Three essays in options pricing: 1. Volatilities implied by price changes in the S&P 500 options and future contracts 2. Price changes in the S&P options and futures contracts: a regression analysis 3. Hedging price changes in the S&P 500 options and futures contracts: the effect of different measures of implied volatility

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THREE ESSAYS IN OPTION PRICING:
1. VOLATILITIES IMPLIED BY PRICE CHANGES IN THE S&P 500 OPTIONS AND FUTURES CONTRACTS
2. PRICE CHANGES IN THE S&P 500 OPTIONS AND FUTURES CONTRACTS: A REGRESSION ANALYSIS
3. HEDGING PRICE CHANGES IN THE S&P 500 OPTIONS AND FUTURES CONTRACTS: THE EFFECT OF DIFFERENT MEASURES OF IMPLIED VOLATILITY

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Abstract

In this work, I develop a new volatility measure; the volatility implied by price changes in option contracts and their underlyings. I refer to this as implied price change volatility. First, I examine the time series behavior of implied price change volatility and investigate possible moneyness and maturity effects. I compare these characteristics to those of the usual implied volatility measure and the historical volatility of the S&P 500 index. Then, I investigate the performance of the implied price change volatility in a regression setup and in hedging applications. I compare the performance of hedges using daily updated implied price change volatility and implied volatility and their averages. Data used in this study are tick-data on pit traded S&P 500 futures options and their underlying from 1998 to 2006.

I find that implied price change volatility has similar time series behavior and moneyness and maturity effects as implied volatility. However, the price change volatility is more disperse than implied volatility. Hedges using daily updated volatilities consistently outperform hedges based on average volatilities. In addition, the delta hedges based on directly estimated implied price change volatility outperform even the delta-gamma and delta-vega hedges for call options. This finding suggests that using volatilities estimated from price changes rather than price levels may result in more effective hedges for call options.

Chapter 1 Introduction

Research in option pricing in the last thirty five years has resulted in many new option pricing models, each of them relaxing one or more of the assumptions in the Black-Scholes model. The most widely cited models include the stochastic volatility models of Heston (1993) and Hull and White (1987), Merton's stochastic interest rate model (1973), Merton (1976) and Bates' jump diffusion models (1991), the stochastic volatility and stochastic interest rates models of Amin and Ng (1993) and Bakshi and Chen (1997) and the stochastic volatility jump diffusion model of Bates (1996). These models are generally able to explain option prices better than the original Black-Scholes model. However, their predictive power depends on the ability to precisely estimate parameters. Parameter estimation errors result in additional pricing errors and this can mitigate their effectiveness. Although the Black-Scholes model has been shown to exhibit consistent pricing biases (Rubinstein 1985, 1994), it is still the most widely used option pricing model on Wall street.

Traders on Wall Street predominantly use the so called Practitioner Black-Scholes (PBS) model. They estimate implied volatility from range of options of different maturity and moneyness. Using interpolation they then create a "volatility surface" relating implied volatility to moneyness and maturity. The result is a continuous surface that can be used to estimate the price of an option with any moneyness and maturity. When recalibrated with sufficient frequency, this method gives surprisingly good results, outperforming more sophisticated models (Christoffersen and Jacobs (2004)).

A fundamental question that I investigate is whether the volatility estimated from price changes produces more efficient hedges than hedges based on volatilities estimated from price levels. Put differently, does the volatility implied by price changes extract information about futures price changes more efficiently than the usual implied volatility estimated from price levels?

My dissertation consists of three essays. In the first essay, I develop the concept of implied price change volatility and obtain estimates using transaction data from 1998 to 2006 on the S&P 500 futures contract. The S&P 500 futures contract closely tracks the price movements of the S&P 500 spot index. These contracts are widely used for speculative as well as hedging purposes. The S&P 500 futures contract is the most liquid derivative contracts in the world with an average of more than one million contracts traded daily in 2006. In the first essay, I also examine the moneyness and maturity effects of implied price change volatility and compare them with these effects of implied volatility.

In the second essay, I examine price changes in the S&P 500 futures options using a regression setup. I investigate how different volatility measures influence the performance of Black's model. Using price changes in evaluation of pricing models mitigates some statistical problems such as heteroskedasticity and autocorrelation of errors that necessarily burden price level models. I also examine the frequency of "wrong signs." For calls, a wrong sign is implied when the call and underlying move in opposite directions. For puts, a wrong sign is implied when the put and its underlying move in the same direction.

In the third essay, I investigate the performance of hedges using different measures of volatility. Hedging is a widely used strategy designed to minimize exposure to unwanted business risk such as changes in the market, currency values, interest rates, or commodity prices. The effectiveness of the hedge depends on the ability to create a hedging portfolio that offsets price changes in the held, or core, portfolio. With respect to options, this means that option price change should be offset by a change in a synthetic portfolio consisting of the underlying and a bond. To create a specific hedge, one has to make a decision on the option pricing model to be used and the method for estimating the model's parameters. In the third essay, I evaluate hedges when volatility is estimated by implied price changes, implied prices, historical volatility and when the hedge ratio is adjusted by regression coefficients.

The rest of the dissertation is organized as follows. Chapter two presents the first essay, chapter three the second essay and chapter four the third essay. Chapter five gives the summary and conclusions.

Chapter 2 Volatilities Implied by Price Changes in the S&P 500 Options and Futures Contracts

Thirty five years after the development of the Black-Scholes-Merton (BSM) model, implied volatility is still the most widely used parameter for option pricing on Wall Street. Traders resisted using more sophisticated models, such as stochastic volatility models (Heston (1993)), jump diffusion models (Merton (1976)) or GARCH models (Duan (1995)), Heston and Nandi (2000)) and rather continued to embrace a version of the Black-Scholes model referred to as the Practitioner Black-Scholes (PBS) model (Berkowitz (working paper)). Using this version of BSM, traders calculate implied volatilities from wide range of options on the underlying asset. Using these implied volatilities, they create a volatility surface that relates implied volatility to moneyness and maturity. Then they use the volatility surface to estimate implied volatility for desired moneyness and maturity to price or hedge a specific option on the underlying. Although very simple, this method gives surprisingly good results. Christoffersen and Jacobs (2004) found that if recalibrated with sufficient frequency, PBS outperforms more complex models, such as Heston's stochastic volatility model (1993). Berkowitz (working paper) gave a theoretical justification for this finding by showing that the PBS model is an approximation to an unknown but correct option pricing formula and with sufficient upgrade frequency, the PBS model gives asymptotically correct prices. In practice, traders re-estimate implied volatilities to create a new volatility surface at least daily.

