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Experimental Sloshing Studies in Sway and Heave Base Excited Square Tanks

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EXPERIMENTAL SLOSHING STUDIES IN SWAY AND HEAVE BASE EXCITED SQUARE TANKS

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Civil Engineering

in

The Department of Civil and Environmental Engineering

by

Wei Peng

B.S., Central South University, China, 2001

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LIST OF NOMENCLATURE

a	- general forcing amplitude of shake table for different excitation [m]
a_i	- acceleration of external forcing, $i = x, y, z$ [m/s ²]
a_{hx}	- horizontal forcing amplitude of shake table in surge direction [m]
a_{hy}	- horizontal forcing amplitude of shake table in sway direction [m]
a_v	- vertical forcing amplitude of shake table in heave direction [m]
b	- inner base dimension of tank in sway direction [m]
$Fr (U_0)$	- Froude number of bore; define as $U_0 / \sqrt{gh_1}$
$Fr (u_y)$	- Froude number of traveling wave; define as $u_y / \sqrt{gh_1}$
g	- gravitational acceleration [m/s ²]
h_0	- still water depth inside the tank [m]
h_1	- developed water depth behind the bore [m]
h_2	- developed water depth in front of the bore [m]
h_3	- developed water depth in front of the upper bore of diagonal propagating bore [m]
h_4	- developed water depth behind the lower bore for diagonal propagating bore [m]
l	- inner base dimension of tank in surge direction [m]
n	- wave number $n = 0, 1, 2 \dots$
p	- pressure of fluid flow [Pa]
S	- wave steepness ($k_n \times \zeta_{\max}$)
t	- time [s]
T	- non-dimensional time, define as $T = t \times \omega_1$

T_i	- free-surface tension, $i = x, y, z$ [N/m]
U_0	- propagation speed of the bore [m/s]
u	- velocity of free-surface wave in x direction [m/s]
u_y	- propagation speed of the traveling wave [m/s]
v	- velocity of free-surface wave in y direction [m/s]
w	- velocity of free-surface wave in z direction [m/s]
x, y, z	- Cartesian coordinate system [m]
Ω_n	- vertical forcing ratio, define as ω_n/ω_v ($n = 1, 2, 3, \dots$)
ε	- non-dimensional initial wave steepness ($a \omega_n^2 / g$).
ρ	- density of water (1000 kg/m ³)
κ_{hx}	- non-dimensional horizontal forcing amplitude of shake table in surge direction.
κ_{hy}	- non-dimensional horizontal forcing amplitude of shake table in sway direction.
κ_v	- non-dimensional vertical forcing amplitude of shake table in heave direction
k_n	- wave number ($2\pi / L$)
ω	- general forcing frequency [rad/s]
ω_{hx}	- horizontal forcing frequency of shake table in surge direction [rad/s]
ω_{hy}	- horizontal forcing frequency of shake table in sway direction [rad/s]
ω_n	- sloshing frequency of water [rad/s] ($n = 1, 2, 3, \dots$)
ω_v	- vertical forcing frequency of shake table [rad/s]
μ	- dynamic viscosity [N/m ²]

ζ	- free-surface elevation [m]
ζ_{max}	- maximum free-surface elevation before wave breaking [m]

Subscripts

hx	- surge motion
hy	- sway motion
v	- vertical motion

ABSTRACT

In this thesis, free-surface water waves are experimentally investigated with a focus on sloshing in tanks. The sloshing problem is selected for its relatively simple geometry, yet it captures the features of nonlinearities including wave breaking. Free-surface motions are monitored in a square tank with base dimensions of $1 \times 1 \text{ m}^2$. Sloshing motions have been studied both off- and at resonance in shallow to deep water to investigate nonlinear free surface effects. The tank has been prescribed to move in sway and / or heave excitation. Interesting hysteresis behavior at the surface is captured at and near resonance. Faraday peaks also exhibit hysteresis behaviors. In the sway test series, linear resonances are observed. In the heave test series, exponentially growing resonances exist with evidence of mode interactions and detuning effects in the time evolutions.

The key contribution to current literature is found when the tank is prescribed to move in sway and heave simultaneously. The effects of vertical excitation on the pure horizontal excitation show complicated wave forms with random mode interaction, detuning effects and wave breaking typically present.

A special case study in shallow water depth ($h_0/b < 0.05$) is carried out in which traveling waves and bores are analyzed with Froude numbers $F_r(h_1) \in [1.1, 1.7]$.

Although this thesis has a focus on the understanding of free-surface water waves in tanks, the impact of the study affects broad areas. Accurate nonlinear wave investigations can benefit other communities in condensed matter physics, naval architecture, oceanography, seismology and ocean / coastal / earthquake engineering.

Keywords: sloshing motion; hysteresis behavior; wave breaking; bore formation

CHAPTER 1. INTRODUCTION

1.1 Background

Free-surface water waves are known to behave unpredictably. The understanding of the nonlinear behavior at the free-surface is important for a range of engineering applications. One way of achieving insight into free-surface behavior is to monitor waves in tanks. At any water depth the waves will typically slosh in some arbitrary manner. The behavior can under some circumstances be extrapolated to local wave behavior in the ocean. These kinds of studies are the focus of this thesis. Sloshing is a wave form in which large free-surface wave motion can occur when the liquid tank is subjected to external force. This can typically result in high dynamic pressures, mode interaction, three-dimensionalities and wave breaking. Sloshing motions in liquid tanks are affected by several variables, e.g., still water depth inside tanks, the natural frequency of sloshing, amplitude and direction of the external forces, tank shape and dimension. The violent sloshing motions of free-surface waves inside liquid tanks may be caused by small initial amplitude external oscillations. It may also result in high pressures on container walls and even damage the whole structure. Thus, predictions of free-surface motions in liquid tanks have great importance in several engineering applications. Some of the important applications are ship stability, e.g. Faltinsen (1974) derived a nonlinear 2-D analytical solution of rectangular shape tanks for ship stability; fuel tank transportation, water pipeline and liquid storage tank under seismic load and hurricane, e.g. Chen *et al.* (1996) investigated the nonlinear finite amplitude sloshing motion induced by seismic excitation in two-dimensional tanks. The stability of liquid tanks in spacecrafts is another application affected by sloshing. One of the early studies was carried out by Abramson

(1966) who made a comprehensive review of the sloshing motion application in aerospace engineering and suggested using liquid as damping. One can also take advantage of the energy dissipation occurring due to waves breaking and utilize the tank sloshing in relation to damping devices to restrain the movement of structures. Isaacson & Premasiri (2001) studied the hydrodynamic damping due to the baffles in a rectangular tank subjected to horizontal excitation and found damping ratio increases with baffle length.

1.2 Motivation

The importance of the studies of free-surface waves in liquid tanks can be helpful in many engineering applications. Sloshing motions of free-surface waves in tanks are important topics for a variety of engineering applications. The present experimental study intends to provide further insight into free-surface behavior. This study is very important to any applications involving partially filled containers with free-surfaces which may experience large liquid sloshing motions and high pressure on container walls. Relatively high dynamic pressure due to the sloshing may damage the wall when the forcing frequency is close to the natural frequency. The present study also applies to any liquid dampers. The sloshing problem is selected for its relative simple geometry, yet it captures the features of nonlinear breaking waves. Sloshing motion problem investigations can provide insight into highly nonlinear processes which today is still not well understood. It is not possible to obtain analytical solutions, especially when the forcing frequency is close to natural sloshing as mode interaction and wave breaking typically occur. Frandsen (2004) investigated the sloshing waves in a 2-D rectangular tank excited in horizontal and / or vertical direction simultaneously using a fully nonlinear potential solver. The present

thesis extends this work dealing with viscous overturning, 3-D waves. Most of the research previously done on sloshing problem is either under horizontal or vertical excitations. In this thesis, the effect of vertical excitation on the pure horizontal excitation has been explored. The behavior of free-surface waves is studied when the liquid tank is excited in horizontal, vertical, and combined horizontal and vertical simultaneously. The combined horizontal and vertical excitation is a new contribution of free-surface studies.

1.3 Scope of Work

The study includes a systematical investigation of the free-surface in shallow to deep water with various external forces. The study includes experimental investigations only. The free-surface is monitored from small to large wave steepnesses by varying the forcing amplitude and frequency. The experimental data is typically categorized (1) outside resonance, and (2) at / near resonance. Wave forms, time evolution and conditions of breaking have been monitored.

There are eight chapters in this thesis. Chapter 1 is a general introduction of the background and motivation of this thesis. Chapter 2 presents a literature review of the sloshing problem. Experimental and analytical methods have been mainly introduced in the previous studies on the sloshing motion topic. Chapter 3 explains the problem formulation of the sloshing motion including the governing equations, boundary conditions and the physical meaning of each parameter. A detailed statement of the experimental setup and the method to analyze the data are shown in Chapter 4. Chapter 5 and Chapter 6 are the core parts of this thesis. In Chapter 5, investigations of the free-surface water waves have been carried out in sway and heave excitation as a function of external forcing in shallow to deep water for different excitations. These test cases

include sway, heave, and combined sway and heave base excitations with depth to width ratio $h_0/b = 0.2, 0.3$ and 0.6 , where h_0 denotes the still water depth and b is the tank width. Chapter 6 shows the study of the behavior of free-surface waves at shallow water depths ($h_0/b = 0.05$). Traveling waves and bores have been explored. A summary of the result is presented in Chapter 7. Chapter 8 shows some suggestions of the future work that may be extended.

CHAPTER 2. LITERATURE REVIEW ON TANK SLOSHING

Free-surface waves in liquid tanks have been investigated by many researchers with a variety of approaches. It can be traced back to Faraday (1831), who was the first researcher to investigate free-surface wave forms in containers subjected to vertical excitations. In the 1950s, experimental and theoretical natured research flourished. Classical work and findings were reported by Taylor (1954), Benjamin / Ursell (1954) amongst others. In the 1960s, the first numerical work on sloshing problems was published. Extensive reviews have been given by e.g. Abramson (1966), Perlin & Schultz (2000) and Ibrahim *et al.* (2001).

2.1 Analytical Work on Sloshing in Tanks

Extensive works on deriving analytical free-surface solution have been undertaken. These are mainly inviscid solutions but viscous solutions also exist, e.g. Wu *et al.* (2001) who presented a second order viscous solution in a 2-D rectangular tank. Analytically, the free-surface has been studied starting from linear solution. Lamb (1932) derived a 3-D linear inviscid solution of first order accuracy for rectangular and circular cylindrical tanks for shallow water. Verhagen & Wijngaarden (1965) derived a first order solution for forced oscillation of fluid from a linearized approximation of shallow water equation in a 2D rectangular tank. Experiments are carried out in a 1.20 m width tank with still water depth $h_0 = 0.09$ m and compared with analytical results, as well. The comparison showed good agreements but in some cases there were discrepancies between analytical and experimental results due to the viscosity. Abramson (1966) derived a linear inviscid first order analytical solution for 3-D rectangular tanks under harmonic excitation. Innovative applications were described of sloshing dampers and vehicle

stability for usage in the spacecraft industry. Faltinsen (1974) derived a third order nonlinear inviscid analytical solution for 2-D rectangular shape tanks with focus on the ship industry. It has been found that this solution did not work well when the water reached the tank top. Faltinsen *et al.* (2000) derived a modal system with the Bateman-Luke variational principle describing nonlinear sloshing in rectangular tanks under three-dimensional motions. Faltinsen *et al.* (2001) investigated two-dimensional nonlinear sloshing of incompressible, irrotational flow in rectangular tanks based on a fifth order polynomial nonlinearity. Stability of frequency domain has been studied. Sloshing motion at frequencies close to the first sloshing frequency in a square base tank has been investigated by Faltinsen *et al.* (2003). Waterhouse (1994) studied the liquid behavior of inviscid 2-D waves in rectangular tanks near critical depth by a fifth order analysis. He found the large-amplitude response behaves as a softening spring oscillator when the water depth is close to either side of the critical depth. Ockendon *et al.* (1996) investigated multiple dimensional responses of a horizontally oscillated tank that depends on tank geometry and wave dispersion by asymptotic method. The sloshing motions induced by earthquakes have also been studied. Cerda & Tirapegui (1998) studied the free-surface behavior of a 2-D viscous fluid prescribed vertical excitation at second order accuracy. Instabilities of free-surfaces are compared between low and high viscosity. They found the Mathieu equation is different from weak viscous fluid when the viscosity increased. Isaacson & Ryu (1998) studied the 3-D inviscid liquid sloshing in a vertical cylindrical tank with arbitrary sections subjected to harmonic and random base excitation. The estimation of maximum hydrodynamic forces was reported. Davis & Weidman (2000) derived useful formulas for natural frequencies of two-dimensional sloshing

modes in channels. Perlin & Schultz (2000) summarized the effects of surface tension on free-surface waves. Nonlinear waves close to and at breaking are discussed. Amundsen *et al.* (2001) studied 2-D inviscid free surface waves in shallow water tanks under harmonic excitation. It was reported that non-periodic solutions and multiple steady solutions can be obtained for the same frequency. Recently, Peregrine (2003) made a comprehensive review of the previous work of violent impacts of waves on walls and reported the fluid mechanics for wave impacts and ‘flip-through’ phenomena where a high speed vertical jet emerged with a high speed.

2.2 Experimental Work on Sloshing in Tanks

Comprehensive experimental studies on sloshing in tanks have been carried out and compared with analytical or numerical solutions. These experimental studies can be classified into two categories: non-overturning wave studies and breaking wave studies. Breaking wave is a frequent but complicated phenomenon for sloshing in tanks. The following reviews describe the previous experimental works on non-overturning waves and breaking waves respectively.

2.2.1 Study on Non-overturning Waves

For non-overturning waves, Abramson (1966) made a comprehensive summary of experimental studies on the sloshing problem in tanks related to space vehicles. Experimental technology was introduced in details. The mode shapes for circular cylinders, spherical tanks and rectangular tanks were compared. Virnig *et al.* (1988) carried out a series of experiments in a $17.78 \times 30.48 \times 25.40 \text{ cm}^3$ rectangular tank to investigate three-dimensionality free-surface waves under vertical excitation. These types of experiments are often referred to as Faraday waves due to its origin (Faraday, 1831).

Miles & Henderson (1990) made an extensive review of investigations on the parametrically forced Faraday wave. The comparisons between analytical and experimental solutions on stability boundary were reported. Chanson & Montes (1995) experimentally studied the undular hydraulic jumps in a $20 \times 0.25 \times 0.27 \text{ m}^3$ rectangular channel and described the flow patterns. Montes & Chanson (1998) tested the velocity, pressure and wave elevation of undular jumps in a 0.25 m wide smooth rectangular channel. Faltinsen *et al.* (2000) carried out experiments in a 2-D rectangular tank with dimensions of $1.73 \times 0.2 \times 1.05 \text{ m}^3$ under surge / pitch harmonic excitation and compared with the analytical solution. It was reported that beating period occurred due to the transient and forced oscillations. Faltinsen *et al.* (2003) conducted experiments in a square tank with dimensions of $0.59 \times 0.59 \times 0.80 \text{ m}^3$ under horizontal longitudinal / diagonal harmonic excitation and classified the possible nonlinear wave forms. Akyildiz & Unal (2005) investigated the pressure distribution on tank walls and 3-D effects on sloshing loads in a $92 \times 62 \times 46 \text{ cm}^3$ under pitch harmonic excitation. It was reported that excitation amplitude had significant effects on sloshing load.

