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Effect of Nuclear Forces on the Cross-Sections of Photonuclear Reactions.

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EFFECT OF NUCLEAR FORCES ON THE CROSS SECTIONS
OF PHOTONUCLEAR REACTIONS

A Dissertation

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Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

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by

Kazuto Okamoto

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ABSTRACT

The effect of nuclear forces, or the effect of the quasi-deuteron model, is discussed for the integrated cross section and the bremsstrahlung weighted cross section for photon absorption by nuclei.

As is verified also in other nuclear reactions, the independent particle model, in which we neglect inter-particle correlations due to nuclear forces, is a good approximation for the photonuclear reaction. In the case of our sum rule calculations knowledge of the wave functions of the excited states of a nucleus is not necessary, because the calculation can be reduced to the expectation value of some operator for the ground state, so that the approximation of the independent particle model is an appropriate one. In fact, many sum rule calculations by Levinger et al. have shown that the results of the independent particle model for the integrated cross section and the bremsstrahlung weighted cross section are not inconsistent with experiment.

On the other hand, it is undoubtedly true that there exists a strong correlation between nucleons due to nuclear forces. In the case of photonuclear reactions this dynamical correlation, or the so-called quasi-deuteron model, is known to be very important if the photon energy is more than $100 M_eV$. This seems to be in contradiction with the facts mentioned above, and no calculation has ever been done to solve this discrepancy.

In order to clarify this situation we shall investigate the effect of this dynamical correlation on the sum rule calculations using the Fermi gas model. We expand the wave function using

perturbation theory and take into account the two body nuclear force. The nuclear force used in our calculation is of partly Majorana exchange character and the potential is a Gaussian type without a repulsive core; the parameters are taken from the effective range theory of Blatt-Jackson.

The results of the calculation are that the integrated cross section is increased by only several per cent and the bremsstrahlung weighted cross section is decreased by the same rate. The effect of the hard core has not been estimated. It may not be very small in the case of the integrated cross section, but is probably very small in the case of the bremsstrahlung weighted cross section.

CHAPTER I

INTRODUCTION

It is now well known that at low energies a nucleus can be regarded as the assembly of approximately free nucleons, although the forces acting between two nucleons is clearly very strong. The validity of this treatment is supported by many theories and experiments.

For instance, the nuclei which have some special numbers (8, 20, 28, 50, 82, and 126) of protons or neutrons are known to be more stable than other nuclei. These numbers are called "magic numbers," and the stability of these magic number nuclei can be well explained by the shell model of Mayer and Jensen¹, in which the nucleons inside the nucleus are assumed to move without any mutual interaction, except that of the common shell model potential. This model has been applied also to the explanation of spin, magnetic moment, and quadrupole moment of the ground state of nuclei, and at least for some nuclei which are at or near magic numbers the treatment of the shell model has been shown to be fairly successful.

Recently the so-called cloudy crystal ball model has been proposed² for the explanation of low energy reactions. In this model a nucleus is described as an over-all potential with a small imaginary part. In other words, the absorption coefficient for the motion of low energy nucleons inside the nucleus is assumed to be fairly small, and a nucleon with a kinetic energy of several Mev outside the nucleus behaves as an approximately free particle. This is essentially the same assumption as that of the shell model. The fact that this cloudy

crystal ball model has been fairly successful for low energy neutron reactions shows that the treatment of the independent particle model (hereafter abbreviated as IPM) described at the beginning of the section holds good also in the region of several Mev.

Also in the field of photonuclear reaction the IPM shows a remarkable success, especially in the calculations of various photonuclear cross sections using the sum rule, as is seen below.

The cross section for photon absorption by a nucleus shows a large resonance at about 20 Mev, and the resonance energy depends on the mass number of the nucleus. This dependence is known to be a decreasing function of mass number, A . This resonance is usually called the giant resonance, and the mechanism of photon absorption is predominantly dipole. Many theories have been proposed to interpret this phenomenon and one of them is the sum rule, which we shall discuss here. Since it is now well known that the electric dipole approximation is fairly good in this case, we shall consider only the dipole sum rules.

We shall define the quantity which is called the dipole oscillator strength, f_{on} , by the following formula:

$$f_{on} = \frac{2M(E_n - E_o)}{\hbar^2} \left| \int \psi_o^* z \psi_n d\tau \right|^2 = \left| \frac{z_{on}}{\kappa_{on}} \right|^2 \quad (1)$$

where E_o and E_n are energies of the ground and excited states, M is the nucleon mass, and z is the component of the displacement along the direction of photon polarization. Then the cross section for dipole absorption of a photon which has the energy $W = E_n - E_o$ is given by:

$$\sigma_{on} = \frac{2\pi^2 e^2 \hbar}{M C} f_{on} \quad (2)$$

According to the Thomas-Reiche-Kuhn sum rule in the case of an atom the sum of f_{on} for all states n equals the numbers of electrons.

$$\sum_n f_{on} = Z \quad (3)$$

(This is derived using closure: $\sum_n A_{on} B_{no} = (AB)_{oo}$)

In the case of a nucleus we introduce an effective charge, which is given as $\frac{N}{A} e$ for a proton and $-\frac{Z}{A} e$ for a neutron, respectively. Then Eq. (3) becomes in this case:

$$\sum_n f_{on} = \frac{NZ}{A} \cong \frac{A}{4} \quad (3')$$

The approximation holds for the case of $N = Z$.

Accordingly the integrated cross section, σ_{int} , which is defined by the following formula

$$\sigma_{int} \equiv \int \sigma dW \equiv \sum_n \sigma_{on} \quad (4)$$

is given by

$$\sigma_{int} = \frac{2\pi^2 e^2 \hbar}{M C} \frac{NZ}{A} \quad (5)$$

Feenberg³, Siegert⁴, and Levinger and Bethe⁵ (hereafter abbreviated as LB) have shown that the sum rule should be changed if there is an exchange force which interchanges the positions of the particles (i.e. Majorana or Heisenberg force). For instance, in the case of the IPM and for Majorana force σ_{int} is given by⁵

$$\begin{aligned} \sigma_{int} &= 60 \frac{NZ}{A} (1 + Cx) \text{ MeV-mb} \\ &\cong 15A (1 + Cx) \text{ MeV-mb} \end{aligned} \quad (6)$$

where x is the fraction of the Majorana exchange force and is usually taken as $1/2$. C is a constant and is usually taken as approximately 0.8 (See also in the discussion after Eq. (9)). This result is not inconsistent with experiment.

LB⁵ also calculated the bremsstrahlung weighted cross section, σ_b , which is defined by

$$\sigma_b \equiv \int \frac{\sigma}{W} dW = \frac{e^2}{\hbar C} \frac{4\pi^2}{3} \left[\frac{N^2}{A^2} \langle (\sum_i r_i) \rangle_{00}^2 + \frac{Z^2}{A^2} \langle (\sum_j r_j) \rangle_{00}^2 \right] \quad (7)$$

where r_i and r_j are the co-ordinates of proton and neutron respectively, and $\langle \rangle_{00}$ means the expectation value for the ground state of the nucleus. If we take the IPM, the result for σ_b is too large as compared with experiment and has a much stronger mass number dependence than observed. Therefore LB concluded that there must be some correlation between the particles and suggested that the alpha particle model could give a result which was not inconsistent with experiment. However, even in the simple IPM in which we assume no correlation due to nuclear forces between particles there does exist a certain correlation due to the Pauli principle, as is true also in the case of electrons⁶. Since the total wave function of a pair of protons (or neutrons) must be antisymmetric, the spatial wave function must be antisymmetric for the spin triplet state, and symmetric for the spin singlet state. Because of this property of the wave function a certain correlation appears between the particles even if we neglect nuclear forces. Let us call this correlation "the Pauli principle correlation". LB⁵ neglected this effect and calculated only the terms which correspond

to $\langle \underline{r}_i^2 \rangle_{\infty}$ and $\langle \underline{r}_j^2 \rangle_{\infty}$ in Eq. (7). However, there are also terms of $\langle \underline{r}_i \underline{r}_{i'} \rangle_{\infty} \langle \underline{r}_j \underline{r}_{j'} \rangle_{\infty}$ ($i \neq i', j \neq j'$) which are due to the Pauli principle correlation. Levinger and Kent⁷ (hereafter referred to as LK) pointed out this fact, calculated these terms, and found that the correction by this correlation was undoubtedly very large and reduced the value of σ_b considerably to fit the experimental data. For instance, for Cu LB⁵ obtained $\sigma_b = 0.34$ barns assuming that the nuclear shape parameter, $\underline{r}_0$, is 1.5×10^{-13} cm (The nuclear radius, R_0 , is taken to be $R_0 = \underline{r}_0 A^{1/3}$.) LK showed that $\sigma_b = 0.12$ barns for $\underline{r}_0 = 1.5 \times 10^{-13}$ cm, which could be reduced to 0.08 barns if we used $\underline{r}_0 = 1.2 \times 10^{-13}$ cm, whereas the experimental data is also 0.08 barns. (This value of $\underline{r}_0 = 1.2 \times 10^{-13}$ cm is now regarded as more appropriate than $\underline{r}_0 = 1.5 \times 10^{-13}$ cm.) LK also showed that the σ_b calculated in this way did not depend so strongly on the mass number of the nucleus as the results of LB. Thus the serious discrepancy between theory and experiment for σ_b has also been removed.

In the calculations of LB and LK, the model of Fermi gas or of the IPM square well potential was used for the nucleus. Levinger⁸ has extended this to the simple harmonic oscillator potential ($\underline{r}_0$ in this case is defined by the relation $\langle \underline{r}^2 \rangle_{\infty} = \frac{3}{5} \underline{r}_0^2 A^{2/3}$) and found for $\underline{r}_0 = 1.2 \times 10^{-13}$ cm

$$\sigma_b = 0.36 A^{\frac{4}{3}} \text{ mb} \quad (8)$$

This agrees fairly well with experiment.

σ_b is calculated also by Khokhlov⁹ for an IPM square well. His results are summarized by Levinger¹⁰ as

$$\sigma_b = 0.30 A^{\frac{4}{3}} mb \quad (9)$$

This is in good agreement with experiment and with the result of Levinger, Eq. (8), although the coefficient is somewhat smaller.

