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Teaching high school geometry with tasks and activities

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TEACHING HIGH SCHOOL GEOMETRY WITH TASKS AND ACTIVITIES

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ABSTRACT

Task-based learning is an instructional method in which students complete coherently-structured activities in order to meet objectives set by the educator. This thesis illustrates learning communities as the ideal environment for a task-based learning classroom. It discusses the teacher's role in a task-based classroom. This paper also describes three examples of tasks performed in my high school geometry classroom along with my observations of students' interactions and discussions.

INTRODUCTION

Task-based learning is an instructional method in which students complete coherently-structured activities in order to meet objectives set by the educator. Tasks are designed to support curriculum standards by engaging students in meaningful work that fosters the development of understanding. The task-based strategy is ideal for addressing the Standards for Mathematical Practice put forward in the Common Core State Standards, which refer to attitudes and habits of mind that are characteristic of effective mathematics-users. Task-based learning is similar to project-based learning. The main difference is that projects tend to bring together a broader range of learning objectives, lead to more complex activities, call for more independence and self-direction on the part of students, and take a much longer period of time to complete. The length of time it takes to complete a task may range from a half of an hour or sometimes even less to several class periods, depending on the goal, the nature and complexity of the mathematics involved.

The goal of the action-research project on which this thesis is based was to develop lessons that encourage understanding. As a first attempt, I tried to incorporate project-based learning in the geometry course I was teaching. This proved to be impractical in the school environment at hand. I found task-based instruction to be a more practical option due to time constraints and the demands of enforced pacing and testing. In recounting what occurred in my classroom, I will illustrate how task-based learning is linked to the Standards of Mathematical Practice. During this project, I planned and tested several task-based lessons that develop a deep understanding of topics in high school geometry. I will describe three of these tasks, relating in detail what was observed and how the students reacted.

Stepanek (2000) argues that understanding develops best in an environment where students become part of a learning community. In a learning community students are expected to take responsibility for their own learning and the learning of their peers, respect the ideas of others, and collaborate with each other. Successful task-based instruction possesses the same requirements. A mathematically proficient student should be capable of working in a task-based environment as a member of a learning community. In a traditional classroom, teachers do most of the talking, but in a classroom centered on tasks in a learning community, students do most of the talking; they feel comfortable interacting with their peers and expressing their ideas, and they become directly responsible for their learning and understanding. "[T]he classroom becomes a place where students can be themselves and where their ways of knowing, thinking, and expressing themselves are valued" (Stepanek 2000).

It is the job of the educator to facilitate the learning that goes on in the classroom. In a task-based classroom, the teacher's role is to prepare well-planned activities that guide students to a conclusion whose general nature the teacher has anticipated. What Hiebert says about developing understanding in classrooms applies here: "If teachers tell too much, students will not need to develop their own problem-solving abilities; if teachers tell too little, students will not make much progress." (Hiebert 1997) It is important that teachers allow students to collaborate and make their own discoveries. When students take ownership of their learning, they are much more likely to take pride in what they learn and commit it to long term memory.

I am a high-school mathematics teacher in the Ascension Parish Public School System. I work with a group of teachers very closely, and together we draw up a calendar of topics to be covered in one semester, based on the Ascension Parish Comprehensive Curriculum. Though I am able to teach the topics with my own style, it is important that I follow the established schedule. I had to bring together all these features of good teaching aimed at developing understanding and do this in a setting where I had to meet many requirements set by others. My thesis reports on what I was able to accomplish.

CHAPTER 1: LITERATURE REVIEW

In the book, *Making Sense: Teaching and Learning Mathematics with Understanding*, author James Hiebert and his coauthors offer a number of insights about teaching for understanding. The analyses presented in the book were based on data and observations obtained in four research and development projects that the coauthors directed. Though the research that went into these projects took place in various elementary-schools around the world, the authors state their intent of preparing and presenting conclusions that may be applied to any mathematical topic. Findings from the projects were compared and examined over a period of five years. In this time, the authors created a consensus concerning the features that make up a classroom that encourages understanding.

The authors argue that tasks are an important way for students to learn, because they aid them in thinking and discovering on their own. They offer the following set of criteria that help to define how tasks should be conceived:

> First, the tasks must allow the students to treat the situations as problematic, as something they need to think about rather than as a prescription they need to follow. Second, what is problematic about the task should be the mathematics rather than other aspects of the situation. Finally, in order for students to work seriously on the task, it must offer students the chance to use skills and knowledge they already possess. (Hiebert 1997)

Students get involved in problems that are interesting to them. A problem becomes interesting when students are able to claim it as their own. Students will work toward solving a problem if they have a personal investment in the situation. They should not be enticed to solve problems for an outside reward, like candy or free time; they should have self-interest in the outcome of the problem.

The most interesting part about a task should be the math that goes into it and not the task itself. The authors explain that tasks should also allow students to use tools they are familiar with. They describe tools as tangible things that students work with, or methods they are comfortable with. Either way, tasks should encourage students to use previous knowledge to solve new problems or create new outcomes. Students do not do this by watching others, they learn by personal experience. They must therefore be allowed to play around with and explore the tools they have. Educators should prepare suitable problems for the tools that are available.

Hiebert et al. explain that it is important that tasks leave behind a “residue” of learning (Hiebert 1997). Thus, tasks should be strategically planned to lead students to think and reflect about important topics as they pursue the goals of the task. Students who make connections and leave a task with an understanding are more likely to make other connections later on. In contrast, students who learn through memorization have a difficult time connecting concepts and procedures. When creating a task, the teacher must think of the processs his or her students will be involved in and what they will ultimately take away.

One of the most difficult things about task-based learning, according to the authors, is that the teacher has to change his/her role in the classroom. It is important for teachers to step aside and allow students to struggle and work through rough patches in the task. Teachers need to find a balance, where they can guide discussion without getting too involved. A total hands-off approach is unrealistic. If students are left on their own, there could be little discovery taking place and a great deal of frustration. There is no rule for teachers to follow in order to create this balance, each classroom is different and has to develop its own environment.

The main role of the teacher is to plan and create tasks that produce understandings that can be used in the future. Teachers should not only plan based on the goals that are to be achieved in the present task, they must also keep in mind future tasks and goals and how they will tie together and be meaningful for the student. Since student understanding is a growing process, students should be able to build from one task to the other and use the “tools” from previous tasks to complete future ones. If teachers are successful at planning tasks in this manner, students will see the flow of concepts throughout the course instead of distinct individual activities.

Teachers select tasks based on knowledge of mathematics together with knowledge of their students. Knowing the material means that teachers are comfortable moving around the curriculum and are able to connect topics. Teachers must also be familiar with how their students think. This is important because teachers must be aware of the tools that their students possess, how their students will approach a task and what their students will take away from it.

The authors take on the big question of how much should educators tell. Their answer is, teachers should share information as long as it does not get in the way of student problem solving. Students should have room to reflect and explore without teacher interference. One way in which teachers can share information is by providing useful methods for recording and communicating mathematical ideas. Students have their own language and terminology, even in the mathematics classroom, but teachers can provide symbols and terminology that make the writing and communicating process shorter and more efficient. Teachers must be careful in how they present these things. What is provided should act as an aid and should not burden students.

Another way teachers can intervene is by presenting alternate methods for solving problems. Teachers must be careful in how they do this, so that students do not view the teacher’s method as the “right method”. Teachers can avoid this by introducing methods from other students and suggesting modifications for offerings that may be flawed or in need of improvement. Students should be able to use their own methods and should not need to worry about reproducing the teacher's way. A third way teachers can share information is by clarifying the major concepts involved in a task. Students may begin to grasp these concepts, but the teacher can shape this early understanding into something students are able to use. Again, it is important that the teacher be very careful in how they approach this. Students should value their findings as their own, so it is important that the teacher clarify methods and not make students feel like they are changing their own method.

Hiebert and coauthors also discuss the characteristics of a classroom environment that is conducive to task-based learning and the manner in which students and teachers should behave socially in this environment. Students should respect one another and value one another's ideas Communication and reflection should be encouraged. Three chapters of the book are dedicated to portraying cognitively guided classrooms, conceptually based classrooms, and problem-centered classroom, three different styles of instruction that incorporate task-based learning.

Stepanek (2000) considers the problem of building effective teaching and learning practices that work in harmony with state standards. The intent of her article is to provide teachers with research-based strategies to create classrooms conducive to learning with understanding. The article acknowledges input and guidance from many educators in science and mathematics. The main theme is the importance of "learning communities" within a classroom in creating an environment suitable for rigorous learning. The author details the different kinds of relationships that can be created between teacher and student. According to this article, to increase learning with understanding, it is important for students to be in a classroom where the teacher is not the center of discussion and learning, but the students share control over the conversation.

Stepanek emphasizes importance of establishing classroom norms, developing relationships suitable for learning, and maintaining respect. She describes a number of strategies: the democratic classroom, the caring classroom, and the ecological classroom. Stepanek goes into great detail on the options teachers have in developing learning communities. She describes reflection and collaboration as a great tool. The way a teacher collaborates with his/her students will change every year as a new group of students arrives. It is important that he/she reestablish positive collaborative relationships each year. It is also important that teachers keep a collaborative relationship with their colleagues as well. Teaching can be challenging, so it is important to keep a learning community among teachers to aid each other as situations arise.

