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Dynamic econometric modeling of the U.S. wheat grain market

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DYNAMIC ECONOMETRIC MODELING OF THE U.S. WHEAT GRAIN MARKET

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ABSTRACT

Structural-time series models have not gained much ground in commodity market modeling despite the overwhelming popularity of time series approaches in forecasting and dynamic analyses. This dissertation contributes by applying developments in seasonal cointegration and structural-time series analysis (e.g., Zellner and Palm (1974); Hsiao (1997); Lee (1992); Franses and Kunst (1999); Ghysels and Osborn, 2001) to the study of agricultural commodity markets. The focus is on three research themes. The first theme investigates the role of cointegration and seasonal cointegration for market data, an issue considered timely because most applications assume deterministic seasonal components. The second issue breaks new ground in agricultural commodity modeling by introducing a new dynamic simultaneous equation model (DSEM) that accounts for seasonal cointegration. Lastly, the research compares the out-of-sample forecasting performance and impulse responses of four multi-equation models for the U.S. wheat market. The forecasting comparisons apply recent developments on testing for differences in mean-squared-errors.

The study adopts a structural model for the U.S. wheat market and estimates four econometric specifications: a vector error-correction model without seasonal cointegration (VECM), a VECM with seasonal cointegration (SVECM), a DSEM with cointegration (CDSEM), and a DSEM with seasonal cointegration (SCDSEM). The conclusions may be summarized as follows. First, quarterly data in the U.S. wheat market (1975:03-1999:04) have seasonal unit roots, therefore, a VECM or DSEM should be specified. Second, in a forecasting context, seasonally cointegrated VECMs perform uniformly better that their nonseasonal counterpart. DSEM with seasonal cointegration, however, perform better than VECMs at longer forecast horizons. Lastly, the impulse response analysis and dynamic multiplier comparisons lead to one salient conclusion, omission of seasonal cointegration components when significant generates unexpected response functions and dynamic multipliers.

Of particular interest for future research is an assessment of the small sample properties of impulse response functions for structural-time series models with seasonal cointegration. From a more pure economic perspective, a similar structural-time series analysis to other agricultural markets seems timely given the new finding that these models may outperform other multiple time series models that are often used in empirical work.

CHAPTER 1

INTRODUCTION

Recent changes in U.S. agricultural policy have emphasized the development of a more market oriented agricultural sector. Prior to the enactment of the Federal Agriculture Improvements and Reform (FAIR) Act of 1996, federal agricultural subsidies were pegged to market price volatility. Government payments to producers increased when commodity prices were depressed, and decreased when commodity prices rebounded. The FAIR Act changed that fundamental relationship by decoupling U.S. agricultural markets from this traditional subsidy system. More over, the recently passed Farm Security and Rural Investment Act of 2002 (FSRI Act) reauthorize through 2007 the trade programs designed in the 1996 farm bill to develop and expand commercial outlets for U.S. commodities, although target prices were reinstated in the form of counter-cyclical payments. The FSRI Act also introduces new programs to address nontariff barriers to U.S. exports, the provision of information to assist exporters, and the preparation of a long-range agricultural trade strategy that identifies export growth opportunities.

These policy changes pose fundamental challenges and opportunities to economists who formulate commodity models to generate information useful to grain market agents. Academic and private institutions working in market modeling research could be very instrumental in providing accurate and timely market information for various market participants. Farmers and agribusiness, for example, benefit from economic information because profitability and survival in the grain industry substantially depends on having a good understanding of market trends and changes. Policy makers also benefit because they usually need price and production projections to better assess the impact of actual and alternative policies on agriculture. The uncertainty surrounding new agricultural policies also represents an exceptional opportunity for commodity modelers to provide timely information to policy discussions.

Various questions arise regarding the quality of grain market information generated from existing econometric models. For example, the high grain prices in spring 1996 were not forecasted accurately the previous summer because the relevant levels of the supply and demand factors were not accurately forecasted either. Actual crop size was smaller than expected, and use of this crop turned out to be larger than expected. Thus, ending stocks for the 1995-1996 crop year for grains were small (Tomek, 1997). In trying to address these problems, economic theory of commodity markets may help explain the dynamic nature of these markets and why changes occurring in the markets are gradual, with responses taking time to stabilize (Garcia and Leuthold, 1997).

Parallel to the changes in the agricultural policy environment, recent developments in the econometrics literature offer a means of improving the specification and estimation of existing models. Some recent advances in time series econometrics are appealing for empirical commodity modeling because they bring together economic theory with data properties coherently. Thus, these new techniques may contribute to the development of market econometric models that better explain the short and long run dynamics of market equilibrium (Hsiao, 1997a, 1997b; Choi and Phillips, 1997) in a theory-data-coherent framework for forecasting and simulation analyses.

Early in the history of commodity modeling, economic theory was the backbone of model specification. These models played an important role in structural analyses and were formulated as simultaneous equation models (SEMs). In spite of their economic appeal, their historical forecasting performance was poor, giving way to the use of time series models (TSM). TSMs for forecasting, however, have been accompanied by much criticism because of their "ad hoc" nature, and for decades, after the introduction of Box-Jenkins models, researchers have been making progress on the marriage between the SEMs and TSMs philosophies. These two types of approaches were known in other fields to have good forecasting performance. Garcia and Leuthold (1997) and Tomek (1997) provide a comprehensive review of commodity models and their history. Efforts to expand and improve SEMs are continual and often driven by the availability of more extensive and accurate data, by the introduction of new methods to capture either market structure or data properties, and by improved computers and software that facilitate the computational aspects of modeling. However, recent studies still find that fairly simple time-series models, with a limited basis in economic theory, can outperform SEMs forecasts and simulation results (Tomek, 1997). How can this be explained? Is there a way to combine SEMs and time-series models to capitalize on the advantages of each?

In searching for possible responses to these and other related questions, a critical review of the econometric assumptions of well-known SEMs may shed some light. The Food and Agricultural Policy Research Center (FAPRI) or the U.S. Department of Agriculture (USDA) wheat models provide good examples of SEMs (see for example Devadoss *et al*., 1993; Bailey, 1989). A distinguishing feature of SEMs is the endogenous and exogenous causal flow. Typically, SEMs are estimated by either two-stage (2SLS) or three-state least squares (3SLS). An implicit assumption in adopting these estimation techniques has been that 2SLS/3SLS work well for a wide range of time series data. Therefore, nonstationarity[1] of commodity market time series has been of little concern. This implicit assumption has been questioned and new results have been introduced in the time-series literature to uncover how inference, forecasting, and dynamic simulation may be impacted by the presence of nonstationarity.

It is also known that markets often depart from the supply-demand equilibrium suggested by the theory of pure competition. From a time-series perspective, the deviations from equilibrium as revealed by available empirical data, particularly of the nonstationary type, have been more complex to model than initially suspected. It has been proven that when variables are nonstationary, such as in the "random walk" behavior, it is still possible to observe long-run market equilibrium. Also, some theoretical efforts that deal with nonstationary data in SEMs may be found in the literature. The seminal papers by Hsiao

[1] Statistically speaking, this concept distinguishes between stationary and nonstationary time series. Let y_t be a stochastic process of dimension $n \times 1$. Then y_t is said to be *covariance stationary*, simply referred as *stationary* for our purposes, if its first and second moment are time invariant. In other words, a stochastic process y_t is stationary if (i) $E(y_t) = m$ for all *t*, and (ii) $E\{(y_t - m)(y_{t-h} - m)'\} = \Gamma_y(h) = \Gamma_y(h)'$ for $h = 0,1,\ldots,$ and all *t*. Condition (i) means that all y_t have the same finite mean vector m and condition (ii) requires that the autocovariances of the process do not depend on *t* but just on the time period *h* the two vectors y_t and y_{t-h} are apart. By this definition, a *white noise* or *innovation process* u_t, which satisfies $E(u_t) = 0$, $E(u_t u_t') = \Sigma_u$, and $E(u_t u_s') = 0$ for $s \neq t$, is a stationary process. In the context of this study a time series is defined as *nonstationary* if it is integrated of order one, denoted as $I(1)$. A series y_t is $I(1)$ if its first difference is stationary. One common example of a $I(1)$ process is the *pure random walk* process, defined as $y_t = y_{t-1} + u_t$, where u_t is a stochastic term. Finally, $I(0)$ will denote stationarity.

(1997a, 1997b) and Choi and Phillips (1997), for instance, provide estimators of a SEM specification that accounts for the nonstationary property of the time-series involved.

On the other hand, the data available for the U.S. wheat market possess special challenges since calls for modeling procedures that must take into consideration the seasonal (quarterly) nature of the time series. Although seasonal nonstationarity, more specifically, seasonal integration and seasonal cointegration[2] have been profusely studied (Hylleberg *et al.*, 1990; Lee, 1992; Johansen and Schaumburg, 1999), the existing structural U.S. wheat market models found in the literature disregard the seasonal stochastic properties of the time series involved.

By approaching the U.S. wheat market, this research pursues to evaluate the accuracy of forecasts and simulation information constructed with a new generation of models that blend the traditional way of modeling agricultural commodity markets with recent advances in modeling the various forms that nonstationarity may adopt. The study and evaluation of the impacts of such a new modeling approach provide unique research opportunities to fine-tune existing commodity market models or to develop new ones.

1.1 Problem Definition

Agricultural economists have used a wide array of models to systematically analyze the behavior of markets. With improvements in data collection, computers and statistical programs, the area of model-based research has expanded. Choosing an adequate model is not a trivial task. The envisioned use for the model, availability and sources of data, and the current state of knowledge in relevant analytical procedures are prerequisites to model building. In essence, the practice of building empirical models is a blend of economic theory, econometric procedures, and simplified representations of agricultural commodity markets.

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[2] For present purposes, without loosing generality, it suffices to state that a process y_t is *seasonally integrated* if presents unit roots at seasonal frequencies. A formal definition of seasonal integration will be presented in Chapter 3. For example, an integrated quarterly time series has unit roots at the zero, semi-annual, and/or annual frequencies. As a $I(1)$ process has the unit root at the zero frequency, it will be referred as a *nonseasonal integrated* process.

In reviewing the extensive literature on commodity modeling, it is found that recent econometric contributions to time series analysis regarding specification, estimation, and forecasting with "structural-time series models" have not made inroads into market model building. Most commodity market models can be classified into two broad econometric types. The first type is often termed "structural (simultaneous)" equation models. These models are consistent with the logic and theory underlying the commodity sectors being analyzed and are used primarily for the estimation of elasticities. In forecasting, however, these structural models usually performed poorly. This has led commodity modelers to frequently use a second type of model, time series models of which ARIMA and vector autoregressive models are two examples. In forecasting exercises, these time series models perform well and repeatedly outperform their structural counterparts. The recent econometric contributions bridge the gap between these two types of models (modeling philosophies) by balancing the use of economic theory with the time series properties of market data. One methodological problem that is of interest to this dissertation is to evaluate whether a market model that is theory and data coherent (a structural time series model) performs better in forecasting and impulse response analysis than existing time series models.

Another important recent contribution to the econometrics literature relates to the specification of nonstationary seasonal components in multiple time series models. Seasonality is a repeating pattern often found in demand and price data. Most applications in the literature assume that seasonal components can be modeled as deterministic, however, this practice may be a potential source of misspecification in vector autoregressions or error-correction type of models. The dissertation also breaks fresh ground on the application of these econometric procedures to commodity market modeling.

Commodity market models are often used for forecasting. A true test of forecasting performance is how well the model forecasts out-of-sample. A standard quantitative evaluation typically measures nominal differences in mean squared errors. Recent contributions to the literature on assessing predictive ability of models suggest that such nominal comparisons may not be appropriate in the sense that if forecast error uncertainty is accounted for, nominal MSE differences may not be significant. The dissertation will shed light on the usefulness of tests of predictive ability in assessing out-of-sample forecast performance of alternative market models. Given the wide array of models being evaluated, it would seem useful to assess whether “structural time series models” have added predictive power when compared to competing models that appear frequently in the commodity modeling literature.

In the context of prerequisites for building empirical models, these new econometric developments would fit well and have appeal for forecasting and dynamic simulation in commodity markets. This dissertation provides initial empirical evidence on these inquiries using a structural model for the U.S. wheat market.

1.2 Justification

Outlook information and simulation or policy analyses are widely used in U.S. agriculture and play a central role in evaluating the economy of the agricultural sector. Researchers at public and private organizations use econometric techniques to generate forecasts and conduct dynamic analysis of commodity markets. It is natural to comprehend, then, the concerns that arise in the agricultural sector when outlook information may be biased and/or inaccurate.

The commodity modeling literature is void of applications that integrate structural characteristics of commodity markets with the often-reported nonstationary behavior in market data. New econometric models and techniques have been developed which offer an opportunity to re-assess or expand the set of models available for forecasting and impulse response analysis. Forecasting practitioners rarely have practical guidelines on the application of the new models and techniques.

The value of accurately measuring and eliminating forecasting uncertainty is sizeable. Typically, finding a model that minimizes a criterion such as the mean square forecasting error (MSFE) makes model choice easier as the model with the lower MSFE is considered better for forecasting. But a decision-maker considering the purchase of forecast information or wishing to adopt a new forecasting system may want to know whether the incremental gain in nominal forecasting accuracy (the nominal difference in MSFEs) is significant. That is, how large should a MSFE difference be for it to justify the adoption of a new model. In the context of this research, of importance is to determine the extent to which the complexity in model specification may warrant the adoption of a model, compared to simpler parsimonious structures, given any incremental gain in forecasting accuracy.

Also of importance is to measure the extent to which these new models may offer improved forecasts and impulse response functions for the U.S. wheat market. Potential beneficiaries in the U.S. wheat industry include farmers, agribusinesses, market analysts, and government policymakers. Researchers at the land grant universities, the USDA, and other public and private research institutes on agricultural commodity markets may benefit from the methods and knowledge this study is expected to generate.

1.3 Research Objectives

The general objective of this study is to determine the forecasting and simulation analysis capabilities of alternative dynamic econometric nonstationary time series model specifications for the U.S. wheat market.

The specific objectives of this research are:

1. To develop alternative dynamic-structural nonstationary time-series econometric models for the U.S. wheat market;
2. To determine and rank the forecast ability of the models proposed in objective (1).
3. To evaluate the impulse responses and dynamic multipliers constructed with the models proposed in objective (1).

1.4 Methodology

1.4.1 Objective 1

The dynamic structural econometric model to be developed for the U.S. wheat market has its roots in the structural model of Chambers and Just (CJ) (1981), because it provides a simple and aggregated structure of the U.S. wheat market demand and supply. The CJ model, as well as the USDA and FAPRI U.S. wheat models, assumes the data and the error terms have means and variances that do not change as time passes, although it is known that the data used by these models are nonstationary in mean and/or variances.

From the econometrics point of view, two different broad types of models are used in the study. The first type of models are *reduced forms* (RF) of structural representations (i.e., Judge *et al*., 1988), and the second type adopt *structural forms* (SF). The independent variables in the RF and the endogenous variables in the SF are the endogenous variables of the CJ specification, while the exogenous variables of the CJ model enter into the RF and SF models as exogenous regressor variables.

The RF and the SF models will take into consideration the nonstationarity property of the time series available for the U.S. wheat market. The nonstationary behavior of the time series grossly depends on the frequency with which the data is collected. When data are observed quarterly, as is the case of the U.S. wheat market, the likelihood is that nonstationarity (integration and possible cointegration) varies across seasons.

For these reasons, two alternative scenarios will be considered in this study of the U.S. wheat market (Table 1.1). The first scenario will assume that nonstationarity does not vary across seasons, i.e., integration if present is nonseasonal (Granger, 1981; Hsiao, 1997a, 1997b) and will be referred to as *nonseasonal integration*. It will be considered a minor relaxation of the standard assumption of stationarity in the theory and applications of existing U.S. wheat market models (i.e., Devadoss, 1993). The second scenario will assume *seasonal integration* (i.e., Hylleberg *et al*., 1990; Lee, 1992; Johansen and Schaumburg, 1999).

When all types of representative forms and scenarios are combined, four models are selected as potential candidates to help improve the quality of forecasts and impulse response information generated for traditional U.S. wheat market models. These models are the vector error correction model (VECM), the seasonal vector error correction model (SVECM), the cointegration dynamic simultaneous equation model (CDSEM) and the seasonal cointegration dynamic simultaneous

Table 1.1 Models proposed to conduct the study

Time-Series Property Scenarios	**Model Representation**	
	Reduced-Form	*Structural Equations*
Nonseasonal integration	VECM: Vector error correction model (Granger, 1981)	CDSEM: Cointegration dynamic simultaneous equation model (Hsiao 1997a, 1997b)
Seasonal integration	SVECM: Seasonal vector error correction model (Johansen and Schaumburg, 1999)	SCDSEM: Seasonal cointegration dynamic simultaneous equation model

equation model (SCDSEM). The non-shaded cells in Table 1.1 highlight the fact that the econometric models are adopted from previous works. The VECM is a model that has it roots in the work of Granger(1981). Its basic characteristic is that it is an unrestricted multiple time series model for cointegrated variables that decomposes the short-run dynamic effects from the long-run equilibrium. The CDSEM is a model proposed by Hsiao (1997a, 1997b). Hsiao's original contribution accounts for non-seasonal nonstationarity using a structural specification introduced by Zellner and Palm (1974); which may be considered the first serious approach to blend simultaneous equation models with time series. The SVECM model developed from the seminal papers of Hylleberg *et al*. (1990) and Lee (1992), and generalized by Johansen and Schaumburg (1999). This model is an improved version of the VECM in the sense that it allows for decomposing the short-run dynamics from the long-run equilibrium, which in turn allows varying across seasons. For nonstationary quarterly data, this may be the case. Finally, the SCDSEM, in the shaded cell, refers to a model that is developed in this dissertation. Since the CDSEM proposed by Hsiao does not allow for handling seasonal integrated and cointegrated variables, one contribution of this research is to develop a CDSEM that properly takes into account the seasonal nonstationary property of quarterly time series data. This new modeling approach allows for decomposing the structural model into short-run dynamics and seasonal equilibrium relationships.

1.4.2 Objective 2

Forecasts of domestic consumption, inventories, exports, production, and market prices of U.S. wheat will be generated using the four models in Table 1.1. The forecast accuracy of the models will then be assessed. Accuracy measures are usually defined using forecast errors (i.e., the difference between the observed data and the forecast). Examples of such measures are the mean error (ME), the error variance (EV), the mean square error (MSE), and the mean absolute error (MAE), as defined, for example, in Diebold (1998).

In agricultural economics literature, qualitative forecast measures play an important role. For instance, Naik and Leuthold (1986) used a 4 x 4 contingency table to evaluate four different types of forecast turning points. But none of these approaches take into consideration the sample variability and uncertainty of the measures. Recent work revisited the concept of evaluating forecasts. McCracken and West (1999), for example, provide a comprehensive review on inference about a model's ability to predict. The Diebold and Mariano (1995) test will be used to compare the forecasting ability of any two of the four models. Diebold and Mariano proposed a test statistic for the null hypothesis of equality of two mean square forecast errors (MSFE) , which is asymptotically normal distributed. As there are four MSFE to contrast, there are six possible pairs of MSFEs to compare. To attain a given global significance protection level, say $a = 0.05$, the Bonferroni's criteria (Johnson and Wichern, 1998) will be adopted to select the significance level used to conduct each comparison of any pair of MSFE, which is $a/6 \simeq 0.01$. In this way, a ranking of the four models may be empirically constructed.

1.4.3 Objective 3

The analysis of the response of one market endogenous variable to a unit change in another variable, or a policy instrument, is generally called impulse response (IR) or dynamic multiplier (DM) analysis. Although the terms IR and DM are used interchangeably, we are going to adopt the following distinction. IR will refer to the analysis of responses to a unit change in some endogenous variable (Lütkephol, 1993). In contrast, DM will refer to the analysis of responses to a unit change in some exogenous variable (Theil, 1971). The works of Lütkepohl and Reimers (1992b) and Phillips (1998) provide the basis for conducting the IR analysis for nonseasonal nonstationary systems. These bases will support the extension of IR for the seasonal nonstationary systems considered in this study. The well known approach of shocking the exogenous variables to obtain the DM, as described in Theil (1971), will be followed for all four models in Table 1.1. A small Monte Carlo experiment will be designed to compare the IR and DM constructed with the four models in Table 1.1 when the data are generated under a known data-generating process. Phillips (1998), who compared the IR obtained under different models when the data were generated with a known nonseasonal integrated data generating process, followed this approach.

1.5 Overview of the Research

The following chapters of this dissertation are structured as follows. Chapter 2 conducts a literature review structured in three main sections. The first section reviews the economic background of agricultural commodity markets; the second section details the econometrics issues associated with agricultural commodity market models, and the third section introduces the specific U.S. wheat market models. Chapter 3 presents the econometric methods and their application to the U.S. wheat market. The economic model and the econometric procedural aspects are thoroughly described. Chapter 4 introduces the data, the statistical properties of the four U.S. wheat market models, the forecast ability comparison of the models, and the impulse responses and dynamic multipliers evaluation. The dissertation finishes in chapter 5 with conclusions and future research.

CHAPTER 2

LITERATURE REVIEW

The use of structural econometric models in commodity modeling has a long history in the agricultural economics literature. Most of this work emerged soon after the various works of the Cowles Commission on structural modeling were published in the 1940s and 1950s (Christ, 1994). An excellent and recent treatment of this literature is found in Garcia and Leuthold (1997). What has been a dormant empirical area of research has been the formulation of models that blend structural characteristics of commodity markets with time series properties of the variables. Fortunately, much econometric progress has been made over the past decade on the theory of estimation, testing and forecasting with blend models. This review of literature condenses both bodies of work, commodity modeling and econometric developments, with a specific empirical focus, the development of a dynamic model for the U.S. wheat market that allows for nonstationarity and cointegration.

This chapter is organized as follows. Section 2.1 introduces the essentials of demand and supply theory for commodity modeling reviews of previous U.S. wheat models. Section 2.2 reviews the econometric literature on nonstationarity in structural models. This section emphasizes the analysis of seasonal nonstationary time series ; these developments are fairly new in the econometrics literature and form the foundation for theoretical and empirical contributions of the dissertation research. Last section discusses the relevance of this literature review.

2.1 Survey of U.S. Wheat Market Models

2.1.1 The Economic Framework

The study of grain markets has been the subject of considerable research since the early 1940s. The studies of Labys (1973), Labys and Pollak (1984), Tomek and Myers (1993), Lord (1991), Allen (1994), Garcia and Leuthold (1997), and Tomek (1997) provide a balanced progress report of the different stages of evolution of this vast literature. Over time agricultural commodity models have changed to reflect more accurately the nature of the data.

The main components of a domestic market are represented in Labys and Pollak (1984). Figure 2.1 illustrates the interdependence among commodity demand, supply, inventories and prices in a domestic market. The demand for a commodity depends on its price as well as other external influences, such as per capita income and export demand. The end-use demand for raw materials may depend on the level of economic activity or the level of technology. Similarly, supply is responsive to commodity prices, crop yields, technology, levels of exports/imports, and weather conditions. As inventories may be held for precautionary, speculative and transactions purposes, they are influenced both by demand and supply. The effect of inventories on market prices depends on the elasticities of supply and demand for the commodity.

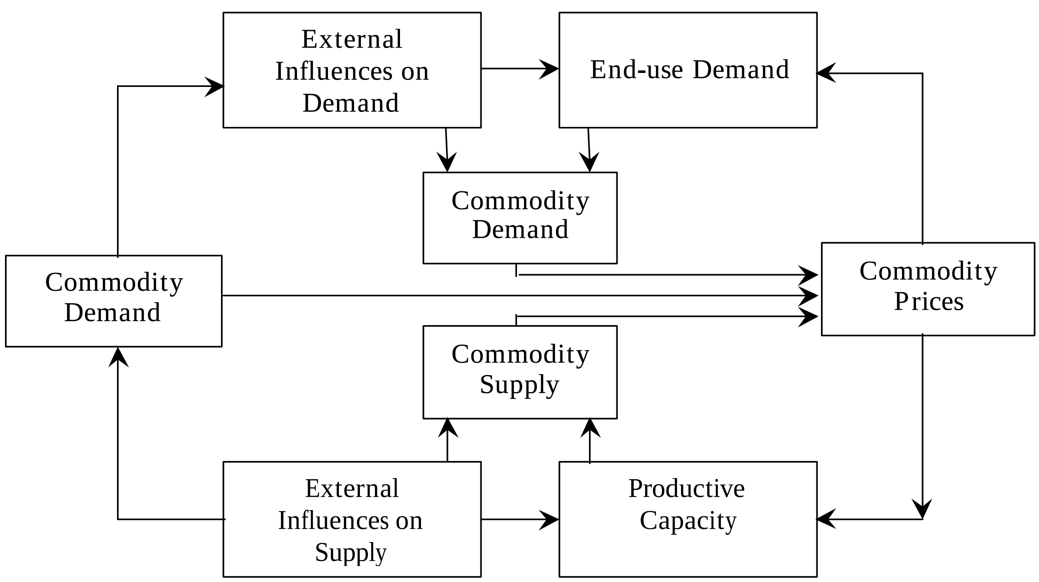

Figure 2.1. Model Representation of a Commodity Market

The specification of *static* relationships to explain commodity demand and supply is derived from the economic theory of consumer demand and production, inventory relationships are derived from the partial adjustment to equilibrium theory (Nerlove, 1958), and price relationships are derived from the competitive or noncompetitive nature of the markets. The vast literature in commodity market modeling shows that the behaviors of agents in these markets are more appropriately described *dynamically*. When income or prices change, for example, commodity consumers do not respond immediately, nor do they delay their responses. Rather, they spread their response over some period of time.

Consumer demand theory explains demand based on the maximization of consumer utility subject to budget constraints. Solution of the maximization problem through differentiation leads to a set of demand equations of the form $q_{i,t}^{(d)} = f_i(p_{i,t}, p_{j,t} ..., p_{n,t}, y_t),\ i = 1,...,n$, which relate consumption $q_{i,t}^{(d)}$ of the *i*-th commodity to its price $p_{i,t}$, the prices of other commodities $p_{j,t},..., p_{n,t}$, and income y_t. A relevant assumption for the commodity models is that demand behavior is assumed to be time variant (Labys, 1973).

The *market demand function* for a commodity is a statement of the relation between the aggregated quantity demanded and all factors that affect this quantity. In functional form, a market demand function may be expressed as $Q_t^{(d)} = f(p_t, p_{j,t},..., p_{n,t}, y_t, Pop_t, z_t)$, where $Q_t^{(d)}$ is total market demand in period *t* for the commodity of interest, Pop_t is total population in the market in period *t*, z_t represent other variables, such as tastes and preferences, advertising expenditures, etc. Other variables are as already defined.

Fulfillment of these conditions and assumptions is normally assumed in bridging the gap between the above theoretical model and the empirical equations, which are estimated in commodity demand studies. The equation that might be estimated for a single commodity would be of the form $Q_t^{(d)} = f(p_t, p_{j,t},..., p_{k,t}, y_t, Pop_t, z_t, u_t^{(d)})$, which includes the price of the commodity of interest, prices of only one or two complementary or substitute commodities, income, population, and possible other explanatory variables z_t. A stochastic disturbance term u_t is added relevant to statistical estimation, assumed to be independent of the explanatory variables, independently and identically distributed, with expected zero mean and constant variance.

Consumer reactions to a change in prices of a commodity are not instantaneous, rather it is spread over some period of time. A change in price, therefore, influences consumption not only in the short run but also over the long run. Within the context of agricultural commodity market, the *dynamic theory* of Nerlove (1958) was widely accepted to explain consumer behavior. This theory makes a marked distinction between short run and long run demand. Demand in any period is assumed to adjust only partially towards equilibrium. The *long run demand* for a commodity may be expressed, if no demand rigidities are present, as a function of income and prices, *ceteris paribus*, i.e., $q_t^{*(d)} = a_0 + a_1 y_t - a_2 p_t$, where $q_t^{*(d)}$ represents the long run or equilibrium demand, while other variables are as previously defined. Note that actual values of equilibrium demand cannot be observed, therefore, the parameters a_0, a_1, and a_2 cannot be estimated directly. To resolve this problem, it is possible to assume that change in current consumption varies proportionally to the difference between long run consumption and past consumption, i.e., $q_t^{(d)} - q_{t-1}^{(d)} = \boldsymbol{d} \times (q_t^{*(d)} - q_{t-1}^{(d)})$, where δ describes the rate or speed of adjustment. This last equation is known as *adjustment equilibrium* and may be used to explain actual consumption as a function of past equilibrium consumption, i.e. $q_t^{(d)} = \sum_{i=0}^{t} \boldsymbol{d}\,(1-\boldsymbol{d})^i q_{t-i}^{*(d)}$. A dynamic equation suitable for estimation can be obtained by substituting the adjustment equilibrium into the long run demand and elimination of the unobservable variable $q_t^{*(d)}$. The values of the coefficient of adjustment and the short and long run elasticity of price and income could be obtained from this relationship, yet one must be careful in their interpretation. Values of $\boldsymbol{d}$ close to 1 may imply that changes in the determining variables do not influence demand in the future as well as in the present. Values of $\boldsymbol{d}$ greater than 1 may imply that market participants overreact (Nerlove, 1956).

The adjustment equilibrium equation may be improved to explain demand for commodities that are semidurable (Witherell, 1967), to allow for the possibility that purchases might be deferred or made earlier if income temporarily declines or increases, respectively.

The empirical specification of demand relationships based on the theory of demand presented before is referenced in the literature as *ad-hoc* or *partial demand specification*. The early history of empirical demand analysis is marked not by an attention to theory but by the extensive use of single equation methodology centered on the measurement of elasticities. This is because elasticities are easily understood and can be directly measured as the parameters of a regression equation linear in the logarithms of purchases, outlay, and prices (Keynes, 1933; Garcia and Leuthold, 1997).

Some challenges that demand for agricultural commodities modeling pose are related to the following issues. First, agricultural commodities have multiple end uses and the scope of the markets is often global. Second, demand for inventories is difficult to specify because holders may be numerous and their motivations varied. The speculative dimensions of inventory activities have been modeled using extrapolative procedures such as adaptive expectations (see Labys, 1973), or more recently via rational expectations (Miranda and Glauber, 1993). The modeling of demand for stocks is further complicated by the presence of government stock programs that change frequently, and often are designed to set price supports or stabilize prices, making econometric estimation difficult due to concerns with structural change. Third, and finally, some consumer activities are measured at retail or wholesale levels while producer activities are quantified at the farm level, which implies the existence of marketing margins that usually change over time and in response to changes in input usage (Garcia and Leuthold, 1997).

Just as the static demand function derives from a set of maximization conditions under constraint, the static supply relationship stems from the maximization of profits for a producing unit subject to the production function constraint. For the derivation of the supply schedule it is required that each producing unit be in competitive equilibrium with the real cost of a factor equal to its marginal productivity. The *supply function* is more concerned with the responses of output to one or more prices, as opposed to the production function, which describes the relationship between output and various inputs. The *static* supply schedule of an individual firm, derived from the theory of production to describe commodity behavior is, in general, of the form $q_t^{(s)} = f(p_{1,t}, p_{2,t}, w_{1,t},..., w_{k,t}, u_t^{(s)})$, where $p_{1,t}$ is the price of the commodity of interest, $p_{2,t}$ refers to the prices of inputs to the production process or to prices of other commodities closely related in production, $w_{1,t},..., w_{k,t}$ are noneconomic determinants, such as technological or institutional factors, and u_t is a stochastic disturbance term (Labys, 1973).

The *market* or *industry supply curve*, if defined in terms of the underlying cost relationships, represents the summation of that portion of the marginal cost curve lying above the average variable cost curve for individual producers. The aggregated version of the supply function may be derived as a summation of the supply of all the producer agents in the industry, i.e., $Q_t^{(s)} = \sum_i q_{i,t}^{(s)}$.

Certain commodity models require that a distinction be made between *country* or *domestic demand/supply* and *country exports*. Some models may focus their attention on the demand of a given commodity for exports, while others on the supply for exports. Where the model builder prefers to concentrate on export behavior, export equations can be included following the theories of demand or supply behavior, as outlined before. Where the desire is to concentrate on the demand for exports, the specification for such equations would obviously require that the explanatory variables be demand oriented, including factors such as exchange rates, prices of competing countries, stocks in the competing exporter countries, etc. The export demand approach to specification has been used more frequently, as more models become trade oriented (Labys, 1973).

A possible specification for supply is $q_t^{(s)} = b_0 + b_1 p_t^* + b_2 z_t + b_3 w_t + b_4 q_{-1}^{(s)} + u_t$, where p_t^* is the expected future price of the commodity of interest, and all other variables are as already defined. This expression is based on the partial equilibrium adjustment theory of Nerlove.

The only variable left in expectation form is price, which can be substituted by making assumptions about the manner in which producers form price expectations. The simplest model is the *naive expectation*, where the current expected price equals the previous actual price (Labys, 1973). Other models are the *extrapolative expectations model* (Goodwin, 1947), the *adaptive expectations model* (Cagan, 1956) and the *rational expectations model* (Muth, 1961).

For the supply side of agricultural commodities, the agronomic and biological nature of commodities heavily influence the specification of relationships useful for market models. Yet, the analysis of agricultural supply is firmly rooted in microeconomic theory: farm managers are assumed to maximize expected discounted profits (or utility of profits) subject to various constraints (Tomek and Myers, 1993). Important dimensions are the stages of growth and development of the commodity, physical and economic factors that affect output, and the existence of lags between production and sale decisions, among others. These dimensions, in turn, raise questions about the dynamic specification involved, such as expectations, risks in production and prices, changes in technology, the existence of government programs to stimulate or control production, etc. (Lord, 1991).

Agricultural supply models have some features in common. Since a lag exists between the decision to produce and actual production, expected profits are often assumed to be a function of expected output prices and expected yields. Input prices are assumed known and exogenous at the initial decision time (Tomek and Myers, 1993). Among farm commodities, annually produced crops are perhaps the simplest to model, because supply responses can be measured via acreage planted and yield equations.

However, supply analysis of annual crops presents some difficulties. Many of the major crops (wheat, corn, soybeans, rice, and cotton) have been influenced by government programs, thus, considerable effort has been devoted to estimating supply response in the presence of government programs. Broadly speaking, two approaches may be found in the literature. In the first approach, the variables that consider farm programs are included in the model (Devadoss *et al.*, 1986; Bailey, 1989). The second approach attempts to model the free market and farm program time periods as separate regimes, allowing for varying parameters, as opposed to the first approach (Lee and Helmberger, 1985). The second alternative has the potential to provide more information and avoid the possible bias of assuming constant parameters over the entire sample period, but this advantage is obtained at the cost of estimating more parameters (Tomek and Myers, 1993).

Since production in agriculture is not instantaneous, and is dependent upon past investment decisions, the production observed in any period tends to be affected greatly by decisions made in the past. Incorporation of expectational variables for prices into supply functions, as was already introduced, represents an *ad-hoc* method or allowing for the role of investment in supply response (Colman, 1983). The study of Antonovitz and Greene (1990) gives an example, for the supply of fed beef, in which futures and rational expectations give different empirical results. Under some specifications, rational and adaptive expectations models result in identical or similar empirical models (Eckstein, 1985). However, the use of naive expectations is common in the literature related to annually produced, continuous inventories commodities, because current observations on cash, nearby, and distant futures prices are highly correlated (Tomek and Gray, 1970).

Technological change (seeds, machinery, fertilizers, etc.) has shifted supply schedules for many agricultural commodities (Chambers, 1988). Unfortunately, there is no direct measure of "changes in technology". The most common practice of using a proxy variable, such as a trend variable, assumes a smooth change in technology of equal amounts each period. Sometimes general measures of changes in productivity are used, such as the Divisia index of changes in productivity (Chambers, 1988). No simple answer exists for the question of whether to include a proxy variable, though the usual practice is to include it (Maddala, 1977; Tomek and Robinson, 1990).

A final topic that deserves attention is the fact that current production usually is highly correlated with production in the previous period. Tomek and Robinson (1990) clearly point that production in particular regions is influenced by physical and climatic conditions, thus, many acres devoted to wheat, for example, simply have no viable alternative over a wide range of prices. Further, large changes in production tend to be restricted by factors such as resource fixity, managerial ability of farmers, and habitual production patterns. As a result, some researchers have specified the dependent variable lagged one time period as an independent variable to model the fact that current production is influenced by the level of production in the previous period. In this way, current production may be viewed as changing from the previous level in response to various prices and other factors. Unfortunately, the lagged dependent variable often tends to be highly correlated with the influence of technology and other trending explanatory factors. In such cases, the lagged variable becomes, in part, a proxy for other variables. A possible consequence then is that the effect of the lagged variable may be overstated (Tomek and Robinson, 1990).

Integrating the concepts of demand and supply establishes a framework for understanding how they interact to determine market prices and quantities. When the quantity demanded and the quantity supplied of a product are in perfect balance at a given price, the market for the product is said to be in *equilibrium.* Equilibrium is stable when the factors underlying demand and supply remain unchanged in both the present and the foreseeable future. During those instances when the factors underlying demand and supply are dynamic rather than constant, a change in current market prices and quantities is likely. Temporary market equilibrium of this type is often referred to as an unstable equilibrium. The forces that drive market prices and quantities either up or down to achieve equilibrium are the concepts of *surplus* and *shortage*. A surplus is created when producers supply more of a product at a given price that buyers demand. Such a condition is one of excess supply. Conversely, a shortage is created when buyers demand more of a product at a given price than producers are willing to supply. Shortage describes a condition of excess demand. Neither surplus nor shortage will occur when a market is in equilibrium. Surplus and shortage describe situations of market disequilibrium because either will result in changes in prices and quantities offered in the market(Tomek and Robinson, 1990).

The development of a *theory of commodity inventory behavior* is most essential to describing the workings of commodity markets (Labys, 1973). Inventory adjustment represents an important mechanism whereby short run price equilibrium is reached for commodities where consumption and production or both are price inelastic within a given time period. If the period considered is so short that consumption and production cannot be varied, for example, a high price would motivate some stockholders to sell inventories, thus driving prices downward until markets reach equilibrium. Then, a theory that explains the levels of inventories held is needed. One of the theories that explains the inventory behavior of producers is the *accelerator* theory, according to which inventories vary directly and proportionately with output. The accelerator can be stated in two different forms: First, that commodity inventories should rise or fall with sales or manufacturing activity, say $s_t = a\,y_t$, where s_t represent appropriate stock and y_t is output. Second, that the rate of change of stock holding should vary directly with the rate of change of this activity, i.e. $\frac{ds_t}{dt} = a\,\frac{dy_t}{dt}$. A second theory of inventory behavior is the *flexible accelerator*, which resolves some of the simple accelerator theory problems (Goodwin, 1947; Nerlove, 1958).

The role played by *producer inventories* in market adjustment could possibly be used to explain export quantities and prices around their equilibrium levels. Goodwin (1947) proposes that producer inventories may be modeled as $s_t = b_0 + b_1 p_t + b_2 \Delta p_t + b_3 s_{t-1} + b_4 y_t + u_t$, where all variables are as before. This model may be derived by applying the partial equilibrium theory in a similar manner as it was proposed for consumption or supply by Nerlove (1958).

The basis of *commodity price theory* has roots in the works of Working (1931), Shepherd (1966), Kendall (1953), and Samuelson (1965), as outlined in Labys (1973). A competitive market organization has been traditionally assumed in formulating commodity price relationships. The simplest form of price relationship which can be specified to reflect competitive behavior, assuming either consumption and production may be price responsive but that inventories do not vary[1] or do not exist, is (Labys, 1973),

$$\begin{aligned} q_t^{(d)} &= f(p_t, y_t, z_t, u_t^{(d)}) \\ q_t^{(s)} &= g(p_{t-1}, z_t, u_t^{(s)}) \\ q_t^{(d)} &= q_t^{(s)}, \end{aligned} \tag{2.1}$$

where production is determined by past prices, and consumption equals production in the same period.

[1] As is the case of perishable commodities not suitable for storage.

If the data interval is short relative to consumption and production lags and inventories vary considerably, price behavior can be explained in terms of adjustments in inventories. Several theories of specification can be adopted for this situation. A general model is

$$\begin{aligned} q_t^{(d)} &= f(p_{t-1}, y_t, z_t, u_t^{(d)}) \\ q_t^{(s)} &= g(p_{t-1}, z_t, u_t^{(s)}) \\ p_t &= h(\Delta s_t, z_t, u_t^{(p)}) \\ \Delta s_t &= q_t^{(d)} - q_t^{(s)}. \end{aligned} \tag{2.2}$$

The price relationship given in (2.2) shows prices as a function of changes in inventories, Δs_t, but the final specification adopted would depend on whether the underlying price structure reflects a *flow adjustment* or a *stock adjustment* process. For example, the system (2.2) reflects a flow adjustment process, embodying conventional equilibrium theory. It explains that excess demand or excess supply leads to an increase or decrease in prices and that the associated price difference will be reduced as consumers and producers react to the new market situation. Prices, eventually, will return then to their equilibrium level.

The price relationship (2.2) can be specified as a stock adjustment process by replacing the change in inventory variable with one describing inventory levels, as follows,

$$\begin{aligned} q_t^{(d)} &= f(p_{t-1}, y_t, z_t, u_t^{(d)}) \\ q_t^{(s)} &= g(p_{t-1}, z_t, u_t^{(s)}) \\ p_t &= h(s_t, z_t, u_t^{(p)}) \\ s_t &= s_{t-1} + q_t^{(d)} - q_t^{(s)}. \end{aligned} \tag{2.3}$$

A final consideration is *export* (import) *demand* (supply). It frequently happens that total quantity demanded (supplied) of a commodity is not resolved within the domestic market and a portion of this quantity is exported (imported) to foreign markets. Questions arise as to whether separate export (import) behavioral equations should be added to the model or whether exports (imports) should be determined based on identities. If export (import) fluctuations are determined by factors other than those explaining demand (supply) it might be useful to include a behavioral equation into the system that describes price relationships. Examples of such price relationship specification may be found in the Houck and Mann (1961) soybean study and in the Chambers and Just (1981) wheat study.

2.1.2 U.S. Wheat Market Models

Benchmark models for the U.S. wheat market are the models developed by Mo (1968), Chambers and Just (1981), and those developed for the *World Wheat Trade Model* of FAPRI (Devadoss et al, 1986, 1990, 1993) and the *World Wheat Market Model* used by the USDA (Bailey, 1989).

The work by Mo (1968) provides a basic conceptual framework to model the flow of U.S. wheat supply and utilization. Mo's model is a typical commodity model based on the theory of commodity markets already outlined. It consists of a set of equations that model total demand (consumption, inventories, and exports) and a set of equations that model total supply (production, inventories, and imports). An identity closes the system, total supply equals total demand, which allows determining the equilibrium price and total quantities supplied and demanded that clear the U.S. wheat market.

The FAPRI and the USDA specifications for the U.S. wheat market (Devadoss *et al*., 1993; Bailey, 1989) are very similar models, which also specify the domestic U.S. supply and demand of wheat according to neoclassical producer and consumer theory. These models provide excess supply and demand schedules, with the main characteristic of these two models being the high number of variables and equations involved compared with other specifications, i.e. Mo (1968) or Chambers and Just (1981).

Another important reference that provides a dynamic model for three commodities (wheat, corn, and soybeans) is that of Chambers and Just (1981). The model contains four equations (production, disappearance, inventories and exports) and an identity that clears the markets for each of the agricultural commodities (wheat, corn, and soybeans). It incorporates predetermined endogenous variables (lagged variables) to model the dynamics of the system, and links the U.S. macro economy with the U.S. commodities market. This link is provided by the exchange rate, an exogenous variable that explains the U.S. exports for each commodity. The model of Chambers and Just (1981) appeared in a period where the U.S. was experiencing tight fiscal and monetary policies at the macro economy level, affecting the exchange rate, which in turn raised concerns about the effects on the U.S. agricultural sector. They used the U.S. real support price to model government programs, while the U.S. exchange rate was used as a proxy to model the U.S. macroeconomic environment. In and Mount (1994) used exchange rates to model the macroeconomic environment, also.

The econometrics of the U.S. wheat market models surveyed reveal that these models adopt the classical assumptions of stationarity with error terms that are normally distributed, homoskedastic, and serially uncorrelated (see for example Bailey, 1989 and Devadoss et al., 1993). Consequently, these structural models are commonly estimated via ordinary least squares (OLS) if the system is recursive or via instrumental variable estimators (two or three-stage least squares) if not. Alternatively, OLS is applied to the reduced form of the system (Judge et al, 1989). A different econometric approach is shown in Chambers and Just (1981), who report the use of 2SLS to estimate the parameters in the SEM for wheat, corn and soybeans, although no residual analysis is reported. A similar situation may be found in SEM specifications for other agricultural commodities, as in the work of Watanabe *et al.* (1990), who developed a structural model for the U.S. rice industry.

2.2 The Econometrics of Commodity Models

A literature review on econometric commodity modeling in presented in this section. The traditional approaches of structural and time series models are reviewed in subsections 2.2.1 and 2.2.2, respectively. Subsection 2.2.3 reviews the literature on modeling market dynamics, and subsection 2.2.4 presents current developments in the arena of time series econometrics.

The econometric review of literature on commodity models is summarized in Table 2.1. The shaded block on the right hand side of this table lists the most recent developments in modeling systems of equations and forms the foundation for theoretical and empirical analyses developed in this research.

Table 2.1. Econometric commodity models topics

Econometrics Time Series of Commodity Models

Structural approach (Section 2.2.1)	Stochastic processes: (Section 2.2.2)	Dynamic simultaneous equation models (Section 2.2.3)	New Developments (Section 2.2.4)
– Structural Models: • Ordinary least squares • Indirect least squares • Two-stage least squares • Three-stage least squares • Limited information maximum likelihood • Full information maximum likelihood – Reduced forms – Recursive models	– Univariate time series: • Deterministic linear trends • ARIMA models • Stochastic trends tests: (a) Unit roots (b) Stationarity (c) Seasonal unit roots • ARCH and GARCH – Multivariate time series: • Vector autoregressive processes • Vector autoregressive and moving average processes	– Under the stationarity assumption: • ARMAX models • Zellner-Palm final form • DSEM & expectations of economic agents • DSEM & exogeneity • DSEM & causality • SVAR – Nonstationary DSEM (No short-run & long-run decomposition effects): • Fully modified-OLS, -FM-VAR, FM-VARX – Nonstationary reduced form (nonseasonal unit roots): • Error correction model	- ***Nonstationary DSEM: (Short-run & long-run decomposition effects):*** • *Nonparametric transformation and parametric estimation* (Choi and Phillips, 1997) • *Parametric transformation and estimation* (Hsiao, 1997a and 1997b) – ***Nonstationary reduced form (seasonal unit roots):*** • *Seasonal error correction model* (Johansen and Schaumburg, 1999)

The components of Table 2.1 are discussed in detail in the sections that follow. These new results are mainly focused on estimating the structural parameters as advocated by the Cowles Commission, but allowing for their decomposition into direct estimates of the short-run dynamics and long-run equilibrium relationships. More over, the nature of these new results stimulates their extension in an innovative way, as will be presented in Chapter 3.

2.2.1 The Traditional Structural Approach

The quantitative analysis of agricultural commodity models has been mainly conducted using econometrics procedures. Econometric analyses of commodity market models can provide market participants and policy makers with a clearer vision of the economic environment in which they operate, by systematically identifying the characteristics of agricultural demand and supply. Thus, *econometric commodity models* help quantify the relationships that explain economic behavior of a market or system of markets (Garcia and Leuthold, 1997). Econometric structural commodity models are predominantly found in the literature, which are used to represent and quantify the relationships and factors that influence the market (Garcia and Leuthold, 1997). In practice, these relationships and factors are specified as a set of equations, following the *simultaneous equations models* (SEM) approach developed during the late 1940s and early 1950s at the Cowles Commission, University of Chicago (Labys, 1973; Judge *et al.*, 1985; Maddala, 1988; Christ, 1994). The econometricians at the Cowles Commission recognized early that the error terms in a SEM specification are correlated with some of the endogenous variables, and showed that the ordinary least squares parameter estimates are inconsistent[2].

The estimated parameters from SEMs can be used to solve for equilibrium price and quantity values, given the values of the exogenous variables and the error processes. Mathematically, the solutions

[2] This problem is related to the problem of *identification* (Maddala, 1988). If we can somehow obtain consistent estimates of the parameters in a single equation of a SEM, we say that the equation is *identified*. But getting consistent estimates is just a necessary condition for identification, not a sufficient condition. Roughly speaking, if we can get unique estimates for the structural parameters of an equation from the reduced-form parameters, we say that the equation is *exactly identified*. When we get multiple estimates, we say that the equation is *overidentified*, and when we get no estimates, we say that the equations is *not identified*. Necessary and sufficient conditions are presented in Judge *et al.* (1985) and Maddala (1985) for stationary SEMs . For stationary dynamic SEMs, see Judge *et al.* (1985). For nonstationay dynamic SEMs, see Hsiao (1997a, 1007b) and Choi and Phillips (1997). These concepts are considered with further details in Chapter 3.

are based on finding the simultaneous solution for price and quantity expressed in terms of the exogenous variables and the error terms (i.e. Intriligator, 1978). These expressions are referred to as the *reduced form*.

The literature on parameter estimation of SEMs is impressive. It has been summarized in most of the econometric advanced textbooks, such as that of Intriligator (1978), Judge *et al.* (1985), Judge *et al*. (1988), Maddala (1988), and Greene (2000), among others. The traditional procedures of ordinary least squares (OLS), indirect least squares (ILS), two-stages least squares (2SLS), three-stage least squares (3SLS), limited information maximum likelihood estimator (LIML), and full information maximum likelihood estimator (FIML) are extensively presented in these textbooks.

Structural models have the potential to provide useful information, but they have been subjected to growing criticism: 1) Basically, the maintained hypotheses – the information that can be treated as unquestionably correct – are in fact questionable (Tomek and Myers, 1993); 2) The classification of variables into endogenous and exogenous is sometimes arbitrary (Maddala, 1988). Moreover, in general the “exogeneity” of the exogenous variables is in general not tested (Tomek and Myers, 1993); 3) An important consideration in simultaneous models is the ability to extract information of the structural parameters from the reduced form (the identification problem). Usually, many variables should be included in the equation that are excluded to achieve identification, an argument that is known as the *Liu critique* (Liu, 1960) but did not receive much attention. Yet, as pointed out by Sims (1980), over identifying restrictions are seldom tested; 4) One of the main purposes of SEM estimation is to forecast the effect of changes in the exogenous variables on the endogenous variables. However, if the exogenous variables are changed and profit-maximization agents anticipate the change, they would modify their behavior accordingly. Thus, the coefficients in the SEM cannot be assumed to be independent of changes in the exogenous variables (Lucas, 1976), a critique now called the *Lucas critique*; 5) Also, simultaneous systems often have a large number of predetermined variables relative to the number of observations. Consequently, a few observations may have a large influence on the estimates (Tomek and Myers, 1993).

Different solutions have been proposed to these criticisms, mainly to those presented in 1) and 2) before. Redefinition of the concepts of exogeneity and causality and related tests have been suggested by Granger (1969), Engle, Hendry and Richard (1983), among others.

Second forms of econometric commodity models are the *recursive models*, which are also predominant. Agricultural product markets are commonly assumed to be competitive and in equilibrium. Given the biological lags between decisions to produce and the realization of output, models of these markets are often recursive (Gallagher et al., 1981; Spriggs, 1981; Tomek and Myers, 1993; Garcia and Leuthold, 1997). In a recursive framework, quantity and price are determined sequentially through time, assuming one-way causality from independent and predetermined variables to the dependent variable. One of the advantages of recursive models over structural models is that they are appropriately identified systems. A recursive framework reflects one-way causality, thus, reflecting the sequential flow in the endogenous variables through time (Intriligator, 1978).

An important consideration in choosing a recursive or a structural model is the temporal unit of observation, e.g. weekly, monthly, quarterly or yearly data, relative to the biological or adjustment lags in the system. As the lag structure increases, important variables may be viewed as predetermined by past occurrence, hence, the structure can be specified in a recursive framework (Waugh, 1964; Hallam, 1990; Garcia and Leuthold, 1997). On the other hand, recursive models are not recommended when variables are determined simultaneously; For example, price in the current period and quantities allocated to the various markets (e.g., the allocation of grains to inventories, human consumption, feed, or export markets). The nature of the causality in the model depends highly on the underlying characteristics of the market. In general, as the model becomes richer in the details of demand, the degree of simultaneity increases (Thurman, 1987; Garcia and Leuthold, 1997). An important attribute of structural models is that they may be used for structural, predictive and policy analyses.

2.2.2 The Traditional Time Series Approach

The idea of capturing and modeling the dynamics of commodity models was recognized early (Labys, 1973). Primordial studies of the dynamic nature of commodity models were strictly focused on the stability and the multiplier analysis of a static structure. Efforts on modeling the dynamics of the markets started to evolve soon after the Labys work. The works by Lord (1991), Tomek and Myers (1993) and Allen (1994) provide a complete reference on the subject. This approach switched the focus of interpreting and handling the dynamics of the markets from the static approach dominating the 1970s to a new paradigm for which dynamics are built-in to the models.

Agricultural markets have been modeled, although in a few instances, as being in disequilibrium (Baumes and Womarck, 1979; Ziemer and White, 1982). When prices are influenced by government programs, it is reasonable to model such markets as having at least two regimes, one when government programs are influential and one when they are not (Liu et al., 1990). Yet, previous works argue that little appears to be gained from disequilibrium specifications (Ferguson, 1983).

2.2.2.1 Univariate Time-Series Methods in Commodity Models

The study of the time-series properties of commodity prices has been widely used. One property of commodity price time-series that was recognized early in the literature is the high degree of positive correlation in price levels (Tomek and Robinson, 1990). Another feature of price movements over time is that they have occasional spikes that perhaps are associated with a major regime shift in the underlying market. A truncation of the underlying price distribution is indicated when commodity prices, subject to government support programs, sometimes bounce around a price floor. Dynamic models allow for autocorrelation modeling, though not necessarily spikes in commodity prices. Advances in time-series methods have provided new insights into the behavior of commodity prices, casting doubts on some assumptions made in traditional models[3] (Tomek and Myers, 1993). It should be noted that time-series

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[3] A review on the assumptions of some benchmark models for the U.S. wheat markets is presented in a later section.

methods are commonly applied to data observed at high frequencies (e.g. daily or weekly) while structural models are commonly applied to data observed at lower frequencies, e.g. quarterly or annually (Tomek and Robinson, 1990).

Early attempts to model the autocorrelation in commodity prices with a time-series approach involved the simple approach of fitting *deterministic linear trends*, with predictions that soon were considered inaccurate. A possible approach to resolve this problem is to assume a *stochastic trend*, which changes by a given amount on average and in a particular period the deviation from the average is given by some unpredictable amount[4] (Stock and Watson, 1988).Modeling commodity prices assuming stochastic trends (as in Baillie and Myers, 1991) is consistent with ARMA models (Box and Jenkins, 1976) with order of integration one[5], also called ARIMA models[6] (Beveridge and Nelson, 1981).

2.2.2.2 **Unit Roots**

Space constraints preclude an extensive review of the literature on testing for stochastic trends (or unit roots). An extensive overview on unit root tests is provided by Maddala and Kim (1998), with the parametric tests of Dickey and Fuller (1979) and the non-parametric tests of Phillips and Perron (1988) being the most cited in applied works. Dickey and Fuller (1979) derived the distributions –DF distributions, for various unit roots data generating process (DGP) assuming identical distributed and independent (*iid*) error terms. Phillips and Perron (1988) proposed a nonparametric correction of the DF test to account for errors terms that are not *iid.* The limiting distributions of the statistic proposed by Phillips and Perron (PP) under different DGPs are identical to the DF distributions.

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[4] The notion of a *stochastic trend* can be modeled as a random walk with drift:

$$w_t - w_{t-1} = m + e_t$$

where the drift parameter m is the average predictable change in w_t and e_t is a serially uncorrelated random shock to the trend. When a commodity price p_t contains a stochastic trend the price can be written as the sum of a random walk w_t representing the stochastic trend and a stationary component z_t representing deviations or cyclical swings away from trend.

[5] A series y_t is said to be *integrated of order d*, denoted as $y_t \sim I(d)$, if it becomes stationary after differencing d times. The operator Δ will be used to denote the differencing operator, that is, $\Delta y_t = y_t - y_{t-1}$.

[6] The ARIMA representation helps explain why stochastic trends are also called *unit roots*: the autoregressive polynomial in the lag operator representation of ARIMA models integrated of order one has a root that is equal to one.

The DF, ADF, and PP tests have been criticized because of size distortion problems (Schwert, 1989), mainly if the series has a moving average portion of high order. These tests have also been criticized because of the low power showed in Monte Carlo simulations, as in the PP test case which generally is less than 0.10 (DeJong *et al.*, 1992). To solve these problems, modifications have been suggested to the DF and the PP tests, such as the augmented-DF test, and the tests of Perron and Ng (1996), and Elliot, Rothenberg, and Stock (1996).

On the other hand, tests for the null hypothesis of stationarity have been proposed as alternatives that are more powerful. Some of these are Tanaka (1990), Park (1990), Kwiatkowsky *et al.* (1992) – known as KPSS test, Choi (1994), and Arellano and Pantula (1995). Phillips and Xiao (1998), and Maddala and Kim (1998) provide a survey of theory, procedures, and an extensive list of references on unit roots. Kwiatkowsky *et al.* (1992) and Choi (1994) suggested that the most fruitful approach is to test the null hypothesis of stationarity for confirmatory analysis, i.e., to confirm the conclusion about unit roots. Various Monte Carlo studies show that using the ADF-KPSS combination provides similar confirmations as the PP-KPSS.

For the unit root test, the frequency and the span of observations are of importance. Shiller and Perron (1985) using Monte Carlo experiments found that power depends more on the span of the data rather than on the number of observations. When considering time aggregation problems, the distinction between *flow* data and *stock* data is important[7] (Maddala and Kim, 1998). Choi (1992) finds that using data generated by aggregating subinterval data results in lower power of unit root tests. Thus, using quarterly data is better than using annual data, and using monthly data is better than using quarterly data. Choi also finds that for aggregated data, PP tests are more powerful than the ADF test.