2.2.2 Study on Breaking Waves

For breaking waves, Jiang *et al.* (1998) investigated Faraday wave resonances in a $60 \times 6 \times 48.3 \text{ cm}^3$ tank who discussed wave breaking mechanisms and period tripling (three recurrent modes) due to the mode interaction. Anastasiou & Bokaris (2000) studied the 2-D and 3-D wave breaking criteria in a $10 \times 0.32 \times 0.23 \text{ m}^3$ wave flume and $6 \times 11 \text{ m}^2$ wave basin. Utku & Basco (2002) investigated the wave breaking mechanism for shallow water using a relative Froude number. The experiments were carried out in an

$18.29 \times 0.91 \times 0.91 \text{ m}^3$ wave flume with a 1:20 slope. The critical Froude number for wave breaking is approximately 1.36.

2.3 Tank Sizes and Scale Effect

Analytical, experimental and numerical methods have been applied to the studies of free-surface waves in liquid tanks. Numerical method developments have been investigated since 1960s. For example, Peregrine (1966) modeled bores with a finite difference approach. Hirt & Nichols (1981) created a new method based on the concept of a fractional Volume of Fluid (VOF) dealing with complicated free boundary conditions. The numerical work is useful but it is beyond the scope of this thesis. So it has not been reviewed in detail.

Generally, the previous studies of free-surface waves in tanks have been carried out mainly under pure horizontal or vertical excitation. The tank shapes are mainly rectangular and cylindrical. Although many studies have been carried out on liquid sloshing, many questions still remain unanswered e.g. three-dimensionality and wave breaking. When the wave steepness becomes sufficiently large, mode interactions, hysteresis, and detuning effects may occur. Therefore, it is a challenge to predict free-surface wave motions in containers. The new contribution of this thesis is the investigation of vertical excitation effects on the pure horizontal excitation for viscous 3-D waves. Table 2.1 shows the scales of experimental tanks. To the author's knowledge, the tank applied in this thesis is the largest square base sloshing tank which is appropriate for three dimensionality study and also avoids scale effects problems for engineering application.

Table 2.1. Tank dimensions of experimental investigations.

Investigators	Tank Shape	Length (m)	Width (m)	Height (m)	Excitation Direction
Virnig <i>et al.</i> (1988)	Rectangular (3-D)	0.18	0.30	0.25	Heave
Jiang <i>et al.</i> (1988)	Rectangular (2-D)	0.60	0.06	0.48	Heave
Faltinsen <i>et al.</i> (2000)	Rectangular (2-D)	1.73	0.20	1.05	Sway
Faltinsen <i>et al.</i> (2003)	Square (3-D)	0.59	0.59	0.80	Sway/Surge/Roll/Pitch
Akyildiz & Unal (2005)	Rectangular (3-D)	0.92	0.62	0.46	Roll
Peng (present)	Square (3-D)	1	1	1.22	Sway/Heave/Sway + Heave

The scale effect has been compared between two different scale tanks. The dimension of the large tank is $1 \times 1 \times 1.22 \text{ m}^3$ and of the small tank is $0.28 \times 0.28 \times 0.20 \text{ m}^3$. The water depth and forcing amplitude are scale down proportionally in the small tank. Froude number is kept same value in order to generate the same type of flow. From the response curves shown in Fig. 2.1, the free surface elevation has been under predicted in the small tank. Full scale tank size with dimension $1 \times 1 \times 1.22 \text{ m}^3$ is recommended.

Fig. 2.1. Scale effect comparison at center of wall ($x=0, y=-l/2$), $a_{hy}/b = 0.006$. —, large tank ($1 \times 1 \text{ m}^2$ base); and —, small tank ($0.28 \times 0.28 \text{ m}^2$ base) in sway excitation. (a) $h_0/b = 0.2$, (b) $h_0/b = 0.4$. •, wave breaking.

CHAPTER 3. PROBLEM FORMULATION

Investigations of free surface flow in a 3-D square tank with base dimensions of $1 \times 1 \text{ m}^2$ have been carried out. A 3-D incompressible viscous Navier-Stokes Equation problem has been considered in this thesis. The coordinate system has been chosen as shown in Fig. 3.1. The Cartesian coordinate system (x, y, z) has its origin on the center of the bottom of the tank. The system is prescribed external harmonic excitations from the base.

Fig. 3.1. Schematic coordinate system.

$$\rho \frac{\partial u}{\partial t} + \rho(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}) = -\frac{\partial p}{\partial x} + \rho a_x + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) + T_x, \quad (1)$$

$$\rho \frac{\partial v}{\partial t} + \rho(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}) = -\frac{\partial p}{\partial y} + \rho a_y + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) + T_y, \quad (2)$$

$$\rho \frac{\partial w}{\partial t} + \rho(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}) = -\frac{\partial p}{\partial z} + \rho(a_z + g_z) + \mu(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) + T_z, \quad (3)$$

where in the Navier-Stokes equation, T_i ($i = x, y, z$) is the contribution of surface tension and μ is dynamic viscosity. ζ is the free-surface elevation. ρ is the density of water. u, v, w are the velocities correspond to x, y, z axis respectively. And a_i ($i = x, y, z$) is the

acceleration of external forcing in x, y, z axis respectively. The governing equations are solved experimentally.

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 . \quad (4)$$

The kinematic boundary condition at the surface ($z = h_0 + \zeta$) is

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} = 0 . \quad (5)$$

The dynamic boundary condition at the surface ($z = h_0 + \zeta$) is

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(u^2 + v^2 + w^2) + (a_z + g_z)\zeta = -ya_y . \quad (6)$$

Because there is no flow across the wall, the boundary conditions at the walls are:

$$u = 0 , \quad x = -b/2, b/2 , \quad (7)$$

$$v = 0 , \quad y = -l/2, l/2 , \quad (8)$$

$$w = 0 , \quad z = 0 . \quad (9)$$

Time histories and frequency spectra of the free-surface are non-dimensionalized

with the sloshing frequency ω_n , which is defined as: $\omega_n = \sqrt{g \frac{n\pi}{b} \tanh(\frac{n\pi}{b} h_0)}$, wave

number $n = 0, 1, 2, \dots$. The 3-D version of sloshing frequency $\omega_{mn} = \sqrt{gk \tanh(kh_0)}$, where

$k = \sqrt{\pi^2(\frac{m^2}{l^2} + \frac{n^2}{b^2})}$ was given by Lamb (1932).

CHAPTER 4. EXPERIMENTAL SETUP AND INSTRUMENTATION

4.1 Experimental Setup

Extensive sloshing wave studies have been carried out experimentally in a base excited square tank without a lid. Fresh water has been used in all the test series. The square tank is made of Plexiglas and its inner dimensions are 1 m (length) $\times$ 1 m (width) $\times$ 1.2 m (height). The wall thickness of the sloshing tank is 0.02 m to ensure negligible deformation of the tank walls. Therefore, it is assumed that the tank walls are stiff and that no local displacement will interact with the free-surface waves. In order to avoid the scale effect, the sloshing tank is built up large enough for real engineering applications. The tank is mounted on a six degrees-of-freedom (DOF) shake table, as shown in Fig. 4.1(a). The shake table is firmly mounted onto a 0.15 m thick concrete slab located on the ground floor and thus no local vibrations are generated. The sloshing motions inside tanks are generated by the shake table. It is an electrically powered six DOF (roll, pitch, heave, sway, yaw and surge) shake table which can be set up to provide generic harmonic, seismic or random excitations.

The time dependent variables of interest are the free-surface (ζ), the velocity field (u_x, u_y, u_z) and the pressures (P). The experiments are limited to report on the free-surface at selected locations, as shown in Fig. 4.1(b). Ten capacitance wave gauges are utilized to monitor the free-surface elevations. The wave gauges are accurate to ± 0.003 m when the depth to width ratio h_0/b varies between 0.24 (20% full length of wave gauges) and 0.96 (80% full length). When the depth to width ratio is outside this range, the accuracy of wave gauges is ± 0.012 m. It should be noted that the wave gauges are not accurate when the depth to width ratio is $h_0/b < 0.03$.

Fig. 4.1. Experimental setup. (a) sloshing tank on 6 DOF shake table and (b) plan view of tank and definition of coordinate system; ● sensor location.

Fig. 4.2. Sketch of experimental setup: A. controller, B. sloshing tank, c. wave sensor, D. shake table, E. data acquisition system, F. digital video camera.

Wave gauges are located inside the tank as shown in Fig. 4.1(b) with adjustable steel frames to measure the free-surface elevation at any desirable location. The origin of the coordinate is located at the center of the tank bottom. Wave profiles have been recorded with a high speed camera. The high speed camera can record a sequence of images at speed of 1000 fps. A detailed description of instrumentation is attached in Appendix A.

The data acquisition system allows for simultaneous measurements. Typical sampling frequency for the initial wave steepness, $\varepsilon = a \omega_n^2 / g < 0.013$ is 30 Hz (where a is the forcing amplitude, ω_n is the natural frequency and g is the acceleration due to gravity). And $\varepsilon = a \omega_n^2 / g > 0.013$ is 125 Hz. Fig. 4.2 shows the overview sketch of experimental setup.

4.2 Initial Conditions and Transients

The unsteady free-surface behavior is extremely sensitive to its initial conditions, as would be expected for any dynamical system. When a large base excitation force is applied, the surface elevation may undergo a shock and occur in form of a discontinuity. In this record, the concern is addressed at the surface by applying two different initial base excitation conditions. The first base excitation method involves ramping the forcing amplitude relatively slowly. The second method, referred to as the single step initial condition, involves keeping the function $\sin(\omega_h t) = 0$. There are mechanical shocks of the shake table when the forcing amplitude (a) is varied if the actuators are not at the original position. In order to avoid the initial shock of the shake table, a proper time t has been chosen to keep the function $\sin(\omega_h t) = 0$. The function is $\sin(\omega_h t) = 0$ by assuming $\omega_h t = n\pi$. For the shallow water case, $h_0/b = 0.05$, $\omega_l = 2.1913$ rad/s and the period is 2.87 s. Take $n = 4$, it can be obtained that $t = 4\pi/\omega_h = 5.213$ s ($t \times \omega_l = 5.213 \times 2.191 = 11.4$). There are several transient phases in connection with initiation of movement of the shake table as shown in Figs 4.3-4.4. These transients occur when (1) the wave gauge is activated (L_0); (2) the shake table raises in a vertical movement into the operating position (L_1); (3) keeping the shake table still with $a_h/b = 0$ (L_2); (4) the shake table moves in a swaying direction with a_h (L_3); (5) the shake table moves in a swaying

direction with $a_h(L_4)$; (6) the shake table moves in a swaying direction with $a_h(L_5)$ and (7) the shake table moves in a swaying direction with the desirable $a_h(L_6)$ i.e. true physical transient. Zero defines the time when the shake table begins to move with the desirable a_h . A sway base excitation example is used. The forcing motion is $y = a_h \sin(\omega_h t)$, where $h_0/b = 0.05$, $a_h/b = 0.02$ and $\omega_h = 1.10\omega_I = 2.410$ rad/s.

Fig. 4.3. Forcing amplitude a_h/b for two initial conditions shown. ‘— —’, ramping; ‘—’, single step.

Fig. 4.4. Comparison of free-surface elevations at the wall. Transient phases for two initial conditions shown. ‘— —’, ramping, ‘—’, single step.

For the ramping initial condition, a non-physical free-surface spike occurred near $t \times \omega_I = 2$ in the transient phases. Comparing the free-surface time history starting from near $t \times \omega_I = 0$, i.e. the two different initial base excitations eventually yield similar free-surface behavior. From the series shown in Figs 4.3-4.4, it is concluded that the ramping

initial condition is not necessary but could however be applied due to avoiding longer experimental test series. The base excitation initial condition with a single step ramp is chosen throughout this thesis.

4.3 Filters

In the following some functions used to filter the physics and noise will be described. A Gaussian and a Low-pass filter as shown in Fig. 4.5 have been applied. The Gaussian filter is described as:

$$G(\zeta) = e^{-(\zeta^2 / 2\sigma^2)} / (\sqrt{2\pi} \times \sigma) \quad (10)$$

where σ is the corresponding standard deviation and ζ is the free-surface time history. The low-pass filter will stop the relative high frequency component.

Fig. 4.5. Filter comparison: (a) Gaussian; (b) ideal Low-pass.

Figs. 4.6 – 4.7 shows a typical comparison of free-surface time histories and spectra using the two filter functions for off-resonance and at resonance cases. Generally, the physical frequency range is $\omega_h/\omega_1 \in (0, 5)$ for the experiments series in this thesis. Typically free-surface time histories are filtered at frequencies higher than 3.75 Hz ($\omega_h/\omega_1 > 5$). Compared to the Gaussian filter, the time history display some high frequency components in peaks/troughs of the time history filtered by the low-pass filter which resemble the trough physics of high frequencies due to mode interactions. In order

to avoid filtering the high frequency component of true sloshing motion, the Low-pass filter with 3.75 Hz limit has been chosen throughout this thesis.

Fig. 4.6. Filter comparison: time history and spectra of free-surface elevation across the tank at (a,b) the left wall; (c,d) center. $h_0/b = 0.05$; $a_h/b = 0.006$, $\omega_h/\omega_1 = 0.85$. ‘—’, Gaussian filter; ‘- -’, low pass filter, the frequency higher than 3.75 Hz was filtered.

Fig. 4.7. Filter comparison: time history and spectra of free-surface elevation across the tank at (a,b) the left wall; (c,d) center. $h_0/b = 0.05$; $a_h/b = 0.006$, $\omega_h/\omega_1 = 1.09$. '—', Gaussian filter; '---', low pass filter, the frequency higher than 3.75 Hz was filtered.

CHAPTER 5. SLOSHING

5.1 Introduction

In this chapter, water sloshing motion studies in a square tank with base dimensions $1 \times 1 \text{ m}^2$ (without lid) have been carried out with prescribed base motions. The free-surface behavior is investigated as a function of external harmonic forcing in shallow to deep water. These test cases include sway, heave, and combined sway and heave base excitations with depth to width ratio $h_0/b = 0.2, 0.3$ and 0.6 , where h_0 denotes the still water depth and b is the tank width.