In their article, Lucia Grugnetti and Francois Jaquet (1996) discuss the importance of problem solving in the mathematics classroom. The emphasis is placed on how students should solve problems and how teachers can prepare their students as problem solvers. It is important to present students with a problem that is just within their reach, but that challenges the way they think. How a problem is picked apart by a student varies, but the use of strategies, such as group work and technology, can aid the process. One principle for problem solving is the use of a “knowledge-constructing activity” (Grugnetti 1996). There are a series of steps a student should undergo to complete the thought process required for the problem. Grugnetti and Jaquet also discuss the French practice of “situations-problemes.” There are five steps that the students must take: the student begins alone, constructs new knowledge, makes a number of trials and conjectures, self-corrects, and develops new knowledge expected by the teacher (Grugnetti 1996).

There is an immense amount of literature that can be found on learning that encourages understanding and the appropriate techniques that can be used when implementing this strategy.

There are several components and points of view that should to be taken into consideration when planning tasks. These pieces of literature were beneficial for my planning and reflection as a teacher in a task-based classroom. These articles helped focus my attention on the most important aspects. It would be interesting to continue reading on this topic, as there are a number of situations and strategies that can be learned from.

CHAPTER 2: SAMPLE HIGH SCHOOL GEOMETRY ACTIVITIES

2.1 "The Baseball Project"

2.1.1 Introduction

The baseball project was done at the beginning of the geometry course. This project required students to construct a scale model of a high school baseball field and involved a lot of work with similar figures. The unit which covers similar figures does not occur until after midterms, but when planning for this project, I was looking for a preparatory assignment that would introduce my students to deadlines, get them used to working with others, and familiarize them with the basic tools needed in geometry. I expected students to need a good sense of the basic units of length. The lesson would ask them to use a ruler to draw many specific lengths. The students would also need to know how to apply a scale factor to setting up a proportion. It would be important for the students to know how to use a scale factor when determining the measurements in a scale model. To prepare the students, I created lessons to refresh their understanding. My goal for this project was for my students to gain a more concrete understanding of units. In achieving this they would need to know how to use a ruler, draw complete figures, name parts of figures, and identify corresponding parts of similar figures. I created this project based on inspiration from my father, a professional land surveyor, who has instilled in me the importance of accurate measurements.

2.1.2 Lesson #1

The first lesson was to test my students' understanding of different units of measure. This lesson took place over a period of four days, during the ten to fifteen minute warm up exercises for the day. Each day, they were given a piece of paper with four horizontal lines spaced a couple inches apart. They were given a list of lengths (e.g. one millimeter, two centimeters, four inches, et cetera) and were instructed to estimate the lengths and mark their lines to show their estimates. Every day I changed the list of assigned lengths to prevent them from using memorization. After marking their lines, they then used rulers to “grade” themselves. The grading process was interesting. Some students would claim to have marked their measurements exactly right, but investigation by their classmates and myself showed their measurements were off by a few millimeters or as much as a quarter of an inch. The students did not think this was significant, and claimed they were “close enough”. I explained to them that in the project we were preparing for, a measurement that is off by a quarter of an inch could convert to an error of several feet (or even miles). The students’ initial work showed most of them to be very poor at estimating. At the beginning about one sixth of them were unsure of the size of a millimeter. Some students noticed that on the rulers they were using inches were divided into sixteenths but centimeters into tenths. This was something they had never paid close attention to before.

We held a discussion on the proper way to mark the segments to show estimates. I expected students to use dots to mark the assigned distances, but was pleasantly surprised to find that the majority of them used tick marks instead. When I asked them about their choice of markings, the common reply was, “That is how it is marked on the ruler.” This may seem insignificant, but when the lesson is focused on accuracy and precision of measures, it is important to use a marking that leaves little room for debate. If the students use dots, there is a question of whether to measure from the middle, outside, or inside of the dot, and depending on the choice, results may vary drastically. By the end of the week, they grasped the importance of the ability to estimate. Without this introductory lesson, I can imagine there might have been many more questions about units of measure, how to use rulers, and how to draw a line whose measure is not a whole number.

2.1.3 Lesson #2

The second lesson was on scale factors and scale drawings. Many of the students lacked experience with scale factors and constructing scale drawings. In this lesson, we discussed how to read a scale factor as a ratio of the scaled measurement to the actual measurement and then discussed notation for scale factor (see Appendix A). Our discussion on setting up proportions was brief, since this was something they were familiar with from their previous math courses. Then they created scale drawings of geometric shapes (see Appendix B) using several scale factors (figure 2.1).

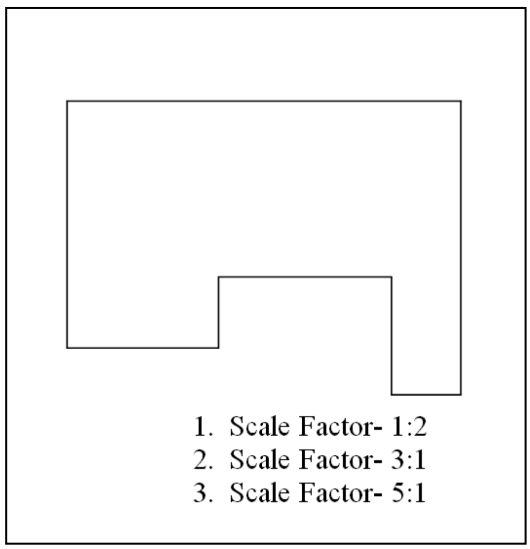

Figure 2.1 Example of Practice Scale Drawing Activity

They were to measure the length of each side of the original figure to the nearest tenth of a centimeter and then use each scale factor to calculate the side lengths of the scaled image. After calculating the scaled measurements, they were to construct each scale model on a piece of blank paper using a ruler and a drafting triangle. The students were given the majority of the class period to work on their scale drawings and play around with the tools they were given. This lesson took a total of one whole class period and by the end of the class, the students had much more experience working with scale factors and scale drawings.

2.1.4 The Project

The baseball project was presented to the students as a work order for a design team at an engineering company. The students were given a letter from a fictional engineer explaining the details and expectations of the project (see Appendix C). The students were to work in pairs and were allowed to choose their own partner. As a checkpoint, the students were given the weekend to research and record the dimensions of a high school baseball field. I graded the recording of the dimensions as a homework assignment. We held a class discussion on what the proper scale factor would be for this particular drawing. I allowed the students ample time to work with their partner to figure out which scale factor would work best for the dimensions of a baseball field. The final decision on the choice of scale factor was left to the student teams.

After students decided on a scale factor we discussed their choices. My concern was that some students would choose scale factors that would cause the drawing to extend off the board. In our discussion we talked about what an appropriate scale factor would be based on the size of the poster board. Since a poster board is approximately 71 by 55 centimeters, the scale factor had to allow the largest part of the baseball field, the distance from home plate to the outfield, to fit within the dimensions of the poster. Some of the students noticed that their scale factor made the model too large and had to make a change, others saw that their scale factor made the baseball field fit onto the poster, but the measurements were too small to accurately draw using the tools we had available.

How to change the scale factor brought up an interesting conversation. If their original scale factor made the drawing extend off the poster board some of the students' initial reaction was to make the denominator of the scale factor smaller. After scaling their measurements with this new scale factor, they quickly noticed that a smaller denominator only made the drawing bigger. Then they tried a larger number in the denominator and re-calculated the scaled measurements. Other students made the observation that when you use a larger number in the denominator it means you are covering a longer distance. For example: if the original denominator was five feet, then the distance represented by one scaled unit would be five feet, but if the denominator is changed to two feet, then the distance represented by one scaled unit would only be two feet. Once the students determined and appropriate scale factor they were to determine the scaled measurements of all their baseball field dimensions.

The blueprint was to be completed using a ruler, drafting triangle, and compass. The students had one school week to complete their project. Because of other demands made by my school and district, I was not able to give additional class time to complete their project. Having to work outside of class would test the students' ability to collaborate and manage their time to meet deadlines. At the end of the deadline the projects would be checked and returned for editing. Because the students were ninth-graders (fourteen or fifteen years old), I felt a lot of uncertainty about their ability to manage their time outside of class. On the one hand this was a new level of responsibility that they had never had before. Moreover they were not able to drive themselves to and from a meeting place to complete their project with their partner. I predict these issues would not be as significant in a class of juniors or seniors who have easier access to transportation and have more experience with responsibility.

After the students turned in their first attempt, I examined it. I wrote comments and returned the work to the students for editing. "Reflecting and communicating are the processes through which understanding develops." (Hiebert 1997) Allowing students to reflect on their errors gave them the opportunity to look at the mistakes they made and make a decision on how to change those mistakes. Reflection is a step students should practice in all the work they do. By exposing them to this strategy early in the course, I hoped they would continue applying it throughout the remainder of the semester. The ability to reflect is also a beneficial lifelong strategy. There is a similar process that takes place in the work force. Professionals are held accountable for the work they turn in, so to ensure an adequate outcome, it is common for professionals to experience an editing and critiquing process before turning in a final product. Once the students' blueprints were checked and edited the students presented their final product and explained the process they underwent to create the scale model.