In the simple harmonic oscillator model mentioned above Levinger⁸ also calculated the value of C in Eq. (6) for He⁴, O¹⁶, Ca⁴⁰. It should be pointed out here that although C represents the effect of the exchange force between the two nucleons, the wave function used in the calculation does not include any effect of nuclear forces. In other words, our model is still regarded as the IPM, even after we introduce C, the effect of the two body forces.

His results for the simple harmonic oscillator model⁸ and the results of LB⁵ for the Fermi gas model indicate that the value of C depends on the nucleus, on the model, and on the values of the parameters used in the calculation, but is of the order of unity. For instance, for $r_0 = 1.2 \times 10^{-13}$ cm it has values in the range

$$0.70 \lesssim C \lesssim 1.07 \quad (10)$$

It should be pointed out that the value of C also depends on the character of the nuclear forces. It is only the forces which interchange the positions of the particles that affect the value of C. The above results are for Majorana exchange force. In the case of Heisenberg exchange force the results change¹¹, but C is still of the order of unity. Other forces (the ordinary, tensor, and Bartlett forces) do not change the value of C at all.

On the other hand, Migdal¹² calculated the so-called second moment, σ_{-2} which is defined by the following formula:

$$\sigma_{-2} \equiv \int \frac{\sigma}{W^2} dW \quad (11)$$

and showed that

$$\sigma_{-2} = \frac{\pi^2}{20} \frac{e^2}{\hbar c} \frac{R_0^2 A}{k} \quad (12)$$

where k is a constant which appears in the Weizsäcker mass formula¹³ for a nucleus in the symmetry energy term $k(N - Z)^2/A$.

Levinger¹⁰ analysed the experimental data using this result and found

$$\sigma_{-2} = 2.25 A^{\frac{5}{3}} \mu b / \text{MeV} \quad (13)$$

for $r_0 = 1.2 \times 10^{-13}$ cm and $k = 23$ Mev, whereas the experimental data show

$$\sigma_{-2} = 3.5 A^{\frac{5}{3}} \mu b / \text{MeV} \quad (14)$$

It should be mentioned here that Migdal's results can be obtained also by the Fermi gas model¹⁴ although he calculated using the collective model. This fact suggests that the agreement sometimes found between the IPM and the collective model in the case of photonuclear reaction is not necessarily fortuitous.

As we have seen, various cross sections of photonuclear reaction can be calculated by sum rules using the IPM. It should be pointed out here that in the sum rule the calculation is reduced to expectation value of some operator for the ground state, as is seen in Eq. (3) or (7). In other words, knowledge of the wave functions of the excited states is not necessary. This is a great advantage of the sum rule method, and the fact that the calculations mentioned above

agree fairly well with experiment shows that the IPM is a good approximation for the nucleus. This result is consistent with the statement at the beginning of this section.

However, it is also true that the forces between two nucleons are very strong and of short range. There must be some correlation between nucleons due to these forces in addition to the Pauli principle correlation discussed already. This we call the "dynamical correlation". In fact, there is evidence that this dynamical correlation becomes effective in the case of photonuclear reaction. In the high energy region of more than 100 Mev the energy distribution of emitted particles has a much larger high energy component than is expected by the model with no dynamical correlation. Also the angular distribution of the particles in the laboratory system shows a strong forward maximum which is inconsistent with the model without correlation. In order to explain this discrepancy Khokhlov¹⁵ and Levinger¹⁶ proposed the so-called quasi-deuteron model, in which a proton and a neutron form a deuteron-like sub-unit inside the nucleus, absorb a photon of high momentum, and are emitted from the nucleus. This model could remove the discrepancy mentioned above, and since that time a large number of experiments have been performed and the results support the validity of the quasi-deuteron model (hereafter abbreviated as qd model). It was also proposed by Yoshida¹⁷ that the dynamical correlation among n particles inside the nucleus becomes effective in the high energy photonuclear reaction and, if we change n from 12 to 2 as the energies of emitted protons

goes from 40 to 90 Mev, we can explain their angular distributions fairly well. Of course, $\underline{n} = 2$ corresponds to the qd model.

This means that at least at the high energy of more than 100 Mev, the effect of the dynamical correlation plays an important role for the mechanism of photon absorption, while on the other hand this effect seems to be negligible from the results of the sum rule calculations. At first sight these results seem to be inconsistent with each other, but this discrepancy might be explained if we evaluate the effect of the qd model in the sum rule calculation.

Levinger¹⁸ tried to evaluate this effect in two different ways. He used the IPM wave function for energy less than E_{qd} and the qd wave function above; or alternatively he multiplied the qd wave function by a damping factor $\exp(-\delta r)$. His results for σ_b are sensitive to the values of δ or E_{qd} ; but for $E_{qd} = 50$ Mev or $\delta = 3 \times 10^{12} \text{ cm}^{-1}$ he found that σ_b decreased by about 10% of the value of the IPM. This result is not inconsistent with the fact that the qd model plays an important role for the high energy photonuclear reaction, because in the case of σ_b , a main contribution comes from the low energy part, as can be understood from the definition of σ_b .

Since Levinger's calculation was only preliminary, we shall investigate this problem in a somewhat different way. We shall neglect the hard core of the nuclear forces so that we can expand the wave function by perturbation theory. We shall assume a Gaussian potential between a neutron and a proton and calculate both σ_{int} and σ_b . We shall see that the results are that σ_{int} will increase by about several per cent, and σ_b will decrease by about the same amount.

Therefore, we conclude that the IPM is a good approximation for

photonuclear reaction. However, it should be mentioned that Brueckner found an appreciable increase for σ_{int}^{19} . The reason for the discrepancy between his result and ours is not yet clear. If we consider the hard core for the potential, we must use a different potential; therefore, it is difficult to predict the effect of the hard core exactly. This may not be negligible for σ_{int} , but probably negligible for σ_b because of the same reason stated above.

CHAPTER II

CALCULATION OF THE INTEGRATED CROSS SECTION

As is shown by Eqs. (1), (2), and (4) the integrated cross section is given by

$$\begin{aligned}\sigma_{int} &\equiv \int \sigma dW \\ &= \frac{2\pi^2 e^2 \hbar}{Mc} \sum_n f_{on} \\ &= 4\pi^2 \frac{e^2}{\hbar c} \sum_n (E_n - E_0) |\langle \chi_0 | \chi_n \rangle|^2\end{aligned}\quad (15)$$

In the case that there are no exchange forces $\sum_n f_{on} = \frac{NZ}{A}$ as is given by Eq. (4). For $N \cong Z$ this can be approximated by $A/4$, and σ_{int} is, as is shown by Eq. (5)

$$\sigma_{int} = 15 A \text{ MeV-mb}$$

According to Feenberg³, Siegert⁴, and LB⁵, this result is modified if there is an exchange force. The summed oscillator strength is given by

$$\sum_n f_{on} = \frac{NZ}{A} - \frac{2M}{\hbar^2} x \frac{1}{6} \int \psi_0^* \sum_i \sum_j r^2 V P_{ij} \psi_0 d\tau \quad (16)$$

where x is the fraction of the Majorana exchange forces which, as is explained after Eq. (6), is usually taken as $1/2$; i and j refer to a proton and a neutron, respectively; r is the distance between them; V is the two body potential between nucleons; P_{ij} is the Majorana operator which exchanges the positions of the particles i and j .

LB⁵ took a product of plane wave functions for ψ_0 , calculated the second term of Eq. (16) and showed that it was approximately

$0.8\underline{x}$, as is already discussed after Eq. (9). This value is different for different models, but at least the coefficient of $\underline{x}$, C , is of the order of unity.

Here we shall calculate this term again taking the dynamical correlation into account. We shall take the Fermi gas model for infinite nuclear matter, which is now assumed to be a fairly good approximation for the centre of a large nucleus. Since we are dealing with a pair of a neutron and a proton, the antisymmetrization of the wave function is not necessary. We shall also neglect the hard core of the two body nuclear forces, and expand the wave function as follows

$$\psi = \psi_0 + \sum_n \frac{(V_{in})_{on}}{E_0 - E_n} \psi_n \quad (17)$$

$$\psi_0 = \frac{1}{\Omega} e^{i\mathbf{k}_i \cdot \mathbf{r}_i} e^{i\mathbf{k}_j \cdot \mathbf{r}_j} \quad (17')$$

$$\psi_n = \frac{1}{\Omega} e^{i\mathbf{k}'_i \cdot \mathbf{r}_i} e^{i\mathbf{k}'_j \cdot \mathbf{r}_j} \quad (17'')$$

where Ω is the volume of a sphere containing A particles and is given by

$$\Omega = \frac{4\pi}{3} r_0^3 A \quad (18)$$

$\mathbf{k}'_i, \mathbf{k}_i, \mathbf{k}'_j, \mathbf{k}_j$ and $\mathbf{k}'_i, \mathbf{k}'_j$ are wave numbers of a proton i and a neutron j , the notations $\mathbf{k}_i$ etc. represent a vector and $\mathbf{k}_i \cdot \mathbf{k}_j$ etc. represent a scalar product of two vectors. Since our problem now is an infinite nucleus, these wave numbers must satisfy the following relation of momentum conservation

$$\mathbf{k}_i + \mathbf{k}_j = \mathbf{k}'_i + \mathbf{k}'_j \quad (19)$$

For the sake of convenience for later calculations let us define the following quantities.

$$\underline{q} = k_i - k_i' = k_j' - k_j \quad (20)$$

$$\underline{q}' = k_j' - k_i = k_j - k_i' \quad (20')$$

$\underline{q}$ is the difference of the momenta between initial and final states in the case of an ordinary force, and $\underline{q}'$ is the corresponding quantity in the case of an exchange force.