Initial findings are reported in Frandsen & Peng (2005), who investigated the free-surface in a $100 \times 100 \text{ cm}^2$ square base sloshing tank subjected to horizontal and vertical excitations. The hysteresis of free-surface was observed.

In the following, test series in the tank which is prescribed in sway, heave, and combined sway and heave base excited motions are presented respectively. Harmonic forced motions are prescribed. Shallow to deep water sloshing test series are carried out varying the tank aspect ratio and external force ($\kappa = a\omega^2/g$), where a is the external forcing amplitude, ω is the external forcing frequency and g is the acceleration due to gravity. The hysteresis existing at the free-surface at large wave steepness is demonstrated below and above critical water depth when the tank is at prescribed horizontal excitation. There exists instability regions governed by the Mathieu's equation when the tank is excited in a pure vertical motion (Benjamin & Ursell, 1954). It is shown, that the free-surface elevations exhibit exponentially growing resonances and, in some cases, detuning effects. All results are non-dimensionalized. Time histories and frequency

spectra are non-dimensionalized with the sloshing frequency ω_n , which is defined as:

$$\omega_n = \sqrt{g \frac{n\pi}{b} \tanh\left(\frac{n\pi}{b} h_0\right)}, \text{ wave number } n = 0, 1, 2, \dots$$

5.2 Sway Excitation

In these test series, the tank is excited in a sway direction, as shown in experimental set-up Fig. 4.1. The fluid motion is initially at rest. The tank is prescribed to move harmonically $y = a_{hy} \sin(\omega_{hy} t)$ where a_{hy} is the forcing amplitude in the sway direction and the forcing frequency is ω_{hy} . In the following tests, the forcing frequency is varied outside and at resonance. Resonance is defined as when the external forcing frequency equals the sloshing frequency; the free-surface would be largely excited and maximum free-surface elevation would occur.

5.2.1 Off-Resonance

Fig. 5.1 shows the free-surface elevation at the center of the left tank wall ($(x,y)=(0,-l/2)$) for an off-resonance case with forcing frequency ratio $\omega_{hy}/\omega_1 = 0.85$. A beating response of the surface can be observed, as expected, characterized by two distinct frequencies corresponding to the forcing frequency and the first sloshing frequency. The wave form has a first mode sloshing form. The shallow water case ($h_0/b = 0.2$) displaying nonlinearity due to mode interaction which can be seen by the irregular peaks and troughs compared to the moderate / deep water cases, as shown in the spectra in Fig. 5.1b. In order to capture sloshing waves at small to large amplitudes, the free-surface behavior is used as explored by varying the external force κ_{hy} , which is a measure of nonlinearity.

(a)

(b)

Fig. 5.1. Free-surface elevation (a) and spectra (b) at center of wall ($x=0, y=-l/2$) in sway excitation with moderate forcing amplitude ($a_{hy}/b = 0.006$) off-resonance ($\omega_{hy}/\omega_1 = 0.85$). —, $h_0/b = 0.2$, $\kappa_{hy} = 0.008$; — • —, $h_0/b = 0.3$, $\kappa_{hy} = 0.01$; — — —, $h_0/b = 0.6$, $\kappa_{hy} = 0.013$. Superscripts in spectra denote water depth to width ratio h_0/b .

5.2.2 At Resonance

The next test series including the free-surface elevations at resonance ($\omega_{hy}/\omega_I \approx 1$) at the left tank wall ($(x,y)=(0,-l/2)$) for water depth $h_0/b = 0.2, 0.3, 0.6$ are shown in Figs. 5.2-5.6. The water depths below, at and above critical water depth are chosen. The critical water depth is found to be $h_c/b = 0.3$, where h_c is the critical depth. This agrees well with Gu *et al.* (1988) who found $h_c/b \approx 0.3368$ in a rectangular tank with dimensions of $17.78 \times 30.48 \times 25.40$ cm³. The time history for at resonance cases for sway excitation at the three different water depths are shown in Fig. 5.2. The time history is divided into two parts for clarity. At the first part when the dimensionless time $t \times \omega_I = 0-100$, linearly growing resonances can be observed for all the three water depths. The free-surface waves are nearly two-dimensional except at the tank walls where breakings are observed. The shallow water case illustrates nonlinear free-surface behavior where the peaks become higher and troughs decrease as time grows. This is the consequence of high frequency component ($\omega_h + \omega_I$). For $h_0/b = 0.2$, the maximum free-surface elevation ($\zeta_{\max}/a_h = 50$) is observed at $t \times \omega_I \approx 102$. At the time exactly before the first wave breaks, the wave steepness is $S_I = 0.956$ where the wave steepness is defined as $S_n = k_n \times \zeta_{\max}$, where k_n is the wave number and $\zeta_{\max}$ is the maximum free surface elevation. The associated snapshots of wave profiles are shown in Fig. 5.3 for $h_0/b = 0.2$ at resonance. The breaking of the wave would cause the amplitude to decay and three-dimensionality of the free-surface to occur. The first mode in the sway direction would interact with the forcing frequency, yielding a third mode in a surge direction. At $t \times \omega_I \approx 150$, the free-surface waves transform into swirling motions. For the water $h_0/b = 0.3$, the maximum free-surface elevation ($\zeta_{\max}/a_h = 34.75$) is observed at

(a)

(b)

Fig. 5.2. Free-surface elevation at center of wall ($x=0, y=-l/2$) in sway excitation with moderate forcing amplitude ($a_{hy}/b = 0.006$) at resonance. (a) $t \times \omega_1 = 0-125$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 1.03$, $\kappa_{hy} = 0.011$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.014$; - - - , $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.95$, $\kappa_{hy} = 0.017$.

Fig. 5.3. Wave profile of wave generated in sway excitation at moderate forcing amplitude ($a_{hy}/b = 0.006$) in shallow water ($h_0/b = 0.2$) at $\omega_{hy}/\omega_1 = 1.03$.

$t \times \omega_I \approx 99$. The wave steepness is $S_I = 0.655$, and there is no breaking wave observed at this instant. After this point, mode interaction occurs and yields swirling motions followed by decreasing free-surface amplitudes. Later, wave breaking is observed ($t \times \omega_I \approx 111$) yielding $\zeta_{\max}/a_h = 29.77$ and $S_I = 0.561$. For the deep water case $h_0/b = 0.6$, the first point of wave breaking is observed at $t \times \omega_I \approx 173$ with free-surface elevation $\zeta_{\max}/a_h = 25.38$. The associated wave steepness is $S_I = 0.479$. The maximum free-surface elevation ($\zeta_{\max}/a_h = 75.18$) occurs later at $t \times \omega_I \approx 696$ after the first wave breaking point. The wave steepness is $S_I = 1.417$. It is notable that the deep water case generated the maximum wave steepness for all three of the selected water depths. Also, as expected, it takes longer for the deep water case to reach resonance due to the large natural frequency. The spectra in Fig. 5.4 show the coincidences with the time histories. For $h_0/b = 0.2$, when the first mode in the sway direction interacted with the forcing frequency, a third mode in a surge direction was identified as ω_{3x} . The nonlinearity of the free-surface wave is clearly illustrated by the wave phase planes in Figs. 5.5. For $h_0/b = 0.2$, again the nonlinearity can be observed from the higher peaks and the lower troughs. For $h_0/b = 0.3$, it represents a near linear solution (equal peaks and troughs). For $h_0/b = 0.6$, it displays a trajectory of linear solution at $t \times \omega_I \approx 0-125$. Later, at $t \times \omega_I > 125$, some nonlinearity is evident. It is notable that $\zeta_{\max}$ is found relatively late in the time evolution.

5.2.3 Constant Forcing

The free-surface behavior is studied by varying the water depth while the external force $\kappa_{hy} = a_{hy} \omega_{hy}^2 / g$ is kept constant: $\kappa_{hy} \in [0.005, 0.05]$. The free-surface time histories are shown in Fig. 5.6. It can be observed that the free-surface elevation at shallow water

(a)

(b)

Fig. 5.4. Spectra at wall in sway excitation with moderate forcing amplitude ($a_{hy}/b = 0.006$) at resonance. (a) $t \times \omega_1 = 0-125$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 1.03$, $\kappa_{hy} = 0.011$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.014$; — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.95$, $\kappa_{hy} = 0.017$. Superscripts in spectra denote water depth to width ratio h_0/b . Subscripts x denote the frequency corresponding to surge direction.

Fig. 5.5. Phase plane at wall in sway excitation with moderate forcing amplitude ($a_{hy}/b = 0.006$) at resonance at $t \times \omega_1 = 0-125$. (a) —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 1.03$, $\kappa_{hy} = 0.011$; (b) — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.014$; (c) — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.95$, $\kappa_{hy} = 0.017$. And $t \times \omega_1 = 125-715$. (d) —, $h_0/b = 0.2$, $\kappa_{hy} = 0.011$; (e) — • —, $h_0/b = 0.3$, $\kappa_{hy} = 0.014$; (f) — —, $h_0/b = 0.6$, $\kappa_{hy} = 0.017$.

(a)

(b)

Fig. 5.6. Free-surface elevation at center of wall ($x=0, y=-l/2$) in sway excitation with constant small forcing $\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$. (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-300$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 0.98$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 0.85$; — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.75$.

is larger than deep water displaying a highly nonlinear solution with high peaks and low troughs. The associated snapshots of wave profiles are shown in Fig. 5.7 for water depth $h_0/b = 0.2, 0.3$ and 0.6 , respectively. Fig. 5.8 shows a typical first sloshing mode when the external force $\kappa_{hy} = 0.005$ at water depth $h_0/b = 0.2$. The nonlinearity is introduced due to the mode interaction ($\omega_h + \omega_l$) as shown in the spectra. The vertical velocity at the wall and associated phase plane are shown in Figs. 5.9-5.10. It is clearly shown that when the water depth is shallow, the free surface behavior is highly nonlinear.

Fig. 5.7. Wave profile of wave generated in sway excitation at constant forcing $\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$, at water depth $h_0/b = 0.2, 0.3$ and 0.6 , respectively.

The next test series include an increase in external force $\kappa_{hy} = 0.05$ (Figs. 5.11-5.14). Linearly growing resonances are observed for all the three water depths at the initial part of the time evolution ($t \times \omega_l = 0-60$) as shown in Fig. 5.11. For $h_0/b = 0.2$, a third mode wave form is observed. When $t \times \omega_l \approx 69$, wave breaking is observed with wave steepness $S_3 = 0.742$. This also can be observed in the spectra shown in Fig. 5.12.

(a)

(b)

Fig. 5.8. Spectra at wall in sway excitation with constant small forcing $\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$. (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-300$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 0.98$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 0.85$; — — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.75$. Superscripts in spectra denote water depth to width ratio h_0/b .

(a)

(b)

Fig. 5.9. Vertical velocity history at wall in sway excitation with constant small forcing $\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$. (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-300$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 0.98$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 0.85$; — • —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.75$.

(a)

(b)

Fig. 5.10. Phase plane at wall in sway excitation with constant small forcing $\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$. (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-300$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 0.98$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 0.85$; — — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 0.75$.

(a)

(b)

Fig. 5.11. Free-surface elevation at center of wall ($x=0, y=-l/2$) in sway excitation with constant moderate forcing $\kappa_{hy} = 0.05$, $a_{hy}/b = 0.006$. (a) $t \times \omega_1 = 0-80$, and (b) $t \times \omega_1 = 80-200$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 2.18$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.90$; - - - , $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 1.67$.

(a)

(b)

Fig. 5.12. Spectra at wall in sway excitation with constant moderate forcing $\kappa_{hy} = 0.05$, $a_{hy}/b = 0.006$. (a) $t \times \omega_1 = 0-80$, and (b) $t \times \omega_1 = 80-200$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 2.18$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.90$; - - -, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 1.67$. Superscripts in spectra denote water depth to width ratio h_0/b . Subscripts x denote the frequency corresponding to surge direction.

The free-surface elevation decreases due to a new mode in surge direction after breaking. The equivalent test series for moderate and deep water show typical beating at the free-surface at tank walls. The vertical velocity at the wall is shown in Fig. 5.13. The phase plane diagram shown in Fig. 5.14, for the shallow water case, three-dimensionality becomes evident at the surface after wave breaking is introduced.

Table 5.1 outlines the key findings of the sway excitation at different water depths for at resonance case.

Table 5.1. Key results of at resonance case of sway excitation.

h_0/b	ω_I	ω_{hy}/ω_I	$\zeta_{\max}$	S	$t \times \omega_I$	Mode
0.2	4.14	1.03	0.30	0.956	102	First Mode
0.3	4.76	1	0.21	0.655	99	First Mode and Swirling Mode
0.6	5.42	0.95	0.47	1.417	696	First Mode and Swirling Mode

In summary, the maximum free-surface elevations at the wall as a function of the forcing frequency are shown (Fig. 5.15). The response curves are generated with a small forcing amplitude $a_{hy}/b = 0.003$ and a moderate forcing amplitude $a_{hy}/b = 0.006$. A data point for off-resonance is extracted when $t \times \omega_I \approx 100$, whereas at resonance $t \times \omega_I \approx 300$, respectively. The external forcing parameters κ_{hy} varies from 0.004 to 0.022. Hysteresis in the solution is found as the maximum free-surface elevation does not have to occur at $\omega_{hy}/\omega_I = 1$ (which represents a linear solution). Softening oscillator behavior is the maximum free-surface elevation occurring at a frequency lower than the sloshing frequency, whereas hardening oscillator behavior yields the maximum free-surface elevation at a higher frequency than the sloshing frequency. For the water depth

$h_0/b = 0.2$ as shown in Fig. 5.15a, the maximum free-surface elevation occurs at $\omega_{hy}/\omega_I = 1.03$ for a small forcing amplitude of $a_{hy}/b = 0.003$. This is equivalent to hardening oscillator behavior of the free-surface where the amplitude increases along with the frequency when $\omega_{hy}/\omega_I \geq 1$. The hardening fluid behavior becomes stronger with increasing forcing amplitude. For the water depth $h_0/b = 0.3$, a linear solution is captured regardless of whether the forcing amplitude is small or large, which yields a maximum free-surface elevation at $\omega_{hy}/\omega_I = 1$ as shown in Fig. 5.15b. This also indicates that the critical water depth for the sloshing tank is near $h_c/b = 0.3$. For the deep water case $h_0/b = 0.6$, the free-surface behaves as a softening liquid spring, yielding the maximum free-surface elevation at $\omega_{hy}/\omega_I = 0.97$ and 0.96 for small and large forcing amplitudes respectively, as shown in Fig. 5.15c.