2.1.5 Observations and Outcomes

This project was an eye opener. Students' initial understanding of measurement was inadequate for their grade level. In spite of the lessons taught to familiarize students with units of measure, there were a number of mistakes made when it came to mapping out a specific length. My assumption that ninth graders had experience working with centimeter and inch rulers and compasses was flawed. Many students were unfamiliar with how to hold a compass. Some attempted to control the compass with two hands, one hand for each leg; others would hold the compass with one hand and rotated their paper with the other hand. Up to this point, the students have not had much practice with geometric tools. Although there is some measuring done in previous mathematics courses, there is no unit where students practice using these tools at length. The blueprint gave students an opportunity to gain a great deal of experience with their geometric tools. They had to pay close attention to details, such as the thickness of the line their pencil produced or the way their units were divided on a ruler. The features of their tools determined the precision and accuracy of their model. When grading the projects, I measured each required dimension with a leniency of three millimeters. I found some students missing their measures by up to one whole centimeter. I feel this goes back to the "good enough" mentality that the students showed at the beginning of the lesson. I observed that the students gained a better sense of units of measure, but still needed help using a ruler to draw an accurate unit of measure that was not a whole number.

In the future, I would like to familiarize my students with the ruler, compass, and drafting triangle early in the semester by scheduling "play times" for them to gain practice. It would be beneficial for me and my students if I allow them time to practice drawing geometric sketches of circles, right angles, and lines of a fixed length. I will probably change my lessons to spend more time using a ruler to mark specific lengths, and allow the students to do peer grading and discuss the accuracy of their markings. I would also like to do the same with a compass, assign circles of a given radius to be drawn and checked for accuracy.

2.1.6 Conclusion

When reflecting on the main focus of this project, I find myself most interested in familiarizing students with how to use their basic geometric tools and the importance of accurate measurements. Although using a scale factor was an important aspect of this project, the majority of the flaws that took place on the final project were a result of their lack of familiarity. Aesthetically the products were very well put together, but as I investigated, I found their work to be very flawed because they simply did not know how to use their tools properly. My students showed signs of frustration and confusion in the creation of their final products. I believe this was due to their inexperience working on this type of assignment. The students became more comfortable with using geometric tools and drawing scale models, but did not master the use of these tools accurately. One of the most commonly asked questions in a mathematics classroom is, “When will I ever use this?” Most of my students may never be faced with drafting a baseball field by hand, but the importance of accurate measurements will most likely follow them for the rest of their lives. I believe I emphasized this to them through the completion of this project.

2.2 “What Can I Learn From my Shadow?”

2.2.1 Introduction

In order to set the stage for this activity, I will first explain how it fit into my course. At this point in our curriculum the students were being introduced to similar figures, the Pythagorean Theorem, special right triangles (30-60-90° and 45-45-90°), and trigonometric ratios. There are a total of three activities that I will discuss which covered this unit in the comprehensive curriculum. The goal of the last activity was to give my students direct experience of the concept of angle of elevation so that they could apply this experience to real life problems. The design was to have students use trigonometry to find the height of an object based on its shadow length. They would do this by calculating the angle of elevation of the sun using the shadow length of a person with a known height. This main activity is to be performed toward the end of the unit. The students should know how to set up a trigonometric ratio and find the measures of the sides and angles of a triangle using trigonometry. The first two activities covered these topics, and once the students were familiar with these concepts they would be ready for the “shadow activity”. This activity was created by Austin James of Wake Forest University and was presented at the North Carolina Council of Teachers of Mathematics Annual Meeting in 2007. It can be found at the Wake Forest University website (http://www.wfu.edu/~mccoy/outdoor/aj.pdf).

2.2.2 Lesson #1

The lesson began with a lecture during which the students noted that the sine of an angle is the ratio of the opposite side to the hypotenuse, the cosine of an angle is the ratio of the adjacent side to the hypotenuse, and tangent of an angle is the ratio of the opposite side to the adjacent side; this was followed by routine practice problems similar to figure 2.2. First, the students practiced writing trig ratios working from given side lengths of various triangles. Then we used the calculator to find the values of trig ratios, using given angle measures. There was one significant problem with the structure of my lesson. When the students typed a number into their calculators followed by a trigonometric function, a decimal was produced, but they did not relate this decimal to a triangle.

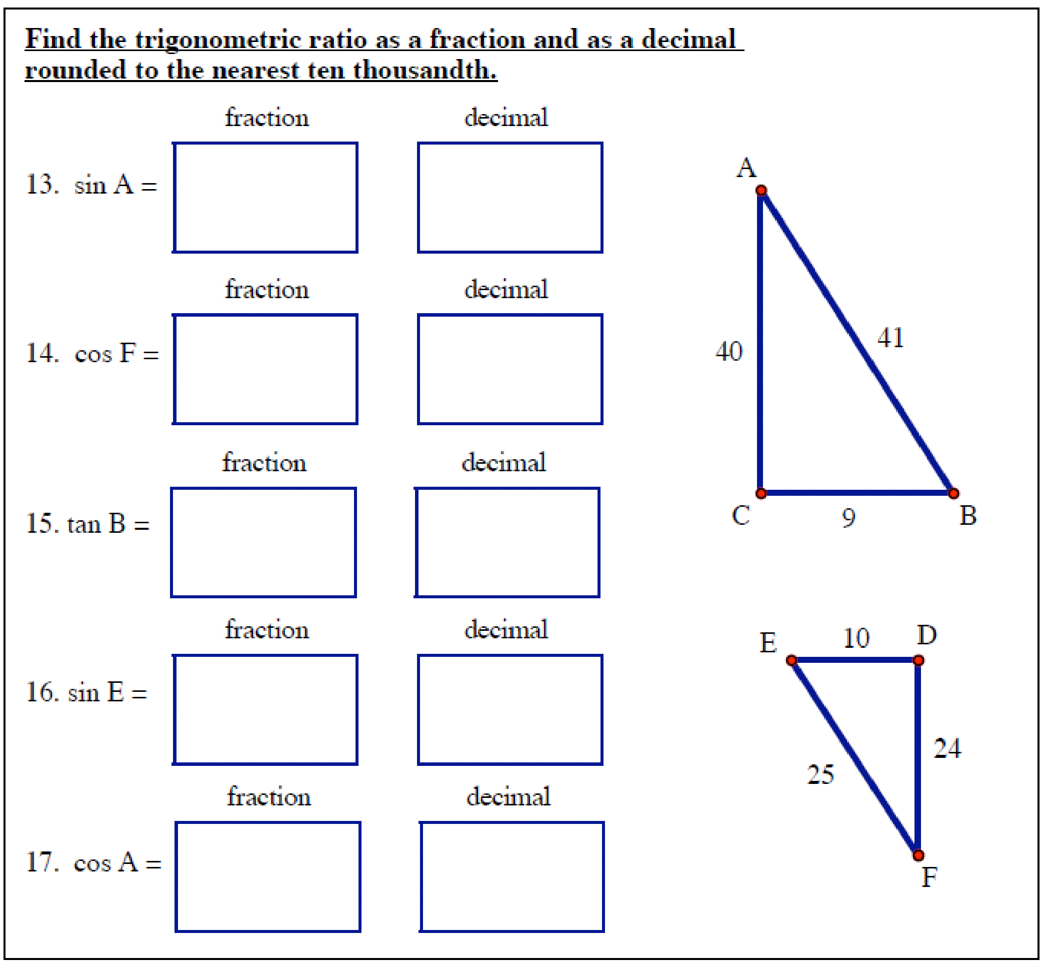

Figure 2.2 Examples of Trigonometric Ratio Practice Problems

When teaching this lesson, the first step we took was to look at how trig ratios are set up. The students were given a triangle with known side lengths and unknown angle measures. They were told to write several trig ratios using a variable to represent the angle measure. In other words, they were substituting the measure of side lengths into a formula. This was followed by typing an angle measure in to a calculator followed by a trig function, which produced a decimal. In one case they were looking at the sides lengths of a triangle and in the second case they were looking at an angle measure, but there was no connection to show how these two tasks were related. This was the flaw in the design of the lesson. To mend the confusion, I turned to special right triangles. Since the students knew the measures of the angles and the relationship between the sides of special right triangles, I asked them to use their calculators to find the sine of 60°. This produced a very long decimal, which they did not connect to a triangle. I then asked them to find the sine of a 60° angle in a 30-60-90° triangle with given side lengths. They were to write their answer using a fraction and, since they are familiar and comfortable using their calculators, they then type this fraction into the calculator to see its decimal notation. Three or four of the students explained that the decimal made by typing sine of 60° in the calculator was the same as the decimal made by typing in the fraction from their own work. We performed the same activity with the 30° angle and had the same results, the decimal produced by typing sin 30° was the same as the decimal notation produced from typing in the ratio of the opposite side length to the hypotenuse. I could then see that the students were making the connection between what the calculator produced from and angle measure and a triangle. In hindsight, using special right triangles to make this connection would have best fit between writing ratios and using a calculator to produce the ratio of a trig function.