Then Eq. (16) becomes

$$\begin{aligned} \sum_n f_{on} &= \frac{NZ}{A} - \frac{M}{3\hbar^2} \alpha \int \psi^* \sum_i \sum_j r^2 V P_{ij} \psi d\tau \\ &= \frac{NZ}{A} - \frac{M}{3\hbar^2} \alpha \int (\psi_0^* + \sum_n \frac{(V_{ij})_{on}}{E_0 - E_n} \psi_n^*) \sum_i \sum_j r^2 V P_{ij} \\ &\quad \times (\psi_0 + \sum_n \frac{(V_{ij})_{on}}{E_0 - E_n} \psi_n) d\tau \\ &= \frac{NZ}{A} - \frac{M}{3\hbar^2} \alpha \int \psi_0^* \sum_i \sum_j r^2 V P_{ij} \psi_0 d\tau \\ &\quad + \frac{M}{3\hbar^2} \alpha \int \sum_i \sum_j r^2 V \sum_n \frac{(V_{ij})_{on}}{E_n - E_0} (\psi_n^* P_{ij} \psi_0 + \psi_0^* P_{ij} \psi_n) d\tau \\ &\quad - O[(V_{ij})_{on}^2] \end{aligned} \quad (21)$$

Both $\underline{V}$ and $\underline{V}_{ij}$ are the potential between a neutron and a proton and the relation between $\underline{V}$ and $\underline{V}_{ij}$ will be given later in Eq. (24). For the sake of later discussion we reverse the order of

E_0 and E_n in the third term. Since we shall restrict ourselves to the first order perturbation calculation, we neglect the last term of order V^2 . The first and second terms correspond to the first and second terms in the bracket of Eq. (6), respectively and were already calculated by LB⁵. The third term corresponds to the dynamical correlation which we shall calculate in this paper. Let us denote the third term as $\Delta \sum_n f_{on}$.

Terms in $\Delta \sum_n f_{on}$ including P_{ij} are given by

$$\psi_n^* P_{ij} \psi_0 = \psi_0^* P_{ij} \psi_n = \frac{1}{\Omega^2} e^{i\mathcal{R}' \cdot \mathbf{r}} \quad (22)$$

Then $\Delta \sum_n f_{on}$ is given by

$$\begin{aligned} \Delta \sum_n f_{on} &= +x \frac{2M}{3\hbar^2} \frac{1}{\Omega^2} \iint \sum_i \sum_j r^2 V \sum_n \frac{(V_{ij})_{on}}{E_n - E_0} e^{i\mathcal{R}' \cdot \mathbf{r}} d^3\mathbf{r} d^3\mathcal{R} \quad (23) \\ &= x \frac{2M}{3\hbar^2} \frac{1}{\Omega} \int \sum_i \sum_j r^2 V \sum_n \frac{(V_{ij})_{on}}{E_n - E_0} e^{i\mathcal{R}' \cdot \mathbf{r}} d^3\mathbf{r} \end{aligned}$$

where $\mathcal{R}$ is the co-ordinate of the centre of mass of the two nucleons. The integral with respect to $\mathcal{R}$ gives Ω . As a potential we take a mixture of ordinary and Majorana exchange forces:

$$V_{ij} = V(1 + x P_{ij}) \quad (24)$$

Then, $(V_{ij})_{on}$, the fourier transform of the potential, is given by

$$\begin{aligned} (V_{ij})_{on} &= (V)_{on} + x (V P_{ij})_{on} \\ &= \frac{1}{\Omega} F(\mathcal{Q}) + \frac{1}{\Omega} x F(\mathcal{Q}') \quad (25) \end{aligned}$$

where $F(\mathcal{Q})$ is given by

$$F(\mathcal{Q}) = \int_0^\infty V e^{i\mathcal{Q}r} d^3r \quad (26)$$

The summations with respect to $\underline{i}, \underline{j}$ in Eq. (21) correspond to those with respect to k_i, k_j and the summations with respect to $\underline{n}$ correspond to those with respect to k'_i, k'_j , but can be reduced to the summation with respect to k'_i only, because if k_i, k_j, k'_i are specified, k'_j is uniquely determined by Eq. (19).

Then Eq. (23) becomes

$$\sum_n f_{0n} = x \frac{4}{3} \left(\frac{M}{\hbar^2} \right)^2 \frac{1}{\Omega^2} \int_0^\infty \sum_{k_i} \sum_{k_j} \sum_{k'_i} \frac{F(\mathcal{Q}) + x F(\mathcal{Q}')}{k_i'^2 + k_j'^2 - k_i^2 - k_j^2} e^{i\mathcal{Q}r} r^2 V d^3r \quad (27)$$

The term involving $F(\mathcal{Q})$ is difficult to evaluate. However, the value of the integral involving $F(\mathcal{Q}) e^{i\mathcal{Q}r}$ is smaller than that of the integral involving $F(\mathcal{Q}') e^{i\mathcal{Q}'r}$ because of the "interference" between $\mathcal{Q}$ and $\mathcal{Q}'$. Therefore, in order to get the upper limit of $\Delta \sum_n f_{0n}$ we replace $F(\mathcal{Q})$ by $F(\mathcal{Q}')$

$$\Delta \sum_n f_{0n} \leq x(1+x) \frac{4}{3} \left(\frac{M}{\hbar^2} \right)^2 \frac{1}{\Omega^2} \int_0^\infty \sum_{k_i} \sum_{k_j} \sum_{\mathcal{Q}'} \frac{F(\mathcal{Q}') e^{i\mathcal{Q}'r}}{k_i'^2 + k_j'^2 - k_i^2 - k_j^2} \times r^2 V d^3r \quad (28)$$

The summation with respect to k'_i in Eq. (27) is again replaced by the summation with respect to $\mathcal{Q}'$, because after all there are three independent wave numbers among those which appear in Eqs. (19), (20), (20') and we can choose any three of them.

Using the relations of Eqs. (19), (20) and (20') we can simplify the energy denominator in the integral of Eq. (24)

$$k_i'^2 + k_j'^2 - k_i^2 - k_j^2 = 2 q' (q' + k_i - k_j) \quad (29)$$

The three dimensional summation with respect to the wave number can be transformed into the integral using the following relation

$$\sum_{\mathbf{k}} = \frac{\Omega}{(2\pi)^3} \int d^3 k \quad (30)$$

Furthermore, we must take into account the Pauli principle. Since the nucleons in the initial states must have the wave numbers lower than k_F , the Fermi wave number, and the transition must be to the unoccupied states because of the Pauli principle, we have the following relations

$$\left. \begin{aligned} |k_i| &< k_F \\ |k_j| &< k_F \\ |k_i + q'| &> k_F \\ |k_j - q'| &> k_F \end{aligned} \right\} \quad (31)$$

$$k_F = \frac{1}{r_0} \frac{3}{2} \left(\frac{\pi}{3} \right)^{\frac{1}{3}} = \frac{1.52}{r_0} \quad (32)$$

Using Eqs. (29), (30), (31), Eq. (28) can be written in the following form (we omit the inequality in Eq. (28) for the sake of simplicity).

$$\begin{aligned}
\Delta \sum_n f_{on} &= x(1+x) \frac{4}{3} \left(\frac{M}{\hbar^2} \right)^2 \frac{1}{\Omega^2} \left[\frac{\Delta \Omega}{(2\pi)^3} \right]^3 \\
&\quad \times \int_0^\infty r^2 V d^3 r \int d^3 k_i \int d^3 k_j \int_0^\infty q' \frac{F(q') e^{i q' r}}{2 q' (q' + k_i - k_j)} \\
&= x(1+x) \frac{2}{3} \left(\frac{M}{\hbar^2} \right)^2 \frac{\Delta \Omega}{(2\pi)^9} \\
&\quad \times \int_0^\infty r^2 V d^3 r \int d^3 k_i \int d^3 k_j \int_0^\infty q' \frac{F(q') e^{i q' r}}{q' (q' + k_i - k_j)}
\end{aligned} \tag{33}$$

where the limit of the integration for k_i or k_j is given by Eq. (31). We also express the wave numbers in unit of the Fermi wave number, k_F , to simplify the limit of integration

$$\left. \begin{aligned} k_i &= k_F p \\ k_j &= k_F n \\ q' &= k_F s' \end{aligned} \right\} \tag{34}$$

then Eq. (31) becomes

$$\left. \begin{aligned} |p| &< 1 \\ |n| &< 1 \\ |p + s'| &> 1 \\ |n - s'| &> 1 \end{aligned} \right\} \tag{34'}$$

where p , n and s' are dimensionless quantities. Eq. (33)

then becomes

$$\begin{aligned}
\Delta \sum_n f_{on} &= x(1+x) \frac{2}{3} \left(\frac{M}{\hbar^2} \right)^2 \frac{\Delta \Omega}{(2\pi)^9} k_F^7 \int_0^\infty r^2 V d^3 r \\
&\quad \times \int d^3 p \int d^3 n \int_0^\infty s' \frac{F(s') e^{i k_F s' r}}{s' (s' + p - n)}
\end{aligned} \tag{35}$$

the limit of the integration is given by Eq. (34').

The integrals with respect to $\mathbb{P}$ and $\mathbb{M}$ were already done by Euler²⁰ for the problem of the binding energy of the nucleus: see the Appendix.

$$\int d^3\mathbb{P} \int d^3\mathbb{M} \frac{1}{S'(S'+\mathbb{P}-\mathbb{M})} = \frac{4\pi^2}{15} \frac{P(S')}{S'} \quad (36)$$

where $\underline{P}(\underline{S}')$ is the polynomial given in the Appendix.