5.3 Heave Excitation

In the following test series, the tank is excited vertically when $h_0/b = 0.2, 0.3$ and 0.6 . The initial fluid motion is not at rest. In order to excite the free-surface, a small perturbation of horizontal base excitation ($a_{hy}/b = 0.002$) is applied to the tank in a duration of about 2 seconds (only). Then the tank is prescribed to move harmonically $z = a_v \sin(\omega_v t)$ in a heave direction where a_v is the forcing amplitude. The forcing frequency in the heave direction (ω_v) is varied in order to capture small to large amplitudes of sloshing waves. In this thesis work, the nonlinear free-surface behavior is studied in both stable and unstable regions. Benjamin & Ursell (1954) analytically investigated the linear stability of the free-surface based on Mathieu's equation, $y'' + (p - 2q \cos(2x))y = 0$. The corresponding stability map is shown in Fig. 5.16. The free-surface behavior in vertical excitation can be described through the parameters

(a)

(b)

Fig. 5.13. Vertical velocity history at wall in sway excitation with constant moderate forcing $\kappa_{hy} = 0.05$, $a_{hy}/b = 0.006$. (a) $t \times \omega_1 = 0-80$, and (b) $t \times \omega_1 = 80-200$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 2.18$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.90$; — — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 1.67$.

(a)

(b)

Fig. 5.14. Phase plane at wall in sway excitation with constant moderate forcing $\kappa_{hy} = 0.05$, $a_{hy}/b = 0.006$. (a) $t \times \omega_1 = 0-80$, and (b) $t \times \omega_1 = 80-200$. —, $h_0/b = 0.2$, $\omega_{hy}/\omega_1 = 2.18$; — • —, $h_0/b = 0.3$, $\omega_{hy}/\omega_1 = 1.90$; — — —, $h_0/b = 0.6$, $\omega_{hy}/\omega_1 = 1.67$.

(a)

(b)

(c)

Fig. 5.15. Maximum free-surface elevation at center of wall ($x=0, y=-l/2$) in sway excitation with small forcing amplitude (—, $a_{hy}/b = 0.003$) and moderate (—, $a_{hy}/b = 0.006$). (a) $h_0/b = 0.2$; (b) $h_0/b = 0.3$; (c) $h_0/b = 0.6$. •, wave breaking.

$\kappa_v = a_v \omega_v^2 / g$ and $\Omega_n = \omega_n / \omega_v$. The first three instability regions for $\Omega_n = 0.5, 1$ and 1.5 are shown in Fig. 5.16. The free-surface may experience exponentially growing resonances, when any of the pairs of parameters (κ_v, Ω_n) are located in the instability regions. The free-surface elevation in the first instability region would have the fastest growing speed. Many discussions have been reported about the wave forms occurring when the tank is excited by twice the first sloshing frequency, e.g. Jiang *et al.* (1996). Compared to the horizontal excitation, the free-surface behaves very differently in vertical excitation test series due to the exponentially growing resonances yielding interesting nonlinear wave forms.

Fig. 5.16. Stability map for the linear solution of Mathieu equation.

The series cover the rectangular area shown in Fig. 5.16 in both stable and unstable regions. Four different test series will be introduced in details which correspond to the data points 1 to 4 labeled in Fig. 5.16.

5.3.1 Stable Regions

The first test series are conducted in stable regions at $(\kappa_v, \Omega_n) = (0.5, 0.7)$. The free-surface time histories are shown in Fig. 5.17. Irregular peaks and troughs are

(a)

(b)

Fig. 5.17. Free-surface elevation at center of wall ($x=0, y=-l/2$) in heave excitation in stable region $(\kappa_v, \Omega_n) = (0.5, 0.7)$ corresponding to data point no. 1 in Fig. 5. 16. (a) $t \times \omega_1 = 0-150$, and (b) $t \times \omega_1 = 150-250$. $\omega_v/\omega_1 = 1.429$, —•—, $h_0/b = 0.3$, $a_v/b = 0.106$; — — —, $h_0/b = 0.6$, $a_v/b = 0.082$.

observed in the time histories. For $h_0/b = 0.2$, first wave breaking is observed at $t \times \omega_I = 111.5$ following by a fourth mode wave in surge direction. Fig. 5.18 shows the wave profiles for heave excitation in stable regions $(\kappa_v, \Omega_n) = (0.5, 0.7)$ at water depth $h_0/b = 0.2, 0.3$ and 0.6 , respectively. Second mode wave forms with sloshing frequency $\omega_v/2$ and ω_v are observed at each water depth. The latter frequency relates to deep water. Fig. 5.19 shows the associated spectra. For $h_0/b = 0.2$, a frequency component ω_{2x} is introduced due to the wave breaking. The vertical velocity at the wall and associated phase plane are shown in Figs. 5.20-5.21. It can be observed that the nonlinearity becomes dominant earlier in the time evolution for the shallow water depth than deep water depth.

Fig. 5.18. Wave profile of wave generated in heave excitation in stable region $(\kappa_v, \Omega_n) = (0.5, 0.7)$ corresponding to data point no. 1 in Fig. 5. 16, at water depth $h_0/b = 0.2, 0.3$ and 0.6 , respectively.

In the following test series, the forcing amplitude and forcing frequency are kept constant ($a_v/b = 0.02$; $\omega_v/\omega_I = 0.85$) corresponding to large forcing. The κ_v varies due to the changes in water depth. The time histories and spectra of free-surface elevations at the center of the tank wall for $h_0/b = 0.2, 0.3, 0.6$ are displayed in Fig. 5.22 corresponding to test series in a stable region ($\Omega_I = 1.177$). With respect to the linear stability map (Fig. 5. 16), these tests are located on the region just above the second instability region (data point 3). The free-surface elevation show non-growing amplitudes as time evolves, as expected. The free-surface elevations are small even for large forcing amplitude, which is due to the stable characteristics embedded in the Mathieu's equation. The low energy frequency contents at the surface are also shown in the associated spectra (Fig. 5.22b). The shallow water case has enhanced irregularity at the free-surface due to the mode interaction component ($\omega_2 - \omega_I$) compared to other water depths.

5.3.2 Unstable Regions

The following test series are carried out in unstable regions at $(\kappa_v, \Omega_n) = (0.5, 0.5)$. The free-surface time histories are shown in Fig. 5.23. Rapidly exponentially growing amplitudes are observed in the time histories. From the superimposed time histories shown in Fig. 5.23, the amplitude grows relatively slower for the shallow water depth ($h_0/b = 0.2$) than the other two water depths. For $h_0/b = 0.3$, first wave breaking is observed at $t \times \omega_I \approx 80$ following by a second mode wave in surge direction. The amplitudes at the corners ($x = -b/2, y = -l/2$) and ($x = b/2, y = l/2$) are amplified due to the interaction of second modes in sway and surge directions. Fig. 5.24 shows the wave profiles for heave excitation in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$ with forcing frequency $\omega_v/\omega_I = 2$ at water depth $h_0/b = 0.2, 0.3$ and 0.6 , respectively. Diagonal mode shapes with swirling

(a)

(b)

Fig. 5.19. Spectra at wall in heave excitation in stable region $(\kappa_v, \Omega_n) = (0.5, 0.7)$ corresponding to data point no. 1 in Fig. 5. 16. (a) $t \times \omega_1 = 0-150$, and (b) $t \times \omega_1 = 150-250$. $\omega_v/\omega_1 = 1.429$, —, $h_0/b = 0.2$, $a_v/b = 0.140$; — • —, $h_0/b = 0.3$, $a_v/b = 0.106$; — —, $h_0/b = 0.6$, $a_v/b = 0.082$. Superscripts in spectra denote water depth to width ratio h_0/b . Subscripts x denote the frequency corresponding to surge direction.

(a)

(b)

Fig. 5.20. Vertical velocity history at wall in heave excitation in stable region (κ_v, Ω_n) = (0.5, 0.7) corresponding to data point no. 1 in Fig. 5. 16. (a) $t \times \omega_1 = 0-150$, and (b) $t \times \omega_1 = 150-250$. $\omega_v/\omega_1 = 1.429$, —, $h_0/b = 0.2$, $a_v/b = 0.140$; — • —, $h_0/b = 0.3$, $a_v/b = 0.106$; — — —, $h_0/b = 0.6$, $a_v/b = 0.082$.

(a)

(b)

Fig. 5.21. Phase plane at wall in heave excitation in stable region $(\kappa_v, \Omega_n) = (0.5, 0.7)$ corresponding to data point no. 1 in Fig. 5. 16. (a) $t \times \omega_1 = 0-150$, and (b) $t \times \omega_1 = 150-250$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; - - -, $h_0/b = 0.6$.

(a)

(b)

Fig. 5.22. Free-surface elevation and spectra at center of wall ($x=0, y=-l/2$) in heave excitation in stable region ($\Omega_1 = 1.177$) with forcing amplitude ($a_v/b = 0.02$) corresponding to data point no. 3 in Fig. 5. 16. —, $h_0/b = 0.2$, $\kappa_v = 0.025$; — • —, $h_0/b = 0.3$, $\kappa_v = 0.033$; — —, $h_0/b = 0.6$, $\kappa_v = 0.043$. Superscripts in spectra denote water depth to width ratio h_0/b .

(a)

(b)

Fig. 5.23. Free-surface elevation at center of wall ($x=0, y=-l/2$) in heave excitation in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$ corresponding to data point no. 2 in Fig. 5. 16. (a) $t \times \omega_1 = 0-200$, and (b) $t \times \omega_1 = 200-300$. $\omega_v/\omega_1 = 2$, —, $h_0/b = 0.2$, $a_v/b = 0.071$; — • —, $h_0/b = 0.3$, $a_v/b = 0.054$; — — —, $h_0/b = 0.6$, $a_v/b = 0.042$.

Fig. 5.24. Wave profile of wave generated in heave excitation in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$ corresponding to data point no. 2 in Fig. 5. 16, at water depth $h_0/b = 0.2, 0.3$ and 0.6 , respectively.

waves have been observed. The associated spectra are shown in Fig. 5.25. The first sloshing frequency and twice sloshing frequency are identified in the spectra, which led to diagonal mode shapes with swirling waves. The vertical velocity at the wall and associated phase plane are shown in Fig. 5.26 and Fig. 5.27. The free-surfaces behave nonlinearly soon after the tank is excited.

In the following test series, where the forcing amplitude and forcing frequency are kept constant ($a_v/b = 0.02$; $\omega_v/\omega_1 \approx 2$). Fig. 5.28 shows examples of surfaces in the first instability region. The time history is divided into two parts. In the first part, when $t \times \omega_1 = 100-260$ in Fig. 5.28a, exponentially growing amplitudes are observed. The exponentially growing resonance for $h_0/b = 0.3$ corresponds in a first sway mode until

(a)

(b)

Fig. 5.25. Spectra at wall in heave excitation in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$ corresponding to data point no. 2 in Fig. 5. 16. (a) $t \times \omega_1 = 0-200$, and (b) $t \times \omega_1 = 200-300$. $\omega_v/\omega_1 = 2$, —, $h_0/b = 0.2$, $a_v/b = 0.071$; — • —, $h_0/b = 0.3$, $a_v/b = 0.054$; — — —, $h_0/b = 0.6$, $a_v/b = 0.042$.

(a)

(b)

Fig. 5.26. Vertical velocity history at wall in heave excitation in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$ corresponding to data point no. 2 in Fig. 5. 16. (a) $t \times \omega_1 = 0-200$, and (b) $t \times \omega_1 = 200-300$. $\omega_v/\omega_1 = 2$, —, $h_0/b = 0.2$, $a_v/b = 0.071$; — • —, $h_0/b = 0.3$, $a_v/b = 0.054$; - - - , $h_0/b = 0.6$, $a_v/b = 0.042$.

(a)

(b)

Fig. 5.27. Phase plane at wall in heave excitation in unstable region
 $(\kappa_v, \Omega_n) = (0.5, 0.5)$ corresponding to data point no. 2 in Fig. 5. 16. (a) $t \times \omega_1 = 50-100$,
and (b) $t \times \omega_1 = 200-250$. $\omega_v/\omega_1 = 2$, —•—, $h_0/b = 0.3$, $a_v/b = 0.054$; — — —, $h_0/b = 0.6$, $a_v/b = 0.042$.

(a)

(b)

Fig. 5.28. Free-surface elevation at center of wall ($x=0, y=-l/2$) in heave excitation in the first instability region with forcing amplitude ($a_v/b = 0.02$) at resonance corresponding to data point no. 4 in Fig. 5. 16. (a) $t \times \omega_1 = 80-260$, and (b) $t \times \omega_1 = 240-400$. —, $h_0/b = 0.2$, ($\kappa_v = 0.139$, $\Omega_1 = 0.501$); — • —, $h_0/b = 0.3$, ($\kappa_v = 0.180$, $\Omega_1 = 0.508$); — — —, $h_0/b = 0.6$, ($\kappa_v = 0.221$, $\Omega_1 = 0.521$).

$t \times \omega_I = 144.7$. Then first point of breaking wave is observed at $t \times \omega_I \approx 164$ with elevation ($\zeta_{\max}/a_v = 15.05$) yielding a wave steepness $S = 0.945$. The amplitude of free-surface elevation then decays in a combined first sway and surge mode. The maximum free-surface of intermediate water depth is $\zeta_{\max}/a_v = 17$, which occurred at $t \times \omega_I \approx 171$, and is the fastest growing surface elevation compared to other water depths. At $t \times \omega_I \approx 235$ and 233, the first wave breaking occurred for water $h_0/b = 0.2$ and 0.6 yielding wave steepnesses in order of 0.63. In Fig. 5.28b, for water $h_0/b = 0.2$ and 0.6, the wave steepness grows large enough to introduce the third mode in a surge direction and contribute to the detuning behavior of the free-surface. This is also shown in the spectra in Fig. 5.29. The associated phase planes are shown in Figs. 5.30. Nonlinearity of the free-surface is illustrated clearly in the phase plane at all water depth.

Table 5.2 outlines the key findings of the heave excitation in unstable regions at different water depths.

Table 5.2. Key results of heave excitation in unstable regions.

h_0/b	ω_I	ω_v/ω_I	$\zeta_{\max}$	S	$t \times \omega_I$	Mode
0.2	4.14	1.995	0.20	0.628	235	Diagonal First Mode
0.3	4.76	1.97	0.34	1.068	171	First Mode and Third Mode
0.6	5.42	1.92	0.20	0.627	233	First Mode and Third Mode

In summary, the maximum free-surface elevations at the selected water depth $h_0/b = 0.2, 0.3, 0.6$ as a function of forcing frequency are summarized in Fig. 5.31. The test series are carried out for small and large forcing amplitudes ($a_v/b = 0.005, 0.02$). The data points represent both the stable regions and the first and second instability regions

(a)

(b)

Fig. 5.29. Spectra at wall in heave excitation in the first instability region with forcing amplitude ($a_v/b = 0.02$) at resonance corresponding to data point no. 4 in Fig. 5. 16. (a) $t \times \omega_1 = 80-260$, and (b) $t \times \omega_1 = 240-400$. —, $h_0/b = 0.2$, ($\kappa_v = 0.139$, $\Omega_1 = 0.501$); — • —, $h_0/b = 0.3$, ($\kappa_v = 0.180$, $\Omega_1 = 0.508$); — — —, $h_0/b = 0.6$, ($\kappa_v = 0.221$, $\Omega_1 = 0.521$). Superscripts in spectra denote water depth to width ratio h_0/b . Subscripts x denote the frequency corresponding to surge direction.