2.2.3 Lesson #2

My second lesson was focused on finding the measure of an angle. I explained to them that in order to find the measure of an angle from the measures of the sides, you must use inverse trigonometric functions on the calculator. I roughly defined inverse as working a problem backwards. I also introduced them to the words "arcsin", "arccos", and "arctan" as alternatives to using "inverse". There was a question about the exponent of negative one found on the calculator used to take the inverse of a trigonometric function. They associated an exponent of negative one with a reciprocal. I explained to them that the negative one exponent used in naming the inverse trig function has no relationship to exponentiation of real numbers. Otherwise, the students were successful in calculating the measure of an angle. The students used practice problems that required finding missing angle measures similar to the ones in figure 2.3.

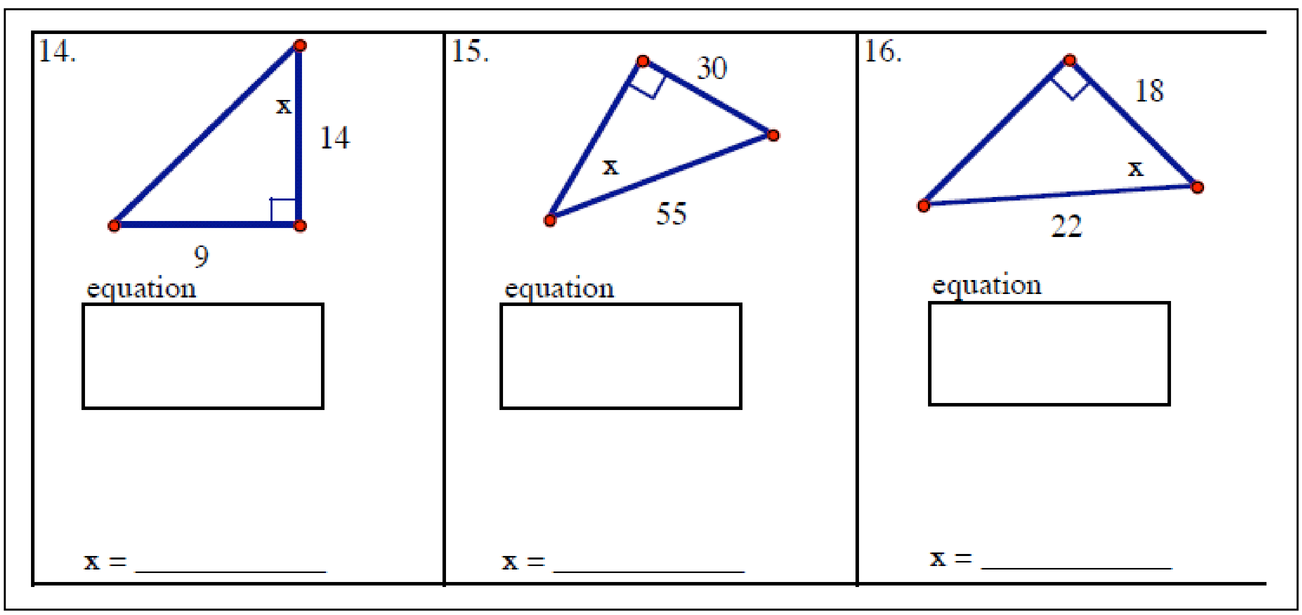

Figure 2.3 Examples of Missing Angle Problems

2.2.4 The Main Activity

After the students were comfortable working with basic problems such as these, I introduced a variety of word problems which required them to create their own visual representations. When working with these, I found my students struggling. In many cases I would present a word problem similar to the following:

> The sun shines on a tree 25 m tall, so that a shadow 18 m long is cast. To the nearest degree, find the angle of elevation of the sun's rays with the ground.

As with all problems similar to this the students were to draw a pictorial representation and then use their knowledge of trigonometric functions to find angle measurements. Most of the students understood they were to use a right triangle in their representation, but in many cases, they would label the parts incorrectly. A very common mistake was that they drew the shadow as the hypotenuse of the triangle, as if it were floating in the air. Placing the shadow in the right position seemed to be a very elementary concept to me, since we all have shadows that are obviously found on the ground, but for some, this was not so clear. Studies have shown that students may have a "bank" of images in their minds, but have a difficult time putting those images together. It is also important that students understand why a diagram is important and how to use them (Duthie 1985). Based on my observations, diagramming a math problem was unknown territory for my students. I could see that the practice of diagramming and depicting images from a mathematical word problem was not something these students were used to, and therefore needed both guidance and practice to achieve this important idea. This was my inspiration for the "What Can I Learn From my Shadow" activity. Before conducting this activity I first prepared a picture representation of angles of depression and elevation (see Appendix E).

Once they became familiar with the two dimensional representation and the terminology, I put the students into groups of three and equipped them with a tape measure and a worksheet to organize their thoughts and measurements (see Appendix F). On this worksheet they wrote reminders of the definitions of trigonometric functions. One person from the group was elected to have their height measured and the length of their shadow to be measured, these measurements were recorded. They were then to sketch a picture representation of the person and their shadow. They were then assigned an object in the school parking lot whose shadow was to be measured, but not the height of the object. Some of the students measured the shadow of very tall objects such as a light pole or a school building; others were assigned shorter objects such as a stop sign or fence post. After collecting data outside, we brought our results into the classroom to calculate the angle of elevation of the sun using the person's height and shadow length. The students noticed quickly that they all had to use the tangent function to find the angle of elevation since the person was the side opposite the angle and their shadow was the side adjacent the angle. Using the same angle of elevation calculated the students were to find the height of their assigned object.

2.2.5 Observations and Outcomes

One frustration that the students faced is that they were disappointed that their numbers were not exactly the same. For instance, though the class took measurements at the same time of day, the angles of elevation that were calculated were different from each other. The same was true when students were trying to find the height of the same object. The shadow of a light pole was measured by three different groups and when they used trigonometry to find the actual height of the light pole they came up with different measurements. This may be due to inaccurate measurements or something as simple as rounding differently. This was a great opportunity to talk about the range of error that occurs in real life experiments. In one class the difference in angle measures was five degrees while in another class the angle measures were within .6 degrees of each other. The students had a lot of questions as to why this took place. To answer their questions the different groups presented their data. I asked the “outliers” how their data was so different from others. Some of the reasons were very simple, such as it was hard to tell where the end of the shadow was, while others had difficulty in their calculations.

Some of the students also observed that finding the height of an object could be solved using similar triangles and proportions rather than trigonometry. We discussed the steps needed to find the height of the object using proportions instead of trigonometry. We discussed and accepted that in order to use proportions both triangles had to be similar, and that similarity is could be recognized from angles alone. In both situations, with the student and the object, there was a right angle formed with the ground, the first pair of congruent angles, and the angles of elevation were the same, the second pair of congruent angles. Knowing two pairs of angles to be congruent, we could use the third angle theorem to see that the last angles are congruent as well. To set up a proportion, the students had to take one side of the first triangle and the corresponding side of the second triangle and place them in the numerator. Then they had to take a second side of the first triangle and the corresponding side of the second triangle and place them in the denominator. We discussed that the person’s height would correspond with the unknown variable, the objects height, and the shadow length of the person would correspond with the shadow length of the object.

For the remainder of the week we continued to look at real life scenarios that use trigonometric ratios. In all of these practice problems I required students to organize their work by drawing a diagram to use as a reference when trying to solve the problem. I could see that they had a better understanding of how to draw problems similar to our shadow activity, but they still struggled with other scenarios, such as a descending submarine or a ski slope. For the future, I can see that it is important for students to understand how to use diagrams to solve word problems, but it is equally important to choose scenarios that can be related to by my students. This is evident by the effectiveness of the activity. After performing this hands-on activity, the students had no problem recreating the scenario over again in their heads. If they were able to act out any word problem given to them, their experience with the situation would allow them to recreate the scenario in their heads therefore increasing the level of understanding and visualization. As seen here, even the simplest tasks can increase the understanding in a student. The ability to visualize and produce an affective diagram requires both experience and practice. When students are unable to relate what they are learning to their own lives they are more likely to lose interest and be less willing to problem solve.

The difficulties that students had drawing shadows have interesting connections to topics in psychology about which much has been written. Diezmann and English (2001), are concerned with student proficiency with diagrams. Having collected data from several elementary students, they explore the difficulties students have with diagrams and the knowledge they need to become diagram-literate. The authors clarify that diagrams are distinct from pictures and drawings. Diagrams are not focused on details, but representations. Pictures or drawings pay more attention to details. The authors first emphasize that a student must understand why a diagram is useful, which diagram is appropriate, and how to use the diagram to solve problems. The authors define diagram literacy as knowing about diagram use and being able to use diagrams appropriately.

Freeman and Cox (1985) are concerned with picture production of children and students. Students' ability to represent depth and three dimensional figures does not come naturally. This is a skill that requires training and guidance (Duthie 1985). How something is described can affect the outcome of the drawing. Depending on the student's point of view and experience, the drawings can be created very differently.

In light of the significant problems that my students had in learning to use mathematical diagrams, it would be very interesting to pursue these connections further.