Then Eq. (35) becomes (we denote S' as S for the sake of simplicity).

$$\Delta \sum_n f_{on} = x(1+x) \frac{2}{45} \left(\frac{M}{\hbar^2}\right)^2 \frac{\Delta\Omega}{(2\pi)^7} \hbar_F^7 \times \int_0^\infty r^2 V d^3\mathbb{R} \int_0^\infty 4\pi S^2 dS \frac{F(S)P(S)}{S} e^{i\hbar_F S \mathbb{R}} \quad (37)$$

$$= x(1+x) \frac{4}{45} \left(\frac{M}{\hbar^2}\right)^2 \frac{\Delta\Omega}{(2\pi)^6} \hbar_F^7 \times \int_0^\infty r^2 V d^3\mathbb{R} \int_0^\infty F(S)P(S) e^{i\hbar_F S \mathbb{R}} S dS$$

The first integral with respect to $\mathbb{R}$ can be carried out very easily if we take the potential $\underline{V}$ as a Gaussian type. According to the effective range theory²¹, $\underline{V}$ for the Gaussian potential is given by

$$V = -s_0 V_0 e^{-\frac{r^2}{\lambda^2}} \quad (38)$$

$$V_0 = \frac{229.21}{b^2} \text{ MeV} \cdot 10^{-26} \text{ cm}^2 \quad (39)$$

$$\lambda = \frac{b}{(2.06)^{\frac{1}{2}}} \quad (40)$$

where $\underline{b}$ is called the intrinsic range of nuclear forces, and is approximately 2×10^{-13} cm. The results of our calculation depend on

the ratio of b/r_0 which will be discussed in Sec. III. S_0 is called the well depth parameter, which determines the depth of the potential. (In the original paper of Blatt-Jackson²¹, this is denoted as $\underline{S}$, but we use S_0 because we already used $\underline{S}$). According to Blatt-Jackson's effective range theory, if $S_0 = 1$, b is equal to the effective range, which can be determined by experiment and is given by²²

$$r_{os} = (2.40 \pm 0.23) \times 10^{-13} \text{ cm} \quad (41)$$

$$r_{ot} = (1.704 \pm 0.028) \times 10^{-13} \text{ cm} \quad (42)$$

for the singlet and triplet states respectively. (It should be noted that the effective range is usually denoted as r_0 , but in our case r_0 is the nuclear shape parameter and is in this paper taken to be $1.2 \times 10^{-13} \text{ cm}$). The relation between the intrinsic and effective ranges is discussed in great detail in the paper of Blatt-Jackson²¹. For the present we shall restrict ourselves to the triplet state, since we are now dealing with the problem of a quasi-deuteron.

For the sake of simplicity of calculation we introduce the following quantity

$$\frac{M}{\hbar^2} V \equiv -S_0 W = -S_0 W_0 e^{-\frac{r^2}{a^2}} \quad (43)$$

$$W_0 \equiv \frac{M}{\hbar^2} V_0 = \frac{5.53}{a^2} \quad (44)$$

Then Eq. (37) becomes

$$\Delta \sum_n f_{on} = \chi(1+\chi) \frac{4}{45} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^2} k_F^3 S_0 \int_0^\infty r^2 W e^{i k_F S r} \frac{1}{a^3 r} \times \int_0^\infty F(s) P(s) s ds \quad (45)$$

The first integral is evaluated as follows

$$\begin{aligned}
\int_0^\infty r^2 W e^{i k_F S r} d^3 r &= W_0 \int_0^\infty r^2 e^{-\frac{r^2}{\lambda^2}} e^{i k_F S r} d^3 r \\
&= 2\pi W_0 \int_0^\infty r^4 e^{-\frac{r^2}{\lambda^2}} dr \int_{-1}^1 e^{i k_F S r \omega} d\omega \\
&= 4\pi W_0 \frac{1}{k_F S} \int_0^\infty r^3 e^{-\frac{r^2}{\lambda^2}} \sin k_F S r \cdot dr \\
&= \frac{\pi^{\frac{3}{2}}}{4} W_0 \lambda^5 (6 - k_F^2 \lambda^2 S^2) e^{-\frac{\lambda^2}{4} k_F^2 S^2}
\end{aligned} \tag{46}$$

Inserting Eq. (45) into Eq. (44) we get

$$\begin{aligned}
\Delta \sum_n f_{on} &= -S_0 x (1+x) \frac{\pi^{\frac{3}{2}}}{45} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^6} k_F^7 \lambda^5 W_0 \\
&\quad \times \int_0^\infty (6 - k_F^2 \lambda^2 S^2) e^{-\frac{\lambda^2}{4} k_F^2 S^2} F(S) P(S) S dS
\end{aligned} \tag{47}$$

$\underline{F}(s)$ in the integral is defined by Eq. (26) and for the potential of Eq. (43) it is given as

$$\begin{aligned}
\frac{M}{\hbar^2} F(S) &= \frac{M}{\hbar^2} \int_0^\infty V e^{i \underline{q} \cdot \underline{r}} d^3 r \\
&= -S_0 \int_0^\infty W e^{i k_F S r} d^3 r \\
&= -S_0 W_0 \int_0^\infty e^{-\frac{r^2}{\lambda^2}} e^{i k_F S r} d^3 r \\
&= -S_0 W_0 \pi^{\frac{3}{2}} \lambda^3 e^{-\frac{\lambda^2}{4} k_F^2 S^2}
\end{aligned} \tag{48}$$

Then Eq. (47) becomes

$$\Delta \sum_n f_{on} = + S_0^2 x(1+x) \frac{\pi^3}{45} \frac{\Omega}{2^6 \pi^6} k_F^7 \lambda^8 W_0^2 \times \int_0^\infty (6 - k_F^2 \lambda^2 S^2) e^{-\frac{\lambda^2 k_F^2}{2} S^2} P(S) S dS \quad (49)$$

If we put $\underline{S} = 2u$ Eq. (49) becomes

$$\Delta \sum_n f_{on} = S_0^2 x(1+x) \text{const} \times 4 \cdot \int_0^\infty (6 - 4k_F^2 \lambda^2 u^2) e^{-2\lambda^2 k_F^2 u^2} P(u) u du \quad (50)$$

Let us denote $4 \times \text{const.}$ as $\underline{K}$. Then inserting Eqs. (18), (32), (40), and (44), $\underline{K}$ is given by

$$\begin{aligned} K &= \frac{4}{45} \frac{1}{2^6 \pi^3} \frac{4\pi}{3} r_0^3 A \frac{1}{r_0^7} \frac{3^7}{2^7} \left(\frac{\pi}{3}\right)^{\frac{7}{3}} \frac{b^8}{(2.06)^4} \frac{(5.53)^2}{b^4} \quad (51) \\ &= 0.006 A \underline{\epsilon}^4 \quad (\underline{\epsilon} = b/r_0) \end{aligned}$$

Eq. (50) then becomes $\underline{S}_0^2 x(1+x) \underline{K} \underline{J}$ where $\underline{J}$ is defined by

$$\begin{aligned} J &\equiv \int_0^\infty (6 - \beta u^2) e^{-\alpha u^2} P(u) u du \quad (52) \\ \alpha &= 2\lambda^2 k_F^2 \quad \beta = 2\alpha \end{aligned}$$

where $\underline{P}(u)$ is the polynomial given in the Appendix. If we expand

$\underline{P}(u)$ into the power series and take up to a certain term as is shown in the Appendix, $\underline{J}$ is

$$J = J_1 + J_2 \quad (53)$$

$$J_1 = \int_0^1 (6 - \beta u^2) \left[40(1 - \log 2) u^3 - 10u^5 + \frac{4}{3} u^7 + 0.21 u^9 \right] \times e^{-\alpha u^2} du \quad (53')$$

$$J_2 = \int_1^\infty (6 - \beta u^2) \left[\frac{10}{3} + \frac{1}{3} \frac{1}{u^2} + 0.16 \frac{1}{u^4} \right] e^{-\alpha u^2} du \quad (53'')$$

$\underline{J}_1$ can be calculated analytically

$$\begin{aligned}
J_1 = & 36.822 \alpha^{-2} - (12.274\beta + 60) \alpha^{-3} + (30\beta + 24) \alpha^{-4} (16\beta - 15.12) \alpha^{-5} \\
& - e^{-\alpha} [(-1.804\beta + 11.452) \alpha^{-1} + (0.059\beta - 8.658) \alpha^{-2} \\
& + (9.726\beta - 28.44) \alpha^{-3} + (14\beta + 39.12) \alpha^{-4} \\
& + (-16\beta + 15.12) \alpha^{-5}]
\end{aligned} \quad (54)$$

If we use Eqs. (32) and (40), α is given by

$$\begin{aligned}
\alpha & \equiv 2\lambda^2 k_F^2 \\
& = 2 \times \frac{k^2}{2.06} \frac{1}{r_0^2} \frac{9}{4} \left(\frac{\pi}{3}\right)^{\frac{2}{3}} \\
& = 2.25 \xi^2
\end{aligned} \quad (55)$$

Using Eq. (55) and the relation $\beta = 2\alpha$ of Eq. (52), Eq. (54) becomes

$$\begin{aligned}
J_1 = & 2.42 \xi^{-4} (1 - 0.128 \xi^{-4} + 0.108 \xi^{-6}) \\
& + e^{-2.25 \xi^2} (3.61 - 5.03 \xi^{-2} - 2.13 \xi^{-4} + 0.0385 \xi^{-6} \\
& - 0.277 \xi^{-8} - 0.261 \xi^{-10})
\end{aligned} \quad (56)$$

J_2 is evaluated numerically

$$J_2 = \int_1^\infty [15 \xi^2 u^2 + (1.50 \xi^2 - 20) + (0.72 \xi^2 - 2) u^{-2}] e^{-2.25 \xi^2 u^2} \alpha u \quad (57)$$

Of course, the values of J_1 and J_2 are dependent on the value of ξ . However, as we shall see in Sec. III, ξ is usually greater than 1.5 and for this value of ξ all other terms except the first one of J_1 are very small and can be neglected. In other words, we can safely approximate

$$J = J_1 + J_2 \cong 2.42 \xi^{-4} \quad (58)$$

Combining Eqs. (51) and (58) we get

$$\begin{aligned}
\Delta \sum_n f_{0n} & = 0.0145 A S_0^2 x (1+x) \\
& \cong 0.058 \frac{NZ}{A} S_0^2 x (1+x)
\end{aligned} \quad (59)$$

In this approximation $\Delta \sum_n f_{0n}$ is independent of ξ . In the second approximation we put $N = Z$ as we did in Eqs. (3'), (5), and (6). If we choose the well depth parameter $\underline{S}_0 = 1$, the coefficient of $NZ/Z \cdot x^2$ becomes simply 0.058. Let us denote this coefficient as C' . Combining Eqs. (5) and (18) we get

$$\sigma_{int} = \frac{2\pi^2 e^2 \hbar}{M C} \frac{NZ}{A} \{1 + (C + C')x + C'x^2\} \quad (60)$$

The value of C for the Fermi gas with a two body Gaussian potential, which corresponds to our present case, is given by Levinger⁸

$$C = 1.07 \quad \text{for} \quad r_0 = 1.2 \quad (61)$$

From our calculation we know

$$C' \cong 0.06 \quad (62)$$

We shall see in the next section that the value of C' does not depend so strongly on the value of $\underline{r}_0$ provided that the latter lies in the range which is regarded as reasonable from our present knowledge concerning nuclei.