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[7] The *stock* variable concept is relevant in the context of skipping samples at an interval of m periods. With a stock variable y_t and sample values y_t^* we have $y_t^* = y_t$ for $t = m, 2m, 3m, \ldots$ In the case of a *flow* variable, however, it is a problem of time aggregation and we have $y_t^* = (1 + L + L^2 + \cdots + L^{m-1})y_t$. In this second type of variable, for example, if the quarterly model is $y_t = r y_{t-1} + u_t,\ t = 1, \ldots, T$ then the yearly model is $x_s = r^4 y_{s-1} + v_s,\ s = 1, 2, \ldots, T/4$, where $x_s = y_{4,s} + y_{4,s-1} + y_{4,s-2} + y_{4,s-3}$ and v_s is a moving-average of u_t.

2.2.2.3 **Seasonal Unit Roots**

All the tests on unit roots mentioned before assume there is only one root of interest, associated with a peak at the zero-frequency in the spectrum. Thus, this unique unit root describes the long memory properties of the series. However, quarterly or monthly economic time series usually exhibit strong seasonality, mainly characterized by the existence of (seasonal) unit roots associated to peaks at some seasonal frequencies in the spectrum (Hylleberg *et al.*, 1990). The definition of integration (Engle and Granger, 1987) can be generalized to include seasonal integration[8] (Hylleberg *et al.*, 1990; Lee, 1992). Strong evidences of unit roots at some seasonal frequencies have been found in a large number of seasonally unadjusted macroeconomic time series (Lee and Siklos, 1991; Ghysels, Lee, and Noh, 1994; Hylleberg, 1995). No applied work in agricultural commodity market modeling has yet reported the use of seasonal integration, although quarterly or monthly time series are used frequently. Tests for stochastic trends have been applied to high frequency commodity price data, showing evidence of stochastic trends (see for example Ardeni 1989; Baillie and Myers, 1991; Goodwin, 1992). For annual data, the evidence is not clear, perhaps because of the smaller number of observations available and/or in the low power of these tests (Kwiatkowsky *et al*., 1992).

A stochastic characteristic frequently shown by commodity price series is time-varying volatility, i.e., a tendency for the price series to move between periods where relatively large price changes are observed and periods where price movements or changes are relatively small. This behavior is especially

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[8] Lee (1992) defines seasonal integration as follows. Let $S(B)$ have a root with modulus 1 at frequency θ [i.e., $S(B) = (1-e^{i\theta}B)$] for $\theta \in (-\pi,\pi)$, and let $D(B)$ have all the unit roots at seasonal frequencies as well as the zero frequency, if any. A series x_t with no deterministic component is said to be seasonally integrated of order d at frequency θ, denoted as $x_t \sim I_\theta(d)$, if d is the smallest integer for which the representation $S(B)^d D(B)x_t = C(B)\varepsilon_t$ has the following properties: (i) the spectrum of $C(B)\varepsilon_t$ is bounded away from zero and infinity at all frequencies; (ii) $\{\varepsilon_t\}$ is a sequence of serially uncorrelated random vectors with finite and constant unconditional variance; (iii) the series is taken to be initialized by ε_t, $x_t = 0$ for $t < 0$. For simplicity, only the value $d = 0$ and $d = 1$ will be considered in this study. Some examples of integrated seasonal processes are in HEGY (1990). Consider for quarterly data, for example, the process $(1\text{-}B^4)x_t = \varepsilon_t$. Using the definition above, x_t can be said to be seasonal integrated for any $\theta = 0, \pi, \pm\pi/2$. That is, it has four roots with modulus one: one at the zero frequency ($\omega = 0$), one at two cycles per year [i.e., a half cycle per quarter ($\omega = 1/2$)], and a pair of complex roots at one cycle per year [i.e., a quarter cycle per quarter ($\omega = 1/4$)].

obvious in commodity prices sampled at high frequencies (i.e., daily, weekly, and monthly intervals), but seems less important in quarterly and annual data. Two ways of modeling volatility are the autoregressive conditional heteroskedastic (ARCH) model (Engle, 1982) and the generalized ARCH (GARCH) model (Bollerslev, 1986). Multivariate applications of these models to agricultural prices may be found in Holt and Aradhyula (1990).

2.2.2.4 **Multivariate Time-Series Models**

The issue of avoiding fragile empirical econometrics in the 1970s stimulated developments in topics other than estimation of structural models, such as *time-series econometrics* (Tomek, 1997). One process that has proven to be useful to describe dynamic relationships between economic variables is the *vector autoregressive moving average* (VARMA) process[9] (see for example Lütkepohl (1993) and Hamilton (1994) for a broad and detailed presentation).

Sims (1972, 1980) and others proposed vector autoregressive models[10] (VAR) as an alternative to structural models, an approach that was adopted and used more frequently than the more general and complex *(*VARMA) setup in agricultural economics. Few studies have adopted a VARMA representation, and when adopted, VAR models have outperformed VARMA models. Park (1990) evaluates the forecasting performance of five multivariate time-series models for the U.S. cattle sector, showing that the VAR related models outperform the VARMA model. Several criticisms concerning the classical way of building econometric models motivated the used of VAR models. The main criticism is that many

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9 We will say that a vector $\mathbf{y}_t$ is a *k-dimensional vector autoregressive-moving average process of order p and q*, VARMA(p,q), if it is generated as $\mathbf{y}_t = \Theta_1 \mathbf{y}_{t-1} + \ldots + \Theta_p \mathbf{y}_{t-p} + \mathbf{v}_t + \mathbf{A_1 v}_{t-1} + \ldots + \mathbf{A_q v}_{t-q}$, where $\Theta_n = \{\theta_{i,j}\}_n$, $i,j = 1,\ldots,k$ and $n = 1,\ldots,p$ (p is assumed to be finite and known), $\mathbf{A}_n = \{\alpha_{i,j}\}_n$, $i,j = 1,\ldots,k$ and $n = 1,\ldots,q$ (q is assumed to be finite and known) and $\mathbf{v}_t$ is $(k \times 1)$, a identically and independently distributed white noise disturbance, with mean zero and covariance matrix $\mathbf{S_u}$ (positive definite). In shorthand notation, the VARMA(p,q) representation can be written as $\Theta_p(L)\mathbf{y}_t = \mathbf{A}_q(L)\mathrm{v}_t$, where $\Theta_p(L) = \mathbf{I} - \Theta_1 L - \ldots - \Theta_p L^p$, $\mathbf{A}_q(L) = \mathbf{I} - \mathbf{A}_1 L - \ldots - \mathbf{A}_q L^q$, and L the lag operator such that $L^n \mathbf{y}_t = \mathbf{y}_{t-n}$. A VARMA($p$,$q$) is *stationary* if $\det[\Theta_p(z)] = |\Theta_p(z)| = |\mathbf{I} - \Theta_1 z - \ldots - \Theta_p z^p| \neq 0$ for $|z| \leq 1$, which implies that there exist a (possibly infinite order) MA representation $\mathbf{y}_t = \mathbf{v}_t + \mathbf{M_1 v}_{t-1} + \ldots = M(L)\mathbf{v}_t$, with $M(L) = \Theta_p(L)^{-\mathbf{1}}\mathbf{A}_q(L)$. A VARMA($p$,$q$) is *invertible* if $\det[\mathbf{A}_q(z)] = |\mathbf{A}_q(z)| = |\mathbf{I} - \mathbf{A}_1 z - \ldots - \mathbf{A}_q z^q| \neq 0$ for $|z| \leq 1$, which implies that there exist and AR representation $\mathbf{y}_t - \Phi_1 \mathbf{y}_{t-1} - \ldots - \Phi_p \mathbf{y}_{t-p} = \Phi(L)\mathbf{y}_t = \mathbf{v}_t$, with $\Phi(L) = \mathbf{A}_q(L)^{-\mathbf{1}}\Theta_p(L)$.

10 A *vector autoregression* process is a k-variate VARMA(p,q) process with $q = 0$.

untested assumptions are introduced, such as exogeneity assumptions, identification restrictions, or assumptions on the pattern of distributed lag coefficients. In this context, Sims proposes considering VAR models in which the *a priori* assumptions are much weaker. VAR models can be used to test theories on causality relationships (Granger, 1969), to test hypothesis about the time-series properties of a variable, and to generate conditional and unconditional forecasts, among other applications.

Since the mid-1970s, a number of studies have used VAR models to evaluate the magnitude and timing of macroeconomic impacts on agriculture. Use of the VAR method has been viewed as a way to obtain empirical evidence about these impacts that might not emerge from traditional structural but less dynamic econometric models. Empirical VAR results support the hypothesis that agricultural prices respond faster than manufactured product prices to a change in money supply (Barnett *et al.*, 1983; Devadoss and Myers, 1987; Saunders, 1988; Orden and Fackler, 1989; In and Mount, 1994).

The attractiveness of VAR models has an obvious counterpart, i.e. the number of parameters to be estimated quickly rises with the size of the model. Critics of VAR models have argued that it has the unfortunate consequence of obscuring some important identification issues (Cooley and Leroy, 1985).

Finally, if economic theory is not able to provide a fully specified DSEM, it will be necessary to use sample information for model specification. Sims (1980) has argued that economic theory is not likely to provide a completely and uniquely specified model. For example, in dynamic models with lagged endogenous variables and temporally correlated error terms where the exact lag lengths are not known *a priori*, the conditions for identification require distinguishing between lagged dependent variables and exogenous variables. To determine this, it will be necessary to use sample information for model specification. A number of criteria and procedures are discussed in the literature for estimating the order of a VAR process (Akaike, 1971, 1974; Schwarz, 1978; Quinn, 1980). A comparison of these criteria and further references can be found in Lütkepohl (1993).

2.2.3 Dynamic Simultaneous Equations Models (DSEM)

2.2.3.1 DSEMs Under Stationarity

The fact that a stationary VARMA representation is not unique[11] has received considerable attention in the literature and the model has been used in empirical multiple time series analysis. This property motivated the specifications of dynamics into SEMs (Zellner and Palm, 1974; Wallis, 1977; Chan and Wallis, 1978; Bohara and McNown, 1992; Lütkepohl, 1993). *Dynamic simultaneous equations model*[12] (DSEM) can be interpreted as parts of VARMA processes. The equations in a DSEM look very much like a SEM. The main differences are that lagged endogenous and exogenous variables are present in a DSEM, while the error terms are serially correlated (Judge *et al*, 1985). DSEMs are often called ARMAX models.

The DSEM specification contains a built in *structural form*, corresponding to the SEM specification. It is possible to derive the *final form*[13] from the structural form (Zellner and Palm, 1974; Prothero and Wallis, 1976; Wallis, 1977, Harvey, 1981). As the final form of the lag polynomials in the

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[11] Note that if $\mathbf{y}_t$ is a stationary VARMA(p,q), i.e. $\mathbf{y}_t$ has the representation $\Theta_p(L)\mathbf{y}_t = \mathbf{A}_q(L)\mathbf{A}_q(L)\mathrm{v}_t$, and $\Theta_p(L) = \mathbf{I} - \Theta_1 L - \ldots - \Theta_p L^p$ satisfies the condition $\det[\Theta_p(z)]\neq 0$ for $|z| \leq 1$, then if we left-multiply the given representation by the adjoint of $\Theta_p(L)$, i.e., by $\Theta_p^*(L) = |\Theta_p(L)|\ \Theta_p^{-1}(L)$, this results in

$$|\Theta_p(L)|\,\mathrm{y}_t = \Theta_p^*(L)\ \mathbf{A}_q(L)\mathbf{v}_t$$

which although is a new and different representation, but with the same autocovariance structure.
There are various other representations of VARMA processes that are useful occasionally. For instance, define a triangular matrix $\mathbf{P}$ such that $\mathbf{P}\Sigma_{\mathbf{v}}\mathbf{P}'$ is a diagonal matrix (decomposition known as *Choleski decomposition*), then

$$\mathbf{P}\mathbf{y}_t = \mathbf{P}\Theta_1\,\mathbf{y}_{t-1} + \ldots + \mathbf{P}\Theta_p\,\mathbf{y}_{t-p} + \mathbf{w}_t + \mathbf{P}\mathbf{A}_1\mathbf{P}^{-1}\mathbf{w}_{t-1} + \ldots + \mathbf{P}\mathbf{A}_q\mathbf{P}^{-1}\mathrm{w}_{t\text{-}q}$$

is an equivalent representation, where the white noise process $\mathbf{w}_t = \mathbf{P}\mathbf{v}_t$ has a diagonal variance-covariance matrix $\Sigma_{\mathbf{w}}$.

[12] The ARMA representation of a *dynamic SEM* is characterized by two equations (see *first* and *second* equations below) as follows. Let $\mathbf{y}_t' = (\mathbf{z}_t', \mathbf{x}_t')$, $\mathbf{z}_t$, $\mathbf{x}_t$ be k endogenous and m exogenous variables of a system, respectively, and $\mathbf{v}_{1t}$, $\mathbf{v}_{2t}$ be independent white noise processes.

Then $\Theta_p(L)\mathrm{y}_t = \begin{bmatrix}\Theta_{11}(L) & \Theta_{12}(L)\\ \mathbf{0} & \Theta_{22}(L)\end{bmatrix}\begin{bmatrix}\mathbf{z}_t\\ \mathbf{x}_t\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{11}(L) & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_{22}(L)\end{bmatrix}\begin{bmatrix}\mathbf{v}_{1t}\\ \mathbf{v}_{2t}\end{bmatrix}$,

which in turn can be written as $\Theta_p(L)\mathrm{y}_t = \begin{cases}\Theta_{11}(L)\mathbf{z}_t + \Theta_{12}(L)\mathbf{x}_t = \mathbf{A}_{11}(L)\mathrm{v}_{1t} & \text{(first equation)}\\ \Theta_{22}(L)\mathbf{x}_t = \mathbf{A}_{22}(L)\mathrm{v}_{2t} & \text{(second equation)}\end{cases}$.

[13] The *structural form* of a DSEM is what we called first equation in footnote 12, which written without the lag operator is

$$\Theta_{11,0}\mathbf{z}_t + \ldots + \Theta_{11,p}\mathbf{z}_{t-p} + \Theta_{12,0}\mathbf{x}_t + \ldots + \Theta_{12,q}\mathbf{x}_{t\text{-}q} = \mathbf{e}_t.$$

The corresponding *reduced form* is

$$\mathbf{z}_t = -\Theta_{11,0}^{-1}(\Theta_{11,1}\mathbf{z}_{t-1}\cdots + \Theta_{11,p}\mathbf{z}_{t-p}) - \Theta_{11,0}^{-1}(\Theta_{12}(L)\mathbf{x}_t + \Theta_{11,0}^{-1}\mathbf{e}_t$$

and the *final form* is

$$\mathbf{z}_t = -\Theta_{11}^{-1}(L)\Theta_{12}(L)\mathbf{x}_t + \Theta_{11}^{-1}(L)\,\mathbf{e}_t.$$

VARMA representation of a DSEM is usually not known from economic theory, then data have to be used to determine these operators. This is an important new idea in multiple time series modeling. As the VARMA is not unique, in the context of a DSEM it is desirable to identify the first equation without referencing the second one. Identification conditions for a stationary DSEM are given by Hatanaka (1975), and extended by Deistler (1976, 1978) and Deistler and Schrader (1979).

An extension of the DSEM is obtained when expectations of economic agents formed in a previous period are assumed to have an impact on the system[14] (Wallis, 1980). The expectations are called *rational* if they are formed using all information available at time t–1 (Muth, 1961). Theoretically, this extension can include expectations several periods ahead or expectations formed in earlier periods (Wallis, 1980). As was cited for the SEM models, the *Lucas critique* also applies here.

Another issue that has deserved strong attention in the literature is the discussion about a meaningful definition of a exogenous variable. The definition of exogeneity in the foregoing is closely related to the concept of causality in Granger's (1969) sense. Since many controversial economic theories exist, statistical tests on causality were developed (Granger, 1969). Broadly speaking, the concept of *causality* is defined as restrictions on the MA and AR coefficients of a VARMA specification, which contains past and present information of the variables. Standard statistical techniques can be used to test these restrictions, being the main virtue of the popularized *Granger-causality* concept. An updated revision on testing exogeneity is offered in Ericson and Irons (1994).

Maximum likelihood procedures are recommended to estimate the parameters of a VARMA process, as they result in consistent, asymptotically efficient, and normally distributed estimators. After a tentative model has been specified, checks for its adequacy have to be carried out (Judge *et al*., 1985). The significance of the individual parameters should be tested and insignificant parameters should be deleted in accordance with the principle of parsimony (Quenouille, 1957). A residual analysis can be based on a multivariate version of the portmanteau test. Further details and references on model checking procedures, such as criteria for determining the VAR order, whiteness of the residuals, normality, and structural changes may be found in Lütkepohl (1993).

A concluding remark to this review on VARMA is that multiple time series analysis is far too big to be covered completely in a review of this size. Many interesting problems for the stationary case remain untouched, but the traditional topics reviewed provide a background that is good enough for the purposes of reviewing new developments in this arena, and therefore, for this study, as provided next.

2.2.3.2 **DSEMs and Nonstationarity Conditions**

Although popular in the 1950s and 1970s, econometric research with SEMs has attracted less attention in the 1980s and 1990s, partly due to a perceived failure in large macroeconomic models and criticism of SEMs following the work of Sims (1980). But SEMs are still used for forecasting and policy analysis as in FAPRI (Devadoss et al, 1993) and the USDA (Bailey, 1989). One explanation is that it is hard to find methods that work better for policy analysis (Choi and Phillips, 1997). The statistical properties of traditional estimators of SEMs when time series are nonstationary have not attracted much attention from researchers either. Exceptions are the works of Hsiao (1997a, 1997b) and Choi and Phillips (1997), bringing new theoretical results that may be promising in commodity modeling.

Most of the theoretical and applied studies that deal with the analysis of nonstationary time series have taken a “time series” approach, namely, all the variables are treated as jointly dependent and prior information is rarely imposed (Hsiao, 1997). This approach raised serious questions concerning the use of the OLS parameter estimator of a VAR specification in the presence of nonstationarity in applied econometric works (Granger and Newbold, 1974). In practice, it has been found that the unrestricted VAR model gives very erratic estimates, because of high multicollinearity among the explanatory

[14] Rational expectations are easily incorporated in a DSEM, by adding a term $\Phi\mathbf{z}_t^*$ to the right-hand side of the first equation of the VARMA representation, where Φ is a $(k\times k)$ matrix with unknown parameters and $\mathbf{z}_t^*$ represent expectations of $\mathbf{z}_t$ in period $t-1$.

variables, and several restricted versions have been suggested. If some variables are I(1), then first-differences must be used. If some of these I(1) variables are cointegrated[15] then further restrictions on the parameters of the VAR model must be imposed. A viable alternative to the VAR model when variables are cointegrated is the *error correction model* (ECM[16]) first introduced into the econometric literature by Sargan (1964) and popularized by Davidson *et al.* (1978). Maddala and Kim (1998) report that the revival in popularity of the ECMs has been based on the demonstration by Granger and Weiss (1983) that if two variables are I(1) and are cointegrated then they can be modeled as having been generated by an ECM. The main characteristic of ECMs compared with the VARs is the notion of a long run equilibrium relationship and the introduction of past disequilibrium as explanatory variables in the dynamic behavior of current variables. Updated literature reviews on general applications of ECMs may be found in Sarker (1995) and for agriculture in Zapata and Gil (1998).

In a multivariate and parametric framework, cointegration estimation and testing are commonly done using the vector autoregressive restricted maximum likelihood (VAR-RML) procedure of Johansen (1988) and Johansen and Juselius (1990). Although the Johansen approach is very popular, it has some problems that must be taken into consideration. It is well known that the VAR-RML rank test for the number of cointegrating vectors and the VAR-RML estimates of the cointegrating matrix have nonsymmetrical distributions in the presence of non-identical and independent errors. Johansen and Juselius (1990) provide tables with selected percentage points of its asymptotic distribution. Toda and

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15 A $(k\times1)$ vector $\mathbf{y}_t \sim I(d)$ is said to be *cointegrated of order* (d,b) if there exists a vector $\mathbf{a}$ different from $\mathbf{0}$ such that $\mathbf{a}'\mathbf{y}_t$ is $I(d\text{-}b)$. In practice, $d = b = 1$.

16 For a single time series y_t, the ECM links the realized value y_t to its long run or equilibrium value $y_t^* = b' z_t$. Then, the simplest form of an ECM is $\Delta y_t = l_1 \Delta y_t^* + l_2 (y_{t-1}^* - y_{t-1})$, where λ_1 and λ_2 are both greater than zero, and the last term represents past disequilibrium. The *partial adjustment* model is given by $\Delta y_t = l(y_t^* - y_{t-1}) = l \Delta y_t^* + l(y_{t-1}^* - y_{t-1})$. Thus, the partial adjustment model corresponds to the ECM with $\lambda_1 = \lambda_2$. In matrix notation and for a VAR model as noted in footnotes 10 and 9, a possible way to write the ECM is $\Delta\mathbf{y}_t = \Pi\mathbf{y}_{t-1} + \Gamma_1\Delta\mathbf{y}_{t-1} + \ldots + \Gamma_k\Delta\mathbf{y}_{t-k+1} + \mathbf{u}_t$, where $\Pi = -\mathrm{I} + \sum_{i=1}^{k}\Theta_i$ and $\Gamma_j = -\sum_{i=j}^{k}\Theta_i$ for $j = 1,\ldots,k$. Since $\mathbf{y}_t$ is I(1), and $\Delta\mathbf{y}_{t-1}, \ldots, \Delta\mathbf{y}_{t-k+1}$ are all I(0), in order that this equation be consistent, Π should not be of full rank. More over, letting its rank be denoted by r, then we can write $\Pi = ab'$, where α is a $n\times r$ matrix that contains the error correction terms measuring the *speed of* (*partial*) *adjustment* of the system, and b' is an $r\times n$ matrix containing the cointegrating vectors, with $b'\mathbf{y}_{t-1} = \mathbf{y}_t^*$ explaining the long run equilibrium.

Phillips (1993) show that the asymptotic distribution of the test in the unrestricted VAR has nuisance parameters and is nonstandard.

In agricultural commodity market modeling, the theory of cointegration has been applied extensively. Examples of early works are the studies on the comovement that different commodity prices exhibit (Ardeny, 1989; Zapata and Garcia, 1990; Pindyck and Rotemberg, 1990; Tomek and Myers, 1993), the dynamics of price linkages and international commodity markets (Mohanty *et al.*, 1995), and the relationship between agricultural productivity and exports (Arnade and Vasavada, 1995), among other applications. The contribution of these works is the use of a nonstationary and cointegrated framework to better understand the long and short run dynamics involved in the problems studied.

It is well known that a parametric framework imposes too much structure, mainly in terms of the underlying distribution of the data generating processes. So non-parametric approaches for cointegrating regressions may be found in the literature. Phillips and Hansen (1990) introduced a nonparametric methodology that provides optimal estimates of cointegrating regressions. They named the method as fully modified OLS (FM-OLS) regression, because it modifies OLS to account for serial correlation effects and for the endogeneity in the regressors that results from the existence of a cointegrating relationship.

Phillips (1995) provides a general framework to study the asymptotic behavior of the FM-OLS in models with full rank, and extends the results to VAR models where there are possible some unit roots and some cointegrating relations (FM-VAR). The main difference between the VAR-RML procedure and the FM-VAR is that FM-VAR does not employ reduced rank regression, using no knowledge or pre-test information about the rank of the cointegrating space. Thus, FM-VAR is a good alternative to unrestricted levels VAR estimation, because it may be used without regard to the number of unit roots in the system. It is worth mentioning that the papers by Quintos (1997) and Choi and Phillips (1997) heavily rely in FM-OLS. Quintos (1997), for example, applies the fully modified (FM) corrections of Phillips and Hansen (1990) to the framework of Johansen and Juselius (1990) to test for the number of cointegrating relationships among multivariate time series. Phillips (1995) shows that the FM-VAR test for the "null of cointegration" has a standard χ^2 distribution, in contrast to the nonstandard and possibly nonsymmetric distribution of the LR test of Johansen. However, under less restrictive assumptions on the errors, the FM-VAR test is degenerate for the "null of no cointegration". The FM-VARX approach solves the problem of degeneracy, with rank tests for the number of cointegrating vectors and a test for causality that are chi-squares distributed under a large class of errors. The procedure basically consists in augmenting the VAR specification by an I(0) exogenous or predetermined variable (VARX), and then using the FM correction approach (FM-VARX). A nice property of FM-VARX is that it does not require knowledge of the number of lagged variables in the VAR, an improvement over the Toda and Yamamoto (1995) approach. FM-VARX does not require errors to be independent and identically distributed.

2.2.4 New Developments in Time Series Econometrics

Many quarterly economic time series display trending and seasonal patterns that do not appear to be constant over time. A representation of time series that accounts for time-varying trends and seasonals assumes the presence of stochastic trends at the zero and seasonal frequencies. Thus, given a set of economic time series, it is of interest to study whether these have stochastic trends at certain frequencies in common. A usual next step in analyzing a set of seasonal (weekly, monthly, or quarterly) time series involves testing for cointegration at the nonseasonal and seasonal frequencies[17]. Lee (1992) proposes tests for seasonal cointegration based on a fully specified multivariate time series model, while Engle *et al.* (1993) suggest so-called residual-bases tests. The seminal papers of Hylleberg *et al.* (1990) and Lee(1992) triggered an important amount of applied and theoretical research. Some of these are the works by Engle *et al.* (1993), Kunst (1993), Lee and Siklos (1995), Franses (1996), Kunst and Franses (1998), Franses and Kunst (1999), and Johansen and Schaumburg (1999).

[17] HEGY (1990) and Lee (1992) generalize the idea of cointegration in Granger (1986) and Engle and Granger (1987) to define seasonal cointegration as follows: Let all components of x_t be seasonally integrated of order 1 at frequency θ, i.e., $x_t \sim I_\theta(1)$. The components of the vector x_t are said to be seasonally cointegrated at frequency θ, denoted $x_t \sim CI_\theta(1,1)$, if there exists a vector α ($\neq 0$) so that $z_t = \alpha' x_t \sim I_\theta(0)$.

More recently, Hsiao (1997a, 1997b) and Choi and Phillips (1997) provide alternative models for SEMs that take into account nonstationarity and cointegration (COI-SEM), an approach that is considered relevant for the purposes of our study. A first glance at these works immediately recalls the Zellner and Palm (1974) SEM-Time Series approach, which was constructed on the assumption of stationary series. The idea supporting the COI-SEM procedure of Hsiao, very similar to that of Choi and Phillips, is to re-write a given SEM in such a way that the nonstationary and the possible cointegrated structure of the system are built-in. The work by Hsiao (1997) presents an updated perspective of the Cowles Commission structural equations that account for the advances of time series on regression analysis with integrated regressors. Important questions that Hsiao studies include: What is the relationship between the multiple time series model and the structural equation model with or without cointegration? Are the concepts of identification for nonstationary data relevant? Does the separation of long run and short run relationships require separate sets of identification conditions? Does the super-consistency result of Phillips and Durlauf (1986) and Stock (1987) render the issue of "simultaneity bias" irrelevant for models involving integrated regressors? Does cointegration call for a new estimation method in structural equations? Hsiao's main conclusions on these important issues may be summarized as follows: (1) If one takes a SEM approach of dichotomizing variables into endogenous and exogenous variables, the presence and absence of cointegration is pre-assumed from the way the model is written down. (2) That identification conditions derived for stationary variables hold for integrated variables under appropriate assumptions. (3) There is only one set of conditions that simultaneously identifies both the short-run dynamics and long-run relations. (4) Simultaneity bias remains a legitimate concern when regressors are integrated, which must be considered to derive the limiting distribution of SEM estimators under cointegration. (5) Despite the fact that variables may be integrated, standard SEM estimation and testing procedures can still be applied. For these purposes, Hsiao derives the limiting distributions of the conventional SEM estimators (OLS, 2SLS, 3SLS) under cointegration and Wald type test statistics involving integrated regressors.

On the other hand, Choi's and Phillips (1997) approach does not involve lagged variables. Instead, the short-run dynamics are nonparametrically incorporated into the regression errors (as in Phillips and Hansen, 1990). Once the transformation is done, a new expression of the SEM is obtained, and the standard techniques of OLS and 2SLS may be used for estimation purposes.

2.2.5 Impulse Response Functions and Forecasts

Econometric commodity market models have been constructed for *market analysis*. This involves structural parameter estimation and derivation of elasticities, for use in *economic policy analysis* or forecasting purposes (Garcia and Leuthold, 1997).

For economic policy analysis, outcome variables sensible for the objectives of economic policies and control variables, which are the instruments of such policies, must be identified. By varying the levels of control variables, the impact of policies on the outcome variables can be observed. Control variables are usually exogenous variables generated independently of the model (instruments) and outcome variables are a subset of the endogenous variables (whose values are solved in the model), with the control and outcome variables linked by the model structure. The reduced-form of this model contains on its right hand side the non-policy exogenous variables and variables representing the control exogenous variables. This kind of analysis is often called *dynamic multiplier analysis* (Intriligator, 1978, 1983), recalling that dynamic multipliers have to be carefully interpreted since the VARMA representation is not unique.

In applied work, it is often of interest to know the response of one variable to an impulse in another variable in a vector autoregressive model that also involves a number of other variables. This kind of analysis is often called *impulse response function* analysis (Lütkephl, 1993). A problematic assumption in this analysis is that a shock occurs only in one variable at a time. Such an assumption may be reasonable if the shocks in different variables are independent. If they are not independent, the technique of responses to orthogonal impulses must be adopted (Lütkepohl, 1993).

Confidence bands for impulse response estimates are often based on asymptotic normal approximation (Lütkepohl, 1990), nonparametric bootstrap methods (Runkle, 1987), or parametric Monte Carlo integration procedures (Doan, 1992). Moreover, if variables in the system are cointegrated then the procedure described by Lütkepohl and Reimers (1992a) can be followed, which verifies well known asymptotic properties. In the presence of unit roots, Phillips (1998) describes the impulse response asymptotic distribution that closely resembles the cointegrated case.

Impulse response functions are often used in agricultural economics to study price dynamics, market integration, and linkages between the macro economy and agriculture, among other areas. However, macroeconomic data sets for the postwar era are comparatively short, which has led observers to question the statistical reliability of impulse response estimates from unrestricted VARs (Lutz, 1998).

On the other hand, models built for forecasting purposes usually start as market analysis models, in the sense that the model is specified according to proper economic theory and market conditions. Forecasts can then be generated from either the structural model or its reduced-form version. In the case of forecasting models, however, the primal objective (model performance or forecasting accuracy) differs from the market analysis objective (model specification). A model for forecasting is preferred, in general, to be more parsimonious in both the number of variables and equations (Tomek and Myers, 1993; Tomek, 1997; Garcia and Leuthold, 1997). Diebold (1998) provides a comprehensive presentation and discussion on various issues that ought to be considered when building forecasting models.

The evaluation of forecasting accuracy via measures of point estimates is a well-established practice in the forecasting literature. The mean error (ME), the error variance (EV), the mean square error (MSE), and the mean absolute error (MAE) are often used for evaluating forecasting performance. The usual practice for choosing among alternative forecasting models has been to select a model that shows a lower accuracy measure, but with no attempt in general to assess its sampling uncertainty. In this sense, the work by Parks (1990) is a good exception. More recently, the sampling uncertainty of point estimates of forecast accuracy has received considerable attention in econometric and forecasting literature (Diebold and Mariano, 1995; West, 1996; Stock and Watson, 1999; West and McCracken, 1998). This set of rich contributions allows for the evaluation of alternative forecasting models and assessing their sampling uncertainty simultaneously, one of the specific objectives of this research.

2.3 Relevance of the Literature Review

This chapter overviewed the economic theory and econometric methods that support the modeling of agricultural commodity markets. In particular, it reviewed some specific well-known models for the U.S. wheat market. The model of Chambers and Just (1981) has several appealing features that make it a good candidate for blending structural commodity models with multiple time series. First, it is a simple market commodity model, which allows for determining the equilibrium price and total quantities supplied and demanded that clear the U.S. wheat markets. It provides a link between the U.S. wheat sector, U.S. macroeconomic conditions, and international wheat markets, through six handy exogenous variables. Second, it incorporates lagged endogenous variables that allow for modeling the dynamics of the system. Finally, its simplicity makes it tractable for implementing new alternative time series econometric specifications and evaluating their forecast abilities and simulation analysis capabilities. The Chambers and Just model is used in this research.

This literature review provides a broad perspective of the historical developments and contributions that shaped the modeling of commodity markets, and it introduces current developments in nonstationary time-series analysis. These later developments allow for innovative applications in search of agricultural commodity models that handle more coherently the nature of the data and the economic theory of these models. More specifically, until the present day, the nonstationary property of some variables of these models have only been handled via reduced forms that sometimes prove inconsistent with economic theory, yielding contradictory results. The seminal works of Hsiao (1997a, 1997b) and Choi and Phillips (1997) effectively teach how to hold the flavor and elegance of the structural models for inferential, dynamic simulation, or forecasting purposes. Yet these works also account for the short-run dynamics and the long-run relationships after imposing the exclusions on some endogenous and exogenous variables in each structural equation in the system. Hsiao's relatively new approach may offer an alternative to the widely-used error correction model (Engle and Granger, 1987) in segmenting short-run dynamics and long-run equilibrium relationships in structural-time series modeling.

Another important contribution to the theory of unit roots is that of seasonal unit roots, an aspect that is often ignored in commodity modeling. The developments in the field of seasonal integration and seasonal cointegration (Lee, 1992; Johansen and Schaumburg, 1999) promise an improved way to model agricultural commodity markets. Naturally, the concept of seasonal cointegration emerged also.

The question of how to use SEMs when time series available are nonstationary has not attracted attention from researchers modeling agricultural commodity markets, although SEMs are still used for forecasting and policy analysis (i.e. Devadoss *et al.*, 1993). Essentially, this review of literature has stimulated the extension of the results of Hsiao on modeling cointegration within the framework of structural models advocated by the Cowles Commission, and the modeling of seasonal cointegration. This innovation, a contribution of the research presented in Chapter 3, merges the modeling of seasonal cointegration within structural models.

CHAPTER 3

ECONOMETRIC THEORY AND THE APPLICATION

This chapter provides an overview of the econometric developments in dynamic-structural time-series modeling and illustrates how they can be used for impulse-response and dynamic multiplier analyses and forecasting in commodity modeling. The models presented in this chapter can be interpreted as a new-generation of economic-theory and data-coherent market models. The relevance of such models in forecasting and dynamic analyses is unknown. In the agricultural economics literature, vector autoregressive (VAR) and error correction models are frequently encountered. However, most applications to date have ignored the role of integration and cointegration at seasonal frequencies. These VAR-type models are also included in the empirical comparisons.

The plan of this Chapter is as follows. Section 3.1 presents a general self-contained overview of the econometric modeling strategy adopted for this research. Section 3.2 fully describes a new method proposed for modeling seasonal cointegration within a dynamic simultaneous equation model. This development is an extension of a model proposed by Hsiao (1997a, 1997b) for nonseasonal cointegrated systems. Section 3.3 presents the estimation procedures for four selected methods. Sections 3.4 presents the methodology adopted for the construction and evaluation of forecast accuracy of the selected models and the generation and evaluation of the impulse responses and dynamic multipliers, respectively. Finally, Section 3.5 describes the U.S. wheat market model derived from the model of Chambers and Just (CJ) (1981) specification.

3.1 Modeling Strategy

A general framework that links dynamic simultaneous equations models (DSEM) and time series models, and the implications of cointegration and seasonal cointegration for structural equation modeling are presented in this subsection. The econometric scope of the methods selected to conduct this research

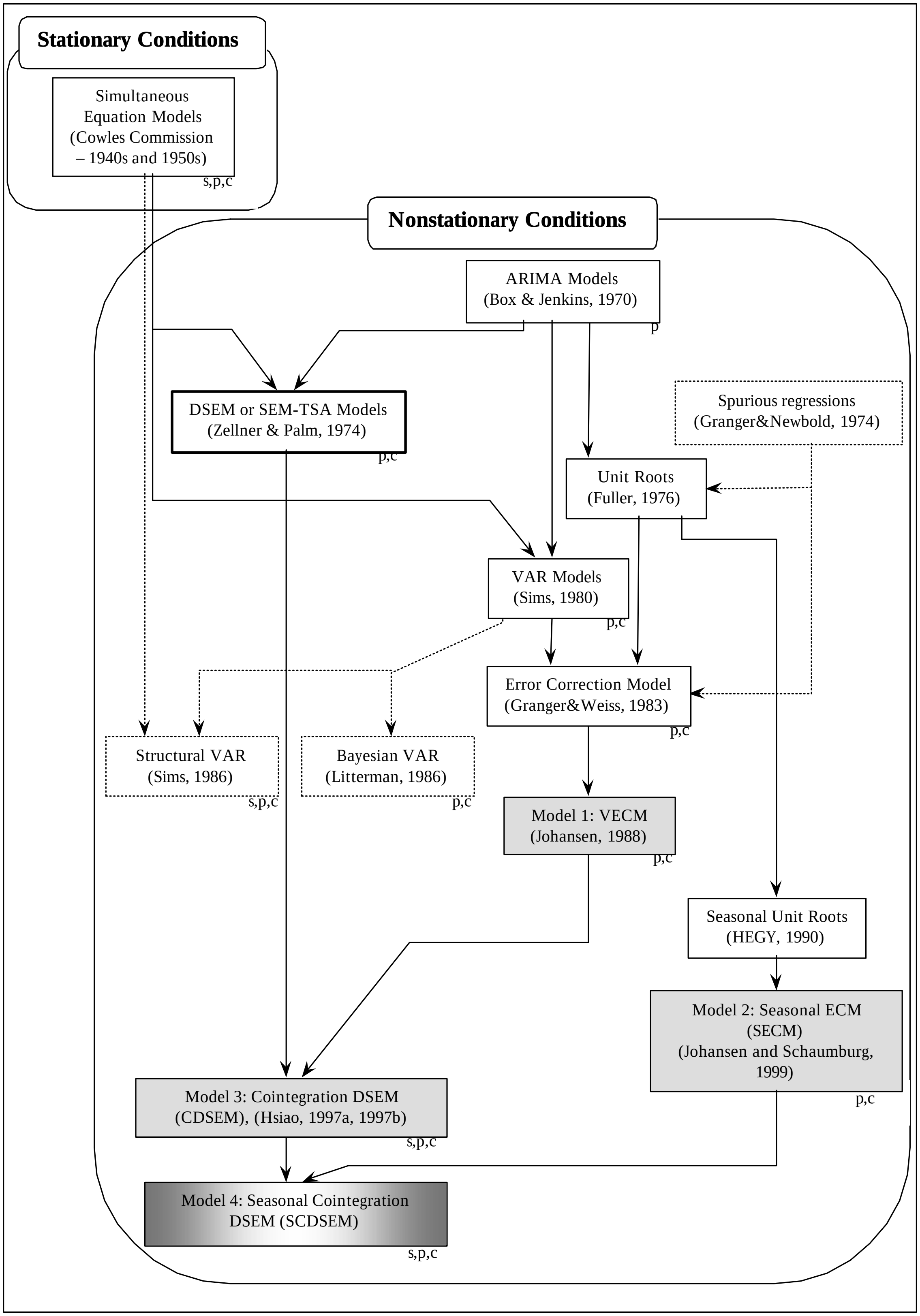

Figure 3.1. Time series, dynamic simultaneous equation models, cointegration and seasonal cointegration parametric topics

Letters on the bottom and right of a box indicate possible uses of that method: structural analysis(s), prediction(p), control(c) is summarized in Figure 3.1, which depicts the main concepts and antecedents to economic modeling with multiple time series. This figure provides a broad overview of the main econometric and time series contributions to date since the time tested works of the Cowles Commission in the 1940s. The selected models are highlighted as shaded boxes, and include the vector error correction model (VECM), the seasonal ECM (SVECM), the cointegration dynamic simultaneous equation model (CDSEM), and the seasonal cointegration DSEM (SCDSEM). As can be seen in this figure, the SCDSEM –a new model developed in this research, is constructed on the basis of the CDSEM of Hsiao (1997a, 1997b) and the SVECM of Johansen and Schaumburg (1999). The CDSEM, in turn, blends the concepts of the VECM (i.e., Johansen, 1988) and the DSEM (Zellner and palm, 1974), while the DSEM of Zellner and Palm (bold box) represents the first serious attempt for combining the simultaneous equation modeling approach advocated by the Cowles Commission with the ARIMA time series modeling approach of Box and Jenkins (1976). The major frame entitled "nonstationary conditions" contains methods that are able to account for the presence of unit roots in the data. In this sense, the ARIMA models of Box and Jenkins (1976), the works on spurious regressions (Granger and Newbold, 1974), on unit roots (Fuller, 1976), on error correction models (Granger and Weiss, 1983), and on seasonal unit roots (Hylleberg *et al.*, 1990), are framing the modeling strategy adopted in this study.

The analysis of simultaneous equation models (SEM) using multiple time series (MTS) has received limited empirical investigation. Most MTS models found in the empirical literature have used vector autoregressive (VAR) specifications, introduced by Sims (1980), in forecasting and dynamic multiplier analyses. Despite their popularity, VAR-type models have the limitation of treating all variables as jointly dependent, ignoring, therefore, prior information prescribed by economic theory. The first modern treatment of combining time series with simultaneous equation models (SEM) was introduced by Zellner and Palm(1974). They derived associated reduced form and transfer function equation systems similar to the single-equation models of Quenouille (1957) and to the widely known autoregressive-integrated-moving average (ARIMA) models of Box and Jenkins (1970). Structural equation modeling in economics, on the other hand, dates back to Haavelmo (1943) and the various works of the Cowles Commission since ending the 1930s (for a summary, see Christ, 1994). Zellner and Palm's analysis went to the core of what today still is an area of much needed research, how to best combine economic theory models with time series data. Their idea was to formulate models that provided a blend between economic theory, that is, models that incorporate structural characteristics (endogenous-exogenous relationships) of simultaneous equation models (SEM) and that are also coherent with nonstationary properties of economic data. In this framework, dynamics are driven by the data since economic theory provides little guidance. The analysis generated a new class of hybrid models that transformed ARIMA structures into a dynamic SEM (DSEM) in reduced form. In previous econometric work, these reduced form specifications were known as final equations (Theil and Boot, 1962; Kmenta 1971) or transfer functions (Box and Jenkins, 1970).

A unique feature of DSEMs is the accounting for prior information derived from economic theory in the structure of the parameter matrices of ARIMA models. This re-specification of economic-theory consistent ARIMA models resulted in the *Zellner and Palm form* of a dynamic simultaneous equation model (Zellner and Palm, 1974),

$$\Gamma(L)\Delta\mathbf{y}_t + \mathrm{B}(L)\Delta\mathbf{x}_t = \Theta_{11}(L)\mathbf{e}_{1t}, \tag{3.1}$$

$$V_{22}^{*}(L)\Delta\mathbf{x}_t = \Theta_{22}(L)\mathbf{e}_{2,t}, \tag{3.2}$$

where $\mathbf{y}_t$ and $\mathbf{x}_t$ are the endogenous and the exogenous variables of dimension G and K, $\Gamma(L)$ and $\mathrm{B}(L)$ are lag polynomial matrices, respectively. Equation (3.2) describes an independent process for the exogenous variables (Hsiao, 1997a). In the terminology of modern time series econometrics, if $\mathbf{y}_t$ and $\mathbf{x}_t$ are not cointegrated, then they could be modeled by a process such as (3.1). This was an improvement over traditional vector autoregressive models (VAR) because model (3.1) maintains the endogenous-exogenous characteristics of SEM and has a separate process (3.2) explaining the exogenous variables.

The adequacy of model (3.1)-(3.2) as a general specification for economic modeling with variables that may be integrated and possibly cointegrated was not resolved until cointegration theory appeared (Engle and Granger, 1987). In a cointegration framework, (3.1) would represent a misspecified model because of the omission of long-run relationships of the variables. Additionally, conditions for identification of SEM were settled by the Cowles Commission, but in the context of structural-time series models, questions remained regarding identification when data are nonstationary and the relationship between short and long run dynamics for identification (Johansen and Juselius, 1995). Similarly, distributional results for estimators and test statistics were the subject of much inquiry. The works of Hsiao (1997a, 1997b) provide answers to the above questions by blending cointegration and dynamic SEM. As will be discussed below, Zellner and Palm's model is one possible model in the more general specification approach of Hsiao.

Hsiao begins the analysis by assuming that endogenous and exogenous variables of SEM, $\mathbf{y}_t$ and $\mathbf{x}_t$, of dimension $G \times 1$ and $K \times 1$, respectively, are generated by an autoregressive model of the form:

$$\Phi(L)\mathbf{w}_t = e_t, \tag{3.3}$$

where $\mathbf{w}'_t = (\mathbf{y}'_t, \mathbf{x}'_t)$ is a $(G+K)\times 1$ vector of I(1) random variables, $\Phi(L)$ is $(G+K)\times(G+K)$ matrix of polynomials in the lag operator L, $\Phi(L) = \sum \Phi_j L^j$, and e_t is a $(G+K)\times 1$ vector of independently, identically distributed random variables with mean zero and covariance matrix Ω^*. This representation can be reparameterized by imposing a normalization that $M(L)$ is diagonal in the factoring of $\Phi(L) = M(L)V(L)$, where the roots of $|M(L)| = 0$ and $|V(L)| = 0$ are equal to one or lying outside the unit-circle, respectively. By diagonalization, $|M(L)| = (1-L)^d$ where d is the number of linearly independent I(1) processes in $\mathbf{w}_t$ and $G+K-d$ denotes the number of linearly independent cointegration processes. Using this notation, (3.3) can we rewritten as:

$$\begin{bmatrix} M_{11}(L) & 0 \\ 0 & M_{22}(L) \end{bmatrix} \begin{bmatrix} V_{11}(L) & V_{12}(L) \\ V_{12}(L) & V_{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{e}_{1t} \\ \mathbf{e}_{2t} \end{bmatrix} \tag{3.4}$$

where $M_{ij}(L), V_{ij}(L)$, $\mathbf{e}_{1t}$, and $\mathbf{e}_{2t}$ are conformable partitions. The specification is general enough to allow for the following cases:

Case 1: *No cointegration*. If $\mathbf{w}_t$ is not cointegrated then $M(L)$ is a first-differencing operator and has dimension $G+K$, that is, $M(L)=\Delta\mathbf{I}_{G+K}$ and $\Delta=(1-L)$ is the difference operator.

Case 2: *Cointegration between* $\mathbf{y}_t$ *and* $\mathbf{x}_t$. If the cointegrating rank is G, then $M_{11}(L)=\mathbf{I}_G$, and $M_{22}(L)=\Delta\mathbf{I}_K$.

Case 3: *Dynamic simultaneous equation model of Zellner and Palm* (1974). Under certain conditions (existence of common factors in the coefficient matrices of the lag polynomial) and no cointegration, if $\mathbf{x}_t$ is exogenous (i.e., $V_{12}(L)=0$) and the covariance between the equation errors is zero, then equation (3.4) reduces to equations (3.1)-(3.2). If $\mathbf{y}_t$ and $\mathbf{x}_t$, are cointegrated, however, equation (3.4) becomes

$$\Gamma(L)\mathbf{y}_t + B(L)\mathbf{x}_t = \Theta_{11}(L)\mathbf{e}_{1t}, \tag{3.5}$$

$$V_{22}^{*}(L)\Delta\mathbf{x}_t = \Theta_{22}(L)\mathbf{e}_{2t}. \tag{3.6}$$

Note that models (3.1)-(3.2) and (3.5)-(3.6) are the conventional DSEM with moving average errors but that (3.5)-(3.6) brings cointegration theory in the specification. This is what explains the use of levels of $\mathbf{y}_t$ and $\mathbf{x}_t$ in models in equation (3.5). This specification is very different from the typical expression for error-correction models that have appeared in the econometrics literature. However, Hsiao proves that is possible to re-specify this model in error-correction form through the following:

$$\Gamma^{*}(L)\Delta\mathbf{y}_t + B^{*}(L)\Delta\mathbf{x}_t + \Gamma(1)\mathbf{y}_{t-1} + B(1)\mathbf{x}_{t-1} = \mathbf{u}_t, \tag{3.7}$$

$$V_{22}^{*}(L)\Delta\mathbf{x}_t = \Theta_{22}(L)\mathbf{e}_{2t}. \tag{3.8}$$

In equations (3.7)-(3.8), $\Gamma(1)\mathbf{y}_{t-1} + B(1)\mathbf{x}_{t-1}$ and $\Gamma^{*}(L)\Delta\mathbf{y}_t + B^{*}(L)\Delta\mathbf{x}_t$ may be viewed as the implied long run and short run relations between $\mathbf{y}_t$ and $\mathbf{x}_t$, where $\mathbf{u}_t = \Theta_{11}\mathbf{e}_{1t}$. This specification is what makes clear that if $\mathbf{y}_t$ and $\mathbf{x}_t$ are cointegrated, then the levels of $\mathbf{y}_t$ and $\mathbf{x}_t$ along with the differences should appear in the model. Comparing this model to that in (3.1)-(3.2) makes explicit its misspecification in the presence of cointegration. For the exposition that follows, Equation (3.1)may be written without using the lag polynomials, assuming p and q are the orders of $\Gamma(L)$ and $\mathrm{B}(L)$, respectively,

$$\sum_{j=0}^{p} \Phi_j^{\bullet} \mathbf{y}_{t-j} + \sum_{j=0}^{q} \Theta_j^{\bullet} \mathbf{x}_{t-j} = f_0 + \mathbf{e}_t, \quad t = 1,...,T \tag{3.9}$$

in which f_0 is a general $G \times 1$ vector of unknown constants, $\Phi_j^{\bullet}$ and $\Theta_j^{\bullet}$ are $G \times G$ and $G \times K$ unknown matrices. A more convenient way of writting (3.9), for inference purposes, is,

$$\mathbf{y}_t = f_0 + \sum_{j=1}^{p} \Phi_j \mathbf{y}_{t-j} + \sum_{j=0}^{q} \Theta_j \mathbf{x}_{t-j} + \mathbf{e}_t, \qquad t = 1,...,T\ , \tag{3.10}$$

where $\Phi_j = -(\Phi_0^{\bullet})^{-1} \Phi_j^{\bullet}$ and $\Theta_j = -(\Phi_0^{\bullet})^{-1} \Theta_j^{\bullet}$ must be estimated from the data, as well as f_0, which in turn may be written as,

$$f(L)\mathbf{y}_t - q(L)\mathbf{x}_t = \mathbf{e}_t\ , \tag{3.11}$$

where $f(L) = \mathbf{I}_G - \Phi_1 L - \Phi_2 L^2 - \cdots \Phi_p L^p$ and $q(L) = -(\Theta_0 L^0 + \Theta_1 L^1 + \cdots + \Theta_q L^q)$.

Throughout this theoretical overview, error-correction models (VECM) of the vector autoregressive type have been omitted because those models are more common in the literature. In section 3.2, however, the theory of VECM will be discussed first in the context of seasonal integration and cointegration, so that these more familiar models can be put in perspective with the above developments.

To sum up this brief overview, the dissertation will apply the above cointegration and dynamic simultaneous equation models to the U.S. wheat market. VECM with and without seasonal integration and cointegration will also be evaluated in an effort to incorporate the most recent developments in the econometrics literature on the subject of specifying multiple time series models. Table 3.1 presents the functional forms of the selected models and the definition of terms and notation. In the top two rows of this table, the class of error-correction models is presented and in the two bottom rows the class of structural time series models are introduced. A brief outline of the two general classes of models presented in Table 3.1 and the specific models used in the application are provided below.

Table 3.1 Functional form, terms, and assumptions of the selected econometrics models

Model name and functional form	Description of terms and assumptions
Model 1: ***Nonseasonal vector error-correction model (VECM)*** $\Delta \mathbf{y}_t = f_0 + \sum_{j=1}^{p-1}\Phi_j^*\Delta\mathbf{y}_{t-j} + \sum_{j=0}^{q-1}\Theta_j^*\Delta\mathbf{x}_{t-j}$ $-\Phi^*\mathbf{y}_{t-1} - \Theta^*\mathbf{x}_{t-1} + f^*\mathbf{S}_t + \mathbf{u}_t,\ t=1,\dots,T$	f_0 is a general $G\times 1$ vector of unknown constants, $\Phi_i^* = \sum_{j=2}^{p}\Phi_j,\ i=1,\dots,p-1$, $\Theta_i^* = \sum_{j=2}^{p}\Theta_j,\ i=1,\dots,q-1$, $\Phi^* = \mathbf{I}_G - \Phi_1 - \cdots - \Phi_p$, $\Theta^* = \Theta_0 + \Theta_1 + \cdots + \Theta_q$, and $\Phi^*\mathbf{y}_{t-1} + \Theta^*\mathbf{x}_{t-1}$ and $\sum_{j=1}^{p-1}\Phi_j^*\Delta\mathbf{y}_{t-j} + \sum_{j=0}^{q-1}\Theta_j^*\Delta\mathbf{x}_{t-j}$ may be viewed as the implied long run and short run relations between $\mathbf{y}_t$ and $\mathbf{x}_t$, and $\mathbf{u}_t \sim\sim MVN(\mathbf{0},\Sigma_{\mathbf{u}})$
Model 2: ***Seasonal vector error-correction model (SVECM)*** $\Delta_4\mathbf{y}_t = m + d\,\mathbf{S}_t + p_1\mathbf{y}_{1,t-1} + p_2\mathbf{y}_{2,t-1} + p_3\mathbf{y}_{3,t-2} + p_4\mathbf{y}_{3,t-1} +$ $P_1\mathbf{x}_{1,t-1} + P_2\mathbf{x}_{2,t-1} + P_3\mathbf{x}_{3,t-2} + P_4\mathbf{x}_{3,t-1} +$ $A_1\Delta_4\mathbf{y}_{t-1} + \cdots + A_{p^*-4}\Delta_4\mathbf{y}_{t-p^*+4} +$ $B_0\Delta_4\mathbf{x}_t + B_1\Delta_4\mathbf{x}_{t-1} + \cdots + B_{p^*-4}\Delta_4\mathbf{x}_{t-p^*+4} + \mathbf{u}_t$	ì is a constant term, $d = (m_1, d_1, d_2, d_3)$, $\mathbf{S}_t = \left(1, \cos p(t-1), \cos\frac{p}{2}(t-1), \cos\frac{p}{2}(t-2)\right)$, $\mathbf{y}_{1,t-1} = S_1(B)\mathbf{y}_t$, $\mathbf{x}_{1,t-1} = S_1(B)\mathbf{x}_t$, $S_1(B) = (B + B^2 + B^3 + B^4)$, $\mathbf{y}_{2,t-1} = S_2(B)\mathbf{y}_2$, $\mathbf{x}_{2,t-1} = S_2(B)\mathbf{x}_t$, $S_2(B) = -(B - B^2 + B^3 - B^4)$, $\mathbf{y}_{3,t-2} = S_3(B)\mathbf{y}_t$, $\mathbf{x}_{3,t-2} = S_3(B)\mathbf{x}_t$, $S_3(B) = -(B^2 - B^4)$, $\mathbf{w}'_{3,t-2} = (\mathbf{y}'_{3,t-2}, \mathbf{x}'_{3,t-2})$. By letting let $p^* = \max(p,4)$ and $q^* = \max(q,4)$, and collecting matrices in the following way, $p_j^\circ = [p_j \vdots P_j]$, $j=1,\dots,4$ in which p_j is $G\times G$ and P_j is $G\times k$, and $A_i^\circ = [A_i \vdots B_i], i=1,\dots,p^*-4$, where A_i is $G\times G$ and B_i is $G\times k$, the following definitions applies: $p_1^\circ = \frac{-j(1)}{4} = -\frac{1}{4}[f(1) \vdots q(1)]$, $p_2^\circ = \frac{-j(-1)}{4} = -\frac{1}{4}[f-(1) \vdots q(-1)]$, $p_3^\circ = -\frac{1}{2}\mathrm{Real}[j(i)] = -\frac{1}{2}\mathrm{Real}[f(i) \vdots q(i)]$, $p_4^\circ = -\frac{1}{2}\mathrm{Im}[j(-i)] = -\frac{1}{2}\mathrm{Im}[f(-i) \vdots q(-i)]$, and $A_i^\circ = \sum_{j=1}^{[(p^*-i)/4]} j_{i+4j} = \sum_{j=1}^{[(p^*-i)/4]} [f_{i+4j} \vdots q_{i+4j}],\ i=1,\dots,p^*-4$.
Model 3: ***Cointegration dynamic simultaneous equation model (CDSEM)*** The gth equation takes de form: $y_g = Z_g\tilde{M}_g\tilde{M}_g^{-1}d_g + e_g$ $= Z_g^* d_g^* + e_g$ $= Z_{g1}^* d_{g1}^* + Z_{g2}^* d_{g2}^* + e_g$	$Z_{g1}^* = (\Delta\boldsymbol{Y}_g, \Delta\tilde{\boldsymbol{Y}}_{g,-1},\dots,\Delta\tilde{\boldsymbol{Y}}_{g,-p+1}, \Delta\boldsymbol{X}_g,\dots,\Delta\boldsymbol{X}_{g,-q+1})$ consists of linearly independent stationary variables and $Z_{g2}^* = (\boldsymbol{X}_{g,-1}, \tilde{\boldsymbol{Y}}_{g,-1})$ consists of linearly independent I(1) variables. $\boldsymbol{y}_g = (y_{g1},\dots,y_{gT})'$, $e_g = (\mathbf{e}_{g1},\dots,\mathbf{e}_{gT})'$, $Z_g = (\boldsymbol{Y}_g, \tilde{\boldsymbol{Y}}_{g,-1},\dots,\tilde{\boldsymbol{Y}}_{g,-p}, \boldsymbol{X}_g,\dots,\boldsymbol{X}_{g,-q})$ and $d_g = (\Phi'_0, \Phi'_1,\dots, \Phi'_p, \Theta_0,\dots,\Theta'_q)'$. The matrices $\tilde{\boldsymbol{Y}}_g = (\boldsymbol{Y}_g, \boldsymbol{y}_g)$ and $\boldsymbol{X}_g$ denote the $T\times G_g$ and $T\times K_g$ included joint dependent and exogenous variables of the $T\times G$ joint dependent variables $\boldsymbol{Y} = (\tilde{\boldsymbol{Y}}_g, \tilde{\boldsymbol{Y}}_{g\nabla})$ and $T\times K$ exogenous variables $\boldsymbol{X} = (\boldsymbol{X}_g, \boldsymbol{X}_{g\nabla})$ in the system. The matrices $\tilde{\boldsymbol{Y}}_{g\nabla}$ and $\boldsymbol{X}_{g\nabla}$ denote the excluded joint dependent and exogenous variables, respectively, and $\tilde{\boldsymbol{Y}}_{g,-j}$, $\boldsymbol{X}_{g,-j}$ denote the $T\times G_g$ and $T\times K_g$ included joint dependent and exogenous variables lagged by j period. Finally, it is assumed that the initial observations $\mathbf{y}_0,\dots,\mathbf{y}_{-p+1},\mathbf{z}_0,\dots,\mathbf{z}_{-q+1}$ are available, thus treated as fixed constants.
Model 4: ***Seasonal cointegration dynamic simultaneous equation model (CDSEM)*** The gth equation takes de form: $y_g = Z_g\tilde{M}_g\tilde{M}_g^{-1}d_g + e_g$ $= Z_g^* d_g^* + e_g$ $= Z_{g1}^* d_{g1}^* + Z_{g2}^* d_{g2}^* + e_g$	$Z_{g1}^* = [\Delta_4\boldsymbol{Y}_g, \Delta_4\tilde{\boldsymbol{Y}}_{g,-1},\dots,\Delta_4\tilde{\boldsymbol{Y}}_{g,-p+4}, \Delta_4\boldsymbol{X}_g, \Delta_4\boldsymbol{X}_{g,-1},\dots,\Delta_4\boldsymbol{X}_{g-q+4}]$, $Z_{g2}^* = [\boldsymbol{X}_{g2_{1,t-1}}, \boldsymbol{X}_{g2_{2,t-1}}, \boldsymbol{X}_{g2_{3,t-2}}, \boldsymbol{X}_{g2_{3,t-1}}, \tilde{\boldsymbol{Y}}_{g2_{1,t-1}}, \tilde{\boldsymbol{Y}}_{g2_{2,t-1}}, \tilde{\boldsymbol{Y}}_{g2_{3,t-2}}, \tilde{\boldsymbol{Y}}_{g2_{3,t-1}}]$, Z_{g1}^* consists of linearly independent I(0) variables, and Z_{g2}^* consists of linearly independent I(1) variables, more specifically. All the definitions provided for Model 2 and Model 3 applies for Model 4.