2.2.6 Conclusion

In project-based learning the Buck Institute for Education suggests, "Projects range from shorter, more contained projects of one to two weeks to open-ended explorations lasting much longer" (Buck Institute for Education 2003). The project covers the main concepts in the subject while small lessons are taught in between to fill in the information that is unknown to students. While I began this semester with an ambitious goal of teaching with project-based learning, based on my experience with the baseball project, I found that performing smaller, step-by-step activities a much more practical option. In a school where students are not accustomed to working on long complex projects, I found myself spending too much time instructing them on how to communicate with each other and manage their time wisely. After completing the "Baseball Project" I saw that a lot of time was being wasted teaching how to do a project rather than getting them involved in the math, which is when I decided to switch my focus to tasks. By this point in the semester, the students' had more experience working together and their attitude toward this type learning was much improved from the baseball project. My main goal for the entire course was to give students a more conceptual understanding of geometry. With my schedule and outside demands it became clear to me that task-based learning just made more sense. The goal of the main activity was to show students a real life trigonometry situation so that they could in turn apply what they learned to draw sketches of other similar situations. This task-based learning structure proved to be more affective for my students than project-based learning.

2.3 "Transformations"

2.3.1 Introduction

The transformation task was the first lesson conducted for the last unit in geometry. Transformations are studied in our curriculum using coordinate geometry. This task focused on isometries, which are transformations that preserve distance, and hence preserve the size and shape of figures. The isometries covered in this task were reflections across the x-axis, y-axis, and y = x line, and rotations of 90°, 180°, and 270° counterclockwise about the origin. The only background knowledge I expected students to have to complete this task was how to plot and name ordered pairs on a coordinate grid. In the task, the students would use transparencies to reflect and rotate ordered pairs on a coordinate grid. Based on their observations, the students were to generate rules to describe these transformations. My geometry students had some experience with transformations from eighth grade math and Algebra I, but I found they were not very strong at visualizing some of the transformations. The goal of this task was to give students a visual understanding of transformations.

2.3.2 Reflections

This task was given to the students to be done independently. The materials were collected by the students as they completed a quiz. On this day as students turned in their quizzes, they collected four small pieces of transparency, three pieces of scotch tape, an instructions page, a coordinate grid page, and a notes page (see Appendix F). The instructions page gave them step by step instructions on how to prepare their transformation apparatus. The grid page had four separate coordinate planes labeled "reflection on the x-axis", "reflection on the y-axis", "reflection on y = x", and "rotations". On the first grid labeled "reflection on x – axis" the students were to tape the edge of one transparency onto the x-axis, then plot and label three ordered pairs on the transparency. The students were to reflect their transparency over the x-axis to find the image and write down the new ordered pairs. The same was to be done for the grids labeled "reflection on the y-axis" and "reflection on y = x". Many students were not sure how to properly tape their transparency on the given axis or line. Some of them taped the transparency before the axis or over the axis, and showed confusion when they tried to reflect their ordered pairs. I wanted the students to experiment and struggle with this task, so I was careful not to answer questions that gave away too much information. The lack of immediate approval was very difficult for them to withstand. They wanted to know if what they were doing was right or wrong, so if I did not answer their questions, they would try to gain help from a classmate. I did not want to interfere with students working together, so after everyone had a chance to work on the reflections activity alone, I allowed them to work with a neighbor to collaborate on their findings. On their notes page, students were to write in words what they noticed happening as they reflected the ordered pairs over the given lines. After describing with words, they wrote an algebraic rule to describe the transformation. I walked around the classroom as this activity took place and when everyone was finished we held a class discussion on what was observed and the rules they created. Most of the students described reflections as a "flip" or mirror image. Once they positioned their transparencies correctly, the students had no trouble writing the ordered pairs of the image. When it came to writing the rule, I noticed the students using the term "opposite" to describe a number being negated. They wrote the rule for reflection across the x-axis as “(keep x, opposite y)” and the rule for reflection across the y-axis as “(opposite x, keep y)”. I explained to them that instead of writing “opposite x” or “opposite y” they could use a negative sign as notation (“-“). I also explained to them that the word “keep” was unnecessary. They could simply leave the variable as is. Their first question about the negative sign notation was, “What if the number is already negative?” This question was answered by other students in the class who remembered that minus a negative is equivalent to a positive. “Teachers can provide several kinds of useful information: One is the conventions that are used in mathematics for recording and communicating actions and ideas. These include names and written symbols for numbers, operations, and relationships…“ (Hiebert 1997) The recording process that students undergo can be shortened with the use of this symbol I introduced. Though we did not practice reflections about the origin in this activity, I showed the students that this meant a reflection across the x-axis and y-axis.

2.3.3 Rotations

Once the students were comfortable with reflections we looked at rotations. For this geometry course we were only focused on rotations about the origin. To prepare their rotation apparatus, the students were to draw an x- and y-axis in the center of their transparency. This would act as a guide when rotating about the origin. Then they were to plot and label ordered pairs on their transparency and rotate them, counterclockwise, by 90°, 180°, and 270°. The students had less trouble setting up the apparatus for rotations and seemed to have a very good idea how to rotate their transparencies for each degree measure. On their notes page they were to write a rule to describe each of the rotations. Generating rotation rules took a little more trial and error than the reflections. Most students experimented with several points and their images before drawing conclusions about the rule. With the 90° and 270° rotations the students did not initially realize that the x and y coordinates were exchanging places and changing signs. By plotting and rotating several points and comparing the results with classmates, they began to make this connection. After the students showed a better grasp of rotations I asked them why they thought I did not include a rotation of 360° on the instruction page. The students were quick to answer that a 360° rotation would not change the orientation of the image, so it would be the same as not rotating at all. When looking at a rotation of 180° some of the students noticed it produced the same outcome as reflecting about the origin. This observation was a great way to introduce the last step of our task.

2.3.4 Comparing Rotations and Reflections

The last step in this task was challenging the students to compare the 90° and 270° rotation to a reflection or a set of reflections across the x-axis, y-axis, or y = x line. They were allowed to use their transparencies and to work with a partner. Their results were to be written on their notes page. Some students noticed that a reflection could easily be made by folding the ordered pair of the image and pre-image of one of the rotations on top of each other. Though this reflection was not across the x-axis, y-axis or y = x line, a couple of students attempted to write the equation of this line of reflection. This was an unexpected step, but I encouraged them to explore this method. Most of the students struggled, folding their paper across the x and y axes only. They realized it would take more than one reflection to make the rotation, but they did not initially see how it was possible. Reflecting only across the x- and y-axes would not produce the image of a 90° or 270° rotation. After collaborating with their classmates, the students began to realize that reflecting across the x- or y-axis had to be combined with a reflection across the y = x line as well, in order to produce a 90° or 270° rotation. At this point in the class we started to run short on time, so I allowed students to present their discoveries as they were made.

2.3.5 Observations and Outcomes

As an educator, I really enjoyed watching my students complete this task. It was interesting to see them struggle through the hurdles they faced. Though the students thought they knew how to use reflections and rotations, it was easy to see that they had never thought of them in concrete physical terms. I realized the effectiveness of this lesson when the students asked if they could use their transparencies on the unit test. I also noticed some students using their hands and turning their test around to visualize the transformations. The students seemed to enjoy working with this task, especially the rotations. There were no rules or guidelines; they were simply expected to experiment and make discoveries. These observations show the success of the task. I view success as students interacting in a meaningful way and sharing conversations that are mathematically sound. Both of these goals were met in this task.

In the future, I would like to allow more time for this task and on transformations in general. I would like to expand the task so that students can observe a variety reflections and rotations. Because of the demands of the comprehensive curriculum, we looked at transformations in coordinate geometry only. If time allows in the future, I would like to work with students on transformations that do not require a coordinate system. Transformations can apply to a number of topics in geometry, but by following the curriculum, students are only exposed to transformations briefly at the end of the course. Geometry requires a great amount of visualization, and I feel a greater concentration on transformations would increase the students' ability to see things from a different point of view.

2.3.6 Conclusion

Although the transformations unit is small in my geometry curriculum, I feel the students gained a concrete understanding of how reflections and rotations operate. This task required a lot of preparatory work, but ran very smoothly in the class. I feel the amount of struggle the students experienced was as I predicted. Of all the tasks I assigned in geometry, this was my favorite. I could see the students struggling and working through their confusions. Recognizing their struggles and having a willingness to work through them showed me that they were becoming more comfortable with problem solving. The goal of this task was for students to gain a visual understanding of isometries. Based on the discussions held in class and how students handled other transformation problems, I feel this task was a success.

The new Common Core State Standards call for developing geometry based on transformations, introduced in middle school through the use of transparencies and developed in high school using transformations as an entry point and foundation. The experience I had with transformations confirms the wisdom of this plan.

CHAPTER 3: CONCLUDING THOUGHTS

The goal of my thesis was to create and evaluate lessons that would promote a deep understanding of high school geometry. The lessons I experimented with included both project-based learning and task-based learning. In both styles, the goal is for students to gain a deeper understanding of the material by active, productive involvement in work that requires planning and organization. The difference between project-based learning and task-based learning is that the former has a time-frame that may be up to several weeks. Task-based learning, encourages a deeper understanding, but focuses on smaller concentrated activities that can be completed in one or two days. One aspect that both types of learning have in common is that they both require student collaboration and encourage open classroom discussion.