Substituting, we obtain as a final result

$$\sigma_{int} = 15A (1 + 1.13x + 0.06x^2) \quad (63)$$

Using the usual value $\underline{x} = 1/2$, we see that C in Eq. (6) is increased from 1.07 to 1.16. In other words, the effect of the qd model is about 9% increase of C , but if we put $\underline{x} = 1/2$, the coefficient of $15A$ is increased from 1.54 to 1.58. In other words, the final result is only 3% increase. Furthermore, as is stated after Eq. (27) our results should be regarded as an upper limit of the qd effect. Therefore, the true value of this effect is less than this. We conclude that the effect of the qd model on the integrated cross section is to increase it by only a few per cent.

CHAPTER III

THE DEPENDENCE OF THE CORRECTION OF THE INTEGRATED CROSS

SECTION ON THE VALUE OF $\xi = b/r_0$

As is stated after Eq. (57) we neglected many terms of Eqs. (56) and (57). Of course, this approximation should be justified. Since all quantities are functions of ξ , we shall investigate the variation of $\underline{J}$ with respect to ξ , i.e. the dependence of C' on

$$\xi = b/r_0.$$

1. $\xi = 0$

From Eqs. (48) and (51) $\alpha = \beta = 0$

Inserting $\alpha = \beta = 0$ into Eqs. (53') and (53'') we find that $\underline{J}_1$ and $\underline{J}_2$ have finite values. Since $\Delta \sum_n f_{0n}$ is proportional to $\underline{K} \underline{J}$ and $\underline{K}$ is proportional to ξ'' (Eq. 51) C' in this case is zero.

$$C' = 0 \quad \text{for} \quad \xi = 0 \quad (64)$$

2. $\xi \rightarrow \infty$

Combining the results of Eqs. (51), (56), and (57) we find

$$C' = 0.06 \quad \text{for} \quad \xi \rightarrow \infty \quad (65)$$

3. $0 < \xi < \infty$

In this case we must evaluate C' by numerical calculations using Eqs. (51), (56), and (57). The calculations are performed for $1 \leq \xi \leq 2$ and the results are shown in Figure 1.

In order to facilitate the comparison with Levinger's results⁸, which are shown in Figure 2, we plot it with respect to η . ($\eta = \xi^{-1} = r_0/b$)

As seen in Figure 1 the effect of the dynamical correlation is constant for $0 < \underline{r}_0/\underline{b} < 1$. For larger η , the effect rapidly decreases and approaches zero asymptotically. In other words, this effect is roughly constant as long as the particle lies in the range of nuclear forces of an average nearest particle and rapidly vanishes if it goes outside the range. This seems to be a reasonable conclusion. The value of $\underline{r}_0$ is taken to be 1.2×10^{-13} cm and since we choose $\underline{S}_0 = 1$, the value of $\underline{b}$ equals to $\underline{r}_{ot}$, 1.7×10^{-13} cm. Then the value of η is about 0.7. The value of $\underline{b}$ will change if we change $\underline{S}_0$, and also the value of $\underline{r}_0$ is different from different experiments; but at least from our present knowledge the value of their ratio, η , lies in the region shown in Figure 1, and in this region the value of C' is constant and is given by Eq. (62). In other words, the approximation of Eq. (57) is justified.

CHAPTER IV

CALCULATION OF THE BREMSSTRAHLUNG WEIGHTED CROSS SECTION

a. Damping Factor Method

The Bremsstrahlung weighted cross section is given by the following formulae

$$\begin{aligned}\sigma_b &\equiv \int \frac{\sigma}{W} dW \\ &= \frac{2\pi^2 e^2 \hbar}{M C} \sum_n \frac{f_{on}}{E_n - E_0} \\ &= 4\pi^2 \frac{e^2}{\hbar C} \sum_n \frac{\hbar^2}{2M} \frac{f_{on}}{E_n - E_0}\end{aligned}\tag{66}$$

Inserting Eq. (1) we get

$$\sigma_b = 4\pi^2 \frac{e^2}{\hbar C} \sum_n \left| \left(\frac{N}{A} \sum_i x_i - \frac{Z}{A} \sum_j x_j \right)_{on} \right|^2 \tag{67}$$

Here $\underline{i}$ and $\underline{j}$ are referred to a proton and a neutron respectively as before. We introduced an effective charge of $\frac{N}{A} e$ for a proton and $\frac{Z}{A} e$ for a neutron.

Eq. (67) can be transformed as follows⁷

$$\begin{aligned}\sigma_b &= 4\pi^2 \frac{e^2}{\hbar C} \left\langle \left(\frac{N}{A} \sum_i x_i - \frac{Z}{A} \sum_j x_j \right)^2 \right\rangle_{00} \\ &= 4\pi^2 \frac{e^2}{\hbar C} \left[\frac{N^2}{A^2} \sum_i \langle x_i^2 \rangle_{00} + \frac{Z^2}{A^2} \sum_j \langle x_j^2 \rangle_{00} + 2 \frac{NZ}{A^2} S_t \sum_{i \neq j'} \langle x_i x_{j'} \rangle_{00} \right. \\ &\quad \left. + 2 \frac{Z^2}{A^2} S_t \sum_{j \neq j'} \langle x_j x_{j'} \rangle_{00} - 2 \frac{NZ}{A^2} S_t \sum_i \sum_j \langle x_i x_j \rangle_{00} \right] \\ &= 4\pi^2 \frac{e^2}{\hbar C} \left[\frac{NZ}{A} \frac{1}{3} \langle r^2 \rangle_{00} + 2 \left(\frac{N^2}{A^2} + \frac{Z^2}{A^2} \right) S_t \sum_{i \neq j'} \langle x_i x_{j'} \rangle_{00} \right. \\ &\quad \left. - 2 \frac{NZ}{A^2} S_t \sum_i \sum_j \langle x_i x_j \rangle_{00} \right] \tag{68} \\ &= 4\pi^2 \frac{e^2}{\hbar C} \frac{NZ}{A} \left[\frac{1}{3} R_o^2 + \frac{2}{A} \left(\frac{N}{Z} + \frac{Z}{N} \right) S_t \sum_{i \neq j'} \langle x_i x_{j'} \rangle_{00} \right. \\ &\quad \left. - \frac{2}{A} S_t \sum_i \sum_j \langle x_i x_j \rangle_{00} \right]\end{aligned}$$

where S_t is the statistical factor for the singlet and triplet states. The first term is originally given by LB⁵ and can be taken from the experimental data of the nuclear radius. The second term $\langle \sum_{i \neq j} z_i z_j \rangle_{\infty}$ corresponds to the Pauli principle correlation and is calculated by LK⁷. The third term $\langle \sum_i \sum_j z_i z_j \rangle_{\infty}$ corresponds to the dynamical correlation which we are now going to discuss. If we put $N = Z$, we obtain

$$\sigma_c = \frac{\pi^2}{137} A \left[\frac{1}{3} R_0^2 + \frac{4}{A} S_t \sum_{i \neq j} \langle z_i z_j \rangle_{\infty} - \frac{2}{A} S_t \sum_i \sum_j \langle z_i z_j \rangle_{\infty} \right] \quad (69)$$

Therefore, the change of σ_b due to the dynamical correlation, or the qd effect is given by

$$\Delta \sigma_b = - \frac{2\pi^2}{137} S_t \sum_i \sum_j \langle z_i z_j \rangle_{\infty} \quad (70)$$

If we assume the charge independence of nuclear forces (Nuclear forces do not depend on the charges of nucleons.), the dynamical correlation between the singlet n-p pair cancels exactly with that between n-n and p-p pair. Therefore, we shall consider only the triplet n-p pair. We introduce the following variables to simplify the calculation

$$Z = \frac{1}{2} (z_i + z_j) \quad (71)$$

$$z_{ij} = z_j - z_i \quad (72)$$

Then $z_i z_j$ becomes

$$z_i z_j = Z^2 - \frac{1}{4} z_{ij}^2 \quad (73)$$

In order to calculate $\langle z_i z_j \rangle_{\infty}$ we use the wave function of Eq. (15)

$$\begin{aligned} \langle z_i z_j \rangle_{\infty} &= \langle \psi | z_i z_j | \psi \rangle \\ &= \langle \psi_0 + \sum_n \frac{(V_{in})_{0n}}{E_0 - E_n} \psi_n | z_i z_j | \psi_0 + \sum_n \frac{(V_{in})_{0n}}{E_0 - E_n} \psi_n \rangle \end{aligned}$$

$$= \langle \psi_0 | z_i z_j | \psi_0 \rangle + 2 \langle \sum_n \frac{(V_{in})_{on}}{E_0 - E_n} \psi_n | z_i z_j | \psi_0 \rangle + O[(V_{ij})_{on}^2] \quad (74)$$

Since we are now dealing with the problem of the first order perturbation, we neglect the last term. The first term vanishes because it is the integral of an odd function. Using the relation of Eq. (73) the second term becomes

$$\begin{aligned} & 2 \langle \sum_n \frac{(V_{in})_{on}}{E_0 - E_n} \psi_n | z_i z_j | \psi_0 \rangle \\ &= 2 \iint \sum_n \frac{(V_{in})_{on}}{E_0 - E_n} \psi_n z_i^2 \psi_0 d\tau_i d\tau_j \\ &\quad - \frac{1}{2} \iint \sum_n \frac{(V_{in})_{on}}{E_0 - E_n} \psi_n z_{ij}^2 \psi_0 d\tau_i d\tau_j \end{aligned} \quad (75)$$

The first term of Eq. (75) vanishes because of the orthogonality of the wave functions. Finally, the quantity to be calculated (let us call this quantity I)

$$\begin{aligned} I &\equiv \sum_i \sum_j \langle z_i z_j \rangle_{00} \\ &= -\frac{1}{2} \sum_i \sum_j \iint \sum_n \frac{(V_{in})_{on}}{E_0 - E_n} \psi_n z_{ij}^2 \psi_0 d\tau_i d\tau_j \\ &= +\frac{1}{6} \iint \sum_i \sum_j \sum_n \frac{(V_{in})_{on}}{E_n - E_0} \psi_n r^2 \psi_0 d\tau_i d\tau_j \end{aligned} \quad (76)$$

Here we reversed the order of E_0 and E_n as we did in Eq. (23) to simplify the calculation, and also made use of the relation of $\langle z_{ij}^2 \rangle_{00} = 1/3 \langle r^2 \rangle_{00}$.

