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A Model of the Tonal-Chromatic System and Its Application to Selected Works of Gustav Mahler

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A MODEL OF THE TONAL-CHROMATIC SYSTEM AND ITS APPLICATION TO SELECTED WORKS OF GUSTAV MAHLER

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The School of Music

by
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ABSTRACT

During the latter half of the twentieth century there was a marked shift in the way that many scholars approached analysis of late-nineteenth-century tonality. This shift in approach was motivated by the behavior and interaction of harmonic and melodic entities encountered in the music of the great composers of the nineteenth century, such as Schubert, Schumann, Brahms, Mahler, Wagner, and Strauss. The question arose: did the diatonic system remain at the heart of tonality during the late nineteenth century, or—as some propose—did a new chromatic-based tonality emerge? The acceptance of this new chromatic-based tonality is at the heart of this project.

In order to address the acceptance of this premise, this dissertation brings together two disparate strands of research and posits a system termed the *tonal-chromatic* scale. First, the writings of Mitchell, Marra, and Proctor help define the need for and structure of a chromatic scale that implies tonal function; second, the mechanics of transformational theory, stemming from the work of Lewin underpins the methodology of the system. The recent work of Rings is an influential aspect of the methodology.

Throughout past century, different analysts have used a myriad of methodologies to explore the complex musical language of Gustav Mahler. Mahler’s work represents an important turning point in the evolution of tonal music. Specifically, his work contributed to the onset of and evolution toward the complete breakdown of tonality. The chromatic tendencies in Mahler’s music make it an especially relevant candidate for analysis using this type of system. Even though his music exhibits an inordinate amount of chromaticism, the pillars of functional tonality are still operative. Three selected works of Gustav Mahler serve as the examples of the analytical usefulness of the proposed system: 1) the first movement of the Piano Quartet in A minor, which
was composed as a student at the Vienna conservatory; 2) the fourth movement of Symphony no. 5, Adagietto; and 3) *Kindertotenlieder*, no. 2, “Nun seh’ ich wohl, warum so dunkle Flammen.”
CHAPTER 1
LITERATURE REVIEW

INTRODUCTION

During the latter half of the twentieth century there was a marked shift in the way that many scholars approached analysis of late-nineteenth-century tonality. This shift in approach was motivated by the behavior and interaction of harmonic and melodic entities encountered in the music of the great composers of the nineteenth century, such as Schubert, Schumann, Brahms, Mahler, Wagner, and Strauss. The question arose: did the diatonic system remain at the heart of tonality during the late nineteenth century, or—as some propose—did a new chromatic-based tonality emerge? Of course, theorists answer this question in one of two ways, and thus, fall into one of two main camps: either they reject this premise and believe that there is only one classical diatonic system, or they accept this premise and believe there is a second tonal system, a tonality situated in the chromatic aggregate.¹

The acceptance of this premise is at the heart of this project, and the remainder of this chapter is devoted to the chronicling and exploration of the theories and methodologies that accept the chromatic aggregate as the background structure of late-nineteenth-century tonality. Section one of this chapter is a detailed inspection of four pioneering essays that establish a system of chromatic tonality (Mitchell, 1962; Proctor, 1978; Marra, 1986; McCreless, 1996). Each of these essays details the structural differences between the diatonic and chromatic systems of tonality. The second section of this chapter takes an historical approach and traces the

¹ Perhaps the most outspoken of those who reject this premise is Matthew Brown. Brown (1986) condemns proponents who believe in a chromatic tonality, specifically Gregory Proctor and Patrick McCreless, whose ideas are discussed below. Brown firmly believed that Schenker’s theories of harmony and analysis were fully capable of handling highly chromatic musical structures. In fact, Brown argues that, although Schenker’s system is fundamentally diatonic, chromaticism is not a substitution for or an elaboration of the diatonic, but is generated directly from the tonic triad.
evolution of harmonic dualism and its subsequent affect on harmonic analysis of chromatic music. Specifically, this section focuses on the development of Neo-Riemannian theory. The final section of this chapter examines several analytical methodologies that expand upon the concepts of harmonic dualism and transformation (Harrison, 1994; Kopp, 2002).

Ultimately, Chapter 2 will propose a revision to the previous concepts of a tonal system supported by the chromatic aggregate. This system, termed the Tonal-Chromatic System, will approach the topic of chromatic tonality from a transformational perspective. The pioneering studies by David Lewin (1982; 1984; 1987) on the topics of mathematical music theories and tonal harmonic function act as the soil in which the present project grows. Likewise, the research of Rings (2011), Cohn (1996; 2012), Tymoczko (2012), and Hook (forthcoming) contributes directly to this area of research, and thus has a significant role in the ideas presented below.

The music of Gustav Mahler burgeoned a desire to refine a tonal system situated in the chromatic universe and Mahler’s music serves as the analytical component of this project. Throughout his vast career as a composer in Austria and the United States, Mahler continually developed a unique compositional voice—one that employed a delicate balance between traditional diatonic tonal materials and highly chromatic materials. Even in the most chromatic of passages, the sense of tonal function never left Mahler’s compositional pallet. Thus, Mahler’s music demands a system that speaks to both the functional characteristics of tonality and the chromatic scale as a unit. It is my hope that the subtle refinements of the system developed below will achieve this task and produce analyses of a selection of Mahler’s work that prove to be powerful and sensitive to the detailed features of the musical landscape.
The first landmark essay addressing the structural status of the chromatic scale in late-nineteenth-century music was William Mitchell’s 1962 article, “The Study of Chromaticism.” Mitchell opens with a number of pivotal remarks, including the dismissal of the longstanding assumption that chromaticism consists of “seven tones plus five tone” (Mitchell 1962, 2). He goes on to state, “chromaticism is based ultimately on a scale of twelve tones, each with ascertainable functions related to a diatonicism which, in practice, is sometimes immediately present, sometimes remotely so, but which remains a prime ordering factor” (Mitchell 1962, 2). The objective of his study is to explore music in which the chromatic scale and the diatonic system are cooperative—where functional harmony, inherent in the diatonic system, remains intact while the chromatic scale governs the background.

The next portion of Mitchell’s essay explores the properties of the diatonic and chromatic scales. The first distinction Mitchell acknowledges, and the most obvious difference between the two scales, is that the diatonic scale is an asymmetrical filling of the octave, while the chromatic scale is a uniform scale, an “unremitting succession of half steps” (Mitchell 1962, 5). This basic difference has resounding repercussions on structure and function. As Mitchell goes on to illustrate, the asymmetric quality of the diatonic scale is what gives each tone of the scale a unique character. This uniqueness gives each scale degree a purpose, a direction, and a function. For example, the diatonic (major) scale has one and only one unique tritone. Conversely, the chromatic scale imparts no such individuality upon any one pitch class or group of pitch classes. Each pitch class of the chromatic scale can generate the interval of a tritone, removing the possibility of privileging any single pitch class through the traditional key-defining role of this interval class. The ordering force born of this asymmetry is what provides the diatonic scale with
the ability to individuate scale degrees—the unique position of each tone of the diatonic scale gives it a unique identity. Thus, *degree identification* and *degree progression* are basic to Mitchell’s study. Again, the chromatic scale yields the opposite characteristic—since each tone can generate all possible intervals without leaving the chromatic system, no tone of the chromatic scale is structurally unique.

Thus, melodically, intervallically, and harmonically, the diatonic scale, especially as represented by the major mode, stands as a strong ordering force, while, by the same measurements, the chromatic scale stands as a marked diffusing force. Chromaticism, or the union of the two forces, represents a constant play of the centripetal powers of diatonicism against the centrifugal character of the chromatic scale. (Mitchell 1962, 9)

For Mitchell, chromatic elements appear in diatonic textures in two ways. He terms the first way *interpolation*—the introduction of a chromatic half step between adjacent diatonic tones. An example of interpolation is the use of C# between 1 and 2 in the key of C major. The second source of chromaticism is *replacement*—the substitution of a diatonic element for a chromatic element (Mitchell 1962, 11). Furthermore, he identifies four functions of chromaticism. *Degree identification* and *degree progression* are of diatonic origin and refer to factors of structural orientation (Mitchell 1962, 11). Third, *Degree inflection* is a function of chromaticism which gives identical meaning to chromatic variants of a given pitch name (i.e. D♭, D, D#). Lastly, *degree transformation* is a function of chromaticism whereby a given pitch may express the meaning of more than one pitch name. For example, Cx, D, and E♭♭ represent different aspects and capabilities of the same sounding pitch.

Regarding notation, Mitchell notes that the general rule of thumb is to notate ascending chromaticism with sharps and descending chromaticism with flats; however, this is a general guideline (Mitchell 1962, 15). This rule of thumb breaks down in keys populated by indigenous sharps and flats. For example, in the key of F♯, it is more common to see the chromatic tone
between 1 and 2 notated as G, rather than Fx. This is merely a byproduct of the conventions of our notational system, which is essentially a diatonic system (Mitchell 1962, 15).

The next section of Mitchell’s essay, which I will only briefly discuss, explores tuning and its impact on chromaticism. Perhaps of most importance is that only in the confines of an equally tempered tuning system can a full-fledged system of chromaticism exist (Mitchell 1962, 21). The equal tempered system also influences interval perception and connotation. The interval of a major third between C and E sounds the same as the interval of a diminished fourth between C and Fb (Mitchell 1962, 22). However, the aforementioned intervals assume new responsibility when placed in a musical context and are completely dependent upon their surrounding.

Mitchell’s essay does not present so much of a formal model as it does provide general description of the nature of chromaticism derived from a tonal syntax. It is, though, the first analytically based essay that attempts to understand the chromatic scale as the backbone structure of a tonal environment. Thus, it provides a platform upon which later essays can stand, and over the next several decades, a number of monumental essays emerge that ultimately change the analyst’s perspective of the nature and structure of chromatic tonal music. His own ideas of tonality and the chromatic scale are evident in later essays written by Mitchell. For example, in his 1967 essay “The Tristan Prelude: Techniques and Structure”, Mitchell includes graphics that are indicative of a tonal system where the scale degrees are malleable—the major or minor version of a scale degree may invoke structural status. Example 1.1 recreates Mitchell’s Example 1a from his essay. The example shows Mitchell’s interpretation of the essential features of the linear and harmonic aspects of the piece. There are two relevant aspects of his analysis. First, both the minor and major forms of scale degree 3 represent structural tones in the upper voice of the graph. Second, the natural form of scale degree 2 occupies the linear descent of the
upper voice while the bass voice presents the phrygian form of scale degree 2 in the structural bass line.

Example 1.1. From Mitchell (1967), graphic reduction of the essential features of the Prelude from Wagner’s opera, *Tristan und Isolde*

Using Mitchell’s article as a foundation, Gregory Proctor’s 1978 dissertation, “Technical Bases of Nineteenth-Century Tonality: A Study in Chromaticism,” represents the next significant work in the area of chromatic tonality. Proctor divides his study into two large components, the second of which is most relevant for this project. The first installment of the dissertation defines the *Classical tonal system*, the tonal system constructed with the diatonic scale as its structural background. Proctor asserts that the theories of Heinrich Schenker form the absolute basis for the description of Classical tonality (Proctor 1978, 2). Chromaticism is not a structural component of Classical tonality, but is rather an elaboration of the diatonic scale—a system of seven tones plus five tones.

More relevant to the current project is the second component of Proctor’s dissertation. Here, Proctor establishes a theoretical foundation for a tonal system that has the chromatic scale as its background pitch structure. First, there is an important stipulation: each of the operations in the Classical tonal system remains intact in nineteenth-century chromatic tonality (Proctor 1978,
This stipulation is of utmost importance; without the inheritance of the tonal system from earlier generations, traditional tonal function is not inherent within the equal-tempered chromatic scale. Thus, chromatic tonality is a hybrid tonality. The fabric of this tonal system is contingent upon an equal-tempered tuning system and enharmonic equivalence. Enharmonicism comes to the fore in the nineteenth century and encourages, for example, the reinterpretation of a German augmented sixth chord as a dominant seventh, a procedure beyond the scope of Classical tonality (Proctor 1978, 132). For Proctor, there are two types of enharmonic experiences: *enharmonic equality* and *enharmonic equivalence*. Enharmonic equality indicates a note that has structural significance, but is renotated for ease of reading and performance. Enharmonic equivalence, on the other hand, indicates notes of the same pitch whose independent diatonic possibilities are both separately fulfilled (Proctor 1978, 139). Proctor’s notion of enharmonicism and notation align well with the ideas of Mitchell—that musical context is paramount when making decisions about note spellings, intervals, and other structural musical phenomena.

A number of consequences arise from having the equal-tempered chromatic scale as the background pitch structure of tonality. It is no longer necessary to define chromatic events by some type of diatonic derivation; all types of chromatic motion are now inherent, rather than extrinsic (Proctor 1978, 140). The structural use of other modal collections is also a possibility within the system, and it is common in the nineteenth century to find instances of the church modes without major-minor influence (Proctor 1978, 143). Another significant consequence of the chromatic system is the ability to divide the space of an interval symmetrically. This notion populates the next large portion of Proctor’s dissertation.

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2 Here, Proctor is not using the term operations in the Lewinian sense. There is no mathematical structure underlying the “operations” of the classical tonal systems. Rather, the term simply implies an action that is acting on the tonal system.
Symmetrical division of the octave by way of many intervallic values is a structural possibility that is inherent in the chromatic scale. This is not true of the diatonic scale, where the only symmetrical division of the octave is by the interval of a tritone, and then only between scale degrees 4 and 7 in major, and 2 and 6 in natural minor. Spaces not symmetrically divisible within a diatonic tonality are divisible within chromatic tonality due to enharmonic equivalence. For example, C-D♯ and D- Eb have different intervallic meanings in diatonic tonality; in chromatic tonality, under enharmonic equivalence, both spellings represent the same intervallic meaning (Proctor 1978, 150). Under enharmonic equivalence, there are five simple symmetrical divisions of the octave: division by twelve semitones, division by six whole tones, division by four minor thirds, division by three major thirds, and division by two tritones. In the following statement, Proctor describes the overall effect of symmetrical division on tonality:

Symmetrical divisions, their layering, and even some compounds, usually have reference in nineteenth-century chromatic tonality to some overall diatonic source, or they at least reside within a large-scale diatonic background. Gradually, this underlying tonality disappears in the face of chromatic elements in what would have been the Ursatz; the failure of the Urlinie or Bass-Brechung altogether; or non-diatonic references on the remote levels of the composition...even where a clear background can still be discerned. (Proctor 1978, 155)

This breakdown of diatonic background structure, where symmetrical divisions or other sources of non-diatonic structure replace it, lies at the heart of this project.

Transposition is a recognized transformation in the atonal music of the twentieth century. Proctor, however, submits that transposition is also an active agent in the chromatic tonal music of this period. He states, “The nonconformance of the voice leading in some passages to the operations of traditional counterpoint suggests the positing of some new operation—transposition” (Proctor 1978, 159). Chromatic transposition, as described here, is different from diatonic transposition, as in the transposition of a theme from one key to another; rather, Proctor
is positing the idea that strict transposition acting on the chromatic background structure can occur. Just as in post-tonal analysis, the transposition operation in tonal analysis consists of parallel voice leading. Consequently, the transposition of vertical sonorities typically reflects no single underlying scalar structure, or no scalar structure familiar to tonal music. Proctor recognizes four categories of voice leading, the last of which is the only one likely to involve transposition (Proctor 1978, 63-64):

1. Traditional counterpoint with a diatonic result;
2. Parallel counterpoint with a diatonic result;
3. Traditional counterpoint with a chromate result;
4. Parallel counterpoint with a chromatic result.

Lastly, in the final chapter of his dissertation, Proctor discusses the entanglement of the diatonic and chromatic. As chromatic as the music may be, when the relationships inherent in the classical tonal system exist, it is necessary to maintain some semblance of traditional tonal analysis. On the other hand, it is also necessary to extend the methodologies of traditional tonal analysis in order to allow for the explanation of chromatic events, especially when those events have no relationship with the diatonic source. As an example, Proctor discusses in detail the harmonic structure of Wolf’s “In der Frühe.” Example 1.2 reproduces Proctor’s Example 228, which is a reduction of Example 488 by Felix Salzer in the second volume of *Structural Hearing* (1962, 272). As the example shows, the background structure of the song has no diatonic origin, and no traditional Ursatz. Rather, parallel voice leading governs the harmonic motion between tonal areas. The abbreviation CS means *contrapuntal-structural chords*. This is not to say that diatonic materials are not present inside each section; instead, the diatonic materials involved exist within the habitat of a chromatic background structure.
Like Mitchell, Proctor’s dissertation is one step closer to a formalized model of analysis. He does this by introducing specific criteria for describing harmony and voice leading in certain textures of chromatic music. Specifically, by defining symmetrical division and transposition as specific aspects of this style of music, Proctor delineates between music based on diatonic structures and music based on chromatic structures. As seen in Example 1.2, these structures can coexist, in a nested fashion; the chromatic aggregate defines the background structure while the internal sections exhibit diatonic behavior.

The third significant work developing the field of chromatic tonality is the article “The Tonal Chromatic Scale as a Model for Functional Chromaticism,” by James Marra (1986). This essay, unlike the previous two, takes a study in musical cognition as its point of departure. The essay nonetheless goes on to develop a tonal system supported by a chromatic scale. The main objective is to develop a notation of the chromatic aggregate that orients all twelve pitch classes

Example 1.2. Reduction of Wolf, “In der Frühe,” originally by Felix Salzer

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3 While the cognition aspect of this research is not immediately relevant to this study, it is worth noting its premise. The study by Deutsch and Feroe (1981), “The Internal Representation of pitch sequences in Tonal Music,” sets up a system where the structure of chromaticism is hierarchical and explains that chromaticism has a relationship to a higher-lever diatonic foundation. Marra’s claims that this study, even though it represents an important shift in theoretical orientation, does not satisfactorily orient a twelve-note collection around a single tonal center.
around a single tonal center. Before setting down his own concept of a tonal chromatic scale, Marra notates four previously known ways of notating a chromatic scale, shown in Example 1.3: (1) melodic, (2) intervallic, (3) synthetic, and (4) harmonic chromatic scales (Marra 1986, 73).^4

Example 1.3. Figure 2 from Marra (1986), four spellings of the chromatic scale

Marra notes that it is inherently problematic to recognize such a wide variety of notation for one scale. Different syntactical rules govern each of the four provided notations of the chromatic scale. For example, the melodic chromatic scale uses ascending chromatic notation (#’s) when ascending, and descending chromatic notation (b’ s) when descending. On the other hand, the intervallic chromatic scale either preserves diatonic intervals with respect to the tonic,

^4 (1) Melodic chromatic scale (Forte, 1974); (2) intervallic chromatic scale (Drabkin, “Scale,” in New Grove Online); (3) synthetic chromatic scale (Sessions, 1951); (4) harmonic chromatic scale (Prout, 1903).
or uses the least amount of accidentals. The harmonic chromatic scale preserves the tonic and
dominant scale degrees. The variety in notational schemes obfuscates the relationship between
the chromatic scale and the tonal system. This problem is what ultimately leads Marra to develop
a notation of the chromatic scale that maintains the relationships inherent in the tonal system.

Marra posits his own notation for a chromatic tonal scale, which is akin to both the
synthetic scale and harmonic scale (see Example 1.3). Marra fixes the diatonic scale degrees
within the chromatic gamut, making major and minor distinguishable. Thus, Marra represents
chromatic tonality with two forms of the chromatic scale, a form for major and a form for minor,
shown in Example 1.4. Seven principles regulate the construction of Marra’s tonal chromatic
scales:

1. The scale will be linearly ordered either as a unidirectional ascent or descent;
2. It will be delimited by tonic scale degrees, one octave apart;
3. All twelve tones of the equal-tempered system must be included;
4. A diatonic scale must be a proper subset of the tonal chromatic scale;
5. 1, 3, and 5 may not be chromatically altered;
6. All chromatic degrees must be represented as either ascending or descending
   Semitonal neighbor prolongations of 1, 3, or 5; except #6♭7;
7. #6 and ♭7 in major are chromatic neighbor tones to 7 and 6, respectively.

While there is much value in recognizing the difference between a major or minor tonic, I will
posit that a tonal chromatic scale is representable in one form, a form that absorbs both the major
and minor forms of the tonic triad. I will explore this idea below, in the analytical methodology
of this project.
In 1996, William Kinderman and Harald Krebs edited a volume of essays titled, *The Second Practice of Nineteenth-Century Tonality*. The chromatic foundation of late-nineteenth-century music lies at the heart of these essays. There are three sections of the text, the first of which focuses on directional tonality and tonal pairing. Over the course of the nineteenth century, these two concepts sometimes replaced the standard monotonal design of single-movement works. A breakdown of the diatonic structure of tonality must occur in order for both of these concepts to exist. In the case of directional tonality, one diatonic structure evolves into the second, perhaps with some overlap. In the case of tonal pairing, two tonic triads co-exist as structural pillars.

In his essay, “An Evolutionary Perspective on Nineteenth-Century Semitonal Relations,” Patrick McCreless posits a theory of semitonal relationships that govern the musical structure. McCreless begins his investigation by locating instances of deep-structure semitonal relations,

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5 The music of the late nineteenth century poses a challenge to the traditional analytical methodologies. One such challenge is the phenomenon of musical works beginning in one key and ending in another, demonstrating the practice of directional tonality, tonal pairing, or the double-tonic complex (Kinderman 1996, 1). Most commonly, the interval of a third separates a pair of tonal centers.
and from this, introduces the concept of a chromatic tonal space.\(^6\) A classic example occurs near the end of the final movement of Beethoven’s Piano Trio in C minor, op. 1, no. 3 (McCreless 1996, 88-90). The main theme, having already returned in the tonic of C minor, takes a sudden and unprepared digression to a new tonic a half step lower, B minor. McCreless compares this harmonic shift to Proctor’s transposition operation—rather than shifting the thematic material in a diatonic fashion, it is a real transposition to a distantly related key, one related by a semitone (McCreless 1996, 91). A second example comes from Schubert, an example more complex than the Beethoven except. The passage is from Schubert’s Fourth Symphony in C minor, D. 417. This passage involves a modulation through five different keys, each a semitone higher than the one previous. Beginning in C\(_b\), the passage moves, via a 6-5 voice-leading procedure, through C, D\(_b\), D, and terminating on E\(_b\) (McCreless 1996, 91).

Similar, but not identical, to the juxtaposition of semitone-related keys is the overlap of two semitone-related keys. For examples of this technique, McCreless draws upon two analyses, one by Christopher Lewis (1987) and another by David Lewin (1984). Lewis demonstrates this type of relationship in his analysis of Schoenberg’s song “Traumleben,” op. 6, no. 8. At one point in the song, the keys of E major and F major are projected simultaneously. Likewise, in his analysis of the tonal structure of Wagner’s Parsifal, Lewin shows that passages from the music of Amfortas’s Prayer to Titurel where D minor is sometimes replaced by D\(#\)-minor, and at other times by D\(_b\) major (McCreless 1996, 94). These examples serve to show the growing complexity

\(^6\) McCreless acknowledges two opposing points of view regarding the existence of two tonalities in nineteenth-century music: diatonic and chromatic. Here, McCreless cites Gregory Proctor’s dissertation as a proponent of the existence of two tonal systems (Proctor 1978). However, he also cites Matthew Brown as an example of a scholar who disagrees with this concept (Brown, 1986). For Brown, Schenker’s theory of harmony adequately managed highly chromatic musical structures. From this he deduces no need for a separate tonal system based upon the chromatic scale.
of the harmonic structures in late-nineteenth-century music. McCreless sums this up with the following statement,

One might hypothesize that these examples trace the evolution in the nineteenth century of the concept of a harmonically based chromatic tonal space, a space in which the guiding harmonic point of reference in not a single tonic triad to which all other sonorities are necessarily related but an entire twelve-key system of potentially tonic triads, any two or more of which may be invoked over time to control the large-scale harmonic structure of a given piece. (McCreless 1996, 98)

Furthermore, McCreless directly declares his conclusion of a twelve-tone tonal system when he states,

The real issue that such passages force us to consider is whether the transposition operation necessarily takes us out of our governing diatonic mind-set, even if ever so briefly, and into a chromatic one. I believe that it does and that its doing so raises an even more interesting question: when, historically speaking, does deeper-level chromaticism of this sort become sufficiently pervasive to penetrate to the core of our diatonic mind-set and begin to orient that mind-set toward a Schoenbergian, twelve-note set of harmonic possibilities? Stated more radically, and with a view toward Wagner, Mahler, and Strauss, at what point do such procedures predominate to the extent that we experience diatonicism as a subset of the chromatic spatial universe rather than chromaticism as an inflection of the diatonic one? (McCreless 1996, 101-2)

Stated so directly, this proposed question cuts to the heart of this project, and my theoretical endeavor will attempt to address these technical nuances. First, I want to add a further question: if we accept McCreless’s twelve-tone system (not to be confused with Schoenberg’s dodecaphonic serialism, which is, however, clearly descended from it) as the governing background structure of this music, than under what pretense can we justify functional tonal relationships, particularly because they are not inherent in the twelve-tone system? However, if we accept the inheritance of the tonal system from prior generations, it is possible to build a theoretical system that reconciles the functional harmonic materials from the diatonic tonal system with that of the chromatic tonal system. But, before moving into the technical aspects of
this concept, I would like to explore other theoretical systems that address aspects of chromatic voice leading in tonal music— theories that will be absorbed into the context of my theoretical system.