In my curriculum, there is not enough time to spend weeks working on one project. Each unit in my curriculum takes no more than a week and a half to complete. Because of this, I found task-based learning to be more practical for my classroom setting. In the tasks I pilotted, I noticed a progression of success. My definition of success is evidence that students are actively involved and that the conversations taking place are correct and rich in meaningful mathematics. In order to be successful, students must have an invested interest and should value the outcome of the task.

In the baseball project, there were a number of confusions and uncertainties. I feel this was due to the students' inexperience with a task-based learning environment. As the course went on, students showed signs of increased responsibility and an understanding of how to collaborate affectively with their peers. With the shadow task, the students showed they were more comfortable working together and easily fell into their roles as group members. By the end of the course, I found students very comfortable interacting with each other and working through their struggles. The transformation task was evidence to their confidence. Not only were students willing to work on their own, through their own struggles, but also easily fell into conversation with other students to discuss the outcomes of their work.

As I completed each of these tasks I made notes to guide changes in the future. I realized the importance of giving very clear directions in the activities. I came to the understanding that since my students were not used to working in a task-based learning environment, more direction is needed in the first activity than in the last activity. Toward the middle and end of the course, it was less important for me to explain details such as, "work with your partner" or "show your steps". As students became more familiar with the structure, I was able to assign the steps without much explanation.

I was pleased with the results of task-based learning in my classroom. I enjoyed the rich conversations that engulfed my room and the sharing that took place between students. The students were eager to share ideas within small groups, when they might have been hesitant in front of the entire class. By sharing ideas with each other first, they were able to use their own ideas and the ideas of their peers when asked to present to the entire class.

As a final thought, let me say that as teachers, we may feel compelled to assist our students when we see them struggle. In a task-based classroom, teachers should allow students to work through their struggles. In this sense, we should take the advice of educators who focus on teaching with understanding. As Hiebert et al. write, "Many teachers worry that if they do not step in when a wrong answer is given or a flawed method is presented, students will be led astray and develop misunderstandings." Yet, as he asserts, these fears are unfounded.   "Our experience is if the tasks are appropriately challenging, that is, if they link up with students' thinking and allow students to use familiar tools, and if there is full discussion of various solution methods and solutions by the students after they have completed the task, then sound mathematical thinking and correct solutions eventually carry the day." It may be difficult for teachers to give up the spotlight in their classrooms, but to achieve understanding it is important to give students control of their learning. To summarize my experience with the entire process I have reported upon, I wish to agree. I have witnessed students correcting their own mistakes and the mistakes of others in the classroom. They achieved this by collaborating and reflecting on the thoughts and discoveries shared in the classroom.

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APPENDIX A: NOTES AND PRACTICE PROBLEMS USING SCALE FACTOR

SCALE DRAWINGS

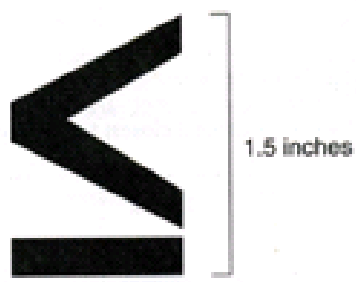
Architects regularly use scale drawings when they design houses and buildings. A scale is a ratio that compares the measurements used in the drawing to the actual measurements. Scale drawings or models are similar to the actual drawing or figure, and therefore the sides are proportional.

The ratio you should use when working with scale drawings or models is

$$\frac{Scale\ measurement}{Actual\ measurement}$$

This is called scale ratio. For example, if a model car measures 2 cm for every 1 ft of the actual car, the scale ratio is 2cm/1 ft. Knowing this ratio allows you to set up a proportion to figure out how long the actual car would be if the scale model was 36 cm: $\frac{2cm}{1ft} = \frac{36cm}{xft}$. By solving the proportion, you can determine that the actual car length is 18ft.

EXAMPLE: The height of the symbol in the accompanying drawing is 1.5 inches. The actual symbol will be 12 inches tall once it is placed on a sign. If the width of this symbol is 1 inch, how wide will the actual symbol be?



Set up a proportion.

$$\frac{Scale}{Actual} = \frac{scale}{actual}$$

$$\frac{1.5}{12} = \frac{1}{x}$$

$$1 \bullet 12 = 1.5x$$

$$12 \div 1.5 = x$$

The actual symbol will be 8 inches wide.

1. A photograph measures 4 inches wide and 5 inches long. If you hav etgh ephotograph enlarged to fit a fram 36 inches long, what is the widest the photograph can be?
   A. 36 inches B. 32 inches
   C. 30 inches D. 28 inches

2. A Florida map has a scale of 1 inch = approximately 22.8 miles. If the distance on the map between Vero Beach and Boynton Beach is 3.5 inches, what is the actual distance?

3. The smallest mammal, Kitti's hog-nosed bat, has a head-body length of ½ inch and a wingspan of about 2 ½ inches. A scale drawing of this little bat is made showing it in full flight. The wingspan on the drawing is 5 inches. What should the length of the bat be in the drawing?

   A. 1 inch B. 10 inches
   C. 2 ½ inches D. 7 ½ inches

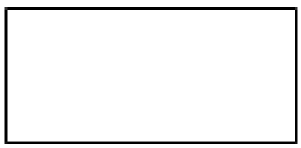
APPENDIX B: PRACTICE SCALE DRAWING ACTIVITY

Scale Drawing Worksheet Name ___________________________

Create multiple scale drawings of each figure using the specified scale factors. Follow the steps listed below.

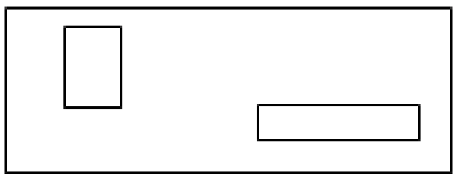
a) Measure the dimensions of the shape to the nearest tenth of a centimeter.
b) Multiply each dimension by the given scale factor.
c) Draw a shape using the new dimensions (answer when multiplied by the scale factor) on your drawing paper. Remember your new drawing should be the same shape as the original shape, just larger or smaller.
d) Label the drawing with the appropriate scale. (ex. 1:2 or 1:5, etc.)
e) Repeat for each given scale factor.

Figure 1



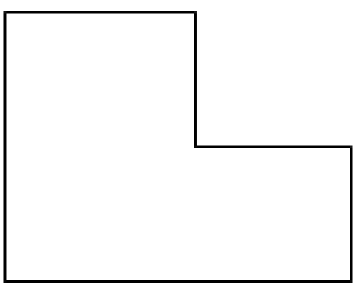
1. Scale Factor- 2:1
2. Scale Factor- 4:1
3. Scale Factor- 5:1

Figure 2



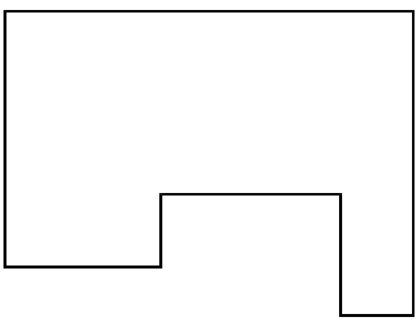
1. Scale Factor- 1:2
2. Scale Factor- 2:1
3. Scale Factor- 4:1

Figure 3



1. Scale Factor- 2:1
2. Scale Factor- 3:1
3. Scale Factor- 4:1

Figure 4



1. Scale Factor- 1:2
2. Scale Factor- 3:1
3. Scale Factor- 5:1

APPLY IT!!!

Make a scale drawing of the textbook under your desk using a scale factor of 1 ½ cm: 1 inch. Use the space below to draw the scaled version of your book and label the sides with the actual length.

APPENDIX C: LETTER TO STUDENTS FROM FICITONAL ENGINEER

St Amant High School Baseball

St Amant, LA

January 20, 2011

Wes Graham, PE

Jacobs Engineering

4949 Essen Ln # 323

Baton Rouge, LA 70809-3481

Dear Mr. Graham:

I spoke with you recently regarding our contract to build a new baseball field. In order to move forward with this project, we will require complete plans.

Included with this letter, you will find the dimensions of our space as well as the dimensions of the field. Once the detailed blueprints have been received, a final draft of our contract will be drawn up. If you foresee any complications due to the specifications, please let me know as soon as possible.

Our season starts in two months, so time is a critical factor here. On the other hand, we would like the best possible field for our team, so accuracy is most important. Thank you very much for your time.

Respectfully,

John Lerner

John Lerner

JE JACOBS

MEMORANDUM

Date: January 21, 2011

To: Research and Development Team

From: Margaret Fazekas

You probably remember that I mentioned this contract at the last departmental meeting. Attached are copies of the letter and basic design specs, along with a list of relevant materials in stock. Start working up a scaled blueprint so we can begin construction as soon as possible. Work in SI, and keep track of significant figures. Present your plan to me before you submit it to the school. I will need it on my desk by Tuesday 1/25/11. Make sure to accurately label the dimensions for each of the areas indicated below.

