream at $x=1.48D_{ho}$, the cascade has weakened. The first mode is still clearly present with its first harmonic, whereas the sub-harmonic and its cascade have faded. In the outer shear layer, $f_o/8$ has not significantly amplified but spreads now in the inner jet.

At the two other levels, the inner shear layer does no longer present a peak related to the frequency $f_i$. 
The outer mode \( f_o/8 \) is dominating and has become more vigorous in the inner than in the outer shear layer.

### 5.2.2 Azimuthal Energy Distribution

Near the exit at \( x = 0.05D_{ho} \), the mode \( f_i \) presents a peak in the inner and middle shear layer in all the figures. This value does not change significantly from one vertical plane to the other but its amplitude gradually weakens in the region of the diagonal plane. The associated frequency cascade is more developed in the plane 150°. Similarly, in this region, the outer mode \( f_o/8 \) is stronger than in the side and diagonal planes.

At the second level, the diagonal is not anymore characterized by a lower fluctuation level but still shows a difference with the side plane. In the former the energy of the fluctuations is shared between the first mode and its cascade, along with the sub-harmonic and its cascade. As we approach the corner, the sub-harmonic and the associated cascade fade but the energy of the first mode and its cascade increases and spreads in the span wise direction. The outer shear layer spreads wider in the side plane. The mode \( f_o/8 \) is still predominant in the neighborhood of the plane 150° and fades in the diagonal planes. For \( x = 1.48D_{ho} \), the same observations apply: the first mode and its first harmonics are stronger in the corner, the mode \( f_o/8 \) predominates near the plane 150°. As we go downstream, the inner and outer shear layer mix, the trend being more pronounced at the corner.

### 5.3 Integral Entrainment

In order to conclude that the square shape of the nozzle presents more mixing advantages than the circular shape with similar hydraulic diameter, the flowrate is
Figure 5.10: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz with respect to the radius $y/D_{ho}$, in planes $120^\circ$, $130^\circ$, $140^\circ$, $150^\circ$, $160^\circ$, $165^\circ$ at level $x/D_{ho}=0.049$, $Re_\omega=16,000$, $\lambda=0.5$, $Au_\omega=9.5881 \times 10^{-4}$, square coaxial nozzle
Figure 5.11: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz with respect to the radius \( y/D_{ho} \), in planes 120°, 130°, 140°, 150°, 160°, 165° at level \( x/D_{ho} = 0.984 \), \( Re = 16,000 \), \( \lambda = 0.5 \), \( Au = 9.5881 \times 10^{-4} \), square coaxial nozzle
Figure 5.12: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz with respect to the radius $y/D_{ho}$, in planes $120^\circ$, $130^\circ$, $140^\circ$, $150^\circ$, $160^\circ$, $165^\circ$ at level $x/D_{ho}=1.476$, $Re_\infty=16,000$, $\lambda=0.5$, $Au_\infty=9.5881 \times 10^{-4}$, square coaxial nozzle
Figure 5.13: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz with respect to the radius $y/D_{ho}$, in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/Dh_o = 3.198$, $Re_o = 16,000$, $\lambda = 0.5$, $Au_o = 9.5881 \times 10^{-4}$, square coaxial nozzle.
Figure 5.14: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz with respect to the radius $y/D_{ho}$, in planes $120^\circ$, $130^\circ$, $140^\circ$, $150^\circ$, $160^\circ$, $165^\circ$ at level $x/Dh_{o} = 4.429$, $Re_{o} = 16,000$, $\lambda = 0.5$, $Au_{o} = 9.5881 \times 10^{-4}$, square coaxial nozzle.
Figure 5.15: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz with respect to the radius \( y / D_{ho} \), in planes 120°, 130°, 140°, 150°, 160°, 165° at level \( x / Dh_o = 0.049 \), \( Re_o = 16,000 \), \( \lambda = 0.5 \), \( Au_o = 9.5881 \times 10^{-4} \), square coaxial nozzle.
Figure 5.16: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz with respect to the radius $y/D_{ho}$, in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho} = 0.984$, $Re_o = 16,000$, $\lambda = 0.5$, $Au_o = 9.5881 \times 10^{-4}$, square coaxial nozzle
Figure 5.17: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz with respect to the radius $y/D_h$, in planes $120^\circ$, $130^\circ$, $140^\circ$, $150^\circ$, $160^\circ$, $165^\circ$ at level $x/D_h=1.476$, $Re_o=16,000$, $\lambda=0.5$, $Au_o=9.5881\times10^{-4}$, square coaxial nozzle.
Figure 5.18: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz with respect to the radius $y/D_{ho}$, in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho}=3.198$, $Re_o=16,000$, $\lambda=0.5$, $Au_o=9.5881\times10^{-4}$, square coaxial nozzle
Figure 5.19: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz with respect to the radius $y/D_{ho}$, in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/Dh_o = 4.429$, $Re_o = 16,000$, $\Lambda = 0.5$, $Au_o = 9.5881 \times 10^{-4}$, square coaxial nozzle
integrated over the surface where measurements were conducted, which represents an octant of the full horizontal plane. At the initial level, and at the 4\textsuperscript{th} and 5\textsuperscript{th} levels where it is assumed that one axis-switching occurrence is completed, the flowrate is integrated over the octant and multiplied by 8. This calculus is valid since the profile of the mean streamwise component is supposed to be symmetrical with respect to the y and z axes.