Inserting the explicit forms of the functions defined by Eqs. (17), (24), and (25) we obtain

$$I = \frac{1}{3} \frac{M}{\hbar^2} \frac{1}{\Omega^3} \int \int \sum_i \sum_j \sum_{i'} \frac{F(\mathbf{q}) + x F(\mathbf{q}')}{k_i'^2 + k_j'^2 - k_i^2 - k_j^2} e^{i\mathbf{q}\mathbf{r}} r^2 d^3\mathbf{r} d^3\mathbf{R}$$

$$= \frac{1}{3} \frac{M}{\hbar^2} \frac{1}{\Omega^2} \int \sum_i \sum_j \sum_{i'} \frac{F(\mathbf{q}) + x F(\mathbf{q}')}{k_i'^2 + k_j'^2 - k_i^2 - k_j^2} e^{i\mathbf{q}\mathbf{r}} r^2 d^3\mathbf{r} \quad (77)$$

The integral with respect to $\mathbf{R}$ reduces the denominator from Ω^3 to Ω^2 as in the case of Eq. (21). The calculations are quite similar to those in Sec. II. However, in the case of σ_{int} we took only the part including $F(\mathbf{q}')$ because the part including $F(\mathbf{q})$ is small due to the interference of $\mathbf{q}$ and $\mathbf{q}'$ as is stated after Eq. (23). In the case of σ_b , $e^{i\mathbf{q}\mathbf{r}}$ appears in the integral instead of $e^{i\mathbf{q}'\mathbf{r}}$ for the case of σ_{int} , therefore, the situation is reversed and the part including $F(\mathbf{q}')$ is expected to be smaller than the part including $F(\mathbf{q})$. Following the same argument after Eq. (27) we replace in this case $F(\mathbf{q}')$ by $F(\mathbf{q})$ and regard the resultant value as the upper limit of the contribution of the qd effect. Namely

$$I = (1+x) \frac{1}{3} \frac{M}{\hbar^2} \frac{1}{\Omega^2} \int \sum_i \sum_j \sum_{i'} \frac{F(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}}}{k_i'^2 + k_j'^2 - k_i^2 - k_j^2} r^2 d^3\mathbf{r} \quad (78)$$

After that the calculation proceeds in a very similar way to that of

$$\sigma_{\text{int}} \cdot$$

$$I = (1+x) \frac{1}{3} \frac{M}{\hbar^2} \frac{1}{\Omega^2} \left[\frac{\Omega}{(2\pi)^3} \right]^3 \int d^3\mathbf{r} \int d^3\mathbf{k}_i \int d^3\mathbf{k}_j \int d^3\mathbf{k}_i' \frac{F(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}} r^2}{k_i'^2 + k_j'^2 - k_i^2 - k_j^2}$$

$$= \frac{1+x}{6} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^7} \int_0^\infty r^2 d^3\mathbf{r} \int d^3\mathbf{k}_i \int d^3\mathbf{k}_j \int d^3\mathbf{q} \frac{F(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}}}{\mathbf{q}(\mathbf{q} + \mathbf{k}_i - \mathbf{k}_j)}$$

$$= \frac{1+x}{6} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^7} k_F^7 \int_0^\infty r^2 d^3\mathbf{r} \int d^3\mathbf{p} \int d^3\mathbf{q} \int d^3\mathbf{s} \frac{F(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}}}{\mathbf{s}(\mathbf{s} + \mathbf{p} - \mathbf{q})} \quad (79)$$

where $k_F S = 2$ and the limit of the integration is given by Eq. (31) or Eq. (34'). The integrals with respect to p and n are exactly the same as the case of σ_{int} and are given in the Appendix.

$$\int d^3 p \int d^3 n \frac{1}{S(S+p-n)} = \frac{4\pi^2}{15} \frac{P(S)}{S} \quad (80)$$

Inserting this into Eq. (79) we obtain

$$I = \frac{1+x}{10} \frac{M}{k^2} \frac{\Omega}{(2\pi)^3} k_F^7 \int_0^\infty r^2 e^{ik_F S r} d^3 r \int_0^\infty d^3 S \frac{F(S) P(S)}{S} \quad (81)$$

However, unlike the case of σ_{int} , the integral with respect to r will diverge. Therefore, we introduce a damping factor $e^{-\frac{r^2}{l^2}}$. Physically speaking this corresponds to regarding the nucleus as having a Gaussian density distribution. l in this case corresponds to a mean square radius of such a nucleus defined by the following formula

$$\frac{\int_0^\infty e^{-\frac{r^2}{l^2}} r^2 d^3 r}{\int_0^\infty e^{-\frac{r^2}{l^2}} d^3 r} = \frac{3}{5} R_0^2 \quad (82)$$

In other words l is given by

$$l = \left(\frac{2}{5}\right)^{\frac{1}{2}} R_0 \quad (83)$$

Then the first integral in Eq. (81) can be calculated in the same way as Eq. (46)

$$\begin{aligned} \int_0^\infty r^2 e^{ik_F S r} d^3 r &\rightarrow \int_0^\infty r^2 e^{-\frac{r^2}{l^2}} e^{ik_F S r} d^3 r \\ &= \frac{\pi^{\frac{3}{2}}}{4} l^5 (6 - k_F^2 l^2 S^2) e^{-\frac{l^2}{4} k_F^2 S^2} \end{aligned} \quad (84)$$

After that the calculation again becomes exactly the same as that of Sec. II except λ is replaced by $\underline{1}$

$$\begin{aligned}
 I &= \frac{1+x}{90} \frac{M}{\hbar^2} \frac{\Omega_1}{(2\pi)^7} \hbar_F^7 \frac{\pi^{\frac{7}{2}}}{4} \left(\frac{2}{5}\right)^{\frac{5}{2}} \mathcal{R}_0^5 \int_0^\infty (6 - \hbar_F^2 l^2 S^2) e^{-\frac{l^2}{4} \hbar_F^2 S^2} \\
 &\quad \times \frac{F(s)P(s)}{S} d^3 S \\
 &= \frac{1+x}{90} \frac{M}{\hbar^2} \frac{1}{(2\pi)^7} \frac{40\pi}{3} \mathcal{R}_0^3 A \frac{1}{\mathcal{R}_0^7} \left(\frac{3}{2}\right)^7 \left(\frac{\pi}{9}\right)^{\frac{7}{2}} \left(\frac{2}{5}\right)^{\frac{5}{2}} \mathcal{R}_0^5 A^{\frac{5}{3}} 4\pi \cdot \frac{\pi^{\frac{3}{2}}}{4} \\
 &\quad \times \int_0^\infty (6 - \hbar_F^2 l^2 S^2) e^{-\frac{l^2}{4} \hbar_F^2 S^2} \frac{F(s)P(s)}{S} S dS \\
 &\quad (85) \\
 &= -\frac{1+x}{90} \frac{1}{(2\pi)^5} \frac{3^6}{2^7} \left(\frac{\pi}{3}\right)^{\frac{7}{2}} \left(\frac{2}{5}\right)^{\frac{5}{2}} \pi^{\frac{3}{2}} \mathcal{R}_0 A^{\frac{8}{3}} S_0 W_0 \pi^{\frac{3}{2}} \lambda^3 \\
 &\quad \times \int_0^\infty (6 - \hbar_F^2 l^2 S^2) e^{-\frac{1}{4}(l^2 + \lambda^2) \hbar_F^2 S^2} \frac{P(s)}{S} S dS
 \end{aligned}$$

If we put $\underline{s} = 2\underline{u}$ as we did in Sec. II

$$\begin{aligned}
 I &= -S_0(1+x) \frac{1}{90} \frac{\pi^3}{(2\pi)^5} \frac{3^6}{2^7} \left(\frac{\pi}{3}\right)^{\frac{7}{2}} \left(\frac{2}{5}\right)^{\frac{5}{2}} W_0 \lambda^3 \mathcal{R}_0 A^{\frac{8}{3}} \\
 &\quad \times 4 \int_0^\infty (6 - 4\hbar_F^2 l^2 u^2) e^{-(l^2 + \lambda^2) \hbar_F^2 u^2} P(u) u du \\
 &\quad (86) \\
 &= -S_0(1+x) \times 2^{-\frac{17}{2}} \cdot 3^{\frac{5}{2}} \cdot 5^{-\frac{7}{2}} \cdot \pi^{\frac{1}{2}} \cdot \frac{5.53}{(2.06)^{\frac{1}{2}}} \cdot \mathcal{R}_0 A^{\frac{8}{3}} \\
 &\quad \times \int_0^\infty (6 - 4\hbar_F^2 l^2 u^2) e^{-(l^2 + \lambda^2) \hbar_F^2 u^2} P(u) u du \\
 &= -S_0(1+x) \times 1.73 \times 10^{-4} \mathcal{R}_0 A^{\frac{8}{3}} J'
 \end{aligned}$$

where

$$J' = \int_0^{\infty} (6 - \beta' u^2) e^{-\alpha' u^2} P(u) u du$$

$$\alpha' = (\ell^2 + \lambda^2) k_F^2$$

$$\beta' = 4 \ell^2 k_F^2$$
(87)

If we choose $\underline{s}_0 = 1$, $\underline{b} = \underline{r}_{ot} = 1.7 \times 10^{-13} \text{ cm}$ (see after Eqs. (42), (43).) Putting $\underline{r}_0 = 1.2 \times 10^{-13} \text{ cm}$ and $\underline{x} = 1/2$ we obtain

$$I = -5.29 A^{\frac{8}{3}} J' \cdot 10^{-30} \text{ cm}^2$$
(88)

The calculation of $\underline{J}'$ is identical to that of $\underline{J}$ in Sec. III, if we replace α' , β' by α , β . However, in the case of $\underline{J}$ the simple relation of $\beta = 2\alpha$ holds, whereas in the case of $\underline{J}'$, α' , and β' are not proportional to each other. If we substitute $\underline{l}$, λ , and $\underline{k}_{ir}$ using Eqs. (83), (41), and (33) we find

$$\alpha' = 0.93 A^{\frac{2}{3}} + 1.13 \Xi^2$$
(89)

$$\beta' = 3.71 A^{\frac{2}{3}}$$
(89')

where $\Xi = \underline{b}/\underline{r}_0$ as defined in Sec. II. From this we see that the relation between α' and β' is very complicated.