**NEO-RIEMANNIAN THEORY**

Hugo Riemann was a prolific music theorist during the latter half of the nineteenth century and the early twentieth century and the revival of his theories of triadic transformations remains one of the fastest growing areas of theoretical research. Riemann developed a system of twenty-four transformations that related triads to one another—twelve transpositions and twelve contextual inversions (discussed below). These twenty-four transformations are independent of diatonic constraints. This characteristic is what attracted its revival in the late twentieth century through the present. As Cohn notes, “Neo-Riemannian theory arose in response to analytical problems posed by chromatic music that is triadic but not altogether tonally unified,” (Cohn 1998, 167). For the sake of space, this chapter will address only the most significant landmarks in the development of neo-Riemannian theory, along with the most significant applications of the theory.

For the entirety of his career, Riemann was convinced of the existence of a harmonic undertone series. For Riemann and his contemporaries, the existence of the undertone series allowed the minor triad to occur in nature. The combination of a tone (prime) with its (major) upper-third and upper-fifth generates the *overclang* (major chord); the combination of a tone (prime) with its (major) under-third and under-fifth generates the *underclang* (minor chord)

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7 Cohn (1996, 13) recognizes that consonant triads have two sets of unique properties. The first has to do with acoustics and their “syntactic routines of diatonic tonality.” The second property concerns the voice-leading potential between triads in group-theoretic terms. This idea removes triads from any diatonic restraints and elevates the notion of efficient voice leading over the syntax of functional relationships.
(Riemann, 10). By this logic, C, E, G = C overclang (C+) and A, C, E = E underclang (*e). The significance in this perspective lies in the fact that Riemann recognized major and minor triads in the same position were inversionally symmetrical. Example 1.5 illustrates this symmetry using E as the inversional axis.

![Diagram of inversional symmetry between major and minor triads, E = inversional axis]

Example 1.5. Inversional symmetry between major and minor triads, E = inversional axis

In Riemann’s terminology, two types of transformations move triads from one to another: Schritts and Wechsels. A Schritt is a transposition and a Wechsel is a reflection that projects a major triad onto a minor triad and a minor triad onto a major triad—the root of a major triad transforms into the fifth of a minor triad and the fifth of a minor triad transforms into the root of a major triad. All together, there are twenty-four transformations, including the identity element, which leaves a triad unchanged. These transformations account for all of the possible motions between consonant triads. Example 1.6 shows each triadic transformation: twelve schritts and twelve wechsels.

In his 2002 dissertation, subsequently condensed and published as an article in the Journal of Music Theory, “Uniform Triadic Transformations,” Julian Hook addresses the mathematical properties of all types of triadic relationships. The twenty-four Riemannian
<table>
<thead>
<tr>
<th>Transformation</th>
<th>Interval</th>
<th>Example</th>
</tr>
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<tbody>
<tr>
<td><strong>Schritt</strong></td>
<td></td>
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<tr>
<td>1</td>
<td>Identity</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Gegenleittonschritt</td>
<td>m2</td>
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<tr>
<td>3</td>
<td>Ganztonschritt</td>
<td>M2</td>
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<tr>
<td>4</td>
<td>Gegenterzschritt</td>
<td>m3</td>
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<tr>
<td>5</td>
<td>Terzschnitt</td>
<td>M3</td>
</tr>
<tr>
<td>6</td>
<td>Gegenquintschritt</td>
<td>P4</td>
</tr>
<tr>
<td>7</td>
<td>Tritonuschritt</td>
<td>a4/d5</td>
</tr>
<tr>
<td>8</td>
<td>Quintschritt</td>
<td>P5</td>
</tr>
<tr>
<td>9</td>
<td>Gegensextschritt</td>
<td>m6</td>
</tr>
<tr>
<td>10</td>
<td>Sextschritt</td>
<td>M6</td>
</tr>
<tr>
<td>11</td>
<td>Gegenganztonschritt</td>
<td>m7</td>
</tr>
<tr>
<td>12</td>
<td>Leittonschritt</td>
<td>M7</td>
</tr>
<tr>
<td><strong>Wechsel</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Seitenwechsel</td>
<td>Invert around prime</td>
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<tr>
<td>14</td>
<td>Gegenleittonwechsel</td>
<td>Gegenleittonschritt + Seitenwechsel</td>
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<td>15</td>
<td>Ganztonwechsel</td>
<td>Ganztonschritt + Seitenwechsel</td>
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<td>16</td>
<td>Gegenterzwechsel</td>
<td>Gegenterzschritt + Seitenwechsel</td>
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<td>17</td>
<td>Terzwechsel</td>
<td>Terzschnitt + Seitenwechsel</td>
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<td>18</td>
<td>Gegenquintwechsel</td>
<td>Gegenquintschritt + Seitenwechsel</td>
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<td>19</td>
<td>Tritonuswechsel</td>
<td>Tritonuschritt + Seitenwechsel</td>
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<td>20</td>
<td>Quintwechsel</td>
<td>Quintschritt + Seitenwechsel</td>
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<td>21</td>
<td>Gegensextwechsel</td>
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<td>22</td>
<td>Sextwechsel</td>
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<td>23</td>
<td>Gegenganztonwechsel</td>
<td>Gegenganztonschritt + Seitenwechsel</td>
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<tr>
<td>24</td>
<td>Leittonwechsel</td>
<td>Leittonschritt + Seitenwechsel</td>
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</tbody>
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Example 1.6. Riemannian triadic transformations
transformations earn their own section. This group of transformations has a special characteristic: under Hook’s system, the total transposition of each transformation is 0. Under this system, ordered triples represent each transformation. The first place holder of each triplet indicates whether the transformation is mode preserving (+) or mode reversing (−); the second indicates the transposition level of major triads under the transformation; the third indicates the transposition level of minor triads under the transformation. In the case of all twenty-four Riemannian operations, the two transposition values sum to 0 (modulo 12). In other terms, given a transformation, whatever the value of transposition is for a major triad, the transposition value for a minor triad is the mod 12 inversion. For example, using this notation, the Leittonwechsel is ⟨−, 4, 8⟩, and the Terzwechsel is ⟨−, 9, 3⟩.

For contemporary theorists, the three transformations that have efficient voice leading—transformations that hold two common tones—have significance with regard to chromatic tonality. The neo-Riemannian tradition of extracting these three triadic transformations from Riemann’s original system has its origins in the work of David Lewin. In 1987, Lewin published his pioneering treatise, Generalized Musical Intervals and Transformations (GMIT), after which the field of transformational music theory was born. In chapter 8 of GMIT ("Transformation Graphs and Networks (2): Non-Intervallic Transformations"), Lewin introduces graphic networks that relate triads with one another. The networks consist of “operators” that transform one triad into another. Of all the operations introduced by Lewin, the three operations, leading-tone exchange (L), parallel exchange (P), and relative exchange (R), will ultimately carry the greatest significance for later theoretical work on chromatic tonality.

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8 Some of the concepts in chapter 8 of GMIT, such as the diatonic operations dominant (D) and subdominant (S), have their origins in Lewin (1982).
The three above-mentioned triadic transformations each involve the movement of only one pitch class; two transformations involve motion by semitone and the other by a whole step. All three operations are mode-changing operations, meaning that each sends a major triad to a minor triad and vice versa. The leading tone exchange, for example, transforms a C-major triad into an E-minor triad; the parallel exchange transforms a C-major triad into a C-minor triad; the Relative exchange transforms a C-major triad into an A-minor triad. Each transformation is an involution, meaning that each operation is its own inverse—two consecutive applications of the operation yields the original triad, as shown in Example 1.7.

Example 1.7. Neo-Riemannian operations

Perhaps the most prolific author on the subject is Richard Cohn. Over the course of a number of articles and the 2012 publication of his book, *Audacious Euphony*, Cohn has developed a system under which he is able to explain unusual and uncanny harmonic relationships, specifically in the music of Schubert. Most notable from Cohn’s early research using neo-Riemannian operations is his development of the hexatonic system (Cohn 1996); this system is the topic of a number of later articles as well (Cohn 1999; Cohn 2012). For Cohn, the

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9 The leading tone exchange bears its name from the Riemannian tradition of dualism. In the case of moving from a C-major triad to an E-minor triad, the leading tone of each chord is substituted during the transformation. From C major to E minor, the C is replaced by the leading tone B. From E minor to C major, the B is substituted by its “upper” leading tone C. Since, for Riemann, the root of a minor triad is the fifth of the chord, the leading tone in minor is the half-step neighbor to that chord tone. Thus, the leading tone in a minor key is what diatonic tonal theory considers scale degree 6.
greatest musical symbol of the Das Unheimliche, the uncanny, is the hexatonic pole, a relationship between two triads, one major and one minor, such that the minor triad’s root is a major third below that of the major triad. This relationship yields a special voice-leading condition: all three voices move by half step in contrary motion. Like the other three neo-Riemannian operations, the hexatonic pole relationship is an involution. One must apply one of two ternary operations, PLP or LPL, in order to achieve a hexatonic relationship using the traditional neo-Riemannian operators as generators. Example 1.8(a) demonstrates this ternary operation in musical notation. As seen in the example, the ternary operator, whether starting with L or P, yields the same output. There are a total of four hexatonic systems, each containing six triads, which are related by alternating P and L operations (Example 1.8(b)). The term “hexatonic” comes from the composite scale formed by the six triads that comprise each system, and when taken as a scale, each system forms a set of six pitch classes, a hexatonic scale, as seen in Example 1.8(c).10

Considerable theoretical inquiry and analysis has adopted these operations as a means of explaining nineteenth-century tonality. Two studies are worth discussion here, Harrison (1994) and Kopp (2002). These studies use Riemannian concepts in different ways. Both studies focus on the chromatic relationships in nineteenth-century tonality—Kopp focuses heavily on the relationship between triads, while Harrison focuses more on the individual tendencies of pitch classes. These two studies represent highly developed systems of analysis of chromatic music. My goal is to observe their strengths and weaknesses, while in the effort of discerning what parts of each system can be adapted into my own.

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10 A hexatonic scale is formed by the symmetrical alternation of semitones and minor thirds. As defined in Straus (2005, 149), there are four possible hexatonic scales, and the pitch class on which it begins labels each scale. Thus, the four hexatonic scales are labeled as follows: HEX0,1, HEX1,2, HEX2,3, and HEX3,4.
HARRISON, *HARMONIC FUNCTION IN CHROMATIC MUSIC*

Supporting the foundation of Harrison’s book, *Harmonic Function in Chromatic Music*, is a dualistic perspective of the major and minor key systems of tonal music—major-mode is set against minor-mode. This dualism is the first and governing postulate of the book. In order to achieve the effect of a generalized major versus minor system dichotomy, one must generalize the diatonic scales that represent both major and minor. With the major system, it is simply the traditional major scale (Ionian); however, the minor mode system presents different challenges.
There are three recognized versions of the minor scale, natural, harmonic, and melodic minor. Harrison narrows the structure of the minor-mode complex to the natural minor version of the scale, which is the truly diatonic version (Harrison 1994, 24). In doing this, major-mode music is theoretically distinguishable from minor-mode music based upon the scale structure, as well as the triadic quality of the primary scale degrees. In contrast to Proctor or McCreless, Harrison’s approach depends on the tangible existence of the diatonic scale in order for the system to bear fruit.

For Harrison, the dualism between major and minor extends below the surface of the major and minor scales themselves. For example, there is derived dualism contained in what the author calls the characteristic semitone of each mode. For major, this is 7-8 and its dual in minor is 6-5 (Harrison 1994, 26). Harrison points to Riemann to contextualize this idea in an historical light. For Riemann, in minor, scale degree 6 is the leading tone to the root (conventionally, the fifth) of the tonic triad. At this point, it is worth noting the difference between how Harrison uses the word dualism compared to that of his earlier contemporaries (such as Riemann), from which he is drawing his ideas of dualism. For Riemann, and other theorists who subscribed to his ideas, harmonic dualism was a theory of inherent inversional symmetry—what happened in major occurred in an equal and opposite way in minor. Harrison justifies this difference of perspective when he says,

We do not hold with Riemann’s theory of underchords, so vouchsafing 6-5 as a leading tone is hardly possible. Yet, although the perquisites that come with leading-tone status are lost if classic, Riemannian harmonic dualism is abrogated, the sense that 6-5 is at least as characteristic of minor as 7-8 is of major persists nonetheless. (Harrison 1994, 26).

Therefore, the reader must accept a generic definition of the term dualism, since there is no inversional symmetry when looking at minor as the dualistic opposition to major. Of course, both
scales are diatonic and have identical intervalllic structure; however, minor is not the mirror image of the ordered structure of the major scale. In fact, the diatonic dual to the Ionian mode is the Phrygian mode.

The second postulate of the theory extends to the harmonic dimension: tonal music, both major and minor, revolves around a focal point called the tonic, from this point or origin, two structural scale degrees extend in equal and opposite directions, the subdominant and dominant. These three units, tonic, subdominant, and dominant, constitute the harmonic functions of tonal music (Harrison 1994, 34-5). The categorization of harmonic function is the result of certain chords and tonal combinations that sound and behave alike, even though they may not be analyzed as identical musical objects (Harrison 1994, 37). Harmonic function does not merely result from the quality of a chord, or its position within a scale; rather, harmonic function resides in the composite tendencies of the scale degrees that make up the chord (Harrison 1994, 42).

The idea of disassembling a chord into its constituent parts is a concept that will transfer without difficulty into the methodology established in Chapter 2 of this project. Extending this concept into a chromatic tonality, a tonality where scale-degree attributions still exist, will prove fruitful for understanding complex harmonic sonorities that do not take on the guise of traditional tonal objects, such as triads and seventh chords. The primary triads have a three-part structure: the functional base, which consists of the root of the triad; the functional agent, which consists of the third of the triad; and the associates, which consists of the fifth of the triad (Harrison 1994, 45). This perspective sheds insight into functionally mixed structures—chords with constituent parts that can communicate more than one function. A simple example of a functionally mixed

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11 Harrison makes a distinction between two types of dualism: two-term dualism and three-term dualism. The flanking of the subdominant and dominant about the axis of a tonic is an example of a three-term dualism. Two-term dualism does not have center fulcrum; rather, there are only two objects present that oppose one another.
chord is the supertonic triad, 2, 4, and 6. 4 and 6 belong to the subdominant triad, as the functional base and functional agent, respectively; whereas, 2 is the associate constituent of the dominant (Harrison 1994, 61).

The process of position finding, a term first introduced by Richmond Browne (1981), is the primary means one uses to locate a tonic. In the classical tonal system, certain intervals have inherent tendencies to resolve in specific ways. For example, the tritone formed between C and F#/Gb can serve as the third and seventh of a dominant seventh chord in the keys of G and Db. Once another note of the chord is given, a more accurate understanding of the tonic emerges (Harrison 1994, 73). A diatonic texture is a necessary prerequisite for position finding. In the music of Mahler, even at cadential areas, where diatonicism seems most crucial, the lack of diatonicism can make the location of the tonic a rather difficult task. A brief example from one of Mahler’s earliest works, his unfinished Piano Quartet in A minor, will demonstrate.

Example 1.9 shows mm. 147-151 of the piano quartet, the last four measures of the development section and the first of the recapitulation. Commonly, composers employ some type of dominant structure at this moment, but in this case, the listener hears something far from that. Instead, the F# diminished seventh chord heard in m. 147 shifts to an F dominant seventh chord (F# → F7). The Eb of this chord then moves to the lowest voice in the piano and cello. This chord, an F7 chord in third inversion, is the chord that resolves to the tonic, A minor. There are a number of ways to try to explain this chord as a vertical structure, but for now the most relevant issue is that this cadence throws a wrench in the concept of position finding. The tritone of the penultimate chord is Eb and A, combined with the other notes of the chord, points towards a resolution to Bb major. Of course, there are other means of describing the voice leading of this passage; this discussion will continue in Chapter 3, where this quartet is analyzed in more detail.
In cases such as these, when position finding does not provide insight on the harmonic resolution, it is possible to establish tonic via other processes. *Position Asserting* is the ability of the tonic to establish itself, without the need for intervals. Harrison (1994, 76-81) provides four characteristic behaviors and rhetorical devices of tonic that occur in chromatic music:

1. Tonic function ends a composition;
2. Tonic begins compositional sections;
3. Harmonic stasis and immobility attract tonic function;
4. Thematic exposition is heard in a tonic context.

In the case of the Mahler excerpt, rules two and four are applicable. This does not explain the behavior of the proceeding harmony; it does provide a sense of tonal orientation, however.

The functional identity of the constituent parts of each harmonic unit defines the voice-leading behavior between each chord. Harrison (1994, 91) defines this behavior *tonal motion and*
*functional discharge.* Specific scale-degree identities and relationships will have tendencies to resolve in traditional ways; it is thus possible to categorize these scale-degree motions. For example, Harrison describes the scale-degree motion in a cycle of authentic cadences and plagal cadences. Examples 1.10(a) and (b) reproduce Harrison’s figures 3.4 and 3.5. In Example 1.10(a), one can observe the typical scale-degree motion in an authentic harmonic cycle. From the tonic to subdominant: $3 \to 4$; from the subdominant to dominant: $6 \to 5$; and from dominant to tonic: $7 \to 8$. Likewise, Example 1.10(b) shows the functional scale-degree motion in a cycle of plagal cadences.

![Example 1.10. Harmonic cycles and scale-degree agents; (a) authentic cycle; (b) plagal cycle](image)

There are different categories of harmonic motion—the more scale-degree motion between harmonies, the more active the motion is. Harmonic motion has two broad categories: parallel and contrary motion. Diatonic music tends to preserve generic interval motion, while chromatic music is marked by parallel motion that preserves specific intervals (Harrison 1994,
The preservation of specific intervals recalls Proctor’s transposition operation. It is possible to extend this same concept to contrary motion. Pitch classes that do not represent scale degrees of the governing key put the notion of functional agents in jeopardy. Harrison’s solution to this is to consider chromaticism as a type of alteration of either the individual scale degrees or the chord itself (Harrison 1994, 107). Again, this is a similar concept to those seen in Proctor (1978) and Marra (1986).

In the analytical methods chapter of his book, Harrison describes three different types of analysis: Segmental, Linking, and Accumulative. In its most simple definition, segmental analysis is the process of dividing the musical landscape into discrete units, the surrounding context providing meaning to each unit. The segments can vary in size, depending on the type of analysis. For example, in traditional Roman numeral analysis, each chord comprises a musical segment (Harrison 1994, 128). In functional analysis, the segments are larger, grouping harmonies in categories of function. Linking analysis focuses on individual pitch classes and extracts a constant functional meaning across larger scopes of the work, even if the scale-degree function of a pitch class seems absent during a section (Harrison 1994, 135). Accumulative analysis evaluates functional behavior across different keys—that is, how different keys play functional roles across the entirety of a work. In the Prelude to Act One of Tristan und Isolde, for example, the keys A, C, and E play important roles in the drama (Harrison 1994, 154). Each type of analysis moves further towards the background structure: segmental analysis is the most surface-oriented; accumulative analysis is the most large-scale; and functional analysis occupies an intermediate scale.
The principles of harmonic dualism, stemming from the work of nineteenth-century theorists such as Hauptmann and Riemann, underpin the methodologies in David Kopp’s book, *Chromatic Transformations in Nineteenth-Century Music*. Specifically, Kopp puts forth a formal theory of common-tone tonality, with a specific emphasis on third-related harmonies. In the same way that fifth relations and diatonic third relations are fundamental to functional classical tonality, third relations have independent functional identities in nineteenth-century chromatic music (Kopp 2002, 3). Determining the function of third relations, however, proves more arduous when compared to its fifth-related counterpart. This is mainly due to the sheer volume of third-related chords. For any given chord, there are eight possible third-related triads—major chords, minor chords, diatonically related, and chromatically related—and the best way to categorize these relationships is to take note of the common-tone relationships. There are three categories, those with two common tones, those with one common tone, and those with no common tones. *Relative* mediants with two common tones are of opposite mode and belong to the same diatonic scale. *Chromatic* mediants have one common tone, are of the same mode, and have one or two tones outside of the diatonic scale. *Disjunct* mediants have no common tones, are of opposite mode, and have no common diatonic pitch classes (Kopp 2002, 8-10).

After a lengthy detour on the historical aspects of nineteenth-century harmonic theorists, Kopp proposes a transformational system based on a “common-tone tonality.” Therefore, it is beneficial to conceive of all fifth-related and third-related harmonic relationships as unary.

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12 As Kopp (2002) notes, the term “function” has its origins in the work of Riemann. Function denotes a chord’s harmonic meaning. Function signifies the intrinsic potentiality of a given chord to resolve in a certain way. It also denotes the identity of a chord within a given key, situating the chord’s root onto a certain scale degree. In this case, the relationship between the chords is less important and serves merely as a naming device.
harmonic processes (Kopp 2002, 165). This represents an outright rejection of established neo-Riemannian practices where binary, ternary, and larger compound operations relate harmonic entities. For Kopp, there are two basic types of transformations: mode-preserving and mode-reversing. He further divides these transformations into subcategories based on the interval of root motion. There is a subtle difference of perspective compared to traditional neo-Riemannian theories. In neo-Riemannian systems, the root motion between triads is a consequence of the voice-leading procedures of $P$, $L$, and $R$, whereas, in Kopp’s system, the voice-leading efficiency is a consequence of the root motion.

From Lewin’s system of triad transformations, Kopp keeps two: the identity element, which sends a triad to itself and the parallel operation, which preserves the root of a triad but changes the mode from major to minor, and vice versa. Both of these operations are involutions. Unique to Kopp’s system are transformations for the four chromatic mediant relations, and they define a single transformation type: mediant (Kopp 2002, 166). There are two forms of the mediant transformations: $M$, with root motion of a major third, and $m$, with root motion of a minor third. By default, the root motion is descending. The transformations $M^{-1}$ and $m^{-1}$ represent ascending root motion by major and minor third, respectively. The relative transformation, unlike the mediant transformations, conforms to the diatonic scale. This transformation takes on two forms: $R$, with root motion of a major third, and $r$, with root motion of a minor third. $R$ and $r$ correspond to the $L$ and $R$ transformations of Lewin’s system (Kopp 2002, 167). There are two mode-preserving transformations related by root motion of a fifth, represented by $D$ and $D^{-1}$. In order to obtain a mode-reversing fifth relation, one must apply a binary operation such as $D$ followed by $P$ (Kopp 2002, 169). This need for a binary operation is unsatisfactory for Kopp. Instead, Kopp creates a new category of fifth relations, represented by
$F$. $F$ represents a harmonic motion down by perfect fifth and reversal of mode. $F^{-1}$, on the other hand, represents a harmonic motion up by perfect fifth with a reversal of mode. Lastly, Kopp includes the slide transformation within his system. This transformation maintains the third between two triads. The result is a pair of opposite-mode triads a semitone apart (Kopp 2002, 175). Thus, there are fifteen common-tone transformations, illustrated in Example 1.11.

Mathematically speaking, there are some interesting ramifications of this chromatic transformation system. A transformational system, in the Lewinian sense, typically adheres to the axioms of mathematical group structure. A few characteristics of this system do not agree with these axioms. Before delving into these, one should take note that Kopp, in an attempt to have a simplified system of unary operations between common harmonic progressions, admits to creating a more musically economic system as opposed to rigorous mathematical system (Kopp 2002, 166). It is worth understanding at what point Kopp’s chromatic transformation system breaks from the traditional mathematical principles of Lewinian transformational systems. In particular, there are discrepancies in mappings and commutativity.

In Kopp’s chromatic system, the set of musical objects on which the transformations act are the set of twenty-four consonant triads. In a well-formed transformational system there exists a one-to-one mapping between objects—given objects $a$ and $b$, there is a unique transformation sending $a$ to $b$. In the present scenario, only triads with common-tone relationships have uniquely defined transformations. Because the set of objects in the system are the twenty-four consonant triads, there is no unique transformation sending a triad to another that has no common-tone relationship. In order to move between two triads with no common tones, one must apply a binary transformation. Furthermore, there are no uniquely defined binary operations between
Example 1.11. Kopp’s common-tone transformations

triads with no common tones. Multiple binary operations define motion between two triads with no common tones. Thus, two binary transformations send C+ to B+: \((P \cdot S)\), and \((M^{-1} \cdot D^{-1})\).

The two binary transformations leading from C+ to B- lead to the next formal issue. Transformational systems are either commutative or non-commutative. In commutative systems, the application order of transformations does not affect the outcome. The transposition operator
acting on pitch classes is an example of a commutative system. \((T_3 \ast T_3)C \rightarrow Ab\), and \((T_5 \ast T_3)C \rightarrow Ab\). In non-commutative systems, the application order of transformations does affect the outcome. The \(P, L, \) and \(R\) transformations of the neo-Riemannian system represent an example of a non-commutative system. For example, \((L \ast P)C+ \rightarrow B+,\) whereas \((P \ast L)C+ \rightarrow Ab+.\) Kopp’s system is both commutative and non-commutative. The mode-preserving transformations are commutative while the mode-reversing transformations are non-commutative. Likewise, the application of a mode-preserving transformation with a mode-reversing transformation is non-commutative. The two binary transformations sending \(C+\) to \(B+\) represent both commutative and non-commutative transformations. The first transformation \((P \ast S)\) is non-commutative. \((P \ast S)C+ \rightarrow B+,\) but \((S \ast P)C+ \rightarrow C#+.\) The second transformation, \((M^{-1} \ast D^{-1})\), is commutative. \((M^{-1} \ast D^{-1})C+ \rightarrow B+\) and \((D^{-1} \ast M^{-1})C+ \rightarrow B+\).