The flowrate \( Q \) is defined as: \( Q = \int_0^{2\pi} \int_0^R \overline{U} \cdot dA \), where \( dA \) is the unit area, \( \overline{U} \) is the mean streamwise velocity component and \( R_{5\%} \) is the radius from the center at which the mean velocity value in the outer shear layer becomes inferior to 5\% of the maximum velocity encountered in the plane.

Table 5.1: Integral flowrate over a horizontal plane, for three downstream locations, \( \lambda = 0.5 \), circular and coaxial nozzle

<table>
<thead>
<tr>
<th>( \lambda = 0.5 )</th>
<th>Integral flowrate Q (m(^3)/s) over the 2 jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular coaxial nozzle</td>
<td>Square coaxial nozzle</td>
</tr>
<tr>
<td>X1( -x / D_{ho} = 0.05 )</td>
<td>1.23( \times 10^{-2} )</td>
</tr>
<tr>
<td>X4( -x / D_{ho} = 3.20 )</td>
<td>1.85( \times 10^{-2} )</td>
</tr>
<tr>
<td>X5( -x / D_{ho} = 4.43 )</td>
<td>2.17( \times 10^{-2} )</td>
</tr>
</tbody>
</table>

Thus, \[
\begin{align*}
\left( \frac{dQ}{dX} \right)_{\text{square}} &= \frac{(Q_4 - Q_1)/(X_4 - X_1)}{(Q_4 - Q_1)/(X_4 - X_1)} = 1.33 \\
\left( \frac{dQ}{dX} \right)_{\text{circular}} &= \frac{(Q_5 - Q_1)/(X_5 - X_1)}{(Q_5 - Q_1)/(X_5 - X_1)} = 1.25
\end{align*}
\]
At \( x/D_{ho} = 3.20 \), the global entrainment (or flowrate growth rate) appears to be 1.3 times higher for the square than for the circular coaxial nozzle. Consequently, the square shape clearly results in local mixing enhancement in the near field compared to the circular geometry.

The flowrate was also integrated over the inner jet to give an idea of the mixing occurring in the inner shear layer. The integration is carried over the area of the inner jet.