- Outer boundary line for the infield
- Infield grass line
- Baselines
- Three bases
- Pitcher's mound
- Batter's circle and batter's boxes
- On deck circles
- Dugouts
- Coach's Boxes
- Back Stop

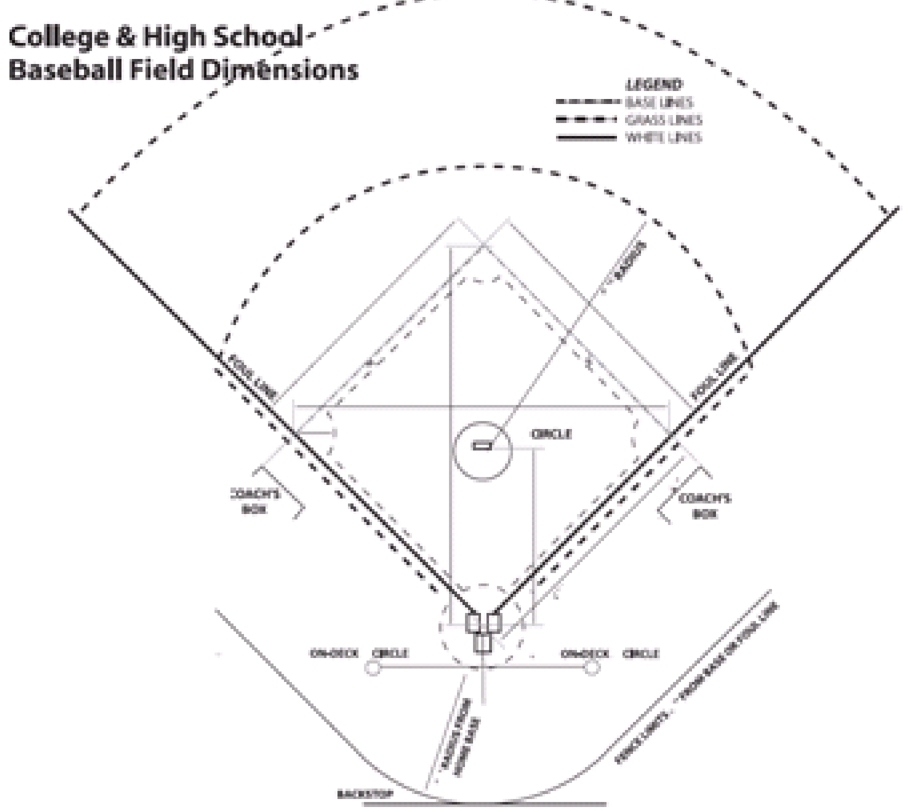
As I said at the meeting, it has been a great year at this company thanks to all of you. Good luck on this project.

4949 Essen Ln # 323 • Baton Rouge, LA 70809-3481

MATERIALS

ITEM	QTY.
✔ poster board	1 sheet
✔ mechanical pencil	1
✔ metric ruler	1
✔ meter stick	1
✔ protractor	1

SPECIFICATIONS



APPENDIX D: PROJECT DESCRIPTION & REFLECTION

St Amant High Baseball Field

Project Description

Step 1: Research the dimensions of a standard baseball field. You will need the following dimensions

- ✓ Distance between each of the bases: ______________________
- ✓ Radius of the infield boundary line: ______________________
- ✓ Radius of the pitcher's mound: ______________________
- ✓ Radius surrounding each base (including home plate) : ______________________
- ✓ Distance between each on-deck circle: ______________________
- ✓ Radius of the on-deck circle: ______________________
- ✓ Distance between home plate and the backstop: ______________________
- ✓ Dimensions of Coaches Boxes: ________________________
- ✓ Dimensions of Batter Boxes: ________________________
- ✓ Dimensions of Umpire Boxes: ________________________
- ✓ Radius of Back Stop: ___________________________

Step 2: Create an accurate scale drawing of the field using the given example as a guide. Include your chosen scale factor and show all calculations on a separate page.

Notes:

Step 3: Use your scale drawing to layout your design to test for accuracy.

Notes:

St Amant High Baseball Field

Project Conclusions and Reflection

1.) Was your layout accurate? Did it look similar to the baseball field at St Amant? Why or why not?

2.) Describe your method for making your scale drawing. Were there any complications? If so, describe them.

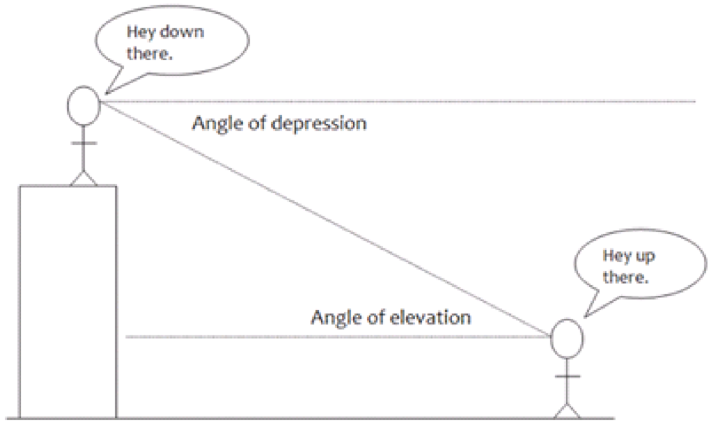
3.) What did you learn about scale factors and their purpose in drafting and design projects?

APPENDIX E: ANGLES OF ELEVATION AND DEPRESSION REFERENCE SHEET

Angles of Elevation and Depression

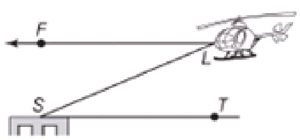
Name:___________________________

Date:________________Block:_____

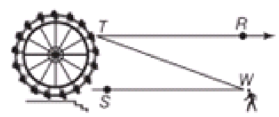


Name the angle of depression AND angle of elevation in each figure.

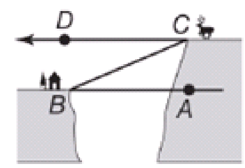
1.



2.



3.



APPENDIX F: SHADOW ACTIVITY RECORDS SHEET

Mathematics Outdoors NCCTM 2007

What can I learn from my SHADOW?

Name: ______________________________ **Date:** ________________

Group Members: ____

Objective: Students will apply trigonometric ratios and other things they know about right triangles to determine the height of an object outdoors.

Trig Ratios:

We have used right triangles to determine some important relationships that you have listed above. Today, you and your group members will use these ratios to determine the height of an object outside. Follow the following steps:

1. Pick one person in the group and measure height: ______________________

   Name of person you are measuring: ______________________________

2. Measure the length of that person's shadow: __________________________

3. Using the appropriate trigonometric ratio, find the angle of elevation (sketch picture):

4. Find the length of the shadow of the object your group has chosen: ____________

5. Using the angle of elevation and the shadow length, find the height of the object:

   ____

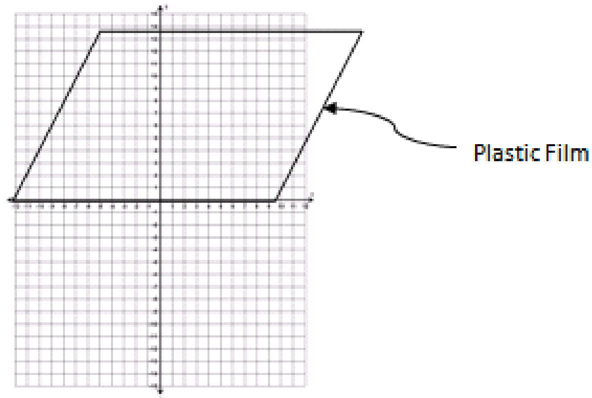
6. Sketch a picture of the object, its shadow, and the angle of elevation.

APPENDIX G: TRANSFORMATION HANDOUTS

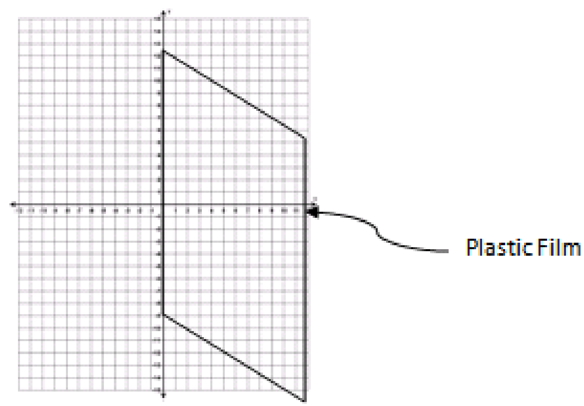
Transformations

Instructions:

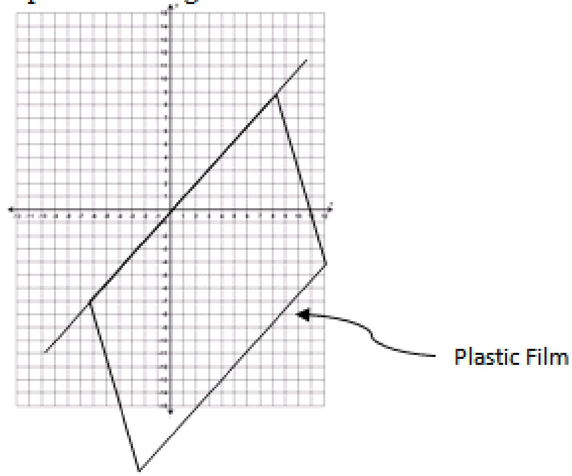
1) Using one of the "reflection" films, tape the edge of the plastic film onto the first grid, labeled "Reflection on x – axis" so that the edge of the film lines up with the x-axis.