**Summary**

Mitchell, Proctor, Marra, and McCreless are correct in stating the existence of the “Second practice of nineteenth-century tonality.” As shown in all of these essays, there is strong evidence that the diatonic scale no longer controls the background pitch-class content of this music and that the chromatic aggregate better serves as the foundation. As profound as this insight is, the idea has all but seemed to vanish from current literature. While many of the ideas of the second practice remain implied when discussing the music of this era, an explicit formalization of the behavior and consequences of such a chromatic tonality remains incomplete. This, necessarily, demands the mechanics of a new system of analysis, one that brings together all of the disparate strands of ideas and methodologies.

All of the aforementioned theoretical inquiry provides invaluable insight into chromatic tonality. The voice-leading relationships between vertical sonorities predominates each of the
author’s notions, particularly the relationship between triads. In fact, triads and seventh chords remain the only two musical objects that the analytical and theoretical literature explores in depth. While there is an abundance of overlap in the concepts, each project presents its own unique ideas, all of which will contribute to the methodology set forth in Chapter 2 of the present project. What analysts need is a system of analysis that combines traditional tonal objects with non-functional musical entities. Instead of attempting to analyze these two aspects of the music in isolation, it is imperative that a system reconciles these differences and exploits the ways in which traditional tonal materials and non-functional materials interact with one another.

\[13\quad \text{Along with traditional neo-Riemannian analysis, which only considers triads, authors such as Childs (1998) and Strunk (2003) have expanded the concepts of neo-Riemannian theory to include seventh-chord structures.} \]
In order to bring together a multitude of theoretical ideas, an abstract language, mathematics, underpins the methodology of this project. The mathematics, on top of which musical objects are placed, allows a clear and concise way to articulate the musical notions. As Rahn (2007) describes it, mathematics is yet another tool through which musical concepts may be described. The mathematics is thus an expressive tool—any specialized language is an expressive tool (Rahn 2007, 8). In the early 1980s, David Lewin changed the landscape of the music theory community with the introduction of his mathematically based transformation theories. This field of research continues to grow yearly, seeing more conference exposure and publications. Over the past two decades, mathematical music theories have expanded beyond the scope of Lewin’s original ideas, some extending and some critiquing Lewin’s original ideas; however, group theory, a branch of abstract algebra, lies at the heart of most transformational theories. While there is a dependence on mathematics, the theory presented below produces intuitive musical analyses. Mathematics imposes a highly structured landscape where the analyst can compare and contrast musical objects and extract meaning not apparent in other analytical methodologies.

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1 The use of geometrical figures to represent musical objects and the motion from one object to another is also a growing subfield in music theory and mathematics. Dmitri Tymoczko is the leading proponent of these ideas. Tymoczko (2011) is the culmination of Tymoczko’s work in the field and claims to effectively explain the harmonic and melodic behavior of tonal music. In part, Tymoczko’s choice of geometry comes from his condemnation of algebraic-based theories. For Tymoczko, direction and distance are not inherent in algebra, which is an elemental aspect of musical relationships. On the other hand, while geometry represents direction and distance, it lacks the group structure that makes the algebra-based theories so eloquent. Below, I will attempt to reconcile these conflicts by introducing aspects of geometry into the algebraic structure.
summarizes the transformational process as, “a process by which one figure, expression, or function is converted into another that is equivalent in some important respect but is differently expressed or represented.”

Transformational theory embodies two different perspectives, but the communicative power of the theory dissolves the dichotomy between the two perspectives. In his pioneering book, *Generalized Musical Intervals and Transformations (GMIT)*, Lewin first defines the term interval in terms of a generalized interval system (GIS), and then in terms of a *simply transitive group action on a set*. The GIS perspective is Cartesian and observer oriented, while the second perspective is gestural and subject-oriented (Satyendra 2004, 100). The *simply transitive group action on a set* (STRANS) captures the idea of interval as distance, as well as interval as transformation. The transformational attitude is an active perspective, as if the listener or performer is carrying out the action of the interval, whereas the GIS perspective is passive. The remainder of this section is devoted to understanding both of these perspectives while also introducing some of the basic mathematical terminology.

In chapter 2 of GMIT, Lewin defines a GIS as an ordered triple (S, IVLS, int), where S, the *space* of the GIS, is a family of elements, IVLS, the *group of intervals* for the GIS, is a mathematical group, and int is a function mapping S × S into IVLS (Lewin 1987, 26). The GIS is subject to the following two conditions:

A. For all r, s, and t in S, int(r, s) int(s, t) = int(r, t).

B. For every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s, t) = i.

A myriad of musical objects can fill the space S of a GIS; the choice of musical objects depends on the musical context. The musical objects can be pitch classes, chords, rhythms, orchestration,
timbres, etc. The space of musical objects dictates the structure of the group of intervals. For example, if one compares a GIS occupied by the diatonic gamut of pitches classes (mod 7) with a GIS occupied by the chromatic aggregate of pitch classes (mod 12), the meaning of interval changes dramatically. In a musical space of the diatonic pitch classes, \( \text{int}(C, D) = 1, \text{int}(E, G) = 2, \) and \( \text{int}(C, G) = 4. \) A stepwise interval in the diatonic system is either a semitone or whole tone depending on its placement in the scale. In the chromatic system, a stepwise interval is always a semitone. Thus, \( \text{int}(C, D) = 2, \text{int}(C, D^\flat) = 1, \) and \( \text{int}(C, G) = 7. \)

Lewin introduces the concept of a STRANS system in Chapter 7 of GMIT. Given any element \( s \) and \( t \) of \( S \), then there exists a unique member \( OP \) of STRANS such that \( OP(s) = t \) (Lewin 1987, 157). Thus, the group of transpositions is simply transitive on the space of any GIS. A simply transitive group refers to a system composed of two items: a space denoted by “S,” and the “group” whose elements transform elements of the space (Satyendra 2004, 100). Both the space \( S \) and the group of intervals are collections of elements. The elements of the space \( S \) may be likened to nouns, whereas the elements of the group of intervals may be likened to verbs (Satyendra 2004, 101). This metaphor reflects the transformational or active attitude. It is possible to imagine the action of the verb acting on the noun, taking it from point \( a \) to point \( b \).

At their core, the GIS and STRANS perspectives, while having different philosophical angles, are identical formalizations. This means that any STRANS system has a matching GIS system and any system that is not simply transitive cannot support a GIS. Putting the formal definitions aside, the transformational approach allows musical analysis to shadow musical performance. When the analysis takes on an active perspective, it is possible to hear the music in a new and fruitful way. Just as the listener hears a performer playing a particular passage, the transformational analyst can reconstruct the same musical passage in order to understand the
musical action occurring during the performance. The following section develops a transformational system of the chromatic scale—a chromatic scale that inherits the functional characteristics of the tonal system. The goal is to glean new insight into the music of the late Romantic era, where tonal function is still active, but the chromatic scale serves as the background structure. The formal structure developed in the following section will bear the name chromatic-tonal scale (or system).

**Pitch Space and the Tonal-Chromatic Scale**

According to the theory that there exists a second practice of nineteenth-century music, tonality had evolved in such a way that the functions of diatonic tonality had been subsumed into a chromatic tonality—a tonality that has the chromatic scale as its background structure, but still retains harmonic relationships inherent in diatonic tonal music. As discussed in Chapter 1, Marra posits that there are two distinct forms of a tonal-chromatic scale, one that represents a major tonic and one that represents a minor tonic, accounting for twenty-four possible tonal-chromatic scales. I propose a refinement to this system: I posit there are only twelve tonal-chromatic scales, each representing the twelve possible tonic pitch classes and each representing both the major and minor forms of the tonic triad. Historically, mode mixture was one of the earliest and most common forms of chromaticism—major keys borrowing structures from minor keys, and vice versa. In many cases, by the late nineteenth century, the mode of a piece of music became nearly indiscernible. In that light, I find it trivial, if not cumbersome, to attempt any segregation of major and minor tonic keys when dealing with highly chromatic music in this repertoire.

Before positing a revised version of a tonal-chromatic system, it is necessary to define the pitch space in which the system resides. First, the present system will reside in discrete pitch space and will assume twelve-tone equal temperament. Discrete pitch space lives on the infinite
one-dimensional plane of continuous pitch space that represents every available frequency. Thus, discrete pitch space is also infinite, reaching the lowest and highest available frequencies of the equal-tempered system. Discrete pitch space is representable by the set of all integers (\( \mathbb{Z} \)). For the purposes of the present system, it is necessary to extract a more concise pitch space, one that recognizes enharmonic and octave equivalence. Equal temperament divides the space of an octave (2:1) into twelve equal portions. Therefore, within the octave between C4 and C5, the notes D#4 and Eb4 represent the same frequency, having no discernable difference in sound. This is true for any enharmonic spelling of any pitch class. Likewise, it is possible to evoke a notion of octave equivalence, where one can group together all pitches that have frequency relationships of 2:1, the octave equivalency class.\(^2\) Examples 2.1(a) and (b) show this space in two forms. Example 2.1(a) shows the infinite line of discrete pitch space wrapped around a vertical axis (helix) and each octave-related pitch lies on a vertical lattice. Example 2.1(b) represents the same space viewed from above, now represented as a spiral.

Example 2.1. (a) Helix representation of pitch-class quotient space; (b) spiral representation of pitch-class quotient space

\(^2\) The octave equivalency class forms one of the five primary equivalency classes in the work of Callander, Quinn, Tymozcko (2008).
Taken together, one can represent enharmonic and octave equivalent pitch space using a quotient map \( (Q_{12}) \) from the set of all integers. If one compresses the helix of Example 2.1(a) in such a way that each level of the spiral maps onto the adjacent level, the resulting figure becomes a circle, where each point represents all octave equivalent pitches. Likewise, it is possible to obtain the same geometry from the spiral of Example 2.1(b) by gluing together each row of the spiral, creating a circle where each discrete point represents all octave equivalent pitches. Each point on the circle represents one of twelve equal-tempered notes in pitch-class space (pc space) and one can assign integers \((\mathbb{Z}_{12})\) to each pitch class. This quotient map \( (Q_{12}) \) associated with the enharmonic and octave equivalency relation \( \equiv \pmod{12} \) maps any integer \( n \) (in \( \mathbb{Z} \)) to its equivalence class in \( \mathbb{Z}_{12} \) (Hook, forthcoming, 2-51).\(^3\) Following convention, 0 represents all iterations of the pitch class C, 1 represents all iteration of C#/D♭, 2 represents all iterations of D, and so on \((0 \ldots 11)\). The circle shown in Example 2.2 represents the quotient space of octave and enharmonic equivalency (the clock-face of atonal pitch-class set theory).

Example 2.2. Cyclic representation of the quotient map \((Q_{12})\) represented by the integers \((\mathbb{Z}_{12})\) of chromatic pitch-class space (pcs)

\(^3\) I would like to thank Julian Hook for sharing his in-progress monograph manuscript with me, and allowing me to cite it. Please note that the page numbers cited from this work are subject to change. For each chapter, the pagination resets. So, 2-12 means chapter 2, page 12.
Discrete pitch-class space is the arena in which the present tonal-chromatic system lives. To aid the understanding of the tonal-chromatic space, however, it is helpful to unwrap the pitch-class space created above. Thus, instead of perceiving pc space as a circle, it is possible to observe it on an infinite line while still maintaining its properties as a quotient space ($\mathbb{Z}_{12}$). Example 2.3(a) illustrates this concept. C represents the middle of the line and leftward motion moves in the flat direction while rightward motion moves in the sharp direction. Enharmonic equivalence is still intact, and the orthography of sharps and flats as opposing directions will aid in the construction of the tonal-chromatic system. Like the semitone, twelve iterations of the interval of a perfect fifth are also capable of generating the chromatic scale. It is possible to get from the chromatic space of example 2.3(a) by multiplying the pc integers by seven (mod 12). For example, $3 \times 7 = 21 = 9_{\text{mod} 12}$. Example 2.3(b) shows this chromatic pitch-class space generated by a series of perfect fifths. Like the semitone-generated space, C occupies the middle of the fifth-generated space with flat spellings protruding to the left and sharp spellings to the right.

Example 2.3. (a) Discrete pitch-class space generated by a series of semitones; (b) discrete pitch-class space generated by a series of perfect fifths
Now, having moved from infinite discrete pitch space to discrete pitch-class space, the quotient space defined by the integers mod 12 ($\mathbb{Z}_{12}$), it is fruitful to reduce this space yet another time, to a space defined by the integers mod 7 ($\mathbb{Z}_7$). Hook (forthcoming, 1-20) calls this space *generic pitch-class space* (GPC), where the integers mod 7 represent generic note names—0 represents any pitch class with the base note name of C, 1 represents any pitch class with the base note name of D, and so on (0 . . . 6). Example 2.4 takes example 2.3(b) and reduces the integer notation to mod 7, generic pitch-class space.

Using the pitch spaces developed above, one can now develop a GIS of the tonal-chromatic scale. Following Lewin’s model, the tonal-chromatic GIS is composed of the Cartesian product ($S \times S$) → IVLS, where $S$ is the space of all twelve transpositions of the chromatic gamut and IVLS is the group of all intervals of the GIS, a mathematical group. $^4, ^5$

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$^4$ A Cartesian product is a construction in which two groups are put together to form a new larger group. For example, the Cartesian product of two sets $X$ and $Y$ (notated $X \times Y$) is the set of all ordered pairs in which the first component is a member of $X$ and the second component is a member of $Y$. This definition also serves to define the direct product between two groups (for this paper, the groups corresponding to $\mathbb{Z}_7$ and $\mathbb{Z}_{12}$), which is also an operation of combining two groups to make a larger group.

$^5$ Four axioms define group structure: (1) the set must be closed under composition, meaning that any interval from IVLS added to another interval from IVLS yields an interval within IVLS; (2) the set must exhibit the associative property, meaning the order in which the operations are performed yields the same result: $a(bc) = (ab)c$; (3) the set must contain an identity element; and (4) the set must contain an inverse operation for each interval within the set.
twelve transpositions of the chromatic scale correspond to the twelve possible tonic pitch classes. This concept is elaborated further in the following section that defines the chromatic scale as a tonal entity. IVLS is isomorphic to the $\mathbb{Z}_{12} \times \mathbb{Z}_{12}$, which is isomorphic to the cyclic group $\mathbb{Z}_{144}$.\textsuperscript{6,7}

IVLS has a simply transitive action on the space $S$—one and only one interval in IVLS exists between any two transposed scale degrees.\textsuperscript{8} The present system varies slightly from that of Rings’s (2011) model. Rings’s transformational model is tethered to the diatonic scale; thus, Ring’s GIS is isomorphic to the cyclic group $\mathbb{Z}_{84}$, the twelve transpositions of the diatonic scale $\mathbb{Z}_{12} \times \mathbb{Z}_7$.

The proposed tonal-chromatic scale has a few requirements. First, like the diatonic scale, there must be seven scale degrees represented. Second, the tonic pitch class must remain as a fixed point of reference, around which all of the other pitches relate. Third, the dominant scale degree, $\hat{5}$, and subdominant scale degree, $\hat{4}$, must remain as fixed scale degrees. Fourth, all other scale degrees, $\hat{2}$, $\hat{3}$, $\hat{6}$, and $\hat{7}$, are malleable—each may be represented by either their major or minor diatonic forms. Lastly, the pitch class located a tritone from the tonic pitch class does not represent a scale degree, but rather serves as a symmetrical division of the octave, which has distinct functions from either scale degree 4 or 5. There are twelve scale-degree identities, each representing one of the twelve semitones dividing the chromatic aggregate. The twelve scale degree locations within a key are: $\hat{1}$, $\hat{-2}$, $\hat{\pm 2}$, $\hat{-3}$, $\hat{\pm 3}$, $\hat{4}$, TT, $\hat{5}$, $\hat{-6}$, $\hat{\pm 6}$, $\hat{-7}$, and $\hat{\pm 7}$. Thus, $\hat{-2}$

\textsuperscript{6} Isomorphic literally means “equal form.” Two groups are isomorphic if there exists a one-to-one bijective mapping between two groups that preserves the group operation.

\textsuperscript{7} A group is called cyclic if there is an element $a$ in $G$ such that $G = \{a^n \mid n \in \mathbb{Z}\}$. Such an element $a$ is called a generator of $G$.

\textsuperscript{8} Simple Transitivity is defined as an action on a group that is both transitive and semiregular (free). Transitive action: for any two elements $x$ and $y$ of $S$, there exists some member $g$ of $G$ such that $g(x) = y$. Semiregular (free) action: No nonidentity element in $G$ fixes an elements $x$ of $S$. Hence $G_x = 1$, for any $x$ in $S$.
represents a distance of a half step from the tonic, while $+2$ represents a whole step from the tonic; likewise, $-3$ represents a minor third from the tonic, while $+3$ represents a major third from the tonic. The enharmonic spellings corresponding to each scale-degree identity exist as adjacent notes in chromatic pc space generated by fifths. If, for example, the tonic pitch class is C (0), then the adjacent notes to the left and to the right (through the interval of a tritone) determine the spelling of the individual scale degrees (shown by the box labeled (1) in Example 2.5(a)). Then, as shown in Example 2.5(b), reorienting these pitch classes into semitone order creates a chromatic scale with the correct spellings that correspond with the scale-degree identities. The same is true for all twelve tonic pitch classes. The box labeled (2) in Example 2.5(a) shows the enharmonic pitch-class spellings for each scale degree in the key of A and these pitch classes correspond to the scale-degree identities shown in Example 2.5(c).

![Diagram of chromatic pitch-class space generated by fifths](image)

Example 2.5. (a) Linear representation of chromatic pitch-class space generated by fifths, each box represents the scale-degree collection of the keys of C (box 1) and A (box 2); (b) cyclic representation of the key of C; (c) cyclic representation of the key of A
The pitch-class space in Examples 2.5(b) and (c) will henceforth be labeled as *chromatic scale-degree space* (CSD space). Returning to Example 2.4, generic pitch-class space, one can further generalize CSD space to a space isomorphic to the integers mod 7. This space represents each scale degree in a more generalized fashion, where the components of the malleable scale degrees ($2, 3, 6,$ and $7$) are indistinguishable. In the same way example 2.4 reduces the integer notation to mod 7 by ignoring accidentals and labeling pitch classes by their letter name alone, one can reduce CSD space by labeling the tonic pitch class with the integer 1 and each adjacent pitch-class letter name with the integers 2-7. This generalized space will be called *generic scale-degree space* (GSD space). The integers 1-7 of GSD space correspond to the integers 0-6 (mod 7); the use of 1-7 correlates to the scale-degree labels of the tonal system. (It would be counterintuitive to label a scale degree with the integer 0). Example 2.6(a) recreates the linear pitch-class space of Example 2.4 with boxes surrounding the generic pitch-class space of the keys of C (box 1) and A (box 2). Examples 2.6(b) and (c) represent these tonal spaces in cyclic form. Notice that the interval of a tritone from the tonic pitch class does not appear in GSD space. This is due to the stipulation that only one pitch class can represent scale degrees $4$ and $5$. Finally, Example 2.7 represents the tonal area of C in musical notation with CSD space and GSD space labeled. Worth noting is the subtle difference between GSD space and diatonic scale-degree space. While in diatonic scale-degree space each scale degree contains one representative pitch class, GSD space has no such restriction. In GSD space, it is possible to transpose to different scale degrees without conforming to any particular diatonic collection. The sections on intervals and transposition below explore this concept in more detail.

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9 Similarly, Santa (1999) shows modular transformations that send chromatic ideas to diatonic ideas and vice verse in the music of Bartók. Here, participation in scale-degree identity is absent, particularly for the chromatic pitch space.
Example 2.6. (a) Linear representation of generic pitch-class space generated by fifths, each box represents the scale-degree collection of the keys of C (box 1) and A (box 2); (b) cyclic representation of the generic scale-degree space of C; (c) cyclic representation of the generic scale-degree space of A

GSD space: \(1 \quad \hat{2} \quad \hat{3} \quad \hat{4} \quad \hat{5} \quad \hat{6} \quad \hat{7}\)

CSD space: \(1 \quad -2 \quad +2 \quad -3 \quad +3 \quad \hat{4} \quad \text{TT} \quad \hat{5} \quad -6 \quad +6 \quad -7 \quad +7\)

PC space: 0 1 2 3 4 5 6 7 8 9 10 11

Example 2.7. Tonal-chromatic scale with PC space, CSD space, and GSD space notated
The notion that each pitch class can provide scale degree value has historical precedence. Heinrich Schenker presents a model nearly identical to the one presented above, albeit in quite a different context. Scale degrees 2, 3, 6, and 7 have two possible identities. For Schenker, combining the major and minor diatonic system, as well as including the Phrygian II, yields a system of eleven possible scale degrees, each of which can represent a simulated key. These simulated keys may extend over large passages of music, or can locally inflect the diatonic scale degrees (Schenker 1954, 298). Example 2.3 recreates the graphic showing the eleven possible scale degrees presented in Schenker (1954, 298).

\[
\begin{array}{cccccccc}
C & D-flat & D & E-flat, E & F & G & A-flat, A & B-flat, B \\
I & b\text{II} & b\text{III} & \text{III} & \text{IV} & \text{V} & \text{VI} & \text{VII} \\
\end{array}
\]

Example 2.8. Schenker, scale degrees of the tonal system when combining major and minor, as well as the Phrygian II

The tritone is absent from Schenker’s list of possible scale degrees. This is for stylistic reasons; during the classical and early romantic era, the interval of a tritone from the tonic was not a familiar part of the structural harmonic vocabulary. It was not until the latter part of the Romantic era when the tritone from the tonic gained more value. In addition, Schenker did not recognize the musical value in many composers of the late Romantic era, including Mahler. This is evident in the fact that, throughout his significant analytical career, Schenker did not publish one item on the music of Mahler. Thus, it is logical that Schenker would altogether ban the tritone from his scale-degree system.\footnote{Brown, Dempster, and Headlam 1997} A more detailed discussion of the tritone appears in the following section.

\footnote{Brown, Dempster, and Headlam 1997 posit that the tritone cannot relate directly to the tonic in Schenker’s theories. It can only arise from prolongation of a different stufen.}
ENHARMONIC RESPPELLING

In the context of any governing key area, there are circumstances when composers choose enharmonic spellings that fall outside of the present (sd, pc) system. These moments most commonly occur as foreground events written for the ease of the performer or for the appropriate spelling of an underlying harmony. Thus, for analytical purposes, the present system restricts the use of the enharmonic identity of any (sd, pc) to foreground events. These enharmonic equivalences are not permitted to participate in background or deep middleground structure. In the event that an enharmonic spelling manifests itself at the middleground or background level, it is identified by its enharmonic equivalence within the tonal-chromatic system. Many times, when this occurs, it is on the behalf of the performer and the ease of reading.11

Regarding their placement within the tonal-chromatic system, enharmonic respellings do not affect their intervallic identity in relation to the tonic pitch class. It only affects their (sd, pc) notation. Moreover, recognizing the enharmonic respelling ensures accurate understandings of interval quality on the foreground level. When a pitch class is employed that falls outside of the (sd, pc) notation of the governing tonal-chromatic scale, accidentals indicate its position relative to the tonic. For example, in the key of C, E♭ is -3; in the circumstance where D♯ is more appropriate, the scale-degree identity may be #2. One should note, however, that (D♯, #2) does not represent a structural identity in the key of C, but rather a manifestation of enharmonic respelling for the convenience of the immediate harmonic context.

11 Schubert is perhaps the most well known for his enharmonic detours. A ripe example of using an enharmonic spelling for convenience occurs in *Four Impromptus*, D. 935, No. 2. The governing key is A♭ major, the trio moves to D♭ major (IV). During the trio, the music moves to bVI, a common key area explored by Schubert; however, in this context, bVI is B♭, a difficult key for notation. To reconcile this problem, Schubert writes the passage in A major, its enharmonic equivalent. Analytically, the spelling of A major does not effect its aural quality as bVI.
Consider two sets $S$ and $S^1$, $S$ is occupied by the set of twelve pitch classes and $S^1$ is occupied by a set of twelve instruments. Example 2.9(a) shows a mapping between the two sets, where there is a one-to-one correlation. A mapping between these two sets is an example of a cross-domain mapping, when $S \rightarrow S$ where $S \neq S$. Each instrument is paired with a specific pitch class. The mapping is one-to-one because for each pitch class in set $S$, there is exactly one instrument in set $S^1$. In transformational theory, mapping is synonymous with the mathematical term \textit{function} (Satyendra 2004, 104). A function in mathematics is the relation between a set of inputs and a set of permissible outputs; for each input, there is one output. Mappings, or functions, are often noted $f(x) = y$. When one applies the function $f$ to the object $x$, $y$ is the output.

Lewin reserves the terms transformation and operation for mappings between like sets. Therefore, the example above would not be, in the Lewinian sense of the term, a transformation or operation. Early in GMIT, Lewin defines the subtle difference between transformations and operations with the following definition: \textit{A function from a family $S$ into $S$ itself will be called a transformation on $S$. If the function is 1-to-1 and onto, it will be called and operation on $S$} (Lewin 1987, 3). Two brief examples will demonstrate these concepts. First, let $S$ be the space of twelve pitch classes. The input of the mapping will be the twelve pitch classes. The output will be a C major triad (Example 2.9(b)). In this mapping, each of the twelve pitch classes travels to the closest pitch-class member of a C major triad. This is an example of a transformation because both the input and output are like sets (pitch-class space). It is not an operation because the output includes elements that appear more than once and some that are not included at all. Example 2.9(c) shows a transformation that is one-to-one and onto and meets the requirements.
for an operation. Here, the input is pitch-class space, and the function is the transposition operator that acts equally on each member of the set. Like the input, the output contains a representative from each pitch-class member.