Table 5.2: Integral flowrate over the inner jet, for two downstream locations, \( \lambda = 0.5 \), circular and coaxial nozzle

<table>
<thead>
<tr>
<th>( X_1 - x/D_{ho} )</th>
<th>Circular coaxial nozzle</th>
<th>Square coaxial nozzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>1.26*10^{-3}</td>
<td>1.68*10^{-3}</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>1.89*10^{-3}</td>
<td>2.84*10^{-3}</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>2.10*10^{-3}</td>
<td>2.96*10^{-3}</td>
</tr>
</tbody>
</table>
Thus, \[
\frac{(dQ/dX)_{\text{square}}}{(dQ/dX)_{\text{circular, inner jet}}} \approx 1.84 \]
\[
\frac{(dQ/dX)_{\text{square}}}{(dQ/dX)_{\text{circular, inner jet}}} \approx 1.53
\]
This suggests that the mixing between the inner and outer jet in the near field region is more efficient than for the circular case.
Chapter 6 Passive Forcing on Coaxial Jets for Case $\lambda = 1.5$

Piffaut (2003) investigated this case for a large number of stream-wise locations for the side (corresponding to $120^\circ$) and the diagonal ($165^\circ$) planes. Again, results are based on the assumption of a two-dimensional flow. This chapter presents information in additional vertical planes for 5 stream-wise locations.

6.1 Time Space Results

6.1.1 Mean Streamwise Velocity

Results in this chapter were derived in the same manner as in chapter 5. The initial conditions are presented in Table 6.1 The Reynolds number is about $Re_\alpha = 16,000$.

Table 6.1: Initial conditions of the jets

<table>
<thead>
<tr>
<th></th>
<th>Coaxial circular jet (Choy, 2001)</th>
<th>Coaxial square jet (side) (Piffaut, 2003)</th>
<th>Coaxial square jet (diagonal) (Piffaut, 2003)</th>
<th>Coaxial square jet (side) (this study)</th>
<th>Coaxial square jet (diagonal) (this study)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ISL OSL ISL OSL ISL OSL ISL OSL ISL OSL</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial shear layer thickness $\delta_\tau$ (mm)</td>
<td>0.64 0.81 0.56 0.73 0.80 1.45 0.55 0.70 0.65 1.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The evolution of the mean streamwise velocity profiles is presented in Figure 6.1 for 5 different levels. The average of the inner and outer jet streamwise nominal velocities at the exit equals $U_\tau = 17.73$ m/s in the side plane.

The flow shows similar characteristics as described for the low velocity ratio case. In the outer core region in the diagonal plane, the velocity deficit can be observed close to the inner shear layer until a few hydraulic diameters downstream of the nozzle exit.
Again, the inner and middle shear layers form downstream a single shear layer called the inner shear layer (ISL).

Outer and inner core regions are initially distinctly defined. The flow pattern eventually becomes similar to the one observed in the case of a single jet, it is then said to be “jet-like”.

### 6.1.2 Shear Layers Evolution

The mean streamwise velocity contours obtained (Piffaut, 2003) for the square case in the side and diagonal planes are shown in Figure 6.2. The inner and outer shear layers are identified through the mean of the traces $Y_{01}$, $Y_{05}$ and $Y_{09}$ as defined earlier, which derivatives $dY_{01}/dx$, $dY_{05}/dx$, $dY_{09}/dx$ are presented in Figures 6.3 and 6.4 respectively, along with the growth rate of the inner and outer shear layer $d\delta/dx$. Unlike the previous case, the evolution is shown until up to 4 outer hydraulic diameters downstream from the exit, since farther downstream the outer and inner shear regions merge.

#### 6.1.2.1 Outer Shear Layer

As for the previous case, the initial growth rate of the outer shear layer is more important in the side plane than in the diagonal plane. Indeed, the outer shear layer grows significantly close to the exit, faster and earlier in the side than in the diagonal plane, with maximum values located in the region between 0.5 and 1.5 outer hydraulic diameters. This trend in the initial region is very similar for low and high velocity ratio cases and stops at around 2 outer hydraulic diameters where the difference between both planes is less pronounced. Farther downstream, the growth rate appears to be more similar.
Figure 6.1: Axial mean velocity evolution for the coaxial jet in the side and diagonal plane of the square coaxial jets, all at $Re_o = 16,000$, $\lambda = 1.5$
Figure 6.2: Mean velocity contour plots in the side (SC) (a) and diagonal (SD) (b) planes for the square coaxial jets, $Re_o=16,000$, $\lambda=1.5$ (Piffaut, 2003).