Implied volatility is used not only to price options but also to calculate hedging ratios. Hedging efficiency depends on having portfolios with offsetting price changes. With respect to options, this means that option price change should be offset by a price change in the synthetic portfolio. The motivation of this paper is to examine the characteristics and behavior of volatility implied by observed option price changes, i.e. the implied price change volatility. It follows that implied price change volatility as an input to the PBS should produce more accurate hedges. The purpose of this study, therefore, is to introduce the concept of implied price change volatility and to explore its relationship to price level implied volatility (hereafter implied volatility) and historical volatility. I also examine the smile and maturity effect in implied price change volatility.

Data used for this study are the S&P 500 futures options and the underlying S&P 500 futures contracts. The findings indicate that implied price change volatility has time series behavior similar to that of the S&P 500 implied volatility and the moving average of S&P 500 historical volatility. However, the dispersion of implied price change volatility is higher than the dispersion of either of these more traditional measures. In other respects, implied price change volatility is similar to implied volatility. For example, there are differences in average magnitude between put and call options. The discrepancy between implied volatility calculated from calls and puts has been documented in the financial literature and has been attributed to the differences in demand curves between calls and puts (Bollen and Whaley (2004)). In addition, moneyness and maturity effects in price change volatility are similar to those found in implied volatility.

The contribution of the concept of implied price change volatility depends in large measure on its performance in hedging applications. The challenge in its implementation lies in finding accurate and meaningful estimates. Large datasets are required since consecutive and equally spaced observations on the same option contract and its underlying are necessary. Moreover, several data screens may be required since many observations do not provide useful information. For example, no information on price change volatility is produced if consecutive option transactions are at the same price. The need for large datasets and data screens can be minimized if an accurate empirical relationship between implied price change volatility and implied volatility can be established.

The paper is organized as follows. Section 2.1 discusses implied volatility. Section 2.2

introduces the model of implied price change volatility and Section 2.3 describes the data. Section 2.4 discusses the time series behavior of implied price change volatility and Section 2.5 examines moneyness and maturity considerations. Section 2.6 discusses some implications for hedging and Section 2.7 concludes the paper.

2.1 Implied Volatility

The implied volatility of an option contract is the volatility implied by the market price of the option based on an option pricing model, typically the BSM model. Implied volatility is calculated by solving the option pricing model for the volatility that sets the market price equal to the model price.

The concept of implied volatility was introduced by Latane and Rendleman (1976). They emphasized that the usefulness of the Black-Scholes model depends on the ability of the researcher to forecast the volatility of returns. They examined the performance of the weighted average implied volatilities in their empirical study. Their results indicate that the weighted average implied volatility is a better predictor of the volatility than the historical estimate. This study had a large impact on the broad use of implied volatilities. The better performance of implied volatility compared to historical volatility was also documented by Chiras and Manaster (1978).

MacBeth and Merville (1979) in their empirical study found that the implied volatility of the option is systematically related to the difference between the stock price and the exercise price and time-to-expiration. The non-constant relation frequently observed between the implied volatility of the option and strike price has been referred to as the volatility smile. Extensive research in this area was also done by Rubinstein (1985). One explanation for volatility smile is based on the more dispersed implied distribution of returns with heavier left tail than the lognormal distribution that is assumed in the BSM model.

Various other aspects of implied volatility have been studied. For example, Day and Lewis (1988) studied the behavior of implied volatility around the quarterly expiration. Stein (1989) examined the term structure of implied volatilities. He hypothesized that since the implied volatility is mean reverting, the change in implied volatility for long term options should be smaller than the corresponding change in volatility of short term options. However, he found that the changes in implied volatility of long term options are larger than expected. He concluded that this is a manifestation of overreaction and inefficiency of option markets.

Schwert (1990) studied the behavior of implied volatility around the stock crash of 1987. He found that the volatility dramatically increased during and after the crash. Then the volatility returned back to its normal levels. This effect has also been documented by Bates (2000) and Arnold, Hilliard, and Schwartz (2007).

Sheikh (1989) examined the behavior of implied volatility around stock splits and their announcements. He found that the implied volatility does not increase around the stock split announcements, but increases at the *ex*-date of the stock splits. Deng and Julio (working paper) compared the implied volatility of options written on splitting stocks that expire before or after the stock split *ex*-date. They found that following the announcement of a split, the implied volatility of options expiring after the split *ex*-date increases significantly relative to the implied volatility of options expiring before the split.

Market professionals use implied volatilities in short term applications since it is forward looking. Specifically, when the underlying process is generalized so that volatility is a deterministic function of time, the BSM implied volatility (σ_I) is the solution to

$$\sigma_I = \sqrt{\frac{1}{T}\int_0^T \sigma^2(s)ds}. \tag{2.1}$$

That is, implied volatility is the square root of the average value of the variance between today and option expiration. The view that implied volatility is superior to estimates based on historical data is not universal, however. Canina and Figlewski (1993) studied implied volatility of S&P 100 index options. They found that the implied volatility is an inefficient and biased forecast of future volatility. They concluded that implied volatility does not reflect all the information contained in historical volatility. Harvey and Whaley (1991) point out that many papers studying implied volatility generally assume that options are European and have a constant dividend yield. These approximations together with time gap between the observed price of the option and the underlying may lead to large errors in estimation of implied volatility and to misleading results. More recently, a number of papers have modeled realized volatility using the GARCH framework (Bollerslev (1986), Heston and Nandi (2000)) and intraday high frequency data. See, for example Andersen and Bollerslev (1998) and the associated econometric literature. While these models offer some improvements in pricing, they have not been shown to substantially improve the effectiveness of hedging models. Bakshi, Cao and Chen (1997) report that the only significant improvement of the stochastic volatility models over Black-Scholes is when hedging out-of-the money calls. In fact, for all classes of models considered by Bakshi, Cao and Chen, they conclude that "...the performances in most cases are virtually indistinguishable."

More complex option pricing models use volatility as an additional stochastic state variable. These models have been developed by Hull and White (1987)[1], Heston (1993)[2], Stein and Stein (1991) and a number of other researchers.

Implied volatility has high popularity among both the academics and professionals. The importance of this measure among investors can be illustrated by their wide reference to the VIX-CBOE, a volatility Index. This index is based on the implied volatilities of a wide range of 30 day S&P 500 index options. Due to the forward looking nature of implied volatilities, the VIX is frequently used as a quantitative measure of market risk or "fear".

1 Hull and White (1987) developed a stochastic volatility model and compared the pricing of the model with the Black-Scholes prices. They showed that when the stochastic volatility is uncorrelated with the stock price, the Black-Scholes model overprices at-the-money or close to-the-money options and underprices deep in- or deep out-of-money options.