The calculation is performed in a very similar way to that of $\underline{J}$ in Sec. II $\underline{J}'$ is decomposed into two parts as in the case of

$$J' = J'_1 + J'_2$$
(90)

$$J'_1 = \int_0^1 (6 - \beta' u^2) [40(1 - \log 2) u^3 - 10 u^5 + \frac{4}{3} u^7 + 0.21 u^9] \times e^{-\alpha' u^2} du$$
(91)

$$J'_2 = \int_1^{\infty} (6 - \beta' u^2) \left[\frac{10}{3} + \frac{1}{3} \frac{1}{u^2} + 0.16 \frac{1}{u^4} \right] e^{-\alpha' u^2} du$$
(92)

However, in the case of $\underline{J}$, $\alpha = 2\lambda^2 k_F^2 = 2.25 \Xi^2$ and is fairly small if Ξ is small. In the case of $\underline{J}'$, α' is given by Eq. (88) and is more than 10, because we are now dealing with a fairly large nucleus. (For a small nucleus the plane wave function which we have been using is not appropriate and our entire calculation becomes invalid in this case). Since α' is fairly large, $\underline{J}_2^1$ can be safely neglected as compared with $\underline{J}_1^1$. The expansion of $\underline{J}_1^1$ is exactly the same as $\underline{J}_1$ given in Eq. (54), if we replace α by α' , and the terms after $e^{-\alpha'}$ are completely negligible because α' is at least more than 10. Therefore, we get similarly to Eq. (54),

$$J' \cong J_1' \cong 36.822 \alpha'^{-2} (12.274 \beta' + 60) \alpha'^{-3} + (30\beta + 24) \alpha'^{-4} - (16\beta - 15.12) \alpha'^{-5} \quad (93)$$

The evaluation of $\underline{J}_1'$ is done numerically. However, as we see from Eqs. (89) and (89') α' is a function of A and Ξ , and β' is a function of A ; hence, the value of $\underline{J}'$ depends not only on Ξ but also on A . This dependence is different from that of $\underline{J}$. Numerical results are listed in Table I.

TABLE I Values of

A	$\Xi = 1$	$\Xi = 1.5$	$\Xi = 2.0$
64	-0.031	-0.009	+0.002
125	-0.013	-0.008	-0.004
216	-0.008	-0.006	-0.004

As we see in Table I, for $A = 64$ the sign of $\underline{J}_1'$ changes for $\Xi = 2.0$: i.e., the results become qualitatively different. This is unreasonable. However, since we assume $\underline{r}_0 = 1.2 \times 10^{-13}$ cm, $\Xi = 2.0$ means $\underline{b} = 2.4 \times 10^{-13}$ cm. which is too large and should be excluded.

Furthermore, we use the plane wave as a wave function, which is a fairly good approximation for a heavy nucleus, but may not be so good for such a light nucleus. Therefore, this discrepancy should not be taken so seriously. From other results of Table I we see that the variation of $\underline{J}_1$ with respect to $\underline{\epsilon}$ is not so strong for large nuclei.

Combining these results with Eqs. (70) and (87) we get the final results. However, we must determine the statistical factor, $\underline{S}_t$, in Eq. (70). Since we are dealing with the triplet state as is stated after Eq. (70), $\underline{S}_t$ is usually taken as $3/4$; but as we see in Eq. (77) and in other equations, we took the sum with respect to the number of states, and each state has four particles. Therefore $\underline{S}_t$ is given by

$$\underline{S}_t = \frac{3}{4} \times 4 = 3 \quad (94)$$

Combining Eqs. (70), (87), (94) we get

$$\begin{aligned} \Delta \sigma_L &= + \frac{6\pi^2}{137} \cdot 5.29 A^{\frac{8}{3}} J' \mu b \\ &= + 2.30 A^{\frac{8}{3}} J' \mu b \end{aligned} \quad (95)$$

Since we assume $\underline{s}_0 = 1$, $\underline{b} = 1.7 \times 10^{-13}$ cm (See after Eq. (86), the value of $\underline{\epsilon}$ is approximately 1.5. Therefore, we take the results of the second column of Table I. The values of $\Delta \sigma_L$ in mb for $A = 64, 125, 216$ are listed in Table II together with the results of previous calculations of the IPM^{7,9,10}, for nuclei with approximately the same mass numbers, and also with the experiments.

TABLE II

Results of $\Delta\sigma_b$ and σ_b in mb

A	64	125	216			
$\Delta \sigma_b$	-0.98	-5.39	-15.58			
$\sigma_{b, \text{calc.}}^*$	77 ^a , 74 ^b	188 ^a	354 ^a			
$\sigma_{b, \text{exp.}}^*$	65 ² , 80 ^c	194 ^a	416 ^a			
$\left \frac{\Delta \sigma_b}{\sigma_b} \right (\%)$	1.2	1.5	2.7	3.0	3.5	4.0

Calculated and experimental values are for Cu^{63,65}, I¹²⁷, Bi²⁰⁹, respectively. All these values are taken from Table II of ref. 10. Calculated values are for $r_0 = 1.2 \times 10^{-13}$ cm, $b = 1.7 \times 10^{-13}$ cm, $S_0 = 1$.

a. Khokhlov, ref. 9

b. Levinger, ref. 7

c. Rustgi and Levinger, ref. 11

From Table II, we see that the qd effect on σ_b is to decrease it by several per cent and this percentage increases with increasing A. Therefore, we shall investigate this effect for infinitely large nuclei. If we put A as infinity in Eq. (86) or (87) we find

$$\beta' = 4\alpha' \quad \text{for } a = \infty \quad (96)$$

The calculation of $\underline{J}'$ becomes very simple and quite similar to that of $\underline{J}$ in Sec. II (In the case of $\underline{J}$, $\beta = 2\alpha$)

$$J \cong J_1 \rightarrow -12.274 \alpha^{-2} \rightarrow -14.253 A^{-\frac{4}{3}} \quad (97)$$

$$\Delta\sigma_b, \text{ lim} = -0.023 A^{\frac{4}{3}} \text{ mb} \quad (98)$$

whereas in the IPM σ_b is given by Eq. (9)

$$\sigma_b, \text{IPM} = 0.30 A^{\frac{2}{3}} \text{mb}$$

Therefore

$$\lim_{A \rightarrow \infty} \left| \frac{\Delta \sigma_b}{\sigma_b} \right| \cong 8\% \quad (99)$$

b. Box Normalization Method

Since we used a damping factor $e^{-\frac{r^2}{e^2}}$ in our calculation of σ_b , the results may not be so reliable. In order to test the validity of this method let us try to evaluate σ_b in some other way.

We shall start from Eq. (81) but replace the integration with respect to $\mathbf{S}$ by a sum with respect to $\mathbf{q}$. In other words, instead of transforming all sums which appear in Eq. (78) into integrals, we shall transform the first two of them and leave the last one in the form of a sum with respect to $\mathbf{q}$.

$$\begin{aligned} I &= \frac{1+x}{90} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^7} \hbar_F^7 \int_0^\infty r^2 e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r} \int_0^\infty d^3\mathbf{S} \frac{F(\mathbf{S})P(\mathbf{S})}{S} \\ &= \frac{1+x}{90} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^7} \hbar_F^5 \int_0^\infty r^2 e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r} \int_0^\infty d^3\mathbf{q} \frac{F(\mathbf{q})P(\mathbf{q})}{q} \\ &= \frac{1+x}{90} \frac{M}{\hbar^2} \frac{\Omega}{(2\pi)^7} \hbar_F^5 \int_0^\infty r^2 e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r} \frac{(2\pi)^3}{\Omega} \sum_{\mathbf{q}} \frac{F(\mathbf{q})P(\mathbf{q})}{q} \\ &= \frac{1+x}{90} \frac{M}{\hbar^2} \frac{1}{(2\pi)^4} \hbar_F^5 \sum_{\mathbf{q}} \int_0^\infty r^2 e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r} \frac{F(\mathbf{q})P(\mathbf{q})}{q} \end{aligned} \quad (100)$$

where the summation with respect to $\mathbf{q}$ is a three dimensional one.

Let us introduce here the box normalization. In other words, we shall assume that the whole system is in a large cubic box and decompose $\mathbf{q}$ into its three components. For each component we have

$$\left. \begin{aligned} q_x &= n_x \cdot 2\pi/L \\ q_y &= n_y \cdot 2\pi/L \\ q_z &= n_z \cdot 2\pi/L \end{aligned} \right\} \quad (101)$$

where L is the dimension of the cube, and $\underline{n}_x, \underline{n}_y, \underline{n}_z$ are positive integers including zero. Physically speaking this corresponds to regarding a nucleus as a cubic box with a periodic wave function.

Then the integral in Eq. (100) becomes

$$\int_0^\infty r^2 e^{i \underline{q} \cdot \underline{r}} d^3 r = 3 \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 e^{i(n_x x + n_y y + n_z z) \frac{2\pi}{L}} dx dy dz \quad (102)$$

The integration with respect to y and z is straightforward and gives us a delta function

$$\begin{aligned} \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{i n_y y} dy &= L \delta_{n_y} \\ \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{i n_z z} dz &= L \delta_{n_z} \end{aligned} \quad (103)$$

When we evaluate the integral we should demand that $\underline{n}_y = \underline{n}_z = 0$ because otherwise the above integrals become zero. Since $\underline{n}_y = \underline{n}_z = 0$, $\underline{n}_x = 0$ means $\underline{q} = 0$. However, since $\underline{P}(\underline{q})$ in Eq. (100) is proportional to q^2 (see Appendix) and $\underline{F}(\underline{q})$ has no singularity at $\underline{q} = 0$ because of its definition, Eq. (100) becomes zero if $\underline{q} = 0$. In other words, we must assume $\underline{n}_x \neq 0$.