<table>
<thead>
<tr>
<th>(a) f: S → S¹</th>
<th>(b) Res(C⁺)</th>
<th>(c) T₇</th>
</tr>
</thead>
<tbody>
<tr>
<td>C → Flute</td>
<td>C → C</td>
<td>C → G</td>
</tr>
<tr>
<td>C# → Clarinet</td>
<td>C# → C</td>
<td>C# → G#</td>
</tr>
<tr>
<td>D → Oboe</td>
<td>D → C or E</td>
<td>D → A</td>
</tr>
<tr>
<td>D# → Basson</td>
<td>D# → E</td>
<td>D# → A#</td>
</tr>
<tr>
<td>E → Horn</td>
<td>E → E</td>
<td>E → B</td>
</tr>
<tr>
<td>F → Trumpet</td>
<td>F → E</td>
<td>F → C</td>
</tr>
<tr>
<td>F# → Trombone</td>
<td>F# → G</td>
<td>F# → C#</td>
</tr>
<tr>
<td>G → Tuba</td>
<td>G → G</td>
<td>G → D</td>
</tr>
<tr>
<td>G# → Violin</td>
<td>G# → G</td>
<td>G# → D#</td>
</tr>
<tr>
<td>A → Viola</td>
<td>A → G</td>
<td>A → E</td>
</tr>
<tr>
<td>A# → Cello</td>
<td>A# → C</td>
<td>A# → F</td>
</tr>
<tr>
<td>B → Bass</td>
<td>B → C</td>
<td>B → F#</td>
</tr>
</tbody>
</table>

Example 2.9. Mapping tables; (a) cross-domain mapping between pitch-class space and a space of twelve instruments; (b) many-to-one mapping between pitch-class space and a C major triad; (c) one-to-one mapping of pitch-class space into pitch-class space

**INTERVALS**

Within the expression int(s, t) = i, s and t may represent a variety of musical objects, including pitches, pitch classes, and sets of pitch classes including triads, seventh chords, and other scalar forms. The interval in a GIS statement is the relationship between s and t. Thus, the primary objective of such a system is to illuminate the relationships between musical structures in order to extract deeper insights into the musical construction. Under the prescribed GIS, the most elemental intervallic relationship is that between two pitch classes, and through this, all other types of intervals obtain their definition. An interval between two pitch classes does not
imply only one type of measurement, such as distance, but rather a multitude of concepts. The idea of interval is better understood as the relationship between two pitch classes. It is this relationship and connection between two tones that is the basic unit of musical construction (Hindemith 1942, 57). Unlike intervals in atonal music, intervals in tonal music demand labels that determine their functional role (Straus 2005, 6). Therefore, it does not suffice in the present context to merely label intervals by the number of semitones between them—that is only one aspect of their relationship. In keeping with the notions introduced above, intervals in the tonal-chromatic system have a two-part identity: (1) the traditional semitonal measurement, and (2) the measurement between scale-degree identities.

In Forte’s atonal system, an interval between two pitch classes (a and b) is defined as the absolute positive value of the difference of a and b (|a – b|) (Forte 1973, 14). Based upon this equation, it is impossible to obtain a negative interval (I will return to this point later). As a result, the highest interval value is six, and as Forte defines it, the integers 0 through 6 represent the seven interval classes. In practice, there are only six interval classes—interval class 0 is a redundancy representing the unison interval. Inverse-related modulo12 intervals are equivalent. This definition, however, does not suffice in the context of the tonal-chromatic scale—interval values larger than 0-6 are a fundamental aspect of tonal theory (i.e., an augmented sixth-chord is impossible to construct under this definition). Another deficiency in this definition is the lack of direction communicated. In the present system, positive and negative interval values indicate ascending and descending motion, respectively. Without negative interval values, it would seem as if all interval motion is ascending.

Example 2.10(a) shows a portion of PC space (octave and enharmonic equivalent) on the infinite line. On this line, it is possible to move between pitch classes in two directions. Example
2.10(b) shows an example of this. In the example, the motion from point $a$ to point $b$ is expressed as a gesture ($z$) moving from left to right; the motion from point $c$ to point $b$ is expressed as a gesture ($y$) moving from right to left. Example 2.10(c) removes the musical connotations of the previous examples; now the set of all integers ($\mathbb{Z}$) represents the discrete points of the line (with C notated as 0). It is clear in Example 2.5c that $z = 8$ and $y = -4$. When visualized on a line, moving either leftward or rightward is not difficult to conceptualize.

![Example 2.10](image)

Example 2.10. (a) Linear representation of the chromatic universe; (b) interval motion on the infinite chromatic line; (c) interval motion represented as integer values on the infinite chromatic line.

In the present system, under the prescribed quotient spaces, it is desirable to keep intact a notion of direction when speaking of interval. Therefore, unlike Forte’s conception of interval in atonal music, where the absolute value mod 12 is the measurement, the interval between two pitch classes in the present GIS may take on either a positive or negative value around the cyclic group $\mathbb{Z}_{12}$. As Hook (forthcoming, 1-17) describes, “The terms ascending and descending, however, may be used without ambiguity in describing the two directions of motion possible in
pitch-class space . . . we can trace either an ascending or a descending path from G to D.” The distance between two pitch classes is further refined when considering both CSD space and GSD space. In CSD space, the ordered pair (CSDint, PCint) represents an interval. Example 2.11 is a table of the twenty-four possible intervals, twelve ascending intervals and their inverses, twelve descending intervals. Also shown in the example are the GSD intervals that occupy a quotient space of CSD space. The motion between cardinal scale-degree identities defines a GSD interval—for example, any representative of $\hat{2}$ moving to a representative of $\hat{3}$ is an interval of a second. In GSD space, ordered pair notation is not necessary; thus, the singular notation 2nd, 3rd, 4th, etc., denotes GSD intervals.

<table>
<thead>
<tr>
<th>$\mathbb{Z}$</th>
<th>(CSDint, PCint)</th>
<th>Inverse (CSDint, PCint)</th>
<th>GSDint</th>
<th>Inverse GSDint</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(e, e)</td>
<td>(e, e)</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>1</td>
<td>(-2nd, 1)</td>
<td>(+7th, 11)</td>
<td>2nd</td>
<td>7th</td>
</tr>
<tr>
<td>2</td>
<td>(+2nd, 2)</td>
<td>(-7th, 10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(-3rd, 3)</td>
<td>(+6th, 9)</td>
<td>3rd</td>
<td>6th</td>
</tr>
<tr>
<td>4</td>
<td>(+3rd, 4)</td>
<td>(-6th, 8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(4th, 5)</td>
<td>(5th, 7)</td>
<td>4th</td>
<td>5th</td>
</tr>
<tr>
<td>6</td>
<td>(TT, 6)</td>
<td>(TT, 6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(5th, 7)</td>
<td>(4th, 5)</td>
<td>5th</td>
<td>4th</td>
</tr>
<tr>
<td>8</td>
<td>(-6th, 8)</td>
<td>(+3rd, 4)</td>
<td>6th</td>
<td>3rd</td>
</tr>
<tr>
<td>9</td>
<td>(+6th, 9)</td>
<td>(-3rd, 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(-7th, 10)</td>
<td>(+2nd, 2)</td>
<td>7th</td>
<td>2nd</td>
</tr>
<tr>
<td>11</td>
<td>(+7th, 11)</td>
<td>(-2nd, 1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 2.11. Table of GIS intervals in CSD space and GSD space
The following example, Example 2.12, is a brief interval analysis of the first eight-measure phrase, m. 2-9, of the first violin from the fourth movement, Adagietto, of Mahler’s fifth symphony. The phrase begins with the main motive of the movement, an ascending perfect fourth filled in with GSD scale steps in the key of F major, shown in Example 2.12(b). In mm. 4-5, the melody traverses a varied version (transposed) of the ascending forth motive, terminating on C5 (Example 2.12(c)). The melodic profile reverses in the second half of the phrase; starting on the C5 in m. 5, the phrase descends and terminates on G4 in m. 9, outlining a descending perfect fourth (Example 2.12(d)). The distinction in direction between example 2.12(c) and 2.12(d) is crucial. The atonal interval model is unsatisfactory—both intervals would simply be (4th, 5). Mathematically, though, it is desirable to keep direction intact. Therefore, instead of conceiving of an interval as the shortest distance between two points on the circle, one determines the direction around the circle, rightward or leftward, based upon the ascending or descending motion in the music, respectively.\(^\text{12}\) In this example, there is no discernable difference between the way CSD space and GSD space represent the intervals. This is because the melody outlines perfect intervals. If, however, one looks at the interval between the target notes of motive \(x\) and motive \(z\) (connected by the dotted line in Example 2.12), then there is a choice between CSD space and GSD space. In CSD scale, the interval between F and G is (+2nd, 2); in GSD space, the interval between F and G is merely (2nd, 2). The difference is subtle, and the musical context helps determine which interval space is most appropriate. The majority of this brief passage is diatonic in the key of F major. Thus, GSD space adequately expresses the interval information. The choice might be different, if say, one analyzes the entire movement.

\(^\text{12}\) In order to maintain direction on the cyclic group, one must maintain aspects of the linear structure of the chromatic gamut. On the infinite line, motion in either direction is a natural process. Once wrapped into a quotient space, in our case, the cyclic group mod 12, direction becomes lost.
Example 2.12. (a) Gustav Mahler, Symphony No. 5, Adagietto, mm. 2-9, first violin; (b-d) interval qualities of each significant melodic motive of the first eight measure melody; (e) linear representation of intervals x, y, and z; (f) cyclic representation of intervals x, y, and z
A second example, which further highlights the importance of interval direction, comes from the first song of Mahler’s *Lieder eines fahrenden Gesellen*, “Wenn mein Schatz Hochzeit mach.” Spanning mm. 5-17, the first vocal phrase pivots about an A4 fulcrum. There are three significant intervals highlighted from the excerpt and shown in Examples 2.13(a), (b), and (c): \(x\), a descending major third; \(y\), a descending minor third; and \(z\), an ascending minor sixth. In the music, each interval is mirrored by its inverse-related interval, labeled \(x'\), \(y'\), and \(z'\), respectively. The objective is to relate each subsequent interval to the original interval, \(x\). The second interval, \(y\), a descending minor third from B♭ → G, is a transposition in GSD space of the first interval (the mechanics of transposition are discussed below).

The relationship between interval \(x\) and interval \(z\) is more relevant to the current discussion. In terms of interval class, these two intervals are of equal distance, the shortest distance between both points is 4 semitones in PC space. This, however, does not suffice—the distinction between a descending major third and an ascending minor sixth is imperative. Without the distinction, it is not possible to highlight the intervallic symmetry of the first two measures and the last four measures of the phrase. This is not symmetry in the common sense of the term. The intervals surrounding the fulcrum point (A4) are not reflections in a mirror; instead, the symmetry is defined by the inverse-related intervals branching out from the fulcrum. In this example, like the previous example, the choice between CSD space and GSD space depends on context. Here, using both spaces to describe different aspects of the music is valuable. One must use CSD space to make a precise distinction between intervals \(x\) and \(y\); while, as noted above, the transposition level between motives \(x\) and \(y\) is best described in terms of GSD space.
Example 2.13. (a) Gustav Mahler, *Lieder eines fahrenden Gesellen*, no. 1, “Wenn mein Schatz Hochzeit mach,” first vocal phrase, mm. 5-17; (b-d) qualities of three significant intervals; (e) linear representation of intervals $x, y,$ and $z$; (f) cyclic representation of intervals $x, y,$ and $z$
Thus far, each example of intervallic motion involved a change in value to both the scale-degree and pitch-class placeholders of the ordered pair. Another category of interval motion keeps fixed one side of the ordered pair—the \( \text{PCint} \) remains constant while the \( \text{CSDint/GSDint} \) changes. Following Rings (2011, 58), this type of transformation is termed a *pivot interval*—an interval that occurs during modulatory passages where a pitch class will assume a new scale-degree identity in a new key. There are two manifestations of this transformation: either the transfer of the scale-degree identity occurs as a common tone between two sonorities, or the scale degree mutation occurs during the duration of a single harmony, during which the role of the scale degree changes. The passage shown in Example 2.14 demonstrates a clear example of a common-tone pivot interval. The excerpt is taken from the second song of Mahler’s song cycle, *Kindertotenlieder*, “Nun seh’ ich wohl, warum so dunkle Flammen,” mm 38-41. C as tonic dominates the majority of this song; however, in the middle of the song, there is a brief passage where Mahler reorients the perception of tonic toward D. Locally, it is possible to hear a modulation to tonic D; in the large-scale perspective, this brief passage has a clear (neighboring) role in the global key of C major.

The four-measure excerpt of Example 2.14 begins in m. 38 of the song. The preceding few measures (before the example begins) situates C as tonic in a densely chromatic harmonic texture. Beginning with the pickup in m. 38, m. 39 sounds a B\( \flat \) dominant seventh chord \((7)\) in the key of C. Historically, this is a relatively common chord in the key of C minor (pointing towards the relative major of E\( \flat \)). However, in the present case, instead of resolving authentically as a dominant seventh chord, \( V \rightarrow I \) in E\( \flat \), or deceptively, \( V \rightarrow \text{vi} \), which would reinforce a tonic of C, it resolves via a common-tone relationship to a new tonic of D. The shift in scale-degree orientation of D is the hinge upon which the entire harmonic progression swings. Other aspects
of efficient voice leading occur in the passage, but for now, the most relevant aspect is the transformation of the common tone’s identity. In the preceding measures, D is functioning as \((+\hat{2}, 2)\); however, when crossing over from m. 40 to m. 41, D becomes \((\hat{1}, 2)\). The intervallic transformation that maps the first CSD identity to the second is \((+2\text{nd}, e)^{-1}\), the pitch class of D changes CSD identity by a descending major second in CSD space.

Example 2.14. Gustav Mahler, Kindertotenlieder, no. 2, “Nun seh’ ich wohl, warum so dunkle Flammen,” mm. 38-41, score reduction; pivot transformation in modulation from tonic C to D

Example 2.15(a) shows an example of the second manifestation of a pivot interval, wherein a pitch assumes a new scale-degree role within the duration of a single harmonic entity. The excerpt comes from Mahler’s song, “Ich atmet’ einen linden Duft,” the second song from his Five Rückert Lieder, mm. 23-25. The prevailing key of the song is F major; however, during a brief passage in the middle of the song, the tonal center shifts up by a half step to Gb major. The shift occurs within one measure. Unlike the previous example, where the common-tone thread occurred in the top register of the example, the shift in scale-degree function occurs in the bass.
Root position tonic harmony occupies measure 23. Still perceiving the key as F, the bass line of m. 24 moves to D, +\( \hat{6} \). During the course of this measure, though, D transforms its role from +\( \hat{6} \) in the key of F to -\( \hat{6} \) in the key of G\( ^b \)—a pivot transformation of (\( \hat{\text{-I}} \), e)\(^{-1} \). In the key of G\( ^b \), the normative spelling for -\( \hat{6} \) is E\( ^b b \); in this case, though, Mahler notates -\( \hat{6} \) as its enharmonic equivalent D\( ^b \), most likely for convenience of reading.

Example 2.15(b) shows the bass line function of this passage in a graphic network. Each node of the graph represents a bass note and connecting each node are arrows representing the transformational intervallic motion between pitch classes. The bottom of the network highlights a second type of pivot transformation. In any modulating situation, the pivot transformation of the form (SD(change), e), there is an equal and opposite ordered pair that expresses a pivot transformation of the form (e, PC(change)). In Example 2.15, the pivot transformation (\(-2^{\text{nd}}, e\)\(^{-1} \), has a parallel transformation expressing the change in tonic via the notation (e, 1). These two ordered pairs are involutions, which will be the case with any pivot transformation, and is the reason it would be redundant to create two categories of pivot transformations. The network of Example 2.15(b) is what scholars call an oriented network, a network arranged so as to reflect the musical surface.\(^{13} \) All networks in this project will be oriented to reflect the order of musical events in time from left to right. Thus, objects on the left side of networks occur prior to those events situated to the right in the networks.

\(^{13} \) A network is a collection of nodes and arrows; nodes are a family of objects and the arrows are a subfamily of objects that connect the nodes. Oriented networks populate all of the network types in this project. Oriented networks are situated on the page such that the temporal time-line of the musical surface is reflected from left-to-right in the network.
In the present system, a set is a finite unordered subset of objects from the space $S$ of the tonal-chromatic GIS (Lewin 1982, 88). Sets can range in size from $n = 1$ to $n = 12$. Of course, Allen Forte compiled an exhaustive list of sets constructed of pitch classes for the purposes of atonal analysis. In atonal set theory, however, tonal harmonic function is not a relevant matter and many sets would have no meaning in the context of tonal. The number of meaningful sets in even the most chromatic tonal music is restricted. This is not to say that there are no unorthodox

Example 2.15. (a) Gustav Mahler, *Five Rückert Lieder*, no. 2, “Ich atmet’ einen linden Duft,” mm. 23-25, score reduction; (b) transformational network of mm. 23-25 showing the modulation from F to G$b$ via a pivot transformation using D (E$b$)

**Sets**

In the present system, a set is a finite unordered subset of objects from the space $S$ of the tonal-chromatic GIS (Lewin 1982, 88). Sets can range in size from $n = 1$ to $n = 12$. Of course, Allen Forte compiled an exhaustive list of sets constructed of pitch classes for the purposes of atonal analysis. In atonal set theory, however, tonal harmonic function is not a relevant matter and many sets would have no meaning in the context of tonal. The number of meaningful sets in even the most chromatic tonal music is restricted. This is not to say that there are no unorthodox
harmonic or scalar constructions, but these are found typically in highly chromatic passages and demand extra-tonal explanation.

It is logical to demonstrate the construction of sets with the most frequent and meaningful musical objects that occur in tonal music, triads and seventh chords. Sets are collections of pitch classes notated in brackets using the (scale degree, pitch class) notational system. The use of this notation allows one to situate any type of set within the context of a key (Rings 2011, 55). Examples 2.16(a) and (b) show two major triads that are of the same pitch-class content, but reside in a different tonal context. 2.16(a) is a D major triad that functions as V in the key of G; 2.16(b) is a D major triad that functions as bII in the key of C#. Example 2.14(c-e) shows three dominant seventh chords that are of the same pitch-class content, but occupy different functional roles in different tonal contexts. Each chord has the pitch-class content of a dominant seventh chord built on G. The scale-degree content, however, tells a different story in each scenario. The chord in Example 2.16(c) is the most standard occupation for this collection—a root position dominant seventh chord on scale-degree 5 in the key of C. On the other hand, the scale-degree locations of the chord in Examples 2.16(d) and (e) have different, less orthodox resolutions. For the sake of space, it is suitable to use the common shorthand notation for triads and seventh chords. In keeping with the present system, + and − represent major and minor triads, respectively. A superscript circle denotes a diminished triad and a superscript A denotes an augmented triad.¹⁴ There is a variety of ways to represent seventh chords. Here, common Roman

¹⁴ There are various systems used to notate triads and seventh chords. Even within publications by a single author, variations exist in the notation. For example, Cohn (1996; 1997; 1999, 2011) uses + and − symbols to denote major and minor; however, in Cohn (1998) capital letters denote major triads and lower-case letters denote minor triads. In Tymoczko (2011), the same system of capital and lower case letters and the + sign is reserved for augmented triads. Using + and − signs, Hyer (1995) and Hook (2002) have a slight variation. Instead of writing C+
numeral analytical symbols are used. A superscript 7 represents a dominant seventh chord; a superscript maj7 represents a major seventh chord; a superscript ø represents half diminished seventh chords; and a superscript 0 represents a fully diminished seventh chord.

\[
\begin{align*}
\text{(a)} & : [(+\flat, A), (+\natural, F\#), (5, D)] \quad \text{V in G} \\
\text{(b)} & : [(+\flat, A), (+\natural, F\#), (+\natural, D)] \quad b\text{II in C}\#
\end{align*}
\]

\[
\begin{align*}
\text{(c)} & : [(4, F), (+\flat, D), (+\natural, B), (5, G)] \quad \text{V}^7\text{in G} \\
\text{(d)} & : [(TT, F), (\natural, D), (1, B), (-6, G)] \quad G^r+6\text{in B} \\
\text{(e)} & : [(+\natural, F), (-6, D), (4, B), (-2, G)] \quad b\text{II}^7\text{in F}\#
\end{align*}
\]

Example 2.16. (a) and (b) Two distinct pitch-class sets involving a D major triad; (c-e) three distinct pitch-class sets involving a G dominant seventh chord

The representation of larger sets, such as extended chords, diatonic scales, and other non-diatonic scalar collections is also possible in the tonal-chromatic GIS. A common subset of the chromatic gamut is the diatonic collection. For historical and structural reasons, the diatonic collection has and will remain a central component of any style of tonal music. This includes all of the permutations of the church modes:

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15. The historical use of the diatonic scale has a significant influence on its continued use into the twentieth and twenty-first century; however, more important are the unique features of the diatonic scale, which set it apart from most other scales. For further details on the structural integrity of the diatonic scale see the following: (Clough 1979; Browne 1981; Carey and
(1) C Ionian: \{(\hat{1}, C), (+\hat{2}, D), (+\hat{3}, E), (\hat{4}, F), (\hat{5}, G), (+\hat{6}, A), (+\hat{7}, B)\}
(2) C Dorian: \{(\hat{1}, C), (+\hat{2}, D), (-\hat{3}, E\flat), (\hat{4}, F), (\hat{5}, G), (+\hat{6}, A), (-\hat{7}, B\flat)\}
(3) C Phrygian: \{(\hat{1}, C), (-\hat{2}, D\flat), (-\hat{3}, E\flat), (\hat{4}, F), (\hat{5}, G), (-\hat{6}, A\flat), (-\hat{7}, B\flat)\}
(4) C Lydian \{(\hat{1}, C), (+\hat{2}, D), (+\hat{3}, E), (TT, F\#), (\hat{5}, G), (+\hat{6}, A), (+\hat{7}, B)\}
(5) C Mixolydian \{(\hat{1}, C), (+\hat{2}, D), (+\hat{3}, E), (\hat{4}, F), (\hat{5}, G), (+\hat{6}, A), (-\hat{7}, B\flat)\}
(6) C Aeolian \{(\hat{1}, C), (+\hat{2}, D), (-\hat{3}, E\flat), (\hat{4}, F), (\hat{5}, G), (-\hat{6}, A\flat), (-\hat{7}, B\flat)\}
(7) C Locrian \{(\hat{1}, C), (-\hat{2}, D\flat), (-\hat{3}, E\flat), (\hat{4}, F), (TT, G\flat\#), (-\hat{6}, A\flat), (-\hat{7}, B\flat)\}

In chromatic music, the diatonic collection enforces a tonal center; hence, a shift in diatonic collection usually shifts the listener’s tonal orientation. All diatonic collections live in GSD space, but the existence of GSD space does not necessarily imply diatonicism.

**Pivot Intervals and Sets**

The pivot intervals discussed earlier also have a significant effect on sets, in particular triads and seventh chords. In the same way that a pivot interval transforms the function of a pitch class, pivot intervals can send a set of pitch classes onto a new functional role. Because the pitch classes do not change through the transformation, the change of scale-degree identity will act equally on each member of the set, in the same direction. If, for example, a hypothetical C+ triad initially serves as the V chord in the key of F but in the course of a given musical passage obtains a new functional status, such as the tonic chord in the key of C, then it is possible to assign a pivot transformation that acts equally on each component of the set. The language of this pivot transformation is identical to that of its single pitch class counterpart. In this case, the transformation would be (4th, e). The transformation (4th, e) acts on each pitch class of the set equally; thus, (\hat{5}, C) \rightarrow (\hat{1}, C), (+\hat{7}, E) \rightarrow (+\hat{3}, E), (+\hat{2}, G) \rightarrow (\hat{5}, G). As Rings (2011) notes, these types of pivot intervals are ubiquitous in tonal music and are examined rarely in much detail. Nonetheless, pivot intervals provide a transformational perspective of common musical features. Pivot intervals shed light on other types of pc collections in CSD space and GSD space. In many

cases, motivic ideas not based on triads and seventh chords undergo the same type of transformation, changing identity between two key areas and within the same key area. The former involves a pivot transformation and the latter involves transposition.16

**Transposition**

Two operations act on the tonal-chromatic system: transposition and inversion. These, however, do not act the same as the operations from atonal pitch-class set theory; the internal scale-degree identities bring an added element of structure. It is possible, using the previously established notational convention, for one to keep track of chromatic transposition and inversion, as well as transposition and inversion of generic scale degrees. The (sd, pc) notation of intervals will prove fruitful as the nomenclature for the transposition of sets. In tonal music, inversion occurs less frequently, and this is evident in the abundance of transposition and minimal use of inversion that occurs in the analyses in the coming analytical chapters.

By definition, to transpose is to transfer to a different place or context.17 In a musical context, this definition needs refining. Not only is transposition transference from one place to another or one context to another, each constituent part of the object under the operation must move in the same direction by the same amount. Throughout the existence of tonal music, as well as post-tonal music, transposition has been a significant mechanism for musical elaboration and development. In the tonal-chromatic system, transposition may act on both CSD space and

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16 Steven Rings (2011, 96) provides an insightful motivic analysis of the first 33 measures of *Das Lied von der Erde* by Gustav Mahler. In this analysis, Rings shows the transformation of a single motivic idea across the musical surface via transposition and chromatic alteration.

17 Mathematical translation is akin to musical transposition. In mathematics, a translation is a slide maneuver where an object moves to a new context without losing any of its defining characteristics.
GSD space; depending on the musical context, one of the two scale-degree spaces will reveal more insightful features of the musical surface.