2 Heston (1993) developed a stochastic volatility model when the stochastic volatility is correlated with the stock price.

2.2 Implied Price Change Volatility

In this paper I examine the behavior of implied price change volatility using Black's model.[3] By implied price change volatility I mean the implied volatility that is calculated by solving Black's model such that observed price *changes* and model price *changes* are equated. Note that, absent perfect model fit, this is not the same as the volatility given when observed price *levels* are equated to model prices.

Implied price change volatility has not, to the best of my knowledge, been studied in the financial literature. This topic is potentially important because of implications for dynamic hedging. That is, it is plausible to argue that hedges computed using price change volatilities will be superior to those computed using price level volatilities.

2.3 The Data

Data used for this study are the S&P 500 futures options, the underlying S&P 500 futures contract and Libor rates (a proxy for the risk free rate). The S&P 500 futures options and the underlying are pit traded on the Chicago Mercantile (CME). The observations are taken from January 1998 to December 2006 from the CME's Time and Sales database (tick data). Each option and its underlying futures are matched such that their trading has to occur within 30 seconds. To be a valid observation, the option price has to be at least $0.25.

The resulting dataset contains 76,544 observations on call options and 101,010 observations on put options (Tables 2.1 and 2.2). It means that put options account for 56.9% of trades in the S&P 500 futures options during this time period. This is consistent with Bollen and Whaley (2004) who documented that put options represent 55% trades in the S&P 500 index options. The trading of both call and put options and their futures was the highest in 1998 and then, because of the emergence of GLOBEX electronic trading, slowly declined during the following years. The average gap between the option trade and the underlying futures is 5 seconds. The dataset

[3] Note that Black's model is essentially the BSM model when the underlying is a futures contract.

Table 2.1: Data Description for Calls according to Years

Data used for this paper were American style options on S&P 500 futures traded on CME from January 1998 to December 2006. Symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where *F* is a price of the futures contract and *K* is a strike price of the option. Calls are in the money for $F/K > 1$.

						Dataset					
		All	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Number of Call Options		76544	15049	10876	9406	7663	9549	7686	5570	5766	4979
Number of Strike Prices		186	79	77	80	103	99	77	52	46	58
Average Difference between the Trade of Option and Future		5 seconds	4 seconds	4 seconds	5 seconds	5 seconds	4 seconds	5 seconds	6 seconds	6 seconds	6 seconds
	Range	1-238 days	1-156 days	1-238 days	1-150 days	1-167 days	1-238 days	1-162 days	1-120 days	1-160	1-149 days
	1-30 days	46901	10221	6958	5806	4726	4901	4500	3024	3576	3189
	31-60 days	20138	3242	2533	2261	1876	3377	2208	1855	1456	1330
	61-90 days	7647	1246	1110	1128	756	1088	764	565	626	364
Maturity	**91-120 days**	1801	323	264	207	298	175	209	126	105	94
	121-150 days	50	16	10	4	6	6	4	0	2	2
	151-180 days	5	1	0	0	1	1	1	0	1	0
	181-210 days	0	0	0	0	0	0	0	0	0	0
	211-240 days	2	0	1	0	0	1	0	0	0	0
	241-270 days	0	0	0	0	0	0	0	0	0	0
	X1	8720	1081	1347	1788	1269	1799	1192	167	56	21
	X2	31711	6257	4774	4277	3377	3807	2950	2429	1823	2017
Moneyness	**At the money**	34519	7394	4495	3221	2892	3564	3295	2889	3849	2920
	X3	1194	256	199	109	102	218	206	56	32	16
	X4	400	61	61	11	23	161	43	29	6	5

Table 2.2: Data Description for Puts according to Years

Data used for this paper were American style options on S&P 500 futures traded on CME from January 1998 to December 2006. Symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where *F* is a price of the futures contract and *K* is a strike price of the option. Puts are in the money for $F/K < 1$.

		Dataset									
		All	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Number of Put Options		101010	20495	14692	12365	10532	11422	9750	7588	6778	7388
Number of Strike Prices		183	92	98	92	98	99	88	67	62	77
Average Difference between the Trade of Option and Future		5 seconds	4 seconds	4 seconds	4 seconds	5 seconds	4 seconds	5 seconds	6 seconds	6 seconds	6 seconds
Maturity	**Range**	1-265 days	1-204 days	1-202 days	1-265 days	1-176 days	1-181 days	1-156 days	1-185 days	1-147 days	1-174 days
	1-30 days	57367	12429	8258	6845	5950	5762	5135	4097	4035	4856
	31-60 days	27801	4952	3918	3352	2619	4043	3034	2331	1800	1752
	61-90 days	12452	2193	2092	1786	1408	1317	1293	947	826	600
	91-120 days	3227	853	408	367	545	272	286	206	115	175
	121-150 days	126	58	22	9	5	20	1	5	2	4
	151-180 days	29	9	3	3	5	7	1	0	0	1
	181-210 days	6	1	1	1	0	1	0	2	0	0
	211-240 days	0	0	0	0	0	0	0	0	0	0
	241-270 days	2	0	0	2	0	0	0	0	0	0
Moneyness	X1	333	47	13	96	116	43	18	0	0	0
	X2	1358	285	119	198	316	254	69	58	16	43
	At the money	29562	5798	3496	3049	2941	3263	2872	2848	2742	2553
	X3	29368	5859	4027	3547	2678	2754	2804	2502	2314	2883
	X4	40389	8506	7037	5475	4481	5108	3987	2180	1706	1909

contains both short and long term options. However, short term options are markedly prevalent. The majority of options traded are out-of-the-money and at-the-money options. The risk-free rate is calculated from Libor rates based on the British Bankers Association Data. The rates are converted to continuously compounded yields. The Libor data are monthly with the shortest maturity overnight, one and two weeks. Daily Libor rates are obtained by interpolation.

Sequences of records on the same contract are required since I investigate price changes. Therefore, datasets containing records with consecutive observations for one, two, three and four day lags are created. For each strike price, the contract traded closest to 10:00 AM is selected[4]. From these contracts only those contracts that trade with one, two, three or four day lags are used in a particular dataset. It means that for the one day lag, a valid observation consists of a trade on the same contract on two consecutive days (Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday or Thursday and Friday). To prevent overlapping data for two day lags, only trades on the same contract on Monday and Wednesday or Wednesday and Friday are used. Similarly, for three and four day lags, the only trades considered are those on the same contract on Monday and Thursday and Monday and Friday, respectively.