Fig. 5.30. Phase plane at wall in heave excitation in the first instability region with forcing amplitude ($a_v/b = 0.02$) at resonance corresponding to data point no. 4 in Fig. 5. 16. At $t \times \omega_1 = 80-260$ (a) —, $h_0/b = 0.2$, ($\kappa_v = 0.139$, $\Omega_1 = 0.501$); (b) — • —, $h_0/b = 0.3$, ($\kappa_v = 0.180$, $\Omega_1 = 0.508$); (c) — —, $h_0/b = 0.6$, ($\kappa_v = 0.221$, $\Omega_1 = 0.521$). At $t \times \omega_1 = 260-400$. (d) —, $h_0/b = 0.2$, ($\kappa_v = 0.139$, $\Omega_1 = 0.501$); (e) — • —, $h_0/b = 0.3$, ($\kappa_v = 0.180$, $\Omega_1 = 0.508$); (f) — —, $h_0/b = 0.6$, ($\kappa_v = 0.221$, $\Omega_1 = 0.521$).

(a)

(b)

(c)

Fig. 5.31. Maximum free-surface elevation at center of wall ($x=0, y=-l/2$) in heave excitation with small (--- , $a_v/b = 0.005$) and large forcing amplitude (--- , $a_v/b = 0.02$). (a) $h_0/b = 0.2$; (b) $h_0/b = 0.3$; (c) $h_0/b = 0.6$. $\bullet$, wave breaking.

Fig. 5.32. Snapshots of wave profiles generated in heave excitation at forcing amplitude ($a_v/b = 0.02$) in shallow water ($h_0/b = 0.2$) at $\omega_v/\omega_1 = 3.4$, ($\kappa_v = 0.405$, $\Omega_1 = 0.294$).

(a)

(b)

(c)

Fig. 5.33. Maximum free-surface elevation at center of wall ($x=0, y=-l/2$) in heave excitation with small (--- , $a_v/b = 0.005$) and large forcing amplitude (--- , $a_v/b = 0.02$). (a) $h_0/b = 0.2$; (b) $h_0/b = 0.3$; (c) $h_0/b = 0.6$. •, wave breaking in first instability region.

Fig. 5.34. Free-surface elevation at center of wall ($x=0, y=-l/2$) in heave excitation in the first instability region at $h_0/b = 0.6$. (a) $t \times \omega_1 = 100-260$, and (b) $t \times \omega_1 = 240-400$. —, $a_v/b = 0.005$, ($\kappa_v = 0.059$, $\Omega_1 = 0.505$); - - -, $a_v/b = 0.02$, ($\kappa_v = 0.221$, $\Omega_1 = 0.521$).

($\Omega_I \in [0.29, 1.18]$). Compared to the horizontal excitation, multiple resonance peaks are observed in the response curves for vertical excitation. Snapshots of profiles are shown in Fig. 5.32 corresponding to the third peak in the response curve in Fig. 5.31 for $h_0/b = 0.2$. Period tripling is found with three recurrent modes which resembles the Richtmyer-Meshkov instability. This was also found by Jiang *et al.* (1998). Close-up views of Faraday wave peaks are shown in Fig. 5.33 (ω_v/ω_I is near 2). Wave breakings are observed when $\kappa_v \in [0.13, 0.14]$ for $h_0/b = 0.2$, $\kappa_v \in [0.17, 0.19]$ for $h_0/b = 0.3$, and $\kappa_v \in [0.22, 0.24]$ for $h_0/b = 0.6$. Higher external forcing is required to generate wave breaking with increasing water depth. These test series also demonstrate the existence of hardening ($h_0/b = 0.2$) and softening ($h_0/b = 0.6$) liquid behaviors embedded in the Faraday peaks themselves. In order to investigate the effect of varying amplitudes of external forces, a comparison test series is shown in Fig. 5.34 ($h_0/b = 0.6$). For $a_v/b = 0.02$ exponentially growing resonance and associated detuning effects found whereas relatively small amplitude motion is observed for $a_v/b = 0.005$.

5.4 Combined Sway and Heave Excitation

In the following test series, in order to investigate the effect of vertical excitation on the pure horizontal excitation, the tank is excited in sway and heave direction simultaneously. The fluid motion is initially kept still. Harmonically forced motion $y = a_{hy}\sin(\omega_{hy}t)$ in a sway direction and $z = a_v\sin(\omega_vt)$ in a heave direction are applied simultaneously. The off-resonance and at resonance horizontal excitations are combined with vertical excitation in both stable and unstable regions, presented in sections 5.2 and 5.3, respectively.

5.4.1 Stable Regions

In the following test series, where the forcing amplitude and forcing frequency are kept constant ($a_{hy}/b = 0.006$; $\omega_{hy}/\omega_I = 0.85$; $a_v/b = 0.02$; $\omega_v/\omega_I = 0.85$) corresponding to large forcing. An off-resonance horizontal excitation ($\omega_{hy}/\omega_I = 0.85$) is combined with a vertical excitation in the stable region ($\Omega_I = 1.177$) shown in Fig. 5.35. The κ_v varies due to the changes in water depth. Beating responses with two distinct frequencies corresponding to the forcing frequency and the first sloshing frequency are observed. In this case, the vertical excitation has little influence on the horizontal excitations compared to the pure sway case (Fig. 5.1). This is due to the vertical excitation parameters are within the stable region, which behaves as a non-growing free-surface elevation. For $h_0/b = 0.2$, the free-surface exhibits relatively irregular peaks and troughs in the beating cycles, which is due to the mode interaction between the forcing frequency and first sloshing frequency. For $h_0/b = 0.3$ and 0.6 , the free-surfaces exhibit near linear solutions with equal peaks and troughs. The interaction becomes stronger for $h_0/b = 0.2$ as time evolves due to the fact that the time history becomes more irregular, as shown in the spectra in Fig. 5.36.

5.4.2 Unstable Regions

In this section, test series are conducted in unstable regions. Figs. 5.37 – 5.38 show time histories of the evolving surface and associated spectra of an at resonance horizontal excitation ($\omega_h/\omega_I = 1.03$) at a moderate forcing amplitude $a_h/b = 0.006$ combined with a vertical excitation in the first instability region at large forcing $a_v/b = 0.02$. For clarity, the time history is divided into two parts: $t \times \omega_I = 0-100$ and

(a)

(b)

Fig. 5.35. Stable region: free-surface elevation at center of wall ($x=0, y=-l/2$) in combined sway and heave excitation ($\Omega_1 = 1.177$, $\kappa_v \in [0.025, 0.043]$), $a_v/b = 0.02$, $\omega_{hy}/\omega_1 = 0.85$, $a_h/b = 0.006$. (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; — — —, $h_0/b = 0.6$.

(a)

(b)

Fig. 5.36. Stable region: spectra at wall in combined sway and heave excitation ($\Omega_1 = 1.177$, $\kappa_v \in [0.025, 0.043]$, $a_v/b = 0.02$, $\omega_{hv}/\omega_1 = 0.85$, $a_h/b = 0.006$. (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; — —, $h_0/b = 0.6$. Superscripts in spectra denote water depth to width ratio h_0/b .

(a)

(b)

Fig. 5.37. Free-surface elevation at wall in combined sway and heave excitation in an unstable region, $a_{hy}/b = 0.006$, $a_v/b = 0.02$. (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$, —, $h_0/b = 0.2$, $\kappa_{hy} = 0.011$, $\kappa_v = 0.139$, $\Omega_1 = 0.501$; — • —, $h_0/b = 0.3$, $\kappa_{hy} = 0.014$, $\kappa_v = 0.180$, $\Omega_1 = 0.508$; — — —, $h_0/b = 0.6$, $\kappa_{hy} = 0.017$, $\kappa_v = 0.221$, $\Omega_1 = 0.521$.

(a)

(b)

Fig. 5.38. Unstable region: spectra at wall in combined sway and heave excitation in an unstable region, $a_{hy}/b = 0.006$, $a_v/b = 0.02$. (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$, —, $h_0/b = 0.2$, $\kappa_{hy} = 0.011$, $\kappa_v = 0.139$, $\Omega_1 = 0.501$; — • —, $h_0/b = 0.3$, $\kappa_{hy} = 0.014$, $\kappa_v = 0.180$, $\Omega_1 = 0.508$; — —, $h_0/b = 0.6$, $\kappa_{hy} = 0.017$, $\kappa_v = 0.221$, $\Omega_1 = 0.521$. Superscripts in spectra denote water depth to width ratio h_0/b .

100-200. In the first part (Fig. 5.37a), exponentially growing resonance is observed. For $h_0/b = 0.2$, a first sway mode is recorded. First point of wave breaking is observed at $t \times \omega_I = 68.2$ with elevation $\zeta/a_h = 63.23$ and associated wave steepness of $S = 1.192$. The free-surface continues to increase until $t \times \omega_I = 74.6$ resulting in $\zeta_{max}/a_h = 110.31$ and $S = 2.079$, respectively. A combined first sway mode and third surge mode dominated at this point in time. Then the free-surface amplitude began to decay due to the introduction of the high frequency components (detuning effects). For $h_0/b = 0.3$, similar characteristics are observed as $h_0/b = 0.2$. The detuning effect begins at $t \times \omega_I = 82.7$, when the wave steepness is large enough ($S = 2.063$) to introduce a third surge mode. For the deep water case $h_0/b = 0.6$, the parameter $(\kappa_v, \Omega_n) = (0.221, 0.521)$ is located at the edge of first instability regions. For this reason, the exponential growing resonance and detuning effect are not as notable as the shallow water depths. This also can be observed in the spectra shown in Fig. 5.38. The shallow water is monitored with higher frequency component compared with other water depths.

In order to investigate the effect of the pure horizontal excitation on vertical excitation, the tank is excited in sway and heave direction simultaneously by varying the external force κ_{hy} . Fig. 5.39 shows the free-surface time history in two parts (for clarity). The amplitudes at the surface are observed to grow rapidly with $h_0/b = 0.2, 0.3$ yielding maximum amplitudes. For $h_0/b = 0.2$, first wave breaking is observed at $t \times \omega_I = 40.6$. Later maximum free-surface elevation is recorded ($t \times \omega_I = 70.4$). Detuning effects are observed and the amplitude decreases due to the increasing wave steepness ($\zeta_{max}/a_h = 234.9$) resulting introduction of higher modes and interactions. For $h_0/b = 0.3$, first wave breaking is observed at $t \times \omega_I = 54.8$ and ($\zeta_{max}/a_h = 220.2$) at $t \times \omega_I = 64.0$.

(a)

(b)

Fig. 5.39. Unstable region: free-surface elevation at wall in combined sway ($\kappa_{hy}=0.005$, $a_{hy}/b=0.003$) and heave excitation ($\kappa_v, \Omega_n)=(0.5,0.5)$. (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b=0.2$; — • —, $h_0/b=0.3$; — • —, $h_0/b=0.6$.

Free-surface detuned and a higher heave mode is introduced. For the deep water case $h_0/b = 0.6$, the first breaking point is observed at $t \times \omega_l = 114.2$ with free-surface elevation $\zeta_{\max}/a_h = 121.9$. Fig. 5.40 shows the snapshots of wave profiles of combined sway and heave excitation when the horizontal external force is small $\kappa_{hy}=0.005$ ($a_{hy}/b = 0.003$) and heave excitation is in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$. Violent wave breaking are observed at each water depth with maximum wave steepness of 1.15. The associated spectra and vertical velocity at the center of tank wall (wave gauge no. 1) are shown in Figs. 5.41 - 5.42. The nonlinearity of the free-surface wave is further illustrated in the wave phase planes as shown in Fig. 5.43. Fig. 5.43a shows the phase plane for $t \times \omega_l = 0 - 100$, and Fig. 5.43b corresponds to $t \times \omega_l = 100 - 200$.

Fig. 5.40. Wave profile of wave generated in combined sway ($\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$) and heave excitation in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$ at water depth $h_0/b = 0.2, 0.3$ and 0.6 .

Fig. 5.41. Unstable region: spectra at wall in combined sway ($\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$) and heave excitation ($\kappa_v, \Omega_n = (0.5, 0.5)$). (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; — —, $h_0/b = 0.6$. Superscripts in spectra denote water depth to width ratio h_0/b .

(a)

(b)

Fig. 5.42. Unstable region: vertical velocity history at wall in combined sway ($\kappa_{hy}=0.005$, $a_{hy}/b=0.003$) and heave excitation ($\kappa_v, \Omega_n)=(0.5,0.5)$. (a) $t \times \omega_1 = 0-100$, and (b) $t \times \omega_1 = 100-200$. —, $h_0/b=0.2$; — • —, $h_0/b=0.3$; — —, $h_0/b=0.6$.

(a)

(b)

Fig. 5.43. Unstable region: phase plane at wall in combined sway ($\kappa_{hy} = 0.005$, $a_{hy}/b = 0.003$) and heave excitation ($\kappa_v, \Omega_n = (0.5, 0.5)$). (a) $t \times \omega_1 = 50-100$, and (b) $t \times \omega_1 = 150-200$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; — —, $h_0/b = 0.6$.

In the following test series, the horizontal external force is increased to moderate $\kappa_{hy}=0.05$ ($a_{hy}/b = 0.006$). Figs. 5.44 – 5.47 show the cases of combined sway and heave excitation when the horizontal external force is $\kappa_{hy} = 0.05$ and heave excitation is in unstable region $(\kappa_v, \Omega_n) = (0.5, 0.5)$. Fig. 5.44 shows the free-surface time history in two parts for clarity. The amplitudes at the surface are also observed to grow rapidly for all water depths. The associated spectra and vertical velocity at the center of tank wall are shown in Figs. 5.45 - 5.46. In Fig. 5.45, a high frequency component at twice the sloshing frequency is monitored. The nonlinearity of the free-surface wave is further illustrated in the wave phase planes as shown in Fig. 5. 47. Compared with the small horizontal forcing, the higher horizontal forcing amplitude does not show evident effect on vertical excitation due to the dominance of vertical excitation in unstable regions.

A horizontal at resonance excitation ($\omega_{hy}/\omega_I=1.03$) at moderate forcing amplitude $a_{hy}/b = 0.006$ is combined to the third peak (“mushroom”) in the response curve in Fig. 5.31 for $h_0/b = 0.2$. Fig. 5.48 shows snapshots of the associated wave profiles which resembles the “Richtmyer-Meshkov Instability”. The vertical generated period tripling wave form is observed to degenerate into violent overturning waves at the center and walls. At $t \times \omega_I = 0-61$ the free-surface is dominated by a first sway mode. Then the wave breaking occurs. It is noted that the free-surface becomes more irregular with enhanced wave breaking than the pure vertical case. The peak of the period tripling wave tilted to the wall back and forth along with the first sway mode, which is due to the interaction of horizontal and vertical excitations. Fig. 5.49 shows a comparison of wave forms between sway, heave and combined sway and heave excitations.