Unlike in diatonically based tonal music, chromatic transposition is a notable feature of tonal music during the late nineteenth century. Like the transposition operator of atonal set-class theory, the transposition operator acting on CSD space moves each constituent part of a set by the same number of semitones in the same direction, and is isomorphic in structure. The two different types of transposition in the tonal-chromatic system have slightly different notation, in order to identify which of the two spaces the operator acts on. Thus, $T_c$ denotes transposition in CSD space and $T_g$ denotes transposition in GSD space.

A simple example will demonstrate how the chromatic transposition operator acts in CSD space. Example 2.17 shows the opening four-measure phrase from Mahler, *Kindertotenlieder* no. 1, “Nun will die Sonn’ so hell aufgehn!.” The overall objective of the phrase is to establish and prolong the tonal center, albeit in an evasive way. Following the pickup measure, the downbeat of measure one presents two notes of the tonic triad (root and third), in first inversion. The phrase terminates in measure four with the same two notes, the root and third of a tonic triad, now in root position; thus, a voice exchange governs the phrase. All of the material in between these two points serves as a vehicle to move from point A to point B. The pitch material of measure two is difficult to situate comfortably into the key of D (major or minor) if one tries to do so analyzing from a diatonic perspective. An analysis in CSD space, though, clarifies matters.

The score in Example 2.17(a) annotates and highlights the ideas discussed above and those discussed below. The first measure of the phrase (m. 1) presents a small motivic cell that is immediately reiterated in m. 2. The cell’s second appearance, though, is at a new pitch level, two semitones below the original statement. In this case, it is a direct transposition in CSD space, each
pitch from the first measure’s statement of the motive moves in the same direction by the same amount (two semitones, or two notches in in CSD space). Example 2.17(b) shows each measure’s collection of pitches as sets, between the two sets an arrow, labeled $T_c2^{-1}$, indicates that the notes from set one are related to the notes of set two by transposition of two descending semitones.

(a)

![Graphical representation of transposition](image)

Langsam und schwermütig, nicht schleppend

Example 2.17. (a) Gustav Mahler, *Kindertotenlieder*, no. 1, mm. 1-5, score reduction; (b) analysis of sets from mm. 1-2

There are times in even the most chromatic tonal music when the musical surface closely resembles the diatonic scale. In some cases, it is a strict adherence to a local diatonic collection; in other cases, it is a close relative to the diatonic collection. In these scenarios, a different type of transposition is best equipped for understanding the relationship between sets: transposition in GSD space. Transposition in GSD space, notated as $T_{g,n}$, denotes a shift of ordinal scale-degree.
location. For example, the transposition operator $T_g2$ acting on the hypothetical set in GSD space, \{\hat{1}, \hat{5}\} would yield the set \{\hat{2}, \hat{6}\}. This type of transposition does not keep track of semitonal motion, as in CSD space; instead, \{\hat{1}, \hat{5}\} moves to any representative of \{\hat{2}, \hat{6}\}.

Example 2.18 demonstrates transposition in GSD space. The three excerpts come from Mahler, *Lieder eines fahrenden Gasellen*, no. 4, “Die zwei blauen Augen”. Throughout the song, a simple motive connects each stanza and each harmonic area. The listener first experiences the motive, \{(\hat{1}, E), (+\hat{2}, F#), (-\hat{3}, G)\}, supported by E minor (Example 2.18(a)). In measure 5, the motive reappears in a new harmonic context—supported by G major, and is transposed in GSD space. As the line connecting Example 2.18(a) to (b) shows, the transposition is by $T_g3$, each note of the original motive moves up three notches in GSD space. Thus, $\hat{1} \rightarrow \hat{3}$, $\hat{2} \rightarrow \hat{4}$, and $\hat{3} \rightarrow \hat{5}$. Transposing in CSD space yields a false outcome. If counting the semitonal motion between each transposed scale degree, the overall transposition in CSD space is uneven: $(\hat{1}, E) \rightarrow (-\hat{3}, G)$ is three semitones, while $(+\hat{2}, F#) \rightarrow (\hat{4}, A)$ and $(-\hat{3}, G) \rightarrow (\hat{5}, B)$ are both four semitones. The motion between Example 2.18(b) and 2.18(c) brings up an interesting point regarding the choice between transposition in CSD space and transposition GSD space. One can connect the motive of Example 2.18(b), \{(-\hat{3}, G), (\hat{4}, A), (\hat{5}, B)\}, to the motive of 2.18(c), \{(-\hat{6}, C), (-\hat{7}, D), (\hat{1}, E)\}, by transposition in either CSD space or GSD space. The choice is clear, though, from the context of the music. The first transposition of the motive appeared in a context nearly entirely diatonic in construction; thus, it is intuitive to continue using the same type of transformation, $T_g5^{-1}$. It would only become necessary to switch scale-degree spaces if the musical context dictates it.\(^\text{18}\)

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\(^{18}\) In a chromatic musical environment, the transformation $T_c7^{-1}$ also takes the motive of Example 2.16(b) to 2.16(c).
Example 2.18. Score reductions: (a) Gustav Mahler, *Lieder eines fahrenden Gesellen*, no. 4, “Die zwei blauen Augen,” mm. 1-2; (b) mm. 5-6; (c) mm. 18-21; arrows indicate the level of transposition (in GSD space) between each presentation of the motive.
INVERSION

Like transposition, inversion acts on both CSD space and GSD space. Following Lewin’s notation, the notation $I_u^v$ identifies all inversional relationships. This notation indicates that the GIS elements $v$ and $u$ map onto one another and all other GIS elements invert with respect to $v$ and $u$.\footnote{Lewin (1987, 50-51) defines the formal properties of this notational system.} In CSD space, $I_{(5,D)}^{(1,G)}$ exchanges $(\hat{1}, G)$ with $(\hat{5}, D)$; the remaining chromatic scale degrees invert as follows:

$$
\begin{align*}
(\hat{1}, G) & \leftrightarrow (\hat{5}, D) \\
(-\hat{2}, A_b) & \leftrightarrow (TT, D_b) \\
(+\hat{2}, A) & \leftrightarrow (\hat{4}, C) \\
(-\hat{3}, B_b) & \leftrightarrow (+\hat{3}, B) \\
(-\hat{6}, E_b) & \leftrightarrow (+\hat{7}, F\#) \\
(+\hat{6}, E) & \leftrightarrow (-\hat{7}, F)
\end{align*}
$$

Those familiar with the inversion operation from twelve-tone theories of analysis will notice that this form of inversion is formally equivalent—it is merely another notation of the $I_n$ and $T_nI$ systems of notation. In the scenario above, one can notate this inversion as each of the pair in the list. For example, $I_{(5,D)}^{(1,G)}$ can also be written as $I_{(5,D)}^{(-3,B_b)}$ or as $I_{(+\hat{6}, E_b)}^{(-\hat{6}, E_b)}$.

There is only a slight modification of the notation for inversion in GSD space. The ordered pair notation is not a necessary component of inversion in GSD space; instead, only the scale degrees (with no appended pitch classes) are listed in the invensional exchange. Using the same starting point as the example above, the notation $I_{\hat{5}}^\hat{1}$ exchanges generic scale degree $\hat{1}$ with generic scale degree $\hat{5}$. The remaining generic scale degrees invert as follows:

$$
\begin{align*}
\hat{1} & \leftrightarrow \hat{5} \\
\hat{2} & \leftrightarrow \hat{4} \\
\hat{3} & \leftrightarrow \hat{3} \\
\hat{6} & \leftrightarrow \hat{7}
\end{align*}
$$
Again, each point of inversion has multiple notations. In the case above, where the generic scale degrees 1 and 5 are exchanged, the notation of this inversion may take the form of $I^5_1$ and $I^3_3$. As Rings (2011, 89) points out, due to the odd cardinality of generic scale-degree space, a single scale degree will always act as a point of inversion. In the above example, $3$ inverts to itself.

The inversion operator, compared with the transposition operator, is not as prolific in music driven by tonality. While, it may not be at the forefront of many analyses, it does occur in the music enough (see Chapter 3) to merit a discussion. Before moving forward, a brief example will suffice to demonstrate the inversion operation in action. Examples 2.19(a), (b), and (c) are three short passages from the first movement of Mahler’s first symphony. In measures 3 and 5 of the slow introduction, Mahler exposes the listener to a small motivic idea of a descending fourth (x) in the key of D (heavily influenced by minor). Then, as shown in Example 2.19(a), Mahler presents the motivic idea in a sequential manner—after the initial descending fourth, the listener hears another descending fourth, beginning a step up in GSD space from the previous note of the initial motive. Taken together, a second motive (y) is present, a descending fourth followed by a half step (y) or a whole step ($y'$). This three-note motive re-enters the musical landscape during the transitional material leading to the secondary thematic area of the exposition.

Example 2.19(b) shows the first reappearance y at the onset of the TR zone. Far removed from its original context, the motive is presented at a faster tempo and in a harmonic area highly influenced by the major mode; the latter makes it more advantageous to understand the transformational structure of the motives in GSD space. Finally, Example 2.19(c) shows the final measures of the transitional material. The harmonic area is now pressing towards a tonic of A. The first measure of 2.19(c) presents the same pitch-class content of 2.19(b), now in a new

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20 TR zone is an abbreviation for transition zone. In this project, Hepokoski and Darcy (2006) provide the abbreviations and terms for all formal design patterns relating to sonata form.
tonal context—the goal of the phrase in 2.19(c) is to land on a tonic of A. In this passage, the listener hears two forms of the motive played in alternation, one form of the motive in m. 151 followed by another form of the motive in m. 152, and so on.

Example 2.19. (a) Gustav Mahler, Symphony No. 1, I, mm. 7-9, score reduction; (b) mm. 135-137, beginning of the transition into second thematic area; (c) mm. 151-155, first violin, end of the transition zone

Example 2.20 highlights the transformational relationships between each presented form of the motive. The first appearance of the motive during the transition is an inversion of the original motive, \{(+2, E), (-3, F), (5, A)\}, in gsd space. Specifically, the transformation \(t^5\) sends the original motive to the first appearance in the TR zone. As the TR zone progresses, and the
harmonic goal becomes A, the pivot transformation (4th, e), assigns the pitch classes of the motive to new scale-degree identities. Finally, the inversion $I_3^3$ sends the motive to its last form of the TR zone. In both Examples, 2.20(a) and (b), the arrow labeled $T_c4$ connects the initial motive to the final appearance of the motive. This arrow highlights the resulting chromatic transposition following the three transformations that occur throughout the P and TR zones.

Example 2.20. (a) Reduction of motives from Gustav Mahler, Symphony No. 1, I, exposition, with transformational analysis; (b) transformational network of motivic sets.
VOICE-LEADING SPACES

Over the past two decades, mathematical approaches to musical voice leading have been a growing and diverse area of research. Although they differ in methodology and underlying mathematics, the varying approaches all share many common characteristics. Here, there is no need to reinvent the wheel; rather, this project will draw upon the most relevant approaches best suited for describing a particular passage of music. In his 2013 article, “Contemporary Methods in Mathematical Music Theory: A Comparative Case Study,” Julian Hook provides a broad overview of the most prevalent methods of describing voice leading in mathematical terms. As Hook explains, some of these approaches are algebraic in nature, others are geometric; all of these methods are linked by many common threads (Hook 2013, 89). Hook demonstrates this through an analysis of a single passage through the lens of each relevant methodology. A brief overview of each applicable methodology follows below.

First, the most prolific mathematical methodology of voice leading is perhaps that of Neo-Riemannian operations and their mapping on the “Tonnetz.” The musical transformations, P, L, R, and S, of the Neo-Riemannian system have their modern origins in the work of David Lewin (1987). The Tonnetz has its origins in the mid-eighteenth-century work of Leonard Euler, and its resurrection in the twentieth- and twenty-first centuries has been the subject of a considerable amount of research.21 In particular, the Tonnetz has particular strengths in showing efficient voice leading between triads and seventh chords.22 This methodology is most fruitful in musical passages where common tones unite the connection between triads and seventh chords.

21 The Tonnetz first appeared in Euler (1739)—a diagram of tonal relationships wherein triadic relationship are distributed along two intersecting lattices (perfect fifths and minor thirds).

When this is the case, mapping the triadic relationship on the *Tonnetz* provides a visual aid in understanding the voice-leading structure of the musical surface.

Geometry lies at the heart of the second major branch of mathematic voice-leading research. In their 2008 publication in *Science*, “Generalized Voice-Leading Spaces,” Callender, Quinn, and Tymoczko brought Geometrical models of voice leading into the mainstream of the music-theoretical community. Best described as voice leading in chord space, two-note chords exist on a two-dimensional plane; three-note chords (triads) exist in three-dimensional space; four note chords (seventh chords) exist in four-dimensional space. In each appropriate space, efficient voice leading connects chords—the more efficient the voice leading, the closer the closer the chords are in *n*-dimensional space. Again, common-tone relationships drive the categorization of closeness between chords. Furthermore, Tymoczko (2012) takes a geometrical approach to understanding tonal relationships in western music and intuitively describes voice leading in music throughout each historical period of musical development.

Following Lewin’s model of transformational theory, Rings (2011) takes an algebraic approach to understanding voice-leading relationships with an emphasis on tonal orientation. Unlike the geometrical models of voice leading, which do not take into account the harmonic entities as tonal beings, Rings’s work seeks to situate closeness in voice-leading space within tonal space. Here, triads and seventh chords are not stand-alone entities; rather, each chord is situated within the context of a key area. This perspective adds another element of structure to the voice-leading relationship: the scale-degree relationships between each chord are important along with tracking the overall semitonal motion.

The present work considers each of the spaces as a viable option for illustrating voice leading. The specific musical context will dictate which methodology best illustrates the
relationship between harmonic structures. At many times, in keeping with the tonal-chromatic scale, tracking the scale-degree relationships will be of primary focus. Thus, it will be useful to adapt each of these methodologies to the tonal-chromatic system. Whether using more abstract geometrical voice-leading concepts, or *Tonnetz* relationships, the scale-degree identification system of the tonal-chromatic scale will remain intact.
CHAPTER 3
ANALYSIS I: GUSTAV MAHLER, PIANO QUARTET IN A MINOR

HISTORY AND BACKGROUND

Only a single movement (the first movement) from Mahler’s only piano quartet survives. Mahler composed the quartet as a student at the Vienna Conservatory. There are many unknowns surrounding the years during which Mahler was a student at the conservatory. In 1875, despite hesitation from his father, Mahler traveled to Vienna and enrolled as a piano student at the conservatory.\(^1\) During his studies, Mahler studied piano with Julius Epstein, who introduced Mahler to the music of Mozart, Schubert, and Brahms. When Mahler auditioned, Epstein was not only impressed with his abilities at the piano, but also by his compositions, some of which Mahler played for him during the audition (La Grange 1973, 29-30). For three years, the organist Franz Krenn was Mahler’s composition teacher, and later, also his counterpoint teacher (La Grange 1973, 32; 38). During the years that Mahler attended the school, his most well known accomplishments were in the area of composition.

As a student of composition, Mahler certainly composed for the conservatory and for his own pleasure, but only short fragments have survived (La Grange 1973, 35). As La Grange (1973, 35) notes, Mahler told Natalie Bauer-Lechner that he never finished a composition while he was a student at the conservatory; according to Mahler, he was either impatient to start a new piece or had moved passed the piece he was currently composing. Of these remaining fragments, the opening movement of a piano quartet (piano, violin, viola, and cello) survives. The exact date of composition of the Piano Quartet is unknown; Along with the title of the work and composer,

\(^1\) Gustav Mahler’s father, Bernhard Mahler, desired for his son to eventually take over his spirit business. After Julius Epstein proclaimed that Gustav was a born musician, Bernhard Mahler willingly let his son move to Vienna and enroll at the university (La Grange 1973, 27-30).
the title page of the manuscript has 1876 written on it. The actual date of composition is still a matter of speculation; the year written on the score remains the only viable source.

Presently, the piano quartet has not attracted much analytical or musicological interest, not dwelling on any of the musical details of the work, but simply referencing it as a work composed while a young student at the Vienna Conservatory. Donald Mitchell, though, provides one of the only substantive analyses of this work in the fourth chapter of his book, *Gustav Mahler: The Early Years.* Nearly all of Mitchell’s analysis is quite critical of the piece, pointing out the immaturities of the young composer—the simplicity of the thematic material, the ambiguities in the formal design, lack of thematic development, and the “over organization” of the development section (Mitchell 1980, 124-25). While the immaturities of the composer are evident, it remains a relevant part of the overall oeuvre of Mahler. In fact, buried in the oddities of the piece is interesting harmonic terrain—Mahler explores distant harmonic relationships and places unique and sometimes unexpected harmonies at formal boundaries, alluding to a common practice in his mature compositions.

The remainder of this chapter is dedicated to exploring the intricacies of this youthful piece. The goal is not to explore every crevice of the work in fine detail; rather, the analysis is going to focus on a few isolated, but significant passages—passages that illustrate Mahler’s clear

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2 In the second and most recent edition of Susan Filler’s book, *Gustav and Alma Mahler: A Research and Information Guide*, the piano quartet occupies only three bibliographic entries, two of which are published editions of the scores. The third entry is a digital recording of the work by Dika Newlin, who, in the booklet with the recording, discusses the piece in its historical context and claims the work has substantive value.

3 Most, if not all of Mahler’s modern biographers (La Grange 1973; Newlin 1978; and Mitchell 1980) allude to Mahler’s unfortunate habit of trashing [literally or figuratively] his finished and unfinished works of his younger years. The mere fact that the movement from the *Piano Quartet* remains perhaps testifies to Mahler’s confidence in the work. Regardless why the work survives, it nonetheless represents Mahler as a composer and merits investigation.
use of the chromatic scale as a tonal entity. The tonal-chromatic scale serves as a structural backbone of the formal design of the movement. More specifically, the analysis will focus on the interval of a tritone from the tonic pitch class. This interval has structural significance on the foreground, as well as at middle ground structural levels. Along with other features of this system, such as transposition and inversion, the structural use of the tritone in this movement will demonstrate the effectiveness of a tonal system that employs the chromatic scale as its background. In order to understand how these passages fit into the work as a whole, the analysis will begin with an overview of the formal and harmonic structure of the piece.

**FORMAL AND HARMONIC OVERVIEW**

The formal structure of the piece is, at the very least, an elusive matter. It is certainly a derivative of a sonata-allegro design, albeit with significant modifications to the formal and harmonic structure. Better yet, the musical texture and motivic content dictate the formal structure, as opposed to the harmonic design. Mitchell (1980, 124-26) points to the movement’s formal design as one of its weakest aspects. Mitchell never designates the movement as a sonata form, instead discusses the formal design based upon its motivic content, tempo changes, and textural changes. For Mitchell, there are three expository sections (A, B, and C), a development section, followed by an oddly constructed recapitulation (see Figure 3.1 at the end of this section). The argument here is not to dispute the relative weakness of the formal design—Mahler’s youthfulness is evident throughout the movement. Nonetheless, beyond Mitchell’s brief observations of the movement, there are many noteworthy passages that merit attention to detail.

It is possible to reconcile Mitchell’s formal interpretation with sonata form. The movement begins with a brief two-measure piano introduction that establishes the accompaniment rhythm, quarter note triplets, with a tempo marking of *Nicht zu schnell*. The
exposition proper begins in m. 3 with the main motivic idea (x) first introduced as the bass line of the piano—a three-note motive beginning on A, ascending a minor sixth to F and retreats down a half step to an E. This motive \{(1, A), (6, F), (5, E)\} serves as the source of all the thematic material of the primary thematic area (shown in Example 3.1). The only exception occurs in m. 32-33; here, Mahler provides a brief glimpse of the second motivic idea that will define the second thematic area. The treatment of this motive throughout the primary theme serves as the topic of discussion below. The primary theme maintains a tonic of A throughout and comes to its conclusion in m. 41 on its dominant seventh chord (E7).

![Example 3.1. Gustav Mahler, Piano Quartet in A minor, primary theme motive](image)

Beginning in m. 42, a twelve-measure TR zone ensues. Maintaining a tonic of A, the TR zone is marked with a tempo change to Entschlossen (“determined”) and a dramatic change of musical texture—the piano, violin, and viola introduce fluid eighth notes. Thematically, there are no definitive motivic patterns, which aids in this passage’s classification as a transition. The TR introduces the greatest level of chromaticism yet heard in the movement. The ultimate harmonic goal of the twelve-measure phrase is to achieve a C dominant seventh chord (m. 52-53).

The second thematic area begins in m. 54 and coincides with a resolution on F, though it is difficult to make the case that the tonic of A has truly been lost. Example 3.2 shows the main motivic idea of the second theme. Unrelated to the motive from the P theme, the S theme’s
motive is a two-measure phrase that outlines a descending octave (motive ψ). The second thematic area lasts only thirteen measures and oscillates between resolutions on F and A. The final measure of the theme, m. 66, ends firmly on an E dominant seventh chord, reconfirming an A tonic. This is one component of the formal design that is distasteful in Mitchell’s eyes and represents the movement’s biggest departure from traditional sonata form. Typically, the harmonic trajectory of the TR zone should lead to either the dominant key area, or in the case of minor, to the relative minor key. The dominant key area then carries across the formal boundary into the development section. This, of course, is not the case; here, the entire exposition struggles to release its grasp of the tonic A.

Example 3.2. Gustav Mahler, Piano Quartet in A minor, secondary theme motive

The development section, beginning in m. 67, is 84 measures in length and begins in A minor, almost as if the exposition is starting for a third time. The first four bars (m. 67-70) of the development, while reminiscent of the exposition, act more as a transitional launching pad. The dominant triad chord in m. 70 (E) shifts to minor in m. 71, at which point the thematic and harmonic materials begin their true development. Although the passage is quite brief, this emphasis on E is the only moment where the dominant pitch class is emphasized; E does not play a significant role anywhere else in the development section. Instead, Eb ultimately becomes the goal of the cadential bass line. The remainder of the development focuses on the two motivic

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4 This material is akin to what Caplin (1998, 147) calls “pre-core” material. The core material of the development section usually needs preparation. For Caplin, pre-core material is of lesser emotional intensity. Typically, it begins with the tonic of the previous formal section of the piece—in this case, it is still the governing A tonic.
ideas from the exposition; at times they are used in counterpoint with one another, at other times, they are treated independently. Throughout the development, the motives never undergo identity-changing transformations. Rather, it is the harmonic landscape of the development that eventually evolves and deteriorates; it becomes more and more unsettled, producing an unmistakable boundary line between it and the recapitulation. The cadence going into the recapitulation, with an $E_b$ in the bass line, does not point to A as its goal.

The recapitulation, beginning in m. 151, is far from normatively constructed. It begins just like the exposition—the same piano texture, the same motive in the piano’s bass line, and the delayed entry of the strings, which do not enter until the second phrase. Mahler, however, diverges from the exposition’s structure in several ways; first, the recap’s primary theme area is drastically shorter than the exposition’s. Instead of the forty-one-measure primary theme that is repeated during the exposition, the recap’s primary theme is about half that (twenty-three measures) and is not repeated. Second, the twelve-measure transition is omitted from this portion of the recap; immediately following the primary theme, Mahler moves directly into the secondary theme (m. 174). Unlike the exposition, where the secondary theme began in F, the secondary theme of the recapitulation begins in $F_\#$. Of course, the standard procedure is to maintain the tonic key throughout the recapitulation. Just as with F in the exposition, $F_\#$ is only a brief detour from the tonic, and in m. 182, A returns as tonic. The next oddity occurs in m. 190. Following the secondary theme’s return to the tonic, Mahler presents an exact copy of the twelve-measure transitional material from the exposition, which leads to a continuation of the secondary thematic material. Instead of acting as a transition between the two main thematic sections of the recap, the transitional material serves as a digression within the S theme.
From the pickup to m. 219, the first violin is given a four-measure cadenza that leads to a strong cadential moment in the tonic key. Following the cadenza, there is a twelve-measure coda that only uses the S-theme motive. The entirety of the coda has a tonic pedal that supports two harmonies: tonic and dominant. The coda’s texture is scored lightly and gradually diminishes to a solemn A minor conclusion in m. 234. Example 3.3 shows a complete formal overview of the movement, showing formal sections, measure numbers, and significant harmonic areas.

<table>
<thead>
<tr>
<th>EXPOSITION: (66 measures)</th>
<th>Exposition ends with V/i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section:</td>
<td></td>
</tr>
<tr>
<td>Piano (P0)</td>
<td>P1</td>
</tr>
<tr>
<td>Harmony:</td>
<td>Am</td>
</tr>
<tr>
<td>Measure:</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DEVELOPMENT: (84 measures)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Section:</td>
<td></td>
</tr>
<tr>
<td>Episode 1 (P/S)</td>
<td></td>
</tr>
<tr>
<td>Harmony:</td>
<td>Am</td>
</tr>
<tr>
<td>Measure:</td>
<td>67</td>
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</tbody>
</table>

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Harmony:</td>
<td>Am</td>
</tr>
<tr>
<td>Measure:</td>
<td>151</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CODA: (12 measures)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P/S</td>
<td>Harmony: Am (pedal)</td>
</tr>
<tr>
<td>Measure:</td>
<td>223 234</td>
</tr>
</tbody>
</table>

Example 3.3. Formal design of Gustav Mahler, Piano Quartet in A minor

THE EXPOSITION

Even with all of the immaturities that Mitchell (1980) points out, there are still sections of the movement that merit discussion—sections that demonstrate more intentional design than
Mitchell is willing to afford the young Mahler. Besides a general overview of the movement’s formal structure, Mitchell does not discuss the movement’s harmonic continuity—in fact, it seems as though he implies a complete lack of continuity. For example, Mitchell concludes his discussion of the movement with the following statement:

But the successful conformity revealed in the development makes one wonder why, elsewhere in the movement, Mahler’s formal aim and tonal organization should have been so direction-less, patently flouting classical ‘sonata’ practice without, so it appears, a very clear intent to substitute a reasoned alternative. The exposition’s concentration of tonic is enigmatic. (Mitchell 1980, 126)

While these criticisms may hold true at the background level—where an unyielding tonic seems to penetrate all aspects of the formal design—there are middleground and foreground features that demonstrate a more coherent and diverse harmonic strategy.