Error-correction models. These models assume that the data are nonstationary and integrated of order one. Thus, multi-equation error-correction representation is possible. Two are the models in this class, introduced as models 1 and 2 in what follows.

Model 1: *Nonseasonal vector error-correction model (VECM)*. VECMs were introduced in Engle and Granger (1987) and formulated as a system in Johansen and Juselius (1990), and Phillips (1995). This model is the most frequently used in applied economics and agricultural economics. It assumes the existence of a structural model but used in reduced form arguments in its specification. Under certain conditions, model 1 reduces to a classical vector autoregression (VAR) which was used in agricultural economics in the early 1980s.

Model 2: *Vector error-correction with seasonal cointegration (SVECM)*. This model extends the cointegration technique in model 1 to the case where market data have unit roots at both the zero and seasonal frequencies. This model requires knowledge of prior information on which unit roots are present in order to filter out seasonal unit root components and to test for cointegration in the filtered series. This literature is due to Hylleberg, Engle, Granger and Yoo (1990), Engle, Granger, Hylleberg, and Lee (1990), Lee (1992), Johansen and Schaumburg (1999), Frances and Kunst (1999), and nicely summarized in Ghysels and Osborn (2001). With a “top-down” simplification, for instance, assuming that seasonality is deterministic and that there is cointegration, model 2 reduces to model 1.

Structural Time Series Models. This class of models has a long-history in the econometrics literature and was first formally developed in Zellner and Palm (1974). This work presents the first blend of time series techniques and traditional econometric models, which at the time were known as transfer function or final equation models. In the context of modern time series econometrics, however, Zellner and Palm’s contribution remained a bit short of a generalized model specification for multiple time series because it did not allow for unit-roots and cointegration. A new framework for linking multiple time series and dynamic simultaneous equation models under nonstationarity was introduced by Hsiao (1997a; 1997b). The two models in this class are introduced in what follows.

Model 3: *Cointegration and Dynamic Simultaneous Equation Model (CDSEM)*. This model merges recent developments in regression analysis with integrated regressors with classical simultaneous equation models. The approach is very comprehensive in the sense that it addresses most questions of empirical relevance such as the specification of DSEM with and without cointegration, the identification issue, estimation of short and long-run relationships, parameter estimation, limiting distribution of test statistics, and forecasting. The model includes models 1 and 2 as special cases, but this occurs only when the endogenous-exogenous properties of market models disappear.

Model 4: *Seasonal Cointegration and Dynamic Simultaneous Equation Models (SCDSEM)*. This model has empirical appeal in cases where seasonal data (say, monthly or quarterly) are being used. As in model 2, the question is whether seasonal elements should be modeled as stochastic or deterministic. No theoretical model is available in the econometrics literature to use in the context of seasonally cointegrated DSEMs. This research introduces a first attempt to develop such theory and applies it to quarterly data for the U.S. wheat market. In essence, the results reported here are derived from Hsiao's theory for model 3.

Assumptions, estimation theory, forecasting, dynamic simulation, and impulse response functions are discussed in detail in the sections that follow. The reader who may prefer to study the empirical aspects of the research, and who may be somewhat familiar with the extensive econometric literature on the subject, may skip to the application in the last section of this chapter without losing concept of the aim of the research. Readers interested in using the procedures in commodity modeling may benefit by laboring through the theoretical details. A new model is introduced in 3.3.3 and related algebraic derivations in Appendix C.

3.2 Econometrics of Seasonal Nonstationary Systems

This section provides the econometric theory for estimating models 1-4 in Table 4.1. Model 4 is the most generalized specification, however, an introduction to certain principles of stochastic processes is needed to facilitate a quick grasp of the role of unit-roots and cointegration at seasonal frequencies.

Throughout the section, models 1-4 will be cited as needed. As pointed out earlier, models of the VAR-type (Models 1 and 2) will be introduced prior to structural time series models (Models 3 and 4).

3.2.1 Seasonal Time Series Processes

Many economic time series contain important seasonal components. A seasonal series can be described by its spectrum

$$f(w) = \frac{1}{2p} ACOF(e^{-iw}), \tag{3.12}$$

where $ACOF(e^{-iw}) = s^{2} / (j(e^{-iw}) j(e^{iw}))$ is the autocovariance generating function (Hamilton, 1994). Moreover, the spectrum of a seasonal process, assuming that exists, has distinct peaks at the seasonal frequencies $w_s = 2p\, j / s,\ j = 1,...,s/2$, where s is even and represents the number of time periods in a year. In this study, for instance, there is one observation per period and four periods per year, thus $s = 4$.

Following Hylleberg *et al.* (1990), it can be said that three classes of time-series models are frequently used to model seasonality: (a) purely deterministic seasonal processes, (b) stationary seasonal processes, and (c) integrated seasonal processes.

A *purely deterministic seasonal process* is generated by seasonal dummy variables, such us the following quarterly series,

$$x_t = m_t, \text{ where } \quad m_t = m_0 + m_1 S_{1t} + m_2 S_{2t} + m_3 S_{3t}. \tag{3.13}$$

A *stationary seasonal process* can be generated by a potentially infinite autoregression

$$j(B) x_t = e_t, \qquad e_t \text{ are iid,} \tag{3.14}$$

with all roots of $j(B) = 0$ lying outside the unit circle. Some of these roots are complex pairs with seasonal periodicity and its spectrum has peaks at some of the seasonal frequencies w_s.

A series x_t is an *integrated seasonal process* if a seasonal unit root exists in its autoregressive representation. More generally, it is integrated of order d at frequency q if the spectrum of x_t takes the form

$$f(w) = c(w - q)^{-2d}, \tag{3.15}$$

for w near q, and conveniently denoted[1] by

$$x_t \sim I_q(d). \tag{3.16}$$

Assuming that the seasonal pattern of the time series is deterministic, a DSEM like (3.10) then may be updated to account for it as follows,

$$\mathbf{y}_t = f_0 + \sum_{j=1}^{p} \Phi_j \mathbf{y}_{t-j} + \sum_{j=0}^{q} \Theta_j \mathbf{x}_{t-j} + f\mathbf{S}_t + \mathbf{e}_t, \quad t = 1,...,T, \tag{3.17}$$

where $\mathbf{S}_t = (S_{1,t}, S_{2,t}, S_{3,t})'$ is a vector $(s-1)\times 1$ of seasonal dummies[2], f is a $G \times (s-1)$ matrix of unknown constants, and all the variables are defined as before.

Paralleling (3.17), a deterministic seasonal pattern may be incorporated in a VECM as follows,

$$\Delta\mathbf{y}_t = f_0 + \sum_{j=1}^{p-1} \Phi_j^* \Delta\mathbf{y}_{t-j} + \sum_{j=0}^{q-1} \Theta_j^* \Delta\mathbf{x}_{t-j} - \Phi^* \mathbf{y}_{t-1} - \Theta^* \mathbf{x}_{t-1} + f^* \mathbf{S}_t + \mathbf{u}_t, \qquad t = 1,...,T, \tag{3.18}$$

where $\mathbf{S}_t$ is as in (3.17), f^* is a $G \times (s-1)$ matrix of unknown constants, and all the variables defined as in Table 3.1.

3.2.2 Seasonal Integration and Seasonal Error-Correction Models

The unit root tests proposed by Fuller (1976) and Dickey and Fuller (1979), for instance, assumed that the root of interest has a modulus of one and that such root corresponds to a zero-frequency peak in the spectrum. Furthermore, these tests assume that there is no other unit root in the system. Box and Jenkins (1970) implicitly assumed that unit roots may exist at different frequencies when they proposed using the seasonal differencing filter $(1 - B^4)$ for nonstationary quarterly univariate time series. Hylleberg *et al.* (1990) introduced tests for roots in linear time series that have a modulus of one but that correspond to seasonal frequencies. Their seminal paper also presents representations for multivariate processes with

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[1] This work concentrates on the case $d = 1$.

[2] In this study $s = 4$.

combinations of seasonal and zero-frequency unit roots, which lead to a variety of autoregressive and error-correction representations.

Hylleberg *et al.* (1990) and Lee (1992) define *seasonal cointegration* in the following way. Assume that each component of a vector $\mathbf{z}_t$ is seasonally integrated of order 1 at frequency q, i.e. $\mathbf{z}_t \sim \mathrm{I}_q(1)$. Then, the components of $\mathbf{z}_t$ are said to be *seasonally cointegrated*, denoted $\mathbf{z}_t \sim CI_q(1,1)$ if there exists a vector $a\ (\neq \mathbf{0})$ so that $\mathbf{w}_t = a'\mathbf{z}_t$ is stationary at frequency q, i.e. $\mathbf{w}_t \sim \mathrm{I}_q(0)$. This definition can be reduced to the ordinary cointegration when $q = 0$.

For a vector of nonstationary series that has unit roots at some seasonal frequencies as well as at the zero frequency, it is possible that a single cointegrating vector could eliminate all the unit roots in the series. Suppose that each component of a vector $\mathbf{z}_t$ is seasonally integrated of order 1 at some frequencies –not necessarily at the same frequencies for all components. Then, the components of $\mathbf{z}_t$ are said to be *fully cointegrated*, denoted $\mathbf{z}_t \sim CI(1,1)$ if there exists a vector $a\ (\neq \mathbf{0})$ so that $\mathbf{w}_t = a'\mathbf{z}_t$ is stationary.

The implications of seasonal cointegration are not immediately obvious but are quite similar to those of the ordinary cointegration established by Engle and Granger (1987). For instance, seasonal cointegration would mean that an innovation has only a temporary effect on the seasonal behavior of $\mathbf{w}_t = a'\mathbf{z}_t$, while it may have a permanent effect on the seasonal pattern of $\mathbf{z}_t$ (Lee, 1992).

The paper on maximum likelihood inference by Lee (1992) sets the stage for the analysis of multivariate systems. Johansen and Schaumburg (1999) improve Lee's analysis and discuss maximum likelihood estimation, calculation of test statistics, and derivation of asymptotic distributions in the context of the vector autoregressive model.

In agricultural economics, many commodity markets time series exhibit substantial seasonality, therefore, there is a definite possibility that there may be unit roots at seasonal frequencies and that seasonal cointegration may exist. Although the potential these developments have, in terms of forecasts and simulation analysis, they have not been exploited yet in modeling agricultural commodity markets.

It should be noted that the DSEM (3.17) may be rewritten in a VAR form, which is convenient for the analysis of nonstationarity, as

$$\mathbf{w}_t = f_1^* \mathbf{w}_{t-1} + \cdots + f_{p^*}^* \mathbf{w}_{t-p^*} + g^* \mathbf{S}_t + e_t^* , \tag{3.19}$$

where $\mathbf{w}_t = (\mathbf{y}_t', \mathbf{x}_t')'$, $p^* = \max(p, q)$, $[I \vdots -\Theta_0]^-$ is the generalized inverse of $[I \vdots -\Theta_0]$, $f_j^* = [I \vdots -\Theta_0]^- [\Phi_j \vdots \Theta_j]$, $j = 1,..., p*$, are $(G+k) \times (G+k)$ parameter matrices to be estimated from the data, $g^* = [I \vdots -\Theta_0]^- g$, and $e_t^* = [I \vdots -\Theta_0]^- e_t$.

When $\Delta_s \mathbf{x}_t$ is stationary, where $\Delta_s = 1 - B^s$ and s is the number of observations taken per year ($s = 4$), so that the determinant $|f^*(z)|$ has unit roots at the zero frequency $w = 0$ and all seasonal frequencies $w = j/s$ $(j = 1,..., s/2)$, the autoregressive equation in (3.19) is reformulated[3] in the *seasonal vector error-correction model* (SVECM) form (Lee, 1992; Johansen and Schamburg, 1999; Franses and Kunst, 1999),

$$\Delta \mathbf{w}_t = \quad m + d\mathbf{S}_t + \underbrace{p_1^* \mathbf{w}_{1,t-1} + p_2^* \mathbf{w}_{2,t-1} + p_3^* \mathbf{w}_{3,t-1} + p_4^* \mathbf{w}_{3,t-1}}_{Seasonal\ long-run equilibrium terms} + \underbrace{A_1^* \Delta \mathbf{w}_{t-1} + \cdots + A_{p^*-4}^* \Delta \mathbf{w}_{t-p^*+4}}_{Short-run dynamics} + e_t^* , \tag{3.20}$$

where m is a constant term, $d = (m_1, d_1, d_2, d_3)$, $\mathbf{S}_t = \left(1, \cos p\,(t-1), \cos\frac{p}{2}(t-1), \cos\frac{p}{2}(t-2)\right)'$, with cosines are seasonal intercepts as in Franses and Kunst (1999), $\mathbf{w}_{1,t-1} = (B + B^2 + B^3 + B^4)\mathbf{w}_t$ has a unit root at frequency $w = 0$, $\mathbf{w}_{2,t-1} = -(B - B^2 + B^3 - B^4)\mathbf{w}_t$ shows a unit root at frequency $w = 1/2$, $\mathbf{w}_{3,t-2} = -(B^2 - B^4)\mathbf{w}_t$ and $\mathbf{w}_{3,t-1} = -(B - B^3)\mathbf{w}_t$ both have one unit root at frequency $w = 1/4$, the p^*'s are matrices of reduced rank $r_j < G + k$, with $p_1^* = -\dfrac{f^*(1)}{4} = a_1^* b_1^{*\prime}$, $p_2^* = -\dfrac{f^*(-1)}{4} = a_2^* b_2^{*\prime}$,

$p_3^* = -\dfrac{1}{2}\mathrm{Real}[f^*(i)] = -2\left(a_R^* b_R^{*\prime} + a_I^* b_I^{*\prime}\right)$, $p_4^* = -\dfrac{1}{2}\mathrm{Im}[f^*(-i)] = -2\left(a_R^* b_I^{*\prime} - a_I^* b_R^{*\prime}\right)$, $A_i^* = \sum_{j=i}^{[(p^*-i)/4]} f_{i+4j}^*$, $i = 1,..., p^* - 4$, and [.] denotes the largest integer in [].

The seasonal intercepts may be absent or not. If present in the model, they may be restricted or not. Following a similar decomposition of the seasonal intercepts like in Frances and Kunst (1999), sixteen possible specifications of model (3.20) may arise. For ease of exposition, only three of these cases will be present which will exemplify on the role the seasonal intercepts play. The remaining cases not presented arise as variations of the following cases:

(1) Full unrestricted model:

$$\begin{aligned}\Delta \mathbf{w}_t = \; & m^* + a^* \cos p\,(t-1) + b^* \cos\frac{p}{2}(t-1) + c^* \cos\frac{p}{2}(t-2) + \\ & + p_1^* \mathbf{w}_{1,t-1} + p_2^* \mathbf{w}_{2,t-1} + p_3^* \mathbf{w}_{3,t-1} + p_4^* \mathbf{w}_{3,t-1} + \\ & [short - run\ dynamics] + e_t^*,\end{aligned} \tag{3.21}$$

(2) Full-restricted model (each seasonal cointegration space has a linear trend):

$$\begin{aligned}\Delta \mathbf{w}_t = \; & a_1^* \left[b_1^{*\prime} \mathbf{w}_{1,t-1} + m \right] + a_2^* \left[b_2^{*\prime} \mathbf{w}_{2,t-1} + a \cos p\,(t-1) \right] \\ & + a_3^* \left[b_3^{*\prime} \mathbf{w}_{3,t-2} + b \cos\frac{p}{2}(t-1) \right] + a_4^* \left[b_4^{*\prime} \mathbf{w}_{3,t-1} + c \cos\frac{p}{2}(t-2) \right] + \\ & [short - rundynamics] + e_t^*,\end{aligned} \tag{3.22}$$

(3) No restriction at zero frequency, but restrictions on the seasonals:

$$\begin{aligned}\Delta \mathbf{w}_t = \; & m^* + a_1^* b_1^{*\prime} \mathbf{w}_{1,t-1} + a_2^* \left[b_2^{*\prime} \mathbf{w}_{2,t-1} + a \cos p\,(t-1) \right] \\ & + a_3^* \left[b_3^{*\prime} \mathbf{w}_{3,t-2} + b \cos\frac{p}{2}(t-1) \right] + a_4^* \left[b_4^{*\prime} \mathbf{w}_{3,t-1} + c \cos\frac{p}{2}(t-2) \right] + \\ & [short - rundynamics] + e_t^*.\end{aligned} \tag{3.23}$$

The SECM model, as presented in (3.20), may be written in term of the **y**'s and **x**'s, which is more convenient to use if the **x**'s are weak exogenous. This reformulation will play an important role, also, for the development of a DSEM that accounts for seasonal cointegration, as introduced in section 3.1.1.8. To present the re-expression of (3.20), recall that $\mathbf{w}_t = (\mathbf{y}_t', \mathbf{x}_t')'$, then follows that

[3] Appendix B provides insight and details on the derivation of the SVECM from a VAR representation.

$$\Delta_4 \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{bmatrix} = p_1^* \begin{bmatrix} \mathbf{y}_{1,t-1} \\ \mathbf{x}_{1,t-1} \end{bmatrix} + p_2^* \begin{bmatrix} \mathbf{y}_{2,t-1} \\ \mathbf{x}_{2,t-1} \end{bmatrix} + p_3^* \begin{bmatrix} \mathbf{y}_{3,t-2} \\ \mathbf{x}_{3,t-2} \end{bmatrix} + p_4^* \begin{bmatrix} \mathbf{y}_{3,t-1} \\ \mathbf{x}_{3,t-1} \end{bmatrix} + A_1^* \begin{bmatrix} \Delta\mathbf{y}_{1,t-1} \\ \Delta\mathbf{x}_{1,t-1} \end{bmatrix} + \cdots + A_{p^*}^* \begin{bmatrix} \Delta\mathbf{y}_{1,t-p^*+4} \\ \Delta\mathbf{x}_{1,t-p^*+4} \end{bmatrix} + \mathbf{e}_t^*, \quad (3.24)$$

where $p_j^* = \begin{bmatrix} p_{1,j}^{**} & P_{1,j}^{**} \\ p_{2,j}^{**} & p_{2,j}^{**} \end{bmatrix}$, $j = 1,\ldots,4$, $p_{1,j}^{**}$ is $G \times G$, $p_{2,j}^{**}$ is $k \times G$, $P_{1,j}^{**}$ is $G \times k$, $P_{2,j}^{**}$ is $k \times k$, therefore

p_j^* is $(G+k) \times (G+k)$, and $A_i^* = \begin{bmatrix} A_{1,i}^{**} & B_{1,i}^{**} \\ A_{2,i}^{**} & B_{2,i}^{**} \end{bmatrix}$, $i = 1,\ldots, p^* - 4$, $A_{1,i}^{**}$ is $G \times G$, $A_{2,i}^{**}$ is $k \times G$, $B_{1,i}^{**}$ is $G \times k$,

$B_{2,i}^{**}$ is $k \times k$, thus A_i^* is $(G+k) \times (G+k)$. Pre-multiplying (3.24) by $[I \;\vdots\; -\Theta_0]$, follows the reexpression of (3.20) as

$$\begin{aligned} \Delta_4 [I \;\vdots\; -\Theta_0] \begin{bmatrix} \mathbf{y}_t \\ \mathbf{x}_t \end{bmatrix} = p_1^\circ \begin{bmatrix} \mathbf{y}_{1,t-1} \\ \mathbf{x}_{1,t-1} \end{bmatrix} + p_2^\circ \begin{bmatrix} \mathbf{y}_{2,t-1} \\ \mathbf{x}_{2,t-1} \end{bmatrix} + p_3^\circ \begin{bmatrix} \mathbf{y}_{3,t-2} \\ \mathbf{x}_{3,t-2} \end{bmatrix} + p_4^\circ \begin{bmatrix} \mathbf{y}_{3,t-1} \\ \mathbf{x}_{3,t-1} \end{bmatrix} + \\ A_1^\circ \begin{bmatrix} \Delta\mathbf{y}_{1,t-1} \\ \Delta\mathbf{x}_{1,t-1} \end{bmatrix} + \cdots + A_{p^*}^\circ \begin{bmatrix} \Delta\mathbf{y}_{1,t-p^*+4} \\ \Delta\mathbf{x}_{1,t-p^*+4} \end{bmatrix} + \mathbf{e}_t^\circ, \end{aligned} \quad (3.25)$$

where $p_j^\circ = [I \;\vdots\; -\Theta_0] p_j^* = \left[p_j \;\vdots\; P_j\right]$, $j = 1,\ldots,4$, p_j is $G \times G$, and P_j is $G \times k$, therefore

p_j is $G \times (G+k)$, and $A_i^\circ = [I \;\vdots\; -\Theta_0] A_i^* = [A_i \;\vdots\; B_i]$, $i = 1,\ldots, p^* - 4$, A_i is $G \times G$, B_i is $G \times k$, thus

A_i° is $G \times (G+k)$. Moreover, based upon the results in Appendix A follows that $p_1^\circ = \dfrac{-j(1)}{4} =$

$= -\dfrac{1}{4}[f(1) \;\vdots\; q(1)]$, $p_2^\circ = \dfrac{-j(-1)}{4} = -\dfrac{1}{4}[f-(1) \;\vdots\; q(-1)]$, $p_3^\circ = -\dfrac{1}{2}\mathrm{Real}[j(i)] = -\dfrac{1}{2}\mathrm{Real}[f(i) \;\vdots\; q(i)]$,

$p_4^\circ = -\dfrac{1}{2}\mathrm{Im}[j(-i)] = -\dfrac{1}{2}\mathrm{Im}[f(-i) \;\vdots\; q(-i)]$, $A_i^\circ = \displaystyle\sum_{j=1}^{[(p^*-i)/4]} j_{i+4j} = \sum_{j=1}^{[(p^*-i)/4]} [f_{i+4j} \;\vdots\; q_{i+4j}]$, $i = 1,\ldots, p^* - 4$,

which allow for arriving at the desired re-expression of (3.20) in terms of the **y**'s and **x**'s,

$$\begin{aligned} \Delta_4 \mathbf{y}_t = \; & p_1 \mathbf{y}_{1,t-1} + p_2 \mathbf{y}_{2,t-1} + p_3 \mathbf{y}_{3,t-2} + p_4 \mathbf{y}_{3,t-1} + \\ & P_1 \mathbf{x}_{1,t-1} + P_2 \mathbf{x}_{2,t-1} + P_3 \mathbf{x}_{3,t-2} + P_4 \mathbf{x}_{3,t-1} + \\ & A_1 \Delta_4 \mathbf{y}_{t-1} + \cdots + A_{p^*-4} \Delta_4 \mathbf{y}_{t-p^*+4} + \\ & B_0 \Delta_4 \mathbf{x}_t + B_1 \Delta_4 \mathbf{x}_{t-1} + \cdots + B_{p^*-4} \Delta_4 \mathbf{x}_{t-p^*+4} + e_t^\circ, \end{aligned} \quad (3.26)$$

where $B_0 = \Theta_0$ and all matrices, vectors, and indexes are as before. Then,

$$
\begin{aligned}
& p_1 \mathbf{y}_{1,t} + P_1 \mathbf{x}_{1,t} = J_{1,t}, \\
& p_2 \mathbf{y}_{2,t} + P_2 \mathbf{x}_{2,t} = J_{2,t}, \\
& p_3 \mathbf{y}_{3,t} + P_3 \mathbf{x}_{3,t} = J_{3,t}, \text{ and} \\
& p_4 \mathbf{y}_{4,t} + P_4 \mathbf{x}_{4,t} = J_{4,t}
\end{aligned}
\tag{3.27}
$$

are the cointegration relationships at frequencies $w = 0$, w=1/2, w=1/4 respectively, i.e., it denote the implied long-run, semiannual, and annual equilibria of the system. Since the coefficient matrices $p_1,...,p_4, P_1,..., P_4$ may convey the information equilibria, it is needed to investigate the properties of these matrices in order to determine whether or not the components of $\mathbf{y}_t$ and $\mathbf{x}_t$ are seasonally cointegrated in the presence of unit roots at other frequencies. Note that if p_j and P_k have full rank G and k, respectively, then there is no unit root at the corresponding frequency. If the rank of these matrices is zero, a seasonal cointegration relationship at that frequency does not exist. In the intermediate case, where $0 < rank(p_j) = r_{p_j} < G$ and $0 < rank(P_j) = r_{P_j} < K$, it can be shown that $p_j = g_j a'_j$ and $P_j = g^*_j a^{*\prime}_j$ for suitable matrices g_j, a_k, g^*_j, and a^*_j such that $a'_j \mathbf{y}_{j,t-1} + a^{*\prime}_j \mathbf{x}_{j,t-1}$ is stationary even though $\left[\mathbf{y}'_{j,t-1}, \mathbf{x}'_{j,t-1}\right]'$ itself is nonstationary.

3.2.3 Identification of Seasonal Cointegrated Systems

The identification of these seasonal cointegrated DSEMs (SCDSEM) closely resembles the identification of DSEM discussed in Hsiao (1997b). For SCDSEMs, it must be assumed that there exist $0 < r_w \leq G$ linearly independent cointegrating relations at the zero frequency ($w = 0$) and at all the seasonal frequencies showing a unit root ($w = j/s,\ j = 1,2$). This assumption implies for each frequency that if $r_w = G$ for $w = 0,1/2$, and $1/4$ then p_i^{-1}, i =1,…,4. If $0 < r_w < G$ then the corresponding generalized inverse will be taken into consideration. For easy of exposition and without losing generality, p_j^- will denote indistinctly the inverse or g-inverse of matrix p_i, $i=1,\ldots,4$. Then (3.27) is written as,

$$\begin{aligned}\mathbf{y}_{1,t} &= p_1^*\mathbf{x}_{1,t} + v_{1,t},\\ \mathbf{y}_{2,t} &= p_2^*\mathbf{x}_{2,t} + v_{2,t},\\ \mathbf{y}_{3,t} &= p_3^*\mathbf{x}_{3,t} + v_{3,t}, \text{ and}\\ \mathbf{y}_{4,t} &= p_4^*\mathbf{x}_{4,t} + v_{4,t},\end{aligned} \tag{3.28}$$

where $p_j^* = -p_j^- P_j$, $j=1,\ldots,4$ and $v_{j,t} = p_j^- J_{j,t}$, $j=1,\ldots,4$. This last set of equations indicate that each of the cointegrating relations of (3.27) can be written in a way such that it involves at least one distinct seasonal filtered jointly dependent variable and that each seasonal filtered jointly dependent variable is a function of K seasonal filtered exogenous variables. In other words, nonstationarity at frequency zero and each seasonal frequency is driven by nonstationarity in $\mathbf{x}_t$ at various frequencies, while the seasonal filtered exogenous variables can be viewed as the common trends of Stock and Watson (1988).

The unique set of conditions that simultaneously identify both the implied long-run equilibrium and short-run dynamics coefficients in (3.26), that is,

$$Rank[A^*\Phi_g^*] = G-1, \tag{3.29}$$

may be derived by establishing the relationship between the parameters in the VAR representation with the parameters in the SVECM representation, i.e.,

$$A = A^* M_{sc}, \tag{3.30}$$

in which $A = [-\Phi_1,\ldots,-\Phi_p,-\Theta_0,\ \ -\Theta_1,\ldots,-\Theta_q]$, $A^* = [A_1,\ldots,A_{p-4},p_1,p_2,p_3,p_4,B_1,\ldots,B_{q-4},P_1,P_2,P_3,P_4]$, where all terms in A^* are from the seasonal error-correction model (3.26), and M_{sc} is a unique and nonsingular transformation matrix.

3.2.4 Seasonal Cointegration and Dynamic Simultaneous Equation Models

Many macroeconomic, financial, and agricultural commodity market time series are known to be nonstationary. However, certain linear combinations of nonstationary series may be stationary. As already introduced, based upon the nature of seasonal patterns, such series are said to be cointegrated or seasonally cointegrated.

For cointegrated (nonseasonal) series, the focus of the literature appears to have followed a time-series approach by either estimating the error-correction representation (Johansen, 1988) or its long-run relations (Phillips, 1991). However, there are good economic theories that justify following a structural equation approach to construct econometric models, and now it is known that nonstationarity does not reduce the role of prior information, nor complicate statistical inference (Hsiao, 1997a, 1997b). In fact, Hsiao shows that it is much simpler to estimate a traditional autoregressive distributive lag model directly and derive the long-run equilibrium relations and short-run adjustment processes as linear transformations of it. Moreover, the identification and simultaneity issues raised by the Cowles Commission in the 1950's remained legitimate concerns for DSEMs.

For seasonal cointegrated series, since the seminal papers of Hylleberg *et al.* (1990) and Lee (1992), recent literature has concentrated on accounting seasonal integration in the error-correction representation (i.e. Johansen and Schaumburg, 1999). No attempt exists yet to follow a structural equation approach to construct an econometric model that accounts for seasonal nonstationarity. In what follows, a new model is introduced to fill this gap, in which seasonal cointegration is blended within the dynamic simultaneous equation model (3.10). The short and long run relationships at various seasonal frequencies are obtained in this new model as a linear transformation of the gth equation of (3.10).

Similar to the transformation matrix M_{sc} of equation (3.30), if $\mathbf{y}_t$ and $\mathbf{x}_t$ are seasonally cointegrated in this research it is shown that a nonsingular matrix $\tilde{M}_g$ exists such that it transforms the gth equation into the equivalent form[4]

$$\begin{aligned}\boldsymbol{y}_g &= Z_g \tilde{M}_g \tilde{M}_g^{-1} \boldsymbol{d}_g + \boldsymbol{e}_g \\ &= Z_g^* \boldsymbol{d}_g^* + \boldsymbol{e}_g \\ &= Z_{g1}^* \boldsymbol{d}_{g1}^* \ + \ Z_{g2}^* \boldsymbol{d}_{g2}^* \ + \ \boldsymbol{e}_g \\ &= \textit{Seasonal short-run dynamics} + \textit{seasonal long-run equilibrium} + \textit{error}\end{aligned} \tag{3.31}$$

where $Z_g^* = Z_g \tilde{M}_g = (Z_{g1}^*, Z_{g2}^*)$, Z_{g1}^* consists of linearly independent I(0) variables, and Z_{g2}^* consists of linearly independent I(1) variables, more specifically,

$$Z_{g1}^* = [\Delta_4 \boldsymbol{Y}_g, \Delta_4 \tilde{\boldsymbol{Y}}_{g,-1}, \ldots, \Delta_4 \tilde{\boldsymbol{Y}}_{g,-p+4}, \Delta_4 \boldsymbol{X}_g, \Delta_4 \boldsymbol{X}_{g,-1}, \ldots, \Delta_4 \boldsymbol{X}_{g-q+4}], \text{ and} \tag{3.32}$$

$$Z_{g2}^* = \left[\boldsymbol{X}_{g2_{1,t-1}}, \boldsymbol{X}_{g2_{2,t-1}}, \boldsymbol{X}_{g2_{3,t-2}}, \boldsymbol{X}_{g2_{3,t-1}}, \tilde{Y}_{g2_{1,t-1}}, \tilde{Y}_{g2_{2,t-1}}, \tilde{Y}_{g2_{3,t-2}}, \tilde{Y}_{g2_{3,t-1}} \right], \tag{3.33}$$

where the variables in Z_{g2}^* are the seasonal filtered variables,

$$\begin{aligned}
\boldsymbol{X}_{g2_{1,t-1}} &= (B + B^2 + B^3 + B^4)\boldsymbol{X}_g, & \tilde{\boldsymbol{Y}}_{g2_{1,t-1}} &= (B + B^2 + B^3 + B^4)\tilde{\boldsymbol{Y}}_g, \\
\boldsymbol{X}_{g2_{2,t-1}} &= -(B - B^2 + B^3 - B^4)\boldsymbol{X}_g, & \tilde{\boldsymbol{Y}}_{g2_{2,t-1}} &= -(B - B^2 + B^3 - B^4)\tilde{\boldsymbol{Y}}_g, \\
\boldsymbol{X}_{g2_{3,t-1}} &= -(B^2 - B^4)\boldsymbol{X}_g, & \tilde{\boldsymbol{Y}}_{g2_{3,t-2}} &= -(B^2 - B^4)\tilde{\boldsymbol{Y}}_g, \\
\boldsymbol{X}_{g2_{3,t-1}} &= -(B - B^3)\boldsymbol{X}_g, & \tilde{\boldsymbol{Y}}_{g2_{3,t-1}} &= -(B^2 - B^3)\tilde{\boldsymbol{Y}}_g.
\end{aligned} \tag{3.34}$$

The least squares estimator of $\boldsymbol{d}_2^*$ is consistent, but the least squares estimator of $\boldsymbol{d}_1^*$ is not if Z_{g1}^* and $\boldsymbol{e}_g$ are correlated. This result follows from Lemma 1 in Hsiao (1997a). Since the least squares estimator of $\boldsymbol{d}_g$ is simply a linear transformation of the least squares estimator of (3.31) using the transformation matrix $\tilde{M}_g$, then $\boldsymbol{d}_g$ cannot be consistently estimated by the least squares method, despite $\mathbf{y}_t$ and $\mathbf{x}_t$ are seasonally cointegrated. As the interest is focused in the dichotomization of the

[4] Appendix B shows the existence of matrix $\tilde{M}_g$.

long-run and short-run relations at seasonal frequencies, the coefficients of (3.31) and their limiting properties can be derived directly from Hsiao (1997a, 1997b).

3.3 Estimators and Computational Guidelines

The focus of this subsection is in presenting the estimators of the four-selected econometric models (Table 3.1) and aspects relevant to the computation of the estimators. There are non-trivial aspects for the computation of the estimators, which deserve to be addressed, since the parameter estimates are needed for the construction of impulse responses, dynamic multipliers, and forecasts. On the other hand, since the objectives of this study do not focus on conducting structural analysis and testing hypothesis on the parameters of the U.S. wheat market model, the limiting distributions of the estimators are not considered here.

The maximum likelihood estimators of the VECM (Model 1) and the SVECM (Model 3) are presented first. The two-stage least squares (2SLS) and three-stage least squares (3SLS) estimators for the CDSEM (Model 3) and the SCDSEM (Model 4) are presented after.

3.3.1 Maximum Likelihood Estimation Under Cointegration and Deterministic Seasonality

The estimator of the vector error correction model VECM (Model 1) is a maximum likelihood (ML) estimator (Johansen, 1988; Johansen and Juselius, 1990). Below is given a brief description of the estimation procedure.

For notational convenience, let the VECM be written, assuming a finite autoregressive model of order k, as

$$\Delta \mathbf{w}_t = \Gamma_1 \Delta \mathbf{w}_{t-1} + \cdots + \Gamma_{k-1} \Delta \mathbf{w}_{t-k+1} - ab'\mathbf{w}_{t-1} + m + \Psi \mathbf{S}_t + \mathbf{e}_t,\ t = 1,\ldots,T, \qquad (3.35)$$

where $m = f_0, \Psi = f^*$, f_0 and f^* as defined in Table 3.1, the Γ_j's are matrices of appropriate dimensions, and a is a matrix that describes the speed of adjustment to the long-run equilibrium described by $b'\mathbf{w}_{t-1}$, and b is a matrix that contains in its columns the cointegrating vectors.

The likelihood estimation procedure maximizes, under model (3.35), the log-likelihood equation

$$\ln L = -\frac{(G+K)T}{2}\ln 2p - \frac{T}{2}\ln|\Sigma_{\mathbf{e}}| - \frac{1}{2}tr[\mathbf{E}'\Sigma_{\mathbf{e}}^{-1}\mathbf{E}], \tag{3.36}$$

where $\mathbf{E} = \left[\mathbf{e}_1,...,\mathbf{e}_T\right]$ and $\Sigma_{\mathbf{e}}$ represents the nonsingular covariance matrix of the $\mathbf{e}_t$'s. The ML procedure begins by concentrating (3.36) with respect to the parameters matrices $\Gamma_1,...,\Gamma_{k-1},\mathbf{1},m,$ and Ψ by regressing $\Delta\mathbf{w}_t$ and $\mathbf{w}_{t-1}$ on $\mathbf{X}_t = (\Delta\mathbf{w}_{t-1},...,\Delta\mathbf{w}_{t-k+1},\mathbf{1},\mathbf{S}_t)$. By letting R_{0t} and R_{kt} be the residuals of these two regressions, and the residual cross moment matrices be $S_{ij} = T^{-1}\sum_{t=1}^{T}R_{ij}R'_{jt},\ i,\ j = 0,k$, then the concentrated likelihood function has the form of a reduced rank regression,

$$\begin{aligned} R_{0t} &= \Pi R_{kt} + error \\ &= a^* b^{*\prime} R_{kt} + error. \end{aligned} \tag{3.37}$$

By letting P be the lower triangular matrix with positive diagonal satisfying $PS_{11}P' = \mathbf{I}_{(G+K)}$, $\hat{l}_1 \geq \cdots \geq \hat{l}_{G+K}$ be the eigenvalues of $PS_{k0}S_{00}^{-1}S_{0k}P'$, and $\hat{V} = (\hat{u}_1,...,\hat{u}_p)$ be the corresponding orthonormal eigenvectors, then the maximum likelihood estimate for b^* is

$$\tilde{b}^* = P'(\hat{u}_1,...,\hat{u}_r)\,, \tag{3.38}$$

thus, the maximum likelihood estimate for a^*, conditional to $\tilde{b}^*$, is

$$\tilde{a}^*(\tilde{b}^*) = S_{0k}\tilde{b}^*(\tilde{b}^{*\prime}S_{kk}\tilde{b}^*)^{-1}. \tag{3.39}$$

In equation (3.35) the constant term is not absorbed into the cointegration relation. If the constant can be absorbed into the cointegration relation, the model in (3.35) should be written as

$$\Delta\mathbf{w}_t = \Gamma_1\Delta\mathbf{w}_{t-1} + \cdots + \Gamma_{k-1}\Delta\mathbf{w}_{t-k+1} - a(\,b', b_0)(\mathbf{w}'_{t-1},1)' + \Psi\mathbf{S}_t + \mathbf{e}_t,\ t = 1,...,T, \tag{3.40}$$

implying that the first-differenced variables in the error-correction model have a common mean. In this case, the reduced rank regression must use the residuals of the regression of $\Delta\mathbf{w}_t$ and $(\mathbf{w}_{t-1},1)$ on $\mathbf{X}_t = (\Delta\mathbf{w}_{t-1},...,\Delta\mathbf{w}_{t-k+1},\mathbf{S}_t)$.

To determine the number of cointegrating vectors, Johansen suggests two tests, the trace test and the maximum eigenvalue test. The trace test, based on the LR test statistic for the hypothesis of at most r cointegrating vectors, is

$$l_{trace} = -T \sum_{i=r+1}^{G+K} \ln(1-\hat{l}_i)\,, \tag{3.41}$$

where $\hat{l}_{r+1},..., \hat{l}_{G+K}$ are the $G+K-r$ smallest eigenvalues of $PS_{k0}S_{00}^{-1}S_{0k}P'$, in which P is such that $PS_{11}P' = \mathbf{I}_{(G+K)}$.

The maximum eigenvalue test is a LR test statistic for the null hypothesis of r cointegrating vectors versus the alternative of $r+1$ cointegrating vectors. The test is

$$l_{\max} = -T\ln(1-\hat{l}_{r+1}). \tag{3.42}$$

Tables providing quantiles of the asymptotic distributions of the trace test and the maximum eigenvalue test statistics are provided, for instance, in Lütkepohl (1993) and Hansen and Juselius (1995).

For the purposes of this study, the hypothesis of weak exogeneity for the long-run parameters is important. Based upon the economic background underlying our USWMM, there are five endogenous variables and six exogenous. If weak exogeneity is supported by the available data, then an important reduction of parameters in (3.35) can be attained, while getting a model more consistent with the economic tenets of out USWMM.

Weak exogeneity is a hypothesis about the rows of the loading matrix a of equation (3.35) (Ericson and Iron, 1994). Recalling that $\mathbf{w}_t = (\mathbf{y}_t', \mathbf{x}_t')'$, where $\mathbf{y}_t = [y_{1t},...,y_{Gt}]'$ and $\mathbf{x}_t = [x_{1t},...,x_{Kt}]'$, the condition for x_{it}, $i = 1,...,K$, to be weakly exogenous for b is that all the entries in the ith row of a are zeroes, since this implies that the equation Δx_{it} does not contain information about the long-run parameters b.

If all the entries in the ith row of a, $i = 1,...,K$ are zeroes, it is valid to condition on the marginal distribution of $\Delta\mathbf{x}_t$ and continue the analysis of a and b based on the system of $\Delta\mathbf{y}_t$-equations, i.e.

$$\Delta \mathbf{y}_t = \Gamma_0 \Delta \mathbf{x}_t + \tilde{\Gamma}_1 \Delta \mathbf{w}_{t-1} + \cdots + \tilde{\Gamma}_{k-1} \Delta \mathbf{w}_{t-k+1} - ab' \mathbf{w}_{t-1} + \tilde{m} + \tilde{\Psi} \mathbf{S}_t + \tilde{\mathbf{e}}_t,\ t = 1,\ldots,T. \quad (3.43)$$

Conditioning on weakly exogenous variables is sometimes advantageous as a mean of improving the stochastic properties of the model. This might be the case if there have been many interventions during the period, and the weakly exogenous variable exhibits all the "problematic" data features.

In practice, the maximum-likelihood estimates will be obtained using the package CATS (Hansen and Juselius, 1995) for RATS 4.0 (Doan, 1992).

3.3.2 Maximum Likelihood Estimation Under Seasonal Cointegration

Lee (1992) and Johansen and Schaumburg (1999) present the maximum likelihood estimator for the seasonal multivariate cointegration model SCVEM (Model 3) with Gaussian errors, for which the log-likelihood equation adopts the same form as in (3.36).

Restrictions on the seasonal intercepts in model (3.20) play an important role in the estimation procedure and the associated limiting distributions, thus require special consideration. Let $R = (*,*,*,*)'$, where the asterisk is a mask for the j-th entry of vector R, with entries that may be 0, 1, or 2 if the j-th seasonal intercept is not present in the model, is unrestricted, or is restricted, respectively. The first entry of R, i.e. $R(1)$, will be used to represent the general intercept, the second entry the semiannual intercept, the third the first annual intercept, and the forth the second annual intercept. Define now

$$\mathbf{w}_j^* = \begin{cases} \mathbf{w}_{j,t-1} & \text{if } R(1) = 0,1 \\ (\mathbf{w}_{j,t-1}, S_j) & \text{if } R(1) = 2 \end{cases}, \quad (3.44)$$

$$\mathbf{w}_3^* = \begin{cases} \mathbf{w}_{3,t-2} & \text{if } R(1) = 0,1 \\ (\mathbf{w}_{3,t-2}, S_3) & \text{if } R(1) = 2 \end{cases},$$

and

$$\mathbf{w}_4^* = \begin{cases} \mathbf{w}_{4,t-1} & \text{if } R(1) = 0,1 \\ (\mathbf{w}_{4,t-2}, S_4) & \text{if } R(1) = 2 \end{cases}, \quad (3.45)$$

where $S_j = 1, \cos p\,(t-1), \cos\frac{p}{2}(t-1), \cos\frac{p}{2}(t-2)$ for $j = 1,2,3,4$ represent the seasonal intercepts as described in Franses and Kunst (1999).

Once these restrictions are imposed, the estimation procedure and the limiting distribution of the t-statistics at the various frequencies may be described. It should be noted that the estimation procedure at the zero and the semiannual frequencies is based each one on a reduced rank regression, while the estimation procedure at the annual frequency that will be used in this study is an iterated algorithm adjusted from Johansen and Schaumburg (1999). There is no commercial package available to obtain the parameter estimate for the SVECM, as it is the case of the VECM. A special set of routines for RATS 4.0 (Doan, 1992) has been developed that implements the restricted maximum likelihood estimation of the short-run dynamic parameters and the cointegration relations at the zero, semi-annual and annual frequencies. This routines (labeled SCATS[5], which stands for *Seasonal Cointegration Analysis of Time Series*) also allow for (1) testing seasonal for unit roots, (2) determining the cointegration rank at the seasonal and zero frequencies, (3) testing for weak exogeneity, (4) checking the adequacy of the model, (5) estimating the short-run dynamics, (6) constructing *h*-steps forecasts, (7) estimating impulse responses and dynamic multipliers, and (8) constructing graphs that are useful in depicting the seasonal components that may be present in a time series.

The algorithm implemented in SCATS for computing the *maximum likelihood estimator at the zero frequency* is as follows:

(i) Regress $\Delta_4 \mathbf{w}_t$ on $\mathbf{X}_1 = [\mathbf{S}_t, \Delta_4 \mathbf{w}_{t-1}, ..., \Delta_4 \mathbf{w}_{t-p-4}, \mathbf{w}_2^*, \mathbf{w}_3^*, \mathbf{w}_4^*]$ and get the residuals $R_{1,0t}$;

(ii) Regress $\mathbf{w}_1^*$ on $\mathbf{X}_1$ and get the residual $R_{1,kt}$;

(iii) Obtain the restricted maximum likelihood $\tilde{a}_1^*$ and $\tilde{b}_1^*$ as follows:

Let the residual cross moment matrices be

$$S_{1,ij} = T^{-1} \sum_{t=1}^{T} R_{1,ij} R'_{1,jt}, \qquad i, j = 0, k. \tag{3.46}$$

Then, the concentrated likelihood function has the form of a reduced rank regression

[5] The SCATS routines may be requested from the Department of Agricultural Economics and Agribusiness Department, Louisiana State University.

$$R_{1,0t} = \Pi R_{1,kt} + error$$
$$= a_1^* b_1^{*\prime} R_{1,kt} + error. \qquad (3.47)$$

By letting P be the lower triangular matrix with positive diagonal satisfying $PS_{1,11}P' = \mathbf{I}_{(G+K)}$,

$\hat{l}_1 \geq \cdots \geq \hat{l}_{G+K}$ be the eigenvalues of

$$PS_{1,k0}S_{1,00}^{-1}S_{1,0k}P', \qquad (3.48)$$

and $\hat{V} = (\hat{u}_1,...,\hat{u}_p)$ be the corresponding orthonormal eigenvectors, then the maximum likelihood estimate for b_1^* is ($p \leq r_{(w=0)}$, with $r_{(w=0)}$ representing the number of cointegrating relations at the zero frequency)

$$\tilde{b}_1^* = P'(\hat{u}_1,...,\hat{u}_r), \qquad (3.49)$$

thus, the maximum likelihood estimate for a_1^*, conditional to $\tilde{b}_1^*$, is

$$\tilde{a}_1^*(\tilde{b}_1^*) = S_{1,0k}\tilde{b}_1^*(\tilde{b}_1^{*\prime}S_{1,kk}\tilde{b}_1^*)^{-1}, \qquad (3.50)$$

(iv) Rank tests for determining the number of cointegrating vectors are the same as for the ECM model, as presented in the previous subsection 3.3.1.

The algorithm implemented in SCATS for computing the *maximum likelihood estimator at the semiannual frequency* is as follows:

(i) Regress $\Delta_4\mathbf{w}_t$ on $\mathbf{X}_2 = [\mathbf{S}_t, \Delta\mathbf{w}_{t-1},...,\Delta\mathbf{w}_{t-p-4}, \mathbf{w}_1^*, \mathbf{w}_3^*, \mathbf{w}_4^*]$ and get the residuals $R_{2,0t}$;

(ii) Regress $\mathbf{w}_2^*$ on $\mathbf{X}_2$ and get the residual $R_{2,kt}$;

(iii) Obtain the restricted maximum likelihood $\tilde{a}_2^*$ and $\tilde{b}_2^*$ following the same steps as for the zero frequency, but with residuals $R_{2,0t}$ and $R_{2,kt}$.

(iv) Rank tests are the same as for the ECM model, presented in the subsection 3.3.1.

Finally, SCATS computes the *maximum likelihood estimator at the annual frequency* as follows:

(i) Regress $\Delta_4\mathbf{w}_t$ on $\mathbf{X}_3 = [\mathbf{S}_t, \Delta\mathbf{w}_{t-1},...,\Delta\mathbf{w}_{t-p-4}]$ and get the residuals R_{0t};

(ii) Regress $\mathbf{w}_{1,t-1}$ on $\mathbf{X}_3$ and get the residual R_{1t},

(iii) Regress $\mathbf{w}_{2,t-1}$ on $\mathbf{X}_3$ and get the residual R_{2t},

(iv) Regress $\mathbf{w}_{3,t-2}$ on $\mathbf{X}_3$ and get the residual R_{Rt} ,

(v) Regress $\mathbf{w}_{3,t-1}$ on $\mathbf{X}_3$ and get the residual R_{It}

(vi) Regress R_{0t} on $\mathbf{X}_4 = [R_{1t}, R_{2t}]$ and get the residuals U_{0t};

(vii) Regress R_{Rt} on $\mathbf{X}_4$ and get the residuals U_{Rt};

(viii)Regress R_{It} on $\mathbf{X}_4$ and get the residuals U_{It};

(ix) Iterated algorithm to find $\{a_R, b_R, a_I, b_I\}$:

(a) Choose at random $b' = \begin{bmatrix} b_R & -b_I \\ b_I & b_R \end{bmatrix}'$ and calculate $U_{1t} = b' \begin{bmatrix} U_{Rt} \\ U_{It} \end{bmatrix}$;

(b) $S_{00} = \frac{1}{T} U_{0t} U'_{0t}$, $S_{01} = \frac{1}{T} U_{0t} U'_{1t}$, $S_{11} = \frac{1}{T} U_{1t} U'_{1t}$;

(c) $\tilde{a}^N = \frac{1}{2} S_{01} b (b' S_{11}^{-1} b)^{-1} = (a_R^N - a_I^N) = 2(a_R, -a_I)$;

(d) $\tilde{\Omega}_e = S_{00} - S_{01} b (b' S_{11}^{-1} b)^{-1} b' S_{10}$;

(e) $\tilde{U}_{0t} = vec\left(2 \times \tilde{\Omega}_e^{-1/2} \tilde{a} b' \begin{bmatrix} U_{Rt} \\ U_{It} \end{bmatrix} \right)$;

(f) $U_{2t} = \left[\left(U'_{Rt} \otimes a_R^N \right) - \left(U'_{It} \otimes a_I^N \right), \left(U'_{It} \otimes a_R^N \right) - \left(U'_{Rt} \otimes a_I^N \right) \right]$;

(g) $\begin{bmatrix} vec(b'_R) \\ vec(b'_I) \end{bmatrix} = \left(U'_{2t} U_{2t} \right)^{-1} U'_{2t} \tilde{U}_{0t}$;

(h) Set a convergence criteria. Upon convergence stop else return to (c).

(i) Test for ranks are from Johansen and Schaumburg, 1999 (JS) as follows: Use JS's Table 1 if no intercept terms, Table 2 if all seasonal intercepts are restricted, and Table 3 if restricted and unrestricted seasonal constants are present in the model.

SCATS also allows for testing and modeling for weak exogenous variables, in which case, the model specification adopts the form presented in (3.26).

3.3.3 Cointegration, Seasonality and Dynamic Simultaneous Equations Models

The two-stages least square (2SLS) and three-stages least square (3SLS) estimators are presented for the general case of a nonstationary dynamic simultaneous equation, in the next two subsection. After presenting both estimators, practical considerations for the CDSEM (Model 2) and the SCDSEM (Model 3) are presented at the end of this section.

3.3.3.1 The 2SLS Estimator Under Nonstationarity

For ease of exposition, let us recall the notation of the gth equation of a dynamic simultaneous equations model,

$$\boldsymbol{y}_g = Z_g \boldsymbol{d}_g + \boldsymbol{e}_{g,} \tag{3.51}$$

where all terms are defined as in Table 3.1. Then the 2SLS of the gth equation is (e.g. Theil, 1971)

$$\hat{\boldsymbol{d}}_g = [Z_g' W (W'W)^{-1} W' Z_g]^{-1} [Z_g' W (W'W)^{-1} W' \boldsymbol{y}_g] \tag{3.52}$$

where

$$W = (\boldsymbol{Y}_{-1}, \boldsymbol{Y}_{-2} \ldots, \boldsymbol{Y}_{-p^\circ}, \boldsymbol{X}, \boldsymbol{X}_{-1}, \ldots, \boldsymbol{X}_{-q^\circ}, \boldsymbol{S}), \tag{3.53}$$

is the matrix of instruments of dimension $T \times [(G_g \times p^\circ) + K_g(q^\circ + 1)]$, in which $p^\circ = \max(4, p)$, and $q^\circ = \max(4, q)$. It is worth noting that the matrix of instruments is constructed with p lags of the endogenous variables $\boldsymbol{Y}$, q lags of the exogenous $\boldsymbol{X}$, and the dummy variables in $\boldsymbol{S}$, which model seasonal deterministic patterns, if present. Let us also recall that the linear transformation matrix $\tilde{M}_g$ plays the important role of decomposing the structural linear parameters into the short run dynamics and the long run equilibrium,

$$\begin{aligned} \boldsymbol{y}_g &= Z_g \tilde{M}_g \tilde{M}_g^{-1} \boldsymbol{d}_g + \boldsymbol{e}_g \\ &= Z_g^* \boldsymbol{d}_g^* + \boldsymbol{e}_g \\ &= Z_{g1}^* \boldsymbol{d}_{g1}^* + Z_{g2}^* \boldsymbol{d}_{g2}^* + \boldsymbol{e}_g \end{aligned} \tag{3.54}$$

where Z_{g1}^{*} consists of linearly independent I(0) variables, Z_{g2}^{*} consists of linearly independent I(1) variables, and $Z_{g1}^{*}\boldsymbol{d}_{g1}^{*}$ and $Z_{g2}^{*}\boldsymbol{d}_{g2}^{*}$ describe the short-run dynamics and the long-run equilibrium, respectively, i.e. $\boldsymbol{d}_{2}^{*}$ is the cointegration vector of the gth structural equation.

As some of the endogenous and exogenous variables in the system may be excluded from the gth equation, based upon economic tenets, these excluded variables are also not present in the cointegration vector $\boldsymbol{d}_{2}^{*}$. This feature clearly differentiates the modeling approach in (3.54) from the unrestricted cointegration vectors (3.27) implied by the SVECM (3.26), in which all the endogenous and exogenous variables are present in the cointegration relationship.

Similar to matrix $\tilde{M}_g$ presented in (3.54) that transform the nonstationary regressors in Z_g into Z_g^{*}, there exist a transformation matrix $\tilde{M}_w$ that transforms the instrumental variables in W accordingly to the nature of the nonstationarity variables in the system. $\tilde{M}_w$ decomposes the integrated instrumental variables in matrix W into matrices W_1^{*} and W_2^{*},

$$W\tilde{M}_w = (W_1^{*}, W_2^{*}) = W^{*}, \tag{3.55}$$

where W_1^{*} is stationary and W_2^{*} is I(1).

Since $\tilde{M}_g$ and $\tilde{M}_w$ are nonsingular, independent of the nature of seasonality, the 2SLS estimator (3.52) can be rewritten as

$$\begin{aligned}
\hat{\boldsymbol{d}}_g &= \tilde{M}_g\{[\tilde{M}_g' Z_g' W\tilde{M}_w(\tilde{M}_w' W' W\tilde{M}_w)^{-1}\tilde{M}_w W' Z_g \tilde{M}_g]^{-1}\times \\
&\qquad [\tilde{M}_g Z_g' W\tilde{M}_w(\tilde{M}_w' W' W\tilde{M}_w)^{-1}\tilde{M}_w' W' \boldsymbol{y}_g]\} \\
&= \tilde{M}_g\{[Z_g^{*\prime} W^{*}(W^{*\prime}W^{*})^{-1}W^{*\prime}Z_g^{*}]^{-1}\,[Z_g^{*\prime}W^{*}(W^{*\prime}W^{*})^{-1}W^{*\prime}\boldsymbol{y}_g]\} \\
&= \tilde{M}_g\hat{\boldsymbol{d}}_g^{*},
\end{aligned} \tag{3.56}$$

where $\hat{\boldsymbol{d}}_g^{*}$ is the 2SLS estimator of (3.54) using W^{*} as instruments.