2.3.1 Estimation of Implied Price Change Volatility

First, I reproduce the standard formula for completeness and to fix notation. Black's formula for an European call option on a futures contract with price $F \equiv F(t, T)$ is

$$C(F, t) = e^{-r(T-t)} \left(FN(d_1) - KN(d_2) \right), \tag{2.2}$$

where

$$d_1 = \frac{\ln(\frac{F}{K}) + 0.5\sigma^2(T-t)}{\sigma\sqrt{(T-t)}}, \tag{2.3}$$

and $d_2 = d_1 - 0.5\sigma\sqrt{(T-t)}$. The strike price is K , r is the risk-free rate, $T-t$ is the time-to-expiration and $N(x)$ is the standard cumulative normal evaluated at x.

[4] This is to ensure that day lags are close to 24, 48, 72 or 96 hours. The time 10:00 AM is chosen because it is generally a time of heaviest trading activity.

The price change ΔC of the option at time $t+h$ is

$$\Delta C = C(F_{t+h}, T-h) - C(F_t, T). \tag{2.4}$$

The left side of the Equation 2.4 is the observed change in the price of the option, the right side of the equation is calculated by the model. Because the S&P 500 futures options are American type options, I use the binomial tree for American options to calculate the model prices. For comparison, I also report results for European options using the standard Black's formula. The results are almost identical.[5]

The implied price change volatility is estimated using the bisection method. Observed price change of a call or put option is given by the left hand side of Equation (2.4) and model price change is calculated according to the right hand side. Initial guesses inclusive of the root are $\sigma = 0.025$ and $\sigma = 2$. Then the procedure is repeated until the difference between the model and observed price is less than 0.0001. The implied price change volatility is estimated for one, two, three and four day lags.

Some restrictions on options and its underlying futures are imposed. First the restriction of no wrong signs implies that I consider only observations such that $\Delta C \cdot \Delta F > 0$ for calls and $\Delta C \cdot \Delta F < 0$ for puts. This restriction follows findings of Bakshi, Cao and Chen (2000). They examined the S&P 500 options and found that prices of call (put) options often move in

[5] Alternatively, the right hand side of this equation can be computed using the standard Greeks; delta, gamma and theta computed at time t as

$$\Delta C_i \approx C_{F_i}\Delta F_i + C_{t_i}\Delta t_i + \frac{1}{2}C_{FF_i}\left(\Delta F_i\right)^2, \tag{2.5}$$

where ΔC_i is an infinitesimal price change implied by Black's model.
The Greeks are:

$$C_F = e^{-r(T-t)}N(d_1), \tag{2.6}$$

$$C_{FF} = \frac{n(d_1)e^{-rT}}{F\sigma\sqrt{T}}, \tag{2.7}$$

$$C_t = \frac{-Fn(d_1)\sigma e^{-rT}}{2\sqrt{T}} + rFN(d_1)e^{-rT} - rKe^{-rT}N(d_2), \tag{2.8}$$

where $n(x)$ is the standard normal density

$$n(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}, -\infty < x < \infty. \tag{2.9}$$

I find that the direct computation differs little from the approximation in estimating implied volatilities from daily price changes.

opposite (the same) direction with the price of the underlying. They report that call and futures prices move in opposite direction between 7.2% and 16.3% of the time according to the sampling interval. Second, I require that the absolute change in the price of the option cannot be larger than the change in the price of the futures. Third, I require that the absolute change in the price of the option is larger than $0.20. This is because very small option price changes lead to noisy estimation. Fourth, I use only options with time to expiration at least fourteen days to avoid expiration-related biases. This is a common (although *ad-hoc*) procedure to avoid short term maturity biases. Fifth, the time lag between the trading of consecutive daily observations must be between 23 and 25 hours for one day lags. Finally, observations with implied price change volatility larger than 0.7 are omitted.

2.3.2 Description of Time-Series Datasets

The description of the time series datasets for calls and puts is shown in Table 2.3. The dataset of calls with one day lags contained 9,562 observations. However, after applying the above stated filters, the number of observations decreased to 2274 (dataset Calls1). The maturity of calls in this dataset ranges from 14 to 106 days. The majority of options are out-of-the-money options. The amount of data for calls with higher lags decreases sharply; there are 252 observations for two day lags (dataset Calls2), 201 for three day lags (dataset Calls3) and 169 for four day lags (dataset Calls4). Almost all calls with higher lags are at-the-money or in-the-money options. There are only two observations for in-the-money calls in higher lags.

The dataset of puts with one day lag (dataset Puts1) contains 3061 observation. The maturity of put options ranges from 14 to 111 days. The most common are options with maturity up to three months. Similarly as for call options, the majority of put options are out-of-the-money or at-the-money options. Datasets with higher lags contain 410 observations for two day lags (dataset Puts2), 349 observations for three day lags (dataset Puts3) and 268 observations for four day lags (Puts4).

Table 2.3: Data Description for Calls and Puts with One, Two, Three and Four Day Lags

The data used for this study were American style options on S&P 500 futures traded on the CME from January 1998 to September 2006. Records selected according to the description in Section 2.3. Numbers denoting datasets refer to one, two, three and four days lags. The symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where F is a price of the futures contract and K is a strike price of the option.

		Dataset							
		Calls1	**Calls2**	**Calls3**	**Calls4**	**Puts1**	**Puts2**	**Puts3**	**Puts4**
Number of Call Options		2274	252	201	169	3061	410	349	268
Number of Strike Prices		126	95	86	75	119	97	98	88
	Range	14 - 106 Days	15 - 106 Days	14 – 98 Days	17 – 104 Days	14 – 111 Days	15 – 125 Days	14 – 98 Days	17 – 97 Days
Maturity	**1-30 days**	1143	64	41	32	1426	126	93	64
	31-60 days	803	115	90	84	1140	167	139	133
	61-90 days	285	68	60	51	445	107	97	67
	91-120 days	43	5	10	2	50	9	20	4
	121-150 days	0	0	0	0	0	1	0	0
	151-180 days	0	0	0	0	0	0	0	0
	181-210 days	0	0	0	0	0	0	0	0
	211-240 days	0	0	0	0	0	0	0	0
	241-270 days	0	0	0	0	0	0	0	0
Moneyness	X1	271	56	56	47	16	0	1	0
	X2	1213	123	92	89	34	11	6	2
	At the money	770	72	53	32	591	77	54	37
	X3	19	1	0	1	957	108	90	72
	X4	1	0	0	0	1463	214	198	157

2.4 Volatility Time Series

Initially, I compare the time series behavior of implied price change volatility with implied volatility and with historical S&P 500 index volatility based on a 60-day moving average. As shown in Figures 2.1 to 2.5, implied price change volatility, implied volatility and moving average volatility of the S&P 500 index show similar patterns over time. Note two obvious differences. First, implied price change volatility has higher dispersion than implied volatility (the standard deviation for implied volatility for calls is 0.054 while the standard deviation of implied price change volatility is 0.083 (Table 2.4) and 0.091 versus 0.134 for puts (Table 2.5). Although the S&P 500 index futures are heavily traded, the frequency of trading on contracts with different strike prices varies substantially. Therefore I examined whether this large dispersion of implied price change volatility is due to the effect of contracts with lower trading frequency. I did not find any support for this effect. Second, the dispersion of both implied volatility and implied price change volatility is larger for puts than calls.