In such a case the integral with respect to x can be carried out by partial integration:

$$\begin{aligned} \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 e^{i n_x x} dx &= \frac{1}{2\pi^2 n_x^2} L^3 \cos \pi n_x \\ &= \frac{(-1)^{n_x} L^3}{2\pi^2 n_x^2} \end{aligned} \quad (104)$$

Therefore, for $\underline{q} \neq 0$,

$$\begin{aligned} \int_0^\infty r^2 e^{i \underline{q} \cdot \underline{r}} d^3 r &\rightarrow 3 L^2 \delta_{n_y} \delta_{n_z} \frac{(-1)^{n_x} L^3}{2\pi^2 n_x^2} \\ &= \frac{6 L^5 (-1)^{n_x}}{(2\pi)^2 n_x^2} \end{aligned} \quad (105)$$

Substituting this into Eq. (100) we find for $\underline{x} = 1/2$

$$I = \frac{1}{10} \frac{M}{\hbar^2} \frac{\hbar_F^5}{(2\pi)^5} L^5 \sum_{n_x=1}^{\infty} \frac{(-1)^{n_x}}{n_x^2} \frac{F(\underline{x}) P(\underline{x})}{\underline{x}} \quad (106)$$

On the other hand if L is very large, $\underline{q}$ is very small for fixed $\underline{n}$.

Then from the Appendix

$$\begin{aligned} \lim_{\underline{x} \rightarrow 0} \frac{F(\underline{x}) P(\underline{x})}{\underline{x}} &= F(0) \lim_{\underline{x} \rightarrow 0} \frac{P(\underline{x})}{\underline{x}} \\ &= 10(1 - \log 2) F(0) \frac{\underline{q}}{\hbar_F^2} \\ &= 10(1 - \log 2) \frac{F(0)}{\hbar_F^2} \frac{2\pi}{L} n_x \end{aligned} \quad (107)$$

Substituting this into Eq. (106) we get

$$\begin{aligned} I &= (1 - \log 2) \frac{M}{\hbar^2} F(0) \frac{\hbar_F^5}{(2\pi)^5} L^4 \sum_{n_x=1}^{\infty} \frac{(-1)^{n_x}}{n_x} \\ &= (1 - \log 2) W_0 \pi^{\frac{3}{2}} \lambda^3 \frac{\hbar_F^3}{(2\pi)^5} L^4 \log 2 \end{aligned} \quad (108)$$

where λ and W_0 are given in Eqs. (41) and (45).

In order to determine L we shall assume that the volume of the cube is the same as that of a spherical nucleus of radius R_0 .

$$L^3 = 4\pi/3 \cdot R_0^3 \quad (109)$$

$$L = (4\pi/3)^{\frac{1}{3}} r_0 A^{\frac{1}{3}} \quad (110)$$

Then I becomes

$$I = (1 - \log 2) \log 2 W_0 \pi^{\frac{3}{2}} \lambda^3 \frac{\hbar_F^3}{(2\pi)^5} \left(\frac{4\pi}{3} \right)^{\frac{4}{3}} r_0^4 A^{\frac{4}{3}} \quad (111)$$

From Eqs. (70) and (94) we get

$$\Delta \sigma_E = -0.04 A^{\frac{4}{3}} \text{ mb} \quad (112)$$

here again we assumed $\underline{r}_0 = 1.2 \times 10^{-13} \text{ cm}$ and $\underline{b} = 1.7 \times 10^{-13} \text{ cm}$ as

before. Comparing with Eq. (9) we get

$$\left| \frac{\Delta \sigma}{\sigma} \right| \cong 13\% \quad (113)$$

However, we should regard this value as an over-estimate because we assume that the nucleus is a cube and that the nuclearwave function does not damp at all at the boundary. On the contrary in the former method we assume that a nucleus is a sphere and the wave function damps in a Gaussian way. Therefore, these two methods should be regarded as an upper and lower limit respectively. Since the box normalization method gives us about 13% and the damping factor method gives us several per cent we can conclude that the effect of the dynamical correlation on the bremsstrahlung weighted cross section is a decrease of several to ten per cent.

This is not inconsistent with the preliminary result of Levinger¹⁸ that σ_b will decrease by about 10%.

CHAPTER V

SUMMARY

The effect of the dynamical correlation, or the effect of the quasi-deuteron model, on the cross sections of photonuclear reaction has been treated using perturbation theory. The results are that this effect increases the integrated cross section, σ_{int} , by about several per cent, and decreases the bremsstrahlung weighted cross section, σ_b , by about the same percentage.

In the above analyses, however, we did not take into account the effect of the hard core of the nuclear two-body potential. It may affect the results for σ_{int} because σ_{int} is the expectation value of $\frac{r^2 V_{ij} P_{ij}}{ij}$, but this effect probably will not be so large as to change the main conclusion. For example, Gomes, Walecka, and Weisskopf²³ have shown that in the case of an attractive potential with a hard core the wave function outside the core is similar to that of the IPM. This seems to support the above prediction. For σ_b this effect is clearly not so large, because σ_b is the expectation value of $\underline{r}^2$ the main contribution for which comes from the region of large $\underline{r}$, where the effect of the hard core can be regarded as negligible.

APPENDIX

The integrations with respect to p and n for the energy denominator of Eq. (34) have been carried out by Euler²⁰ for the problem of the binding energy of the nucleus. We shall summarize Euler's calculation. We omit the prime and write S instead of S' for the sake of simplicity

$$D \equiv \int d^3 S \int_{|p| < 1} d^3 p \int_{|n| < 1} d^3 n \frac{1}{S(S+p-n)} \quad (A1)$$

$$= \int d^3 S \int_0^\infty e^{-\alpha S^2} d\alpha \int_{|p| < 1} d^3 p \int_{|n| < 1} d^3 n e^{-\alpha(S \cdot p)} e^{+\alpha(S \cdot n)}$$

$|p| < 1 \quad |n| < 1$
 $|p+S| > 1 \quad |n-S| > 1$

where α is the auxiliary variable introduced to transform the integrals.

The integration is divided into two parts. Putting $\alpha S = y$ we get

$$D = \int_0^2 d^3 S \frac{4\pi^2}{S} \int_0^\infty e^{-S^2 y} \left[\frac{1}{y^3} \{ S y e^{-\frac{S}{2} y} + e^{-y} (y+1) (e^{-S y} - 1) \}^2 \right] dy$$

$$+ \int_2^\infty d^3 S \frac{4\pi^2}{S} \int_0^\infty e^{-S^2 y} \left[\frac{1}{y^3} \{ e^y (y-1) + e^{-y} (y+1) \} \right]^2 dy \quad (A2)$$

The integration with respect to y is elementary but tedious. After integration we put $S = 2u$ and get the following results.

$$D = \frac{64}{15} \pi^3 \int_0^1 P(u) u du \quad (A3)$$

where for $u < 1$

$$P(u) \equiv P_1(u)$$

$$= \log(1+u) \left[4 + \frac{15}{2}u - 5u^3 + \frac{3}{2}u^5 \right] + 29u^2 - 3u^4$$

$$+ \log(1-u) \left[4 - \frac{15}{2}u + 5u^3 - \frac{3}{2}u^5 \right] - 40u^2 \log 2$$

$$\cong 40(1-\log 2)u^2 - 10u^4 + \frac{4}{3}u^6 + 0.21u^8$$

(A1)

The error of this approximation is about 0.2%.

for $u > 1$

$$P(u) \equiv P_2(u)$$

$$\begin{aligned} &= \log(u+1) \cdot [4 - 20u^2 - 20u^3 + 4u^5] + 4u^3 + 22u \\ &\quad + \log(u-1) \cdot [-4 + 20u^2 - 20u^3 + 4u^5] + \log u \cdot [40u^3 - 8u^5] \end{aligned} \quad (A5)$$

$$\cong \frac{10}{3} \frac{1}{u} + \frac{1}{3} \frac{1}{u^3} + 0.16 \frac{1}{u^5}$$

The error of this approximation is about 1%.

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FIGURE CAPTIONS

Figure 1. The effect of the dynamical correlation on the integrated cross section as a function of η . C' is the coefficient which appears in Eq. (60) and represents the effect of the dynamical correlation. $\eta = \underline{r}_0/\underline{b}$ where $\underline{r}_0$ is the nuclear shape parameter. (The nuclear radius is given by $R_0 = \underline{r}_0 A^{1/3}$). $\underline{b}$ is the intrinsic range of nuclear forces. The "allowed region" shown by the bracket is the region of η from our present knowledge about nuclei.

Figure 2. The effect of the exchange force on the integrated cross section calculated using the IPM by Levinger (ref. 8). C is the coefficient which appears in Eq. (6) or (60) and represents this effect in the IPM. η is the same as in Figure 1.

VITA

Kazuto Okamoto was born on April 15, 1927, in Tokyo, Japan. He entered The Tokyo Metropolitan First Middle School in 1940 and finished his course in 1944. He entered The First High School (Tokyo) in 1944 and graduated from it in 1948. He entered Tokyo University of Education in 1950, graduated from it in 1953, and was awarded a degree of Master of Science. He then studied in the Graduate School of the same university from 1953 to 1956. After that he left Japan to attend the Graduate School of the Louisiana State University in September, 1956, as a candidate for the Doctor of Philosophy degree in the Department of Physics. He was in the Department of Mathematical Physics in the University of Birmingham, England, from September, 1957, to August, 1958, but he was still enrolled as a graduate student of the Louisiana State University during his stay in England.

EXAMINATION AND THESIS REPORT

Candidate: K. O Kamoto

Major Field: Physics

Title of Thesis: Effect of Nuclear Forces on the Cross
Sections of Photoneuclear Reactions

Approved:

Joseph S. Levinger
Major Professor and Chairman

Richard J. Russell
Dean of the Graduate School

EXAMINING COMMITTEE:

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Date of Examination:

Sept. 19, 1958