Table 5.3 outlines the key findings of the combined sway and heave excitation in unstable regions at different water depths.

Table 5.3. Key results of combined sway and heave excitation in unstable regions.

h_0/b	ω_I	ω_{hy}/ω_I	ω_v/ω_I	$\zeta_{\max}$	S	$t \times \omega_I$	Mode
0.2	4.14	1.03	1.995	0.66	2.079	75	First Sway Mode and Third Surge Mode
0.3	4.76	1	1.97	0.66	2.063	83	First Sway Mode and Third Surge Mode
0.6	5.42	0.96	1.92	0.35	1.104	98	First Mode and Second Mode

(a)

(b)

Fig. 5.44. Unstable region: free-surface elevation at wall in combined sway ($\kappa_{hy}=0.05$, $a_{hy}/b=0.006$) and heave excitation ($\kappa_v, \Omega_n)=(0.5,0.5)$. (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-200$. —, $h_0/b=0.2$; — • —, $h_0/b=0.3$; — —, $h_0/b=0.6$.

(a)

(b)

Fig. 5.45. Unstable region: spectra at wall in combined sway ($\kappa_{hy} = 0.05$, $a_{hy}/b = 0.006$) and heave excitation $(\kappa_v, \Omega_n) = (0.5, 0.5)$. (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-200$.

—, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; — —, $h_0/b = 0.6$.

(a)

(b)

Fig. 5.46. Unstable region: vertical velocity history at wall in combined sway ($\kappa_{hy}=0.05$, $a_{hy}/b = 0.006$) and heave excitation ($\kappa_v, \Omega_n = (0.5, 0.5)$). (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-200$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; - - -, $h_0/b = 0.6$.

(a)

(b)

Fig. 5.47. Unstable region: phase plane at wall in combined sway ($\kappa_{hy} = 0.05$, $a_{hy}/b = 0.006$) and heave excitation ($\kappa_v, \Omega_n = (0.5, 0.5)$). (a) $t \times \omega_1 = 0-120$, and (b) $t \times \omega_1 = 120-200$. —, $h_0/b = 0.2$; — • —, $h_0/b = 0.3$; — — —, $h_0/b = 0.6$.

Fig. 5.48. Wave profile of wave generated in combined sway and heave excitation in shallow water ($h_0/b = 0.2$) at ($a_{hy}/b = 0.006$, $\omega_{hy}/\omega_1 = 1.03$) combined with ($a_v/b = 0.02$, $\omega_{hy}/\omega_1 = 3.4$).

Fig. 5.49. Comparison of wave profiles for wave generated in sway, heave and combined sway and heave excitation in shallow water ($h_0/b = 0.2$), respectively.

CHAPTER 6. BORE FORMATION

6.1 Introduction

In this chapter, the behaviors of free-surface waves in a square tank with base dimensions $1 \times 1 \text{ m}^2$ at shallow water depths ($h_0/b < 0.05$) are investigated. At these shallow water depths, traveling waves and bores are known, as reported by Toro (1998). Three-dimensional free-surface flows under gravity inside a tank have been considered here. The Cartesian coordinate system (x, y, z) is coincident with surge, sway and heave directions respectively, as shown in experimental set-up Fig. 4.1. In the following test series, the free-surface waves are investigated when exciting the tank in sway and in simultaneous sway and surge directions. The forcing parameters are systematically varied.

Fig. 6.1. Definition sketch of bore in tank.

A bore is recognized by two distinct water depths with a propagating front varying from a weak to a fully developed bore front as illustrated in Fig. 6.1. The water depth behind the bore h_1 is deeper than the water depth in front of it h_2 . The propagating speed of a bore is defined as U_0 . In the tank, bores are observed moving between the tank walls periodically. Usually, the tongue of bore is turbulent. A measure of the bore's strength is sought through the Froude number, F_r , defined as:

$$F_r = U_0 / \sqrt{gh_1} \quad (11)$$

where g is the acceleration due to gravity.

It should be noted that the bore in the tanks represent the physics of the bore front observed on beaches. Yeh *et al.* (1989) experimentally investigated run-up velocities of a bore propagating to the beach. Frandsen *et al.* (2005) investigated the free-surface of shallow water in horizontally excited square tanks by Lattice Boltzmann solver. Shortcomings were discussed, in particular, the usages of shallow water equations. Furthermore, the presented investigations of the bore study may also be useful to the tsunami community because the front of fully developed bores in a tank resembles the tongue of a tsunami wave.

The forcing amplitude and the forcing frequency are gradually increased to detect nonlinearities and discontinuities at the free-surface outside resonance and at resonance. Herein, resonance is defined to occur when the external forcing frequency ω_{hy} equals the sloshing frequency, which defined as $\omega_n = \sqrt{g \frac{n\pi}{b} \tanh(\frac{n\pi}{b} h_0)}$, where $n = 1, 2, 3 \dots$. The forcing parameter $\kappa_{hy} = a_{hy} \omega_{hy}^2 / g$ is used as a measurement of nonlinearities.

Free-surface time histories are recorded at various selected location for $a_{hy}/b = 0.003, 0.006, 0.02, 0.05, 0.1$ ranging $\omega_{hy}/\omega_I \in (0.85, 2)$. The maximum free-surface elevations are shown in the response curves for the five different test series in Fig. 6.2. Hardening oscillator behavior of the surface can be observed in all these test series as maximum free-surface elevation occurred at $\omega_{hy}/\omega_I > 1$. The effect strengthens with increasing κ_{hy} . The maximum free-surface elevation is found at forcing ratio $\omega_{hy}/\omega_I = 1.08$ for small amplitude excitation ($a_{hy}/b = 0.003$) whereas the largest amplitude

excitation case ($a_{hy}/b = 0.10$) yielded a maximum surface at $\omega_{hy}/\omega_1 = 1.40$. In the following, these five test series labeled from number 1 to 5 in Fig. 6.2 are described in detail.

Fig. 6.2. Response curve in shallow water. $h_0/b = 0.05$, $\omega_{hy}/\omega_1 \in [0.85, 2.0]$. $\square$, $a_{hy}/b = 0.003$; $*$, $a_{hy}/b = 0.006$; $+$, $a_{hy}/b = 0.02$; $\bullet$, $a_{hy}/b = 0.05$; $\times$, $a_{hy}/b = 0.10$; $\circ$, wave breaking.

6.2 Sway Base Excitation with Small to Moderate Forcing

In these test series, the tank is excited in a sway excitation only. The water depth is kept constant at $h_0/b = 0.05$ while varying the forcing amplitude and frequency. The water in the tank is initially still. A harmonic motion $y = a_{hy} \sin(\omega_{hy} t)$ is applied to the base of the tank; where a_{hy} is the forcing amplitude and ω_{hy} is the forcing frequency in sway direction, respectively.

Data point no. 1 corresponds to test series with $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_1 = 0.85$. A two-dimensional small amplitude traveling wave combined with the first mode small

Fig. 6.3. Traveling waves. Transient period (a-c) and steady state (d-f) time histories of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.003$.

amplitude sloshing is observed in this test (no wave breaking occurred). The frequency is increased to near resonance, where $\omega_{hy}/\omega_I = 1.0$ while the forcing amplitude is kept constant $a_{hy}/b = 0.006$. Fig. 6.3 shows the time history of free-surface elevation at transient and steady state phases for test case, data point no. 2 (Fig. 6.2). The associated snapshots of profiles for data points no. 1 and 2 are shown in Fig. 6.4.

Fig. 6.4. Wave profile of traveling wave for (a) point 1 in Fig. 6.2. $h_0/b = 0.05$, $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_I = 0.85$, $\kappa_{hy} = 0.003$. And traveling wave for (b) point 2 in Fig. 6.2. $h_0/b = 0.05$, $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_I = 1.0$, $\kappa_{hy} = 0.003$. Traveling waves propagating from left to right.

Traveling waves are observed to form without any wave breaking occurring. Initially, the free-surface would slosh back and forth in a first mode ($T = t \times \omega_I \in [0, 18]$). Furthermore, two consecutive traveling waves have been observed in this case when $T > 20$. From observations, it is found that the traveling waves are two-dimensional except when interacting with the tank walls. The associated frequencies are shown in Fig. 6.5. At tank center, there exist two distinct frequencies, $2\omega_I$ and a higher frequency component $4\omega_I$. The frequencies represent two consecutive traveling waves. When the free surface reaches steady state, sloshing frequency is observed. The associated vertical velocity has been shown in Fig. 6.6. Due to the impact on the wall, the vertical component is larger than those at the center of the tank. These components are, however, negligible when compared with the propagating velocities of the traveling waves which is in the order of $u_y = 0.72$ m/s. The Froude number is estimated $Fr(h_I) = u_y / \sqrt{gh_I} = 1.04$, where $h_I = 0.049$

Fig. 6.5. Traveling waves. Transient period (a-c) and steady state (d-f) spectra of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.003$.

Fig. 6.6. Traveling waves. Transient period (a-c) and steady state (d-f) vertical velocity history of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.003$.

Fig. 6.7. Traveling waves. Transient period (a-c) and steady state (d-f) phase plane diagram of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, $a_{hy}/b = 0.006$, $\omega_{hy}/\omega_1 = 1.0$, $\kappa_{hy} = 0.003$.

m. The surface resembles similar features as observed in undular hydraulic jumps. The associated phase plane diagrams, in Fig. 6.7, show the near linear behavior of the free-surface displaying same scale peaks and troughs for traveling waves being generated by moderate forcing.

6.3 Sway Base Excitation with Large Forcing

In these test series, the forcing amplitude is further increased to investigate bores in tank with prescribed sway base excitation ($h_0/b = 0.05$). The forcing amplitudes are chosen to be large enough ($a_{hy}/b > 0.006$) to generate bores and gradually increased to explore the strength of the bores developing from weak to strong. The bore front becomes highly turbulent in fully developed bores. It should be noted that the wave gauges capture and record non-breaking surface. In this chapter, the turbulence and breaking are only described through observations via snapshots of selected time instances.

It is identified that the minimum external forcing parameter to generate a bore is $\kappa_{hy} = 0.012$ corresponding to a weak bore, while the forcing parameter $\kappa_{hy} = 0.096$ corresponds to a fully developed bore. Time histories of the free-surface elevation of bores are shown in Figs. 6.8 for increasing $\kappa_{hy} = 0.012, 0.048$ and 0.096 , respectively. The magnitudes of free-surface elevations are dependent on the strength of bores, as expected. The free-surface elevations are increasing when the Froude number grows, especially at the walls. Maximum free-surface elevation is observed when a fully developed bore is generated. Initially, the water in the tank would slosh back and forth for all the bore cases. However, this period would shorten for larger forcing parameters. Snapshots of wave profiles of weak to fully developed bores are shown in Fig. 6.9, when the external forcing parameter $\kappa_{hy} = 0.012, 0.048$ and 0.096 , respectively. Initially, the free-surface is

Fig. 6.8. Bores. Transient period (a-c) and steady state (d-f) time history of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, —, $a_{hy}/b = 0.02$, $\omega_{hy}/\omega_1 = 1.10$, $\kappa_{hy} = 0.012$; — •, $a_{hy}/b = 0.05$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.048$; — —, $a_{hy}/b = 0.10$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.096$.

observed to slosh between the tank walls in a first mode for these cases too. A weak bore is observed when $\kappa_{hy} = 0.012$. Wave breaking is observed when the front of the bore interacted with the wall, as illustrated in the first row of Fig. 6.9 at $T = t \times \omega_I = 28.55$. The propagating speed of this weak bore is about 0.83 m/s ($F_r(h_I) = 1.10$). Increasing the sway force by a factor four ($\kappa_{hy} = 0.048$) results in a stronger bore, as shown in the second row of Fig. 6.9. Wave breaking is observed, especially at the walls. The propagating speed increases to about 1.15 m/s. Then increase the base force further by a factor two ($\kappa_{hy} = 0.096$). Fully developed bore are observed (Fig. 6.9). Initially, the free-surface is first mode sloshing; however a bore would form rapidly. For $a_{hy}/b = 0.10$, the bore is fully developed displaying a highly turbulent bore front with propagating speed $U_0 = 1.52$ m/s. In these test series, two distinct water levels of bores are recorded, as shown in table 6.1. The water depth ratio is $h_1/h_2 \in [1.6, 6.9]$ with $F_r(h_1) \in [1.1, 1.4]$. It should be noted that $h_2 \rightarrow 0$ for the $F_r(h_1) > 1.4$.

Fig. 6.9. Wave profile of weak to fully developed bores. Sway excitation at moderate forcing amplitude corresponding to data point 3,4 and 5 labeled in Fig. 6.2: $\kappa_{hy}=0.012, 0.048$ and 0.096 , respectively. Bores propagating from left to right.

In the spectra shown in Fig. 6.10, the frequency $1.4\omega_I$ is dominant. This is due to the large forcing parameter κ_{hy} . A high frequency component bore frequency has been captured at the left center wall which contributes to the nonlinearity at the free-surface.

Fig. 6.10. Bores. Transient period (a-c) and steady state (d-f) spectra of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, —, $a_{hy}/b = 0.02$, $\omega_{hy}/\omega_1 = 1.10$, $\kappa_{hy} = 0.012$; — • —, $a_{hy}/b = 0.05$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.048$; — — —, $a_{hy}/b = 0.10$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.096$.

Fig. 6.11. Bores. Transient period (a-c) and steady state (d-f) vertical velocity history of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, —, $a_{hy}/b = 0.02$, $\omega_{hy}/\omega_1 = 1.10$, $\kappa_{hy} = 0.012$; — •, $a_{hy}/b = 0.05$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.048$; — —, $a_{hy}/b = 0.10$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.096$.

Fig. 6.12. Bores. Transient period (a-c) and steady state (d-f) phase plane diagram of free-surface elevation across the tank. (a, d) left center wall ($x=0, y=-l/2$); (b, e) quarter point ($x=0, y=-l/4$) and (c, f) center ($x=0, y=0$). $h_0/b = 0.05$, —•, $a_{hy}/b = 0.02$, $\omega_{hy}/\omega_1 = 1.10$, $\kappa_{hy} = 0.012$; —•, $a_{hy}/b = 0.05$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.048$; —•, $a_{hy}/b = 0.10$, $\omega_{hy}/\omega_1 = 1.40$, $\kappa_{hy} = 0.096$.

Fig. 6.13. Wave profile of weak bore. Sway excitation at forcing amplitude $a_{hy}/b = 0.02$, $\omega_{hy}/\omega_1 = 1.00$, at water depth $h_0/b = 0.09$. Bore propagating from left to right.