Figure 6.3: Streamwise evolution of $d\delta/dx$ (a), $dY_{o1}/dx$ (b), $dY_{o5}/dx$ (c), $dY_{o9}/dx$ (d) in the axisymmetric plane (CC) for the coaxial annular case and in the side (SC) and diagonal (SD) planes for the square coaxial jets $Re_o=16,000$, $\lambda=1.5$, outer shear layer (OSL) (Piffaut, 2003).
Figure 6.4: Streamwise evolution of $d\delta / dx$ (a), $dY_{o1} / dx$ (b), $dY_{o5} / dx$ (c), $dY_{o9} / dx$ (d) in the axisymmetric plane (CC) for the coaxial annular case and in the side (SC) and diagonal (SD) planes for the square coaxial jets $Re_o=16,000$, $\lambda=1.5$, inner shear layer (ISL) (Piffaut, 2003)

Moreover, the evolution of the half-width growth in the outer shear layer appears not to be dependent on the velocity ratio close to the exit. As for the low velocity ratio case, the outer shear layer develops downstream the location $x = 0.5D_{ho}$ outwards the nozzle center in the side plane only, with a growth rate gradually increasing, reaching a maximum at 1.7 outer hydraulic diameters. It then starts to gradually decrease. On the other hand, in the side plane, the outer shear layer develops towards the center of the inner jet, faster close to the exit, and then slower as the distance from the exit increases. At $x = 2D_{ho}$, the outer shear layer does not show any growth in any specific direction but ultimately it starts as in the side plane to expand outwards. Again, the evolution in the diagonal plane is very similar to the one for the other case.
6.1.2.2 Inner and Middle Shear Layers

Resemblance between low and high velocity ratio cases is less pronounced for the inner shear layer, suggesting that the inner shear region is sensitive to the velocity ratio, unlike the outer shear region. For \( \lambda = 1.5 \), after one outer hydraulic diameter, the growth rate of the inner shear layer width is positive and more significant in the side plane than in the diagonal one as compared to the outer shear layer. After 3.2 outer hydraulic diameters, the trend changes. The evolution of the velocity half-width growth shows again that the inner shear layer develops outwards in the side plane and in the opposite direction in the diagonal plane. This trend is conserved until up to 6 outer hydraulic diameters.

6.1.3 Axis Switching

The event of the axis switching is clear for both the inner and outer shear layers. As for the low velocity ratio case, one occurrence only is observed since one crossover only between the side and diagonal planes is visible in the evolution of the half-width locations (Figure 6.5). The occurrence of the axis switching is again more evident in the outer shear layer than in the inner shear layer, as the difference between the locations of the half-width is more significant for the outer shear layer. Clearly, the axis-switching event present in the outer shear layer occurs close to the exit at 1.5 outer hydraulic diameters. A similar distance was found for the case \( \lambda = 0.5 \), suggesting that the early development of the outer shear layer does not depend on the velocity ratio. However, beyond this initial region, the flow appears to be more sensitive to this parameter.

On the other hand, the crossover in the inner shear layer occurs earlier as in the other case (\( \lambda = 0.5 \)), at about \( x = 1.5 D_{ho} \), which is also the distance found for the outer
shear layer. Consequently, the development of the inner shear layer in the initial region is sensitive to the velocity ratio, and becomes more pronounced with increasing velocity ratio even if it remains less vigorous as in the outer regions in the cases presented.