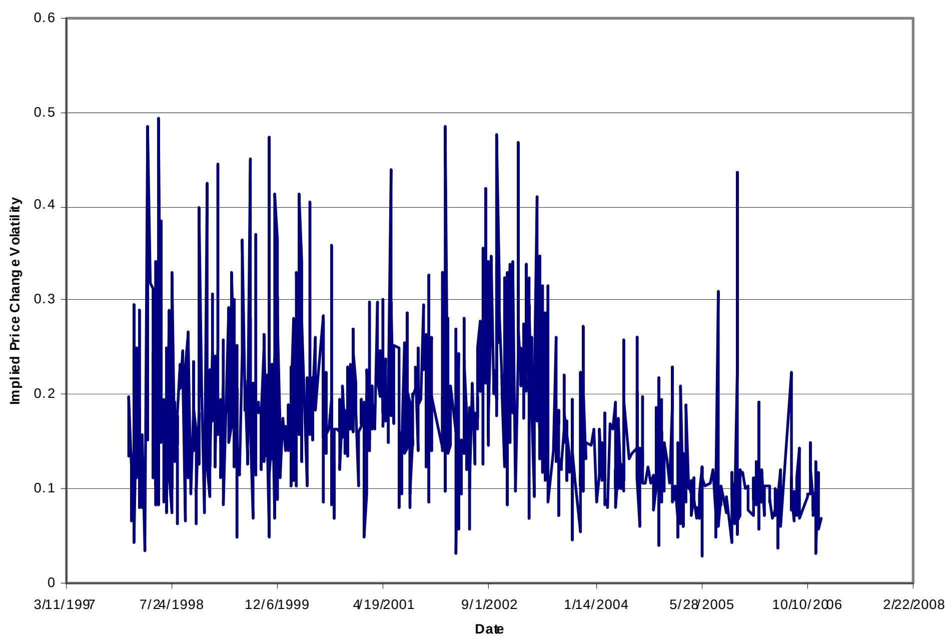

Figure 2.1: Daily Average of Implied Price Change Volatility

The period of investigation (from January 1, 1998 to December 31, 2006) is divided to three