Fig. 6.14. Velocity history of the bore. Sway excitation at forcing amplitude $a_{hy}/b = 0.02$, $\omega_{hy}/\omega_1 = 1.00$, at water depth $h_0/b = 0.09$.

The associated vertical velocities are shown in Fig. 6.11. Fig. 6.12 shows the phase plane diagrams for transient and steady state. The free surface become steady with shorter time compared with small κ_{hy} case. Fig. 6.13 shows the wave profile of a weak bore at water depth $h_0/b = 0.09$ when $\kappa_{hy} = 0.017$. The associated horizontal velocity time history is shown in Fig. 6.14. The horizontal velocity becomes steady around $t \times \omega_1 = 80$. The horizontal velocity for water depth $h_0/b = 0.05$ has not been reported due to the accuracy of instrument.

In summary, for the $1 \times 1 \text{ m}^2$ tank, it is found that traveling waves are generated when the forcing amplitude $a_{hy}/b < 0.02$; whereas bores are observed when the forcing amplitude $a_{hy}/b \geq 0.02$. The bore becomes stronger with the increasing κ_{hy} . Table 6.1 outlines wave forms and F_r for increasing κ_{hy} in the test series undertaken.

Table 6.1. Wave forms and Froude number at $h_0/b = 0.05$. Sway base excitation.

h_1 (m)	h_2 (m)	U_o (m/s)	$F_r(h_1)$	$F_r(h_2)$	$F_r(h_0)$	$(a_{hy}/b, \omega_{ny}/\omega_1)$	Wave form
0.049	0.048	0.72	1.0385	1.0492	1.0280	(0.006,1.00)	Traveling wave
0.058	0.036	0.83	1.1003	1.3967	1.1851	(0.02,1.10)	Weak bore
0.074	0.033	1.15	1.3497	2.0212	1.6420	(0.05,1.40)	Intermediate bore
0.117	0.017	1.52	1.4188	3.7221	2.1703	(0.10,1.40)	Strong bore

6.4 Sway and Surge Case

In the following, the free-surface behavior is investigated when forcing the tank in sway and surge directions simultaneously. A dominating bore would typically form diagonally at certain forcing frequencies. The definition sketch of a diagonally propagating bore is shown in Fig. 6.15.

Fig. 6.15. Definition sketch for diagonally propagating bores. ●, Sensor location.

The water depth $h_0/b = 0.05$ is kept constant, while the forcing amplitude and forcing frequency are varied. Harmonic base excitation $x = a_{hx} \sin(\omega_{hx} t)$ and $y = a_{hy} \sin(\omega_{hy} t)$ are applied to the tank; where a_{hx} , a_{hy} are the forcing amplitudes and ω_{hx} , ω_{hy} are the forcing frequencies in surge and sway directions, respectively. Symmetric base excitation is applied, i.e., the forcing amplitudes and frequencies are prescribed to be the same for the test series presented ($a_{hx} = a_{hy}$, $\omega_{hx} = \omega_{hy}$). The additional surge base excitation is applied in combination with the sway test series 1 to 5 (Fig. 6.2). In general, three distinct water levels h_1 , h_2 , $h_3 = h_4$ are observed, as shown in Fig. 6.15. Besides the diagonal bore, there exist bores propagating in sway and surge

Fig. 6.16. Weak diagonal bores. Transient period (a-c) and steady state (d-f) time history of free-surface elevation across the tank. (a, d) diagonal corner ($x=-b/2, y=-l/2$); (b, e) center ($x=0, y=0$) and (c, f) off-diagonal corner ($x=b/2, y=-l/2$). $h_0/b = 0.05$, $a_{hx}/b = a_{hy}/b = 0.006$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.0$, $\kappa_{hx} = \kappa_{hy} = 0.003$.

directions as well. Moreover, two consecutive bores are observed to form and dominate in the diagonal direction when $\kappa_{hx} = \kappa_{hy} = 0.096$. This corresponds to a fully developed bore.

6.4.1 Sway and Surge Case with Small to Moderate Forcing

The first test case included moderate base excitation for $a_{hx}/b = a_{hy}/b = 0.006$, $\kappa_{hx} = \kappa_{hy} = 0.003$. Fig. 6.16 shows the time histories of this test at transient and steady state phases. It can be seen that the free-surface elevation at the diagonal corner location $(x,y) = (-b/2,-l/2)$ (wave gauge no. 4 in Fig. 4.1) is larger than other points which are not on the diagonal line. Two small amplitude consecutive traveling waves are observed in each forcing direction as shown in Fig. 6.17. While at the diagonal of tank, two weak bores are observed to dominate the flow.

Fig. 6.17. Wave profile of weak diagonal bores. $h_0/b = 0.05$, $a_{hx}/b = a_{hy}/b = 0.006$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.0$, $\kappa_{hx} = \kappa_{hy} = 0.003$.

The frequency spectra corresponding to the location of the diagonal corners contains more energy compared to other locations as shown in Fig. 6.18. In all the spectra at different location, there exist two distinct frequencies, $2\omega_1$ and a higher frequency component of bore frequency $4\omega_1$. Compared to the pure sway excitation (Figs. 6.8), the

Fig. 6.18. Weak diagonal bores. Transient period (a-c) and steady state (d-f) spectra of free-surface elevation across the tank. (a, d) diagonal corner ($x=-b/2, y=-l/2$); (b, e) center ($x=0, y=0$) and (c, f) off-diagonal corner ($x=b/2, y=-l/2$). $h_0/b = 0.05$, $a_{hx}/b = a_{hy}/b = 0.006$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.0$, $\kappa_{hx} = \kappa_{hy} = 0.003$.

Fig. 6.19. Combined sway and surge double bores. Transient period (a-c) and steady state (d-f) time history of free-surface elevation across the tank. (a, d) diagonal corner ($x=-b/2, y=-l/2$); (b, e) center ($x=0, y=0$) and (c, f) off-diagonal corner ($x=b/2, y=-l/2$). $h_0/b = 0.05$; —, $a_{hx}/b = a_{hy}/b = 0.02$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.10$, $\kappa_{hx} = \kappa_{hy} = 0.012$; — •, $a_{hx}/b = a_{hy}/b = 0.05$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.40$, $\kappa_{hx} = \kappa_{hy} = 0.048$; —, $a_{hx}/b = a_{hy}/b = 0.10$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.40$, $\kappa_{hx} = \kappa_{hy} = 0.096$.

frequency spectra at steady state at the center contain more energy for high frequency peaks, as expected.

6.4.2 Sway and Surge Case with Large Forcing

Time histories shown in Fig. 6.19 correspond to $\kappa_{hx} = \kappa_{hy} \in [0.012, 0.0096]$ for the resonance data point 3, 4 and 5 (labeled in Fig. 6.2). The free-surface elevation at corner location $((x,y) = (-b/2,-l/2))$ is larger than any other locations inside the tank. There exist phase lags in the time histories for different forcing due to the difference of propagating speed of the bores.

Fig. 6.20. Wave profile of double bores. Combined sway and surge excitation at moderate forcing amplitude. $\kappa_{hx} = \kappa_{hy} = 0.012, 0.048$ and 0.096 , respectively. Bore propagating diagonally.

The wave profiles shown in Fig. 6.20 are associated with the additional surge forcing ($\kappa_{hy} \in [0.012, 0.096]$). Initially, the free-surface would slosh between tank walls in both surge and sway directions in a first mode for the combined cases. For $\kappa_{hx} = \kappa_{hy} = 0.012$ ($a_{hx}/b = a_{hy}/b = 0.02$), some breaking waves are observed at the front of the bore propagating diagonally as illustrated in the first row of Fig. 6.20. The propagating speed of these two intermediate bores are 1.06 m/s, yielding a Froude number $F_r(h_1) = 1.43$.

Increasing the amplitude to $\kappa_{hx} = \kappa_{hy} = 0.048$ ($a_{hx}/b = a_{hy}/b = 0.05$), two consecutive strong bores are generated as shown in the second row of Fig. 6.20. This was also found by Wu *et al.* (1998). Enhanced breaking waves are observed at the front of bores and corners ($U_0 = 1.53$ m/s, $F_r(h_1) = 1.70$ and $h_1/h_2 = 4.61$). The third row of Fig. 6.20 shows the wave profiles at resonance case for $\kappa_{hx} = \kappa_{hy} = 0.096$ ($a_{hx}/b = a_{hy}/b = 0.10$). Two fully developed strong bores are formed, propagating diagonally. Breaking waves with strong turbulence at the bore fronts and violent wave breaking at the corners are observed. The propagating speed increases up to 1.77 m/s yielding $F_r(h_1) = 1.44$ ($h_1/h_2 = 30.80$). In the spectra shown in Fig. 6.21, the frequency $2.8\omega_l$ is dominant in the tank. A high frequency component of bore frequency has been captured at each location.

Table 6.2 shows a summary of the wave forms and associated data found for bores propagating diagonally. The propagating speed of the bore is found to be amplified by a factor $\sqrt{2}$ compared to the pure sway case. Table 6.3 and table 6.4 show bores propagating in sway and surge, respectively. In summary, two consecutive bores form when forcing amplitude $a_h/b \geq 0.006$, propagating diagonally.

Table 6.2. Wave forms and Froude number at $h_0/b = 0.05$. Combined base excitation.

h_1 (m)	h_2 (m)	$U_0 = \sqrt{U_{0x}^2 + U_{0y}^2}$ (m/s)	$F_r(h_1)$	$F_r(h_2)$	$F_r(h_3)$	$(a_{hx}/b, \omega_{hx}/\omega_1)$	Wave form
0.049	0.028	$0.91 = \sqrt{0.64^2 + 0.64^2}$	1.313	1.736	0.985	(0.006,1.00)	Two weak bores
0.056	0.022	$1.06 = \sqrt{0.75^2 + 0.75^2}$	1.430	2.282	1.183	(0.02,1.10)	Two intermediate bores
0.083	0.018	$1.53 = \sqrt{1.08^2 + 1.08^2}$	1.696	3.641	1.724	(0.05,1.40)	Two strong bores
0.154	0.005	$1.77 = \sqrt{1.25^2 + 1.25^2}$	1.440	7.992	2.047	(0.10,1.40)	Two strong bores

Table 6.3. Wave forms and Froude number at $h_0/b = 0.05$. Combined base excitation. Bore propagating in surge.

h_1 (m)	h_3 (m)	U_{ox} (m/s)	$F_r(h_1)$	$F_r(h_3)$	$F_r(h_0)$	$(a_{hx}/b, \omega_{hx}/\omega_1)$	Wave form
0.049	0.043	0.64	0.923	0.985	1.022	(0.006,1.00)	Weak bore
0.056	0.041	0.75	1.012	1.183	1.197	(0.02,1.10)	Intermediate bores
0.083	0.040	1.08	1.197	1.724	1.724	(0.05,1.40)	Strong bore
0.154	0.038	1.25	1.017	2.047	1.996	(0.10,1.40)	Strong bore

Table 6.4. Wave forms and Froude number at $h_0/b = 0.05$. Combined base excitation. Bore propagating in sway.

h_3 (m)	h_2 (m)	U_{oy} (m/s)	$F_r(h_4)$	$F_r(h_2)$	$F_r(h_0)$	$(a_{hx}/b, \omega_{hx}/\omega_1)$	Wave form
0.043	0.028	0.64	0.985	1.2211	1.0217	(0.006,1.00)	Weak bore
0.041	0.022	0.75	1.183	1.6144	1.1973	(0.02,1.10)	Intermediate bores
0.040	0.018	1.08	1.724	2.5701	1.7241	(0.05,1.40)	Strong bore
0.038	0.005	1.25	2.047	5.6440	1.9955	(0.10,1.40)	Strong bore

Fig. 6.21. Combined sway and surge double bores. Transient period (a-c) and steady state (d-f) spectra of free-surface elevation across the tank. (a, d) diagonal corner ($x=-b/2, y=-l/2$); (b, e) center ($x=0, y=0$) and (c, f) off-diagonal corner ($x=b/2, y=-l/2$). $h_0/b = 0.05$; —, $a_{hx}/b = a_{hy}/b = 0.02$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.10$, $\kappa_{hx} = \kappa_{hy} = 0.012$; — •, $a_{hx}/b = a_{hy}/b = 0.05$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.40$, $\kappa_{hx} = \kappa_{hy} = 0.048$; —, $a_{hx}/b = a_{hy}/b = 0.10$, $\omega_{hx}/\omega_1 = \omega_{hy}/\omega_1 = 1.40$, $\kappa_{hx} = \kappa_{hy} = 0.096$.

CHAPTER 7. CONCLUSIONS

Investigations of free-surface water wave behavior in a relatively large square tank with base dimensions of $1 \times 1 \text{ m}^2$ have been carried out. Harmonic base excitations in shallow to deep water are prescribed. The test series includes sway, heave and combined sway and heave base excitations with depth to width ratio $h_0/b \in [0.05, 0.6]$. Especially, the latter investigations are considered to be a contribution to the current literature.

In pure sway excitation of the tank, the free-surface waves are captured off- and at resonance. The off-resonance case displays a typical beating response characterized by two distinct frequencies corresponding to the forcing frequency and the first sloshing frequency. The resonance cases typically exhibit linearly growing amplitudes at the walls. For the water depth $h_0/b = 0.3$, a linear free-surface time history is captured independent of forcing amplitude yielding a maximum free-surface elevation of $\zeta/b = 0.26$ at $\omega_h/\omega_l = 1$; before wave breaking occurs. This defines the critical water depth as $h_0/b = 0.3$. The peaks for $h_0/b = 0.2$ and 0.6 are observed near the forcing frequency ratio one. For deep water, the free-surface behaves as a softening oscillator yielding the maximum free-surface elevation at $\omega_h/\omega_l < 1$ whereas the shallow water resembles a hardening spring oscillator with maximum free-surface elevation at $\omega_h/\omega_l > 1$. For wave steepness $S \geq 0.48$, high frequency components exist and are responsible for the nonlinearities at the surface. The higher frequencies are typically generated due to mode interaction. The shallow water case is highly nonlinear. The time evolutions show low troughs and high peaks with higher frequency components embedded in the troughs. Mode interaction is also found at off resonance cases. For the test series herein, this

surface behavior is not captured in deep water. Generally, wave breaking occurs when wave steepness $S \geq 0.48$. Wave breaking is associated with three-dimensionality typically in swirling motions in shallow and deep water. For the intermediate water depth or rather the critical depth $h_0/b = 0.3$, wave breaking also results in swirling modes. The observations in sway test series follows what is already reported in the literature.