Figure 6.5: Streamwise evolution of the half-width location in the side (SC) and diagonal (SD) planes of the inner (ISL) and outer (OSL) shear layers, $\text{Re}_o=16,000$, $\lambda=1.5$ (Piffaut, 2003)

The mean velocity contours (Figure 6.6) show several trends in agreement with the ones discussed above. It should be noted that the contours presented for the first level show discontinuities that are not physical. The resolution in the azimuthal direction, i.e. the number of vertical planes of measurements, is too low for the interpolation to be
Figure 6.6 (a): Mean streamwise and radial velocity contours in an octant at 5 levels, 
\( \text{Re}_a =16,000, \ \lambda =1.5, \ U_e =17.73 \text{ m/s} \)
Figure 6.6 (b): Mean streamwise and radial velocity contours in an octant at 5 levels, $\text{Re}_o = 16,000$, $\lambda = 1.5$, $U_c = 17.73$ m/s

On the contrary, the inner shear layer show that the evolution towards a complete axis switching occurrence is faster and more pronounced in the case $\lambda = 1.5$. The square flow pattern is also clearly visible in the Reynolds stresses contours depicted in Figures 6.7 and 6.8, which are showing results close to the one presented in the previous case.
Figure 6.7: Contours of the dimensionless Reynolds stresses $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$ in an octant at 5 levels, $Re_o=16,000$, $\lambda =1.5$, $U_c=17.73$ m/s
Figure 6.8: Contours of the dimensionless Reynolds stresses $\overline{u'v'}$ and $\overline{u'w'}$ in an octant at 5 levels, $Re_\lambda =16,000$, $\lambda =1.5$, $U_\infty =17.73$ m/s
accurate in the shear layer regions. Nevertheless, the other levels are very acceptable. The axis-switching occurrence is vigorous in the outer shear layer is very well defined, as the square shape can be observed at the 1st level with the corner in the diagonal plane, and in the 4th and 5th levels with the corner in the side plane. On the contrary, the square shape is less evident downstream the exit for the inner shear region, but one can see that the shear layer is located farther from the nozzle center in the side plane, suggesting an axis switching occurrence. Again, the outer shear layer widens more in the side plane and spreads outwards. If we look at the radial component of the mean velocity, one can see that the region close to the diagonal plane is characterized by negative values whereas they are positive in the side plane. By comparing the low and high velocity ratio cases, one can see that the outer shear layers have a very similar behavior, suggesting again that the outer shear region do not depend strongly on the velocity ratio.

6.2 Fourier Space Results

Results are derived in a similar way as in the previous case. The same observations apply.