3.3.3.2 **The 3SLS Under Nonstationarity**

If contemporaneous correlation (CC) exists among the equations of the system, the 3SLS estimator must be used, which is consistent and efficient for this case. In presence of CC, the 2SLS is consistent but not efficient (Theil, 1971). For the presentation of the 3SLS estimator, let us recall that it is needed to combine the G equations as

$$\boldsymbol{y} = Zd + \boldsymbol{u}, \tag{3.57}$$

where $\boldsymbol{y}_g = (y_1',...,y_G')'$, $d = (d_1',...,d_G')'$, $\boldsymbol{u} = (e_1',...,e_g')'$, and $Z = diag(Z_1,...,Z_G)$. Then the 3SLS estimator of (3.57) is

$$\hat{d}_{3SLS} = [Z'[\hat{\Omega}^{-1} \otimes W(W'W)^{-1}W']Z]^{-1}[Z'[\hat{\Omega}^{-1} \otimes W(W'W)^{-1}W']\boldsymbol{y}], \tag{3.58}$$

where $\hat{\Omega}$ is an estimate of the matrix of variances and covariances of the error terms Ω based on the 2SLS residuals, and $\otimes$ represents the Kronecker product of matrices (e.g. Judge *et al.*, 1988).

To get around the issue of asymptotic multicollinearity, let W and Z be transformed by the nonsingular matrices $\tilde{M}_w$ and $\tilde{M}$ into $W^* = (W_1^*, W_2^*)$ and $Z^* = (Z_1^*, Z_2^*)$, where W_1^* and Z_1^* are linearly independent I(0) variables, and W_2^* and Z_2^* are linearly independent I(1). Then,

$$\begin{aligned} \boldsymbol{y} &= Z\tilde{M}\tilde{M}^{-1}d + \boldsymbol{u} \\ &= Z^*d^* + \boldsymbol{u}, \end{aligned} \tag{3.59}$$

with $\boldsymbol{d}^* = (d_1^{*\prime},...,d_G^{*\prime})'$, and $d^* = (d_1^{*\prime}, d_2^{*\prime})'$ with d_1^* and d_2^* the coefficients of Z_{g1}^* and Z_{g2}^*.

As it was presented for the 2SLS estimator, since $\tilde{M}_w$ and $\tilde{M}$ are nonsingular, independent of the nature of the seasonal pattern, it follows that

$$\hat{d}_{3SLS} = \tilde{M}\hat{d}_{3SLS}^*, \tag{3.60}$$

where

$$\hat{d}_{3SLS}^* = [Z^{*\prime}[\hat{\Omega}^{-1} \otimes W^*(W^{*\prime}W^*)^{-1}W^{*\prime}]Z^*]^{-1}[Z^{*\prime}[\hat{\Omega}^{-1} \otimes W^*(W^{*\prime}W^*)^{-1}W^{*\prime}]\boldsymbol{y}] \tag{3.61}$$

is the 3SLS estimator of the model $\boldsymbol{y} = Z^*d^* + \boldsymbol{u}$.

One of the most important results in Hsiao (1997b) is that whatever the speed of convergence of the 2SLS or 3SLS estimator, as presented in (3.56) and (3.61) and their limiting distributions ,nothing needs to be changed in applying Wald-type test statistics for hypothesis testing. This is an important result, since it implies that the election between the 2SLS and 3SLS estimators may be conducted using the well-known Hausman (1978) specification test.

3.3.3.3 **2SLS and 3SLS Under Cointegration and Nonstochastic Seasonality**

For estimating Model 2 (CDSEM), the instrumental variables must be specified. Following Hsiao (1997a), they are

$$
\begin{aligned}
W_w^* &= W\tilde{M}_w \\
&= (\boldsymbol{Y}_{-1}, \boldsymbol{Y}_{-2} ..., \boldsymbol{Y}_{-p}, \boldsymbol{X}, \boldsymbol{X}_{-1}, ..., \boldsymbol{X}_{-q}, \boldsymbol{S})\tilde{M}_w \\
&= (\Delta\boldsymbol{Y}_{-1}, ..., \Delta\boldsymbol{Y}_{-p+1}, \Delta\boldsymbol{X}, \Delta\boldsymbol{X}_{-1}, ..., \boldsymbol{X}_{-q+1}, \boldsymbol{S}, \boldsymbol{Y}_{-1} - \boldsymbol{X}_{-1}\Pi^{*\prime}, \boldsymbol{X}_{-1}) \\
&= (W_1^*, W_2^*),
\end{aligned}
\tag{3.62}
$$

where $W_1^* = (\Delta\boldsymbol{Y}_{-1}, ..., \Delta\boldsymbol{Y}_{-p+1}, \Delta\boldsymbol{X}, \Delta\boldsymbol{X}_{-1}, ..., \Delta\boldsymbol{X}_{-q+1}, \boldsymbol{S}, \boldsymbol{Y}_{-1} - \boldsymbol{X}_{-1}\Pi^{*\prime})$ is stationary and $W_2^* = \boldsymbol{X}_{-1}$ is I(1).

In order to implement the 2SLS estimator in (3.56) and the 3SLS estimator in (3.61), it is needed first to obtain the residuals of the regression of $\boldsymbol{Y}$ on $\boldsymbol{X}$, as an estimate of $\boldsymbol{Y}_{-1} - \boldsymbol{X}_{-1}\Pi^{*\prime}$ in (3.62). The computation of the 2SLS or the 3SLS estimator is straightforward, in which the regressor variables in Z_g^* are obtained form its definition in Table 3.1.

3.3.3.4 **2SLS and 3SLS Under Seasonal Cointegration**

For estimating Model 4, the SCDSEM specification developed for the study and given in (3.31), it is needed to specify matrix $\tilde{M}_w$, a specification that depends on the seasonal patterns. Similar to matrix $\tilde{M}_g$ that transforms the nonstationary regressors in Z_g into Z_g^*, there exists a matrix $\tilde{M}_w$ that appropriately transforms the instrumental variables in W accordingly to the nature of the seasonal patterns. For estimating this new model, this matrix must be found. Appendix C shows the existence of this matrix for the case of seasonal cointegration, being such that $W\tilde{M}_w = (W_1^*, W_2^*) = W^*$, where

$$W_1^* = \quad [\Delta_4 \boldsymbol{Y}_{-1}, \ldots, \Delta_4 \boldsymbol{Y}_{-p^\circ+4}, \Delta_4 \boldsymbol{X}, \Delta_4 \boldsymbol{X}_{-1}, \ldots, \Delta_4 \boldsymbol{X}_{-q^\circ+4},$$
$$\boldsymbol{Y}_{1,t-1} - \boldsymbol{X}_{1,t-1}\Pi_1^{*\prime}, \boldsymbol{Y}_{2,t-1} - \boldsymbol{X}_{2,t-1}\Pi_2^{*\prime}, \boldsymbol{Y}_{3,t-2} - \boldsymbol{X}_{3,t-2}\Pi_3^{*\prime}, \boldsymbol{Y}_{3,t-1} - \boldsymbol{X}_{3,t-1}\Pi_4^{*\prime}] \qquad (3.63)$$

is stationary and

$$W_2^* = [\boldsymbol{X}_{1,t-1}, \boldsymbol{X}_{2,t-1}, \boldsymbol{X}_{3,t-2}, \boldsymbol{X}_{3,t-1}] \qquad (3.64)$$

is I(1).

The OLS residuals of the regressions of $\boldsymbol{Y}_{1,t-1}$ on $\boldsymbol{X}_{1,t-1}$, $\boldsymbol{Y}_{2,t-1}$ on $\boldsymbol{X}_{2,t-1}$, $\boldsymbol{Y}_{3,t-2}$ on $\boldsymbol{X}_{3,t-2}$, and $\boldsymbol{Y}_{3,t-1}$ on $\boldsymbol{X}_{3,t-1}$, respectively, must be used as instruments in W_1^*, as long as the $\boldsymbol{Y}_{1,t-1} - \boldsymbol{X}_{1,t-1}\Pi_1^{*\prime}$, $\boldsymbol{Y}_{2,t-1} - \boldsymbol{X}_{2,t-1}\Pi_2^{*\prime}$, $\boldsymbol{Y}_{3,t-2} - \boldsymbol{X}_{3,t-2}\Pi_3^{*\prime}$, and $\boldsymbol{Y}_{3,t-1} - \boldsymbol{X}_{3,t-1}\Pi_4^{*\prime}$ terms are unobservable. Once the instruments in (3.63) are ready, 2SLS and 3SLS may be computed using (3.56) and (3.61), respectively.

3.4 Forecast Evaluation and Dynamic Analysis

The focus of this subsection turns to the issues of central interest in this study, namely the construction and evaluation of forecasts, impulse responses, and dynamic multipliers, based on the cointegrated models VECM (Model 1) and CDSEM (Model 3) and the seasonal cointegrated models SVECM (Model 2) and SCDSEM (Model 4), as proposed for the USWMM.

It is known that predictions from unrestricted systems with some unit roots do not converge to the optimal predictors over long forecast horizons (Phillips, 1998). The construction and evaluation of forecasts for cointegrated and seasonal cointegrated systems have received special attention in the applied literature. Recent citations are the works of Clements and Hendry (1997), Osborn *et al*, 1999, and Löf and Lyhagen (2002), among others. The results reported in the literature on Monte Carlo experiments favor the specification of models that properly account for cointegration or seasonal cointegration, if unit roots exist at the zero and/or at seasonal frequencies. Yet, no clear cut evidence is found in the empirical examples reported. The generality of these results, as reported by Osborn *et al*. (1999) and Löf and

Lyhagen (2002), are still open to question. The forecast evaluation of the selected econometric models of this study is considered next in a first subsection of this section.

Impulse responses and dynamic multipliers are also shown to be inconsistent at long horizons in unrestricted systems with some unit roots (Lütkephol and Reimers, 1992b; Phillips, 1998). In contrast, reduced rank regressions produce impulse responses that are consistent, provided the cointegrating rank is correctly specified or consistently estimated by order statistical selection criteria, such as the bayesian information criteria (BIC)(Schwarz, 1978). The Monte Carlo experiments of Phillips (1998) show these results may be relevant in finite samples in systems with unit roots and cointegration. The computation of the impulse responses and a small-scale simulation exercise is described, in a second subsection, to assess the sensitivity of impulse responses and dynamic multipliers to specific design features of the selected econometric models.

3.4.1 Forecasting and Forecast Evaluation

Before presenting the methods for forecast evaluation, it is necessary to present how the forecasts are constructed for the different models in the study. For ease of presentation, the construction of forecasts with the VECM and SVECM models are presented first and the DSEM and SCDSEM after.

3.4.1.1 **Forecasting Vector Autoregression Models**

A typical representation of a $VAR(p)$ process is given by

$$\mathbf{w}_t = f_1 \mathbf{w}_{t-1} + \cdots + f_p \mathbf{w}_{t-p} + \mathbf{e}_t , \tag{3.65}$$

with the optimal *h*-step forecast (minimal mean square error) given by the conditional expectation[6]

$$E_t(\mathbf{w}_{t+h}) = E(\mathbf{w}_{t+h} \mid \Im_t), \tag{3.66}$$

where $\Im_t$ is the information set available at time *t*, i.e., $\Im_t = \{\mathbf{w}_s \mid s \leq t\}$. The optimality of the conditional expectation (3.66) is still valid for nonstationary systems (Lütkephol, 1993), like the ones

[6] Provided that expectation exists, even if $\det(f(z))$ has roots on the unit circle , with $f(z) = \det(\mathbf{I}_G - A_1 z - \cdots - A_p z^p)$.

considered in this study. Assuming that $\mathbf{u}_t$ is independent white noise, the optimal *h*-step forecast at origin t $\mathbf{w}_{t+h|t}$ is

$$\mathbf{w}_{t+h|t} = m + f_1 \mathbf{w}_{t+(h-1)|t} + \cdots + f_p \mathbf{w}_{t+(h-p)|t} , \tag{3.67}$$

where $\mathbf{w}_{t+j|t} = \mathbf{w}_{t+j}$ for $j \le 0$, just as in the stationary, stable case (Lütkephol, 1993).

A more convenient way to accommodate the optimal *h*-step forecast in (3.67) is in the companion matrix form, which simplifies notation and exposition of concepts. The companion matrix form arranges a $VAR(p)$ into a $VAR(1)$ format, as follows (Hamilton, 1994). Let

$$\mathbf{z}_t = \begin{bmatrix} 1 \\ \mathbf{w}_t \\ \mathbf{w}_{t-1} \\ \vdots \\ \mathbf{w}_{t-p+1} \end{bmatrix}; \quad \mathbf{W} = \begin{bmatrix} m & f_1 & \cdots & f_p \\ \mathbf{0} & \mathbf{I}_G & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_G \end{bmatrix}; \quad \mathbf{u}_t = \begin{bmatrix} \mathbf{0} \\ \mathbf{e}_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \tag{3.68}$$

then the *companion matrix form* of a $VAR(p)$ process is

$$\mathbf{z}_t = \mathbf{W}\mathbf{z}_{t-1} + \mathbf{u}_t , \tag{3.69}$$

from which follows that the *h*-step ahead forecast at origin t for $\mathbf{w}_{t+h|t}$ is the second entry in

$$\mathbf{z}_{t+h|t} = \mathbf{W}^h \mathbf{z}_t . \tag{3.70}$$

As the parameters in $\mathbf{W}$ are in practice unknown, they must be estimated from the data. By using carets to denote estimates, then (3.70) becomes

$$\hat{\mathbf{z}}_{t+h|t} = \widehat{\mathbf{W}}^h \mathbf{z}_t . \tag{3.71}$$

In the case of nonstochastic-seasonal integration and cointegrated systems, the parameters in $\mathbf{W}$ may be estimated from the parameter estimates of the VECM specification. Suppose that the model does not allow for linear trends in the data, but for intercepts in the cointegration relations, i.e., $d = 0$, $m_2 = 0$ and m_1 unrestricted, then the vector error-correction takes the form

$$\Delta\mathbf{w}_t = \sum_{j=1}^{p-1} \Gamma_j \Delta\mathbf{w}_{t-j} + ab' \begin{bmatrix} m \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{e}_t . \tag{3.72}$$

Let's denote by $\hat{m}, \tilde{\Gamma}_1, \ldots, \tilde{\Gamma}_{p-1}, \tilde{a}$, and $\tilde{b}$ the restricted maximum likelihood matrix parameter estimates, then the parameter estimates needed for $\mathbf{W}$ are derived from the following recursion formulae,

$$\begin{aligned}\hat{\Phi}_1 &= \hat{\Gamma}_2 - \hat{\Gamma}_1 - \hat{\Gamma}_p, \\ \hat{\Phi}_j &= \hat{\Gamma}_{j+1} - \hat{\Gamma}_j, \ 2 \le j < p, \\ \hat{\Phi}_p &= \hat{\Gamma}_1 + \hat{\Gamma}_{p-1} - \hat{\Gamma}_p.\end{aligned} \tag{3.73}$$

Under seasonal cointegration, the error-correction model derived from the $VAR(p)$ representation takes the form given in (3.20). After restricting the seasonal constants appropriately, without losing generality, the seasonal error-correction models adopt the form

$$\Delta \mathbf{w}_t = \quad m + p_1 \mathbf{w}_{1,t-1} + p_2 \mathbf{w}_{2,t-1} + p_3 \mathbf{w}_{3,t-1} + p_4 \mathbf{w}_{3,t-1} + \mathbf{e}_t \,, \tag{3.74}$$

then by letting $\hat{m},\ \hat{p_1},\ \hat{p}_2,\ \hat{p}_3,$ and $\hat{p}_4$ be the restricted maximum likelihood matrix estimates, the estimates needed for $\mathbf{W}$ are derived from

$$\hat{\Phi} = \mathbf{H}^{-1}\hat{\Pi} \,, \tag{3.75}$$

where,

$$\hat{\Pi} = \begin{bmatrix} \hat{p}_1 + \frac{1}{4}\mathbf{I}_G \\ \hat{p}_2 + \frac{1}{4}\mathbf{I}_G \\ \hat{p}_3 + \frac{1}{4}\mathbf{I}_G \\ \hat{p}_4 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \end{bmatrix} \otimes \mathbf{I}_G, \text{ and } \hat{\Phi} = \begin{bmatrix} \hat{f}_1 \\ \vdots \\ \hat{f}_4 \end{bmatrix}. \tag{3.76}$$

3.4.1.2 **Forecasting Dynamic Simultaneous Equation Models**

A DSEM, which is linear in its endogenous and exogenous variables, can be forecast using methods similar to those for VAR's and ARIMA models (Doan, 1992).

To compute forecasts out-of sample, some assumptions about the future of the exogenous variables must be made. Following Doan (1992), in a first step, the exogenous may be forecast using a vector error-correction model provided the exogenous are integrated and cointegrated. In a second step, then, the forecast of the endogenous may be performed. These steps must be repeated h times, to get the

h-step ahead forecast. For the case of the DSEM considered in this study, the optimal h-step forecast at origin t of the gth equation, denoted as $\boldsymbol{y}_{g,t+h|t}$, is

$$\begin{aligned}\boldsymbol{y}_{g,t+h|t} &= Z_{g,t+h|t}\tilde{M}_g \tilde{M}_g^{-1}\boldsymbol{d}_g \\ &= Z^*_{g,t+h|t}\boldsymbol{d}^*_g \\ &= Z^*_{g1,t+h|t}\boldsymbol{d}^*_{g1} + Z^*_{g2,t+h|t}\boldsymbol{d}^*_{g2}.\end{aligned} \tag{3.77}$$

For the nonseasonal cointegrated CDSEM Model 3, the transformed regressor variables are

$$Z^*_{g1,t+h|t} = (\Delta\boldsymbol{Y}_{g,t+h|t},\ \Delta\tilde{\boldsymbol{Y}}_{g,,t+h-1|t},\ldots,\Delta\tilde{\boldsymbol{Y}}_{g,t+h-p+1|t},\Delta\boldsymbol{X}_{g,t+h|t},\ldots,\Delta\boldsymbol{X}_{g,t+h-q+1|t})\,, \tag{3.78}$$

and

$$Z^*_{g2,t+h|t} = (\boldsymbol{X}_{g,t+h-1|t},\tilde{\boldsymbol{Y}}_{g,t+h-1|t})\,. \tag{3.79}$$

For the seasonal cointegrated SCDSEM Model 4, the transformed regressor variables are

$$Z^*_{g1,t+h|t} = [\Delta_4\boldsymbol{Y}_{g,t+h|t},\Delta_4\tilde{\boldsymbol{Y}}_{g,t+h-1|t},\ldots,\Delta_4\tilde{\boldsymbol{Y}}_{g,t+h-p+4|t},\Delta_4\boldsymbol{X}_{g,t+h|t},\Delta_4\boldsymbol{X}_{g,t+h-1|t},\ldots,\Delta_4\boldsymbol{X}_{g,t+h-q+4|t}], \tag{3.80}$$

and

$$Z^*_{g2,t+h|t} = \left[\boldsymbol{X}_{g2_{1,t+h-1|t}},\boldsymbol{X}_{g2_{2,t+h-1|t}},\boldsymbol{X}_{g2_{3,t+h-2|t}},\boldsymbol{X}_{g2_{3,t+h-1|t}},\tilde{Y}_{g2_{1,t+h-1|t}},\tilde{Y}_{g2_{2,t+h-1|t}},\tilde{Y}_{g2_{3,t+h-2|t}},\tilde{Y}_{g2_{3,t+h-1|t}}\right]. \tag{3.81}$$

In both models $Z^*_{gi,t+j|t} = Z^*_{gi,t+j}$, $i = 1,2$ and $j \le 0$.

3.4.1.3 **Comparison of Forecast Accuracy between Models**

The question that this subsection addresses is “which model forecasts better and how to make that decision?.” The goal is to determine which of the four selected forecasting models provides improved forecasts. Let $\hat{\mathbf{u}}_{1,t+1}$ and $\hat{\mathbf{u}}_{2,t+1}$ denote the forecast errors at time $t+1$, which are vectors of dimension $(G \times 1)$, where G is de number of endogenous variables in the system. The subscripts 1 and 2 denote two alternative forecasting equations used to get $\hat{\mathbf{u}}_{1,t+1}$ and $\hat{\mathbf{u}}_{2,t+1}$, respectively. The scalar entries of the forecast errors $\hat{\mathbf{u}}_{1,t+1}$ and $\hat{\mathbf{u}}_{2,t+1}$ are $\{\hat{u}_{1\,i,t+1}\}$ and $\{\hat{u}_{2\,i,t+1}\}$, $i = 1, \ldots, G$. In what follows, to simplify the notation, the subscript i will be dropped in the notation of $\hat{u}_{1\,i,t+1}$, and $\hat{u}_{2\,i,t+1}$. Thus $\hat{u}_{1,t+1}$ and $\hat{u}_{2,t+1}$ will represent the i-th entry of $\hat{\mathbf{u}}_{1,t+1}$ and $\hat{\mathbf{u}}_{2,t+1}$, respectively. To determine whether one forecasting model is more accurate than the other, and letting $u_{1,t+1}^2$ and $u_{2,t+1}^2$ be the square of $u_{1,t+1}$ and $u_{2,t+1}$, the following hypothesis is tested,

$$H_0 \ : \ E\{u_{1,t+1}^2\} = E\{u_{2,t+1}^2\} \ \text{vs.} \ H_0 \ : \ E\{u_{1,t+1}^2\} \neq E\{u_{2,t+1}^2\}. \qquad (3.82)$$

Diebold and Mariano (1995) reaccommodated the hypothesis in (3.82) as

$$H_{0,DM} \ : \ E\{u_{1,t+1}^2\} - E\{u_{2,t+1}^2\} = 0 \ \text{vs.} \ H_{1,DM} \ : \ E\{u_{1,t+1}^2\} - E\{u_{2,t+1}^2\} \neq 0. \qquad (3.83)$$

To test the null in (3.83) Diebold and Mariano (1995) showed that,

$$DM = \frac{P^{-1/2}\sum_{t=R}^{T-1}\hat{d}_{t+1}}{[\hat{\mathbf{V}}_1]^{1/2}} \xrightarrow{d} N(0,1)\ , \qquad (84)$$

where $\hat{d}_{t+1} = \hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2 = g_1'\begin{pmatrix}\hat{u}_{1,t+1}^2 \\ \hat{u}_{2,t+1}^2\end{pmatrix}$, $g_1 = (1,-1)'$, and $\hat{\mathbf{V}}_1$ is a consistent estimator of $\mathbf{V}_1 = g_1'\mathbf{V}g_1$.

For the case of no ARCH behavior, the consistent estimator of $\mathbf{V}$ suggested by West and McCracken (1998) may be used. For the case of ARCH-type behavior the Newey and West (1987) serial correlation consistent estimator version may be used.

In the study, there are four forecasting models to be compared; therefore, there are six possible comparisons to perform using the DM test statistic. To obtain a general protection level a of size 0.05, the Bonferroni's criteria (i.e. Johnson and Wichern, 1998) will be adopted, which suggests adopting a protection level of $a/6 \doteq 0.00833 \approx\sim 0.01$ for conducting each of the six pair-wise comparisons. This procedure will allow for ranking the methods in their forecast accuracy.

3.4.2 Impulse Responses and Dynamic Multipliers Estimation and Evaluation

This subsection of the chapter presents the estimation of the impulse response functions and the calculation of the impact, interim, and total multipliers. In applied work it is often of interest to know the response of one variable to a shock in another variable in the system. If there is a reaction of one variable to an impulse of another variable, it is possible to call the latter causal for the former. This subsection presents how to conduct this type of causality analysis by tracing out the effects of an exogenous shock or innovation in one of the variables on some or all of the other variables. This kind of dynamic analysis is often called impulse response analysis or multiplier analysis, which clearly depends on the underlying model assumed. The evaluation of the impulse response and multiplier analysis will be conducted on the basis of a small-scale simulation experiment, as in Phillips (1998).

3.4.2.1 **Estimation of Impulse Responses**

For ease of exposition, consider the effect of an innovation in the U.S. wheat market prices on the endogenous variables of the USWMM, i.e., disappearance (y_1), inventories (y_2), exports (y_3), production (y_4), and market prices (y_5). To isolate such an effect, suppose that all five variables assume their mean value prior to time $t=0$, $y_{it}=m_i$ for $i=1,\ldots,5$ and $t<0$, and price increases by one unit in period $t=0$, that is, $u_{5,0}=1$. Now it is possible to trace what happens to the system in periods $t=1,2,\ldots$ if no further shocks occur, that is, $u_{1,0}=\cdots=u_{4,0}=0$ and $u_{1,t}=\cdots=u_{5,t}=0$. Since the interest is not in the mean of the system but just in the variation of the variables around their means, then it is possible to set $m_i=0$. For instance, consider a $VAR(1)$ of the form[7] $\mathbf{y}_t=\Phi_1\mathbf{y}_{t-1}+\mathbf{u}_t$, where $\mathbf{y}'_t=(y_{1t},\ldots,y_{5t})'$ and $\mathbf{u}'_t=(u_{1t},\ldots,u_{5t})'$. Tracing a unit shock in prices in period $t=0$ in the $VAR(1)$, shows that

$$\mathbf{y}_0=\begin{bmatrix} y_{1,0} \\ y_{2,0} \\ \vdots \\ y_{5,0} \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\mathbf{y}_1=\begin{bmatrix} y_{1,1} \\ y_{2,1} \\ \vdots \\ y_{5,1} \end{bmatrix}=\Phi_1\mathbf{y}_0,$$

$$\mathbf{y}_2=\begin{bmatrix} y_{1,2} \\ y_{2,2} \\ \vdots \\ y_{5,2} \end{bmatrix}=\Phi_1\mathbf{y}_1=\Phi_1^2\mathbf{y}_0.$$

---

[7] This model is adopted for easy of exposition and without loss of generality since any $VAR(p)$ may be represented as a $VAR(1)$ using the companion matrix form, as previously presented.

Continuing the procedure, it turns out that $\mathbf{y}_i$ is the fifth column of Φ_1^i. An analogous line of argument shows that a unit shock in the *j*th variable in $\mathbf{y}_t$ at $t=0$, after *i* periods, results in a vector $\mathbf{y}_i$ that it is the *j*th column of Φ_1^i. Thus, the elements of Φ_1^i represent the effects of unit shocks in the variables of the system after *i* periods, and are called *impulse responses*.

In this context, an important result due to Wold (1938) is of interest. Wold has shown that every stationary process $\mathbf{x}_t$ can be written as the sum of two uncorrelated processes $\mathbf{z}_t$ and $\mathbf{y}_t$, where $\mathbf{z}_t$ is a deterministic process that can be forecast perfectly from its own past and $\mathbf{y}_t$ is a process with MA representation

$$\mathbf{y}_t = \sum_{i=0}^{\infty} q_i \mathbf{u}_{t-i}, \tag{3.85}$$

where $q_0 = \mathbf{I}_k$, and the $\mathbf{u}_t$ constitute a white noise process. This result is often called *Wold's Decomposition Theorem.* An important implication of the Wold's Decomposition Theorem is that any stable $VAR(p)$ process, i.e. $\det(\mathbf{I}_k - \Phi_1 z - \cdots - \Phi_1 z^p) \neq 0$ for $|z| \leq 1$, has a MA representation, since stability implies stationarity (Lütkepohl, 1993).

For a $VAR(1)$ process, the relationship between the parameter matrix and the matrices q_i of its MA representation is $q_i = \Phi_1^i$. For a $VAR(p)$ process, using the companion representation form (3.69), it is possible to show that

$$q_i = J\mathbf{W}_1^i J', \tag{3.86}$$

with $J = (\mathbf{I}_G, \mathbf{0}, \ldots, \mathbf{0})$ a $(G \times Gp)$ matrix. The *jk*th element of matrix q_i in (3.86), say q_{jki}, represents the reaction of the *j*th variable of the system to a unit shock of variable *k*, *i* periods ago, provided the effect is not contaminated by other shocks to the system.

The response of variable *j* to a unit shock (forecast error) in variable *k* is sometimes depicted graphically to get a visual impression of the dynamic inter-relationships within the system. If the variables have different scales, it is more informative to consider innovations of one standard deviation rather than unit shocks.

A problematic assumption in this type of impulse response analysis is that a shock occurs only in one variable at a time, an assumption that may be reasonable if shocks in different variables are independent. If they are not independent, the error terms may consist of all the influences and variables that are not directly included in the system. Correlation of the error terms, on the other hand, may indicate that a shock in one variable is likely to be accompanied by a shock in another variable. In that case, setting all other residuals to zero may provide a misleading picture of the actual dynamic relationships between the variables. This problem may be resolved by using *orthogonal impulses*. If $\Sigma_{\mathbf{u}}$ represents the variance-covariance matrix of the error terms $\mathbf{u}_t$ and P is the lower triangular matrix in the Choleski decomposition of $\Sigma_{\mathbf{u}}$, i.e. $\Sigma_{\mathbf{u}} = PP'$, then the elements in $\mathbf{v}_t = P^{-1}\mathbf{u}_t$ are orthogonal (uncorrelated) and the elements in

$$\Theta_i = q_i P \tag{3.87}$$

are interpreted as responses of the system to such orthogonal innovation. Thus, the *jk*th elements of Θ_i are assumed to represent the effect on variable *j* of a unit innovation in the *k*th variable *i* periods ago.

Integrated and cointegrated $VAR(p)$ systems must be interpreted cautiously, because they are unstable and do not possess a valid MA of the type needed for the Wold's Decomposition Theorem. Yet, the q_i matrices and their orthogonal counter part matrices Θ_i can be computed as presented in this section, with similar interpretations of impulse responses. For stable processes, the responses taper off to zero as $i \to \infty$. This property does not necessarily hold for unstable systems where the effects of a one-time impulse may not die out asymptotically.

The estimates of the orthogonal impulse response coefficients in Θ_i, say $\hat{\Theta}_i$, will be calculated using (3.87), where P will be computed as the Choleski decomposition of $\hat{\Sigma}_{\mathbf{u}} = T^{-1}\sum_{t=1}^{T}\hat{\mathbf{u}}_t\hat{\mathbf{u}}_t'$. The estimate of f_i needed in (3.87), denoted as $\hat{f}_i$, will be computed using $\hat{\mathbf{W}}_1$ in (3.86), where $\hat{\mathbf{W}}_1$ is the same matrix already calculated for the construction of forecasts for the VAR models, as presented in subsection 3.4.1.1.

3.4.2.2 **Multiplier Analysis**

Goldberger (1959) recognized that reduced forms are more convenient than structural forms for calculating the effects of exogenous changes on the behavior of endogenous variables. Yet, the reduced form is actually not good enough for this purpose when there are lagged endogenous variables (Theil, 1971). This is the case of the DSEM considered in this study, which for easy of exposition is repeated here,

$$\mathbf{y}_t = f_0 + \Phi_1 \mathbf{y}_{t-1} + \cdots + \Phi_4 \mathbf{y}_{t-4} + \Theta_0 \mathbf{x}_t + f\mathbf{S}_t + \mathbf{e}_t, \quad t = 1,...,T \,. \tag{3.88}$$

In the multiplier terminology, which is adopted from Goldberger (1959) and Theil (1971), the matrices $\Phi_1,...,\Phi_4$, and Θ_0 are matrices of *multiplicative reduced-form coefficients*.

Equation (3.88) shows that the effect of an exogenous change on an endogenous variable in the same period is determined by the appropriate elements of matrix Θ_0, which are known as the *impact multipliers* of the system. They describe the immediate (current) effect of exogenous changes.

On the other hand, matrices $\Phi_1,...,\Phi_4$ do not provide the effect one period later because of their indirect effects via the terms $\Phi_1 \mathbf{y}_{t-1},...,\Phi_4 \mathbf{y}_{t-4}$. This can be made explicit, by assuming $\Phi_2 = \Phi_3 = \Phi_4 = 0$, $f = 0$, and by replacing $\mathbf{y}_{t-1}$ by the right-hand side of (3.88) lagged one period,

$$\begin{aligned} \mathbf{y}_t &= f_0 + \Phi_1 (f_0 + \Phi_1 \mathbf{y}_{t-2} + \Theta_0 \mathbf{x}_{t-1} + \mathbf{e}_{t-1}) + \Theta_0 \mathbf{x}_t + \mathbf{e}_t \\ &= (\mathbf{I}_G + \Phi_1) f_0 + \Phi_1^2 \mathbf{y}_{t-2} + \Theta_0 \mathbf{x}_t + \Phi_1 \Theta_0 \mathbf{x}_{t-1} + \mathbf{e}_t + \Phi_1 \mathbf{e}_{t-1}. \end{aligned} \tag{3.89}$$

By applying this substitution *s* times, the following expression is obtained,

$$\mathbf{y}_t = (\mathbf{I}_G + \sum_{j=1}^{s} \Phi_1^j) f_0 + \Phi_1^{s+1} \mathbf{y}_{t-s-1} + \sum_{j=0}^{s} \Phi_1^j \Theta_0 \mathbf{x}_{t-j} + \sum_{j=0}^{2} \Phi_1^j \mathbf{e}_{t-j}. \tag{3.90}$$

The elements of matrices $\Phi_1^j\Theta_0,\ j=1,\ldots,s$ describe the effects in period *j* of a shock in period $j=0$; they are known as the *interim multipliers*. The total effect of an exogenous change from now until the very end is found by adding all matrices $\Phi_1^{s+1},\ \Phi_1^j\Theta_0,\ j=0,1,\ldots,s$. Let $\mathbf{G}$ be the matrix that is the summation of all these matrices. It is possible to show that $\mathbf{G}=(\mathbf{I}_G-\Phi_1)^{-1}\Theta_0$ if and only if the roots of Φ_1 are less than one in absolute value. The elements of $\mathbf{G}$ are the *total multipliers* of the system.

When the system is integrated and cointegrated, $\mathbf{G}$ does not converge almost surely, therefore the total multipliers are not computed. A major difficulty is posed in terms of deriving analytical calculations of the interim multipliers in the case where there is a desire for the multipliers to capture the short run dynamic and long run equilibrium effects implied by cointegrated or seasonal cointegrated variables. To circumvent this problem, Doan's (1992) computational approach will be adopted in this study. He suggests computation of the multipliers by computing forecasts with and without a set of changes to the paths of the exogenous variables, and then subtracting them. In this way, the direct and the interim multipliers will be calculated and plotted against time.

3.4.2.3 **Evaluation of Impulse Responses and Dynamic Multipliers**

A small-scale simulation based on $N=1000$ Monte Carlo experiment is proposed to assess the accuracy of the impulse responses and dynamic multipliers (IRF) of the four selected econometric models. The experiment will use a data generating process (DGP) that will focus attention on the effects for a variable that is endogenous or exogenous determined within a system that has unit roots at zero, semiannual, and annual frequencies. The DGP is based on the following structural relationship,

$$\begin{aligned} y_t &= f(z_t) \\ x_t &= f(y_t) \\ z_t &= z_{t-4}, \end{aligned} \tag{3.91}$$

where the variables y_t and x_t are endogenous and z_t exogenous. The DGP, presented in (3.92), has two cointegrating vectors at the zero frequency, two at the semiannual frequency, and four at the annual frequency, of which two are for the root i, and the second ones for the root $-i$,

$$
\begin{aligned}
\begin{bmatrix}\Delta_4 y_t\\ \Delta_4 x_t\\ \Delta_4 z_t\end{bmatrix} = \begin{bmatrix}m_y\\ m_x\\ 0\end{bmatrix} &+ \sum_{j=1}^{4} l_{1,j}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}-1 & 0 & x_{1(1,j)}\end{bmatrix}\begin{bmatrix}S_j(B)y_t\\ S_j(B)x_t\\ S_j(B)z_t\end{bmatrix} \\
&+ \sum_{j=1}^{4}\frac{l_{2,j}}{1-x_{2(2,j)}x_{1(2,j)}}\begin{bmatrix}0\\ x_{2(2,j)}\\ 0\end{bmatrix}\begin{bmatrix}-1 & x_{1(2,j)} & 0\end{bmatrix}\begin{bmatrix}S_j(B)y_t\\ S_j(B)x_t\\ S_j(B)z_t\end{bmatrix} + \begin{bmatrix}e_{y,t}\\ e_{x,t}\\ e_{z,t}\end{bmatrix},
\end{aligned} \tag{3.92}
$$

where $\mathbf{e}_t = (e_{y,t}, e_{x,t}, e_{z,t})' \sim N(\mathbf{0}, \mathbf{I}_3)$, $l = \{l_{ij}\} = \begin{bmatrix}-.2 & .25 & -.25 & -.25\\ -.25 & -.25 & -.25 & -.25\end{bmatrix}$,

$x_1 = \{x_{1(j)}\} = \begin{bmatrix}.6 & -.4 & .4 & .6\\ .4 & -.6 & .6 & .4\end{bmatrix}$, $x_2 = \{x_{2(i,j)}\} = \begin{bmatrix}0 & 0 & 0 & 0\\ 1 & -1 & 1 & -1\end{bmatrix}$, $S_1(B) = \frac{1}{4}(B + B^2 + B^3 + B^4)$,

$S_2(B) = \frac{1}{4}(-B + B^2 - B^3 + B^4)$, $S_3(B) = \frac{1}{2}(-B^2 + B^4)$, $S_4(B) = \frac{1}{2}(-B + B^3)$, with B the backshift

operator, $l_{1,j}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}-1 & 0 & x_{1(1,j)}\end{bmatrix} = a_{1,j}b'_{1,j}$, and $\frac{l_{2,j}}{1-x_{2(2,j)}x_{1(2,j)}}\begin{bmatrix}0\\ x_{2(2,j)}\\ 0\end{bmatrix}\begin{bmatrix}-1 & x_{1(2,j)} & 0\end{bmatrix} = a_{2,j}b'_{2,j}$. Each of

the matrices $a_{1,j}b'_{1,j}$ and $a_{2,j}b'_{2,j}$, $i = 1,2$, $j = 1,\ldots 4$, has two eigenvalues, one at zero and one at $l_{i,j}$, such that the "strength" of attraction to the equilibrium vector does not depend on $x_{1(i,j)}$, $x_{2(i,j)}$, only on $l_{i,j}$. The DGP (3.92) has been motivated by a DGP used by Franses and Kunst (FK)(1999), who used a bivariated DGP with a unique unit root at the semi-annual frequency.

The theoretical IRF of the DGP (3.92) will be compared against the median of the $N = 1000$ IRFS estimated with the VECM, SVECM, CDSEM, and SCDEM models. The use of the median rather than mean responses is proposed so that the results are less affected by occasional very large responses that may occur in the simulations. Phillips (1998) has used this approach, for instance.

3.5 The Application

3.5.1 The Structural Model for the U.S. Wheat Market

The specification adopted for this study closely resembles the models of Chambers and Just (1981). The selected structural model is substantially aggregated compared with much less aggregated models used by Bailey (1989), Devadoss *et al.* (1993), and In and Mount (1994). For instance, it is useful to domestic disappearance of wheat into food disappearance and feed disappearance, or to differentiate U.S. exports by country of destination, or to partition inventories into government-held inventories and privately-held inventories, approaches that are avoided in the present study. The aggregated specification adopted is justified by the fact that interest is centered on the net effects of fluctuations in the U.S. wheat market for simulation analysis –multiplier and impulse-response analysis, and forecasting purposes rather than on each particular component of the market. All the equations are specified in a linear form because the linear relationship is considered to reflect actual economic behavior.

The U.S. wheat market structural model (USWMM) specifies relationships for U.S. wheat production, disappearance, inventories, exports, and prices, variables that are considered endogenously determined in the system. An identity that clears the U.S. wheat market is also specified. Other variables present in the model are exogenous, which are assumed to condition the outcome values of the endogenous variables. These exogenous variables are U.S. real disposable income, the U.S. exchange rate, the European threshold price of wheat, the stocks of wheat held by other major exporters, the U.S. real farm price of wheat, the U.S. wheat price support, and seasonal dummies that account for deterministic seasonal effects.

The dynamic structural USWMM takes the following form:

Disappearance: $$PWD_t = f(PWD_{t-1},...,PWD_{t-s},\ RPW_t,\ RPDI_t, \Delta RPDI_t, FALL_t, WINT_t, SPRI_t) \qquad (3.93)$$

Inventory: $$PWI_t = f(PWI_{t-1},...,PWI_{t-s}, RPW_t, \Delta RPW_t, FALL_t,\ WINT_t,\ SPRI_t), \qquad (3.94)$$

Exports: $$\begin{aligned} PWX_t = f(&PWX_{t-1},...,PWX_{t-s}, RPW_t,\ SDR_t,\ THPW_t, \\ &FALL_t,\ WINT_t,\ SPRI_t, DS_t, DX_t), \end{aligned} \qquad (3.95)$$

Production: $$PWPR_t = f(PWPR_{t-1},...,PWPR_{t-4},\ RWAP_{t-2},\ RWSP_{t-2}, D1_t, D2_t), \qquad (3.96)$$

Prices: $$\begin{aligned} RPW_t = f(&PWD_t, PWI_t, PWX_t, PWPR_t, RPW_{t-1},...,RPW_{t-s},\ SDR_t,\ THPW_t, \\ &FALL_t,\ WINT_t,\ SPRI_t), \end{aligned} \qquad (3.97)$$

Identity: $$PWPR_t + PWI_{t-1} = PWD_t + PWI_t + PWX_t, \qquad (3.98)$$

where PWD is per capita US wheat disappearance (bushels per person), $RPDI$ is real per capita disposable income (Base 1996), $FALL$, $WINT$, $SPRI$ are dummy variables for the seasons of the year, PWI is per capita US wheat inventories (bushels per person), RPW is real wheat price, PWX is per capita US wheat exports (bushels per person), SDR is the exchange rate (SDR per dollar), $THPW$ is the European Union threshold price of wheat (units of account per metric ton), $PWPR$ is per capita US wheat production (bushels per person), $RWAP$ is the real average price of U.S. wheat received by farmers, and $RWSP$ is the real support price of U.S. wheat.

Each functional relationship in the USWMM is assumed to be linear to simplify estimation and facilitate the evaluation of the forecasting and simulation analysis capabilities of the model. All variables that are flows or stocks are in *per capita* units so as to preserve the linearity of the system and to allow the straightforward induction of a linear (in parameters) reduced-form from the structural estimates.

The economic fundamentals supporting the USWMM are the partial adjustment to equilibrium of Nerlove, the Marshallian consumer and producer optimization behavior, the flexible accelerator theory of Goodwin, the equilibrium price theory of Working –advocated by Shepherd, Kendall, Samuelson, among other prominent economists, and the formation of expectations.

CHAPTER 4

IMPULSE RESPONSES AND FORECAST PERFORMANCE OF THE SELECTED DYNAMIC ECONOMETRIC MODELS

The main focus of this chapter is in presenting the results of the evaluation of the forecast and impulse responses performance of the four selected econometric models for the U.S. wheat market. The models are reported following the sequence outlined in Table 3.1.

The models studied in this dissertation have not appeared elsewhere in the commodity modeling literature. The results reported here shed light on their appeal for forecasting and impulse response analysis. Well known agricultural commodity market models used to provide this type of information, for instance the USDA and the FAPRI U.S. wheat models, have for the most part ignored that the data may not support stationarity, empirical evidence of which is found in section 4.1 of this chapter. This section also describes the statistical properties of the four econometric models in an empirical setting.

The results presented in sections 4.2 and 4.3 are useful to provide answers to questions that remain open on possible gains or benefits of using a more data-coherent representation in dynamic econometric models of agricultural commodity markets for simulation analysis and forecasting purposes. For instance, no empirical evidence has been provided in the literature on the impacts of the use of seasonal cointegration models for the estimation of impulse responses and the calculation of forecasts. The last section 4.4 summarizes and discusses the results presented in the whole chapter.

4.1 The U.S. Wheat Market Data

4.1.1 Sources and Descriptive Statistics of the Time Series

The sources of the data are as follows. The U.S. real disposable income and the U.S. exchange rate time series are from the Bureau of Economic Analysis and the CitiBase databank, respectively. All the other variables needed for the U.S. wheat market model (USWMM) are from the *Wheat Situation and Outlook Yearbook 2002* published by the Market and Trade Economics Division, Economic Research

Service, U.S. Department of Agriculture. The series are quarterly observed for the period 1975:03-1999:04, and are presented in Appendix D.

Variable definition and terminology are shown in Table 4.1. The top block in this table identifies the endogenous variables, the middle one distinguishes the exogenous variables and the bottom block contains the deterministic variables entering the USWMM. An uppercase letter P in the acronyms indicates the variable is measured in *per-capita* units, while an uppercase letter R indicates the variable is measured in *real dollars*.

The *demand sector* variables entering into the USWMM are U.S. wheat disappearance (*PWD*), U.S. wheat inventories (*PWI*), U.S. wheat exports (*PWX*), U.S. wheat market price (*RPW)*, U.S. domestic income (*RPDI)*, U.S. exchange rate (*SDR*), European Union wheat threshold price (*THPW*), and wheat stocks in other major exporting countries (*WSTOCKW)*. The *supply sector* variables that enter in the USWMM are U.S. wheat production (*PWPR*), U.S. wheat farm price (*RWAP*), and U.S. wheat support price (*RWSP*). The variables that are endogenous are *PWD*, *PWI*, *PWX*, *PWPR*, and *RPW*, while the exogenous variables are *RPDI*, *SDR*, *THPW*, *WSTOCKW*, *RWAP*, and *RWSP*. U.S. wheat production (PWPR) is represented as a quarterly proxy variable by assigning the realized U.S. wheat production to quarter 3 and setting quarters 1, 2, and 4 equal to zero.

The summary statistics for all the variables used are provided in Table 4.2. The upper block of this table presents the descriptive statistics of the endogenous variables, while the lower block in this table presents those of the exogenous. The statistics provided are the mean, the standard deviation, the minimum and maximum observed, and the coefficient of variation. In parallel, the individual panels of Figure 4.1 and Figure 4.2 plot each of the time series with a line that describes the general trend of the variable. In the next two subsections, the information conveyed by Table 4.2 and Figure 4.1 and Figure 4.2 are used in conjunction in briefly describing each of the variables in the U.S. wheat model.

4.1.2 Demand Sector Variables

The USWMM recognizes three main sources of demand for the U.S. wheat: domestic consumption or disappearance, inventories, and exports. The quarterly per-capita average of wheat disappearance (*PWD*) in the period 1975:03-1999:04 is 1.84 bushels (Table 4.2). The plot depicted in the

Table 4.1. Variable definition and terminology used for the econometric U.S. Wheat market model.

Type of Variable	Variable Acronym and Description		Algebraic Notation
Endogenous (Jointly determined)	*PWD* :	Disappearance	$\mathbf{y}_1=\{y_{1t}\}$, $t=1,\dots,T$
	PWI:	Inventories	$\mathbf{y}_2=\{y_{2t}\}$
	PWX:	Exports	$\mathbf{y}_3=\{y_{3t}\}$
	PWPR:	Production	$\mathbf{y}_4=\{y_{4t}\}$
	RPW:	Real own-price	$\mathbf{y}_5=\{y_{5t}\}$, $G=5$
Exogenous (Predetermined)	*RPDI*:	Real disposable income	$\mathbf{x}_1=\{x_{1t}\}$, $t=1,\dots,T$
	SDR:	Exchange rate	$\mathbf{x}_2=\{x_{2t}\}$
	THPW:	European threshold price	$\mathbf{x}_3=\{x_{3t}\}$,
	WSTOCKW:	Wheat stocks in other major exporting countries	$\mathbf{x}_4=\{x_{4t}\}$
	RWAP2:	Lagged average price received by U.S. farmers($RWAP2_t=RWAP_{t-2}$)	$\mathbf{x}_5=\{x_{5t}\}$
	RWSP2:	Lagged support price ($RWSP2_t=RWSP_{t-2}$)	$\mathbf{x}_6=\{x_{6t}\}$, $K=6$
Deterministic variables	*Trend*:	change in technology trend	$\boldsymbol{Trend}=\{t\}$, $t=1,\dots,T$
	SUMM:	Seasonal dummy variable for Quarter III (June-August)	$\mathbf{S}_1=\{s_{1t}\}$
	FALL:	Seasonal dummy variable for Quarter IV (September-November)	$\mathbf{S}_2=\{s_{2t}\}$
	SPRI:	Seasonal Dummy variable for Quarter I (December-February)	$\mathbf{S}_3=\{s_{3t}\}$
	DS:	Dummy variable for period 1981:03-1999:04 (available information for ***WSOCKW***)	$\mathbf{DX}=\{DX_t\}$
	DX:	Dummy variable for period 1985:02-1987:02 (unexpected low U.S. wheat exports)	$\mathbf{DX}=\{DX_t\}$
	D1:	Dummy variable for period 1980:03-1985:03 (unexpected low U.S. wheat production)	$\mathbf{D}_1=\{D_{1t}\}$
	D2:	Dummy variable for period 1990:03-1991:02 (unexpected low U.S. wheat production)	$\mathbf{D}_2=\{D_{2t}\}$

panel entitled "PWD: Disappearance" of Figure 4.1 shows that *PWD* presents an increasing trend line but also shows increasing variability in the 1981-1990 years. *PWD* is highly variable, showing a coefficient of variation close to 45%. The minimum disappearance of 0.54 bushels per-capita was registered in quarter 1 of 1989, while the maximum was observed in quarter 2 of 1998, with 2.41 bushels per-capita.

Inventories are much higher on average than disappearance and exports, with a mean of 7.13 bushels per-capita. The observed relative variability of inventories is similar to disappearance, with a coefficient of variation close to 45%. The general trend in U.S. inventories is downward, although it is showing a positive trending pattern for the 1975-1985 period, a decreasing trend for the 1985-1996 period, and an increasing trend for the 1996-1999 years (panel "PWX: Inventories", Figure 4.1). The minimum level of inventories was registered in quarter 1982:03 and the maximum in quarter 1996:02, with levels of 1.42 and 13.90 bushels per-capita, respectively.

Table 4.2. Summary statistics of the US wheat market model time series. Period: 1975:03-1999:04.

Series[a]	Units[b]	Obs	Mean	Std Deviation	Minimum	Maximum	CV[c]
PWD:Disappearance	bsh/pc	98	1.08	0.49	0.55	2.41	44.99
PWI:Inventories	bsh/pc	98	7.13	3.17	1.42	13.90	44.75
PWX:Exports	bsh/pc	98	1.29	0.31	0.67	2.10	24.38
PWPR:Production	bsh/pc	25	9.43	1.214	7.39	12.10	12.87
RPW:Chicago prices	$/bsh	98	3.50	0.58	2.12	4.85	16.62
RPDI:Domestic income	1000$/pc	98	18.64	2.35	14.40	22.82	12.62
SDR:Exchange rate	units/$	98	122.15	17.15	97.27	172.08	14.04
THPW:EU threshold price	EU/mton	98	218.29	118.74	0.00	352.99	54.39
WSTOCKW:Stocks Maj.Exp.	mton/pc	98	25.17	7.45	13.30	42.93	29.59
RWAP:US wheat farm price	$/bsh	98	3.39	0.54	2.29	4.98	15.82
RWSP:US support price	$/bsh	98	3.60	0.74	2.05	4.95	20.43

[a] Sources: PWD, PWI, PWX, PWPR, RPW, THPW, WSTOCKW, RWAP, and RWSP are from the "Wheat Situation and Outlook Yearbook 2002", Market and Trade Economics Division, Economic Research Service, U.S. Department of Agriculture; RPDI is from the Bureau of Economic Analysis; and SDR is from the CitiBase databank.
[b] bsh: bushels; pc: per-capita; $: dollars; EU: euros; mtons: metric tons.
[c] CV: coefficient of variation.

Exports (PWX) were on average higher than disappearance, with an average of 1.29 bushels per-capita. The relative variability of exports is lower than disappearance, with a coefficient of variation close to 24%. The general trend in U.S. exports is downward, although exports show an increasing trend until 1985, and a decreasing trend after (panel "PWX: Exports", Figure 4.1). The minimum level of exports was registered in 1997:02 and the maximum in 1981:04, with levels of 0.67 and 2.41 bushels per-capita respectively.

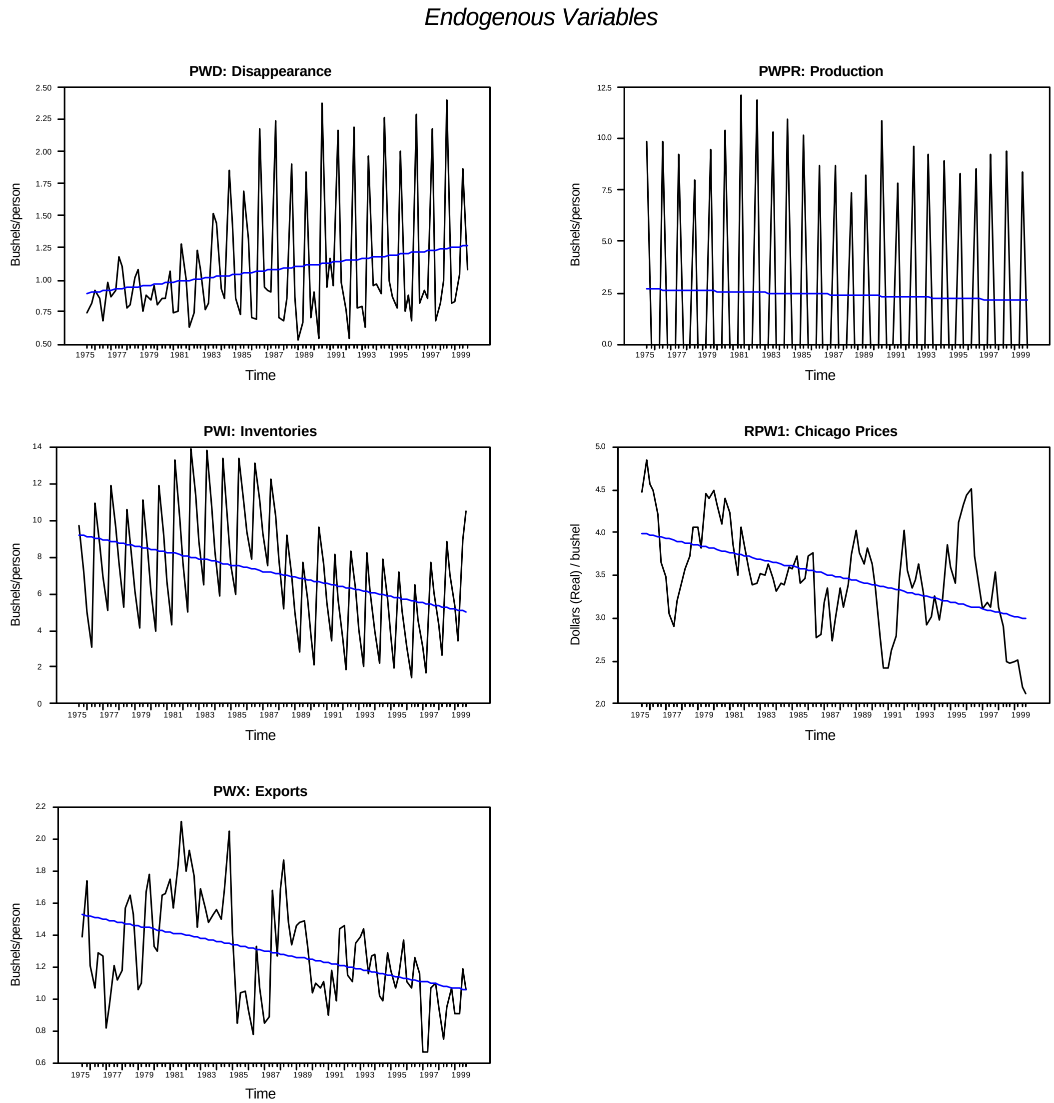

Figure 4.1. U.S. wheat endogenous time series and trends: disappearance (*PWD*), inventories (*PWI*), exports (*PWX*), production (*PWPR*), and Chicago market prices (*RPW1*).

The wheat market real price at Chicago (*RPW1*) averaged 3.50 dollars/bushel, with the minimum price of 2.12 dollars/bushel observed in 1994:04 and the maximum of 4.85 dollars/bushel in 1975:04. The general trend in real wheat prices at Chicago is downward (panel "RPW1: Chicago Prices", Figure 4.1), with an average decline of almost 1 dollar/bushel in the period 1975:03-1999:04. The relative variability of the wheat market price is fairly low (coefficient of variation close to 16%) compared with those shown by disappearance, inventories and exports.

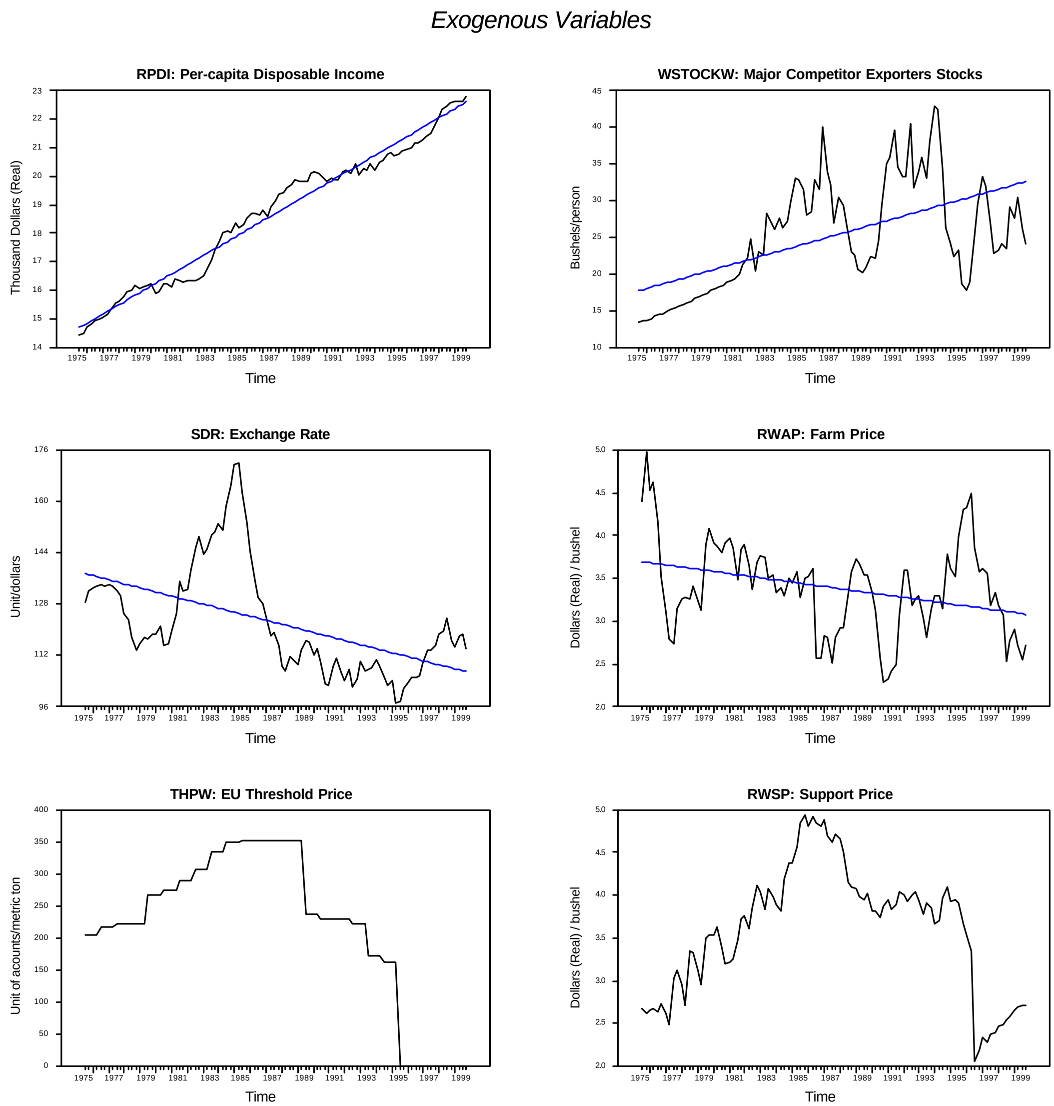

Figure 4.2. U.S. wheat exogenous quarterly time series and trends: U.S. domestic income (*PWDI*), U.S. exchange rate (*SDR*), EU threshold price (*THPW*), major competitors exporters stocks (*WSTOCKW*), U.S. farm prices (*RWAP*), and U.S. wheat support prices (*RWSP*).