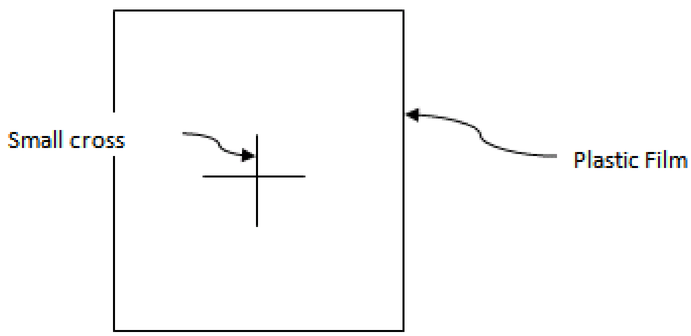
2) Use a second "reflection" film; tape the edge of the film to the right side of the second grid, labeled "Reflection on y – axis", so that the edge of the film lines up with the y-axis.



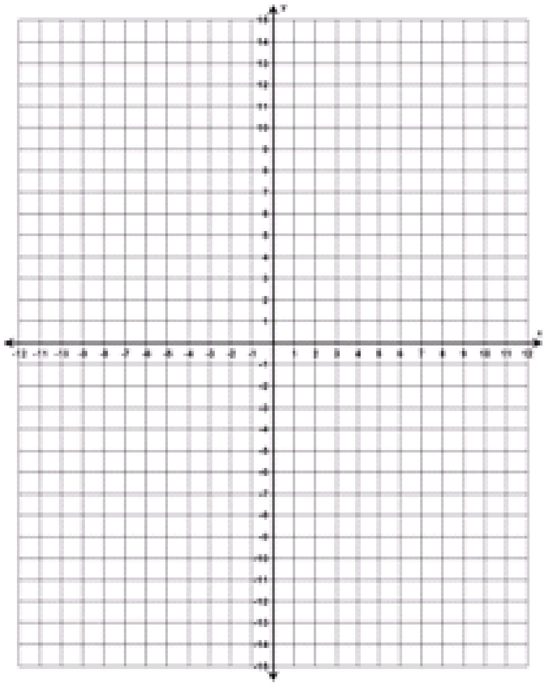
3) Use the last "reflection" film; tape the edge of the film to the right side of the diagonal line drawn on your third grid so that the edge of the film lines up with the diagonal line.



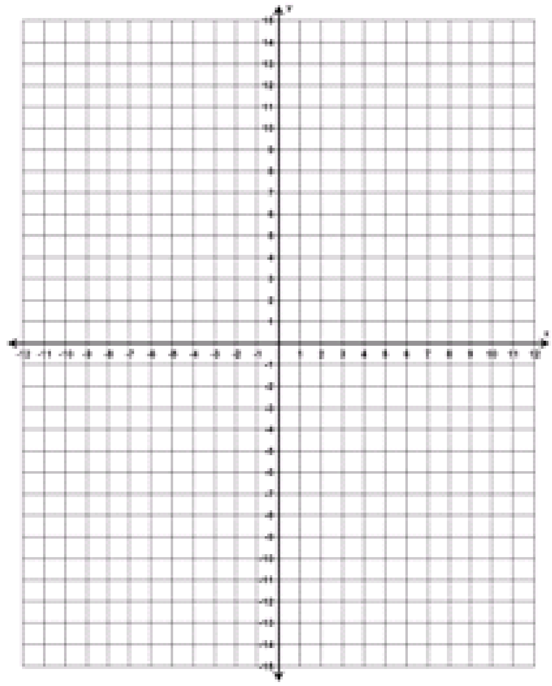
4) On the "rotation" plastic film draw a small cross in the middle to use as a guide when rotating the film around the fourth grid. Line the plastic film up with the x & y axis of the grid provided. Push a thumb tack through the origin into the piece of cork provided.



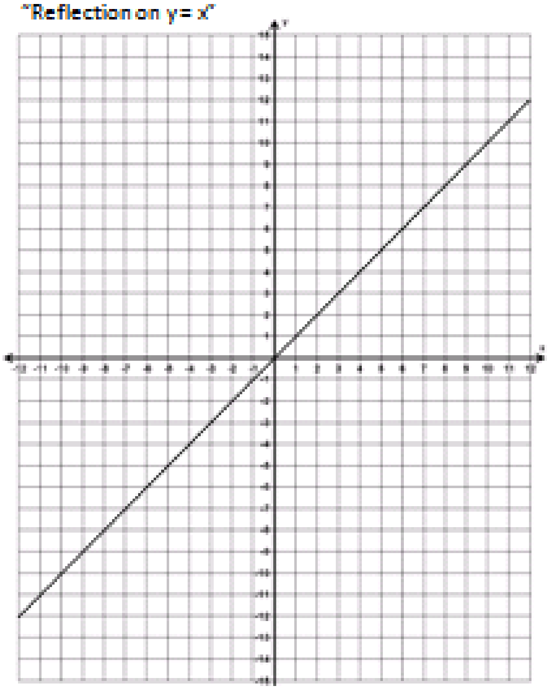
"Reflection on x-axis"



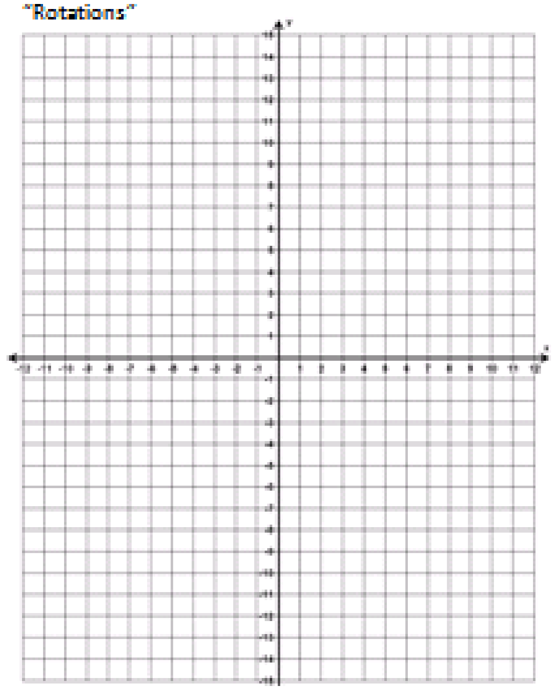
"Reflection on y-axis"



"Reflection on y = x"



"Rotations"



Transformations Notes Page

1) Plot these points on the plastic film of your "Reflection on x – axis" grid using a pen, then flip the points over the x-axis to find x' and y'. Fill out the following table:

Fill out the table:

x	y	x'	y'
(3	2)	(	)
(-1	3)	(	)
(1	0)	(	)

In your own words, what is happening to the points as they reflect across this line?

Rule: (x, y) → (__________, _________)

2) Plot these points on the plastic film of your "Reflection on y – axis" grid using a pen, then flip the points over the y-axis to find x' and y'. Fill out the following table:

Fill out the table:

x	y	x'	y'
(3	2)	(	)
(-1	3)	(	)
(1	0)	(	)

In your own words, what is happening to the points as they reflect across this line?

Rule: (x, y) → (__________, _________)

3) Plot these points on the plastic film of your "Reflection on y=x" grid using a pen, then flip the points over the y = x line to find x' and y'. Fill out the following table:

Fill out the table:

x	y	x'	y'
(3	2)	(	)
(-1	3)	(	)
(1	0)	(	)

In your own words, what is happening to the points as they flip across this line?

Rule: (x, y) → (__________,_________)

4) Line up the cross on your "rotation" film with the intersection of the x and y axis on your "Rotation" grid. Plot these points then rotate the film by putting your pencil at the center of the cross and moving the film in a counter clockwise motion. Follow the rotations in the table and write the new points.

Fill out the table:

x	y	Rotate	x'	y'
(4	1)	90°	(	)
(-2	3)	90°	(	)
(4	1)	180°	(	)
(-2	3)	180°	(	)
(4	1)	270°	(	)
(-2	3)	270°	(	)

What seems to be the rule?

Rotate 90° (x, y) → (__________, _________)

Rotate 180° (x, y) → (__________, _________)

Rotate 270° (x, y) → (__________, _________)

Can you use reflections to replace a 90° rotation? Explain why or why not.

Can you use reflections to replace a 180° rotation? Explain why or why not.

Can you use reflections to replace a 270° rotation? Explain why or why not.

VITA

Margaret Ann Fazekas was born in Baton Rouge, Louisiana, to David B. Fazekas and Deborah S. Fazekas. She is the third of four daughters. She has taught in the Ascension Parish School System for three years as an algebra I and geometry teacher. She is currently teaching algebra I and geometry at St. Amant High School in Ascension Parish. She received her Bachelor of Science degree in mathematics with a concentration in secondary education at Louisiana State University in 2008.