In pure heave excitation of the tank, often referred to as Faraday experiments, the test series are carried out in both stable and unstable regions. In the stable region, non-growing amplitude time history is observed, as expected. Typical beating time evolutions of the free-surface are captured when the external forcing κ_v is small. In the unstable region, the free-surface has exponentially growing amplitudes. Typical detuning effects are observed when the wave steepness grows large enough introducing higher modes and mode interactions. Hysteresis effects are observed in the Faraday peaks when the forcing frequency ratios ω_v/ω_1 are near 2. These Faraday peaks exhibit hardening ($h_0/b = 0.2$) and softening ($h_0/b = 0.6$) spring oscillator behavior of the water. At certain time instants, the associated wave forms take the shape of the classical “mushroom” which resembles the Richtmyer-Meshkov instability and the period tripling phenomena, mentioned in the literature.

The effect of the heave excitation on the pure sway excitation of the tank is investigated when the tank is exciting simultaneously in sway and heave directions. The off resonance and at resonance test series reported for sway excitations test cases are combined with heave excitations in both stable and unstable regions. In the unstable region, the effects of vertical excitation component become dominant which is responsible for typically mode interactions and detuning effects at the free-surface.

In the shallow water case ($h_0/b = 0.05$), formation of traveling waves and bores are observed. This has also been reported by other researchers. For pure sway excitation, a traveling wave is observed at moderate external forcing $a_{hy}/b = 0.006$. It is identified that the minimum external forcing parameter to generate a bore is $\kappa_{hy} = 0.012$ in the tank with base dimensions of $1 \times 1 \text{ m}^2$. This test series correspond to a weak bore; while the forcing parameter $\kappa_{hy} = 0.096$ generates a fully developed bore. The bore test series typically involves generation of two distinct water levels. The test series include water depth ratios in the order of $h_1/h_2 \in [1.6, 6.9]$ and associated Froude numbers $F_r(h_1) \in [1.1, 1.4]$. Other test series at $h_0/b = 0.05$ are undertaken in which the tank is excited in sway and surge simultaneously. As a result, three distinct water levels are generated with a dominating bore forming diagonally for $F_r(h_1) \in [1.3, 1.7]$. Besides the dominating diagonal bore, two other bores in sway and surge directions are monitored. The propagating speed of the diagonal bore is found to be amplified in the order of a factor $\sqrt{2}$ when compared to the pure sway case.

CHAPTER 8. FUTURE WORK

Future studies on sloshing in tanks involving other tank sizes are recommended. A detailed study of breaking wave mechanism is also recommended with information on velocities (also in the entire flow domain) and pressure distributions on walls. Base excitations should be included taking advantage of the six degrees-of-freedom shaking table. Examinations of the free-surface with baffles inside the liquid tanks are also suggested leading into a study on tuned liquid damper performance.

REFERENCES

- Abramson, H. N. 1966 *The dynamics of liquids in moving containers*. NASA SP-106, National Aeronautics and Space Administration, Washington D.C.
- Akyildiz, H. & Unal, E. 2005 Experimental investigation of pressure distribution on a rectangular tank due to the liquid sloshing. *Ocean Engineering* **32**, 1503-1516.
- Amundsen, D. E., Cox, E. A. Mortell, M. P. & Reck, S. 2001 Evolution of nonlinear sloshing in a tank near half the fundamental resonance. *Studies in Applied Mathematics* **107**, 103-125.
- Anastasiou, K. & Bokaris, J. 2000 Physical and numerical study of 2-D wave breaking and non-linear effects. *Coastal Engineering*, 382-395.
- Benjamin, T. B. & Ursell, F. 1954 The stability of the plane free-surface of a liquid in a vertical periodic motion. *Proceedings of the Royal Society of London, Series A* **225**, 505-515.
- Cerda, E. A. & Tirapegui, E. L. 1998 Farady's instability in viscous fluid. *Journal of Fluid Mechanics* **368**, 195-228.
- Chanson, H. & Montes, J. S. 1995 Characteristics of undular hydraulic jumps: experimental apparatus and flow patterns. *Journal of Hydraulic Engineering* **121**, 129-144.
- Chen, W., Haroun, M. A. & Liu, F. 1996 Large amplitude liquid sloshing in seismically excited tanks. *Earthquake Engineering and Structural Dynamics* **25**, 653-669.
- Davis, A. M. & Weidman, P. D. 2000 Asymptotic estimates for two-dimensional sloshing modes. *Physics of Fluids* **12**, 971-978.
- Faltinsen, O. M. 1974 A nonlinear theory of sloshing in rectangular tanks. *Journal of Ship Research* **18**, 224-241.
- Faltinsen, O. M., Rognbakke, O. F., Lukovsky, I. A. & Timokha, A. N. 2000 Multidimensional modal analysis of nonlinear sloshing in a rectangular tank with finite water depth. *Journal of Fluid Mechanics* **407**, 201-234.
- Faltinsen, O. M. & Timokha, A. N. 2001 An adaptive multimodal approach to nonlinear sloshing in a rectangular tank. *Journal of Fluid Mechanics* **432**, 167-200.
- Faltinsen, O. M., Rognbakke, O. F. & Timokha, A. N. 2003 Resonant three-dimensional nonlinear sloshing in a square-base basin. *Journal of Fluid Mechanics* **487**, 1-42.

- Faraday, M. 1831 On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces. *Philosophical Transactions of the Royal Society of London* **121**, 299-340.
- Frandsen, J. B. 2004 Sloshing motions in excited tanks. *Journal of Computational Physics* **196**, 53-87.
- Frandsen, J. B. & Peng, W. 2005 Experimental sloshing studies in sway and heave base excited square tanks. *ASCE Proceeding of Civil Engineering in The Ocean VI* Baltimore, USA.
- Frandsen, J. B., Tubbs, K. R. & Peng, W. 2005 Free-surface lattice Boltzmann simulation of shallow water in horizontally moving tanks. *The 5th International Symposium: Ocean Wave Measurement & Analysis* Madrid, Spain.
- Gu, X. M., Sethna, P. R. & Narain, A. 1988 On three-dimensional nonlinear subharmonic resonant surface waves in a fluid: Part I – theory. *Journal of Applied Mechanics* **55**, 213-219.
- Hirt, C. W. & Nichols, B. D. 1981 Volume of fluid (VOF) method for the dynamics of free boundaries. *Journal of Computational Physics* **39**, 213-219.
- Ibrahim, R. A., Pilipchuk, V. N. & Ikeda, T. 2001 Recent advances in liquid sloshing dynamics. *Applied Mechanics Reviews* **54**, 133-199.
- Isaacson, M. & Ryu, C. 1998 Earthquake-induced sloshing in vertical container of arbitrary section. *Journal of Engineering Mechanics* **124**, 158-166.
- Isaacson, M. & Premasiri, S. 2001 Hydrodynamic damping due to baffles in a rectangular tank. *Canadian Journal of Civil Engineering* **28**, 608-616.
- Jiang, L., Perlin, M. & Schultz, W. W. 1998 Period tripling and energy dissipation of breaking standing waves. *Journal of Fluid Mechanics* **369**, 273-299.
- Jiang, L., Ting, C. L., Perlin, M. & Schultz, W. W. 1996 Moderate and steep faraday waves: instabilities, modulation and temporal asymmetries. *Journal of Fluid Mechanics* **329**, 275-307.
- Lamb, S. H. 1932 *Hydrodynamics*. Six Edition, Dover Publication, New York.
- Miles, J. & Henderson, D. 1990 Parametrically forced surface waves. *Annual Review of Fluid Mechanics* **22**, 143-165
- Montes, J. S. & Chanson, H. 1998 Characteristics of undular hydraulic jumps: experiments and analysis. *Journal of Hydraulic Engineering* **124**, 192-205.

- Ockendon, H., Ockendon, J. R. & Waterhouse, D. D. 1996 Multi-mode resonances in fluids. *Journal of Fluid Mechanics* **315**, 317-344.
- Peregrine, D. H. 1966 Calculations of the development of an undular bore. *Journal of Fluid Mechanics* **25**, 321-330.
- Peregrine, D. H. 2003 Water-wave impact on walls. *Annual Review of Fluid Mechanics* **35**, 23-43.
- Perlin, M. & Schultz, W. W. 2000 Capillary effects on surface waves. *Annual Review of Fluid Mechanics* **32**, 241-274.
- Taylor, G. I. 1954 An experimental study of standing waves. *Proceedings of the Royal Society of London, Series A* **218**, 44-59.
- Toro, E. F. & Clarke, J. F. 1988 *Numerical methods for wave propagation*. Kluwer Academic Publishers, Manchester, U.K.
- Utku, M. & Basco, D. R. 2002 The relative trough Froude number for wave breaking (laboratory experiments). *Proceedings of the 28th International Conference on Coastal Engineering*, 258-268.
- Verhagen, H. G. & Wijngaarden, L. V. 1965 Non-linear oscillations of fluid in a container. *Journal of Fluid Mechanics* **22**, 737-751.
- Virnig, J. C., Berman, A. S. & Sethna, P. R. 1988 On three-dimensional nonlinear subharmonic resonant surface waves in a fluid: Part II – Experiment. *Journal of Applied Mechanics* **55**, 220-224.
- Waterhouse, D. D. 1994 Resonance sloshing near critical depth. *Journal of Fluid Mechanics* **281**, 313-318.
- Wu, G. X., Ma, Q. A. & Eatock Taylor, R. 1998 Numerical simulation of sloshing waves in a 3D tank based on a finite element method. *Applied Ocean Research* **20**, 337-355.
- Wu, G. X., Eatock Taylor, R. & Greaves, D. M. 2001 The effect of viscosity on the transient free surface flow in a two dimensional tank. *Journal of Engineering Mathematics* **40**, 77-90.
- Yeh, H., Ghazali, A. & Marton, I. 1989 Experimental study of bore run-up. *Journal of Fluid Mechanics* **206**, 563-578.

APPENDIX A. SHAKE TABLE

A shake table has been used in the physical experiments to provide the required excitation, as shown in Fig. A.1. The shake table is a six DOF (roll, pitch, heave, surge, yaw and lateral) motion system that is electrically powered by six individual electro-mechanical actuators. A triangular steel frame is designed for a maximum payload of 1000 Kg.

Fig. A.1. 6 DOF shake table system with wave gauges.

Limits of the degree of freedoms are:

Table A.1. Limit of each degree of freedom of shake table.

Degree of Freedom	Minimum Value	Maximum Value
Roll	-33°	33°
Pitch	-29°	29°
Yaw	-29°	29°
Sway	-0.381 m	0.381 m
Surge	-0.381 m	0.381 m
Heave	0 m	0.457 m

The motion system has been mounted to the floor with the base frame by high strength bolts. The power controller can be fed with operation signals from the customer's system controller through the shake table computer. By this method, users may preprogram the joystick files in order to control the motion in whichever way they require. The joystick files can be either a delimited ASCII file which is coincident with each DOF in orders or a motion wave profile created with Matlab. The amplitude of motion for each DOF should not exceed the limit described above. A typical joystick file is a matrix to specify the forcing amplitude of each DOF as:

```
[ Roll1 Pitch1 Heave1 Sway1 Yaw1 Surge1
  Roll2 Pitch2 Heave2 Sway2 Yaw2 Surge2
  Roll3 Pitch3 Heave3 Sway3 Yaw3 Surge3
  Roll4 Pitch4 Heave4 Sway4 Yaw4 Surge4
  etc...                               ]
```

Each column stands for one DOF in the order of roll, pitch, heave sway, yaw and surge.

APPENDIX B. WAVE GAUGES

Ten wave gauges (as shown in Fig. B.1) are used to monitor the free-surface inside the tank. The wave gauge is a capacitive sensor which can measure the free-surface when the depth to width ratio h_0/b fluctuates between 0.03 and 1.22. It is accurate to ± 0.003 m when the depth to width ratio h_0/b varies between 0.24 (20% full length of wave gauges) and 0.96 (80% full length). When the depth to width ratio is outside this range, the accuracy of wave gauges are ± 0.012 m. It has been designed to measure the liquid surface height at frequencies up to 125 Hz for analog output and 30 Hz for digital output. It requires from 5.5 volts to 40 volts of direct current power supply. The wave gauge can display the measurement of any liquid level in containers with a continuous surface. The full length of wave gauge is 1.22 m which is the exact height of water tank. Through serial ports, the wave gauge can sample data with the Wave Gauge Interface by digital output and be reconfigured conveniently. An alternate analog output (0 to 5 volts) is offered which can be sampled with the data acquisition system. The peak analog voltage output noise is 5 mv.

Fig. B.1. Wave gauge.

APPENDIX C. DATA ACQUISITION SYSTEM

A USB data acquisition system has been applied for the data sampling. It provides 16 single-ended inputs or 8 differential-ended inputs up to 100,000 Hz sampling rate of the internal clock frequency for the whole system. A 16 channel-backplane can accommodate up to 16 input/output connections from 16 different sensors simultaneously. Fig. C.1(a-b) shows the depiction of USB system and backplane with modules respectively. Low-noise measurement conditions have been offered by its 500 volts of isolation, and theoretically no ground loops are required. Several 5B serial input/output modules have been applied to simplify the conditioning for different sensors into modular solutions. Ten analog voltage input modules plugging into the backplane provide conditioning for the wave gauges. The power supply of the backplane should be 5.5 v to 40 v direct of current. The wire connection between the wave gauge and the data acquisition system has been shown in Fig. C.2.

(a)

(b)

Fig. C.1. Data acquisition system. (a) USB data acquisition system and (b) 16 channel backplane with modules.

Fig. C.2. Wire connection between wave gauge and data acquisition system

First, connect the positive end of the power supply to the red wire from the wave gauge. Then, connect the negative end of the power supply to the second connector of the backplane term block. Pick up the blue wire (analog output signal) from the wave gauge and hook it up to the third connector of the term block. Finally, connect the black wire (ground line) from wave gauge to the second connector of the term block of Backplane to set up the circuit.

APPENDIX D. HIGH SPEED CAMERA

A high speed digital imaging camera, which can record a sequence of images of an event at a frame rate of 60 to 1000 frames per second (fps), is used to record the wave breaking processes. The camera lens focuses the subject onto a CCD imager. Any C-mount lens is suitable for this camera. Image resolution is 292 x 220 pixels per frame for 60 to 500 fps, and 292 x 110 pixels on a half-height frame at 1000 fps. The frame capacity is 2048 for 60 through 500 fps and 4096 for 1000 fps.

VITA

Wei Peng was born in Changsha, Hunan, People's Republic of China. He graduated from Central South University, Changsha, Hunan, P. R. China, with a bachelor degree in civil engineering. At Central South University, Wei joined in the student government and organized many public service activities. He won the Xiangyin prize and honor graduate of Hunan province. At 2001, he joined a research program dealing with the reinforced concrete behavior in high temperature. Then Wei entered the graduate program in Civil and Environmental Engineering Department working as a research assistant at Louisiana State University under the guidance of Dr. Frandsen at 2003. He investigated the free surface behavior and wave breaking mechanism in tanks. While at Louisiana State University, he published two preceding papers on sloshing studies as a co-author.