U.S. real domestic income (*PWDI*) averaged 18.64 thousand dollars per person, with the minimum income observed in 1975:04 of 14.40 thousand dollars and the maximum in quarters 1999:04 of 22.82 thousand dollars. The general trend of this variable is steadily upward (Panel "PWDI: Per Capita Disposable Income", Figure 4.2), showing an average increase of at least 8 thousand dollar/per-capita in the period 1975:04-1999:04 and a low relative variability around the mean (coefficient of variation of 13%).

The U.S. exchange rate (SDR) averaged 122.15 units/dollar[1], with the minimum exchange rate (lower value of the U.S. currency) observed in 1995:02 at the level of 97.27 units/dollar and the maximum (higher value of the dollar) in 1985:02 with a level of 172.08 units/dollar. The general trend in the exchange rate is downward (panel "SDR: Exchange Rate", Figure 4.2), that is, the American currency depreciated in value by approximately 20 units in the period 1975:04-1999:04, showing a low relative variability around the mean (coefficient of variation of 14%). The exchange rate clearly shows the appreciation process occurred since the beginnings of the 80s until 1985.

The EU threshold price $THPW$ shows an average of 218.30 euros/metric ton. The general trend of $THPW$ (panel "THPW: EU Threshold Price", Figure 4.2) is upward in the period 1975:04-1985:01, reaching its maximum of 352.99 euros/metric tons in 1985:01. Then, $THPW$ remained flat at the maximum between 1985:01-1989:04, and then decreased from 19899:04-1995:04. In 1996 the EU eliminated the threshold prices, when was at its lower level of 162.87 euros/metric tons.

Finally, the stocks of major U.S. competitor wheat exporters, $WSTOCKW$, show an average of 25.17 bushels per-capita. Stocks carried by the main U.S. competitor exporters show an upward trend being highly unstable, with a relative high variability (coefficient of variation of 54%).

4.1.3 Supply Sector Variables

The annual U.S. wheat production averages 9.43 bushels per-capita, which is close to balance, in mean, to the aggregated demand (exports, inventories, and domestic disappearance). The relative variability in production is lower than the one shown by the different sources of demand, with a coefficient of variation of 13%, which indicates a highly stable level of production. The general trend of U.S. wheat production is almost flat (pane; "PWPR: production", Figure 4.1). The minimum level of production was registered in 1988:03 and the maximum in 1981:03, with levels of 7.39 and 12.10 bushels per-capita respectively.

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[1] The U.S. exchange rate describes the amount of a "pool of foreign currencies" a dollar can buy (real, base 1992). This quarterly series is the "EXRUS" series from the City Base database.

The price received by farmers *RWAP* (panel "RWAP: Farm Price", Figure 4.2) closely resembles prices at the market level (panel "RPW!: Chicago Prices", Figure 4.1). The difference between market price and farm price represents the market margin. Farm prices have a mean of 3.39 dollars/bushel, which is lower on average than wheat price at the market level *RPW1*, which was 3.50 dollars/bushel. The observed relative variability of farm wheat prices is low, with a coefficient of variation of 15.82, a little bit smaller than the relative variability of wheat market prices, with a coefficient of variation of 17%. The general trend of farm wheat prices is also downward, similar to the wheat market prices. The minimum farm price observed corresponds to quarter 1994:04 (2.05 $/bushel) and the maximum in 1975:04 (4.95 $/bushel), similarly to those observed at the market level.

Finally, the U.S. wheat price support *RWSP* received was on average 3.5868 dollars/bushel, higher than average market prices and prices received by farmers. This is a clear result of the policy adopted in general by the U.S. for the wheat sector. The general trend of farm wheat prices can be split in two periods. The first period, which has an increasing trend (1975:03-1985), lasted until the 1985 farm bill was passed. After reaching its peak in 1985:04, with a level of 4.95 dollars/bushel, by ending the period of the 1981 farm bill, *RWSP* started declining, with the most dramatic drop along the year the 1996 farm bill was enacted. The minimum farm price observed corresponded to quarter 1996:03 with a level of 2.05 dollars/bushel.

4.1.4 A Graphical Analysis of Seasonality and Nonstationarity

One of the time series properties of economic data that are of main importance to this study is nonstationarity and seasonality. In order to stress these points and illustrate some of the characteristics of the time series involved in this study, consider the graphs depicted in Figure 4.3 to Figure 4.7. Each of these five figures depicts nine graphs, which allow for diagnosing possible seasonal patterns and nonstationary behavior of each endogenous variable of the USWMM, as is explained next.

Panel A of Figure 4.3-Figure 4.7 graphs the levels of each time series, while Panels B and C present the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the series in levels. The ACF and the PACF may convey important information about nonstationarity of the series because if these functions do not tend to zero, the series is nonstationary. This appears to be the case for disappearance, inventories, and production, since the ACFs of theses series did not approach zero after 25 quarters (one fourth of the size of the sample size). On the contrary, exports and Chicago prices do not present strong evidence of nonstationarity.

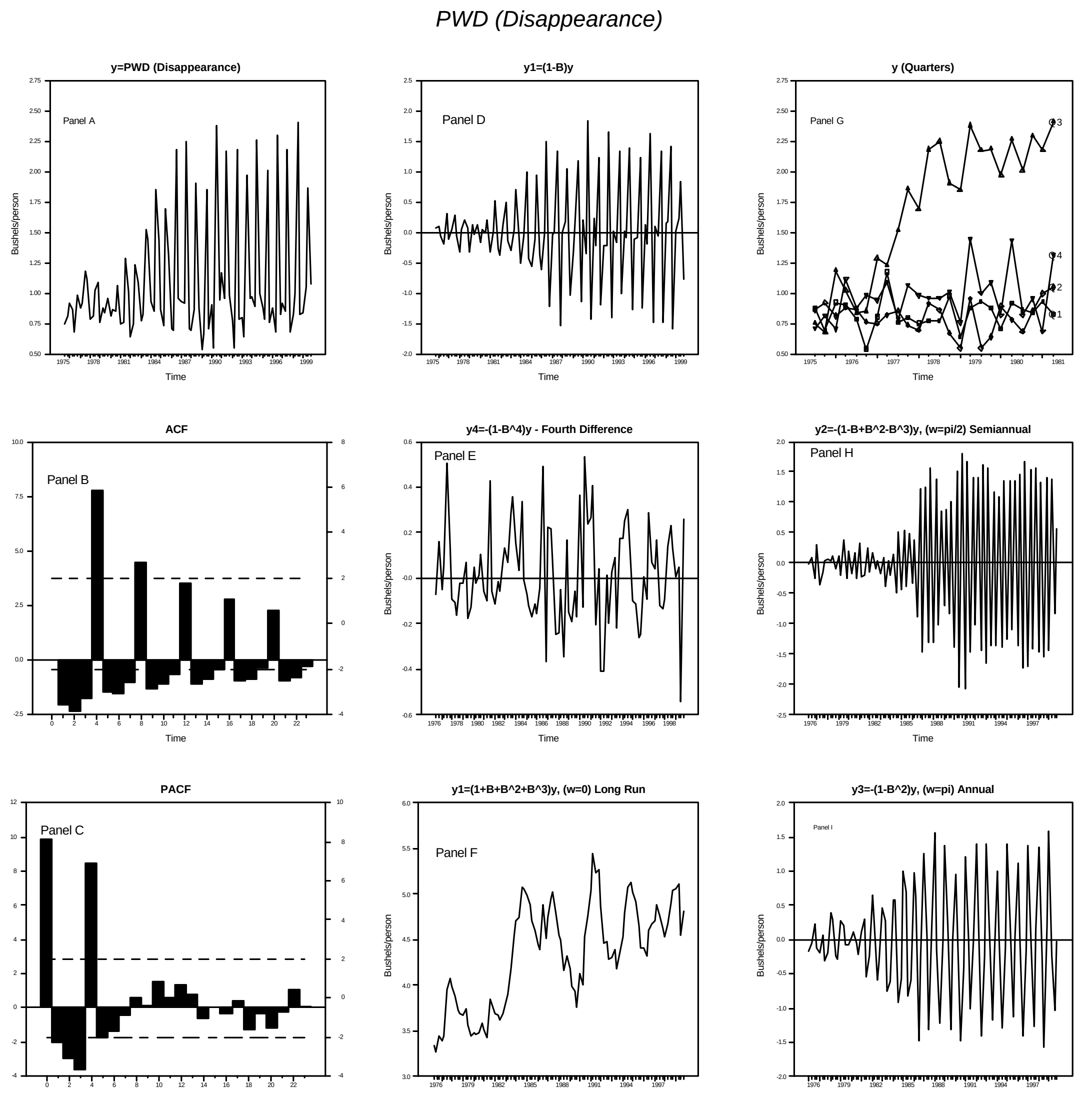

Figure 4.3. Graphical representation of seasonality in the U.S. wheat disappearance.

Panel D presents the differenced series; this panel is entitled as “ y1 = (1-B)y”, to highlight the fact that if the series in levels is nonseasonal integrated of order one, the differenced series should show a stationary behavior. Disappearance shows a stationary behavior around the mean, but it is clearly not stationary in its variance, since the differenced series is showing an increasing dispersion in the period 1980-1986, with a small dispersion prior to 1980, and a big although stable dispersion since 1986. Inventories and exports, after first differences, present a soft decreasing trend in variability, while production and prices appear to be stable in mean and variance.

PWI (Inventories)

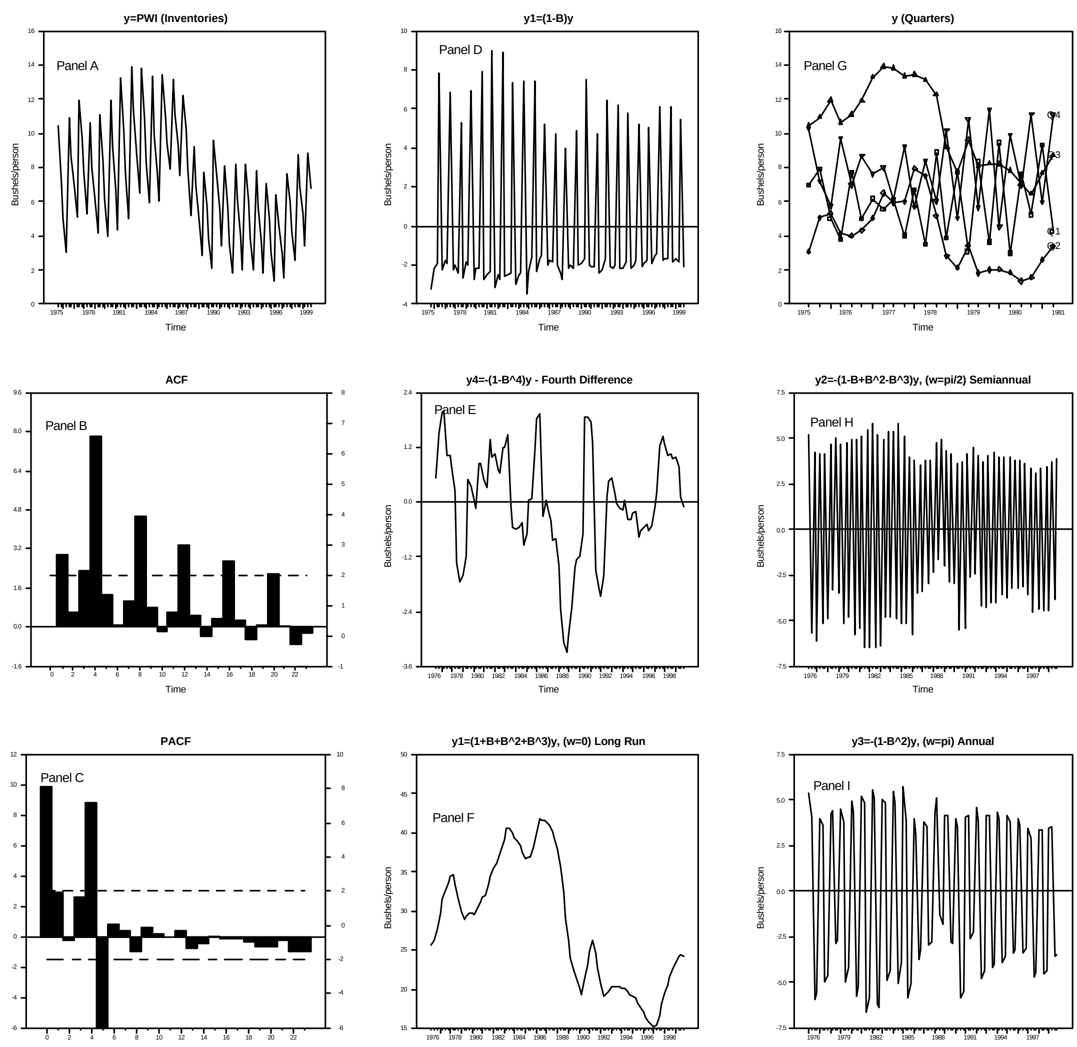

Figure 4.4. Graphical representation of seasonality in the U.S. wheat inventories

Panel E, entitled " y4 = (1-B ^ 4)-Fourth Difference" , presents the Δ_4 -filtered series. Recalling from chapter 3 that this filter sweeps all possible unit roots at the seasonal frequencies of quarterly series, then this graph should show a stationary behavior of the filtered series if the series in levels is seasonal integrated of order 1. From these graphs, it is possible to observe a stationary behavior in mean and variances of all the fourth-differenced endogenous variables.

PWX (Exports)

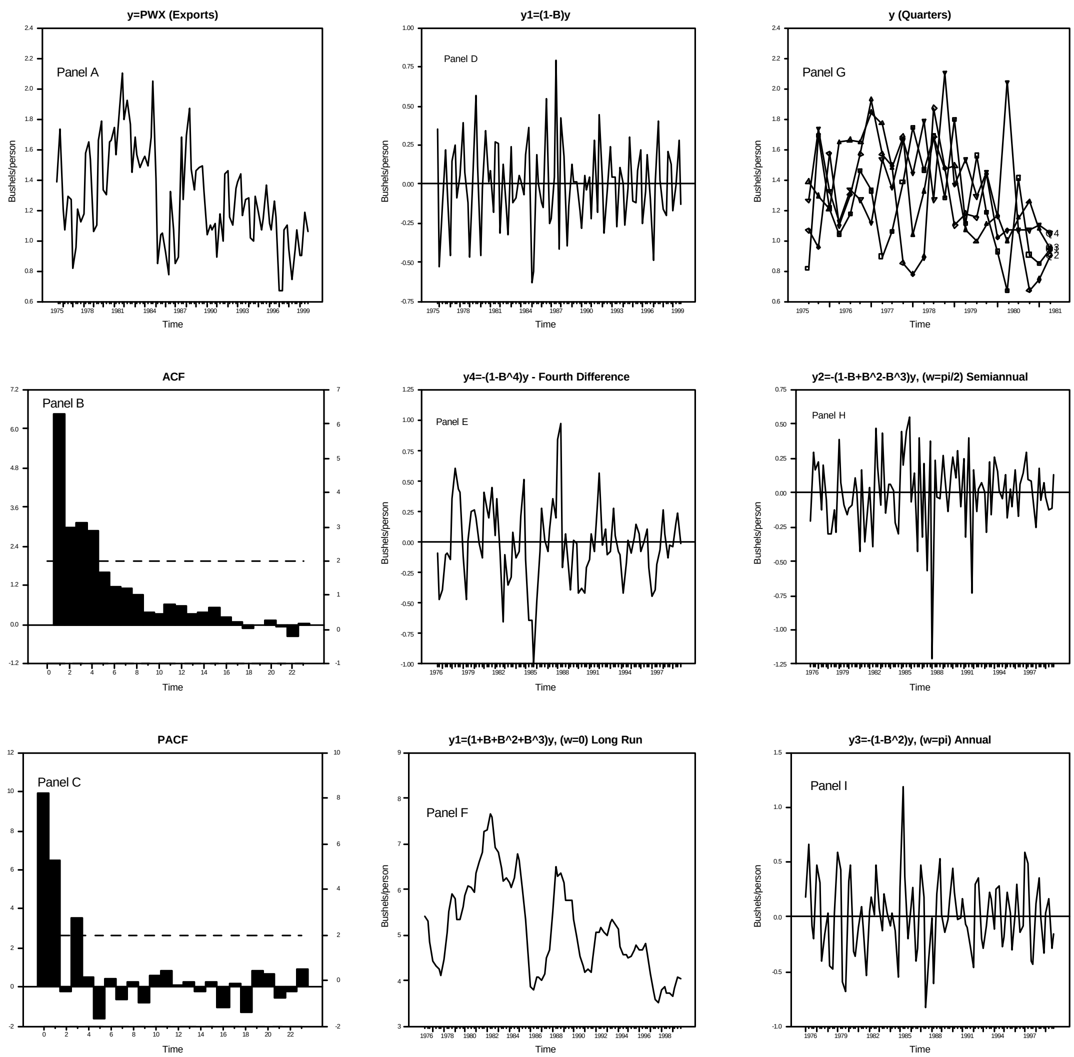

Figure 4.5. Graphical representation of seasonality in the U.S. wheat exports .

Panel F, entitled " $y1 = (1 + B + B \wedge 2 + B \wedge 3)y$, $(w = 0)$ Long Run" depicts the series filtered by the lag-polynomial $(1 + B + B^2 + B^3)$, which eliminates all seasonal unit roots except the one at the zero frequency (i.e. in the long run) if this unit root exists; if this graph shows a stochastic or deterministic trend then the series in levels should have a unit root at this frequency. For instance, disappearance shows a clear positive trend; inventories present a positive trend from 1995-1987 and a negative trend thereafter; exports and production both present a positive trend from 1995-1981 and a negative trend thereafter; while prices have a downward trend. Therefore, all endogenous variables may have a unit root at the zero frequency.

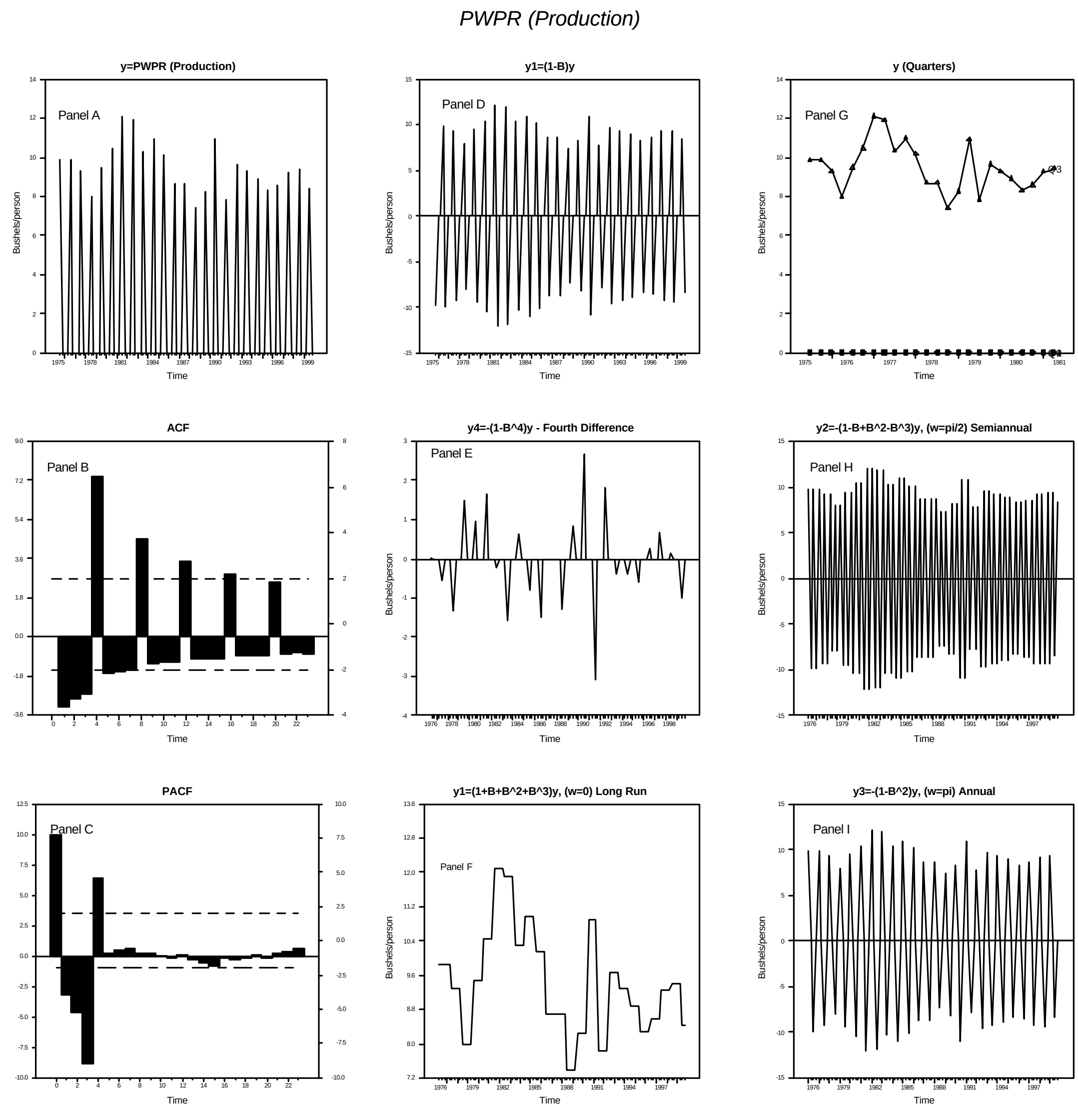

Figure 4.6. Graphical representation of seasonality in the U.S. wheat production.

Panel G, entitled “y(Quarters)”, is one of the most informative because it depicts the series split in its four quarters. The plot of each quarter in this panel is identified as Q1, Q2, Q3, and Q4, respectively. If the series is seasonally integrated then at least one of the quarters should show a nonstationary

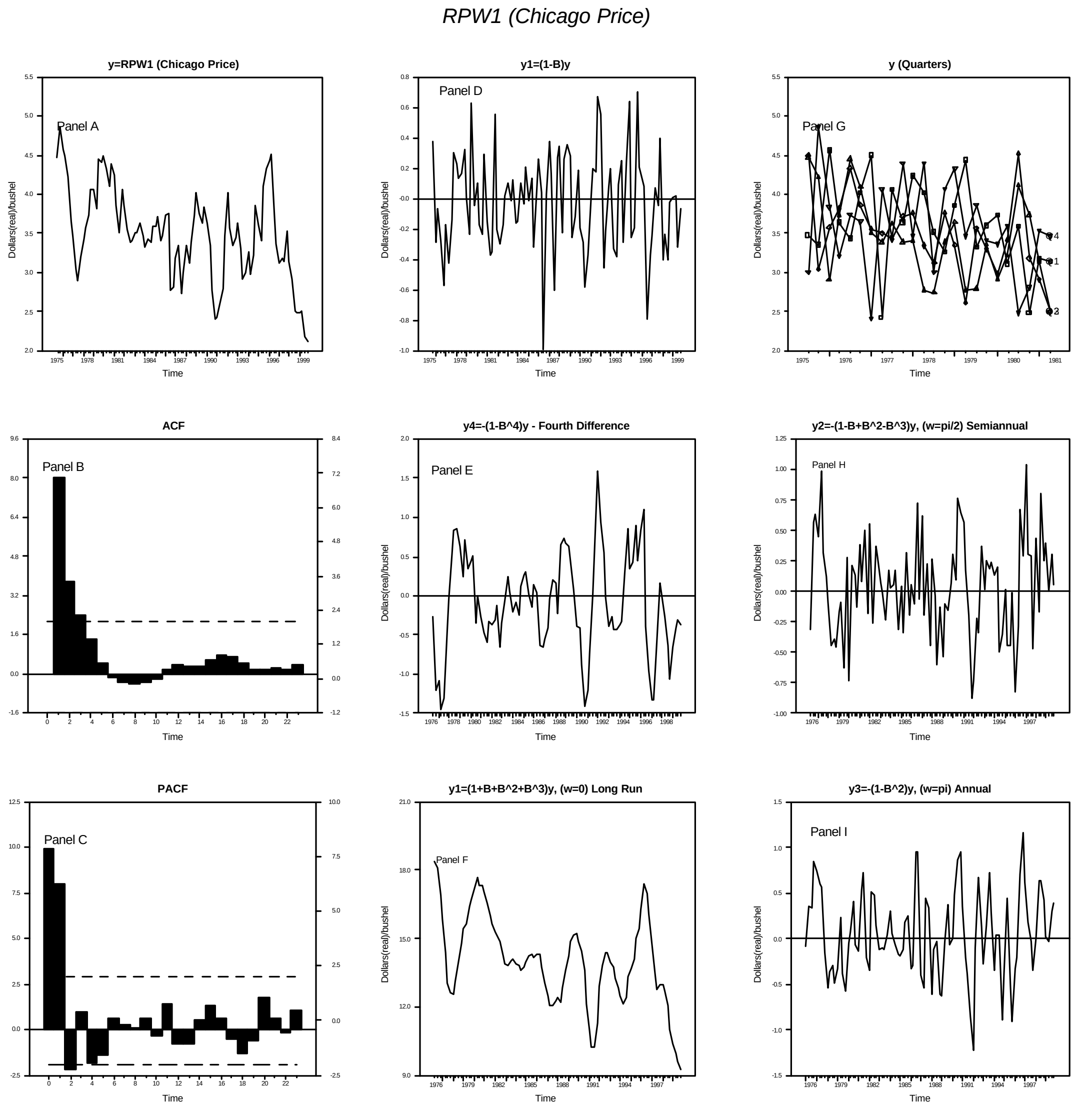

Figure 4.7. Graphical representation of seasonality in the U.S. wheat prices at Chicago.

behavior. For instance, this is the case of quarter 3 for domestic disappearance, which shows an upward trend, while quarters 1, 2, and 3 of disappearance show a stationary behavior. Inventories present two possible nonstationary quarters, Q3 and Q2, in its means, while Q1 and Q4 appear to be nonstationary in their variances since both are presenting an increasing dispersion. Exports present a quadratic trend for all quarters except for quarter 2, which is a kind of stochastic trend, thus seasonal nonstationarity may be present. Production is clearly seasonal (it only occurs in quarter 3), but stationary in mean and variances, while Chicago prices do not present a strong evidence of a seasonal nonstationary behavior.

Panel H, entitled " y2 = -(1-B + B ^ 2-B ^ 3)y, (w = pi/2) Semiannual" , shows the series in levels filtered with the lag-polynomial $-(1-B+B^2-B^3)$, a filter which eliminates all unit roots at all frequencies except the one that may exist at the semiannual frequency (i.e. two cycles per year). If y2 presents a nonstationary behavior, the series in levels may have a unit root at this frequency. For example the behavior depicted by domestic disappearance, inventories, and exports are clearly nonstationary in variances. Production presents a stationary behavior at this frequency, while Chicago prices seem to have a stochastic trend. In the case of Chicago prices, a positive trend is observed in the 1978-1981 period, then prices change to a negative trend in 1981-1984, followed by a positive one for the 1984-1986 period. The above changing trends repeats themselves through 1999.

The ninth Panel, Panel I, entitled " y3 = (1-B ^ 2)y, (w = pi) Annual", shows the series in levels filtered with the lag-polynomial $-(1-B^2)$, a filter which eliminates all unit roots at all frequencies except the one that may exist at the annual frequency (one cycle per year). If this panel presents a nonstationary behavior of y3, then the series in levels may have a unit root at the annual frequency. This seems to be the case for domestic disappearance, which shows this type of behavior. All the other endogenous variable are not presenting a clear nonstationary behavior at this frequency.

In summary, domestic disappearance (*PWD*), inventories (*PWI*), exports (*PWX)*, and production (*PWPR*) are depicting a seasonal nonstationary behavior, while Chicago prices (RPW1) follows a nonseasonal nonstationary behavior.

4.1.5 Testing for Nonseasonal and Seasonal Unit Roots

Visual inspection of Figures 4.3-4.7 provides a very clear insight into the seasonal and nonstationary properties of the wheat market data. More formal tests of unit roots at various frequencies (seasonal unit roots), however, must be conducted.

One of the most common and oldest prescriptions for the treatment of seasonality is to consider the difference between a given quarter or month of the current year and the same quarter or month of last year. This has lead to the use of seasonal differences such as $\Delta_4 = 1 - B^4$, where B is the lag operator (Box and Jenkins, 1970). In the aftermath of the developments of tests for unit roots at the zero frequency, Dickey, Hasza, and Fuller (DHF) (1984) developed the distributional properties and suggested a test following the tradition of the so-called Dickey-Fuller test (Dickey and Fuller, 1976). The test is based on an auxiliary regression of the form $(1 - B^s) y_t = p\, y_{t-2} + e_t$, $s = 2,4,12$, and the test statistic is the 't-value' corresponding for p. Due to the non-standard distributional properties of the t-value under the null hypothesis $H_0 : p = 0$, DHF provides the fractiles of simulated distributions, which may give the critical values to be applied when testing the null against the stationary alternative $H_1 : p < 0$. To whiten the errors the auxiliary regression may be augmented by lagged values of $(1 - B^s) y_t$ and with deterministic parts as intercept, seasonal dummies, and trend.

The limitation of the DHF test is that is is a joint test for roots at long-run and seasonal frequencies, and its alternative is a specific *s*th order autoregression. For instance, the polinomium $1 - B^4$ can be decomposed as $1 - B^4 = (1 - B)(1 + B)(1 - iB)(1 + iB)$, which clearly shows that $1 - B^4$ has roots $B = 1, -1, -i,$ and i, all of length 1. These roots correspond to the zero ($B = 1$), the semi-annual ($B = -1$), and the annual ($B = \pm i$) frequencies if the data are quarterly. This is the basis of the extension of the DHF test by Hylleberg, Engle, Granger, and Yoo (HEGY) (1990). They proposed a test for the quarterly case based on the auxiliary regression

$$(1 - B^4) y_1 = p_1 y_{1,t-1} + p_2 y_{2,t-1} + p_3 y_{3,t-2} + p_4 y_{3,t-1} + e_t \qquad (4.1)$$

where $y_{1t} = (1 + B + B^2 + B^3) y_t$ removes the seasonal unit roots and leave in the zero frequency unit root, $y_{2t} = -(1 + B + B^2 + B^3) y_t$ and $y_{3t} = -(1 - B^2) y_t$ leave in the root at the semi-annual frequency and the annual frequency, respectively.

The existence of unit roots at the zero (long-run), semi-annual, and annual frequencies implies that $p_1 = 0$, $p_2 = 0$, $p_3 = p_4 = 0$, respectively. The t-values on p_1 and p_2 are shown to have a Dickey-

Fuller distribution under the null hypothesis of $p_1 = 0$ or $p_2 = 0$, while the t-value on the p_3 has a DHF distribution with $s = 2$ conditional on $p_4 = 0$. A joint test of $p_3 = p_4 = 0$ is based on the F-value and the critical values of the distribution are presented in HEGY.

Table 4.3 presents the results of the HEGY test, at the zero, semi-annual and annual frequencies, for all the variables in the USWMM. The augmented Dickey-Fuller (ADF) test, and the Phillip-Perron (PP) test for testing unit roots at the zero frequency are presented in the first two columns of Table 4.3. As tests for the null hypothesis of stationarity have been proposed as more powerful alternatives to tests having the null hypothesis of unit roots at the zero frequency, like the ADF and PP tests, the Kwiatkowsky *et al.* (KPSS) 1992) test is also presented in the last column of Table 4.3. In this table, I(1) in a given cell indicates the test concludes that the variable has a unit root at the significance level of 0.05 at the corresponding frequency and I(0) indicates that the test concludes the series is stationary at that frequency. The terms following the colon represent an intercept(I), seasonal dummies (SD), and a trend (Tr).

Table 4.3. Seasonal unit roots results for the ADF, PP, KPSS and HEGY tests.

VARIABLE	ADF Zero freq.	PP Zero freq.	KPSS[a] Zero freq.	HEGY Zero freq.	HEGY Semi-annual freq.	HEGY Annual freq.
PWD:Disappearance	I(1)	I(0):I,Tr	Trend St.	I(1):I,SD,TR	I(1):I,SD,Tr	I(1):I,SD,Tr
PWI:Inventories	I(1)	I(0):I,Tr	I(1)	I(1):I,SD,Tr	I(1):I,SD,Tr	I(1):I,SD,Tr
PWX:Exports	I(1):I	I(0):I,Tr	I(1) *	I(1):I,SD,Tr	I(0)	I(0)
PWPR:Production	I(1)	I(0):I,Tr	Trend St.	I(1):I,SD,Tr	I(1):I,SD,Tr	I(1):I,SD,Tr
RWP:Chicago prices	I(0):I	I(0):I	I(1) *	I(1):I,SD,Tr	I(0)	I(0)
RPDI:Domestic income	I(1):I	I(1):I	I(1)	I(1):I,SD,Tr	I(0)	I(0)
SDR:Exchange rate	I(1):I	I(1):I	I(1)	I(1):I,SD,Tr	I(0)	I(0)
THPW:EU threshold price	I(1):I	I(1):I	I(1)	I(1):I,SD,Tr	I(0)	I(0)
WSTOCKW:Stocks Maj.Exp.	I(1):I	I(1):I	I(1)	I(1):I,SD,Tr	I(0)	I(0)
RWAP:US farm price	I(1):I	I(0)	I(1) *	I(1):I,SD,Tr	I(0)	I(0)
RWSP:US support price	I(1):I	I(1):I	I(1)	I(1):I,SD,Tr	I(0)	I(0)

[a] An asterisk in this column indicates a significance level of 0.10

Based upon the results of Table 4.3 it is possible to conclude about what Figure 4.1 -Figure 4.7 suggested, that is, U.S. wheat disappearance (*PWD*), inventories (*PWI*), and production (PWPR) have significant seasonal unit roots. This conclusion hinges on the results shown for these three endogenous variables by the HEGY tests at the zero, semi-annual, and annual frequencies. U.S. wheat exports (*PWX*) and Chicago prices (*RWP1*) have significant unit roots at the zero frequency based on the HEGY and

KPSS tests. It is worth noting that the PP test fails to detect the presence of a unit root at the zero frequency for all the endogenous variables. It may be the case that seasonal integration lowers the power of the PP test, as pointed out by Maddala and Kim (1998). This does not seem to influence the KPSS test, which rejects the null of stationarity, a conclusion which is in line with the econometric literature that claims that testing the null of stationarity is more powerful that testing the null of nonstationarity (Maddala and Kim, 1998).

Regarding the exogenous variables, it is concluded that all have a significant unit root at the zero frequency, but with no unit roots at the seasonal frequencies. All the tests coincide in their results about unit roots at the zero frequency for the exogenous variables, with the only exception of PP test for the price received by U.S. wheat farmers (*RWAP*).

4.2 Statistical Properties of the Econometric Models

Dynamics in multiple time series models are generally identified from the data. In forecasting and impulse response analyses, this often requires the use of statistical selection criteria to identify how many lags to include in the system

4.2.1 Lag Order Models Selection

Among the various statistical selection criteria available in the literature, the Bayesian statistical criteria (BIC) proposed by Schwarz (1978) is used, since it is a criteria that does not assume a true, but unknown, data-generating process, and is given by

$$BIC(p) = \ln|\tilde{\Sigma}_u(p)| + \frac{2\ln\ln T}{T} pG^2, \qquad (4.2)$$

where p is the number of lags of the endogenous variables $|\tilde{\Sigma}_u(p)|$ is de determinant of the matrix of variance and covariances of the residuals of the model of interest when estimated with p, T is the sample size, and G represents the number of endogenous variables. The estimated $\hat{p}$ for p is chosen so that the BIC is minimized.

0 presents the BIC for the four selected models. The minimum BIC for the vector error correction model (VECM) of –16.9418 is observed when the model uses 5 lags and for the seasonal vector error correction model (SVECM) the minimum BIC of –11.2347. For the cointegration dynamic simultaneous equation model (DSEM) the minimum BIC is of -22.9287, when calculated using 6 lags. For the seasonal cointegration dynamic simultaneous equation model (SDSEM) the minimum BIC of –22.5577 is also observed for 6 lags.

Table 4.4. BIC values for the selected models for the U.S. wheat market[a].

	Lags					
Models	**2**	**3**	**4**	**5**	**6**	**7**
VECM	-12.9536	-12.9564	-12.9348	**-16.9418**	‾	‾
SVECM	*	*	-6.5028	**-11.2347**	-10.9762	‾
CDSEM	•	•	-20.4404	-21.3266	**-22.9287**	•
SCDSEM	*	*	-21.320	-22.1523	**-22.5577**	•

[a] BICs in bold indicates a minimum. Some characteristics in the models avoid the calculation of the BIC: A * indicates the model specification assumes four or more lags, • indicates that some instrumental variables are linearly independent over the regression range, and ‾ indicates a non-invertible matrix in the reduced-rank regression.

In synthesis, the BIC identifies a vector autoregressive model of order 5 as the underlying model for the VECM and the SVECM models, while it identifies that the variables must enter with 6 lags for the DSEM specification and with 5 lags for the SDSEM.

4.2.2 Empirical Adequacy of the Selected Econometric Models

The selection of the lag order, as performed in the previous subsection, may be interpreted as a method for determining a filter that transforms the given data into a white noise series (Lütkepohl, 1993). As long as the residuals of a given model are close enough to white noise, that model may be regarded as appropriately specified (Judge *et al.* 1987). For the forecasting purposes, it may not be of prime importance whether the residuals are really white noise as long as the model forecast well (Lütkepohl, 1993). On the other hand, non white noise residuals may indicate that important variables are omitted from the system, which may lead to distortions in the impulse responses and make them worthless for dynamic analysis.

4.2.2.1 **Testing for Residual Autocorrelation**

The presence of correlation in the estimated residuals may indicate the specification of the model should be modified to include more lag terms or that some important variable is not been included. The autocorrelation function of the estimated residual can be used to show that the population equivalent of the residuals are asymptotically uncorrelated. Ljung and Box (LB) (1979) propose a joint test for the significance of the first m residual autocorrelations $r_k = \sum_{t=k+1}^{T} \hat{e}_t \hat{e}_{t-k} \Big/ \sum_{t=1}^{T} \hat{e}_t^2$, which is given by

$$LB(m) = T(T+2)\sum_{k=1}^{m}(T-k)r_k^2 \xrightarrow{a} c^2_{(m-p)} . \tag{4.3}$$

Table 4.5 reports the LB statistics for the four econometric models. Using a significance level of 0.01, Model 1 (VECM) is the only model that shows residuals with significant autocorrelation. It is clear form the results reported in this table that the other models bypass the autocorrelation problem that Model 1 has, therefore, that this model is possible misspecified.

4.2.2.2 **Autoregressive Conditional Heteroskedasticity**

Another useful property of an estimated error process for time series model is that it is not conditionally heteroskedastic. An example of a process for which the assumption of homoskedasticity is not valid is the autoregressive conditional heteroskedasticity (ARCH) process (Engle, 1982; Bollerslev, 1986). The simple ARCH(k) process is given by

$$E(e_t^2 \mid \Im_{t-1}) = h_t = a_0 + \sum_{i=1}^{k} a_i e_{t-i}^2 , \tag{4.4}$$

where $a_j \geq 0$ ($j = 0,1,...,k$) and $\Im_{t-1}$ denotes the information set at time t. A simple Lagrange multiplier (LM) test statistic for ARCH of order k is given by TR^2, where the R^2 is from the auxiliary regression

$$\hat{e}_t^2 = a_0 + \sum_{i=1}^{k} a_i \hat{e}_{t-i}^2 + u_t . \tag{4.5}$$

Table 4.5. Ljung-Box Q statistics for each equation of the U.S. wheat market selected models.

Model	Statistic	Equations PWD Disappearance	PWI Inventories	PWX Exports	PWPR Production	RPW Chicago Prices
Model 1 (VECM)	Q(4)	**17.54**	**13.82**	**15.18**	**33.21**	6.70
		(0.002)	(0.008)	(0.004)	(<0.001)	(0.153)
	Q(8)	**21.27**	**17.93**	**24.20**	**46.81**	7.22
		(0.006)	(0.022)	(0.002)	(<0.001)	(0.513)
	Q(12)	**24.99**	**28.22**	**27.21**	**70.79**	13.44
		(0.015)	(0.005)	(0.007)	(<0.001)	(0.338)
	Q(16)	**31.35**	**39.50**	31.13	**88.78**	16.99
		(0.012)	(0.002)	(0.013)	(<0.001)	(0.386)
	Q(20)	**37.35**	**41.84**	**34.21**	**93.76**	19.66
		(0.011)	(0.003)	(0.025)	(<0.001)	(0.479)
Model 2 (SVECM)	Q(4)	8.48	4.03	5.10	5.13	7.32
		(0.075)	(0.402)	(0.277)	(0.274)	(0.120)
	Q(8)	8.98 (	4.90	12.83	5.33	14.72
		0.344)	(0.769)	(0.118)	(0.722)	(0.065)
	Q(12)	13.04	8.20	16.57	7.96	19.95
		(0.366)	(0.770)	(0.166)	(0.788)	(0.068)
	Q(16)	15.34	9.04	17.98	8.79	28.11
		(0.499)	(0.912)	(0.325)	(0.922)	(0.031)
	Q(20)	19.39	15.70	24.30	13.00	30.21
		(0.497)	(0.735)	(0.230)	(0.877)	(0.067)
Model 3 (CDSEM)	Q(4)	1.34	2.81	3.89	5.39	**15.44**
		(0.855)	(0.590)	(0.421)	(0.249)	(<0.01)
	Q(8)	1.71	7.60	6.45	7.97	**23.49**
		(0.989)	(0.473)	(0.597)	(0.436)	(<0.01)
	Q(12)	4.64	13.84	6.69	9.61	25.96
		(0.969)	(0.311)	(0.877)	(0.651)	**(0.011)**
	Q(16)	11.03	17.54	15.73	22.69	29.18
		(0.808)	(0.352)	(0.472)	(0.122)	(0.023)
	Q(20)	13.42	27.39	18.41	28.45	31.53
		(0.859)	(0.125)	(0.560)	(0.099)	**(0.049)**
Model 4 (SCDSEM)	Q(4)	2.15 (0.708)	3.25	0.50	1.79	6.5085
		3.77 (0.877)	(40.516)	(0.973)	(0.775)	(0.164)
	Q(8)	6.51 (0.888)	9.43	2.67	3.66	14.0449
		11.04	(80.308)	(0.953)	(0.887)	(0.081)
	Q(12)	(0.807)	11.06	3.04	4.72	19.4458
		14.25	(10.524)	(0.995)	(0.967)	(0.078)
	Q(16)	(0.818)	12.85	15.78	25.58	25.5852
			(10.684)	(0.469)	(0.060)	(0.060)
	Q(20)		16.64	21.34	34.62	27.1313
			(20.676)	(0.377)	(0.022)	(0.132)

Under the null hypothesis of no ARCH, this LM statistic asymptotically follows a $c^2_{(k)}$ distribution. Table 4.6 reports in the column entitled "ARCH" the LM statistics and the *p*-values for the null hypothesis of no ARCH effects for the residuals of each equation and all models. Model 2 presents significant ARCH effects in the residuals of the first equation (U.S. wheat disappearance). Model 4 also presents ARCH effects in the second (U.S. wheat inventories) and fourth equation (U.S. wheat production). Broadly described, the models that account for seasonal cointegration (Models 2 and 4) present evidence of ARCH effects, while the models that assume seasonality is deterministic (Models 1 and 3) do not present ARCH effects.

4.2.2.3 **Normality**

Although the focus of this research is not in interpretation of, say, parameter estimates and *t*-ratios, it is desired that the residuals should be approximately normal, or at least symmetric. Usually, the rejection of normality may indicate that there are some outlying observations or that the error process is not homoskedastic. The well-known Jarque-Bera (1987) test is reported in Table 4.6 under the column entitled “Normality”. This test statistic is given by

$$JB = (SK^2 + K^2) \xrightarrow{a} c^2_{(2)}, \tag{4.6}$$

where $SK = (T/6)^{1/2}(\hat{m}_3^2/\hat{m}_2^3)^{1/2}$ is the statistics for skewness, $K = (T/24)^{1/2}(\hat{m}_4/\hat{m}_2^2 - 3)$ is the statistics for kurtosis, and $\hat{m}_j = T^{-1}\sum_{t=1}^{T}\hat{e}_t^j$.

Table 4.6. Tests of no ARCH effects and normality of the residuals and coefficients of determination of the selected models for the U.S. wheat market.

Model	Equation	ARCH	Normality	R^2
Model 1 (VECM)	**PWD**:Disappearance	2.98 (0.70)	0.33 (0.84)	0.99
	PWI:Inventories	7.54 (0.18)	0.83 (0.65)	0.99
	PWX:Exports	9.74 (0.08)	0.95 (0.62)	0.99
	PWPR:Production	7.87 (0.16)	0.04 (0.97)	1.00
	RWP:Chicago prices	5.85 (0.32)	0.49 (0.78)	0.99
Model 2 (SVECM)	**PWD**:Disappearance	**17.08 (<0.01)**	1.99 (0.36)	0.85
	PWI:Inventories	2.67 (0.75)	3.55 (0.16)	0.95
	PWX:Exports	1.31 (0.93)	0.07 (0.96)	0.93
	PWPR:Production	3.20 (0.67)	1.39 (0.49)	0.87
	RWP:Chicago prices	2.21 (0.82)	**7.36 (0.03)**	0.96
Model 3 (CDSEM)	**PWD**:Disappearance	3.06 (0.69)	0.23 (0.88)	0.88
	PWI:Inventories	9.85 (0.08)	5.58 (0.06)	0.99
	PWX:Exports	1.91 (0.86)	0.03 (0.98)	**0.62**
	PWPR:Production	9.15 (0.10)	5.63 (0.05)	0.99
	RWP:Chicago prices	1.3 (0.93)	0.20 (0.90)	**0.55**
Model 4 (SCDSEM)	**PWD**:Disappearance	2.15 (0.83)	0.05 (0.97)	0.87
	PWI:Inventories	**13.87 (0.02)**	**38.78 (0.01)**	0.99
	PWX:Exports	8.12 (0.15)	0.06 (0.96)	**0.62**
	PWPR:Production	**16.17 (<0.01)**	**12.45 (<0.01)**	0.99
	RWP:Chicago prices	388 (0.57)	0.84 (0.65)	**0.77**

Based upon the results in Table 4.6, the seasonal cointegration models, Models 2 and 4, present some departure from normality for the cases of the last equation and the second and fourth equations, respectively, while the nonseasonal cointegration models, Model 1 and 3, do not.

4.2.2.4 **Variability Explained by the Models**

The coefficient of determination (R^2) of each equation in each model is presented in the last column of Table 4.6. Models 1 and 2 explain in general more than 85% of variability of the endogenous variables, while Models 3 and 4 have some equations with a regular R^2 (less than 80%). In general, the ECM models, Models 1 and 2, present higher coefficients of determination that those shown by their counterpart, the dynamic simultaneous equation models, Models 3 and 4. These results are in line with empirical evidence reported in the literature showing that unrestricted VAR-type models usually outperform the structural-type models in their predictive capabilities. In Model 3, for example, the R^2 of the equation for exports is 0.65 and Chicago prices 0.55. In Model 4, the R^2 of the equation for exports is 0.62 and Chicago prices is 0.77. This is a second and important result, since there is an increase of almost 20% in the variability explained for Chicago prices in favor of Model 4 with respect to that explained by Model 3. This result may be explained by the fact that Model 3 misspecifies the nonstationary nature of the seasonal components.

4.2.2.5 **Conclusions on Diagnostic Measures**

The most important conclusion of the analysis conducted on the empirical adequacy of the models is that autoregression is not present in the residuals of Model 2, 3, and 4. Due to autocorrelation being present in the residuals of Model 1, it is diagnosed that this model is misspecified. One possible source of misspecification is the stochastic nonstationary seasonal nature of the data, a property that Model 1 does not capture.

Second, the VAR-related models, Models 1 and 2, successfully explain higher levels of variability, in general, than the structural models, Models 3 and 4. This is a result that may be important for forecasting purposes, since structural model impose restrictions across equations on the presence of some variables, which naturally decrease the predictive capabilities of these models. The forecast ability comparison of the models conducted in section 4.3 will shed light on these issues.

Third, Models 3 and 4 present residuals in some of their equations that may be heteroskedastic and non-normal. This result is important for inference purposes on the parameters of the models, but not on their estimation. This is a question not addressed in this research, since the main interest is in constructing forecasts and impulse response functions.

4.3 Forecasting Performance Evaluation

One of the themes of this dissertation was to evaluate the out-of-sample forecasting performance of various multiple time series and structural-time series models using data for the U.S. wheat market. Estimation results are presented in Appendix E for the endogenous variables (U.S. wheat disappearance, inventories, exports, prices and production) and four models[2]. Out-of-sample forecasts for the above five variables were generated for the period 1996:1-1999:4 using a fixed scheme. Each variable was forecasted from one to eight quarters ahead using the four models: a vector error-correction model (VECM), a VEC with seasonal cointegration (SVECM), a dynamic simultaneous equation model with cointegration (CDSEM), and a DSEM with seasonal cointegration (SCDSEM). The estimated mean-squared errors are reported in Table 4.7.

The first comparison is based on nominal differences in MSEs. In typical empirical evaluations, a model with smaller MSE is judged superior in forecasting. There are six possible pairs of MSE to contrast from these four forecasting models; these models are shown on the last four columns of Table 4.7. The forecast horizons are reported in the first column; for simplicity, only the 2, 4 and 8 steps ahead MSEs are included. The variable being forecasted is found in the second column of Table 4.7. The model generating the smallest MSE for a variable and forecast horizon is highlighted in bold. Concentrating on the top block of in Table 4.7, MSEs for 2 step-ahead forecasts, the VECM attains the minimum MSE in forecasting wheat disappearance, the cointegration DSEM (CDSEM) model is best for forecasting wheat prices at the Chicago market, but the seasonally cointegrated DSEM (SCDSEM) model is best in

[2] Estimated coefficients are of little relevance from a forecasting perspective.

forecasting inventories, exports, and production. As the forecast horizon lengthens to four steps ahead (middle block in in Table 4.7), the structural-seasonal cointegrated model (SCDSEM) becomes the superior contender as it reaches minimum MSEs for all variables. Somewhat surprisingly, however, in forecasting farther into the future (eight steps ahead), the SCDSEM is uniformly superior to the VECM models with and without seasonal cointegration, but only superior in forecasting inventories relative to the cointegrated DSEM.

The recent literature on evaluating forecasting performance suggests that it may be best to conduct such evaluations by testing differences in MSEs. Diebold and Mariano (1995) introduce one such procedure, hereafter referred to as DM. The DM statistic is asymptotically distributed as a standard normal random variable under the null hypothesis of equal MSEs between two competing

Table 4.7. Mean square errors of the U.S. wheat market forecasting models.

		Models			
Horizons	**Forecasting Model**	**M1:VECM**	**M2:SVECM**	**M3:CDSEM**	**M4:SCDSEM**
2	**PWD**:Disappearance	**0.011**	0.090	0.167	0.167
	PWI:Inventories	165.228	11.130	0.314	**0.032**
	PWX:Exports	12.474	1.822	0.099	**0.068**
	PWPR:Production	148.508	17.288	0.078	**0.002**
	RWP:Chicago prices	57.408	0.657	**0.354**	0.511
4	**PWD**:Disappearance	4.187	0.412	0.111	**0.097**
	PWI:Inventories	7.402	4.061	0.618	**0.113**
	PWX:Exports	3.459	1.260	0.191	**0.146**
	PWPR:Production	34.937	14.486	0.113	**0.011**
	RWP:Chicago prices	2.463	1.041	4.450	**0.193**
8	**PWD**:Disappearance	5.625	0.959	**0.044**	0.097
	PWI:Inventories	52.917	85.385	0.578	**0.149**
	PWX:Exports	4.692	3.149	**0.086**	0.266
	PWPR:Production	66.443	25.350	**0.091**	0.102
	RWP:Chicago prices	18.303	3.281	**0.152**	1.358

models. A Bonferroni approach (Johnson and Wichern, 1998; Lutkepohl, 1993) is also used to evaluate predictive ability. These testing results are reported in Table 4.8. Note that variables and models are reported in columns 1 and 2; forecast horizons and models are reported in columns 3-11. To facilitate evaluation of these results, a new row labeled “Ranking” has been added to identify model superiority in

Table 4.8. Diebold-Mariano statistics for U.S. wheat forecasting models

		Horizons								
		2			4			8		
Variable	**Model**	M2	M3	M4	M2	M3	M4	M2	M3	M4
PWD: Disappearance	M1: VECM	**-3.670**	**-2.248**	**-2.945**	**3.276**	**3.367**	**3.266**	**2.354**	**2.648**	**2.625**
	M2: SVECM		-0.848	-1.040		**2.549**	1.564		**2.542**	**2.387**
	M3: CDSEM			-0.031			0.160			**-2.364**
	Ranking	M2=M3=M4<M1			M1<M2<M3=M4			M1<M2<M4<M3		
PWI: Inventories	M1: VECM	**4.867**	**6.540**	**6.594**	0.694	1.898	**1.954**	-1.196	**2.180**	**2.202**
	M2: SVECM		1.678	1.679		**2.558**	**3.259**		**2.037**	**2.049**
	M3: CDSEM			1.710			**3.087**			**3.137**
	Ranking	M1<M2=M3=M4			M1<M2<M3<M4			M2=M1<M3<M4		
PWX: Exports	M1: VECM	1.831	**2.313**	**2.325**	1.008	1.340	1.335	0.600	**2.094**	**1.956**
	M2: SVECM		**3.683**	**3.641**		**2.110**	**2.154**		**2.750**	**2.460**
	M3: CDSEM			**2.214**			0.696			-1.313
	Ranking	M1=M2<M3<M4			M1<M2<M3=M4			M1<M2<M3=M4		
PWPR: Production	M1: VECM	1.187	1.434	1.434	1.444	**2.267**	**2.267**	1.128	**1.978**	**1.980**
	M2: SVECM		**2.460**	**2.489**		**4.991**	**5.010**		**4.431**	**4.414**
	M3: CDSEM			1.440			1.944			-0.340
	Ranking	M1<M2<M3=M4			M1<M2<M3=M4			M1=M2<M3=M4		
RWP: Chicago prices	M1: VECM	**6.147**	**6.363**	**6.424**	1.299	**-2.301**	**2.190**	**2.215**	**2.122**	**2.031**
	M2: SVECM		1.138	0.388		**-4.510**	**2.849**		1.301	0.883
	M3: CDSEM			-1.427			**4.546**			**-2.683**
	Ranking	M1<M2=M3=M4			M3<M1=M2<M4			M1<M2=M4<M3		

forecasting. The symbol "<" is used in this context to indicate that the model to the left of this symbol has a significantly lower predictive capacity than the model to the right. The symbol "=" is used to indicate that the model to its left has the same predictive ability as the model to its right. Using this ranking, M1 < M2 implies that Model 1 is inferior to Model 2, and M1 = M2 implies equal forecast performance. Significant DM statistics are reported in bold which indicates that the null hypothesis of equal predictive ability of the related models is rejected at a level $a^* = 0.02$. If a DM statistic is positive and highlighted in bold, the model in the associated row has a lower forecasting accuracy than the model in the corresponding column. If the DM statistic is negative, the converse is true.

In reading the left-upper corner of the results in Table 4.8 (two-step-ahead forecasts), there are six comparisons to be made: VECM and SVECM (DM statistic of –3.67), VECM and CDSEM (DM statistic of –2.25), VECM and SCDSEM (DM statistic of –2.95), SVECM and CDSEM (DM statistic of –0.85), SVECM and SCDSEM (DM statistic of –1.04), and CDSEM and SCDSEM (DM statistic of –0.31). The first three results generate highly significant Z-scores, and since these values are negative, they point to a superior alternative to the base model (the VECM). As a result, the VECM model is assumed superior to the other three models. The results for the next two-pairs comparison of seasonal VECM to structural-time series models generate insignificant Z-scores, suggesting, therefore, equal forecast performance for the SVECM, CDSEM, SCDSEM in forecasting two steps ahead. Lastly, the test results for the two structural-time series models (CDSEM and SCDSEM) generates an insignificant statistic, making the addition of seasonal cointegration unnecessary once cointegration at the zero frequency has been added to a DSEM. These results are summarized as $M1 < M2 = M3 = M4$ in the ranking row of table 4.8 to denote the final results for wheat disappearance at two-steps ahead: model 1 is superior to models 2-4 and models 2-4 have equal forecasting performance. In this instance, the nominal evaluation of MSE differences in Table 4.7 generates the same conclusion as the evaluation based on testing such differences. Pairwise comparisons for 4 and 8 step-ahead forecasts for wheat disappearance are shown on the next two blocks in the top section of Table 4.8. Observe, however, that all the Z-scores are positive and significant when the VECM model is compared to a seasonally cointegrated VECM and to the two structural-time series models. Thus, the VECM model is inferior in forecasting beyond two steps ahead. An additional result at 4 steps-ahead is that a cointegrated DSEM (CDSEM) performs better than either of the VECM counterparts, but it is not superior to the DSEM with seasonal cointegration. In fact, at 8 steps-ahead, the DSEM with seasonal cointegration outperforms the cointegrated DSEM.