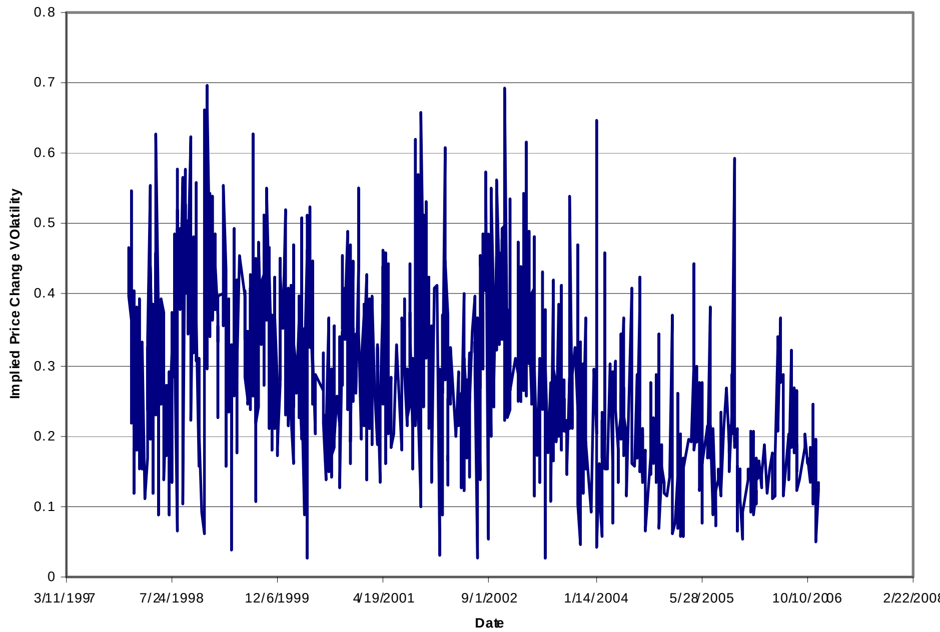

Figure 2.2: Daily Average of Implied Price Change Volatility for Puts

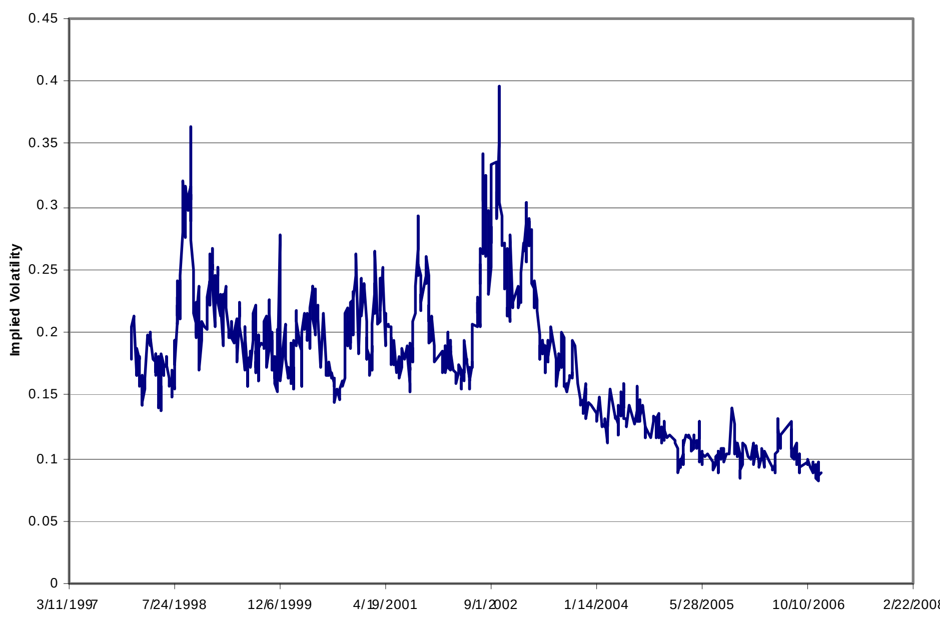

Figure 2.3: Daily Average of Implied Volatility for Calls

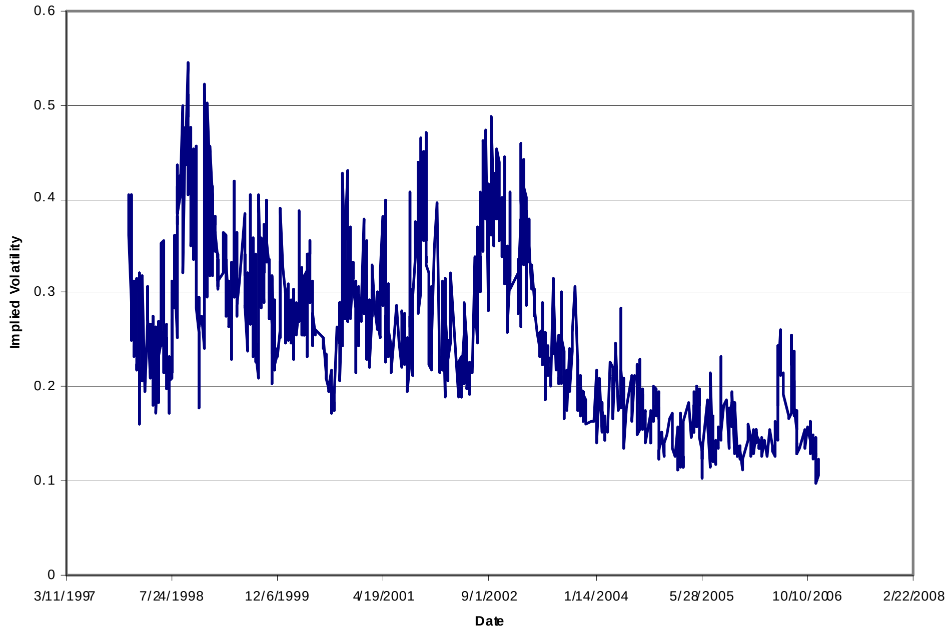

Figure 2.4: Daily Average of Implied Volatility for Puts

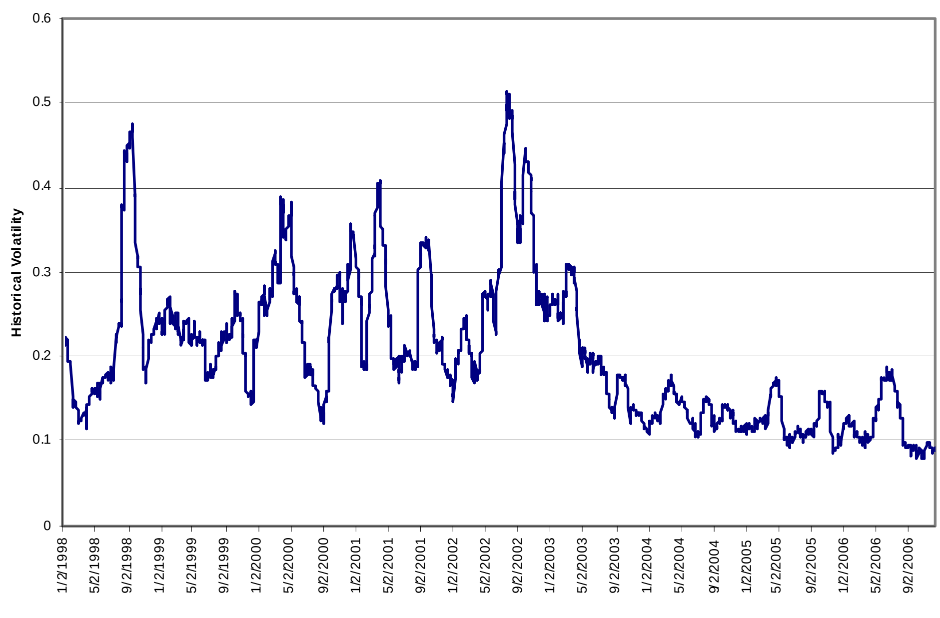

Figure 2.5: Moving Average of S and P 500 Index Volatility

subperiods according to the behavior of S&P 500 index (Figure 2.6). The first subperiod is a period of increasing value of the index (from January 1, 1998 to August 31, 2000). During the second subperiod (from September 1, 2000 to March 6, 2003), the value of the index was decreasing and during the third subperiod (from March 7, 2003 to December 31, 2006), the value of the index was increasing again.

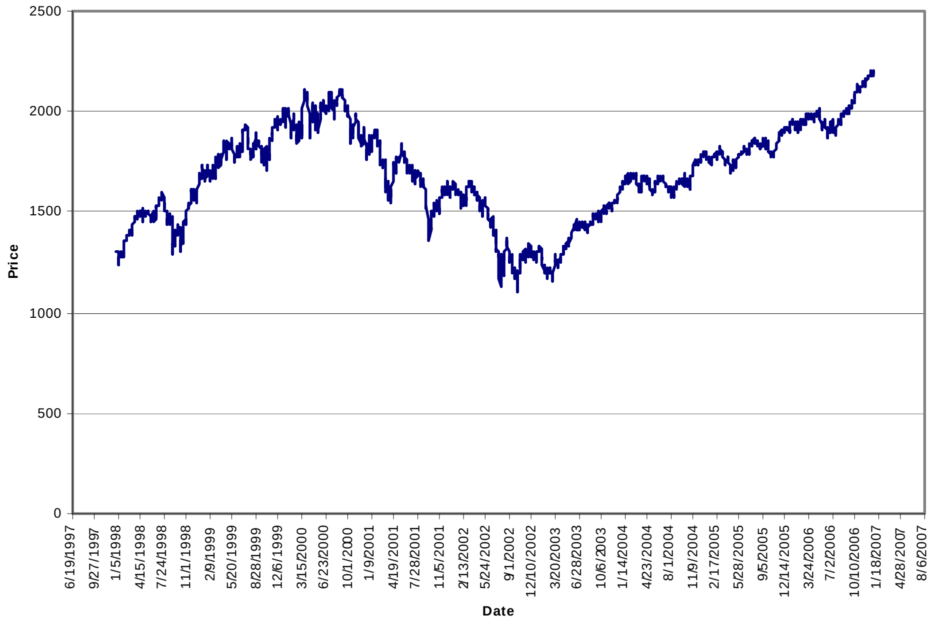

Figure 2.6: The Value of S and P 500 Index from January 1, 1998 to December 31, 2006.