6.2.1 Streamwise Energy Distribution

Close to the exit, the inner and middle shear layers show a dominance of the preferred mode $f_i=3550$ Hz approximately. This is in agreement with the value presented by Piffaut (2003) ($f_i=3700$ Hz). The turbulence level is lower in the outer shear layer. At a lower frequency scale, a frequency $f=68$ Hz is spreading through the inner jet core, dominating the frequency $f_o/8=155$ Hz (found equal to 165 Hz by Piffaut) that spreads over the outer core region.
Figure 6.9: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho}=0.049, \text{Re}_{ho}=16,000, \lambda=1.5, Au_{ho}=2.3830\times10^{-3}$, square coaxial jets
Figure 6.10: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho} = 0.787$, $Re_{ho} = 16,000$, $\lambda = 1.5$, $Au_{ho} = 2.383 \times 10^{-3}$, square coaxial jets
Figure 6.11: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/Dh_o=1.476$, $Re_o=16,000$, $\lambda=1.5$, $Au_o=2.3830 \times 10^{-3}$, square coaxial jets
Figure 6.12: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/Dh_o = 2.805$, $Re_o = 16,000$, $\lambda = 1.5$, $Au_o = 2.3830 \times 10^{-3}$, square coaxial jets.
Figure 6.13: Contours of streamwise velocity spectra for the frequency range [0-1000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho} = 7.381$, $Re = 16,000$, $\lambda = 1.5$, $Au = 2.3830 \times 10^{-3}$, square coaxial jets.
Figure 6.14: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz, with respect to the radius $y/D_h_o$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_h_o =0.049$, $Re_o=16,000$, $\lambda=1.5$, $Au_o=2.3830 \times 10^{-3}$, square coaxial jets
Figure 6.15: Contours of streamwise velocity spectra for the frequency range \([0-10000]\) Hz, with respect to the radius \(y/D_{ho}\) in planes \(120^\circ, 130^\circ, 140^\circ, 150^\circ, 160^\circ, 165^\circ\) at level \(x/D_h=0.787\), \(Re=16,000\), \(\lambda=1.5\), \(Au_z=2.383\times10^{-3}\), square coaxial jets
Figure 6.16: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz, with respect to the radius $y/D_{ho}$ in planes $120^\circ$, $130^\circ$, $140^\circ$, $150^\circ$, $160^\circ$, $165^\circ$ at level $x/Dh_o=1.476$, $Re_o=16,000$, $\lambda=1.5$, $Au_o=2.3830\times10^{-3}$, square coaxial jets
Figure 6.17: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho}=2.805$, $Re_{ho}=16000$, $\lambda=1.5$, $Au_{ho}=2.3830 \times 10^{-3}$, square coaxial jets
Figure 6.18: Contours of streamwise velocity spectra for the frequency range [0-10000] Hz, with respect to the radius $y/D_{ho}$ in planes 120°, 130°, 140°, 150°, 160°, 165° at level $x/D_{ho}=7.381$, $Re_{ho}=16,000$, $\lambda=1.5$, $Au_{ho}=2.383 \times 10^{-3}$, square coaxial jets
At \( x = 0.79 D_{ho} \), the first harmonic \( 2f_i \) is present in the inner shear layer, along with the sub-harmonic \( f_i / 2 \) and its harmonic cascade. In the outer shear layer, the peak observed by Piffaut corresponding to the frequency \( f_o = 1350 \text{ Hz} \) is weaker in our case but still visible. The low frequency peaks observed at the first level are still present, \( f_o / 8 = 155 \text{ Hz} \) being amplified.

Farther downstream, the harmonics and sub-harmonics of \( f_i \) do not appear clearly anymore. The outer shear layer frequency \( f_o / 8 = 155 \text{ Hz} \) is dominating the frequency \( f = 68 \text{ Hz} \) and spreads through the inner and outer jets.

### 6.2.2 Azimuthal Energy Distribution

The evolution of the modes with the angle is less marked as for the case \( \lambda = 0.5 \). At the first level, as for the previous case, the inner preferred mode is present at all angles, but becomes weaker as we approach the corner. The first harmonic is observable in the region close to the side of the nozzle. The frequency \( f = 68 \text{ Hz} \) is present with the same amplitude in every plane. On the contrary, \( f_o / 8 \) is stronger in the region between the side and diagonal planes (around 150°). At the following levels, the frequency peaks appear more clearly for angles close to 150°.

### 6.3 Integral Entrainment

Flowrates are integrated in the same fashion as for the low velocity ratio case, for level 1 and 4 (results in Table 6.2).

\[
\left. \frac{dQ}{dX} \right|_{square} \approx \left. \frac{(Q_4 - Q_1)/(X_4 - X_1)}{Q_4 - Q_1}/(X_4 - X_1) \right|_{circular} = 2.57 \text{. Close to 3 outer hydraulic diameters downstream from the nozzle exit, the entrainment for the square coaxial nozzle}
\]
Table 6.2: Integral flowrate over a horizontal plane, for three downstream locations, $\lambda = 1.5$, circular and coaxial nozzle

<table>
<thead>
<tr>
<th>$\lambda = 1.5$</th>
<th>Integral flowrate Q (m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Circular coaxial nozzle</td>
</tr>
<tr>
<td>X1- $x/D_{ho} = 0.05$</td>
<td>1.51*10^{-2}</td>
</tr>
<tr>
<td>X4- $x/D_{ho} = \begin{cases} 2.95 \ (circular) \ 2.80 \ (square) \end{cases}$</td>
<td>2.06*10^{-2}</td>
</tr>
</tbody>
</table>

equals 2.57 times the one of the circular nozzle. Consequently, the square shape obviously increases the flow entrainment and therefore the mixing in the near field flow. The entrainment is 2 times higher for $\lambda = 1.5$ than for $\lambda = 0.5$. One should note that it was computed closer to the nozzle exit for the high velocity ratio case (level X4 corresponds for the case $\lambda = 0.5$ to $x/D_{ho} = 3.20$), where the mixing is possibly more important.
Chapter 7 Conclusion