A cursory review of the results for testing differences in MSEs in Table 4.8 for the remaining variables (U.S. wheat inventories, exports, production, and Chicago prices) suggests, based on the rankings, that in general the use of a VECM may generate inferior forecasting performance compared to either a VEC that models seasonal cointegration or two structural-time series counterparts with and without seasonal cointegration (with cointegration at the zero frequency included). Based on the DM statistics and the rankings provided in Table 4.7 and Table 4.8, it is possible to say that Model 4, in general, has a forecasting ability that is not improved by the other models. Under a few situations, Model 2 and Model 3 have the same forecast performance as Model 1, as it is the case for disappearance, inventories, and Chicago prices for a forecasting horizon of 2-quarters. The forecasting ability of Models 2 and 3 vary across equations and horizons, but Model 2 never outperforms the predictive performance of Model 3. As an overall ranking, Models 1 and 2 show in general a significantly lower forecasting performance that the structural models, Models 3 and 4, at all horizons. Within each model type (vector-error correction versus structural-time series), the seasonal cointegration models outperform their nonseasonal cointegration counterpart significantly.

4.4 Evaluation of Impulse-Responses and Multipliers of the Selected Models

This section presents results on how design features of the selected models are captured by the impulse responses. The role of prior economic information in the specification of dynamic simultaneous equation models, such as Models 3 and 4, determines whether they are asymptotically unbiased (Phillips, 1991). On the other hand, pure time series models, such as Models 1 and 2, do not impose a priori economic information. The nature of the data, as described and tested in a previous section in this Chapter, also allows for imposing a specification that handles the presence of seasonal integration and cointegration within these models. Although all four models allow for stationary long-run equilibrium among the variables in the system, the adjustment that occurs in these relationships in response to various shocks to a market remain unspecified. There is no previous knowledge on the impacts that seasonal cointegration may have on the dynamics of a system, of multiple equation models. Since the dynamic interactions among the series may be hard to interpret (Orden and Fisher, 1993), a first subsection presents impulse responses constructed using a known seasonal cointegrated system and a Monte Carlo simulation that uses the four models proposed in this research. The results of this small-scale experiment will allow for provide more insight into the accuracy and behavior of the dynamics of a simple seasonal cointegrated structural system. This information will provide an empirical framework to analyze the impulse responses estimated with this new generation of commodity models when applied to the U.S. wheat market, as presented in the last part of this section.

4.4.1 Monte Carlo Simulation Evidence

The main objective of the Monte Carlo simulation presented here is to provide some insight into the role of (1) adopting an unrestricted vector autoregression or a structural modeling approach (2) the exclusion of a variable in a given equation, (3) the presence of nonstationary seasonality in the data, and (4) the misspecification of nonstationary seasonal effects.

The data generating process (GDP) used to implement the Monte Carlo experiment was the GDP presented in section 3.4.2.3. It may be recalled that the GDP is composed of three variables, say y_t, x_t, and z_t. The third variable, z_t, is exogenously generated in the system by the seasonal integrated process $z_t = z_{t-4} + e_t$, while y_t and x_t are endogenously determined, y_t as a function of z_t, and x_t as a function of y_t. A unique property of this GDP is that it contains seasonal unit roots and cointegration relationships at the zero, semiannual, and annual frequency in such a way that z_t is weak exogenous for y_t but not for x_t. This DGP allows, therefore, the study of impulse responses between the endogenous y_t and x_t and the instant and impact multipliers of the exogenous z_t on y_t and x_t.

The models in Table 3.1 were used in the simulation at a lag length of $p = 4$, as implied by the DGP. This setting was adopted in order to avoid unnecessary misspecified dynamics of the models. The numbers of replications were 1000 of 100 sample observations generated by the DGP. For each replication, the parameters of the models were estimated accordingly and the impulse response coefficients were estimated for a horizon of eight periods.

The Monte Carlo simulated impulse responses are shown in Figure 4.8-Figure 4.9. These figures graph the median impulse responses of the 1000 replications for the error correction models (Models 1 and 2) and the dynamic simultaneous equation models (Models 3 and 4), respectively. The impulse responses estimated from the models had so many large responses that the graphs cannot be shown on the same figures without distorting the scale of the graphs. Thus, they are depicted in two separate graphs. The top two panels in both figures show the responses of the endogenous variables y_t and x_t to a shock in

y_t. Both panels in the second row of graphs in these figures present the responses of y_t and x_t to a shock in x_t, while the third row of panels present the multipliers of y_t and x_t due to a shock to the exogenous variable z_t.

Figure 4.8 depicts the estimated impulse responses and multipliers (IR) using the time series models, Models 1 (VECM) and 2 (SVECM) and the true responses and multipliers of the DGP. The overlapping of the graphs of the estimated and true IR allows for describing the accuracy and behavior of the IR in a very effective way. The true IRs in this figure clearly depict the three main characteristics of the DGP: (1) the IRs and multipliers do not tend to zero as it is the case of stationary systems; (2) the quarterly characteristic of the series is present in the oscillating cycles of 4 quarters that all the IRs have; and (3) that the response of y_t is null since y_t is not a function of x_t in the DGP (see panel entitled "Orth.IRFs: X->Y" in Figure 4.8).

A first noticeable characteristic of the IRs estimated with Models 1 and 2 in Figure 4.8 is that the IRs constructed with Models 1 and 2 of y_t on y_t (panel entitled "Orth.IRFs: Y->Y") and x_t on x_t (panel entitled "Orth.IRFs: X->X") closely resemble the true IR. Second, that the IRs of y_t on x_t (panel entitled "Orth.IRFs: Y->X") and x_t on y_t (panel entitled "Orth.IRFs: X->Y") are showing that the IR of Model 2 (SVECM) copies better the true IR than Model 1 (VECM) does. In the case of the IR of x_t on y_t it is clear that Model 1 produces a definitely biased IR (recall that the true IR is null since in the DGP y_t is not a function of x_t). Third, that the multipliers of Model 2 (SVECM) are closer to the true multipliers than those of Model 1 (VECM).

The IRs in Figure 4.9 show the IRs of Model 3 (CDSEM) and Model 4 (SCDSEM). A noticeable result is that the IRs of Models 3 and 4 are very accurate with respect to the true IR. The top four panels in this Figure show this. Note also that the estimated responses of y_t to a shock in x_t are null as in the true GDP. This shows clearly how the IRs are affected by excluding a given variable from an equation on the basis of economic tenets. Second, the multipliers of Model 3 are grossly biased, which are attributed to the fact that this model misspecifies the seasonal cointegration relationships in the system.

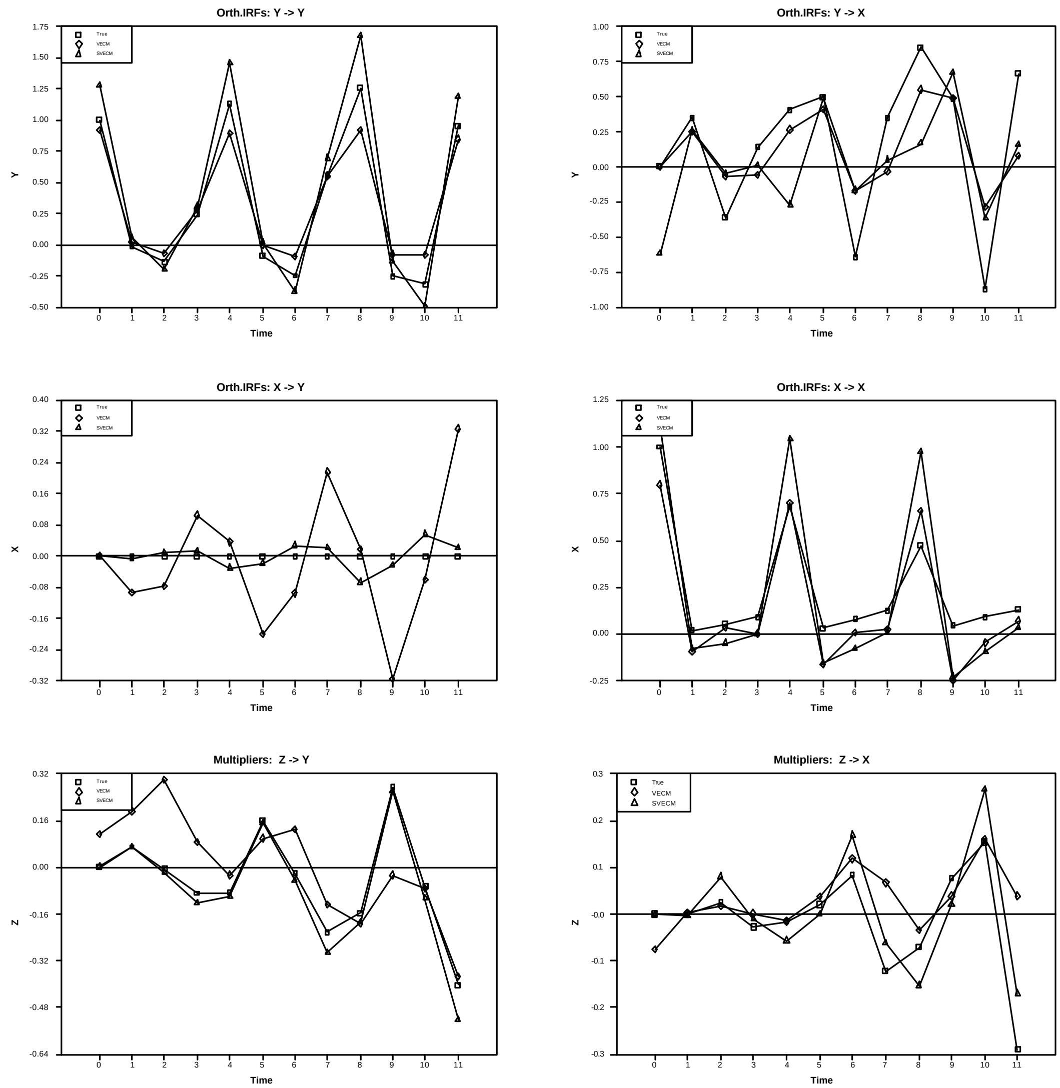

Figure 4.8. True and simulated impulse responses of the vector error correction specifications: Model 1 (VECM) and 2 (SCVECM).

When the IRs and multipliers depicted in Figure 4.8-Figure 4.9 are contrasted the following results may be described. First, it neatly emerges that the IRs of Models 3 and 4 are closer to the true IR than those of Models 1 and 2. This result clearly shows that a dynamic simultaneous equation model may produce more accurate IRs and multipliers than pure time series models. Second, that the models that misspecify seasonal cointegration when it is present in the data, Models 1 (VECM) and 3(CDSEM),

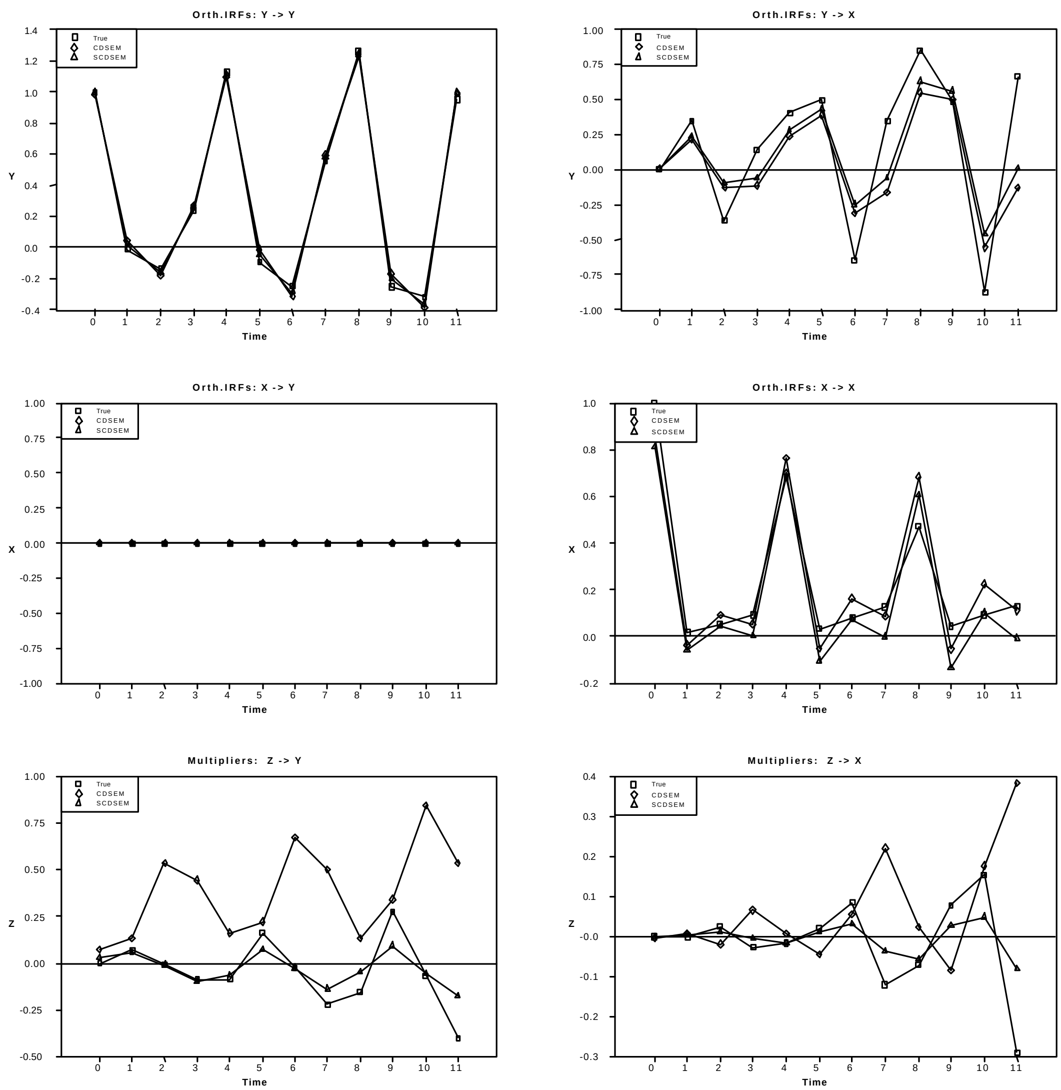

Figure 4.9. True and simulated impulse responses of the dynamic simultaneous equation specifications: Model 3 (CDSEM) and 4 (SCDSEM).

generate IRs and multipliers that are less accurate in two aspects: a higher variability than expected at all periods and/or a departure in the long run from the horizontal axis. This is more clearly depicted in the bottom two panels in both figures, where the responses of the endogenous to a shock in the exogenous are depicted, although in the other panels a similar behavior is shown by the IRs of Models 1 and 3. With these fresh results at hand, now it is possible to analyze the IRs and multipliers of some of the variables of the U.S. Wheat Market.

4.4.2 The Impulse Responses of the U.S. Wheat Exports

In this subsection, the responses of the endogenous variables in the U.S. wheat market (disappearance, inventories, exports, production, and Chicago prices) are analyzed when a shock to U.S. wheat exports occurs. This analysis may be conducted for shocks to each endogenous and exogenous variable in the U.S. wheat market. Yet, this may imply a tedious description and repetition of concepts and ideas, which may bias the focus of this analysis, that is, to evaluate the accuracy of the impulse responses when the models are applied to market data. For the interested readers, Appendix F provides the whole set of IRs and dynamic multipliers for the eleven variables of the U.S. wheat market. The objective of this subsection is to use the results derived from the Monte Carlo simulation as general guidelines for analyzing the IRs and multipliers on the U.S. wheat market, in which seasonal cointegration is supported for the data.

To start the analysis of the IRs and multipliers of the U.S. wheat market endogenous variables to a shock in U.S. wheat exports, the economic relationships between exports and the other endogenous variables is considered first. These relations are implied from the functional form adopted for the U.S. wheat market, as presented in chapter 3, and summarized in Figure 4.10. Exports (PWX) enter into the model as a regressor variable in the fifth equation for Chicago prices (RPW). An unexpected positive shock in exports represents a shift to the right in the demand for wheat in the U.S., thus, an increase in the wheat market price is expected to be observed. If this is the case, the responses of market prices should show an increase that may vanish or not along the quarters, depending on the role of nonstationarity on this variable. The endogenous variable exports does not enter in any of the other equations of the models, but Chicago prices do, which enters in the equations for disappearance (PWD). Therefore, an increase in wheat prices driven by a shock in exports of wheat, should negatively impact disappearance (U.S. domestic consumption of wheat). Collateral effects of a shock to U.S. wheat exports may also be expected, via the contemporaneous correlation with the other variables in the system, on inventories and production of wheat in the U.S. Wheat inventories, for instance, are the unique instantaneous sources of

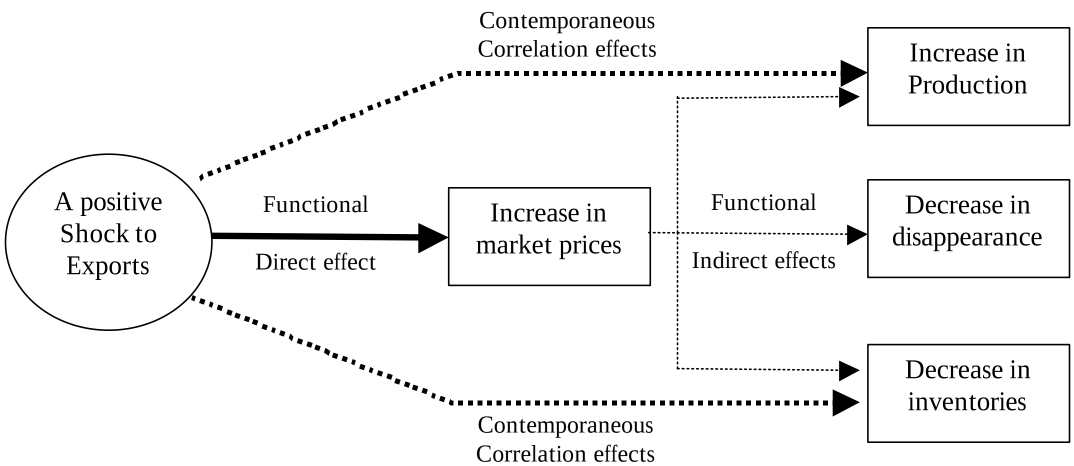

Figure 4.10. Economic causal relationships between U.S. wheat exports and U.S. wheat disappearance, inventories, production, and market prices.

wheat that may satisfy the shock in exports, thus, a reduction in inventories is expected. The effect of a higher price will also send an incentive to wheat stockholders to sell more, thus an effective reduction on inventories should be observed after a shock in U.S. wheat exports. In parallel, farmers will also receive a positive signal to produce more, due to (1) an increase of prices in the market and (2) an expected increased demand to recover inventories (Goodwin, 1947; Labys, 1973). These economic relationships between the U.S. wheat exports and disappearance, inventories, production, and market prices will be used to analyze the rational of the estimated impulse responses in what follows.

The responses of U.S. wheat disappearance to an impulse in U.S. exports of wheat are depicted in the panel entitled “Orthogonal Response of PWD” in Figure 4.11. In this panel, the impulse response constructed with Model 3 (VECM) clearly departs from what it is expected, that is, a negative response in disappearance. All the other models describe a negative effect for period 1, which vanishes across periods. Model 4 (SCDSEM) predicts an immediate negative reaction in disappearance that lasts for almost 5 quarters, and the levels of disappearance return to zero smoothly. This result is in line with the result described by the Monte Carlo simulation for the case of Model 3, which misspecifies seasonal nonstationarity.

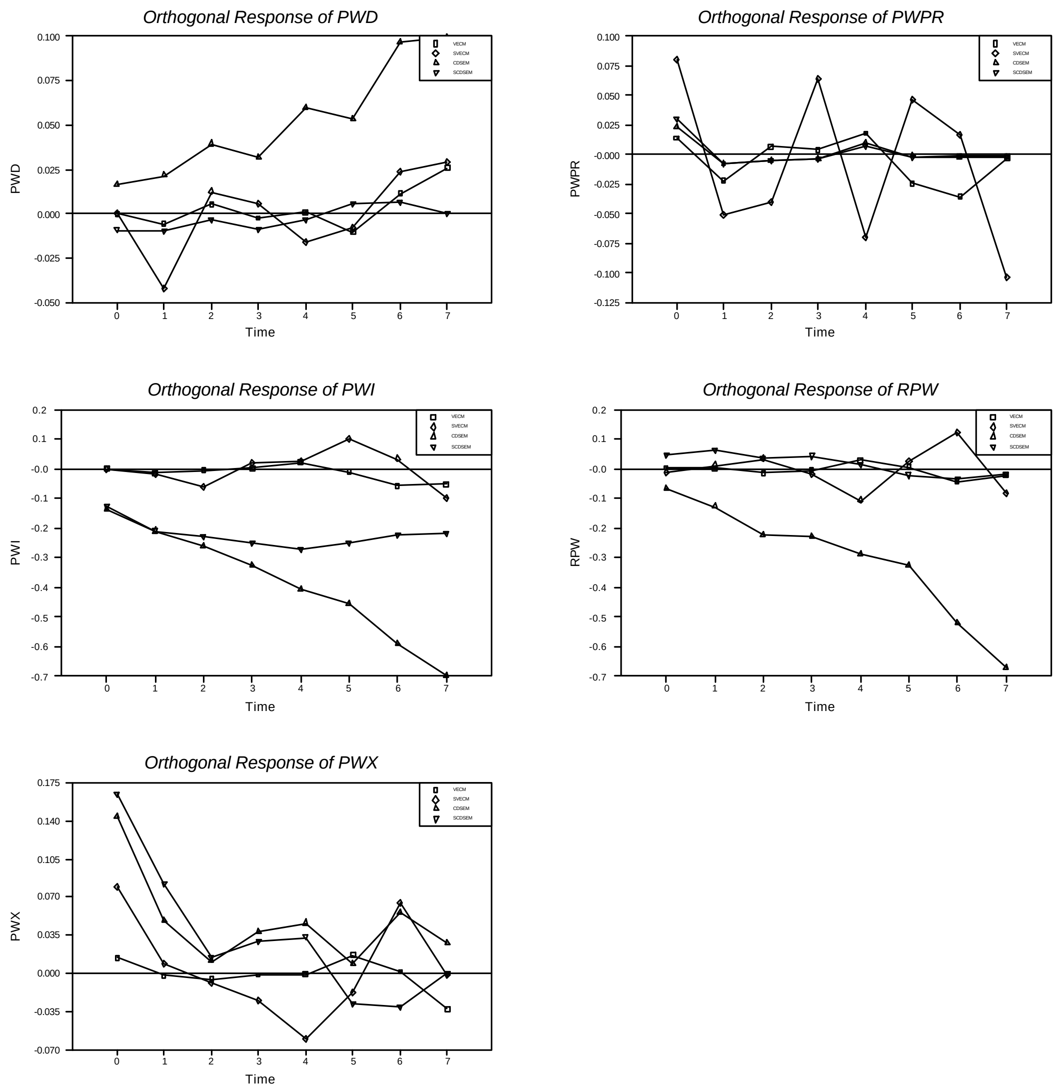

Figure 4.11. Responses of the U.S. wheat market variables to a shock in U.S. wheat exports.

The responses of inventories ("Orthogonal Response of PWI" in Figure 4.11) to a shock in exports estimated by Models 1 and 2 (VECM and SVECM) describes a very timid reaction. In contrast, this is not the behavior described by Models 3 and 4, which predict a similar initial negative response in inventories that is in line with the economic rationale presented above. Yet, Model 3 deviates from the horizontal line at an increasing rate after period 1, while Model 4 predicts that the levels of inventories will not reach their previous levels. The Monte Carlo simulation for the case of Model 3, which misspecifies seasonal nonstationarity, is also in line with this result.

The responses of U.S. wheat exports to an impulse in U.S. wheat exports are depicted in the panel entitled “Orthogonal Response of PWX” in Figure 4.11. All the impulse responses look very similar, in the sense that after the shock, all the IRs approaches to zero after 2 or 3 quarters. This behavior is in line with the results described by the Monte Carlo simulation results, in the sense that a shock of an endogenous to it is well described for all four methods.

The effects of a shock in the U.S. wheat exports to production of wheat in the U.S. are depicted in the panel entitled “Orthogonal Response of PWPR” in Figure 4.11. All models anticipate a positive influence in production, yet the effect does not last and immediately vanishes. Model 2 (SVECM) departs from this description.

Finally Model 4 (SCDSEM) is the only model that describes an increase in the U.S. wheat market prices to a shock in U.S. wheat exports, which lasts for 5 quarters and returns to zero thereafter. The other models describe a behavior that is not in line with the economic rationale, while Model 3 presents the similar inaccurate behavior presented for disappearance and inventories.

In synthesis, what it is clearly shown is that Model 4 (SCDSEM), the structural form that accounts for seasonal cointegration, accurately describes the responses of the U.S. wheat variables to a shock in exports. The other models generate impulse responses that deviate from the expected pattern.

CHAPTER 5

SUMMARY, CONCLUSIONS AND FUTURE RESEARCH

Structural-time series models, a blend between economic theory and data properties of economic time series, have not gained much ground in commodity market modeling despite the overwhelming adoption of other less-theory consistent time series approaches in empirical research. This dissertation contributes to this area of research by applying econometric developments in structural-time series analysis and seasonal cointegration (e.g., Zellner and Palm (1974); Hsiao (1997); Lee (1992); Franses and Kunst (1999); Ghysels and Osborn, 2001) to the study of agricultural commodity markets. An empirical investigation is reported for the U.S. wheat market; this market has been of considerable research interest and is one market where extensive data sets of various frequencies (annual, monthly, and quarterly) are available. Although the apprehension in using time-series models in economic research still exists today, the explosive literature on the subject speaks for its popularity. In the context of commodity modeling, one issue that this dissertation sheds light on is the role of economic theory plays, through simultaneous equation models, in the formulation of theory and data-coherent market models.

This Chapter summarizes the contents of the dissertation, highlights the main conclusions, and outlines areas of immediate research interest. The organization of the Chapter is as follows: Section 5.1 outlines the research themes, which were the subject of Chapter 1. The extensive econometric and market modeling literature is the subject of Section 5.2. The specific conclusions are outlined in section 5.3. Limitations of the work and suggestions for future research are provided in the last section.

5.1 Research Themes

The first research theme investigated the role of testing procedures for cointegration and seasonal cointegration in market data that have unit roots at seasonal frequencies as well as the zero frequency. This issue is considered timely because most applications of cointegration theory and error-correction modeling assume a priori that seasonal components are deterministic. Recent literature beginning with the work of Lee (1992) point out that strong seasonality is a widely observed pattern exhibited by economic time series data. This observation certainly applies to monthly and quarterly data often used in agricultural economics research.

The second theme focuses on applying recent econometric developments on the linkage between multiple time series models and dynamic simultaneous equation models to the development of agricultural commodity market models. This second issue breaks new ground in commodity modeling in two directions: a) it expands classical dynamic simultaneous equation models by introducing cointegration theory and error-correction modeling; and b) it introduces a new dynamic simultaneous equation market model that allows for seasonal cointegration.

Lastly, the research generates empirical comparisons of out-of-sample forecasting performance and impulse response functions of four model specifications for the U.S. wheat market. The forecasting part of this last theme is also timely in the sense that, instead of evaluating model forecasting performance by comparing nominal differences in mean squared errors, the research applies recent developments on the measurement of uncertainty in model prediction by testing for differences in mean-squared-errors.

5.2 The Empirical Literature and Econometric Developments

The empirical analysis adopted a structural model for the U.S. wheat market and estimated two types of specifications. The first specification was the error-correction type models that are usually justified under reduced form assumptions. The specification falls under the umbrella of structural-time series models, which by definition maintain the endogenous-exogenous structure of the U.S. wheat market but make dynamics coherent with the time series properties of the market data. The four specific models used in the empirical comparisons are: a vector error correction model (VECM) without seasonal cointegration, a VECM with seasonal cointegration (SVECM), and dynamic simultaneous equation model with cointegration (CDSEM), and a DSEM with seasonal cointegration (SCDSEM).

Chapter 2 provided a comprehensive review of literature on the economic theory of market models, it reviewed well know works in commodity modeling and discussed econometric specifications that have been published in the literature to date. Although the progress in the adoption of recent econometric developments in time series analysis to commodity modeling is speedy, most published research is formulated in the framework of vector autoregressions or error-correction models with deterministic seasonal variables. What is more relevant from the perspective of this dissertation is the fact that no published research was found on a formal blending between economic theory and time series specifications a la Zellner and Palm (1974).

Chapter 3 provided a nice diagrammatic summary of econometric developments in the area of structural-time series modeling. The Chapter elaborated on the initial efforts by the Cowles commission on estimation, specification, and testing with simultaneous equation models and presented Hsiao's (1997) contribution, which updates the estimation of dynamic simultaneous equation models under conditions of nonstationarity. More specifically, the Chapter summarized the econometric theory on estimation of the models, forecasting and impulse response analysis. Econometric developments on seasonal cointegration are also somewhat recent. The Chapter summarized the main concepts, introduced a VECM with seasonal unit roots and cointegration, and provided theoretical results on a new model, namely, the dynamic simultaneous equation model with seasonal cointegration (SCDSEM). Hsiao's (1977) and Lee's (1992) results form the core of the theory used to arrive at the results presented for the SCDSEM model.

Chapter 4 introduced the data for the U.S. wheat market, which is quarterly beginning in the third quarter of 1975 and extends to the fourth quarter of 1999. The empirical analysis adopted a structural model of the U.S. wheat market (Chambers and Just, 1981) and estimates two types of specifications. The first specification was error-correction type models and the second structural-time series models for the U.S. wheat market. The results on forecast evaluation and analysis of the impulse responses constructed with these models are presented in that chapter, and are summarized in the next section.

5.3 Conclusions and Implications

The general conclusions that emerge from this dissertation research may be summarized in the following areas: integration properties of the U.S. wheat market data, vector-error correction modeling results, and structural-time series findings. The conclusions are drawn from a forecasting experiment and from an analysis of the impulse response functions.

Unit-Root Properties. quarterly data in the U.S. wheat market (1981:01-1999:04) have seasonal unit roots, requiring, therefore, that a VECM or a DSEM should be specified in the framework of seasonal cointegration. This finding is consistent with the often reported use of seasonal dummy variables in the specification of error-correction models in previous research. The finding is far reaching in the sense that it alerts practitioners to be more cautious in the formulation of dynamic models. Seasonal unit-root tests should become part of the tool kit of applied commodity modelers in order to avoid biases specifications and less than optimum forecasts.

Vector Error Correction Modeling. It is found that only in few instances, for example in the two-step ahead forecasts for U.S. wheat disappearance, did a vector error correction model (VECM) perform better than the alternative models. However, the same model (VECM) expanded to allow for seasonal cointegration (SVECM) improved forecasting performance compared to the VECM. This finding should again serve to alert practitioners that mechanical use of deterministic seasonal elements in forecasting serve little purpose. In fact, the attractiveness of the SVECM lies in the fact that seasonal error-correction terms improve forecasting at longer horizons.

Structural Time Series. The comparisons of typical VECMs and DSEM without and with seasonal cointegration in a forecasting context suggest that seasonally cointegrated VECMs perform better that their nonseasonal counterpart, particularly at forecast horizons longer that two quarters ahead. DSEM with seasonal cointegration, however, perform better at longer forecast horizons for production, disappearance, inventories, exports, and prices but not uniformly. Lastly, the impulse response analysis and dynamic multiplier comparisons lead to one salient conclusion, omission of seasonal cointegration components, when significant, generates much different response functions and dynamic multipliers. A typical pattern observed in the wheat data is that impulse responses may not die out when they should.

The research also introduced procedures for evaluating forecasting performance using tests of differences in mean-squared errors (MSE). It is concluded that minor differences in MSEs may not warrant the adoption of a new forecasting model, thus, applied forecasters should carefully assess the reliability of simple structures with these testing procedures.

5.4 Limitations and Future Research

The empirical comparisons generated from this study are based on the use of tests of forecasting performance and impulse response functions. Of much interest to structural analysts may be the estimation of elasticities and/or flexibilities using structural-time series models. Such an area of research is waiting to be addressed.

Of particular interest for immediate future research is an assessment of the small sample properties of impulse response functions for structural-time series models with seasonal cointegration. Preliminary Monte Carlo evidence has been introduced on this issue in this study but a more comprehensive evaluation is needed. The effect of modeling yearly production as a quarterly variable, by assigning the realized production to quarter 3, and setting the other quarters to zero, must also be considered in the Monte Carlo experiment. This may be implemented by adding a forth endogenous variable to the DGP used in this dissertation, with data in quarter 3 and zeros elsewhere.

The focus of the study was on forecasting and impulse response properties of various multivariate models. Perhaps of more immediate empirical relevance may be a more extensive evaluation of various commodity market models using a structural-time series approach similar to the one used in this research.

To conclude, the development of a practical guide for specifying, estimating, forecasting, and impulse responses of the four models considered in this dissertation is in progress.

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APPENDIX A

THE LAGRANGE EXPANSION OF A SEASONAL QUARTERLY LAG POLYNOMIAL

The process $x_t = x_{t-4} + e_t$ is an integrated quarterly process that has four unit roots, one at the zero frequency, one at 2 cycles per year, and two complex pairs at the annual frequency. This fact may be seen by rewriting this process as $(1-B^4)x_t = e_t$. The autoregressive lag polynomial for this example is

$$\begin{aligned} j(B) &= 1-B^4, \\ &= (1-B)(1+B)(1+B^2), \\ &= (1-B)(1+B)(1-iB)(1+iB). \end{aligned} \tag{A.1}$$

For inferential purposes, i.e. estimation or hypothesis testing, the autoregressive polynomial may be conveniently rewritten according to the following proposition given by Lagrange (Franses, 1996):

"*Any (possibly infinite or rational) polynomial* $j(B)$ *which is finite-value at the distinct, nonzero, possibly complex points* $q_1,\ldots,q_p$, *can be expressed in terms of elementary polynomials and a remainder as follows*

$$j(B) = \sum_{k=1}^{p} l_k \Delta(B) \frac{1}{d_k(B)} + \Delta(B) j^{**}(B), \tag{A.2}$$

where $l_k = \dfrac{j(q_k)}{\prod_{j \neq k} d_j(q_k)}$, $d_k(B) = 1 - \dfrac{1}{q_k}B$, *and* $\Delta = \prod_{k=1} d_k(B)$."

To apply this proposition to quarterly data, the autoregressive polynomial $j(B)$ should be expanded about the roots 1, -1, i, and i, for $k = 1,2,3,4,$ as follows,

(i) $k = 1$: $q_1 = 1$;

$$d_1(B) = 1 - \frac{1}{q_1}B = 1 - \frac{1}{1}B = 1 - B;$$

$$1 - d_1(B) = 1 - (1-B) = B.$$

(ii) $k=2$: $q_1=-1$;

$$d_2(B)=1-\frac{1}{q_2}B=1-\frac{1}{-1}B=1+B;$$

$$1-d_2(B)=1-(1+B)=-B.$$

(iii) $k=3$: $q_3=i$;

$$d_3(B)=1-\frac{1}{q_3}B=1-\frac{1}{i}B=1-\frac{i}{i^2}B=1-\frac{i}{-1}B=1+iB;$$

$$1-d_3(B)=1-(1+iB)=-iB.$$

(iii) $k=4$: $q_4=-i$;

$$d_4(B)=1-\frac{1}{q_4}B=1-\frac{1}{-i}B=1+\frac{i}{i^2}B=1+\frac{i}{-1}B=1-iB;$$

$$1-d_4(B)=1-(1-iB)=iB.$$

(vi) $\Delta(B)$

$$=\prod_{k=1}^{4}d_4(B)$$

$$=\left(1-\frac{1}{q_1}B\right)\left(1-\frac{1}{q_2}B\right)\left(1-\frac{1}{q_3}B\right)\left(1-\frac{1}{q_4}B\right)$$

$$=\left(1-\frac{1}{1}B\right)\left(1-\frac{1}{-1}B\right)\left(1-\frac{1}{i}B\right)\left(1-\frac{1}{-i}B\right)$$

$$=(1-B)(1+B)(1+iB)(1-iB)$$

$$=(1-B)(1+B)(1+B^2)$$

$$=1-B^4.$$

(v) Based upon (i)-(iv) follows that

$$j(B)=l_1\frac{\Delta(B)}{d_1(B)}+l_2\frac{\Delta(B)}{d_2(B)}+l_3\frac{\Delta(B)}{d_3(B)}+l_4\frac{\Delta(B)}{d_4(B)}+\text{Remainder},$$

$$
\begin{aligned}
\approx\ & l_1 \frac{(1-B)(1+B)(1+iB)(1-iB)}{1-B} + \\
& l_2 \frac{(1-B)(1+B)(1+iB)(1-iB)}{1+B} + \\
& l_3 \frac{(1-B)(1+B)(1+iB)(1-iB)}{1+iB} + \\
& l_4 \frac{(1-B)(1+B)(1+iB)(1-iB)}{1-iB}
\end{aligned}
$$

$$
\begin{aligned}
=\ & l_1 B(1+B)(1+B^2) + \\
& l_2 (B)(1-B)(1+B^2) + \\
& l_3(-iB)(1-B)(1+B)(1-iB) + \\
& l_4 (iB)(1-B)(1+B)(1+iB).
\end{aligned}
$$

(vi) By letting $p_1 = -l_1$, $p_2 = -l_2$, $2l_3 = -p_3 + ip_4$, and $2l_4 = -p_3 - ip_4$ follows that

$$
\begin{aligned}
j(B) \quad =\ & -p_1 B(1+B+B^2+B^3) \\
& -p_2(-B)(1-B+B^2-B^3) \\
& -(p_4 + p_3 B)(-B)(1-B^2) \\
& + j^*(B)(1-B^4).
\end{aligned}
$$

(vii) Based upon (vi),

$$j(B)x_t = e_t$$

$$
\begin{aligned}
\Rightarrow \qquad j^*(B)(1-B^4)x_t =\ & p_1 B(1+B+B^2+B^3)x_t \\
& -p_2 B(1-B+B^2-B^3)x_t \\
& -(p_4 + p_3 B)B(1-B^2)x_t \\
& + e_t,
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \qquad j^*(B)x_{t-4} =\ & p_1(1+B+B^2+B^3)x_{t-1} + \\
& p_2(-1)(1-B+B^2-B^3)x_{t-1} + \\
& p_3(-1)(1-B^2)x_{t-2} + \\
& p_4(-1)(1-B^2)x_{t-1} + e_t.
\end{aligned}
$$

(viii) By letting

$$y_{4t} = x_{t-4},$$

$$y_{1,t-1} = (1+B+B^2+B^3)x_{t-1},$$

$$y_{2,t-1} = -(1-B+B^2-B^3)x_{t-1},$$

$$y_{3,t-1} = -(1-B^2)x_{t-1},$$

$$\Rightarrow \qquad j^*(B)y_{4t} = p_1 y_{1,t-1} + p_2 y_{2,t-1} + p_3 y_{3,t-2} + p_4 y_{3,t-1} + e_t, \qquad \text{(A.3)}$$

which is the expression given by Hylleberg *et al.* (1990) pp-223, as the Lagrange expansion of an autoregressive seasonal quarterly integrated process;

(ix) By using the definition provided for the l's in (A.2) and the relationships defined in (vi) between the l's and the p's it is possible to provide more specific expressions for the p's,

(a) $l_1 = \dfrac{j(1)}{\prod\limits_{j\neq 1} d_j(1)} = \dfrac{j(1)}{d_2(1).d_3(1).d_4(1)} = \dfrac{j(1)}{2\times(1+i)\times(1-i)} = \dfrac{j(1)}{4}$. As in (vi) it was assumed that $p_1 = -l_1$ then we have that $p_1 = \dfrac{-j(1)}{4}$;

(b) $l_2 = \dfrac{j(-1)}{\prod\limits_{j\neq 2} d_j(-1)} = \dfrac{j(-1)}{d_1(-1).d_3(-1).d_4(-1)} = \dfrac{j(-1)}{2\times(1-i)\times(1+i)} = \dfrac{j(-1)}{4}$. In (vi) it was assumed that $p_2 = -l_2$ then $p_2 = \dfrac{-j(-1)}{4}$;

(c) $l_3 = \dfrac{j(i)}{\prod\limits_{j\neq 3} d_j(i)} = \dfrac{j(i)}{d_1(i).d_2(i).d_4(i)} = \dfrac{j(i)}{(1-i)\times(1+i)\times(1-i^2)} = \dfrac{j(i)}{4}$;

(d) $l_4 = \dfrac{j(-i)}{\prod\limits_{j\neq 4} d_j(-i)} = \dfrac{j(-i)}{d_1(-i).d_2(-i).d_3(-i)} = \dfrac{j(-i)}{(1+i)\times(1-i)\times(1-i^2)} = \dfrac{j(-i)}{4}$;

(e) In (vi) it was established that $2l_3 = -p_3 + ip_4$, and $2l_4 = -p_3 - ip_4$, which in turn implies that $p_3 = -(l_3 + l_4)$ and $p_4 = \dfrac{1}{i}(l_3 - l_4)$;

(f) Using (c)-(e) follows that $p_3 = -l_3 - l_4 = -\dfrac{1}{4}\left(j(i) + j(-i)\right)$, which after some basic algebra, reduces to $p_3 = -\dfrac{1}{2}\mathrm{Real}[j(i)]$, where $\mathrm{Real}[.]$ represents the real part of the argument, a complex number; and

(g) $p_4 = \frac{1}{i}(l_3 - l_4) = \frac{1}{i}\left(\frac{j\,(i)}{4} + \frac{j\,(-i)}{4}\right)$, which after algebraic operations reduces to $p_4 = -\frac{1}{2}\mathrm{Im}[\,j\,(\text{-}i)]$, where Im[.] represents the imaginary part of the argument, a complex number.

Expression (A.3) deserves some comment:

(i) The parameters p_j, $j = 1,\ldots,4$, may be estimated via OLS and the statistics on the p's used for inference;

(ii) The asymptotic distribution of the t-statistics from this regression are from Dickey and Fuller (1979) for p_1 & p_2, and from Dickey, Hazza and Fuller (1984) for $p_3\big|_{p4=0}$. The tests are invariant with respect to nuisance parameters;

(iii) y_1 is asymptotically orthogonal to y_2 since they have unit roots at different frequencies, therefore, the test for $p_1 = 0$ have the same limiting distribution regardless of whether y_2 is included in the regression;

(iv) Similar arguments follow for the other cases;

(v) Deterministic components (intercept and trend) if present in the regression (even if not in the data) influence only the distribution for testing $p_1 = 0$ because they have all spectral mass at zero frequency;

(vi) Seasonal dummies, once the intercept is included, do not affect the asymptotic distribution of $\hat{p}_1$;

(vii) Seasonal dummies do affect the asymptotic distribution of the other statistics;

(viii) Critical values for the one-sided '*t*' tests on p_1, p_2, and p_3 may be found in Table 1a of Hylleberg *et al.* (1990), while critical values of two-sided "t-test" for $p_4 = 0$ and critical values for the F-test on $p_3 \wedge p_4 = 0$ are given in Table 1b of Hylleberg *et al.* (1990).

APPENDIX B

THE TRANSFORMATION MATRIX $\tilde{M}_g$ FOR SEASONAL COINTEGRATION SEM's

Recall that the transformation matrix $\tilde{M}_g$ must allow for the desegregation of the seasonal short-run dynamics and long-run dynamics as follows,

$$\begin{aligned} \boldsymbol{y}_g &= Z_g \tilde{M}_g \tilde{M}_g^{-1} \boldsymbol{d}_g + \boldsymbol{e}_g \\ &= Z_g^* \boldsymbol{d}_g^* + \boldsymbol{e}_g \\ &= Z_{g1}^* \boldsymbol{d}_{g1}^* \;+\; Z_{g2}^* \boldsymbol{d}_{g2}^* \;+\; \boldsymbol{e}_g \\ &= Short-run\ dynamics + Long-run\ equilibrium + error, \end{aligned} \tag{J.1}$$

where

$$\begin{aligned} Z_g &= [(\boldsymbol{Y}_g, \tilde{\boldsymbol{Y}}_{g,-1}, ..., \tilde{\boldsymbol{Y}}_{g,-p}), (\boldsymbol{X}_g, ..., \boldsymbol{X}_{g,-q})] \\ &= [Z_{g_{(1)}}, Z_{g_{(1)}}], \end{aligned} \tag{J.2}$$

$$\begin{aligned} Z_{g1}^* &= [(\Delta_4 \boldsymbol{Y}_g, \Delta_4 \tilde{\boldsymbol{Y}}_{g,-1}, ..., \Delta_4 \tilde{\boldsymbol{Y}}_{g,-p+4}), (\Delta_4 \boldsymbol{X}_g, \Delta_4 \boldsymbol{X}_{g,-1}, ..., \Delta_4 \boldsymbol{X}_{g-q+4})] \\ &= [Z_{g1_{(1)}}^*, Z_{g1_{(2)}}^*], \end{aligned} \tag{J.3}$$

and

$$\begin{aligned} Z_{g2}^* &= \left[(\boldsymbol{X}_{g2_{1,t-1}}, \boldsymbol{X}_{g2_{2,t-1}}, \boldsymbol{X}_{g2_{3,t-2}}, \boldsymbol{X}_{g2_{3,t-1}}), (\tilde{Y}_{g2_{1,t-1}}, \tilde{Y}_{g2_{2,t-1}}, \tilde{Y}_{g2_{3,t-2}}, \tilde{Y}_{g2_{3,t-1}}) \right] \\ &= [Z_{g2_{(1)}}^*, Z_{g2_{(2)}}^*]. \end{aligned} \tag{J.4}$$

Without losing generality, the seasonal deterministic component has been dropped from (J.1), for easy of presentation.

Now, let us consider the following relationships:

(i) between Z_g and $Z_{g1_{(1)}}^*$:

$$Z_{g1_{(1)}}^* = (\Delta_4 \boldsymbol{Y}_g, \Delta_4 \tilde{\boldsymbol{Y}}_{g,-1}, ..., \Delta_4 \tilde{\boldsymbol{Y}}_{g,-p+4})$$

$$=(\boldsymbol{Y}_g-\boldsymbol{Y}_{g,-4},\tilde{\boldsymbol{Y}}_{g,-1}-\tilde{\boldsymbol{Y}}_{g,-5},\ldots,\tilde{\boldsymbol{Y}}_{g,-p+4}-\tilde{\boldsymbol{Y}}_{g,-p})$$

$$=[\underbrace{\boldsymbol{Y}_g}_{T\times(G_g-1)},\quad \underbrace{\tilde{\boldsymbol{Y}}_{g,-1}}_{T\times G_g},\quad \ldots\quad ,\underbrace{\tilde{\boldsymbol{Y}}_{g,-p}}_{T\times G_g}]M_{g,\Delta\tilde{Y}}$$

$$=[(\boldsymbol{Y}_g,\tilde{\boldsymbol{Y}}_{g,-1},\ldots,\tilde{\boldsymbol{Y}}_{g,-p}),(\boldsymbol{X}_g,\ldots,\boldsymbol{X}_{g,-q})]\begin{bmatrix} M_{g,\Delta\tilde{Y}} \\ \cdots \\ \mathbf{0}_{\Delta\tilde{X}} \end{bmatrix}$$

$$=Z_g\begin{bmatrix} M_{g,\Delta\tilde{Y}} \\ \cdots \\ \mathbf{0}_{\Delta\tilde{X}} \end{bmatrix},$$

where $M_{g,\Delta\tilde{Y}}$ is a $[G_g\times(p+1)-1]\times[G_g\times(p-4+1)-1]$ nonsingular matrix

and $\mathbf{0}_{\Delta\tilde{X}}$ is a $[K_g\,(q+1)]\times[G_p\,(p+1)-1]$ matrix of zeroes, and

$$M_{g,\Delta\tilde{Y}}=\begin{bmatrix}
I_{G_g-1} & \mathbf{0}_{(G_g-1)\times G_g} & \cdots & \mathbf{0}_{G_g\times G_g} \\
\mathbf{0}_{G_g\times(G_g-1)} & I_{G_g} & \mathbf{0}_{G_g\times G_g} & \vdots \\
\mathbf{0}_{G_g\times(G_g-1)} & \mathbf{0}_{G_g\times G_g} & \ddots & \mathbf{0}_{G_g\times G_g} \\
\mathbf{0}_{G_g\times(G_g-1)} & \mathbf{0}_{G_g\times G_g} & \cdots & I_{G_g} \\
\begin{pmatrix} -I_{G_g-1} \\ \mathbf{0}_{1\times(G_g-1)} \end{pmatrix} & \mathbf{0}_{G_g\times G_g} & \cdots & \mathbf{0}_{G_g\times G_g} \\
\mathbf{0}_{G_g\times(G_g-1)} & -I_{G_g} & \mathbf{0}_{G_g\times G_g} & \mathbf{0}_{G_g\times G_g} \\
\mathbf{0}_{G_g\times(G_g-1)} & \mathbf{0}_{G_g\times G_g} & \ddots & \mathbf{0}_{G_g\times G_g} \\
\vdots & \vdots & \mathbf{0}_{G_g\times G_g} & -I_{G_g} \\
\mathbf{0}_{G_g\times(G_g-1)} & \mathbf{0}_{G_g\times G_g} & \cdots & \mathbf{0}_{G_g\times G_g}
\end{bmatrix} \qquad \text{(J.5)}$$

(ii) between Z_g and $Z^*_{g1_{(2)}}$:

$$Z^*_{g1_{(2)}}=(\Delta_4\boldsymbol{X}_g,\Delta_4\boldsymbol{X}_{g,-1},\ldots,\Delta_4\boldsymbol{X}_{g,-q+4})$$

$$=(\boldsymbol{X}_g-\boldsymbol{X}_{g,-4},\boldsymbol{X}_{g,-1}-\boldsymbol{X}_{g,-5},\ldots,\boldsymbol{X}_{g,-p+4}-\boldsymbol{X}_{g,-p})$$

$$=[\underbrace{\boldsymbol{X}_g}_{T\times K_g},\quad \underbrace{\boldsymbol{X}_{g,-1}}_{T\times K_g},\quad \ldots\quad ,\underbrace{\boldsymbol{X}_{g,-p}}_{T\times K_g}]M_{g,\Delta X}$$

$$=[(\boldsymbol{Y}_g,\tilde{\boldsymbol{Y}}_{g,-1},\ldots,\tilde{\boldsymbol{Y}}_{g,-p}),(\boldsymbol{X}_g,\ldots,\boldsymbol{X}_{g,-q})]\begin{bmatrix} \mathbf{0}_{\Delta Y} \\ \cdots \\ M_{g,\Delta X} \end{bmatrix}$$

$$= Z_g \begin{bmatrix} \mathbf{0}_{\Delta Y} \\ \cdots \\ M_{g,\Delta X} \end{bmatrix},$$

where $M_{g,\Delta X}$ is a $[K_g \times (p+1)] \times [K_g \times (p-4+1)]$ nonsingular matrix and $\mathbf{0}_{\Delta Y}$ is a $[G_g\,(p+1)-1] \times [K_p\,(q-4+1)]$ matrix of zeroes, and

$$M_{g,\Delta X} = \begin{bmatrix} \boldsymbol{I}_{K_g} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{K_g} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \boldsymbol{I}_{K_g} \\ -\boldsymbol{I}_{K_g} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{I}_{K_g} & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{I}_{K_g} \end{bmatrix}; \tag{J.6}$$

(iii) between Z_g and $Z^*_{g2_{(1)}}$:

$$Z^*_{g2_{(1)}} = (\tilde{\boldsymbol{Y}}_{g_{1,t-1}}, \tilde{\boldsymbol{Y}}_{g_{2,t-1}}, \tilde{\boldsymbol{Y}}_{g_{3,t-2}}, \tilde{\boldsymbol{Y}}_{g_{3,t-1}})$$

$$= (\tilde{\boldsymbol{Y}}_{g,-1} + \tilde{\boldsymbol{Y}}_{g,-2} + \tilde{\boldsymbol{Y}}_{g,-3} + \tilde{\boldsymbol{Y}}_{g,-4},$$

$$-\tilde{\boldsymbol{Y}}_{g,-1} + \tilde{\boldsymbol{Y}}_{g,-2} - \tilde{\boldsymbol{Y}}_{g,-3} + \tilde{\boldsymbol{Y}}_{g,-4},$$

$$-\tilde{\boldsymbol{Y}}_{g,-2} + \tilde{\boldsymbol{Y}}_{g,-4},$$

$$-\tilde{\boldsymbol{Y}}_{g,-1} + \tilde{\boldsymbol{Y}}_{g,-3})$$

$$= [\boldsymbol{Y}_g, \tilde{\boldsymbol{Y}}_{g,-1}, \tilde{\boldsymbol{Y}}_{g,-2}, \tilde{\boldsymbol{Y}}_{g,-3}, \tilde{\boldsymbol{Y}}_{g,-4}, \ldots, \tilde{\boldsymbol{Y}}_{g,-p}] M_{gY}$$

$$= [(\boldsymbol{Y}_g, \tilde{\boldsymbol{Y}}_{g,-1}, \ldots, \tilde{\boldsymbol{Y}}_{g,-p}), (\boldsymbol{X}_g, \ldots, \boldsymbol{X}_{g,-q})] \begin{bmatrix} M_{gY} \\ \cdots \\ \mathbf{0}_X \end{bmatrix}$$

$$= Z_g \begin{bmatrix} M_{gY} \\ \cdots \\ \mathbf{0}_X \end{bmatrix},$$

where M_{gY} is a $[G_g \times (p+1)-1] \times [G_g \times 4]$ nonsingular matrix and $\mathbf{0}_X$ is a $[K_g\,(q+1)] \times [G_g \times 4]$ matrix of zeroes, and

$$M_{gY} = \begin{bmatrix} \mathbf{0}_{(Gg-1)\times G_g} & \cdots & \cdots & \mathbf{0}_{G\times G} \\ I_{G_g} & -I_{G_g} & \mathbf{0}_{G\times G} & -I_{G_g} \\ I_{G_g} & I_{G_g} & -I_{G_g} & \mathbf{0}_{G\times G} \\ I_{G_g} & -I_{G_g} & \mathbf{0}_{G\times G} & I_{G_g} \\ I_{G_g} & I_{G_g} & I_{G_g} & \mathbf{0}_{G\times G} \\ \mathbf{0}_{G\times G} & \cdots & \cdots & \mathbf{0}_{G\times G} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{G\times G} & \cdots & \cdots & \mathbf{0}_{G\times G} \end{bmatrix}; \qquad \text{(J.7)}$$

(iv) between Z_g and $Z^*_{g2_{(2)}}$:

$$Z^*_{g2_{(2)}} = (\boldsymbol{X}_{g_{1,t-1}},\ \boldsymbol{X}_{g_{2,t-1}},\ \boldsymbol{X}_{g_{3,t-2}},\ \boldsymbol{X}_{g_{3,t-1}})$$

$$= (\boldsymbol{X}_{g,-1} + \boldsymbol{X}_{g,-2} + \boldsymbol{X}_{g,-3} + \boldsymbol{X}_{g,-4},$$

$$-\boldsymbol{X}_{g,-1} + \boldsymbol{X}_{g,-2} - \boldsymbol{X}_{g,-3} + \boldsymbol{X}_{g,-4},$$

$$-\boldsymbol{X}_{g,-2} + \boldsymbol{X}_{g,-4},$$

$$-\boldsymbol{X}_{g,-1} + \boldsymbol{X}_{g,-3})$$

$$= [\boldsymbol{X}_g, .\boldsymbol{X}_{g,-1}, .., \boldsymbol{X}_{g,-q}] M_{gX}$$

$$= [(\boldsymbol{Y}_g, \tilde{\boldsymbol{Y}}_{g,-1}, ..., \tilde{\boldsymbol{Y}}_{g,-p}), (\boldsymbol{X}_g, ..., \boldsymbol{X}_{g,-q})] \begin{bmatrix} \mathbf{0}_Y \\ \cdots \\ M_{gX} \end{bmatrix}$$

$$= Z_g \begin{bmatrix} \mathbf{0}_Y \\ \cdots \\ M_{gX} \end{bmatrix},$$

where M_{gX} is a $[K_g \times (q+1)] \times [K_g \times 4]$ nonsingular matrix and $\mathbf{0}_Y$ is a $[G_g\,(p+1) - 1] \times [K_g \times 4]$ matrix of zeroes, and

$$M_{gY} = \begin{bmatrix} \mathbf{0}_{(Gg-1)\times G_g} & \cdots & \cdots & \mathbf{0}_{G\times G} \\ I_{G_g} & -I_{G_g} & \mathbf{0}_{G\times G} & -I_{G_g} \\ I_{G_g} & I_{G_g} & -I_{G_g} & \mathbf{0}_{G\times G} \\ I_{G_g} & -I_{G_g} & \mathbf{0}_{G\times G} & I_{G_g} \\ I_{G_g} & I_{G_g} & I_{G_g} & \mathbf{0}_{G\times G} \\ \mathbf{0}_{G\times G} & \cdots & \cdots & \mathbf{0}_{G\times G} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{G\times G} & \cdots & \cdots & \mathbf{0}_{G\times G} \end{bmatrix}. \qquad \text{(J.8)}$$

(iv) Collecting (i)-(iv) then:

$$\tilde{M}_g = \begin{bmatrix} M_{g,\Delta\tilde{Y}} & \mathbf{0}_{\Delta Y} & \mathbf{0}_Y & M_{gY} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{0}_{\Delta\tilde{X}} & M_{g,\Delta X} & M_{gX} & \mathbf{0}_X \end{bmatrix}.$$

Recall that $d_g = (\Phi_0', \Phi_1', ..., \Phi_p', \Theta_0, ..., \Theta_q')'$, then

$$\begin{aligned} d_g^* &= \tilde{M}_g^{-1} d_g \\ &= \left[(g_0^{*\prime}, \tilde{g}_1^{*\prime}, ..., \tilde{g}_{p-4}^{*\prime}, h_0^{*\prime}, ..., h_{q-4}^{*\prime}), (h_{g,1}^{*\prime}, h_{g,2}^{*\prime}, h_{g,3}^{*\prime}, h_{g,4}^{*\prime}, \tilde{g}_{g,1}^{*\prime}, \tilde{g}_{g,2}^{*\prime}, \tilde{g}_{g,3}^{*\prime}, \tilde{g}_{g,4}^{*\prime}) \right]', \end{aligned} \tag{J.9}$$

where $\tilde{g}_i^* = \sum_{j=1}^{[(p-1)/4]} \Phi_{i+4j}$, $i = 1, ..., p-4$, $h_0^* = \Theta_0$, $h_i^* = \sum_{j=1}^{[(q-i)/4]} \Theta_{i+4j}$, $i = 1, ..., q-4$, $h_{g,1}^*, ..., h_{g,4}^*$ are the parameter vectors associated with the seasonal filtered matrices of exogenous variables having a unit root, i.e. $\boldsymbol{X}_{g_{1,t-1}}, \boldsymbol{X}_{g_{2,t-1}}, \boldsymbol{X}_{g_{3,t-2}}, \boldsymbol{X}_{g_{3,t-1}}$, and $\tilde{g}_{g1}^*, ..., \tilde{g}_{g4}^*$ are the parameter vectors associated with the seasonal filtered matrices of included endogenous variable, having a unit root, i.e. $\tilde{\boldsymbol{Y}}_{g_{1,t-1}}, \tilde{\boldsymbol{Y}}_{g_{2,t-1}}, \tilde{\boldsymbol{Y}}_{g_{3,t-2}}, \tilde{\boldsymbol{Y}}_{g_{3,t-1}}$.