Tables 2.4 and 2.5 summarize basic statistics for implied price change volatilities and implied volatilities for calls and puts during the periods studied. Several observations can be made. First, both implied price change volatility and implied volatility are larger for puts than calls. Under the assumptions of the model, volatilities calculated from prices of call and put options should be the same. Even volatility means are different, however. The average implied volatility for call options is lower at 0.1837 while average implied volatility for put options is 0.2632. Calculation of the normalized difference in sample means gives an ad-hoc t-score of 39.9[6]. This trend is consistent

[6] The t-score is calculated as follows:

$$t = \frac{0.2632 - 0.1837}{\sqrt{\frac{0.0906^2}{3061} + \frac{0.0543^2}{2274}}}$$

Since t-assumptions are not met, the measure should be consider ad-hoc.

Table 2.4: Price Change Implied Volatility and Implied Volatility during Different Time Periods for Calls

The table shows statistics of implied volatility and implied price change volatility for calls with one day lag during different time periods. These periods are based on behavior of S&P 500 index as described in Section 2.4. Data used for this study were American style options on S&P 500 futures traded on CME from January 1998 to September 2006.

Period	Volatility	Mean	Std. Deviation	Minimum	Maximum
All	Implied Price Change Volatility for American Option-1 day lags	0.1717	0.0830	0.0198	0.5442
January 1, 1998 to December 31, 2006	Implied Price Change Volatility for European Option-1 day lags	0.1719	0.0831	0.0198	0.5327
	Implied Price Change Volatility - 2 day lags	0.1709	0.0955	0.0261	0.6309
	Implied Price Change Volatility - 3 day lags	0.1755	0.1029	0.0408	0.6826
	Implied Price Change Volatility - 4 day lags	0.1854	0.0985	0.0526	0.6362
	Implied Volatility	0.1837	0.0543	0.0788	0.3986
Period 1	Implied Price Change Volatility for American Option-1 day lags	0.1829	0.0805	0.0256	0.5236
January1, 1998 to August 31, 2000	Implied Price Change Volatility for European Option-1 day lags	0.1832	0.0809	0.0256	0.5212
	Implied Volatility	0.1972	0.0370	0.1266	0.3986
Period 2	Implied Price Change Volatility for American Option-1 day lags	0.2016	0.0786	0.0291	0.5442
September 1, 2000 to March 6, 2003	Implied Price Change Volatility for European Option-1 day lags	0.2018	0.0785	0.0291	0.5327
	Implied Volatility	0.2201	0.0467	0.1454	0.3947
Period 3	Implied Price Change Volatility for American Option-1 day lags	0.1250	0.0701	0.0198	0.4973
March 7, 2003 to December 31, 2006	Implied Price Change Volatility for European Option-1 day lags	0.1250	0.0699	0.0198	0.4912
	Implied Volatility	0.1270	0.0326	0.0788	0.2417
High Volatility Period	Implied Price Change Volatility for American Option-1 day lags	0.1902	0.0803	0.0256	0.5442
January 1, 1998 to August 31, 2003	Implied Price Change Volatility for European Option-1 day lags	0.1905	0.0805	0.0256	0.5327
	Implied Volatility	0.2056	0.0423	0.1266	0.3986
Low Volatility Period	Implied Price Change Volatility for American Option-1 day lags	0.1140	0.0620	0.0198	0.4973
September 1, 2003 to December 31, 2006	Implied Price Change Volatility for European Option-1 day lags	0.1140	0.0618	0.0198	0.4912
	Implied Volatility	0.1155	0.0210	0.0788	0.1938

Table 2.5: Price Change Implied Volatility and Implied Volatility during Different Time Periods for Puts

The table shows statistics of implied volatility and implied price change volatility for puts with one day lag during different time periods. These periods are based on behavior of S&P 500 index as described in Section 2.4. Data used for this study were American style options on S&P 500 futures traded on CME from January 1998 to September 2006.

Period	Volatility	Mean	Std. Deviation	Minimum	Maximum
All	Implied Price Change Volatility for American Option-1 day lags	0.2860	0.1341	0.0250	0.6975
January 1, 1998 to December 31, 2006	Implied Price Change Volatility for European Option-1 day lags	0.2862	0.1342	0.0259	0.7056
	Implied Price Change Volatility - 2 day lags	0.2772	0.1338	0.0257	0.6915
	Implied Price Change Volatility - 3 day lags	0.2816	0.1342	0.0349	0.6901
	Implied Price Change Volatility - 4 day lags	0.2790	0.1154	0.0512	0.6867
	Implied Volatility	0.2632	0.0906	0.0929	0.6283
Period 1	Implied Price Change Volatility for American Option-1 day lags	0.3243	0.1348	0.0271	0.6964
January1, 1998 to August 31, 2000	Implied Price Change Volatility for European Option-1 day lags	0.3247	0.1351	0.0278	0.7056
	Implied Volatility	0.2953	0.0807	0.1605	0.6283
Period 2	Implied Price Change Volatility for American Option-1 day lags	0.3188	0.1286	0.0250	0.6975
September 1, 2000 to March 6, 2003	Implied Price Change Volatility for European Option-1 day lags	0.3189	0.1286	0.0259	0.6945
	Implied Volatility	0.3054	0.0778	0.1640	0.6176
Period 3	Implied Price Change Volatility for American Option-1 day lags	0.2010	0.0937	0.0262	0.6583
March 7, 2003 to December 31, 2006	Implied Price Change Volatility for European Option-1 day lags	0.2011	0.0937	0.0262	0.6589
	Implied Volatility	0.1785	0.0484	0.0929	0.3669
High Volatility Period	Implied Price Change Volatility for American Option-1 day lags	0.3167	0.1313	0.0250	0.6975
January 1, 1998 to August 31, 2003	Implied Price Change Volatility for European Option-1 day lags	0.3169	0.1314	0.0259	0.7056
	Implied Volatility	0.2951	0.0793	0.1605	0.6283
Low Volatility Period	Implied Price Change Volatility for American Option-1 day lags	0.1937	0.0944	0.0285	0.6583
September 1, 2003 to December 31, 2006	Implied Price Change Volatility for European Option-1 day lags	0.1938	0.0945	0.0281	0.6589
	Implied Volatility	0.1675	0.0420	0.0929	0.3669

without regard to the behavior of the S&P 500 Index, i.e., over all periods. Similar differences have been previously documented in the financial literature by, for example, Bollen and Whaley (2004). The explanation of this inequality between implied volatilities calculated from call and put options is based on different demand curves for calls and puts. Puts are largely demanded by institutional investors for insurance purposes (especially after the crash of October 1987 (Fleming (1999), Rubinstein (1994)). This demand may bid up prices [7]. Second, implied price change volatilities are very stable across different lags. For example, implied price change volatility for calls with one lags is 0.1717, for two day lags 0.1719, three day lags 0.1709 and four day lags 0.1755. Third, the difference between implied volatility and implied price change volatility tends to be larger during a downturn in the market for calls and *vice versa* for puts.