The mean flow of the square coaxial nozzle has been investigated at a low outer-jet Reynolds number of 16,000, for two different inner to outer velocity ratios $\lambda = 0.5, 1.5$. The results obtained are completing previous studies on the same nozzle with a similar Reynolds number accomplished by Bonnafous (2001) and Piffaut (2003) for the low and high velocity ratio cases respectively. For the first time, mean velocity components and Reynolds stresses were obtained in planes other than the side and diagonal already studied, with the mean of a processing technique adapted to measurements in 3-dimensional flows. This method was developed to provide the velocity field in a flow characterized by regions of high velocity gradients. Measurements were achieved with the technique of constant temperature anemometry using a dual-film X-probe orientated in two positions with respect to the flow. The processing technique was found to be accurate for a turbulence level of up to 30% and substantially correct for the velocity gradient effect.

Axis switching has been achieved in both cases and the evolution of the square shape of the flow was visualized in five different horizontal planes, showing evidence of the 45-degree axis switching. Again the phenomenon was more evident in the outer than in the inner shear region even for the high velocity ratio case where the crossover occurred closer from the nozzle exit than for the low ratio case.

A frequency analysis was also conducted using a two-dimensional flow assumption, in order to visualize the evolution of the modes in horizontal planes. Frequency cascades were observed in the inner shear layer.
The mean velocity field was used to calculate the global entrainment over appropriate horizontal surfaces for the square coaxial nozzle. A comparison was done with results obtained by Choy (2001) with a coaxial nozzle of circular geometry with equal hydraulic diameters with similar initial conditions. The global growth rate of the integrated entrainment was found to be 1.30 and 2.57 times higher with the square than the circular coaxial nozzle with $\lambda$ equal to 0.5 and 1.5 respectively, providing the evidence of local mixing enhancement in the near field region. In the case of the low velocity ratio the mixing enhancement factor for the inner nozzle was found to be 1.89. Such advantages can be applicable to gas turbine combustors for mixing purpose, with appropriately designed and scaled coaxial nozzles.

Further work will complete the present by providing a full three dimensional map of the flow and calculating the mixing enhancement factor for a large number of velocity ratios and Reynolds numbers. The processing method will also be used on one hand to show the evidence of multiple axis-switching occurrences with a single jet, on the other hand to study the effect of active forcing for mixing enhancement, using acoustic excitations at appropriate frequencies.
References


Bruun, H. H., Hot-Wire Anemometry, Principles and Signal Analysis, 1995 p93-94


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Vita

Thomas Lagarde was born in Toulouse, France, from the union of Marie-Elisabeth and Jacques Lagarde. His life started very early at 6:50 A.M in the morning of the 18th of March 1981. He spent his childhood with his sister and two brothers in L’Isle-Jourdain, Gers, in the land of D’Artagnan, known as the noble and valorous musketeer of King Louis XIII in the famous novel of Alexandre Dumas.

He left his homeland to obtain his Baccalauréat in the Lycée Pierre de Fermat in Toulouse during the last summer of the 20th century and decided to experience the education “à la française” during two years in the Lycée Saliège of post secondary program leading to competitive examination to “Grandes Ecoles”.

He entered L’Ecole Nationale Supérieure d’Ingénieurs de Constructions Aéronautiques, and was sent after two years over seas in 2003 to complete a Master of Science in Mechanical Engineering degree in the Louisiana State University under the supervision of Dr. Dimitris E. Nikitopoulos in the Mechanical Engineering Department. He submitted his research entitled 3-Dimensional Flow Measurements in Square Coaxial Jets to be graduated in December 2005 and received the engineering diploma from his French school the same year.