The main motive \( (x) \) of the primary theme has a significant influence on two aspects of the harmonic structure throughout the movement; the first resides in the harmonic trajectory of the exposition, the second in the harmonic trajectory of the primary thematic area. The main motive, \( \{(\hat{1}, A), (-6, F), (\hat{5}, E)\} \) is mirrored in the background harmonic trajectory of the exposition. The primary theme projects \( (\hat{1}, A) \); following the brief transition, the secondary theme’s harmony begins firmly with \(-6, F\); the secondary theme (and the exposition proper) end on the dominant, \( (\hat{5}, E) \). Example 3.4 makes this motivic parallelism abundantly clear.\(^5\) Although

\(^5\) Central to Schenker’s theory of tonal organization is the concept of structural levels. Each structural level contains different amounts of musical detail—more detail at the foreground level and less detail as one moves away from the foreground in middleground levels. According to Burkhart (1978), one of Schenker’s most important contributions to musical thought was the idea of hidden repetition, or motivic parallelism. Motivic parallelism is when a melodic cell or “motive” can take different forms on the musical surface, or more importantly, when the motive appears on different structural levels.
the exposition does not follow the traditional harmonic plan, or anything close to that of a sonata form movement, it does have continuity, one generated from internal mechanisms.\(^6\)

Example 3.4. Gustav Mahler, Piano Quartet in A minor, P-theme motivic parallelism during the exposition

Motive x also bears a significant relationship to the primary theme’s harmonic content, perhaps, though, in a less obvious way. Example 3.5 is a reduction of mm. 24-42, the second phrase of the primary theme. Mitchell claims that the movement’s greatest weakness lies in its defective organization (Mitchell 1980, 124). One can only assume this includes not only the thematic organization, but also the harmonic organization. As demonstrated above, there is more organization of the harmonic material than Mitchell is willing to admit. Through the operation of inversion, it is possible to reconcile the motivic and harmonic aspects of this passage.

\(^6\) By the late-nineteenth century, the formal and motivic procedures of sonata form had evolved. Only vestiges of the Classical sonata design of Haydn, Mozart, and Beethoven are operative in this music. As Webster (1978, 18) notes, Romantic composers believed that the exposition was governed more by the contrast between the first and second themes than the tonal polarity between the first and second groups. Therefore, Romantic sonata forms weakened the structural significance of the exposition serving as a large-scale half cadence. By the time Mahler was active, the parameters surrounding sonata form had loosened. Peter Smith (1994) cites a different reason for the marked shift in formal approach between Classical and Romantic composers. Amidst a discussion of Brahms and Schenker’s thoughts on sonata form, Smith acknowledges Brahms’s desire to commit to historically validated formal types, but refused to sacrifice the romantic ideal of an unbroken, goal-directed flow.
As is evident from the reduction, a sequential falling-fifth gesture occurs on several levels. Likewise, Mahler transposes motive \( x \) to conform to each passing harmonic area. A simple voice-leading procedure facilitates these harmonic shifts. The initial tonic triad of A minor is transformed into an A dominant seventh chord—whereby \((-\hat{3}, C)\) moves to \((+\hat{3}, C\#)\) and \((-\hat{7}, G)\) is added—and serves as the dominant seventh chord of the following harmony (D minor). Once D minor is reached and motive \( x \) is heard in accordance with the new harmony, the same voice leading procedure shifts D minor to D dominant seventh, which then serves as the dominant seventh of G minor. Once reached, another iteration of motive \( x \) accompanies G minor at the appropriate pitch level. It is at this point, that Mahler breaks the sequential pattern.

Motive \( x \) generates the harmonic landscape of the passage shown in Example 3.5. An inversion of the motive (in CSD space) brings deeper meaning to the harmonic progression. If one applies the transformation \( I^{(1,A)}_{(1,A)} \) to the original form of the motive, the result is \( \{(1, A), (\hat{+3}, C\#), (\hat{4}, D)\} \). This inverted form of the motive is embedded in the voice leading that leads to D minor. Also, Motive \( x \) outlines an ascending perfect fifth while the harmonic motion outlines a descending perfect fifth. Thus, the harmonic pattern is the inversion of the motivic gesture. As the passage continues, the same relationship exists between each transposition of the motive and the subsequent harmonic change. Once D minor is reached in m. 30, the main motive is
reiterated, now as the three-note collection, \{(\hat{4}, D), (-\hat{2}, Bb), (\hat{1}, A)\}. Applying the inversion operator \(I^{(4, D)}_{(4, D)}\) yields the set \{(\hat{4}, D), (+\hat{6}, F\#), (-\hat{7}, G)\}, which defines the voice leading that leads to G minor in m. 34. Finally, the inversion \(I^{(\hat{7}, G)}_{(\hat{7}, G)}\) applied to the final presentation of the main motive \{(-\hat{7}, G), (TT, Eb), (\hat{4}, D)\} yields the set \{(-\hat{7}, G), (+\hat{2}, B), (-\hat{3}, C)\}. After a brief harmonic detour in m. 36 (detailed below), this inverted form of the motive is present in the voice leading that ultimately leads to C minor in m. 38 (see Example 3.6).

Example 3.6. Oriented network of mm. 24-42

As stated above, Mahler breaks off from the sequential progression following the arrival of G minor. This coincides with the peak of the thematic material. In m. 36, just as the final motivic gesture begins, a deceptive cadence into Eb major occurs—the root of which is a tritone away from the tonic pitch class. The primary thematic area reaches its energetic climax upon the arrival of Eb major, and the following measures (until the TR zone in m. 42) act as a discharge of melodic and harmonic energy and ultimately bring the tonal orientation back to A. After the methodical sequence that brings about Eb, the goal of the quick harmonic transition in m. 37-41 is to restore E7. Example 3.7 shows the overall harmonic trajectory of the P theme, with an
emphasis on the voice leading in m. 37-41. As one can see in the example, there is an emphasis on the relationship between A and Eb—this relationship is a salient feature of the P theme, and plays an important role later in the movement. Following Eb major, the interrupted sequential pattern resumes in mm. 37-38 as a G dominant seventh chord resolves to C minor. The sequence governing the P theme terminates on C minor and the remaining four measures of the theme contain a series of non-functional chords that lead to a cadential ii⁰ – V7 – i, initiating the TR zone.

Example 3.7. Primary theme harmonic trajectory (mm. 1-42); harmonic reduction of mm. 36-42

**THE DEVELOPMENT**

Beginning in the tonic key, the first four measures of the development seem as if the exposition is beginning for a third time. Motive x appears in the same manner as the beginning of the exposition. Rather quickly, though, the harmonic orientation shifts—in m. 69, a D# fully diminished seventh chord leads to E7 in m. 70. At first, this may seem as yet another cadence in the tonic key of A; however, in m. 71, E7 transforms into E minor, accompanied by the main

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Unlike the earlier manifestation of Eb as a structural arrival, the D# in this passage is spelled “correctly” in the key of A, as a leading tone to the dominant.
motive now transposed to begin on E. It is at this point that Mahler brings back the sequential mechanism from the exposition. Based on the inversion of motive $x$, the same progression of descending fifths begins on E minor and governs the harmonic content of the first portion of the development section. Unlike the exposition, Mahler now combines motive $x$ with motive $y$ from the secondary theme, which is also transposed to conform to each new harmonic area. Example 3.8 is a graphic reduction of the thematic material of mm. 71-85 and shows the transformations between each instance of the motive.

Example 3.8. Measures 71-83, thematic reduction showing transformational relationships

Except for the fact that it starts on E, the first section of the development traverses the same harmonic terrain as the exposition—a series a descending fourths, E, A, D, G. As is evident in Example 3.8, the transformations of motive $x$ occur in CSD space, as do the inversion operators acting on $x$, which are embedded in the harmonic structure of the passage. This operates identically to the sequence of the exposition. The manipulation of motive $y$, however, is performed in GSD space; this is because of the underlying dominant seventh chords. This in no way undermines the identification of $y$. For example, the transformation $T_g4$ sends the first instance of motive $y$ to the second:
\{(\hat{5}, E), (\hat{+2}, B), (\hat{1}, A), (\hat{-7}, G), (\hat{-6}, F)\} \rightarrow T_g A \rightarrow \{(\hat{1}, A), (\hat{5}, E), (\hat{4}, D), (\hat{+3}, C\#), (\hat{+2}, B)\}

The same transformation sends the second iteration to the third, and the third iteration to the fourth. There is a level of continuity throughout the passage that is not immediately evident; each presentation of the motives is connected by the same transformations.

Beginning in m. 84, the texture of the music becomes more charged with chromaticism as the passage presses towards the harmonic goal: an E dominant seventh chord. Picking up where the sequence ended, this last phrase of the first part of the development begins on G minor, now supported by a pedal tone of D (which stays in tact through m. 91). In a similar way to mm. 36-42, the goal of mm. 84-91 is to quickly reorient back to the tonic. Over the course of the seven measures, a series of non-functional chords connect G minor and E7. There are no transposition or inversion operators that connect the adjacent harmonies; there are, however, transposition operators that connect the various forms of motive \(y\) that occur in the first and second violins, shown as the top voice of the two-voice counterpoint reduction in Example 3.9. Governing the passage is a chromatic linear progression of sixths (with the exception of one fifth). Beginning with (6th, 8) between the two pitch classes of the set \{(-7, G), (TT, Eb)\}, the next outer interval is (5th, 7), followed by a return of (6th, 8). Once the outer voices return to a sixth, the remaining intervals in the outer voices remain sixths. There is a subtle manipulation of the types of sixths heard throughout the passage—as the passage progresses the sixths increase in size. This is due to the alteration of motive \(y\). With the exception of (5th, 7) the linear progression begins with three intervals of (6th, 8), followed by three intervals of (6th, 9).

The repeated forms of motive \(y\) throughout this passage ultimately deviate from the original form of the motive as the passage progresses. Beginning with the first presentation beginning on Eb, the motive goes unchanged across the first harmonic change. In fact, the first
presentation of the motive begins on a non-chord tone (Eb), against G minor. As the foregoing analysis of the exposition suggests, this is noteworthy. Again, the listener’s attention is drawn to Eb; once again, as the music initiates a transitional passage toward the tonic, Mahler highlights the interval of a tritone. The first three forms of the motive are identical to the original form of the motive. Thus, they are related by transposition in CSD space. As seen in the example, the two arrows connecting the first three forms of the motive are labeled with $T_c^1$. After this, the transposition pattern changes: now the chromatic transposition is heard over the course of two forms of the motive, and is now $T_c^2$. This is evident in the example. The two forms of the motive under the same brackets have an interesting relationship, but one that cannot be expressed in terms of transposition or inversion: all of the notes are the same except that the framing octave moves up by half step.

Example 3.9. Contrapuntal reduction of mm. 84-91

The passage in Example 3.9 precipitates yet another cadence in A. This cadence initiates the second episode of the development section. Motive $x$ and its derivatives drive the melodic motion of this portion of the development section. Beginning on the tonic (major), the first nine measures serve as a melodic and harmonic transition, with a goal of D minor. In m. 102, D minor is reached, as well as the onset of the motivic material that will drive the remainder of the
development. Although the harmonic motion from A to D resembles the exposition’s primary theme and the first episode of the development, once D minor is reached in episode two, it no longer continues the same harmonic trajectory. Instead, an ascending sequential gesture drives the second episode. Beginning on D (m. 102), the harmonic and motivic material climbs upward in a series of whole steps—in m. 106 E prevails, then in m. 110, F#.

Perhaps, though, of most significance is the harmonic material at the end of the second episode and the retransition leading to the recapitulation. Here, the tritone relationship between the tonic (A) and E♭ reaches its most poignant juxtaposition. The ultimate goal is to reach the tonic for the onset of the recapitulation. Following the brief area focusing on F#, the surface of the music quickly shifts back to D, supported by a pedal tone of A. The bass line of the piano in mm. 114-115 is the first direct juxtaposition of E♭ and A, a descending chromatic line from E♭ to A. (see the beginning of the Example 3.10). The tritone relationship is stated even more directly in mm. 130-132. In m. 130, Mahler emphasizes E♭ major through an arpeggiation in the bass line of the piano and its dominant seventh chord (B♭7). Again, this directly resolves to a pedal tone of A, supporting D minor. Finally, in m. 132, the A pedal tone resolves to D (without a dominant seventh chord).

In m. 139, the retransition zone begins with a D dominant seventh chord (an efficient shift from D minor (F → F# and C is added to the harmony)). The D of the D7 chord shifts to E♭ in m. 140. The musical surface briefly undulates between F# fully diminished seventh and G minor until m. 144 when the F# fully diminished chord takes over the musical texture. It is at this point that efficient voice leading and chord inversion produce the final chord of the retransition. In m. 145, F# is articulated strongly as the lowest note in the musical texture (still supporting an F# fully diminished seventh chord. In m. 148, underneath the same harmony, E♭ moves to the
lowest register. On beat three of m. 148, F# moves by semitone to Fs, sounding an F dominant seventh chord in third inversion. This chord is the final chord of the retransition and acts as the cadential harmony leading back to A (minor) for the recapitulation. Example 3.11(a) shows a reduction of the pitch collections from the final measures of this passage. Of particular interest is the efficient voice leading that leads to the final chord of the retransition and ultimately to the tonic. At this point, the A-Eb tritone relationship reaches its most vulnerable state—directly juxtaposed as the cadential bass line. Functionally speaking, it is possible to understand the final harmony as an inverted form of a German augmented sixth chord that resolves directly to the tonic. 8 This “functional” view is, however, problematic: the Eb in the lowest voice, which participates in the interval of an augmented sixth (Eb (D#) to F), resolves by

8 In a passage discussing a generalized system of augmented sixth chords, Tymoczko (2011) states “…Augmented sixth are simply a noteworthy species within the genus of efficient voice leadings from four-note to three-note chords. When thought of this way, the functional relationship is not the primary concern; rather, it is the semitonal voice leading.
tritone to the root of A minor, thus thwarting the resolution of the active interval (augmented sixth). Further, the stepwise resolution of the tritone between A and Eb does not occur either—the A carries over as a common tone upon resolution to tonic. It is more appropriate to understand this cadence in terms of efficient voice leading, of which Example 3.11(b) represents a graphic representation. While the cadence does not abide by any functional norms, it does allude to tonal function in two interesting ways. First, the bass motion at the cadence moves by fifth, albeit a deformed fifth (a half step too low). Second, just as in standard tonal cadences, Mahler employs a dominant seventh chord (F7), but with a root a half step higher than the normative seventh chord.

Looking more closely at Example 3.11(b), the dashed lines connect the dots on the circles that represent semitonal motion between constituent chord members and solid lines connect common tones. The innermost circle is the first chord (D minor). Each adjacent circle (moving away from the center) represents the next chord in the progression. The outermost circle represents the final chord of the progression, A minor, and the onset of the recapitulation. As is evident, semitonal voice leading connects each of the chords—only one note moves between the first two chords, and two notes move by semitone between the remaining adjacent harmonies. This need for two-note motion is partly due to the fact that that last chord change involves a four-note chord moving to a three-note chord. Also, each chord contains two common tones, bringing a high level of continuity to a passage with a diverse collection of chord qualities—minor, dominant seventh, and fully diminished.
As the foregoing analysis shows, Mahler’s early musical language exhibits a proclivity toward highly chromatic harmonic structures rooted in a relatively function tonal landscape—demonstrating the usefulness of a tonal system that emphasizes a chromatic background structure. Furthermore, the emphasis on the interval of a tritone from the tonic pitch class is a musical relationship not inherent in the diatonic tonal system. As seen above, this interval participates in foreground events as well as middleground structures. Likewise, operations not
typically used in common-practice tonal analysis, such as chromatic transposition and inversion, play a significant role in the ways the analyst understands the phrase design and formal structure. In the following chapter, an analysis of the fourth movement of Mahler’s fifth symphony, Adagietto, many of the same issues arise—chromatic motivic materials are present on the foreground as well as the background articulation of form and harmony. To aid the discussion, aspects of Schenkerian analysis work in tandem with the tonal-chromatic transformational system. In Mahler’s music, chromatic structures arise that Schenkerian analysis alone cannot handle with grace. This is not to undermine the insights that Schenkerian analysis can shed on a piece of music. Instead, the marriage of the two systems proves to be a fruitful way to provide an analysis that appeals to both intuition and rigorous examination.
CHAPTER 4
ANALYSIS 2: GUSTAV MAHLER, SYMPHONY NO. 5, IV, ADAGIETTO

INTRODUCTION AND BACKGROUND STRUCTURE

The fourth movement of Mahler’s fifth symphony, known by its subtitle tempo marking Adagietto, is among his best-known works. Today, it remains the only symphonic movement performed independently from its original context (Kaplan, 2005). Evidence shows that this movement was intended as a musical representation of Mahler’s love for Alma, and as Kaplan (2005) so vividly explains, there seems to be a chasm between how the work is performed today and the way Mahler and his contemporaries would have performed it.¹ The Adagietto stands out from the rest of Mahler’s oeuvre for two reasons. First, it is substantially shorter than the typical Mahlerian symphonic movement. Second, it is lightly orchestrated (strings and harp) by comparison not only to its surrounding movements, but also to Mahler’s symphonic movements as a whole.² These two characteristics make this movement manageable for a close examination of Mahler’s harmonic and contrapuntal practice, not only at the middle and background levels, but also at the foreground. The objective of this chapter is to confront some of the problematic voice-leading areas encountered on the surface of the music. This chapter investigates the unique ways Mahler transforms motives, handles dissonance, and engages non-functional sonorities.

¹ In particular, Kaplan discusses two matters: (1) the performance tempo has become drastically slower in more recent recordings in comparison to how Mahler and his contemporaries performed it, and (2) the context in which the movement is performed has been distorted, moving further and further away from projecting a theme of love. The former point directly effects the latter, in so much as that when the tempo of the movement is so drastically altered, the original extramusical import becomes almost impossible to detect.

² Mahler began composing a tenth symphony, but was unable complete it prior to his death in 1911 and left only a draft behind. There were later attempts at completing the orchestration of the symphony, most notably by Deryck Cook, premiered in 1964. Das Lied von der Erde has also been deemed a “symphony” by scholars, but Mahler himself refrained from calling it one.
The chromaticism embedded in the background structure of this movement makes for problematic Schenkerian readings. This remains true at the foreground. This chapter will employ Schenkerian analysis and the transformational approach using the tonal-chromatic scale in tandem, providing a more holistic analysis. Specifically, we are going to look at the manifestation of a chromatic two-note motive that originates on the musical foreground and pervades the middleground and background structure of the movement. Moreover, the analysis will focus on the two modulation zones; here, Mahler employs highly chromatic structures that can only exist if one accepts the chromatic gamut as the background structure. This investigation will begin with an overview of the middleground and background structure of the Adagietto. (See Example 4.1.)

The overall formal design of the movement is best described as a ternary form (ABA) with a TR (transitional) zone between the first A and B sections. Example 4.1 is a graphic reduction of the entire Adagietto, each staff representing a specific formal section of the music. The A section initiates the underlying tonic of F major and is supported by a Kopfton on (5, C). As the graphic reductions depict, the movement’s formal design corresponds closely to the overall tonal scheme: The initial and concluding A sections, both in F major, surround the solitary B section, in Gb major. At the middleground and background levels, the large-scale voice leading is relatively straightforward: 5 is reached in m. 5, which is followed by two descents into an inner voice during the A section; the transitional passage leads the music from F to Gb; the B section prolongs 5 via a neighboring motion on (6, Db); the return of the A section coincides with a regaining of 5, which ultimately descends to 1 in m. 99.

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3 In much the same way, Rings (2007) provides a successful attempt to reconcile two disparate methodologies, combining Schenkerian analysis and neo-Riemannian analysis to provide an account of Schubert’s impromptu in Eb, D. 899, no. 2. The operations of neo-Riemannian theory account for harmonic relationships that Schenkerian theory disregards.
Example 4.1. Middleground reduction of Gustav Mahler, Symphony No. 5, IV, Adagietto.
Moving forward, this analysis will focus on two aspects of the music, both of which are foundational for understanding Mahler’s mature tonal idiom. The first is the effect that motives have on the harmony—how motives inflect, and undermine, and predict the underlying harmony. It is evident in this piece that small motivic cells help generate the large-scale harmonic structure and are embedded in even the most densely chromatic passages. The second aspect of the work involves an investigation of the voice leading that occurs during the modulatory passages, first shifting the music from F major to Gb major and then from Gb major back to F major. Within these two passages, one can find non-functional and highly chromatic harmonic progressions that are best explained using various voice-leading spaces.

THE MOTIVE AND ITS HARMONIC FUNCTION

In Mahler’s mature music, the importance of the melodic motive is a widely recognized component. This holds true for the motivic content in the Adagietto. Furthermore, beyond the melodic role of the motive, there is also a substantial harmonic role. As the analysis shows below, the motivic structure of this movement interacts intimately with the harmonic content. In many instances, the two aspects are inseparable.4 With regards to the motivic content of this movement, two of the most significant theorists of the twentieth century, David Lewin and Allen Forte, have produced substantial analyses. This chapter explores these two analyses in reverse chronological order, beginning with Lewin (2005) and ending with Forte (1984). This ordering stems from the relevance of each article; Lewin posits an interesting idea about the motivic source of this movement, but the present analysis will draw more so upon Forte’s article.

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4 Bruns (1989) discusses this notion in Mahler’s music. Based on an idea posited by Benjamin Boretz (1972), Bruns claims that the late tonality of Mahler, with a particular focus on the tenth symphony, is heavily dictated by the motivic content rather than triadic content.
In his 2005 essay, “Some Theoretical Thoughts about Aspects of Harmony in Mahler’s Symphonies,” David Lewin posits the concept of an ikonic sonority. For Lewin, an ikonic sonority is a “musical Ding an sich, referential as a point of departure and arrival, not necessarily dependent on other sonorities for whatever meaning we sense in it,” (Lewin 2005, 146). Thus, an ikonic sonority’s meaning cannot come from a previously established musical idea; it must be an inherent quality. Applying this concept to the Adagietto, Lewin identifies the three-note figure, shown in Example 4.2, as an ikonic gesture that permeates much of the melodic and harmonic content. The three-note gesture is a downward arpeggiation of the pitch class dyad (C, A). For Lewin, the notes of this ikonic gesture appear as intrinsic characteristics of the melodic landscape. Likewise, the notes of the dyad can serve as the third and fifth of an F major triad or as the root and third of an A minor triad, both of which receive strong cadential moments throughout the movement. The present analysis will make reference to this ikonic sonority, but will not include this sonority as a motivic element of the movement.

Example 4.2. Lewin’s prescribed ikonic sonority in Mahler’s Adagietto

On the other hand, Forte’s 1984 analysis, “Middleground Motives in the Adagietto of Mahler’s Fifth Symphony,” provides an extensive examination of the motivic content throughout the movement. As a means of showing the interaction of motives throughout the movement, Forte uses graphic reductions, somewhat similar in look to Schenkerian reductions. These

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5 Here, the notation of the tonal-chromatic system is not applicable; Lewin explicitly says that an ikonic sonority cannot be affiliated with any scalar, harmonic, or tonal context. Thus, to adhere to Lewin’s concept, only the pitch class note names indicate the ikonic sonority.
graphic reductions, though, do not behave in the same way as Schenkerian reductions. Instead of focusing on the melodic and harmonic structure of the piece, the reductions show the motivic content that occurs at the foreground, middleground, and background levels. Forte is not trying to say that Mahler’s harmonic language is subordinate to the motive, but rather that his harmonic language is directly effected by the motive and that these two aspects of Mahler’s music cannot be examined independently. Agawu (1983, 85-86) defines Mahler’s melodic structure as the process of generation in which an initial melodic kernel or cell is subject to constant manipulation. I will investigate some examples of this technique and look at how Mahler resolves motivically-derived dissonant harmonies. Throughout this analysis, I will draw attention to the most important motives that are embedded within the musical fabric.

Example 4.3 is a reproduction of Example 2 from Forte’s article. It shows the motivic cells that Forte hears as underlying the entire movement. In total there are eleven cells, each derived from the initial stepwise cell, which he labels a. For the purposes of the present analysis, it is possible to organize these motives into a more concise list. This is mainly because the goal of this chapter is not to find every instance of every motivic cell at all structural levels, but rather to show how the basic motive infiltrates and helps determine the movement’s general harmonic structure. To do this, one only needs the main motivic cell and its subsets. The motives needed for this analysis are all present in the first melodic phrase of the movement, reproduced in

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6 Forte does provide a disclaimer at the beginning of the essay that the graphic reductions do not behave like ordinary Schenkerian reductions. Specifically, Forte states, “These graphs do not always follow Schenkerian paradigms and may occasionally bend textbook norms” (Forte 1984, 154).

7 Forte illustrates this point clearly with the following comment regarding the opening section of the Adagietto: “In this situation the pitch a\textsuperscript{1} conflicts with the dominant harmony, a beautiful and idiomatic instance in which a statement of the motive takes precedence over other compositional considerations” (Forte 1984, 157).
Example 4.4(a). Example 4.4(b) then extracts and labels the four prominent motivic cells throughout the movement (labeled a-d).