APPENDIX C

THE TRANSFORMATION MATRIX $\tilde{M}_w$ FOR SEASONAL COINTEGRATION SEM's

In order to derive the transformation matrix $\tilde{M}_w$ it is convenient to express the seasonal error-correction model (S-ECM) in matrix form. Recall that the *t*-th observation of the S-ECM is

$$
\begin{aligned}
\Delta_4 \mathbf{y}_t = \; & \pi_1 \mathbf{y}_{1,t-1} + \pi_2 \mathbf{y}_{2,t-1} + \pi_3 \mathbf{y}_{3,t-2} + \pi_4 \mathbf{y}_{3,t-1} + \\
& P_1 \mathbf{x}_{1,t-1} + P_2 \mathbf{x}_{2,t-1} + P_3 \mathbf{x}_{3,t-2} + P_4 \mathbf{x}_{3,t-1} + \\
& A_1 \Delta_4 \mathbf{y}_{t-1} + \cdots + A_{p^*-4} \Delta_4 \mathbf{y}_{t-p^*+4} + \\
& B_0 \Delta_4 \mathbf{x}_t + B_1 \Delta_4 \mathbf{x}_{t-1} + \cdots + B_{p^*-4} \Delta_4 \mathbf{x}_{t-p^*+4} + \varepsilon_t^{\circ},
\end{aligned}
\qquad \text{(J.10)}
$$

then stacking all T equations we get the S-ECM matrix form expression,

$$
\begin{aligned}
\Delta_4 \boldsymbol{Y} = \; & \boldsymbol{Y}_{1,t-1} p_1' + \boldsymbol{Y}_{2,t-1} p_2' + \boldsymbol{Y}_{3,t-2} p_3' + \boldsymbol{Y}_{3,t-1} p_4' + \\
& \boldsymbol{X}_{1,t-1} P_1' + \boldsymbol{X}_{2,t-1} P_2' + \boldsymbol{X}_{3,t-2} P_3' + \boldsymbol{X}_{3,t-1} P_4' + \\
& \Delta_4 \boldsymbol{Y}_{t-1} A_1' + \cdots + \Delta_4 \boldsymbol{Y}_{t-p^*+4} A_{p^*-4}' + \\
& \Delta_4 \boldsymbol{X}_t B_0' + \Delta_4 \boldsymbol{X}_{t-1} B_1' + \cdots + \Delta_4 \boldsymbol{X}_{t-p^*+4} B_{p^*-4}' + \mathbf{e}_t^{\circ},
\end{aligned}
\qquad \text{(J.11)}
$$

where

$$
\Delta_4 \boldsymbol{Y} = \begin{bmatrix} \Delta_4 \boldsymbol{y}_1' \\ \vdots \\ \Delta_4 \boldsymbol{y}_T' \end{bmatrix}_{T\times G}, \quad \boldsymbol{Y}_{1,t-1} = \begin{bmatrix} \boldsymbol{y}_{1,0}' \\ \vdots \\ \boldsymbol{y}_{1,T-1}' \end{bmatrix}_{T\times G} \quad \text{with} \quad \boldsymbol{y}_{1,t} = (B + B^2 + B^3 + B^4)\mathbf{y}_t, \qquad \boldsymbol{Y}_{2,t-1} = \begin{bmatrix} \boldsymbol{y}_{2,0}' \\ \vdots \\ \boldsymbol{y}_{2,T-1}' \end{bmatrix}_{T\times G}
$$

$$
\text{with} \quad \boldsymbol{y}_{2,t} = (B - B^2 + B^3 - B^4)\mathbf{y}_t, \quad \boldsymbol{Y}_{3,t-2} = \begin{bmatrix} \boldsymbol{y}_{3,-1}' \\ \vdots \\ \boldsymbol{y}_{3,T-2}' \end{bmatrix}_{T\times G} \quad \text{with} \quad \boldsymbol{y}_{3,t-2} = -(B^2 - B^4)\mathbf{y}_t, \quad \boldsymbol{Y}_{3,t-1} = \begin{bmatrix} \boldsymbol{y}_{3,0}' \\ \vdots \\ \boldsymbol{y}_{3,T-1}' \end{bmatrix}_{T\times G}
$$

$$
\text{with} \quad \boldsymbol{y}_{3,t-1} = -(B - B^3)\mathbf{y}_t, \quad \boldsymbol{X}_{1,t-1} = \begin{bmatrix} \boldsymbol{x}_{1,0}' \\ \vdots \\ \boldsymbol{x}_{1,T-1}' \end{bmatrix}_{T\times K} \quad \text{with} \quad \boldsymbol{x}_{1,t} = (B + B^2 + B^3 + B^4)\mathbf{x}_t, \qquad \boldsymbol{X}_{2,t-1} = \begin{bmatrix} \boldsymbol{x}_{2,0}' \\ \vdots \\ \boldsymbol{x}_{2,T-1}' \end{bmatrix}_{T\times K}
$$

with $\boldsymbol{x}_{2,t} = (B - B^2 + B^3 - B^4)\mathbf{x}_t$, $\boldsymbol{X}_{3,t-2} = \begin{bmatrix} \boldsymbol{x}'_{3,-1} \\ \vdots \\ \boldsymbol{x}'_{3,T-2} \end{bmatrix}_{T\times K}$ with $\boldsymbol{x}_{3,t-2} = -(B^2 - B^4)\mathbf{x}_t$, $\boldsymbol{X}_{3,t-1} = \begin{bmatrix} \boldsymbol{x}'_{3,0} \\ \vdots \\ \boldsymbol{x}'_{3,T-1} \end{bmatrix}_{T\times K}$

with $\boldsymbol{x}_{3,t-1} = -(B - B^3)\mathbf{x}_t$, $\Delta_4 \boldsymbol{Y}_{-j} = \begin{bmatrix} \Delta_4 \boldsymbol{y}'_{1-j} \\ \vdots \\ \Delta_4 \boldsymbol{y}'_{T-j} \end{bmatrix}_{T\times G}$, $j = 1,...,p$, and $\Delta_4 \boldsymbol{X}_{-j} = \begin{bmatrix} \Delta_4 \boldsymbol{x}'_{1-j} \\ \vdots \\ \Delta_4 \boldsymbol{x}'_{T-j} \end{bmatrix}_{T\times K}$, $j = 0,1,...,q$.

From (J.11) follows that the "implied" long-run relations for the zero, annual and semi-annual frequencies are

$$\begin{aligned} \boldsymbol{Y}_{1,t-1} p'_1 + \boldsymbol{X}_{1,t-1} P'_1 &= \boldsymbol{v}_{1t}, \\ \boldsymbol{Y}_{2,t-1} p'_2 + \boldsymbol{X}_{2,t-1} P'_2 &= \boldsymbol{v}_{2t}, \\ \boldsymbol{Y}_{3,t-2} p'_3 + \boldsymbol{X}_{3,t-2} P'_3 &= \boldsymbol{v}_{3t}, \\ \boldsymbol{Y}_{3,t-1} p'_4 + \boldsymbol{X}_{3,t-1} P'_4 &= \boldsymbol{v}_{4t}. \end{aligned} \tag{J.12}$$

By denoting $\left(p'_j\right)^-$ and $\left(P'_j\right)^-$ the inverse or generalized inverse of p'_j and P'_j respectively ($j = 1,...,4$), which depend upon the number of cointegrating relationships at the zero and at the seasonal frequencies, and post-multiplying by $\left(p'_j\right)^-$ the j-th equation in (J.12) we have,

$$\begin{aligned} \boldsymbol{Y}_{1,t-1} &= \boldsymbol{X}_{1,t-1} p_1^{*\prime} + \boldsymbol{v}_{1,t}^*, \\ \boldsymbol{Y}_{2,t-1} &= \boldsymbol{X}_{2,t-1} p_2^{*\prime} + \boldsymbol{v}_{2,t}^*, \\ \boldsymbol{Y}_{3,t-2} &= \boldsymbol{X}_{3,t-2} p_3^{*\prime} + \boldsymbol{v}_{3,t}^*, \\ \boldsymbol{Y}_{3,t-1} &= \boldsymbol{X}_{3,t-1} p_4^{*\prime} + \boldsymbol{v}_{4,t}^*, \end{aligned} \tag{J.13}$$

where $p_j^{*\prime} = -P'_j \left(p'_j\right)^-$ are $K \times G$ matrices and $\boldsymbol{v}_{j,t}^* = \boldsymbol{v}_{j,t}\left(p'_j\right)^-$, $j = 1,...,4$.

Now, the transformation $\tilde{M}_w$ can be can fully described as

$$W\tilde{M}_w = (W_1^*, W_2^*) = W^*, \tag{J.14}$$

where $W=(\boldsymbol{Y}_{-1},...,\boldsymbol{Y}_{-p^\circ},\boldsymbol{X},\boldsymbol{X}_{-1},...,\boldsymbol{X}_{-q^\circ})$ is a $T\times(G_g\times p^\circ+K_g\times(q^\circ+1))$ matrix that contains the instrumental variables needed for 2SLS or 3SLS, $p^\circ=\max(4,p)$, $q^\circ=\max(4,q)$, and $W_1^*=[(\Delta_4\boldsymbol{Y}_{-1},...,\Delta_4\boldsymbol{Y}_{-p^\circ+4}),(\Delta_4\boldsymbol{X},\Delta_4\boldsymbol{X}_{-1},...,\Delta_4\boldsymbol{X}_{-q^\circ+4}),(\boldsymbol{Y}_{1,t-1}-\boldsymbol{X}_{1,t-1}p_1^{*\prime},\boldsymbol{Y}_{2,t-1}-\boldsymbol{X}_{2,t-1}p_2^{*\prime},$
$\boldsymbol{Y}_{3,t-2}-\boldsymbol{X}_{3,t-2}p_3^{*\prime},\boldsymbol{Y}_{3,t-1}-\boldsymbol{X}_{3,t-1}p_4^{*\prime})]$ is stationary and $W_2^*=[\boldsymbol{X}_{1,t-1},\boldsymbol{X}_{2,t-1},\boldsymbol{X}_{3,t-2},\boldsymbol{X}_{3,t-1}]$ is I(1).

For the derivation of the transformation matrix $\tilde{M}_g$, the relations between matrix W and different sub-matrices of W_1^* must be established, as follows,

(i) between W and $[\Delta_4\boldsymbol{Y}_{-1},...,\Delta_4\boldsymbol{Y}_{-p^\circ+4}]$:

$$[\Delta_4\boldsymbol{Y}_{-1},...,\Delta_4\boldsymbol{Y}_{-p^\circ+4}]=[\boldsymbol{Y}_{t-1}-\boldsymbol{Y}_{t-5},...,\boldsymbol{Y}_{t-p^\circ+4}-\boldsymbol{Y}_{t-p^\circ}]$$

$$=[\boldsymbol{Y}_{t-1},...,\boldsymbol{Y}_{t-p^\circ}]\begin{bmatrix} I_{G_g} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & I_{G_g} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & I_{G_g} \\ -I_{G_g} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & -I_{G_g} & \ddots & \mathbf{0} \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & -I_{G_g} \end{bmatrix}_{[G_g\times p^\circ]\times[G_g\times(p^\circ-4)]},$$

$$=[\boldsymbol{Y}_{t-1},...,\boldsymbol{Y}_{t-p^\circ}]M_{w,\Delta Y}$$

$$=([\boldsymbol{Y}_{t-1},...,\boldsymbol{Y}_{t-p^\circ}],[\boldsymbol{X},\boldsymbol{X}_{-1},...,\boldsymbol{X}_{-q^\circ}])\begin{bmatrix} M_{w,\Delta Y} \\ \cdots \\ \mathbf{0}_X \end{bmatrix}$$

$$=W\begin{bmatrix} M_{w,\Delta Y} \\ \cdots \\ \mathbf{0}_X \end{bmatrix},$$

with $\mathbf{0}_X$ a $[K_g(q^\circ+1)]\times[G_g(p^\circ-4)]$ matrix of zeroes;

(ii) between W and $[\Delta_4\boldsymbol{X},\Delta_4\boldsymbol{X}_{-1},...,\Delta_4\boldsymbol{X}_{-q^\circ+4}]$:

$$[\Delta_4\boldsymbol{X},\Delta_4\boldsymbol{X}_{-1},...,\Delta_4\boldsymbol{X}_{-q^\circ+4}]=[\boldsymbol{X}_t-\boldsymbol{X}_{t-4},\boldsymbol{X}_{t-1}-\boldsymbol{X}_{t-5},...,\boldsymbol{X}_{t-q^\circ+4}-\boldsymbol{X}_{t-q^\circ}]$$

$$= [\boldsymbol{X}, \boldsymbol{X}_{-1}, \ldots, \boldsymbol{X}_{-q^\circ}] \begin{bmatrix} I_{K_g} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & I_{K_g} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & I_{K_g} \\ -I_{K_g} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & -I_{K_g} & \ddots & \mathbf{0} \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & -I_{K_g} \end{bmatrix}_{[K_g \times (q^\circ + 1)] \times [K_g \times ((q^\circ - 4) + 1)]}$$

$$= ([\boldsymbol{Y}_{t-1}, \ldots, \boldsymbol{Y}_{t-p^\circ}], [\boldsymbol{X}, \boldsymbol{X}_{-1}, \ldots, \boldsymbol{X}_{-q^\circ}]) \begin{bmatrix} \mathbf{0}_Y \\ \cdots \\ M_{w,\Delta X} \end{bmatrix}$$

$$= W \begin{bmatrix} \mathbf{0}_Y \\ \cdots \\ M_{w,\Delta X} \end{bmatrix}$$

with $\mathbf{0}_Y$ a $[G_g \times p^\circ] \times [K_g((q^\circ - 4) + 1)]$ matrix of zeroes;

(iii) between W and

$$[\boldsymbol{Y}_{1,t-1} - \boldsymbol{X}_{1,t-1} p_1^{*\prime}, \boldsymbol{Y}_{2,t-1} - \boldsymbol{X}_{2,t-1} p_2^{*\prime}, \boldsymbol{Y}_{3,t-2} - \boldsymbol{X}_{3,t-2} p_3^{*\prime}, \boldsymbol{Y}_{3,t-1} - \boldsymbol{X}_{3,t-1} p_4^{*\prime}]:$$

$$[\boldsymbol{Y}_{1,t-1} - \boldsymbol{X}_{1,t-1} p_1^{*\prime}, \boldsymbol{Y}_{2,t-1} - \boldsymbol{X}_{2,t-1} p_2^{*\prime}, \boldsymbol{Y}_{3,t-2} - \boldsymbol{X}_{3,t-2} p_3^{*\prime}, \boldsymbol{Y}_{3,t-1} - \boldsymbol{X}_{3,t-1} p_4^{*\prime}] =$$

$$= [\boldsymbol{Y}_{1,t-1}, \boldsymbol{Y}_{2,t-1}, \boldsymbol{Y}_{3,t-2}, \boldsymbol{Y}_{3,t-1}, \boldsymbol{X}_{1,t-1}, \boldsymbol{X}_{2,t-1}, \boldsymbol{X}_{3,t-2}, \boldsymbol{X}_{3,t-1}] \times$$

$$\begin{bmatrix} I_{G_g} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & I_{G_g} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & I_{G_g} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & I_{G_g} \\ -p_1^{*\prime} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & -p_2^{*\prime} & \ddots & \mathbf{0} \\ \vdots & \mathbf{0} & -p_3^{*\prime} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & -p_4^{*\prime} \end{bmatrix}_{[(G_g + K_g)4] \times [G_g \times 4]}$$

$$= [\boldsymbol{Y}_{1,t-1}, \boldsymbol{Y}_{2,t-1}, \boldsymbol{Y}_{3,t-2}, \boldsymbol{Y}_{3,t-1}, \boldsymbol{X}_{1,t-1}, \boldsymbol{X}_{2,t-1}, \boldsymbol{X}_{3,t-2}, \boldsymbol{X}_{3,t-1}] M_{w,D}$$

$$= [\boldsymbol{Y}_{-1}, \ldots, \boldsymbol{Y}_{-p^\circ}, \boldsymbol{X}, \boldsymbol{X}_{-1}, \ldots, \boldsymbol{X}_{-q^\circ}] \times M_{w,C} \times M_{w,D},$$

$$= [\boldsymbol{Y}_{-1},...,\boldsymbol{Y}_{-p^{\circ}}, \boldsymbol{X}, \boldsymbol{X}_{-1},...,\boldsymbol{X}_{-q^{\circ}}] \times M_{w,Y-Xp},$$

where $$M_{w,C} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{0}_{[G_g \times p^{\circ}] \times [K_g \times 4]} \\ \hline \mathbf{0}_{[K_g \times q^{\circ}] \times [G_g \times 4]} & \mathbf{B} \end{array}\right],$$

$$\mathbf{A} = \begin{bmatrix} I_{G_g} & -I_{G_g} & \mathbf{0} & -I_{G_g} \\ I_{G_g} & I_{G_g} & -I_{G_g} & \mathbf{0} \\ I_{G_g} & -I_{G_g} & \mathbf{0} & I_{G_g} \\ I_{G_g} & I_{G_g} & I_{G_g} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \text{ and}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ I_{K_g} & -I_{K_g} & \mathbf{0} & -I_{K_g} \\ I_{K_g} & I_{K_g} & -I_{K_g} & \mathbf{0} \\ I_{K_g} & -I_{K_g} & \mathbf{0} & I_{K_g} \\ I_{K_g} & I_{K_g} & I_{K_g} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

(iv) between W and $W_2^* = [\boldsymbol{X}_{1,t-1}, \boldsymbol{X}_{2,t-1}, \boldsymbol{X}_{3,t-2}, \boldsymbol{X}_{3,t-1}]$:

$$W_2^* = [\boldsymbol{X}_{1,t-1}, \boldsymbol{X}_{2,t-1}, \boldsymbol{X}_{3,t-2}, \boldsymbol{X}_{3,t-1}]$$

$$= [\boldsymbol{Y}_{-1},...,\boldsymbol{Y}_{-p^{\circ}}, \boldsymbol{X}, \boldsymbol{X}_{-1},...,\boldsymbol{X}_{-q^{\circ}}] \times \begin{bmatrix} \mathbf{0}_{G_g \times K_g} & \mathbf{0}_{G_g \times K_g} & \mathbf{0}_{G_g \times K_g} & \mathbf{0}_{G_g \times K_g} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{G_g \times K_g} & \mathbf{0}_{G_g \times K_g} & \mathbf{0}_{G_g \times K_g} & \mathbf{0}_{G_g \times K_g} \\ \mathbf{0}_{K_g} & \mathbf{0}_{K_g} & \mathbf{0}_{K_g} & \mathbf{0}_{K_g} \\ I_{K_g} & -I_{K_g} & \mathbf{0}_{K_g} & -I_{K_g} \\ I_{K_g} & I_{K_g} & -I_{K_g} & \mathbf{0}_{K_g} \\ I_{K_g} & -I_{K_g} & \mathbf{0}_{K_g} & I_{K_g} \\ I_{K_g} & I_{K_g} & I_{K_g} & \mathbf{0}_{K_g} \\ \mathbf{0}_{K_g} & \mathbf{0}_{K_g} & \mathbf{0}_{K_g} & \mathbf{0}_{K_g} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{K_g} & \mathbf{0}_{K_g} & \mathbf{0}_{K_g} & \mathbf{0}_{K_g} \end{bmatrix}_{[G_g \times p^{\circ} + K_g (q^{\circ}+1)] \times [K_g \times 4]}.$$

(v) Collecting (i)-(iv), the final expression is obtained,

$$\tilde{M}_w = \left[\begin{array}{c:c:c:c} M_{w,\Delta Y} & \mathbf{0}_Y & \multirow{2}{*}{$M_{w,Y-Xp}$} & \multirow{2}{*}{$M_{w_2^*}$} \\ \hdashline \mathbf{0}_X & M_{w,\Delta X} & & \end{array} \right], \tag{J.15}$$

which is $[G_g \times p^{\circ} + K_g (q^{\circ} + 1)] \times [G_g \times p^{\circ} + K_g (q^{\circ} + 1)]$.

APPENDIX D

THE U.S. WHEAT MARKET DATA SET

ENTRY	PWD	PWI	PWX	PWPR	RPW1	RPW2	RPDI	SDR	THPW	WSTOCKW	RWAP	RWSP
1975:03	0.7489	10.4171	1.3881	9.8384	4.4731	4.7737	14.4020	128.7600	204.3500	13.3012	4.4035	2.6787
1975:04	0.8147	7.1647	1.7376	9.8384	4.8502	5.1141	14.5150	132.3400	204.3500	13.5120	4.9821	2.6188
1976:01	0.9188	5.0111	1.2129	9.8384	4.5606	4.7033	14.7340	132.8300	204.3500	13.7262	4.5390	2.6585
1976:02	0.8627	3.0652	1.0688	9.8384	4.4921	4.7930	14.8270	133.8500	204.3500	13.9437	4.6186	2.6822
1976:03	0.6782	10.9335	1.2922	9.8461	4.2153	4.5320	14.9300	134.0400	216.1000	14.1647	4.1597	2.6319
1976:04	0.9789	8.6834	1.2672	9.8461	3.6455	3.7299	14.9990	133.3700	216.1000	14.3893	3.5167	2.7308
1977:01	0.8699	6.9706	0.8156	9.8461	3.4735	3.4478	15.0640	134.1800	216.1000	14.6173	3.1146	2.6275
1977:02	0.9160	5.0751	0.9611	9.8461	3.0480	3.0480	15.1250	133.3000	216.1000	14.8490	2.7883	2.4960
1977:03	1.1830	11.9478	1.2106	9.2783	2.9034	3.0801	15.3080	131.8900	221.3000	15.0844	2.7443	3.0359
1977:04	1.1094	9.7084	1.1194	9.2800	3.2053	3.5594	15.5240	130.4000	221.3000	15.3235	3.1463	3.1190
1978:01	0.7806	7.7213	1.1740	9.2800	3.4367	3.6434	15.6340	125.1900	221.3000	15.5663	3.2687	2.9587
1978:02	0.8097	5.3187	1.5717	9.2800	3.5728	3.7382	15.8040	122.9700	221.3000	15.8131	3.2735	2.7062
1978:03	1.0188	10.6046	1.6462	7.9685	3.7331	3.6102	15.9150	117.6100	221.3000	16.0637	3.2607	3.3414
1978:04	1.0865	7.9745	1.5313	7.9685	4.0568	3.8770	16.0240	113.7800	221.3000	16.3183	3.4182	3.3265
1979:01	0.7556	6.1276	1.0623	7.9685	4.0605	3.7081	16.1530	115.4500	221.3000	16.5770	3.2369	3.1290
1979:02	0.8805	4.1279	1.1004	7.9685	3.8242	3.6338	16.0500	117.4900	221.3000	16.8397	3.1205	2.9574
1979:03	0.8398	11.0836	1.6627	9.4723	4.4490	4.3290	16.0980	116.9800	267.5400	17.1066	3.8869	3.4961
1979:04	0.9592	8.3329	1.7821	9.4723	4.3990	4.5616	16.1770	118.7000	267.5400	17.3778	4.0806	3.5303
1980:01	0.8052	6.1646	1.3302	9.4723	4.5001	4.5519	16.2220	118.3800	267.5400	17.6532	3.9224	3.5281
1980:02	0.8558	3.9839	1.3041	9.4723	4.3321	4.2929	15.8850	121.3400	267.5400	17.9330	3.8839	3.6278
1980:03	0.8524	11.9141	1.6468	10.4400	4.0973	4.1804	15.9500	115.3000	273.4000	18.2173	3.7952	3.3868
1980:04	1.0630	9.1837	1.6592	10.4400	4.3909	4.4035	16.1960	115.7800	273.4000	18.5060	3.9236	3.1991
1981:01	0.7460	6.6639	1.7426	10.4400	4.2363	4.2868	16.2050	119.4000	273.4000	18.7993	3.9686	3.2127
1981:02	0.7592	4.3207	1.5676	10.4400	3.8622	4.1997	16.0950	125.2900	273.4000	19.0973	3.8590	3.2477
1981:03	1.2820	13.2844	1.8424	12.1005	3.5098	4.0299	16.4000	134.8600	289.9000	19.4000	3.4938	3.4747
1981:04	1.0077	10.1637	2.1047	12.1005	4.0668	4.4217	16.3580	132.2000	289.9000	19.9000	3.8485	3.7155
1982:01	0.6333	7.7035	1.7941	12.1005	3.8607	4.4605	16.2900	132.6700	289.9000	21.2000	3.8883	3.7539
1982:02	0.7463	5.0154	1.9231	12.1005	3.5584	4.2322	16.3490	138.6300	289.9000	22.1000	3.6447	3.6148
1982:03	1.2272	13.9031	1.7690	11.8979	3.3873	3.8759	16.3420	145.5000	306.4800	24.8000	3.3670	3.8524
1982:04	1.0829	11.3657	1.4473	11.8979	3.4095	4.0310	16.3310	149.2800	306.4800	20.4000	3.6969	4.1065
1983:01	0.7693	8.8870	1.6867	11.8979	3.5146	4.2598	16.4040	143.4600	306.4800	23.1000	3.7618	4.0373
1983:02	0.8169	6.4909	1.5671	11.8979	3.5027	4.1818	16.4840	145.2900	306.4800	22.5000	3.7414	3.8423
1983:03	1.5173	13.7996	1.4784	10.3188	3.6263	3.8540	16.7460	149.6700	334.2000	28.3000	3.4957	4.0683
1983:04	1.4404	10.8165	1.5302	10.3188	3.4681	3.7822	17.0530	150.3700	334.2000	27.0000	3.5336	3.9753
1984:01	0.9260	8.3022	1.5586	10.3188	3.3189	3.6425	17.4150	152.9200	334.2000	26.0000	3.3253	3.8924
1984:02	0.8526	5.9381	1.4949	10.3188	3.4185	3.6702	17.7160	151.0900	334.2000	27.6000	3.3871	3.8211
1984:03	1.8533	13.3568	1.6855	10.9692	3.3879	3.6572	17.9960	158.5800	350.4200	26.2000	3.3002	4.1861
1984:04	1.4295	9.8852	2.0443	10.9692	3.5951	3.9344	18.0670	164.7900	350.4200	27.2000	3.5002	4.3753
1985:01	0.8573	7.5932	1.4103	10.9692	3.5852	3.8264	18.0400	171.4800	350.4200	29.7000	3.4458	4.3837
1985:02	0.7350	5.9918	0.8522	10.9692	3.7187	3.7858	18.3420	172.0800	350.4200	33.1000	3.5737	4.5599
1985:03	1.6855	13.4154	1.0416	10.1562	3.4027	3.5316	18.2030	163.1800	352.6700	32.8000	3.2847	4.8441
1985:04	1.3138	11.0602	1.0476	10.1562	3.4625	3.6017	18.3310	153.6800	352.6700	31.4900	3.5113	4.9452
1986:01	0.7006	9.4187	0.9271	10.1562	3.7253	3.6777	18.5300	144.5900	352.6700	27.9800	3.5165	4.8132

1986:02	0.6942	7.9335	0.7799	10.1562	3.7620	3.8257	18.6770	135.3700	352.6700	28.3500	3.6196	4.9236
1986:03	2.1772	13.1070	1.3240	8.6798	2.7747	2.8630	18.7080	130.3300	352.6700	32.9000	2.5723	4.8355
1986:04	0.9484	11.0895	1.0673	8.6798	2.8037	2.8586	18.6490	128.0400	352.6700	31.5700	2.5695	4.8095
1987:01	0.9234	9.2989	0.8502	8.6798	3.1757	3.0418	18.8300	124.2400	352.6700	39.9300	2.8224	4.8862
1987:02	0.9095	7.5001	0.8922	8.6798	3.3452	3.1555	18.6080	117.8500	352.6700	33.9900	2.8121	4.7011
1987:03	2.2460	12.2552	1.6830	8.6731	2.7396	2.7960	18.9240	119.3300	352.3500	32.2000	2.5037	4.6271
1987:04	0.7060	10.2761	1.2661	8.6731	3.0069	3.0749	19.1190	115.1400	352.3500	26.9900	2.8169	4.7092
1988:01	0.6869	7.8849	1.6919	8.6731	3.3485	3.3910	19.3450	108.6900	352.3500	30.4600	2.9203	4.6512
1988:02	0.8601	5.1493	1.8698	8.6731	3.1240	3.2444	19.4470	107.0500	352.3500	29.3500	2.9178	4.5159
1988:03	1.9018	9.1673	1.4746	7.3894	3.3846	3.5904	19.5710	111.7300	352.9900	26.6000	3.3150	4.1603
1988:04	0.8740	6.9769	1.3381	7.3894	3.7397	3.8456	19.7240	110.8300	352.9900	23.0100	3.5714	4.0950
1989:01	0.5354	4.9823	1.4610	7.3894	4.0169	4.0417	19.8960	109.1600	352.9900	22.6000	3.7251	4.0790
1989:02	0.6723	2.8356	1.4752	7.3894	3.7623	4.0379	19.8000	113.4000	352.9900	20.6600	3.6714	3.9804
1989:03	1.8475	7.7311	1.4892	8.2262	3.6423	4.0242	19.7930	116.7200	236.7400	20.2000	3.5305	3.9404
1989:04	0.7012	5.7226	1.3235	8.2262	3.8262	4.0640	19.8440	115.9900	236.7400	20.8600	3.5473	4.0228
1990:01	0.9018	3.7844	1.0433	8.2262	3.6358	3.8585	20.0920	112.2700	236.7400	22.3300	3.3318	3.8194
1990:02	0.5466	2.1411	1.1024	8.2262	3.3541	3.6339	20.1380	113.8700	236.7400	22.1700	3.1225	3.8173
1990:03	2.3798	9.6095	1.0698	10.9092	2.7747	2.9176	20.1040	110.0900	229.8500	24.5000	2.5711	3.7381
1990:04	0.9372	7.5852	1.1132	10.9092	2.4075	2.6466	19.9000	103.1500	229.8500	29.3900	2.2942	3.8708
1991:01	1.1710	5.5431	0.8962	10.9092	2.4218	2.6494	19.8270	102.4700	229.8500	34.9200	2.3191	3.9453
1991:02	0.9512	3.4270	1.1772	10.9092	2.6204	2.7984	19.9020	108.3700	229.8500	35.8600	2.4267	3.8415
1991:03	2.1730	8.1030	0.9952	7.8290	2.7945	2.9177	19.8860	111.3100	228.6700	39.5000	2.4898	3.8903
1991:04	0.9789	5.7024	1.4427	7.8290	3.4669	3.6013	19.8520	106.7700	228.6700	34.5000	3.0667	4.0351
1992:01	0.7662	3.4798	1.4619	7.8290	4.0200	4.4100	20.1260	104.2400	228.6700	33.1600	3.5900	4.0000
1992:02	0.5449	1.8141	1.1495	7.8290	3.5651	3.9993	20.1900	107.7400	228.6700	33.1900	3.5945	3.9177
1992:03	2.1855	8.2205	1.1052	9.6478	3.3546	3.5610	20.1220	102.3700	221.6800	40.4000	3.1949	3.9936
1992:04	0.7811	6.1674	1.3452	9.6478	3.4413	3.6802	20.4290	104.6500	221.6800	31.6400	3.2698	4.0367
1993:01	0.7956	4.0254	1.3862	9.6478	3.6356	3.8004	20.0210	109.9400	221.6800	33.8600	3.3027	3.9553
1993:02	0.6365	2.0068	1.4348	9.6478	3.3022	3.4123	20.2580	107.2100	221.6800	36.0000	3.0409	3.7775
1993:03	1.9677	8.2033	1.1637	9.2752	2.9199	3.2715	20.2200	107.5500	172.7400	33.1000	2.8125	3.9063
1993:04	0.9561	6.0270	1.2709	9.2752	3.0092	3.3016	20.4300	108.2600	172.7400	38.0300	3.1506	3.8539
1994:01	0.9718	3.8691	1.2801	9.2752	3.2604	3.6480	20.2020	110.4800	172.7400	42.9300	3.2970	3.6633
1994:02	0.8886	2.0508	1.0205	9.2752	2.9747	3.3766	20.4770	108.7300	172.7400	42.5000	3.3024	3.7106
1994:03	2.2655	7.8211	0.9953	8.8971	3.2181	3.5690	20.5620	105.4100	162.8700	34.3000	3.1486	3.9730
1994:04	1.0003	5.6396	1.2932	8.8971	3.8593	4.3033	20.7730	102.8500	162.8700	26.3600	3.7807	4.0984
1995:01	0.8746	3.6426	1.1843	8.8971	3.6038	4.0367	20.8000	104.3500	162.8700	24.1900	3.6203	3.9351
1995:02	0.7794	1.8525	1.0663	8.8971	3.4101	3.9372	20.7170	97.2700	162.8700	22.4300	3.5188	3.9537
1995:03	2.0067	7.0624	1.1490	8.2893	4.1120	4.7190	20.7910	97.6600	0.0000	23.3200	3.9847	3.9162
1995:04	0.7511	5.0262	1.3665	8.2893	4.3171	4.7825	20.8720	101.6000	0.0000	18.7000	4.3079	3.6741
1996:01	0.8775	3.0783	1.1134	8.2893	4.4370	4.8848	20.9570	103.5600	0.0000	17.7300	4.3309	3.5354
1996:02	0.6840	1.3603	1.0691	8.2893	4.5197	5.3674	20.9990	104.9700	0.0000	18.8600	4.5002	3.3459
1996:03	2.2954	6.4358	1.2570	8.5697	3.7329	4.3696	21.1540	105.3300	0.0000	25.0900	3.8602	2.0535
1996:04	0.8174	4.5137	1.1570	8.5697	3.3622	4.0370	21.1640	105.6500	0.0000	29.6200	3.5805	2.1943
1997:01	0.9164	2.9860	0.6717	8.5697	3.1118	4.1784	21.2580	109.5300	0.0000	33.1700	3.6164	2.3384
1997:02	0.8524	1.5535	0.6720	8.5697	3.1834	4.1064	21.3780	113.6600	0.0000	32.0200	3.5626	2.2751
1997:03	2.1736	7.6584	1.0743	9.2486	3.1388	3.5358	21.5140	113.4200	0.0000	26.7800	3.1758	2.3818
1997:04	0.6836	5.9548	1.1003	9.2486	3.5351	3.5877	21.7040	114.9500	0.0000	22.9100	3.3281	2.3918
1998:01	0.8221	4.2518	0.9456	9.2486	3.1379	3.4982	22.0730	118.5100	0.0000	23.2200	3.1953	2.4682
1998:02	0.9938	2.5888	0.7454	9.2486	2.9082	3.3527	22.3370	119.4800	0.0000	24.1300	3.0821	2.4928
1998:03	2.4066	8.7263	0.9501	9.4080	2.5020	2.9288	22.4700	123.6000	0.0000	23.3400	2.5315	2.5414
1998:04	0.8171	6.9176	1.0750	9.4080	2.4788	3.1666	22.5330	116.8300	0.0000	29.0700	2.7811	2.5718

1999:01	0.8316	5.2446	0.9074	9.4080	2.4856	3.3106	22.6280	114.5100	0.0000	27.6000	2.8981	2.6609
1999:02	1.0454	3.3822	0.9049	9.4080	2.5035	3.0858	22.6120	118.2800	0.0000	30.3500	2.7092	2.6987
1999:03	1.8628	8.8463	1.1896	8.4277	2.1865	2.9515	22.6250	118.6500	0.0000	26.0600	2.5479	2.7164
1999:04	1.0746	6.8115	1.0636	8.4300	2.1181	3.0093	22.8180	114.1700	0.0000	24.1900	2.7262	2.7053

APPENDIX E

PARAMETER ESTIMATES OF THE FOUR SELECTED MODELS

Table 1. Model 1 (VECM): Restricted maximum likelihood estimates and standard errors.

Disappearance: PWD(t)

Variable	Coeff.	S.Error
DPWD(t-1)	4.138	4.829
DPWI(t-1)	13.099	15.677
DPWX(t-1)	3.49	4.364
DPWPR(t-1)	-2.273	-2.703
DRPW1(t-1)	1.153	10.431
DPWD(t-2)	6.114	5.003
DPWI(t-2)	3.578	3.442
DPWX(t-2)	6.232	5.137
DPWPR(t-2)	-5.254	-4.421
DRPW1(t-2)	1.428	19.103
DPWD(t-3)	-3.875	-4.098
DPWI(t-3)	-8.74	-8.5
DPWX(t-3)	-3.192	-3.25
DPWPR(t-3)	4.076	4.354
DRPW1(t-3)	1.714	14.245
DPWD(t-4)	-6.358	-8.55
DPWI(t-4)	-1.584	-1.97
DPWX(t-4)	-5.606	-7.466
DPWPR(t-4)	6.212	8.49
DRPW1(t-4)	1.123	11.044
DPWD(t-5)	0.138	2.239
DPWI(t-5)	6.801	9.184
DPWX(t-5)	0.371	6.095
DPWPR(t-5)	-0.185	-7.781
DRPW1(t-5)	0.333	5.627
DRPDI(t)	-0.262	-5.351
DSDR(t)	0.013	6.289
DTHPW(t)	-0.002	-5.85
DWSTOCKW(t)	-0.001	-0.291
DRWAP2(t)	-0.161	-1.872
DRWSP2(t)	-0.035	-0.887
DRPDI(t-1)	0.278	4.596
DSDR	0	0.139
DTHPW(t-1)	-0.002	-4.747
DWSTOCKW(t-1)	0.076	13.55
DRWAP2(t-1)	-0.292	-4.677
DRWSP2(t-1)	-1.397	-15.863
DRPDI(t-2)	1.236	15.891
DSDR(t-2)	0.017	5.238
DTHPW(t-2)	0.002	5.051
DWSTOCKW(t-2)	0.071	16.068
DRWAP2(t-2)	-0.335	-3.99
DRWSP2(t-2)	-1.531	-19.641
DRPDI(t-3)	1.481	14.81
DSDR(t-3)	0.012	4.002
DTHPW(t-3)	0.003	8.237
DWSTOCKW(t-3)	0.067	14.11
DRWAP2(t-3)	0.011	0.165
DRWSP2(t-3)	-1.069	-12.773
DRPDI(t-4)	0.682	7.244
DSDR(t-4)	0.026	8.061
DTHPW(t-4)	-0.006	-18.831

Inventories: PWI(t)

Variable	Coeff.	S.Error
DPWD(t-1)	-9.754	-2.93
DPWI(t-1)	-14.66	-4.517
DPWX(t-1)	-3.852	-1.24
DPWPR(t-1)	7.533	2.306
DRPW1(t-1)	-0.379	-0.882
DPWD(t-2)	7.626	1.607
DPWI(t-2)	15.832	3.92
DPWX(t-2)	11.838	2.512
DPWPR(t-2)	-9.444	-2.046
DRPW1(t-2)	-1.559	-5.367
DPWD(t-3)	17.649	4.804
DPWI(t-3)	7.867	1.97
DPWX(t-3)	20.832	5.46
DPWPR(t-3)	-18.601	-5.116
DRPW1(t-3)	-1.928	-4.125
DPWD(t-4)	20.379	7.055
DPWI(t-4)	1.704	0.546
DPWX(t-4)	22.858	7.836
DPWPR(t-4)	-21.216	-7.464
DRPW1(t-4)	-0.494	-1.251
DPWD(t-5)	-0.138	-0.579
DPWI(t-5)	-22.15	-7.7
DPWX(t-5)	0.734	3.105
DPWPR(t-5)	0.252	2.737
DRPW1(t-5)	0.123	0.533
DRPDI(t)	0.09	0.475
DSDR(t)	-0.011	-1.32
DTHPW(t)	0.005	2.932
DWSTOCKW(t)	-0.031	-3.406
DRWAP2(t)	1.857	5.549
DRWSP2(t)	0.054	0.349
DRPDI(t-1)	-0.578	-2.46
DSDR	0.037	3.042
DTHPW(t-1)	0	-0.216
DWSTOCKW(t-1)	-0.073	-3.349
DRWAP2(t-1)	-1.519	-6.257
DRWSP2(t-1)	2.293	6.702
DRPDI(t-2)	-1.543	-5.105
DSDR(t-2)	0.024	1.887
DTHPW(t-2)	-0.002	-1.759
DWSTOCKW(t-2)	-0.067	-3.901
DRWAP2(t-2)	-1.983	-6.073
DRWSP2(t-2)	2.934	9.689
DRPDI(t-3)	-0.04	-0.103
DSDR(t-3)	0.01	0.849
DTHPW(t-3)	-0.005	-3.937
DWSTOCKW(t-3)	-0.09	-4.909
DRWAP2(t-3)	-2.078	-8.345
DRWSP2(t-3)	2.935	9.029
DRPDI(t-4)	0.906	2.477
DSDR(t-4)	0.055	4.459
DTHPW(t-4)	0.006	4.595

Exports: PWX(t)

Variable	Coeff.	S.Error
DPWD(t-1)	-19.677	-19.528
DPWI(t-1)	-6.09	-6.198
DPWX(t-1)	-17.746	-18.871
DPWPR(t-1)	16.992	17.185
DRPW1(t-1)	-1.643	-12.639
DPWD(t-2)	-20.632	-14.358
DPWI(t-2)	-1.526	-1.249
DPWX(t-2)	-19.053	-13.357
DPWPR(t-2)	18.051	12.918
DRPW1(t-2)	-1.673	-19.026
DPWD(t-3)	-15.577	-14.007
DPWI(t-3)	4.13	3.415
DPWX(t-3)	-14.64	-12.676
DPWPR(t-3)	13.547	12.306
DRPW1(t-3)	-1.817	-12.843
DPWD(t-4)	-9.649	-11.034
DPWI(t-4)	4.924	5.207
DPWX(t-4)	-9.113	-10.321
DPWPR(t-4)	8.3	9.647
DRPW1(t-4)	-1.454	-12.161
DPWD(t-5)	-0.989	-13.676
DPWI(t-5)	8.012	9.201
DPWX(t-5)	-0.708	-9.895
DPWPR(t-5)	0.119	4.246
DRPW1(t-5)	-0.802	-11.524
DRPDI(t)	0.02	0.341
DSDR(t)	0.004	1.567
DTHPW(t)	0.001	3.08
DWSTOCKW(t)	0.028	10.084
DRWAP2(t)	-0.212	-2.09
DRWSP2(t)	-0.033	-0.705
DRPDI(t-1)	0.222	3.127
DSDR	0.036	9.675
DTHPW(t-1)	-0.002	-3.074
DWSTOCKW(t-1)	-0.015	-2.317
DRWAP2(t-1)	1.128	15.345
DRWSP2(t-1)	1.004	9.698
DRPDI(t-2)	0.131	1.434
DSDR(t-2)	0.004	1.041
DTHPW(t-2)	-0.005	-12.663
DWSTOCKW(t-2)	-0.02	-3.824
DRWAP2(t-2)	1.113	11.26
DRWSP2(t-2)	0.961	10.484
DRPDI(t-3)	-0.471	-4.003
DSDR(t-3)	0.016	4.752
DTHPW(t-3)	-0.006	-16.17
DWSTOCKW(t-3)	-0.019	-3.504
DRWAP2(t-3)	1.046	13.878
DRWSP2(t-3)	0.767	7.798
DRPDI(t-4)	-0.281	-2.537
DSDR(t-4)	-0.015	-4.067
DTHPW(t-4)	-0.002	-5.924

Production: PWPR(t)

Variable	Coeff.	S.Error
DPWD(t-1)	-24.988	-11.854
DPWI(t-1)	-8.034	-3.908
DPWX(t-1)	-17.667	-8.98
DPWPR(t-1)	21.861	10.569
DRPW1(t-1)	-0.827	-3.04
DPWD(t-2)	-6.298	-2.095
DPWI(t-2)	18.138	7.092
DPWX(t-2)	-0.291	-0.097
DPWPR(t-2)	2.707	0.926
DRPW1(t-2)	-1.783	-9.696
DPWD(t-3)	-0.793	-0.341
DPWI(t-3)	3.651	1.443
DPWX(t-3)	4.082	1.69
DPWPR(t-3)	-2.01	-0.873
DRPW1(t-3)	-2.022	-6.831
DPWD(t-4)	4.98	2.722
DPWI(t-4)	4.639	2.345
DPWX(t-4)	8.81	4.769
DPWPR(t-4)	-7.328	-4.071
DRPW1(t-4)	-0.791	-3.163
DPWD(t-5)	-0.975	-6.444
DPWI(t-5)	-7.934	-4.355
DPWX(t-5)	0.449	2.996
DPWPR(t-5)	0.169	2.887
DRPW1(t-5)	-0.316	-2.169
DRPDI(t)	-0.144	-1.197
DSDR(t)	0.007	1.4
DTHPW(t)	0.003	3.517
DWSTOCKW(t)	-0.004	-0.693
DRWAP2(t)	1.534	7.24
DRWSP2(t)	-0.032	-0.325
DRPDI(t-1)	-0.055	-0.367
DSDR	0.074	9.623
DTHPW(t-1)	-0.005	-3.974
DWSTOCKW(t-1)	-0.009	-0.682
DRWAP2(t-1)	-0.694	-4.511
DRWSP2(t-1)	1.908	8.807
DRPDI(t-2)	-0.192	-1.002
DSDR(t-2)	0.047	5.755
DTHPW(t-2)	-0.005	-7.02
DWSTOCKW(t-2)	-0.014	-1.332
DRWAP2(t-2)	-1.232	-5.957
DRWSP2(t-2)	2.385	12.436
DRPDI(t-3)	0.946	3.846
DSDR(t-3)	0.039	5.456
DTHPW(t-3)	-0.008	-10.822
DWSTOCKW(t-3)	-0.041	-3.558
DRWAP2(t-3)	-1.025	-6.501
DRWSP2(t-3)	2.633	12.789
DRPDI(t-4)	1.319	5.693
DSDR(t-4)	0.067	8.504
DTHPW(t-4)	-0.002	-3.04

Chicago Prices: RPW(t)

Variable	Coeff.	S.Error
DPWD(t-1)	9.251	8.377
DPWI(t-1)	-3.212	-2.982
DPWX(t-1)	9.962	9.666
DPWPR(t-1)	-8.71	-8.038
DRPW1(t-1)	1.597	11.208
DPWD(t-2)	5.656	3.591
DPWI(t-2)	-3.61	-2.694
DPWX(t-2)	5.649	3.614
DPWPR(t-2)	-4.79	-3.128
DRPW1(t-2)	-0.279	-2.897
DPWD(t-3)	8.778	7.202
DPWI(t-3)	4.003	3.021
DPWX(t-3)	9.004	7.113
DPWPR(t-3)	-8.509	-7.053
DRPW1(t-3)	0.317	2.047
DPWD(t-4)	11.088	11.569
DPWI(t-4)	3.216	3.103
DPWX(t-4)	11.483	11.866
DPWPR(t-4)	-11.586	-12.287
DRPW1(t-4)	-0.117	-0.895
DPWD(t-5)	0.346	4.366
DPWI(t-5)	-10.991	-11.515
DPWX(t-5)	0.269	3.429
DPWPR(t-5)	-0.403	-13.183
DRPW1(t-5)	0.605	7.933
DRPDI(t)	0.278	4.41
DSDR(t)	0.018	6.742
DTHPW(t)	-0.004	-7.032
DWSTOCKW(t)	-0.024	-7.89
DRWAP2(t)	2.36	21.266
DRWSP2(t)	-0.146	-2.843
DRPDI(t-1)	1.262	16.187
DSDR	0.036	8.879
DTHPW(t-1)	0	-0.687
DWSTOCKW(t-1)	-0.023	-3.234
DRWAP2(t-1)	0.682	8.467
DRWSP2(t-1)	-1.196	-10.535
DRPDI(t-2)	0.631	6.297
DSDR(t-2)	0.028	6.628
DTHPW(t-2)	-0.003	-6.41
DWSTOCKW(t-2)	-0.006	-1.069
DRWAP2(t-2)	0.667	6.159
DRWSP2(t-2)	-0.184	-1.829
DRPDI(t-3)	0.948	7.358
DSDR(t-3)	0.047	12.578
DTHPW(t-3)	-0.004	-8.751
DWSTOCKW(t-3)	0.002	0.311
DRWAP2(t-3)	-0.107	-1.301
DRWSP2(t-3)	-0.685	-6.349
DRPDI(t-4)	1.332	10.971
DSDR(t-4)	0.034	8.179
DTHPW(t-4)	0.001	1.609

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DWSTOCKW(t-4)	0.043	10.152	DWSTOCKW(t-4)	-0.032	-1.928	DWSTOCKW(t-4)	0.021	4.12	DWSTOCKW(t-4)	0.032	3.058	DWSTOCKW(t-4)	0.007	1.234
DRWAP2(t-4)	0.615	12.234	DRWAP2(t-4)	-1.289	-6.604	DRWAP2(t-4)	-0.052	-0.879	DRWAP2(t-4)	-0.714	-5.775	DRWAP2(t-4)	0.411	6.35
DRWSP2(t-4)	-0.973	-12.957	DRWSP2(t-4)	2.432	8.335	DRWSP2(t-4)	0.555	6.285	DRWSP2(t-4)	2.028	10.976	DRWSP2(t-4)	-0.434	-4.484
DRPDI	0.041	0.715	DRPDI	0.01	0.047	DRPDI	0.259	3.882	DRPDI	0.268	1.922	DRPDI	1.046	14.315
DSDR	0.022	8.239	DSDR	0.052	5.048	DSDR	-0.023	-7.284	DSDR	0.052	8.011	DSDR	0.015	4.454
DTHPW	-0.004	-10.318	DTHPW	0.001	0.727	DTHPW	0.001	1.719	DTHPW	-0.002	-2.317	DTHPW	0	0.829
DWSTOCKW	0.007	1.868	DWSTOCKW	-0.009	-0.598	DWSTOCKW	0.042	9.097	DWSTOCKW	0.041	4.317	DWSTOCKW	-0.032	-6.485
DRWAP2	0.091	2.002	DRWAP2	-0.363	-2.062	DRWAP2	-0.034	-0.635	DRWAP2	-0.278	-2.492	DRWAP2	0.189	3.242
DRWSP2	-0.265	-5.06	DRWSP2	1.394	6.839	DRWSP2	0.118	1.917	DRWSP2	1.248	9.669	DRWSP2	-0.79	-11.677
CONSTANT	15.399	-	CONSTANT	0.999	-	CONSTANT	-7.43	-	CONSTANT	8.926	-	CONSTANT	23.546	-
PWD(t-1)	5.879	-	PWD(t-1)	-4.653	-	PWD(t-1)	14.822	-	PWD(t-1)	16.357	-	PWD(t-1)	-13.353	-
PWI(t-1)	-0.263	-	PWI(t-1)	0.08	-	PWI(t-1)	0.489	-	PWI(t-1)	0.312	-	PWI(t-1)	-0.311	-
PWX(t-1)	8.459	-	PWX(t-1)	-10.114	-	PWX(t-1)	11.145	-	PWX(t-1)	9.687	-	PWX(t-1)	-13.436	-
PWPR(t-1)	-10.378	-	PWPR(t-1)	5.941	-	PWPR(t-1)	-11.463	-	PWPR(t-1)	-16.133	-	PWPR(t-1)	12.388	-
RPW1(t-1)	-1.458	-	RPW1(t-1)	-0.054	-	RPW1(t-1)	2.008	-	RPW1(t-1)	0.464	-	RPW1(t-1)	-2.628	-
RPDI(t-1)	-0.214	-	RPDI(t-1)	-0.26	-	RPDI(t-1)	0.07	-	RPDI(t-1)	-0.405	-	RPDI(t-1)	-0.624	-
SDR(t-1)	0.009	-	SDR(t-1)	-0.026	-	SDR(t-1)	-0.025	-	SDR(t-1)	-0.042	-	SDR(t-1)	-0.016	-
THPW(t-1)	0.002	-	THPW(t-1)	0.001	-	THPW(t-1)	0.003	-	THPW(t-1)	0.006	-	THPW(t-1)	-0.004	-
WSTOCKW(t-1)	-0.081	-	WSTOCKW(t-1)	0.07	-	WSTOCKW(t-1)	0.045	-	WSTOCKW(t-1)	0.03	-	WSTOCKW(t-1)	-0.016	-
RWAP2(t-1)	-0.589	-	RWAP2(t-1)	4.081	-	RWAP2(t-1)	-0.56	-	RWAP2(t-1)	2.976	-	RWAP2(t-1)	0.275	-
RWSP2(t-1)	1.423	-	RWSP2(t-1)	-1.566	-	RWSP2(t-1)	-1.358	-	RWSP2(t-1)	-1.502	-	RWSP2(t-1)	1.211	-
DS	-0.425	-9.771	DS	1.538	9.096	DS	0.575	11.221	DS	1.682	15.702	DS	-0.641	-11.418
DX	0.032	0.841	DX	0.064	0.424	DX	-0.596	-13.153	DX	-0.492	-5.187	DX	1.042	20.98
D1	0.642	16.22	D1	-0.379	-2.463	D1	-0.15	-3.224	D1	0.073	0.755	D1	0.003	0.059
D2	-0.677	-13.637	D2	1.668	8.648	D2	0.652	11.173	D2	1.677	13.729	D2	-0.206	-3.212
SUMM	-0.181	-0.695	SUMM	-3.539	-3.502	SUMM	-0.57	-1.864	SUMM	-4.407	-6.886	SUMM	-1.246	-3.715
FALL	0.442	1.641	FALL	-5.453	-5.215	FALL	-0.757	-2.393	FALL	-5.849	-8.833	FALL	2.114	6.093
SPRI	0.535	2.221	SPRI	0.776	0.829	SPRI	0.011	0.038	SPRI	1.34	2.261	SPRI	3.027	9.753

Table 2. Model 2 (SVECM): Restricted maximum likelihood estimates.