Example 4.3. The basic motives in the foreground in the Adagietto, as shown by Forte (1984)

Example 4.4. (a) Gustav Mahler, Symphony No. 5, IV, Adagietto, mm. 2-9, first violin; (b) the four main motives throughout the movement

Motive a is the first melodic gesture heard in the movement and is the generator of the primary thematic material. It, along with its inversions, in CSD space and GSD space play a significant role throughout the movement. Motive b is a subset of motive a, an ascending half step. Motive c is the inversion of motive b, a descending half step that is first heard transposed to
conform to the F major diatonic collection. Lastly, motive $d$ is a larger subset of motive $a$, a three-note ascending stepwise gesture. Along with the motive’s melodic contour, the outlining interval content of a motive may also serve as a representative of the motive. For example, the perfect fourth spanning motive $a$, or the half step outlining motive $b$ and $c$.

The opening thematic gesture of the movement foreshadows the overall harmonic landscape. The two violas, cello, and harp begin the movement with a bit of ambiguity. A complete tonic triad is absent in the first two measures; only the pitch classes A and C are present. In fact, this is the ikonic gesture to which Lewin (2005) refers. At this point, determining the tonic pitch class is not possible. In measure 3, though, Mahler confirms the tonic with the pizzicato articulation of an F in the basses and harp. It is at this moment, the downbeat of m. 3, that the motivic material foreshadows the coming events later in the movement. The first three notes of the initial motivic idea, motive $a$, begins as a pickup into m. 3, $\{(\bar{5}, C), (+\bar{6}, D), (+\bar{7}, E)\}$. On the downbeat of m. 3, though, $(+\bar{7}, E)$ is rearticulated, creating the interval of a major seventh between the outer voices. It is not until beat 2 of m. 3 that $(+\bar{7}, E)$ resolves up by half step to $\bar{1}, F)$. For this one beat, there is a strong yearning for resolution up by one half step. Example 4.5 is an oriented network that shows how this motive interacts with the prevailing harmonic background.

Motive $c$, a descending half-step gesture, first appears in mm. 5-6 as a 4-3 suspension over a secondary dominant (G dominant seventh chord). This motive is heard again in the following measures (mm. 6-7) as another 4-3 suspension over another secondary leading-tone seventh chord (F# diminished seventh). The function of these harmonic sonorities is not as important as the prevailing melodic gestures. During the first phrase of the movement, Mahler
presents these two semitonal melodic gestures, one ascending, and the other descending; both of which help unfold the overarching tonal plan of the movement.

As mentioned above, the background harmonic structure is problematic when viewed solely from a Schenkerian perspective. The key areas explored in the movement do not conform to any normative diatonic background structure, and even at the deepest background levels this chromaticism is inevitable. In particular, there are unavoidable parallel fifths in the background level between the two triads of F major and G♭ major, especially when taking into consideration the Urlinie, which moves from (♯5, C) → (b6, D♭).\(^8\) The same problem exists upon the return of F. Example 4.6(a) shows this voice leading in a deep middleground graphic reduction of the entire movement. Although this does not reconcile the overall voice-leading problems, it is beneficial to understand this harmonic motion in terms of motivic structure—motives \(b\) and \(c\) are embedded in this middleground voice leading. Example 4.6(b) is an oriented network showing how these

\(^8\) On a more local level, the consonant skip from C to E♭, the latter of which resolves to D♭ in m. 46, mitigates these fifths. This, along with other aspects of the harmonic structure, is discussed in more detail below.
motives manifest in the background voice-leading structure. The half-step ascent of the structural tones from section A to section B is motive \( b \) and the half-step descent of the structural tones from section B to the return of section A is motive \( c \).

The remainder of this portion of the analysis focuses on the motive’s influence on harmonic behavior at the foreground level. Specifically, motive \( b \) plays an integral role in the most chromatic portions of the movement. Due to the nature of the motive (an ascending half-step gesture), it is necessary to place more constraints on the definition of the motive. It would be reckless to assign every instance of semitonal motion as motivic. Instead, on the foreground, to consider ascending and descending half-step gestures as motivic, they are required to behave as delayed resolutions of dissonances (suspensions and retardations)—in the same way as the opening phrase of the movement. Example 4.7 shows two parallel passages: the first occurs

\[ \text{Example 4.6. (a) Adagietto, deep middleground graph; (b) oriented network of middleground voice-leading structure} \]

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\( ^9 \) This deep middleground manifestation of motives \( b \) and \( c \) is absent in Forte (1984).
Example 4.7. (a) Gustav Mahler, Adagietto, mm. 22-30, score reduction; (b) reduction of mm. 86-95; the enclosed notes are instances of motives $a$ and $b$

during the initial A section, the first climactic approach to the dominant; the second occurs during the recap of the A section and involves the approach to the structural dominant of the
movement. While there are very similar characteristics between these two passages, Mahler does recompose aspects of the passage, which highlights the motive’s subtleties.

As is evident in the example, the harmonic and motivic content is nearly identical in both passages. In both cases, instances of motive b are embedded within the musical fabric, creating tension with the implied underlying harmonic content. Unlike the opening phrase, these two passages involve more complex dissonance. Nonetheless, this passage demonstrates a governing characteristic of Mahler’s mature compositional style—the resolution of dissonance happens well after the expected moment. Due to this characteristic, Mahler is able to employ relatively functional harmonic progressions within the context of a rich chromatic texture. For example, in mm. 26 and 90, the underlying harmony is a G minor seventh chord in first inversion. The cello’s C# heard on the downbeat of the measure is an accented passing tone, a melodic gesture referencing motive b. In this case, the non-chord tone occupies the majority of the measure (three of four beats), resolving upward to D on beat 4 (see the oriented network in Example 4.8(a)).

One should note, however, that the spelling of C# is a matter of notational convenience, the enharmonic respelling of (♭6, Db), now recast as (♯5, C#). As the passage continues, the motivic fragments come into conflict with one another, creating a network of rich chromatic harmonies.

In mm. 27 and 91, the basses and cellos present another instance of motive b in parallel thirteenths (respelled on beat four as a diminished 14th). Here, there are a few interesting voice-leading components. On beat three, the governing harmony becomes an E dominant seventh chord. The E in the cello (participating in the motive) is a chord tone that resolves into a non-chord tone, or it can be thought of as an upper dominant extension. This motivic appearance is backwards from the usual. Thus, the G in the basses is a non-chord tone that finds resolution up
by half step (shown in Example 4.8(b)). Once the G# is reached, the underlying harmony of E7 is clear. The presence of (I, F) in the E7 chord represents a vertical manifestation of motive b. Following the resolution of this chord in mm. 28 and 92, two more instances of the motive appear. This is the point at which the two passages diverge from being nearly identical.

Earlier in the movement, upon resolution to a first inversion F major triad on beat three of m. 28, the violas articulate a form of the motive that resembles the opening gesture of the movement—a half step ascending resolution to A4, above the bass. In the corresponding passage (m. 92), this form of the motive is rescoped in the cello part. In the latter of these two passages, the cellos continue their pressing chromatic ascent, whereas earlier the cello line stopped on F4 (the b9 upper dominant extension of the E dominant seventh chord). This time, upon the arrival of the F major triad in first inversion (beat 3 of m. 92), the cello line presents the form of the motive that was originally scored for the viola, (♯2, G#) → (3, A). In mm. 28-29 (cello) and 92-93 (viola), another version of motive b (C→C#) transforms the tonic triad into an augmented triad. This, however, is only a fleeting harmonic moment, on beat 3 of m. 29 and 93, the bass line continues upward to B♭, producing a non-functional harmonic entity. In the earlier passage, the C# goes unresolved and dissolves into the next adjacent harmony.

In the second excerpt, the purpose of this harmony is to reach the secondary leading-tone seventh chord (B♭7) that occurs on beat 3 of m. 94. The A in the cellos, reached in m. 93 (as well as in the first violin in m. 94), and the F reached on the downbeat of m. 94, are two chord tones that are anticipated prior to the basses reaching B and the violas reaching D on beat 3 of m. 94. Finally, on the downbeat of m. 95, Mahler provides a clearly articulated cadential 6/4 chord, with all chord tones articulated simultaneously. This cadential 6/4 chord is the structural dominant,  

\(^{10}\) Although the rhythmic component is absent in the oriented networks, dashed vertical lines identify measures.
which contains the descent from $\hat{3}-\hat{2}$ of the background *Urline*, and upon its resolution completes the *Ursatz*. Example 4.8(c) shows an oriented network of this portion of the second excerpt. In each of the networks of Example 4.8, only important chord tones and motivic non-chord tones are shown in the network. The curved dashed lines represent ties in the music.

Example 4.8. Three oriented networks; (a) motivic content of mm. 26 and 90; (b) motivic content of mm. 27 and 91; motivic content of mm. 92-95

Another aspect of Example 4.7 worth noting is the linear progressions that underline the entire passage (see example 4.9 below). Beginning in m. 91 and terminating in m. 95, the outer voices and one inner voice project a series of parallel 3rds and 6ths. In the example, the specific interval content of the parallel thirds and sixths are labeled inside of the staff (i.e. (3rd, 3) and
The lower register and alto register of this progression are completely chromatic, while the upper voice (played by the first violin) is diatonic to F major. The linear thirds terminate one chord earlier than the linear sixths in the outer voices. The duration of an entire bar offsets the final 6th of this linear progression—the A of the upper voice is reached on the downbeat of m. 94, while the C of the bass line is not reached until the downbeat of m. 95. This linear progression coupled with the motivic content drives the harmonic content of the passage. In fact, trying to make sense of the chord-to-chord relationships in a functional way is cumbersome, if not frivolous. The following section examines passages where the voice leading between chords is vital. It is important to note, however, the enduring relevance of motivic structure throughout the passage. This will be evident in all of the remaining musical examples.

Example 4.9. Linear progression underlying mm. 91-95.

MODULATION ZONES

Analysis of the foreground harmonic behavior of Mahler’s symphonies occurs less frequently in the literature in comparison to analyses of the song cycles. Harmony at the background and middleground occupy much of the conversation regarding the symphonies,
especially from a Schenkerian perspective. One might speculate that this is due to the complex nature of Mahler’s symphonic counterpoint, the tremendous length of his symphonic works, or his evolving compositional techniques throughout his career. Formal design lies at the center of the literary discussion regarding Mahler’s symphonies. This does not imply that discussions about foreground harmonic behavior do not exist—simply that they do not tend to serve as the primary analytical focus. Below, a detailed discussion of the harmonic content from the modulating passages of this movement will help to reveal the complex nature of Mahler’s symphonic style. The discussions above regarding the motive and its effect on harmony continue in the sections below. In Mahler’s music, it is difficult to escape the influence of the motive.

As noted earlier, Mahler juxtaposes two distantly related keys (F and Gb) for the framework of the movement. Also, as seen in Example 4.1, parallel fifths occur at the middleground level in both modulation areas—from F to Gb, and from Gb to F. The investigation of voice leading in both these passages will explain the ways in which Mahler avoids directly exploiting the parallel fifths between key areas. Example 4.10 is a score reduction of mm. 39-47. In general, the harmonic progressions that lead to Gb are relatively conservative and consist predominately of functional harmonic progressions that point to the subdominant area. As Example 4.10(a) shows, a transposition of motive d \{(\bar{5}, C), (+\bar{6}, D), (-\bar{7}, E\flat)\} is embedded in the melody carried by the first violin. Eb is the pitch on which this passage hinges, and ultimately becomes the note that obviates the problem of parallel fifths.

The first instance of motive d occurs within the first measure of the passage and also helps initiate the instability of the tonic key of F. In m. 40, the dominant of F appears in its minor

\[ \text{Example 4.10 (a)} \]

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\[ \text{Example 4.10 (a)} \]

For example, see Kaplan (1978); Berquist (1980); Lewis (1983); Forte (1984); Kaplan (1981); Williamson (1986); Williamson (1991); Darcy (2001);

Forte (1984, 161) alludes to this emphasis on the subdominant area in his graphic reduction. Measures 39-47 form an auxiliary cadence in Bb: ii\(\text{vii}\) - V\(7\) - bVI.
form (at the completion of motive d). For the remainder of passage, semitonal voice leading drives the harmonic progression. In mm. 41-42, C minor transforms into C diminished via an intermediate Eb augmented triad. This augmented triad is a byproduct of motive b in the second cello that participates in an inverted form of C diminished. Finally, in m. 45, the C diminished sonority progresses to an F dominant seventh chord—via chromatic inner voice motion—that deceptively resolves to Gb major on beat three of m. 46 (V7–VI).

Example 4.10(b) reduces this harmonic progression to one staff and illuminates the efficient voice leading Mahler uses to move from one key area to another. This graphic does not place each note in its exact registral position found within the score; rather, it is an attempt to show the most concise closed-voice position to better demonstrate the voice leading that occurs above the lowest registral voice. The Roman numerals provided highlight the subdominant’s role throughout the passage. Mahler averts the parallel voice leading from (5, C) to (6, Db) by transforming the tonic triad, F major, into a dominant seventh chord, which resolves deceptively to Gb major.

Even more intricate is the voice leading that leads the music back to the structural tonic of F major. Unlike the modulation to Gb, no clear dominant function ushers the return of F major. In fact, the listener’s ear is led astray through a myriad of possible tonal areas, each of which never resolves. F major ultimately emerges without even a semblance of dominant function. The root position Gb major triad encountered upon the modulation on beat 3 of m. 48 is the only instance of that tonic triad heard. The remainder of the passage focuses on the dominant of Gb.

The present analysis focuses on the passage beginning with the final Db dominant seventh chord, encountered in m. 59. Example 4.11(a) is a foreground harmonic reduction spanning mm. 59-72,
Example 4.10. (a) Annotated foreground, mm. 39-47; (b) voice leading, mm. 39-47
beginning with the last dominant chord that sounds in the key of G♭, the start of the transitional harmonic material.

Upon closer examination, two aspects of the passage are of interest. First, the voice leading involves a chromatic descent through a series of dominant seventh chords, terminating on a prolonged A dominant seventh that is denied any proper resolution. Second, Mahler manipulates the augmented sixth relationship that the D♭ dominant seventh has to the structural tonic of F. These two characteristics of the passage are demonstrated in Example 4.11(a) and the middleground reduction of 4.11(b). In 4.11(a), figured bass notation shows how Mahler offsets each dominant seventh chord through a series of suspensions. This technique obviates the direct parallel fifths and octaves that would otherwise occur between each chord. The suspensions also represent instances of motive c, the first time in the movement this motivic fragment drives the contrapuntal texture. This provides a fitting balance for the abundance of motive b, which led to the ultimate ascent up a half step from the structural tonic (refer again to Example 4.10).

The D♭ dominant seventh chord, when first heard in m. 59, functions as V in the key of G♭. A series of descending chromatic dominant seventh chords function as passing chords that lead ultimately to the G♯ in the bass in m. 71, which supports an enharmonically respelled D♭7, now in second inversion (C♯ 4/3). The enharmonic respelling disguises the function of this sonority, but serves to distinguish it from its counterpart in m. 59, which was originally spelled as V in G♭ major. The C♯ dominant seventh chord is best interpreted as a German augmented sixth chord in the global tonic F major. G♯ then descends to G♯, which serves as the bass of a contrapuntal sonority that resolves directly to F major. The oddly spelled, irregularly voiced German augmented sixth is treated in a non-functional manner, since it resolves not to a
dominant, but rather directly to the tonic.\textsuperscript{13} The only glimpse Mahler provides of a dominant function is the passing $G^\#$ on the third beat of m. 71. Thus, the pivot transformation (-2nd, $e$) sends the first spelling of the $Db$ dominant seventh chord to its second enharmonic spelling. In the set analysis of 4.11(c) the enharmonic spelling of the second $Db$ dominant seventh chord (spelled as a C$\#$ dominant seventh) is ignored for the sake of clarity.

The goal of this analysis is to bring continuity to a passage that is otherwise rendered obscure and convoluted through intense chromaticism. When hearing this passage in real time it may be rather difficult to actively hear the motivic and harmonic relationships proposed in the present analysis. Following the final $Db$ dominant seventh chord, any sense of tonic or dominant is abandoned. Throughout the harmonic excursion, retaining a sense of the previous tonic sonority, or clearly predicting the path to the coming tonic return, is quite difficult. Thus, once the enharmonically spelled $Db$ dominant seventh chord is reached (C$\#$ dominant seventh), hearing it as a back-related dominant (of $Gb$) or a German augmented sixth chord (in F) can only occur with advanced knowledge of the passage.

The analytical examples extracted from this movement confirm the importance and role of the motive throughout Mahler’s music. As in his earlier compositions (e.g. the piano quartet analysis of Chapter 3), the motivic content is intimately intertwined with the harmonic content. Not only does the motive serve as the foundation of much of the melodic character of the music, it also underpins its harmonic character. Although Mahler’s melodic process is quite complex and intricate, one can also understand it as an outgrowth of a single melodic idea, which he usually presents at the outset of the piece. While scholars have recognized the harmonic

\textsuperscript{13} Note the similarities between the way this chord functions as a dominant substitute and the way the German augmented sixth chord resolves to tonic at the recapitulation of the piano quartet. Refer to Example 3.11 in Chapter 3.
functionality of Mahler’s tonal language at the middleground and background levels, it seems clear from the selected excerpts that the surface voice leading of Mahler’s works is more complicated and deserves the same attention.\(^{14}\)

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\(^{14}\) For discussion regarding the diatonic structure and harmonic function of Mahler’s music see the following: Agawu (1997); Agawu (1992); Agawu (1986a); Agawu (1986b); Agawu (1983); Berard (2010); Kaplan (1981); Kaplan (1978); Lewis (1984); Williamson (1997).
Perhaps among Mahler’s most beloved and most performed works is the orchestral song cycle *Kindertotenlieder* (“Songs on the death of children”). Identifying an exact date of composition for each song evades even the most respected Mahler scholars. Mitchell (1975, 35) says that the composer’s widow provides the most convincing chronology. Alma indicates in a diary that Mahler composed the final three *Kindertotenlieder* songs in the summer of 1904.\(^1\) Regardless of the exact year of composition, “Nun seh’ ich wohl, warum so dunkle Flammen” falls during Mahler’s second compositional period. This song was selected for the present study because of its dramatic use of chromatic sonorities and its dependency on non-tonal harmonic behavior. The tonal-chromatic system is especially suited for this type of material. Unlike the previous two analyses, where functional harmonic behavior controlled much of the musical surface, this song shows Mahler’s more adventurous compositional style, leading to the eventual breakdown of the tonal system. In this chapter, because tonality becomes more obliterated, the analysis leans heavily upon the transformational operations that connect harmonic and melodic entities, operations that do not necessarily occur in functional tonal music. As demonstrated in the analysis below, the melodic and harmonic characteristics of the song are intimately related, much like the analyses of Chapters 3 and 4. Like the analysis of the Adagietto, aspects of Schenkerian analysis will work in tandem with the tonal-chromatic system to yield an insightful

\(^1\) Indeed, there are many other conflicting accounts of the dates of composition. Mitchell (1975) discusses this in detail. Guido Adler, for example, attributed two of the songs to the years of 1900-1901 and the latter three songs to 1901-1902. This is much earlier than Alma’s recollection. Frau Mahler accounted for a total of six songs that were not finished until 1905.
analysis that investigates the chromatic techniques Mahler employs to accurately depict the dark and unsettling nature of the text setting.

Example 5.1 is an overview of the formal design as it relates to the text. Overall, the work is composed in ternary form (ABA) with an introduction before the first A section and a coda following the restatement of A. As Agawu (1983) notes, the fourteen lines of Rückert’s sonnet break down into two smaller sections of eight lines (octave) and six lines (sestet). Mahler further divides the sestet into two (B), and four (A₁) line units. The rhyme scheme of the octave is ABBA ABBA and that of the sestet CDCDCD. Mahler sets each of the rhyming words to

<table>
<thead>
<tr>
<th>Formal Section</th>
<th>German Text</th>
<th>English Translation</th>
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<tbody>
<tr>
<td>Intro</td>
<td>Nun seh' ich wohl, warum so dunkle Flammen Ihr sprühtet mir in manchem Augenblick. O Augen! [O Augen!] gleichsam, um [voll] in einem Blicke Zu drängen eure ganze Macht zusammen.</td>
<td>Now I see well why with such dark flames your eyes sparkled so often. O eyes! it was as if in one full glance you could concentrate your entire power.</td>
<td>1-4</td>
</tr>
<tr>
<td>A</td>
<td>Doch ahnt' ich nicht, weil Nebel mich umschwammen, Gewoben vom verblendenden Geschicke, Daß sich der Strahl bereits zur Heimkehr schicke, Dorthin, [dorthin], von wamen alle Strahlen stammen.</td>
<td>Yet I did not realize, because mists floated about me, woven by blinding fate, that this beam of light was ready to be sent home to that place whence all beams come.</td>
<td>5-36</td>
</tr>
<tr>
<td>B</td>
<td>Ihr wolltet mir mit eurem Leuchten sagen: Wir möchten nah dir bleiben gerne!</td>
<td>You would have told me with your brilliance: we would gladly have stayed near you!</td>
<td>35-45</td>
</tr>
<tr>
<td>A₁</td>
<td>Doch ist uns das vom Schicksal abgeschlagen. Sieh' [uns nur an], denn bald sind wir dir ferne! Was dir [nur] Augen sind in diesen Tagen: In künf'gen Nächten sind es dir nur Sterne.</td>
<td>But it is refused by Fate. Just look at us, for soon we will be far! What to you are only eyes in these days, in future nights shall be stars to us.</td>
<td>44-67</td>
</tr>
<tr>
<td>Coda</td>
<td></td>
<td></td>
<td>67-74</td>
</tr>
</tbody>
</table>

Example 5.1. Text and formal design of Gustav Mahler, *Kindertotenlieder*, no. 2

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2 The bracketed text in Example 5.1 represents the alterations Mahler made to the original Rückert text.

3 Agawu (1983, 82) and Berard (2010) indicate a similar formal design.
the same type of musical figuration: either an appoggiatura ending with a descending step, a suspension, or an accented descending passing tone. This is true with the exception of stammen, which ends the first eight-line portion, and Sterne, which ends the final six-line portion of the poem. In much the same way as the Adagietto, these accented non-chord tones that punctuate the rhyme scheme of the sonnet derive their meaning from motivic cells established at the outset of the song. Furthermore, three clear cadential 6/4 chords occur throughout the song—each of which involves the three references to “light,” Strahlen, Leuchten, and Sterne (Agawu 1983, 83).

The analysis below explores the different ways that Mahler uses motivic and harmonic content to accent the rhyme scheme of the poem.\(^4\)

**BACKGROUND HARMONIC STRUCTURE**

From a Schenkerian standpoint, this song poses a predicament—a clearly articulated *Urlinie* is absent. In fact, there is no structural descent from the *Kopfton* (\(\hat{8}, \hat{5}, \text{or} \hat{3}\)) to \(\hat{1}\). Nonetheless, Schenker-influenced graphic reductions remain a fruitful way of visualizing the background and middleground structure. In this song, the structural top line begins and ends on the same scale degree (\(\hat{3}\)), not only (\(-\hat{3}, E_b\)), but also (\(+\hat{3}, E\)—emphasizing the chromatic nature of the song from the background to the foreground. Example 5.2 shows a deep middleground graph of the entire song. The *Kopfton* on \(\hat{3}\) alternates between its minor and major form—the minor form of the *Kopfton* frames the songs ABA\(^1\) structure; the major form occurs inside the structure. Amidst tonicizations and dense chromaticism, the background tonic of C never loses its status.

The ascent to the *Kopfton* begins during the introduction (mm. 1-4), with a stepwise octave ascent from \(B_b3\) to \(B_b4\), carried by the cello. In m. 5, the voice enters and carries the rest

\(^4\) For further exploration of the text setting see Agawu (1983) and Rushing (2002).
of ascent from B♭4 to E♭5, the Kopfton, which is not supported by tonic harmony. The supporting harmony for the arrival of the E♭ is an A♭ major triad supported by a G in the bass, which is not a functioning chordal seventh in the bass. At first, this may seem like an arbitrary or distant harmony to support the arrival of such a structural tone. In context, however, it is not too far removed from the tonic. The song begins with an off-tonic opening on G; G is sustained as a pedal tone over the course of the first nine measures. In measure 6, when E♭ is reached, (♭6, A♭) acts as a deceptive cadence supported by the dominant scale degree in the bass. It is not until m. 15 that a root position tonic triad (in its major form) emerges, at which point the top line has worked its way into an inner voice.

This motion in and out of an inner voice is what defines the song’s deep middleground structure. As seen in the graphic reduction (Example 5.2), the Kopfton recurs at salient points throughout the song. In between the iterations of the Kopfton, the structural line descends into an inner voice and then works its way back up. Of more interest, though, are the forms of harmonic punctuation that Mahler uses to approach and leave the tonic areas. Mahler tends to avoid clear dominant-tonic relationships altogether and this is one of the defining aspects of the surface harmonic character. Mahler avoids providing the listener with clear harmonic landmarks; when
the tonic emerges, Mahler tends to occupy it with chromaticism that undermines the tonic’s stability and clear function. It is not until the final measures of the song that the listener gets a stable tonic triad. The following section examines each of these areas in detail. Before examining the harmony and voice leading, it is necessary to discuss the motivic and melodic content of the song, which strongly influences the nature of the harmony.

**Motivic Process**

Referencing this song, Agawu (1983) states that the melodic structure is a process of generation, whereby an initial motivic cell is manipulated constantly throughout the duration of the song. Even more so than the analyses presented in the two previous chapters, the melodic content throughout the song stems from the initial motivic idea presented during the first two measures of the introduction. Forte (1984) provides a brief motivic analysis that shows the main motives and how they relate to one another. A few aspects of this motivic analysis are incorporated into the present analysis, specifically aspects of labeling. One should note, however, that the detailed analyses by Agawu and Forte both separate the melodic and motivic processes from the harmonic process. While it is possible to view them as separately functioning entities, it is more fruitful to understand them in the context of one another. Thus, the motivic content will remain an important aspect of this analysis, even throughout the section that discusses the foreground harmonic content.

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5 As Agawu (1983, 86) points out, this melodic process is not only visible on the score, but Mahler was keenly aware of this process in his compositional process. Agawu provides three quotes where Mahler refers to the role of the generating motivic cell, from which all other melodic content materializes.

6 What Agawu called the process of generation is synonymous to Schoenberg’s term “Developing variation.”
Example 5.3(a) shows the opening of the song. Within the confines of these six measures, Mahler introduces two prominent motives that define the melodic structure of the entire song. Example 5.3(b) extracts the motivic content of this passage and labels each one accordingly. The cello presents the main motivic cell, labeled $a$, in mm. 1-2. The principal motive is an ascending stepwise figure that spans a perfect fourth, five semitones, (4th, 5). Embedded in this motive is a secondary motive ($b$), an ascending stepwise motion that spans a minor second (2nd, 1). It is important to note the circumstances surrounding motive $b$. In order for any stepwise second to be eligible for motivic consideration on the musical surface, it must participate in some type of non-chord tone behavior, for example a suspension, retardation, or appoggiatura. This requisite

Example 5.3. (a) Gustav Mahler, *Kindertotenlieder*, no 2, mm. 1-6, score reduction; (b) motivic content derived from mm. 1-6
behavior is exactly the same as the stipulations placed upon the motivic content in the Adagietto. In mm. 3-4, the cello presents a variation on the main motive (labeled $a'$ in Example 5.3(b)). This presentation of the motive is contracted by a half step—instead of outlining a perfect fourth, it now outlines a diminished fourth (4th, 4). The transformation sending $a$ to $a'$ is $T_g5$.

Beyond the three motivic cells shown in Example 5.2 ($a$, $a'$, and $b$), there is one other motivic derivative that appears as an important feature on the musical surface. This motive, labeled $c$, is the inversion of motive $b$—a descending step. It takes one of two forms, either a whole step (2nd, 2) ($c$) or a semitone (2nd, 1) ($c'$). Again, in order for motivic consideration, this motion must participate in a non-chord tone figuration: a suspension or appoggiatura. Motive $c$ gains prominence as a form of vocal and harmonic punctuation. Together, the three primary motivic forms explain the melodic construction of the song. Likewise, the motives play a considerable role in the harmonic construction. Thus, for now, it will suffice to leave the discussion of the motivic structure and move forward to discussions about the harmonic structure.

**FOREGROUND HARMONIC STRUCTURES**

The deep middleground and background harmonic structure of “Nun seh’ ich wohl, warum so dunkle Flammen” is relatively consonant, maintaining the tonic (C) throughout the duration of the song. The foreground harmonic content is quite different—exploring densely chromatic structures and employing voice-leading procedures that are inherent in Mahler’s mature compositional style. Again, Mahler weaves his network of motivic relationships into the harmonic content. To begin the exploration of this concept, the analysis will focus on harmonic framing and punctuation, beginning with the opening four measures of the introduction, followed by an examination of the closing six measures of the coda. Despite what other analysts have
written about the final cadence, it is possible to extrapolate the ultimate trajectory of the song’s voice-leading content from the opening harmonic structure. Following these two passages, the analysis will examine other forms of harmonic and lyrical punctuation and the different types of cadential processes that Mahler uses to segregate the phrases.

Returning to Example 5.2, the deep middleground graph shows that the song begins off tonic, on the minor form of the dominant (G). The first consonant triad (G -) is not reached until the third beat of m. 4, where it is accompanied by a chordal ninth that resolves upwards, not downwards, due to the demands of motive $a'$. Different analyses label the opening vertical sonority of m. 2 in various ways. For example, Forte (1984) dismisses any discussion of this chord altogether. Berard (2010) labels this chord as a German augmented sixth chord in first inversion; this, however, is not an accurate representation of its function. Further, Agawu (1983) treats everything prior to the first consonant triad as non-chord tones. For the present analysis, Agawu’s perspective is the most relevant. One can reduce the first four measures to one underlying harmony that is elaborated with non-chord tones, all generated by the motivic cells introduced in the opening two measures. Example 5.4 is an orchestral reduction of this passage and shows how the motivic content interacts with the harmony. In the example, the filled-in note heads represent non-chord tones, those foreign to the prevailing G minor triad.

Example 5.4. Voice-leading reduction of Gustav Mahler, Kindertotenlieder, no. 2, mm. 1-4
It is interesting to note the two different ways Mahler treats the dissonance created by the motivic cells. The first and third notes of the first motivic cell, \( a \), are chord tones, while the second and fourth notes are non-chord tones. After rearticulating the chord tone (d), the motive terminates on the second non-chord tone (\( E_b \)). Because of the non-chord tone involvement in m. 2, this two-note cell \( \{(\hat{5}, D), (-\hat{6}, E_b)\} \), is also considered as a member of the smaller motivic cell, \( b \). Due to the dissonance created by the C\# in the lower register, pushing past the chord tone is not an unsatisfactory sound. In fact, both the D and \( E_b \) are involved in dissonant intervals against the prevailing vertical sonority, but moving to \( E_b \) opens up the interval space against the C\#.

The second iteration of motive \( a \) in mm. 3-4 has an complementary format. Here, a non-chord tone initiates the motive and the motive terminates on a chord tone. On beat three of m. 3, the third note of the motive is rearticulated, this time, as a non-chord tone (A) supported by a consonant sonority. Unlike the previous presentation of the motive, the supporting vertical harmony is a complete G minor triad. The only dissonance now comes from the non-chord tone of the motive. Finally, upon its resolution on beat three of m. 4, the first unblemished consonant sonority emerges; it is clear, though, that this harmony governs the duration of the passage, with numerous non-chord tone decorations.

The introductory measures of the song provide musical insights that unfold throughout the remainder of the song. When examining the final cadence of the coda, one can make sense of the voice leading by comparing it to the introductory material. Example 5.5(a) is a score reduction of the final six measures of the song. There are two valid ways of describing the voice-leading structure of this passage, one of which is a stronger interpretation that has precedence from an earlier passage in the song. Analysts traditionally note the lack of a traditional cadence, the absence of any dominant closure. For Agawu (1983) the cadential material begins in m. 72.
The author provides two different readings, both implying no dominant-to-tonic cadence. Example 5.5(b) shows this interpretation. Here, the tonic is achieved in m. 72 but is elaborated by various non-chord tones that resolve into a consonant triad in m. 73. The present analysis will return to this interpretation in a moment. Example 5.5(c) shows the second interpretation that Agawu (1983) provides. Here, the vertical sonority articulated on the downbeat of m. 72 represents a German augmented sixth chord (with the tonic in the bass). This chord resolves directly into the tonic triad. Berard (2010) agrees with the interpretation in Example 5.4(c).

While the interpretations provided in Examples 5.5(b) and (c) show important aspects of the voice leading, they do not necessarily depict the entire cadential landscape. Including the two previous measures (mm. 69-70) provides a more complete cadential procedure. The passage does integrate a dominant substitute; specifically, the active tritone that is inherent in the traditional dominant sorority, \{4, F, (+7, B)\}, is present. This tritone, however, is part of a different harmonic structure, an enharmonically respelled D♭ dominant seventh chord. Example 5.5(d) shows a reduction of the passage that includes mm. 69-74. The inclusion of the D♭7 chord provides a more comprehensive view of the cadence that is absent from other analyses.\(^7\) In order to obviate the problem of parallel fifths and octaves on the musical surface, Mahler resolves the dominant substitute into a different register. Likewise, the problem is avoided by the use of the non-chord tone sonority that occurs on the downbeat of m. 72.

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\(^7\) Some analysts call this chord a tritone substitution. The basis of tritone substitution is that the active interval of the tritone in any dominant seventh chord is shared by another dominant seventh chord whose root is a tritone away (Biamonte, 2008). Traditionally, this terminology is reserved for analysis of jazz harmony and is less applicable to the Viennese canon at the turn of the twentieth century.
Example 5.5. (a) Gustav Mahler, *Kindertotenlieder*, no. 2, mm. 69-74, score reduction; (b) voice leading of the final two vertical sonorities as an elaboration of the tonic; (c) voice leading of the final two sonorities as two independent sonorities; (d) voice leading of mm. 69-74
It should also be noted that the motivic content in mm. 69-74 is a transposition in CSD space of the motivic content from the opening four measures (T:5). The presence of this motivic content lends credence to the idea that the vertical sonority heard in m. 72 is comprised of mainly non-chord tones, in the same fashion as the introduction. Unlike the introductory measures, the incorporation of a second harmonic structure brings closure to the song. It would be unwieldy to try and consolidate all of the vertical material of mm. 69-74 into a single harmonic entity (as with the introduction).

The D♭ dominant seventh chord appears at an earlier moment in the song, offering more context for its use during the coda. The passage in question, mm. 10-15, occurs at the moment Mahler chose to make a small addition to the original Rückert text and is also the first harmonic punctuation that leads to the first root position tonic triad (major). Similarly to the coda, a correctly spelled dominant seventh chord built on (−2, D♭) acts as the cadential harmony leading directly to the tonic. The operative tritone between (4, F) and (+7, B) is present, but is enharmonically recast as a diminished fifth rather than an augmented fourth. Nonetheless, C♭ still aurally provides the essence of the leading tone. Again, to avoid problematic voice leading, the resolution of the individual components of the D♭ dominant seventh chord occurs in different registral positions.

Example 5.6(a) shows a score reduction of the passage. The text, *O Augen! O Augen!* (My eyes! My eyes!), is an expansion of the original text, which only states the phrase one time. The motivic content is similar to the introduction—two iterations of motive $a^j$ drive the melodic and harmonic content. The difference in this passage is its setting in CSD space. The two iterations of the motive are chromatic transpositions, as is the relationship between the two vertical sonorities that lead to the root position tonic. The transformation that sends the material
From mm. 10-11 to mm. 12-13 is $T_3$. Because of the exact transposition in CSD space, both iterations of the motive involve the same treatment of non-chord tones. Each one begins on a non-chord tone, passes through a chord tone, and upon arrival of the downbeat, lands on a second non-chord tone; this second non-chord tone then resolves upward by semitone to a chord tone. As the song continues, the subtle manipulation of this cadential pattern continues. Below, one other exemplary passage is examined to further demonstrate the subtle manipulation of this cadential formula.

Example 5.6. (a) Gustav Mahler, *Kindertotenlieder*, no. 2, mm. 10-15, score reduction; (b) voice-leading reduction of mm. 10-15
The next point of harmonic punctuation occurs at the end of the first quatrain, mm. 17-22, a score reduction of which is shown in Example 5.7(a). Here the harmonic and motivic mechanism used is similar to mm. 10-15. During this cadence, though, the vertical sonorities project a more traditional diatonic progression, albeit in an unusual way. The goal of the harmonic progression is to reach the traditional dominant seventh chord in the key of C (G7). In m. 17, during the presentation of motive \(a\), a semitone lower than its original pitch level of the introduction, the underlying harmony is D minor (ii). During this passage, a new feature occurs—the interjection of a third harmonic sonority in between the two motivic cells. This sonority adds an interesting feature to the overall harmonic progression. Following the D minor harmony is a German augmented sixth chord that would typically resolve to an A major triad or seventh chord (V or V7 of ii). Instead of a traditional resolution, this chord slides to an F minor triad via efficient voice leading. The G\# of the German augmented sixth chord is recast as an A\(_b\), now acting as the third of the F minor triad in first inversion. The F minor triad in first inversion supports the second motivic cell (now \(a'\) because of the transposition in GSD space at T\(_g\)3).

The first-inversion F minor triad is the final sonority of the phrase. The A\(_b\) in the bass resolves across the phrase boundary to G—a motion already encountered in the two cadences involving the semitone motion D\(_b\)-C. What is unusual about this point of punctuation is whether or not it is reasonable to call it a cadence. When the next phrase begins with the text “Doch ahnt’ ich nicht,” the first-inversion F minor triad resolves to a G dominant seventh chord (with an 8-7 and 6-5 suspension). Does this constitute a half cadence on G? No, there is no cadential material preparing the arrival of G, nor does it come to rest on the chord. This is particularly evident when taking the vocal line into consideration. If one looks one measure further, a better explanation of the progression emerges. In m. 23, the G7 resolves deceptively to A\(_b\) major. Beyond the
interesting voice leading, the use of the same motivic procedure, and the same descending semitone in the bass line, the most unusual aspect of this passage is the metric displacement of the cadential resolution. One would typically expect the arrival on V to occur at the end of the phrase. Instead, Mahler displaces the alignment of the text and the harmonic punctuation. As the previous phrase achieves melodic and lyrical closure in mm. 19-21, the harmonic phrasing continues into m. 23; the voice has already begun the next line of the text. In m. 19, one should also note the #4-3 appoggiatura provided by the vocal part that supports the word *zusammen*, which is an important component of the rhyming pattern.

Example 5.7. (a) Gustav Mahler, *Kindertotenlieder*, no. 2, mm. 17-22, score reduction; (b) voice-leading reduction of mm. 17-22
As mentioned at the beginning of the chapter, there are three points in the song where clearly articulated cadential 6/4 chords occur. Unlike the passages discussed above, these three moments all involve the appropriate resolution of German augmented sixth chords. Perhaps more interesting is the fact that none of the cadential 6/4 harmonies participate in cadences—as quickly as they appear, each chord goes unresolved. Example 5.8 is a ten-measure passage that shows the approach to, arrival on, and departure from two of the cadential 6/4 chords in the song. As Agawu (1983) notes, these chords help create the effect of stability and provide tonal orientation, even without their traditional resolutions.

Following a correctly spelled German augmented sixth chord in the key of C, a strongly articulated cadential 6/4 chord occupies m. 34. In m. 35, the cadential 6/4 resolves to an F# diminished seventh chord (with G still in the bass). This alone is not unusual—many times this chord prolongs the cadential procedure as viiº7 of V. This is partially true, here. The next chord in m. 36 is another G dominant seventh chord, in second inversion. It seems at this point that the cadence is dissolving. The cadential harmony gets weaker with each passing measure. In m. 37, the hope for a cadence stemming from the cadential 6/4 in m. 34 has vanished. Via efficient voice leading, the G7 chord in second inversion slides to a Db7 chord—yet another occurrence of this sonority. The Db7 then resolves directly to Bb7, which serves as a correctly resolving German augmented sixth chord to a cadential 6/4 supported by A. One should note the manipulation of the motivic content in this passage is more dramatic than anywhere else in the song. The outer interval of the motivic material in mm. 35-36 and 38-39 is compressed to a minor third. This form of the motive appears only a few times and is labeled aⅠⅠ.

Returning to Example 5.6 (mm. 10-15), these two chords, Db7 and Bb7 have already had direct interaction. Previously, Bb7 acted as a quasi-predominant harmony that resolved to Db7,
which was the cadential harmony leading to the tonic. Examples 5.9(a) and (b) show the transformational relationship between these two sonorities. 5.9(a) is another representation of the transformation shown in Example 5.6(b); 5.9(b) shows how this progression functions in mm. 37-41. In 5.6(b) the transformation sending B♭7 to D♭7 was T₃. 5.9(a), however, shows the motion of the individual pitch-class content—there are two common tones held while two voices
move in contrary motion by semitone. For the latter of the two progressions, a double transformational procedure is necessary due to the coming tonic shift from C to D. The first transformation sends the pitches onto new scale-degree identities, \((\hat{2}, D) \rightarrow (\flat 1, D)\) and \((+\hat{7}, B) \rightarrow (+\hat{6}, B)\) via \((+2nd, 1)\). Note that the \(\flat 1\) is merely an enharmonic respelling of \((+\hat{7}, C\#)\) in the coming key of D. Then, independent interval transformations send each of the notes to a new (sd, pc) location. Like the cadential 6/4 chord of m. 34, the 6/4 chord of m. 41 does not participate in a traditional cadence, let alone does it ever resolve to a V chord in D.

![Example 5.9. (a) Alternative voice-leading analysis of mm. 11-13; (b) voice-leading analysis of mm. 37-40](image)

The examples given above show a few of the ways the musical landscape avoids the use of traditional harmonic punctuation—even at some of the most important points in the overall formal structure. This evasion of consonance also occurs at the important moments of the structural line (Urlinie). Before the final cadence of song, there are three points where the Kopfion emerges in the musical structure. Chromatic insertions interrupt the stability of each

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8 This shift in tonic is a feature of the foreground that disappears at the first middleground level. It is necessary, however, to include this tonic change in the analysis to fully understand the harmonic behavior of the Bb7 chord.
appearance of the Kopfton prior to the final cadence. The emergence of the Kopfton (-\text{3}, E\flat) in m. 6 occurs over a dominant pedal and is harmonized by the non-tonic harmony bVI (see the discussion during the beginning of the section on background harmonic structure). Of more interest are the two other moments during the song where the Kopfton emerges as (+\text{3}, E). At both of these moments, chromatic inflections of the tonic sonority undermine the stability of the structural tone, further destabilizing the overall background structure of the song.

The two passages during the song where the Kopfton appears as (+\text{3}, E) are shown in Examples 5.10(a) and (b). In both passages, the tonic is prepared by the normative dominant seventh chord sonority. The peculiarity of the passage, though, rests in the quality of the tonic triad. At the moment where the listener expects an arrival on a stable tonic sonority, Mahler undermines its stability through the use of a an augmented triad instead of a major triad. Nonetheless, the presence of the Kopfton, the dominant structure preceding it, the root position triad, and their placement in the text all indicate the importance of these two moments in the song. The first moment punctuates the penultimate line of the first half of the poem and the first A section. The Kopfton in m. 30 occurs over the word Schicke, within the phrase, “der Strahl bereits zur Heimkehr schicke,” (that the beam would already be returning home) and is approached via an appoggiatura. During the second moment of the song, the Kopfton in m. 62 appears over the word Tagen (day), again setting up the final line of the poem and the conclusion of the final A section. In both passages, the augmented triad never resolves to a consonant triad. This represents one of the few moments in Mahler’s music where an augmented triad operates as a standalone musical object. In fact, at both moments, (\#5, G\#) lingers in the musical texture beyond the resolution of the augmented triad, denying any sense of dissonant resolution to the tonic sonority.
It is evident from the present analysis that harmonic instability is not only evident in the background and deep middleground structure of the song, but is also a continuing manifestation of the foreground harmonic content. It is not until the final four measures of the music that a stable tonic triad occurs; even then, it is riddled with chromatic structures. This lack of stability speaks to the nature of the text—a monologue-type poem about a parent having to release their child to an early death. In that moment, there is no stability of emotion or in the physical reaction to such a loss. Mahler captures this emotion brilliantly, and the lack of harmonic stability does not undermine the continuity of the song, or its sense of closure.

Example 5.10. (a) Gustav Mahler, Kindertotenlieder, no. 2, mm. 29-31, score reduction; (b) Kindertotenlieder, no. 2, mm. 61-63, score reduction
CHAPTER 6
CONCLUDING REMARKS

The three analytical chapters of this dissertation demonstrate the necessity for a system that explores a tonal language that employs the chromatic universe as its structural basis. The music of Gustav Mahler is especially relevant for exploring these spaces. While the pillars of functional tonality are still operative in his music, much of the musical landscape, from the foreground to the background, employs the chromatic universe as a structural entity. In order to bring together these two systems—functional tonality and the chromatic universe—this dissertation developed a system wherein both systems exist in tandem. It builds upon the work set forth in Mitchell (1962), Marra (1986), Proctor (1978), and McCreless (1996). Likewise, the system theorized in this dissertation combines the ideas posited by the aforementioned authors with the contemporary concepts of transformational theory initiated by Lewin (1987). Perhaps the research that exerts the most influence on the present work is that of Rings (2011). Rings’s tonal GIS is grounded in the diatonic system; using this as a point of departure, the present system expands this concept to a tonal GIS that is occupied by the chromatic system—one where all twelve pitch classes represent scale-degree identities.

The methodology for such a system is set forth in Chapter 2, wherein a GIS for the tonal-chromatic scale is established. This GIS situates the models of functional tonality inside the chromatic gamut. The goal is to demonstrate that the use of chromatic structures does not have to undermine the functionality of tonality. In the Classical tonal system the diatonic scale serves as the backbone and all chromatic inflections serve as derivatives or inflections of some diatonic identity. In the tonal-chromatic system, this is not the case—each member of the chromatic universe has its own independent scale-degree identity, not attached to any other pitch class. Also, in this system, it is possible for different inflections of the same generic scale degree to
exercise structural status. Furthermore, scale degrees that do not occur in the diatonic system, such as $\flat 2$, are able to serve as background harmonic structures. In previous analytical systems of tonality, these scale-degree structures are subsumed into the diatonic system—the further removed from the musical foreground one gets, chromatic structures become derivatives of the diatonic scale.

Over the course of his lifetime, Gustav Mahler produced works that continue to serve as staples in classical-music venues around the world. From the onset of his career as a student at the Vienna Conservatory, Mahler’s music exhibited an innovative use of chromaticism, one that aided in the eventual disintegration of the system of functional tonality. Chapter 3 explores the one of the earliest known works by Mahler, the Piano Quartet in A minor, composed as a student at the conservatory. More so than his later works, the piano quartet exhibits more conventional harmonic behavior. Nevertheless, embedded in the fabric of functional tonal structures, Mahler exploits the chromatic nature of a small motivic cell. The analysis goes on to demonstrate the importance of the symmetrical division of the octave at the interval of a tritone. This occurs in two important places. First, during the primary thematic area, the harmonic trajectory progresses from (I, A) to (TT, E♭), back to (I, A). Likewise, at the return of the recapitulation, the cadence exploits this interval in the bass motion upon the return to the tonic. This motion is coupled with efficient voice leading as a means of replacing a traditional functional dominant.

Chapter 4 explores Mahler’s chromatic techniques in the fourth movement of the fifth symphony, Adagietto. This chapter begins with a look at the way Schenkerian analysis applies to the piece and then compares and contrasts how this perspective and a transformational perspective can combine to provide an insightful analysis. Although the background structure of the movement adheres to many of the Schenkerian principles of tonal structure, much of the
musical surface involves chromatic techniques that Schenkerian reductions do not capture. The analysis also focuses on two important aspects of the music. First, the contrapuntal importance of the motive, and how the motive drives the underlying harmonic behavior; and second, the voice-leading techniques used to modulate from F as tonic to G♭ as tonic and vice versa. As the analysis shows, the motivic cell that is introduced in the opening measures on the foreground is evident even at the deepest middleground levels.

Chapter 5 concludes the analytical portion of the dissertation with an examination of the second song from Kindertotenlieder, “Nun seh’ ich wohl, warum so dunkle Flammen.” In the footsteps of Chapter 4, concepts of Schenkerian analysis work together with the transformational system in order to gain a deep understanding of the tonal structure. As seen in the analysis, this song exhibits some of Mahler’s most entrenched chromaticism. It is not until the very last few measures that the first unblemished form of the tonic triad emerges. The analysis explores the various ways that Mahler evades tonal stability, namely the absence of traditional dominant structures. In this song, a number of different vertical structures serve as the operative dominant, most of which do not involve a root motion of ascending fourth or descending fifth. Even the cadential points where fifth motion is used, the harmony to which they resolve are undermined by chromaticism.

All of Mahler’s oeuvre is ripe for this type of analytical exploration. Furthermore, the works of many of Mahler’s contemporaries exhibit the same chromatic techniques. The goal of this dissertation is to establish one type of methodology that successfully illuminates and demonstrates the voice-leading mechanisms that drive the chromatic behavior. As seen in each of the analytical chapters, the tonal-chromatic system is useful in the creation of engaging analyses, from music that consists of functional tonality immersed in chromaticism to music that veers
away from functional tonality, but still employs melodic and harmonic entities familiar to the
tonal system. Furthermore, the tonal-chromatic transformational system is not intended for use in
isolation. As is evident in Chapters 4 and 5, it pairs well with aspects of Schenkerian analysis.
When used in tandem, the two systems produce fruitful analyses of the chromatic tonal music
from the late nineteenth century. The hope is to invigorate the discussion of tonality and
chromaticism that surrounds the music of the late Romantic era bridging the gap into the
twentieth century.
ABBREVIATIONS


________. “Chromaticism and Tonal Coherence in Liszt’s Sonetto 104 del Petrarca.” In Theory Only 7, no. 3: 3-19.


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VITA

Andrew Nicolette spent the majority of his childhood in Leesburg, Virginia, a suburb of Washington DC, and currently resides in Baton Rouge, Louisiana. He began his musical career by studying classical guitar, and went on to complete a Bachelor of Arts in music composition at Shepherd University in Shepherdstown, West Virginia in 2009. Following his undergraduate studies, Nicolette completed his Master of Arts in music theory from the Catholic University of America in 2011. His master’s thesis, entitled, “An Analytical Study of Leo Ornstein’s Second String Quartet,” was completed under the advisement of Dr. Steven Strunk. Nicolette expects to complete his Doctor of Philosophy in music theory from Louisiana State University in May of 2015, with a minor in music composition. Nicolette has presented his research on Gustav Mahler at regional conferences throughout the United States, and has taught music theory, aural skills, and music fundamentals at Louisiana State University and the Catholic University of America.