Disappearance: PWD(t)		Inventories: PWI(t)		Exports: PWX(t)		Production: PWPR(t)		Chicago Prices: RPW(t)	
Variable	Coeff.	Variable	Coeff.	Variable	Coeff.	Variable	Coeff.	Variable	Coeff.
PWD(1,t-1)	9.8867	PWD(1,t-1)	-6.4141	PWD(1,t-1)	0.2638	PWD(1,t-1)	3.9885	PWD(1,t-1)	-9.0806
PWI(1,t-1)	0.0825	PWI(1,t-1)	0.1114	PWI(1,t-1)	0.2160	PWI(1,t-1)	0.4046	PWI(1,t-1)	0.0101
PWX(1,t-1)	10.7390	PWX(1,t-1)	-6.4656	PWX(1,t-1)	-2.9559	PWX(1,t-1)	1.5139	PWX(1,t-1)	-10.7321
PWPR(1,t-1)	-11.9597	PWPR(1,t-1)	6.2806	PWPR(1,t-1)	1.7688	PWPR(1,t-1)	-4.0958	PWPR(1,t-1)	10.6889
RPW(1,t-1)	-0.0475	RPW(1,t-1)	1.2015	RPW(1,t-1)	0.5060	RPW(1,t-1)	1.6646	RPW(1,t-1)	-1.4343
RPDI(1,t-1)	-0.1857	RPDI(1,t-1)	0.3291	RPDI(1,t-1)	0.0149	RPDI(1,t-1)	0.1633	RPDI(1,t-1)	-0.3241
SDR(1,t-1)	0.0147	SDR(1,t-1)	-0.0266	SDR(1,t-1)	-0.0151	SDR(1,t-1)	-0.0272	SDR(1,t-1)	-0.0120
THPW(1,t-1)	-0.0004	THPW(1,t-1)	0.0010	THPW(1,t-1)	0.0031	THPW(1,t-1)	0.0037	THPW(1,t-1)	-0.0017
WSTOC(1,t-1)	-0.0090	WSTOC(1,t-1)	0.0785	WSTOC(1,t-1)	0.0397	WSTOC(1,t-1)	0.1054	WSTOC(1,t-1)	0.0294
RWAPS (1,t-1)	-0.4938	RWAPS (1,t-1)	1.9791	RWAPS (1,t-1)	0.3834	RWAPS (1,t-1)	1.8708	RWAPS (1,t-1)	0.6085
RWSPS (1,t-1)	0.1930	RWSPS (1,t-1)	-1.0328	RWSPS (1,t-1)	-0.9713	RWSPS (1,t-1)	-1.7886	RWSPS (1,t-1)	-0.1700
PWD(2,t-1)	-6.3374	PWD(2,t-1)	8.2991	PWD(2,t-1)	7.3733	PWD(2,t-1)	10.2201	PWD(2,t-1)	-2.1399
PWI(2,t-1)	-12.0574	PWI(2,t-1)	16.9030	PWI(2,t-1)	15.4182	PWI(2,t-1)	22.0537	PWI(2,t-1)	-5.0526
PWX(2,t-1)	-5.4853	PWX(2,t-1)	6.6520	PWX(2,t-1)	6.1858	PWX(2,t-1)	8.2180	PWX(2,t-1)	-3.2323
PWPR(2,t-1)	6.0306	PWPR(2,t-1)	-8.8910	PWPR(2,t-1)	-7.7965	PWPR(2,t-1)	-11.5601	PWPR(2,t-1)	2.4315
RPW(2,t-1)	0.4341	RPW(2,t-1)	-0.5202	RPW(2,t-1)	-1.0299	RPW(2,t-1)	-1.0948	RPW(2,t-1)	-1.9543
RPDI(2,t-1)	0.2591	RPDI(2,t-1)	-0.6953	RPDI(2,t-1)	-0.0117	RPDI(2,t-1)	-0.4629	RPDI(2,t-1)	-0.9534
SDR(2,t-1)	0.0656	SDR(2,t-1)	0.0723	SDR(2,t-1)	-0.0356	SDR(2,t-1)	0.1019	SDR(2,t-1)	0.0098
THPW(2,t-1)	-0.0095	THPW(2,t-1)	0.0234	THPW(2,t-1)	-0.0009	THPW(2,t-1)	0.0134	THPW(2,t-1)	0.0055
WSTOC(2,t-1)	0.0158	WSTOC(2,t-1)	0.0482	WSTOC(2,t-1)	0.0265	WSTOC(2,t-1)	0.0919	WSTOC(2,t-1)	0.0112
RWAPS (2,t-1)	0.8386	RWAPS (2,t-1)	2.3710	RWAPS (2,t-1)	-0.6016	RWAPS (2,t-1)	2.5209	RWAPS (2,t-1)	0.0175
RWSPS (2,t-1)	-0.9067	RWSPS (2,t-1)	0.1008	RWSPS (2,t-1)	-0.0190	RWSPS (2,t-1)	-0.8225	RWSPS (2,t-1)	1.6705
Const1	-0.0667	Const1	-1.1607	Const1	-0.2462	Const1	-1.4947	Const1	-0.1163
PWD(3,t-2)	4.5867	PWD(3,t-2)	-15.3828	PWD(3,t-2)	2.6877	PWD(3,t-2)	-7.9402	PWD(3,t-2)	1.5714
PWI(3,t-2)	5.9297	PWI(3,t-2)	-4.9938	PWI(3,t-2)	-7.9140	PWI(3,t-2)	-6.5758	PWI(3,t-2)	-9.6070
PWX(3,t-2)	4.7365	PWX(3,t-2)	-14.3708	PWX(3,t-2)	1.9411	PWX(3,t-2)	-7.5232	PWX(3,t-2)	1.4556
PWPR(3,t-2)	-5.0186	PWPR(3,t-2)	15.2428	PWPR(3,t-2)	-2.9989	PWPR(3,t-2)	7.0691	PWPR(3,t-2)	-2.0380
RPW(3,t-2)	-0.1992	RPW(3,t-2)	0.7408	RPW(3,t-2)	0.8421	RPW(3,t-2)	1.3512	RPW(3,t-2)	0.5174
RPDI(3,t-2)	-0.0592	RPDI(3,t-2)	-0.0315	RPDI(3,t-2)	-0.0875	RPDI(3,t-2)	-0.1761	RPDI(3,t-2)	-0.0978
SDR(3,t-2)	0.0106	SDR(3,t-2)	-0.0033	SDR(3,t-2)	0.0074	SDR(3,t-2)	0.0144	SDR(3,t-2)	0.0191
THPW(3,t-2)	-0.0017	THPW(3,t-2)	0.0033	THPW(3,t-2)	0.0056	THPW(3,t-2)	0.0070	THPW(3,t-2)	0.0024
WSTOC(3,t-2)	0.0063	WSTOC(3,t-2)	0.0790	WSTOC(3,t-2)	0.0183	WSTOC(3,t-2)	0.1037	WSTOC(3,t-2)	-0.0855
RWAPS (3,t-2)	0.0559	RWAPS (3,t-2)	1.1240	RWAPS (3,t-2)	0.5721	RWAPS (3,t-2)	1.7065	RWAPS (3,t-2)	1.8120
RWSPS (3,t-2)	0.1108	RWSPS (3,t-2)	-0.5600	RWSPS (3,t-2)	-0.4153	RWSPS (3,t-2)	-0.8570	RWSPS (3,t-2)	0.4009
Const2	-0.1233	Const2	-0.5450	Const2	-0.6178	Const2	-1.2525	Const2	-1.3300
PWD(3,t-1)	-0.7980	PWD(3,t-1)	-10.4937	PWD(3,t-1)	10.7093	PWD(3,t-1)	-0.8285	PWD(3,t-1)	12.3173
PWI(3,t-1)	4.1011	PWI(3,t-1)	-26.1481	PWI(3,t-1)	13.6338	PWI(3,t-1)	-8.4897	PWI(3,t-1)	13.3556
PWX(3,t-1)	-0.3662	PWX(3,t-1)	-10.1909	PWX(3,t-1)	10.8552	PWX(3,t-1)	0.0581	PWX(3,t-1)	11.6416
PWPR(3,t-1)	0.8944	PWPR(3,t-1)	10.7214	PWPR(3,t-1)	-10.6853	PWPR(3,t-1)	1.1644	PWPR(3,t-1)	-11.3206
RPW(3,t-1)	-0.3935	RPW(3,t-1)	0.8344	RPW(3,t-1)	-1.0170	RPW(3,t-1)	-0.5646	RPW(3,t-1)	-0.6861
RPDI(3,t-1)	-0.0256	RPDI(3,t-1)	-0.0562	RPDI(3,t-1)	-0.1058	RPDI(3,t-1)	-0.1857	RPDI(3,t-1)	0.0292
SDR(3,t-1)	0.0018	SDR(3,t-1)	0.0055	SDR(3,t-1)	0.0123	SDR(3,t-1)	0.0191	SDR(3,t-1)	0.0088
THPW(3,t-1)	-0.0049	THPW(3,t-1)	0.0000	THPW(3,t-1)	-0.0025	THPW(3,t-1)	-0.0074	THPW(3,t-1)	0.0025
WSTOC(3,t-1)	-0.0015	WSTOC(3,t-1)	0.0101	WSTOC(3,t-1)	0.0210	WSTOC(3,t-1)	0.0292	WSTOC(3,t-1)	0.0116
RWAPS (3,t-1)	0.1922	RWAPS (3,t-1)	-0.0057	RWAPS (3,t-1)	-0.6701	RWAPS (3,t-1)	-0.4656	RWAPS (3,t-1)	-0.7560
RWSPS (3,t-1)	0.1691	RWSPS (3,t-1)	-0.3092	RWSPS (3,t-1)	0.1982	RWSPS (3,t-1)	0.0552	RWSPS (3,t-1)	0.1859
Const3	0.1316	Const3	1.4660	Const3	0.8106	Const3	2.3739	Const3	0.6923

Table 3. Model 3 (CDSEM): 2SLS estimates and standard errors of the estimates.

Disappearance: PWD(t)

Variable	Coeff.	S.Error
Constant	0.9838	0.4273
DPWD(t-1)	-0.3689	0.2394
DPWD(t-2)	-0.2960	0.2377
DPWD(t-3)	-0.2271	0.2325
DPWD(t-4)	0.4048	0.1947
DPWD(t-5)	0.3443	0.1567
DPWD(t-6)	0.1890	0.1076
DRPW	-0.2525	0.0703
DRPW(t-1)	0.0234	0.0830
DRPW(t-2)	-0.0164	0.0754
DRPW(t-3)	-0.0508	0.0766
DRPW(t-4)	0.0480	0.0725
DRPW(t-5)	0.0882	0.0682
DRPW(t-6)	0.1470	0.0691
DRPDI	0.2043	0.1185
DRPDI(t-2)	0.1346	0.1184
DRPDI(t-3)	-0.1173	0.1219
DRPDI(t-4)	-0.0613	0.1214
DRPDI(t-5)	0.0533	0.1181
DRPDI(t-6)	-0.2952	0.1188
SUMM	0.2949	0.1093
FALL	0.1789	0.0906
SPRI	-0.0200	0.0915
PWD(t-1)	0.2402	0.2467
RPW(t-1)	-0.1399	0.0687
RPDI(t-1)	0.0117	0.0132
DRPDI(t-1)	0.0966	0.1188

Inventories: PWI(t)

Variable	Coeff.	S.Error
Constant	-0.0430	0.0363
DPWI(t-1)	0.8301	0.1166
DPWI(t-2)	0.1485	0.1350
DPWI(t-3)	-0.2496	0.1268
DPWI(t-4)	-0.0392	0.1256
DPWI(t-5)	0.2862	0.1218
DPWI(t-6)	-0.1371	0.1001
DPWPR	0.9944	0.0025
DPWPR(t-1)	0.0010	0.1224
DPWPR(t-2)	-0.1446	0.1097
DPWPR(t-3)	0.1005	0.1015
DPWPR(t-4)	0.1408	0.1001
DPWPR(t-5)	-0.1445	0.1002
DPWPR(t-6)	-0.0059	0.0022
DPWD	-0.9823	0.0073
DPWD(t-1)	-0.0192	0.1194
DPWD(t-2)	0.1124	0.1057
DPWD(t-3)	-0.1302	0.0988
DPWD(t-4)	-0.1744	0.0984
DPWD(t-5)	0.1172	0.1010
DPWD(t-6)	0.0041	0.0080
DPWX	-0.9903	0.0060
DPWX(t-1)	-0.0159	0.1226
DPWX(t-2)	0.1340	0.1091
DPWX(t-3)	-0.1027	0.1013
DPWX(t-4)	-0.1444	0.1006
DPWX(t-5)	0.1337	0.1012
DPWX(t-6)	0.0086	0.0063
DRPW	-0.0039	0.0045
DRPW(t-1)	-0.0129	0.0064
DRPW(t-2)	-0.0130	0.0060
DRPW(t-3)	-0.0049	0.0055
DRPW(t-4)	0.0012	0.0051
DRPW(t-5)	-0.0083	0.0050
DRPW(t-6)	0.0009	0.0051
SUMM	0.0862	0.0349
FALL	0.0722	0.0342
SPRI	0.0483	0.0352
PWI(t-1)	0.9968	0.0010
PWPR(t-1)	0.1621	0.0620
PWD(t-1)	-0.1445	0.0560
PWX(t-1)	-0.1545	0.0624
RPW(t-1)	-0.0033	0.0047

Exports: PWX(t)

Variable	Coeff.	S.Error
Constant	-0.9842	0.5326
DPWX(t-1)	0.1929	0.2140
DPWX(t-2)	0.0038	0.1873
DPWX(t-3)	0.2166	0.1689
DPWX(t-4)	0.1721	0.1547
DPWX(t-5)	0.1008	0.1326
DPWX(t-6)	0.1749	0.1065
DRPW	0.0672	0.0800
DRPW(t-1)	-0.0850	0.0876
DRPW(t-2)	-0.2153	0.0783
DRPW(t-3)	-0.1716	0.0803
DRPW(t-4)	-0.1817	0.0766
DRPW(t-5)	0.0332	0.0787
DRPW(t-6)	-0.1430	0.0803
DSDR	-0.0015	0.0061
DSDR(t-1)	-0.0068	0.0061
DSDR(t-2)	-0.0158	0.0067
DSDR(t-3)	0.0043	0.0066
DSDR(t-4)	-0.0121	0.0062
DSDR(t-5)	-0.0127	0.0055
DSDR(t-6)	0.0024	0.0059
DTHPW	0.0004	0.0009
DTHPW(t-1)	0.0015	0.0010
DTHPW(t-2)	0.0010	0.0010
DTHPW(t-3)	0.0013	0.0009
DTHPW(t-4)	0.0021	0.0009
DTHPW(t-5)	0.0007	0.0009
DTHPW(t-6)	0.0013	0.0008
DWSTOCKW	0.0023	0.0065
DWSTOCKW(t-1)	-0.0034	0.0065
DWSTOCKW(t-2)	-0.0105	0.0068
DWSTOCKW(t-3)	-0.0076	0.0072
DWSTOCKW(t-4)	0.0020	0.0078
DWSTOCKW(t-5)	-0.0033	0.0080
DWSTOCKW(t-6)	-0.0021	0.0072
SUMM	0.2662	0.0757
FALL	0.2438	0.0656
SPRI	0.0799	0.0709
DS	0.2494	0.1191
DX	-0.6276	0.1213
PWX(t-1)	0.1422	0.2307
RPW(t-1)	0.3476	0.0971
SDR(t-1)	0.0061	0.0024
THPW(t-1)	-0.0001	0.0005
WSTOCKW(t-1)	-0.0014	0.0056

Production: PWPR(t)

Variable	Coeff.	S.Error
Constant	2.7267	0.9952
TREW_S	-0.0100	0.0048
DPWPR(t-1)	0.3312	0.3488
DPWPR(t-2)	0.0166	0.3115
DPWPR(t-3)	-0.2917	0.2772
DPWPR(t-4)	-0.0849	0.2185
DPWPR(t-5)	-0.0651	0.1519
DPWPR(t-6)	-0.0291	0.0897
DRWAP2	-0.0475	0.2229
DRWAP2(t-1)	0.1015	0.2756
DRWAP2(t-2)	0.2588	0.2381
DRWAP2(t-3)	-0.0931	0.2425
DRWAP2(t-4)	0.2412	0.2346
DRWAP2(t-5)	0.0437	0.2272
DRWAP2(t-6)	0.8491	0.2043
DRWSP2	0.3700	0.2502
DRWSP2(t-1)	0.2540	0.2435
DRWSP2(t-2)	0.0413	0.2648
DRWSP2(t-3)	0.2161	0.2381
DRWSP2(t-4)	0.1043	0.2402
DRWSP2(t-5)	0.1206	0.2345
DRWSP2(t-6)	-0.3001	0.2475
PWPR(t-1)	-0.6502	0.3853
RWAP2(t-1)	0.0997	0.1985
RWSP2(t-1)	-0.1750	0.0815
SUMM	5.5292	1.0300
D1	0.8209	0.2046
D2	1.0486	0.2656

Chicago Prices: RPW(t)

Variable	Coeff.	S.Error
Constant	0.5934	2.9985
DPWD	-12.3875	7.6306
DPWD(t-1)	10.4134	12.6681
DPWD(t-2)	18.0533	10.1503
DPWD(t-3)	22.8043	7.0867
DPWD(t-4)	18.6287	5.0805
DPWD(t-5)	20.1238	8.2268
DPWD(t-6)	0.5995	0.4850
DPWI	-12.4697	7.4507
DPWI(t-1)	-3.3637	7.2155
DPWI(t-2)	8.5332	8.4171
DPWI(t-3)	5.4153	6.0175
DPWI(t-4)	-3.5733	5.7674
DPWI(t-5)	2.1366	9.3836
DPWI(t-6)	-19.2064	7.9723
DPWX	-12.8688	7.7903
DPWX(t-1)	9.0239	12.1329
DPWX(t-2)	16.9547	9.1275
DPWX(t-3)	22.5058	6.4244
DPWX(t-4)	18.7282	4.9631
DPWX(t-5)	20.6968	8.2999
DPWX(t-6)	0.6190	0.3910
DPWPR	12.5967	7.4710
DPWPR(t-1)	-6.3796	12.9664
DPWPR(t-2)	-15.0385	9.4888
DPWPR(t-3)	-20.4923	6.3537
DPWPR(t-4)	-16.8938	5.3268
DPWPR(t-5)	-19.0741	7.8249
DPWPR(t-6)	0.0230	0.1397
DRPW(t-1)	0.3377	0.4008
DRPW(t-2)	0.1367	0.2970
DRPW(t-3)	0.0954	0.2812
DRPW(t-4)	0.0201	0.2531
DRPW(t-5)	0.3481	0.2434
DRPW(t-6)	-0.1706	0.2244
DRPDI	0.1019	0.5031
DRPDI(t-1)	-0.4854	0.6653
DRPDI(t-2)	-0.0020	0.4072
DRPDI(t-3)	0.4139	0.4678
DRPDI(t-4)	1.0150	0.4260
DRPDI(t-5)	0.6002	0.6113
DRPDI(t-6)	-0.6936	0.6510
DSDR	0.0154	0.0206
DSDR(t-1)	-0.0004	0.0200
DSDR(t-2)	-0.0125	0.0203
DSDR(t-3)	0.0112	0.0144
DSDR(t-4)	-0.0036	0.0261
DSDR(t-5)	0.0055	0.0136
DSDR(t-6)	0.0389	0.0203
DTHPW	-0.0091	0.0039
DTHPW(t-1)	-0.0023	0.0058
DTHPW(t-2)	-0.0048	0.0067
DTHPW(t-3)	-0.0034	0.0048
DTHPW(t-4)	-0.0048	0.0050
DTHPW(t-5)	-0.0006	0.0031
DTHPW(t-6)	-0.0004	0.0024
DWSTOCKW	0.0251	0.0276
DWSTOCKW(t-1)	0.0188	0.0518
DWSTOCKW(t-2)	0.0000	0.0527
DWSTOCKW(t-3)	0.0110	0.0514
DWSTOCKW(t-4)	0.0532	0.0441
DWSTOCKW(t-5)	-0.0100	0.0305

DWSTOCKW(t-6)	0.0027	0.0264
DRWAP	1.6474	0.3618
DRWSP	0.8768	0.4591
SUMM	-3.4877	1.1974
FALL	-0.6588	1.1215
SPRI	-0.8802	0.9855
DX	-0.9662	0.9935
PWD(t-1)	-25.9623	15.2901
PWI(t-1)	0.1318	0.1553
PWX(t-1)	-25.5191	14.1997
PWPR(t-1)	22.1441	14.7659
RPW(t-1)	-0.0771	0.4340
RPDI(t-1)	0.1572	0.1139
SDR(t-1)	0.0044	0.0200
THPW(t-1)	-0.0035	0.0071
WSTOCKW(t-1)	0.0468	0.0477
RWAP(t-1)	1.9341	0.6050
RWSP(t-1)	0.5165	0.6108

Table 4. Model 4 (SCDSEM): 2SLS estimates and standard errors of the estimates.

Disappearance: PWD(t)

Variable	Coeff.	S.Error
Constant	0.4995	0.3987
D4_PWD(t-1)	0.0592	0.1013
D4_RPW	-0.2030	0.0667
D4_RPW(t-1)	-0.0145	0.0689
D4_RPDI	0.1320	0.1203
D4_RPDI{1}	-0.1538	0.1190
RPDI(1,t-1)	0.0099	0.0121
RPW(1,t-1)	-0.0713	0.0611
PWD(1,t-1)	0.4671	0.2316
RPDI(2,t-1)	0.2301	0.4541
RPW(2,t-1)	-0.2003	0.3378
PWD(2,t-1)	0.8291	0.0971
RPDI(3,t-2)	0.0191	0.2954
RPW(3,t-2)	-0.0660	0.1275
PWD(3,t-2)	0.6551	0.0885
RPDI(3,t-1)	-0.3939	0.2950
RPW(3,t-1)	-0.1045	0.1289
PWD(3,t-1)	0.1315	0.0893
SUMM	0.3425	0.0956
FALL	0.1965	0.0833
SPRI	-0.0163	0.0835

Inventories: PWI(t)

Variable	Coeff.	S.Error
Constant	-0.0032	0.0343
D4_PWI(t-1)	0.0116	0.0175
D4_PWPR	0.9980	0.0029
D4_PWPR(t-1)	0.0032	0.0024
D4_PWD	-0.9940	0.0085
D4_PWD(t-1)	-0.0031	0.0075
D4_PWX	-1.0058	0.0069
D4_PWX(t-1)	0.0032	0.0062
D4_RPW	-0.0033	0.0050
D4_RPW(t-1)	-0.0003	0.0046
PWPR(1,t-1)	0.1046	0.0659
PWD(1,t-1)	-0.1043	0.0604
PWX(1,t-1)	-0.1057	0.0661
RPW(1,t-1)	-0.0063	0.0044
PWI(1,t-1)	0.9984	0.0011
PWPR(2,t-1)	1.6778	0.2749
PWD(2,t-1)	-1.6635	0.2739
PWX(2,t-1)	-1.6884	0.2772
RPW(2,t-1)	0.0040	0.0250
PWI(2,t-1)	-2.3571	0.5512
PWPR(3,t-2)	1.0752	0.1363
PWD(3,t-2)	-1.0791	0.1354
PWX(3,t-2)	-1.0998	0.1357
RPW(3,t-2)	0.0056	0.0097
PWI(3,t-2)	0.6718	0.2178
PWPR(3,t-1)	0.7536	0.1594
PWD(3,t-1)	-0.7453	0.1583
PWX(3,t-1)	-0.7253	0.1589
PWI(3,t-1)	-1.8327	0.2016
SUMM	0.0696	0.0378
FALL	0.0584	0.0375
SPRI	0.0334	0.0311

Exports: PWX(t)

Variable	Coeff.	S.Error
Constant	-0.1908	0.4692
D4_PWX(t-1)	0.2303	0.1060
D4_RPW	0.1937	0.0825
D4_RPW(t-1)	-0.1534	0.0781
D4S DR	-0.0021	0.0061
D4_THPW	0.0011	0.0009
D4_WSTOCKW	0.0051	0.0070
SDR(1,t-1)	0.0014	0.0021
THPW(1,t-1)	0.0004	0.0004
WSTOC_1	-0.0016	0.0059
RPW(1,t-1)	0.2048	0.0858
PWX(1,t-1)	0.3032	0.1709
SDR(2,t-1)	-0.0204	0.0311
THPW(2,t-1)	0.0007	0.0032
WSTOC(2,t-1)	0.0259	0.0231
RPW(2,t-1)	-0.2664	0.3731
PWX(2,t-1)	-0.4866	0.3473
SDR(3,t-2)	-0.0001	0.0098
THPW(3,t-2)	0.0005	0.0018
WSTOC(3,t-2)	0.0176	0.0140
RPW(3,t-2)	0.4336	0.1379
PWX(3,t-2)	0.1695	0.1400
SDR(3,t-1)	0.0063	0.0106
THPW(3,t-1)	-0.0016	0.0016
WSTOC(3,t-1)	0.0139	0.0147
RPW(3,t-1)	-0.3088	0.1392
PWX(3,t-1)	0.0697	0.1655
DS	0.1305	0.1237
SUMM	0.2004	0.0731
FALL	0.2308	0.0635
SPRI	0.0197	0.0659
DX	-0.3656	0.1255

Production: PWPR(t)

Variable	Coeff.	S.Error
Constant	2.0865	0.8136
D4PWPR(t-1)	0.0487	0.0838
SUMM	1.6755	1.8310
TREWS	0.0005	0.0044
D1	0.7385	0.1879
D2	0.8448	0.2683
D4RWAPS	0.1353	0.2423
D4RWAPS(t-1)	-0.1879	0.2407
D4RWSPS	0.6792	0.2754
D4RWSPS(t-1)	0.2761	0.2187
RWAPS (1,t-1)	1.3392	0.4887
RWSPS _1	-0.0117	0.2708
PWPR(1,t-1)	-0.6110	0.2661
RWAPS (2,t-1)	0.9812	0.4398
RWSPS (2,t-1)	0.0913	0.2448
PWPR(2,t-1)	0.4404	0.1588
RWAPS (3,t-2)	1.1868	0.3459
RWSPS (3,t-2)	0.1395	0.1916
PWPR(3,t-2)	0.3499	0.1083
RWAPS _4	-0.2055	0.2584
RWSPS _4	-0.0482	0.1576
PWPR(3,t-1)	0.0904	0.1158

Chicago Prices: RPW(t)

Variable	Coeff.	S.Error
Constant	6.3006	1.5514
D4RPW(t-1)	0.2539	0.1719
D4PWD	-0.2743	2.0640
D4PWD(t-1)	0.0715	0.2079
D4PWI	-0.0315	2.2438
D4PWI(t-1)	0.0378	0.3063
D4PWX	0.4970	2.1919
D4PWX(t-1)	-0.1160	0.1612
D4PWPR	0.0427	2.1885
D4PWPR(t-1)	0.0300	0.0575
D4RPDI	0.0540	0.2112
D4RPDI(t-1)	0.2796	0.1657
D4SDR	-0.0001	0.0067
D4SDR(t-1)	0.0127	0.0071
D4THPW	-0.0011	0.0015
D4THPW(t-1)	0.0001	0.0016
D4WSTOC	-0.0021	0.0191
D4WSTOC(t-1)	-0.0017	0.0106
D4RWAPS	0.4875	0.2493
D4RWAPS(t-1)	-0.3010	0.2078
D4RWSPS	0.2469	0.3019
D4RWSPS(t-1)	0.1720	0.1717
RPDI(1,t-1)	-0.1591	0.0405
SDR(1,t-1)	-0.0187	0.0095
THPW(1,t-1)	-0.0027	0.0020
WSTOC(1,t-1)	0.0003	0.0147
RWAPS (1,t-1)	3.4469	1.6096
RWSPS (1,t-1)	1.3037	0.8728
PWD(1,t-1)	-3.7172	5.6247
PWI(1,t-1)	-0.0387	0.0736
PWX(1,t-1)	-3.3815	5.5738
PWPR(1,t-1)	3.9805	5.6122
RPW(1,t-1)	-0.4114	0.4514
RPDI(2,t-1)	-0.0616	0.6403
SDR(2,t-1)	0.0593	0.0399
THPW(2,t-1)	-0.0055	0.0057
WSTOC(2,t-1)	0.0309	0.0366
RWAPS _2	-0.2774	0.3947
RWSPS _2	-0.0540	0.1794
PWD(2,t-1)	3.9238	7.0819
PWI(2,t-1)	8.6533	14.3575
PWX(2,t-1)	4.6003	7.0074
PWPR(2,t-1)	-4.3597	7.1806
RPW(2,t-1)	-0.4957	0.4903
RPDI(3,t-2)	0.9751	0.4565
SDR(3,t-2)	0.0030	0.0146
THPW(3,t-2)	-0.0005	0.0028
WSTOC(3,t-2)	-0.0197	0.0186
RWAPS (3,t-2)	0.1258	0.3087
RWSPS (3,t-2)	0.0238	0.1169
PWD(3,t-2)	-0.2049	0.1867
PWI(3,t-2)	-0.1019	0.1424
PWX(3,t-2)	0.3461	0.1956
PWPR(3,t-2)	-0.0269	0.1011
RPW(3,t-2)	0.0725	0.1833
SUMM	0.1663	1.3411
FALL	-0.5504	0.9839
SPRI	-0.9704	1.1680
DX	0.8986	0.2249

APPENDIX F

IMPULSE RESPONSES OF U.S. WHEAT DISAPPEARANCE, INVENTORIES, PRODUCTION, AND MARKET PRICES

F.1. The Responses to a Shock in U.S. Wheat Disappearance

The economic relationships implied from the functional form adopted for the U.S. wheat disappearance (chapter 3), are sketched in Figure 1. Disappearance (U.S. domestic consumption of wheat) is an endogenous variable that enters into the model as a regressor variable in the fifth equation for Chicago prices (RPW). An unexpected positive shock in disappearance represents a shift to the right in the demand for wheat in the U.S., thus, an increase in the wheat market prices are expected to be observed. Disappearance does not enter in any of the other equations of the models, but Chicago prices does, which enters in the equations for inventories (PWI) and exports (PWX). Therefore, it is expected a negative instantaneous effect on inventories to a shock on disappearance, but the effect after the initial

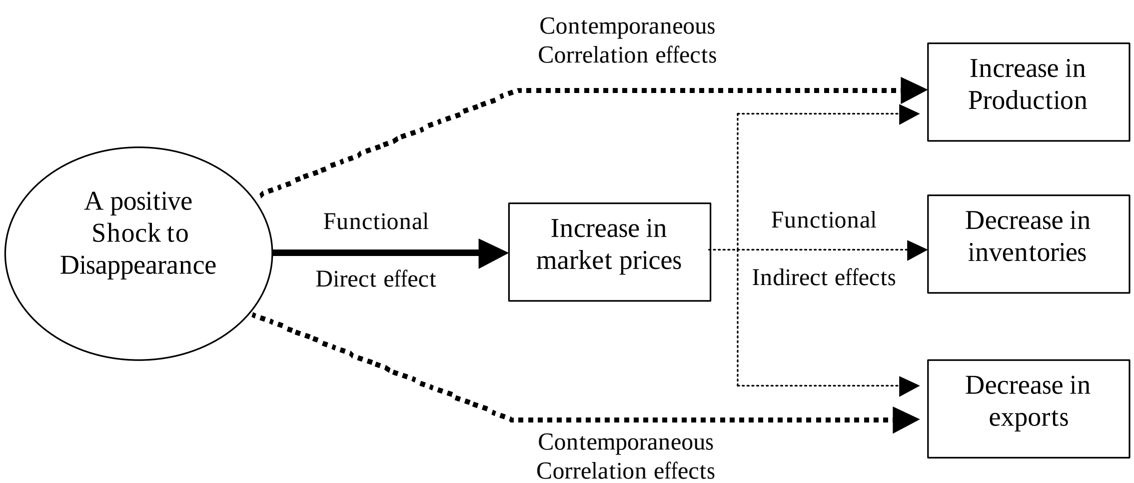

Figure 1. Economic causal relationships between U.S. wheat disappearance and the U.S. wheat disappearance, inventories, production, and market prices.

shock depends on the counterbalancing effect that an increase in market prices may have on inventories and in consumption. Collateral effects of a shock to U.S. wheat disappearance may also be expected, via the contemporaneous correlation with the variables in the system on production of wheat in the U.S. The effect of a higher price will send an incentive to farmers to produce more. These economic relationships between the U.S. wheat disappearance and inventories, exports, production, and market prices are used to analyze the rational of the estimated impulse responses in what follows.

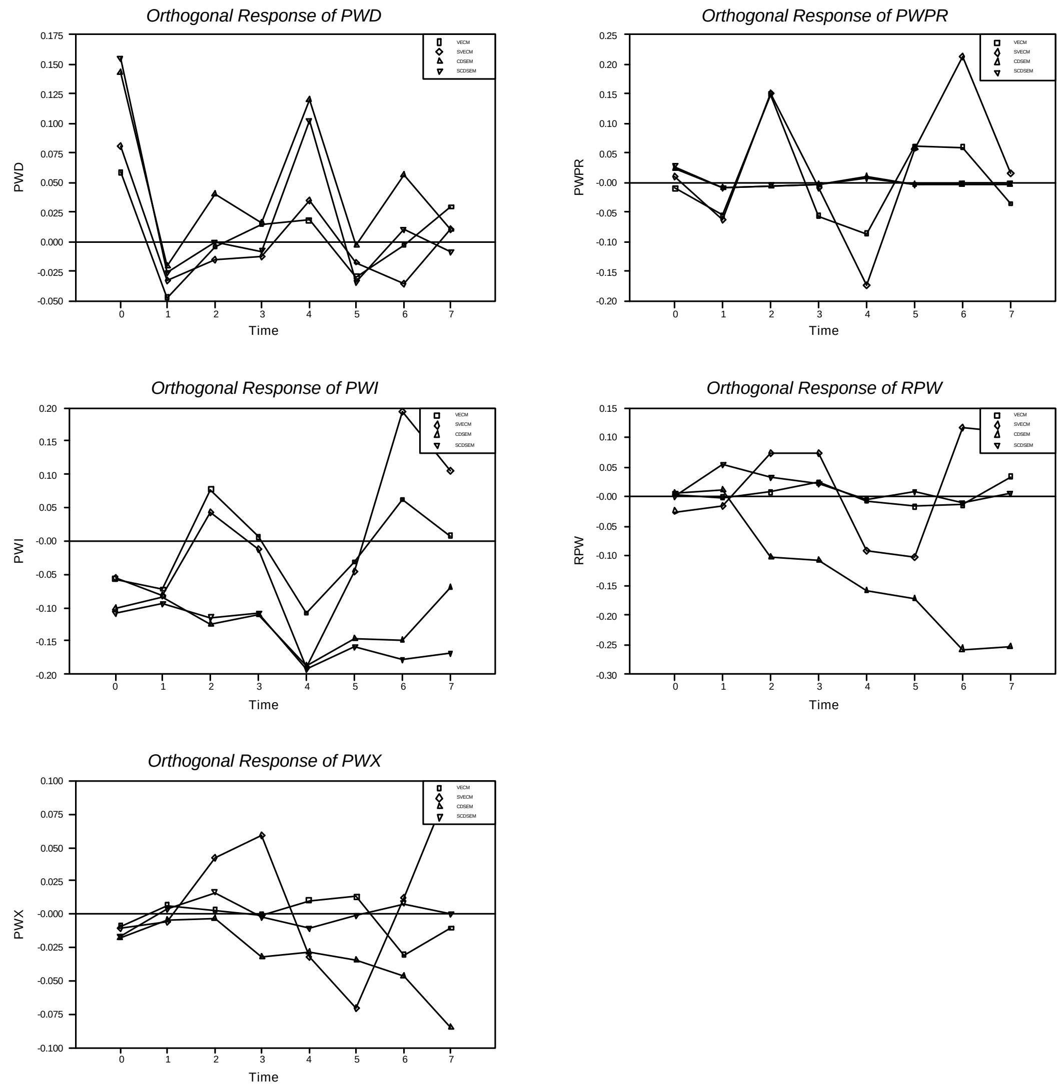

Figure 2. Responses of the U.S. wheat market variables to a shock in U.S. wheat disappearance.

The responses of U.S. wheat disappearance to an impulse in U.S. disappearance of wheat are depicted in the panel entitled "Orthogonal Response of PWD" in Figure 2. All the impulse responses look very similar, in the sense that after the initial shock, all the IRs show a pattern that approach zero after the shock. This behavior is in line with the results described by the Monte Carlo simulation results, in the sense that the responses of an exogenous to a shock on its error term is well described for all four methods.

The instantaneous impact on inventories (“Orthogonal Response of PWI” in Figure 2) to a shock in disappearance is estimated as negative by the four models. Yet, after the initial reaction, the responses evolve in different ways, depending on the models. Models 1 and 2, for instance, show an increasing oscillating behavior, around the horizontal line, while Models 3 and 4 are predicting that the levels of inventories after 8 quarters will not return to the original level. The positive peaks explained by Model 1 and 2 may be observed in the quarter that production realizes. After the shock in production replenishes inventories, the effect of the initial shock in disappearance on inventories four quarters latter will still be decreasing the levels of inventories.

The instantaneous impact on exports (“Orthogonal Response of PWX” in Figure 2) to a shock in disappearance is estimated as negative by the four models. Yet, after the initial reaction, the responses evolve in different ways, depending on the models. Models 1 and 2, for instance, show an increasing oscillating behavior, around the horizontal line, while Model 3 does not converge to zero. Model 4, instead, goes to zero smoothly. These results are in line with the results obtained by the Monte Carlo simulation with Model 3, which misspecifies seasonal nonstationarity, and with Models 1 and 2, which show a wider variability than Models 3 and 4.

The instantaneous impact on production (“Orthogonal Response of PWPR” in Figure 2) to a shock in disappearance is estimated as positive by Models 2, 3, and 4, which is expected on the basis of the economic relationships depicted in Figure 2. Instead, Model 1 predicts a negative instantaneous impact, which is not expected. The impacts after the initial period show that the IRs of Model 1 and 2 keep oscillating around the horizontal line, a behavior already described by the Monte Carlo simulation.

Finally Model 2 (SVECM) is the only model that wrongly describes an instantaneous decrease in the U.S. wheat markets prices to a shock in U.S. disappearance. In the following quarters, Model 3 describes a non-convergent-to-zero behavior. On the contrary, Models 1 and 4 coincide in describing that the initial positive effect will last for almost 4 quarters.

In synthesis, Model 4 is the only model that describes instantaneous impacts that are in line with the expected behavior of all the endogenous variables. Model 3, which misspecifies seasonal cointegration, shows some IRs that are not convergent, while the pure time-series models, Models 1 and 2, show in general IRs more variable than those of Models 3 and 4.

F.2. The Responses to a Shock in U.S. Wheat Inventories

The economic relationships implied from the functional form adopted for the U.S. wheat inventories (chapter 3), are sketched in Figure 3. Inventories (PWI) is an endogenous variable that enters into the model as a regressor variable in the fifth equation for Chicago prices (RPW). An unexpected positive shock in inventories represents a shift to the right in the demand for wheat in the U.S., thus, an increase in the wheat market prices are expected to be observed. Inventories do not enter in any of the other equations of the models, but Chicago prices do enter the equations for disappearance (PWD) and exports (PWX). Therefore, it is expected that an increase in wheat prices

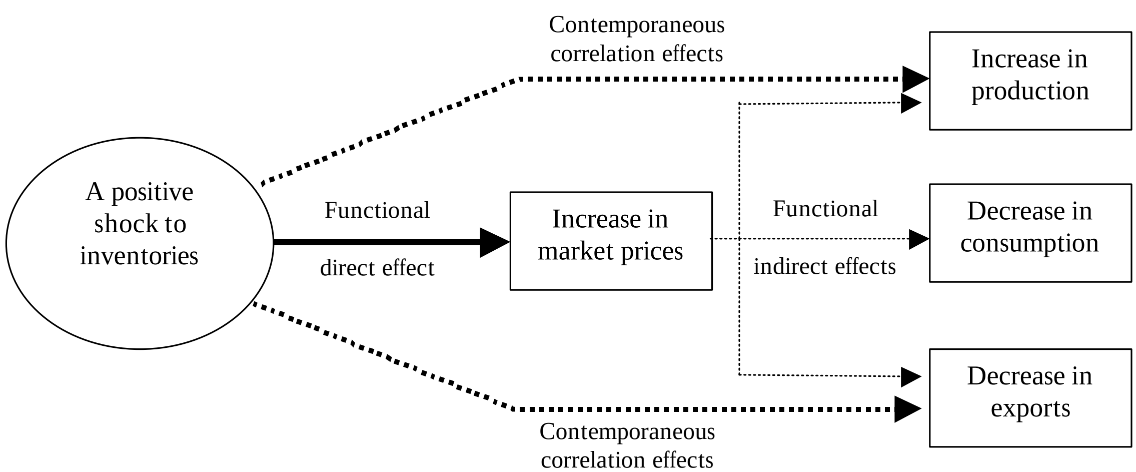

Figure 3. Economic causal relationships between U.S. wheat inventories and the U.S. wheat disappearance, exports, production, and market prices.

driven by a shock in inventories of wheat should have a negative instantaneous effect on disappearance (U.S. domestic consumption) and exports. It may happen that the U.S. wheat market immediately overreact by increasing domestic consumption and/or exports to liquidate as soon as possible the unexpected increase in stocks. Yet, the effect after the initial shock depends on the counterbalancing effect that an increase in market prices may have on the whole system, because high levels of inventories may promote an excess market supply in the near future that may drive a decrease in prices. Collateral instantaneous effects of a shock to U.S. wheat inventories may also be expected, via the contemporaneous correlation with the variables in the system on production of wheat in the U.S. The effect of a higher price will send an incentive to farmers to produce more.

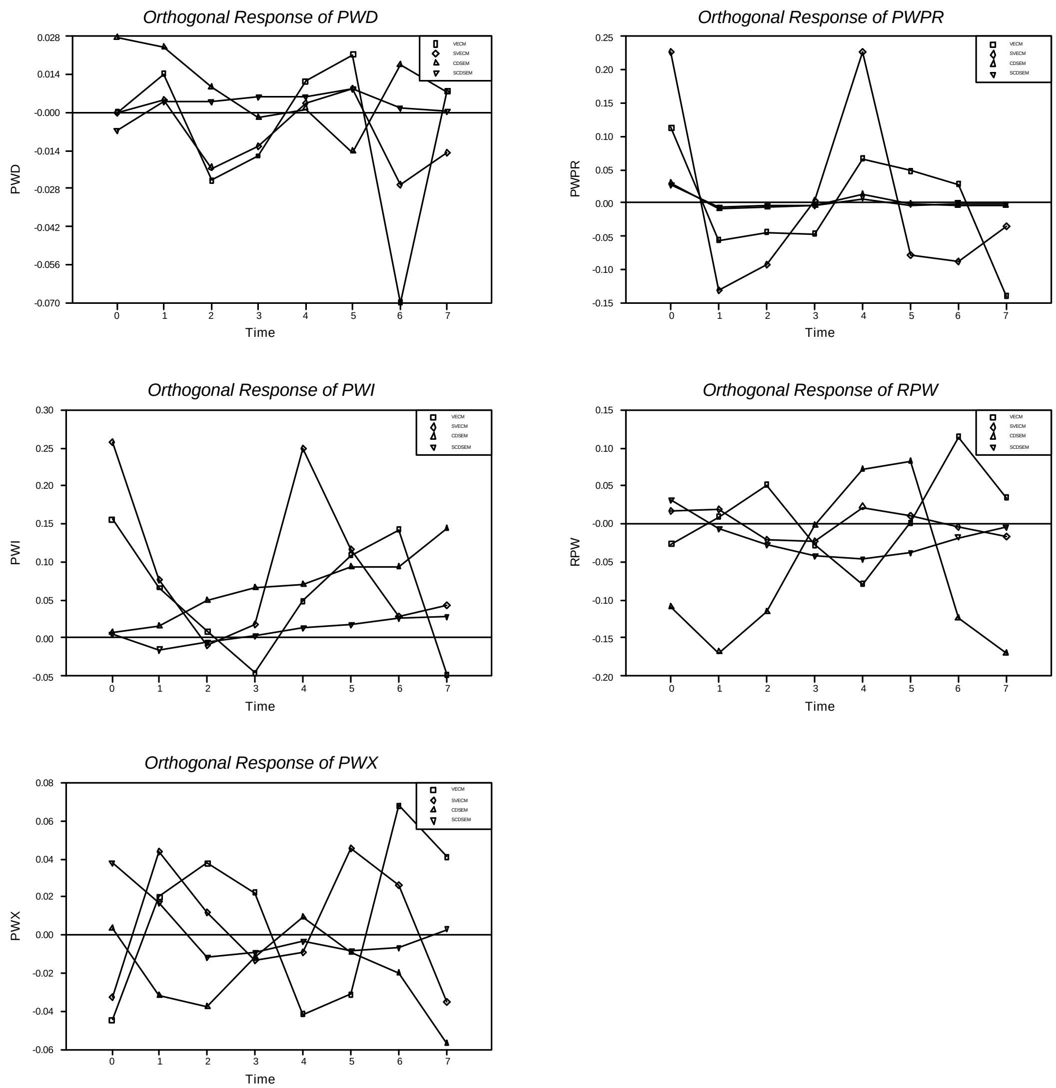

Figure 4. Responses of the U.S. wheat market variables to a shock in U.S. wheat inventories.

The instantaneous impact on disappearance ("Orthogonal Response of PWD" in Figure 4) to a shock in inventories is properly estimated by Model 4. Model 3 estimates a positive instantaneous impact, which is on the contrary to what is expected. Models 1 and 2 do not estimate any instantaneous effect on disappearance, after the shock on inventories. The effect in the following quarters vary across models. Models 1 and 2 show IRs that oscillate and do not converge to zero; Model 3 shows an IR that tends to zero, while Model 4 shows a positive and stable increase in disappearance that last for almost 5 quarters, a behavior that it may be explained by inventories that must be released to the market after the shock, lowering the U.S. wheat market price, and thus increasing domestic consumption of wheat.

The responses of U.S. wheat inventories to an impulse in U.S. inventories of wheat are depicted in the panel entitled "Orthogonal Response of PWI" in Figure 4. Not all the impulse responses present a similar pattern. For instance, Model 3 presents an increasing IR after the initial shock, while Models 1 and 2 present a high variability with respect to Model 4.

The instantaneous impact on exports ("Orthogonal Response of PWX" in Figure 4) to a shock in inventories is estimated as negative by Models 1 and 2, and positive by Models 3 and 4. Both are describing possible outcomes, as explained by the economic relationships between inventories and export (Figure 3). After the initial effect, the IRs present similar patterns as observed for the other IRs, that is, more variability in Models 1 and 2 than in Models 3 and 4.

The instantaneous impact on production ("Orthogonal Response of PWPR" in Figure 4) to a shock in inventories is estimated as positive by all models, in accordance with the economic relationships depicted in Figure 3. The impacts after the initial period shows that the IRs of Model 1 and 2 keep oscillating around the horizontal line.

Finally Models 2 and 4 (the models that incorporates seasonal cointegration) describe an instantaneous increase in the U.S. wheat markets prices to a shock in U.S. inventories. This is in line with the expected response depicted in Figure 3. On the contrary, Models 1 and 3 describe an instantaneous negative impact on market prices, which is not in accordance with the expected reaction explained by the economic rationale of the relationship underlying these variables. In the following quarters, Models 1 and 3 describe a non-convergent-to-zero behavior. On the contrary, Models 1 and 4 coincide in describing that a negative effect on prices may exist after the initial positive increase in market prices. This is a behavior that may be explained also by economics.

In synthesis, Model 4 is the only model that describes instantaneous impacts that are in line with the expected behavior of all the endogenous variables. Model 3, which misspecifies seasonal cointegration, shows some IRs that are not convergent, while the pure time-series models, Models 1 and 2, show in general IRs that are more variable than those of Models 3 and 4.

F.3. The Responses to a Shock in U.S. Wheat Production

The economic relationships implied from the functional form adopted for the U.S. wheat production (chapter 3), are sketched in Figure 5. Production (PWI) is an endogenous variable that enters into the model as a regressor variable in the fifth equation for Chicago prices (RPW). An unexpected positive shock in production represents a shift to the right in the supply curve for wheat in the U.S., thus, a decrease in the wheat market prices are expected to be observed. Production does not enter in any of the other equations of the model, but Chicago prices do, which enters in the equations for disappearance (PWD), inventories (PWI), and exports (PWX). Therefore, it is expected that a decrease

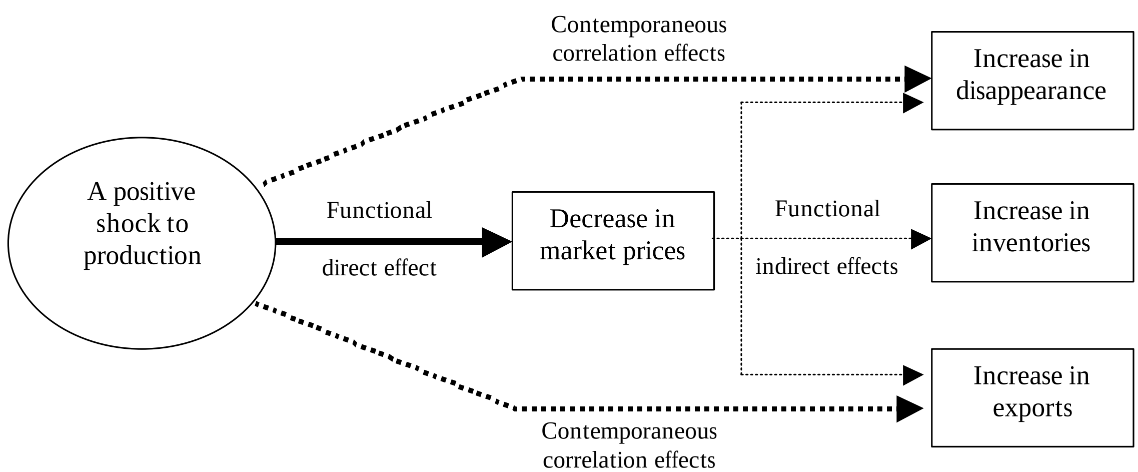

Figure 5. Economic causal relationships between U.S. wheat production and the U.S. wheat disappearance, inventories, production, and market prices.

in wheat prices driven by a shock in production of wheat should have a positive instantaneous effect on the levels of disappearance (U.S. domestic consumption), inventories, and exports.

The IRs in Figure 6, as in the previous equations, show that Model 4 is the only model that describes instantaneous impacts that are in line with the expected behavior of all the endogenous variables. Models 1 and 2 show in general IRs that are more variable than those of Models 3 and 4.

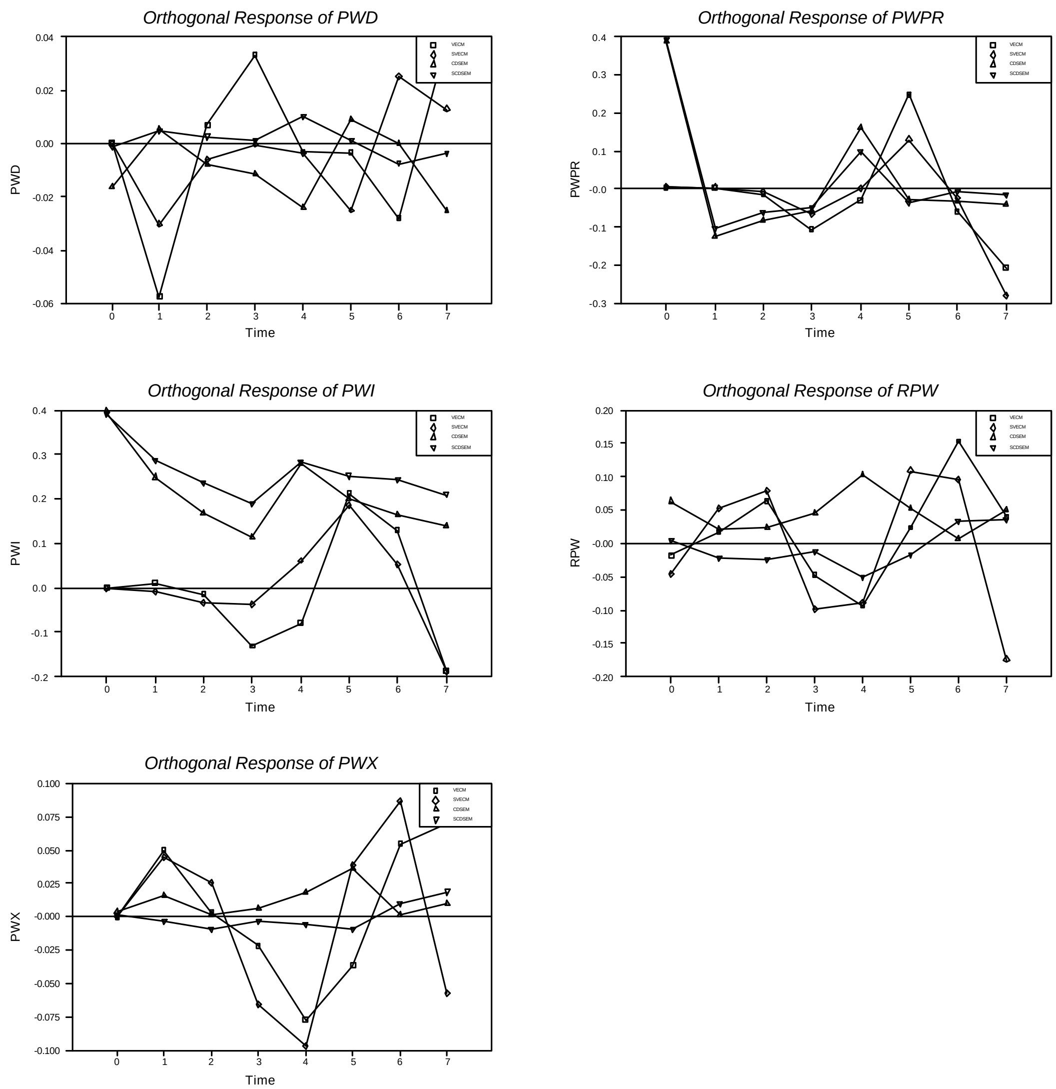

Figure 6. Responses of the U.S. wheat market variables to a shock in U.S. wheat production.

F.4. The Responses to a Shock in U.S. Wheat Prices

The economic relationships implied from the functional form adopted for the U.S. wheat prices (chapter 3), are sketched in Figure 7. Market prices (RPW) is an endogenous variable that enters into the model as a regressor variable in the equations for disappearance (PWD), inventories (PWI), and exports

(PWX). An unexpected positive shock in prices represents a movement along the demand curve, therefore, it is expected a negative impact on disappearance. If prices go up, there will be a signal for stockholders to take advantage of the situation and reduce inventories. Market prices do not enter explicitly in the equation of production, but may affect production via the contemporaneous correlation relationship that may exist among them. If this is true, a positive signal will be received by farmers to produce more.

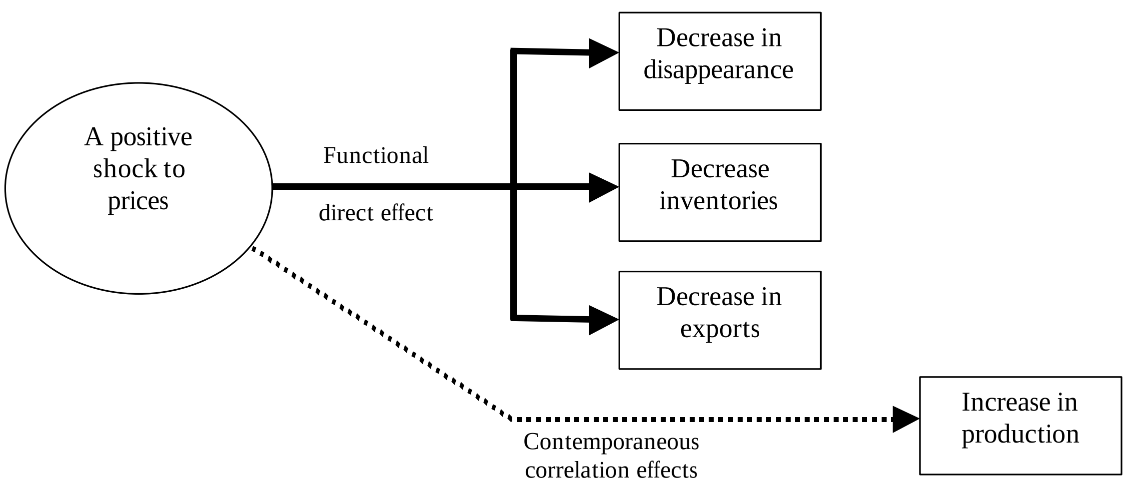

Figure 7. Economic causal relationships between U.S. wheat prices and the U.S. wheat disappearance, inventories, production, and market prices.

The IRs in Figure 8 show that all models fail to explain a negative impact on exports due to an increase in market prices, and that Model 4 is the only model that describes instantaneous impacts that are in line with the expected behavior of all the other endogenous variables. Models 1 and 2 show in general IRs that are more variable than those of Models 3 and 4.

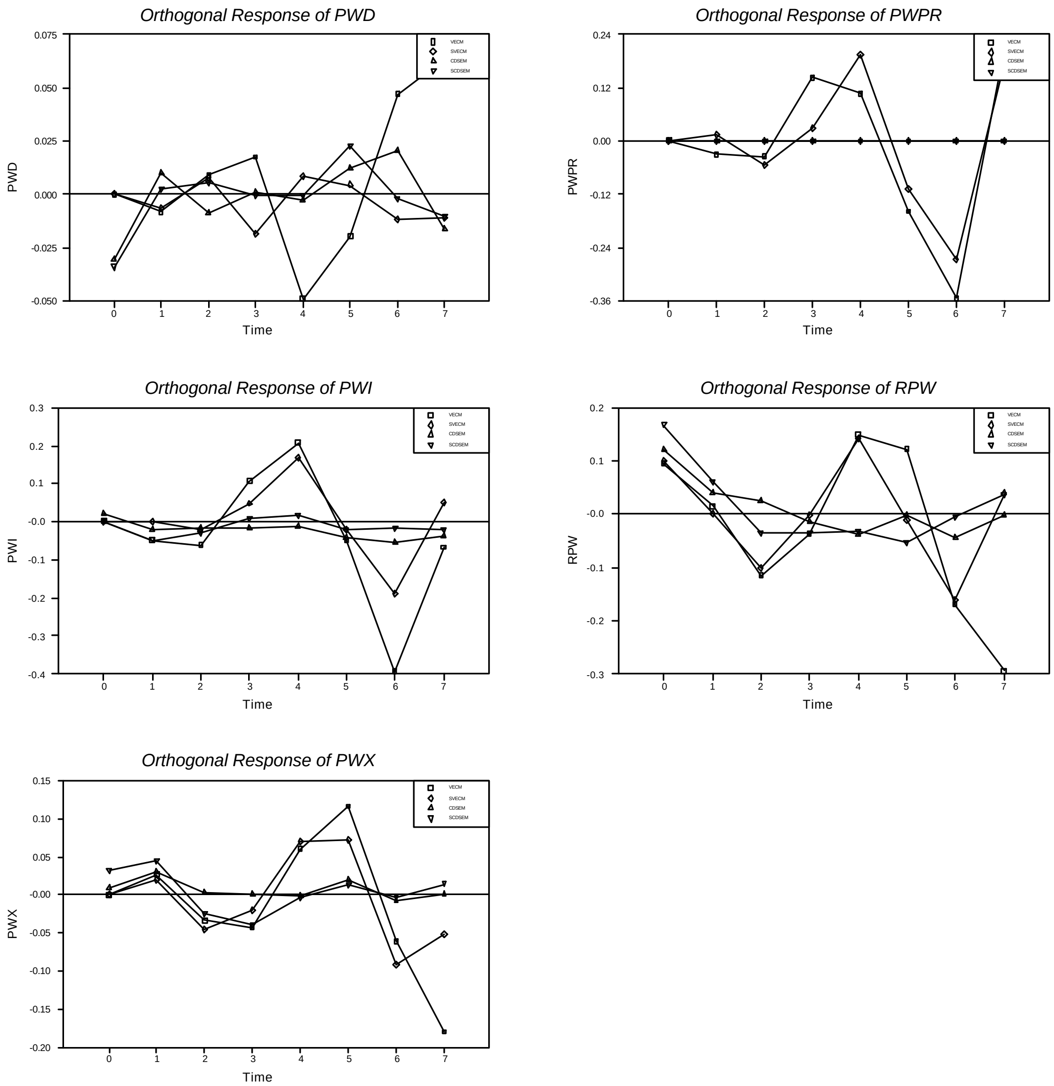

Figure 8. Responses of the U.S. wheat market variables to a shock in U.S. wheat prices.

VITA

Carlos Walter Robledo was born on December 19, 1956, in Córdoba, Argentina. He completed his secondary education at High School Corazón de María in 1974, Córdoba. He enrolled in the College of Agriculture, Universidad Nacional de Córdoba, Argentina, and received the degree of Agricultural Engineer in 1982. He attended Universidad de Buenos Aires, Buenos Aires, Argentina, and earned a Master of Science degree in biometry in 1994. Upon completing the master's degree, he was employed at the College of Agriculture, Universidad Nacional de Córdoba, as an Assistant Professor in Experimental Statistics and Biometry. In the Summer of 1997, he enrolled in the Agricultural Economics doctoral program at Louisiana State University. He is currently a candidate for the degree of Doctor of Philosophy, which will be conferred in December of 2002. Upon completion of his doctoral studies, he will be employed as an Assistant Professor in the Department of Rural Development of the College of Agriculture at Universidad Nacional de Córdoba, Argentina.