The differences between implied volatilities and implied price change volatilities may have hedging implications since Greek deltas using price level implied volatilities differ from Greek deltas based on price change implied volatilities. For example, call hedging deltas, $\frac{\partial C}{\partial F}$, increase with σ

$$\frac{\partial^2 C}{\partial\sigma\partial F} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{\left(\ln\frac{F}{X} + \frac{\sigma^2 T}{2}\right)^2}{\sigma^2 T}\right)\left(\frac{\frac{\sigma^2 T}{2} - \ln(\frac{F}{X})}{\sigma^2\sqrt{T}}\right) \tag{2.10}$$

is positive for $\frac{\sigma^2 T}{2} - \ln(\frac{F}{X}) > 0$ and this includes all cases when $F \leq X$. In the standard application, a larger σ for these options means that a hedge with a synthetic call would need more shares of the underlying asset. The converse is true when $\frac{\sigma^2 T}{2} - \ln(\frac{F}{X}) < 0$, e.g., for calls that are deep-in-the money or $F >> X$.

2.5 Money and Maturity Considerations

To investigate whether implied price change volatility shows moneyness and maturity behavior

[7] This can be the result of market imperfections such as transaction costs, the inability of market makers to fully hedge their positions at all times (Garleanu et. al. (2006)), capital requirements, and sensitivity to risk (Shleifer and Vishny (1997)).

similar to that of implied volatility, I estimate the equation

$$\sigma = a + b(T-t) + c(T-t)^2 + d(\frac{F}{K}) + e\left(\frac{F}{K}\right)^2 + f(\frac{F}{K})(T-t) + \epsilon, \qquad (2.11)$$

using implied price change volatility and implied volatility data. The equation is estimated first for the period from January1, 1998 to December 31, 2006, then separately for three subperiods according to the behavior of S&P 500 index (Figure 2.6). The regression coefficients are reported in Tables 2.6 and 2.7 (implied price change volatility for calls), Tables 2.8 and 2.9 (implied price change volatility for puts) and Table 2.10 and 2.11 (implied volatility for calls and puts). Standard errors are adjusted using the White estimator.

Moneyness and maturity effects in call options are strongly significant at the 99% level for implied volatility for all periods studied. More specifically, note in Table 2.10 that for all periods, the coefficients of moneyness are significant and negative while the coefficients of moneyness squared are significant and consistently positive. Moneyness and maturity effects on implied volatility for puts are more ambiguous. The maturity effect is most notable and is consistently positive and significant for all subperiods. Implied price change volatility for calls also shows a significant dependence on moneyness but does not consistently and significantly depend on maturity. Implied price change volatility for puts shows consistently significant dependence on maturity through all periods but the moneyness effect is weaker. Implied price change volatility yields lower R^2s than implied volatility in these regressions for both calls and puts.

The regression coefficients from equation (2.11) are then used to create surface plots of implied price change volatility as a function of moneyness and maturity (Figures 2.7 and 2.8). As can be seen from Figure 2.7, the implied price change volatility for calls appears to show a smile, while this effect is not noticeable for puts (Figure 2.8).

Table 2.6: Regression Coefficients for Implied Price Change Volatility for Calls with One Day Lags

The equation $\sigma = a + b(T\text{-}t) + c(T\text{-}t)^2 + d((F/K)) + e((F/K))^2 + f((F/K))(T\text{-}t) + \varepsilon$ is estimated using implied price change volatility *(σ)* for calls with one day lags. The *(F/K)* represents moneyness, (*T-t)* time to maturity. For details, see section 2.5.
*** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

Period	**Dates**	**Number of Records**	***a***	***b***	***c***	***d***	***e***	***f***	**Adj.** R^2
All	*January 1, 1998 to December 31, 2006*	2274	12.61***	-5.42***	1.41**	-25.09***	12.65***	5.19***	0.0506
Period 1	*January1, 1998 to August 31, 2000*	910	8349***	-1.55	0.44	-17.41***	9.11***	1.52	0.0236
Period 2	*September 1, 2000 to March 6, 2003*	699	8.04***	-2.25	0.88	-15.96***	8.14***	1.92	0.0366
Period 3	*March 7, 2003 to December 31, 2006*	665	25.12***	-2.63	-1.43**	-51.53***	26.51***	3.20	0.0631
High Volatility Period	*January 1, 1998 to August 31, 2003*	1722	8.66***	-2.59**	0.90	-17.39***	8.93***	2.39*	0.0250
Low Volatility Period	*September 1, 2003 to December 31, 2006*	552	40.84***	-5.66**	-0.88	-83.77***	43.04***	6.14**	0.0644

Table 2.7: Regression Coefficients for Implied Price Change Volatility for Calls with Two, Three and Four Day Lags

The equation $\sigma = a + b(T\text{-}t) + c(T\text{-}t)^2 + d((F/K)) + e((F/K))^2 + f((F/K))(T\text{-}t) + \varepsilon$ is estimated using implied price change volatility *(σ)* for calls with two (Calls2), three (Calls3) and four (Calls4) day lags. The (*F/K)* represents moneyness, (*T-t)* time to maturity. For details, please see section 2.5.

*** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

Dataset	**Dates**	**Number of Records**	**a**	**b**	**c**	**d**	**e**	**f**	**Adj. R^2**
Calls2	January 1, 1998 to December 31, 2006	252	12.94***	-6.53	2.14	-26.28***	13.53***	6.34	0.0554
Calls3	January 1, 1998 to December 31, 2006	201	7.89**	-7.82**	1.55	-15.06*	7.30*	7.91**	0.0529
Calls4	January 1, 1998 to December 31, 2006	169	9.29**	-4.02	0.16	-19.43**	10.34**	4.31	0.0751

Table 2.8: Regression Coefficients for Implied Price Change Volatility for Puts with One Day Lags

The equation $\sigma = a + b(T\text{-}t) + c(T\text{-}t)^2 + d((F/K)) + e((F/K))^2 + f((F/K))(T\text{-}t) + \varepsilon$ is estimated using implied price change volatility *(σ)* for puts with one day lags. The (*F/K)* represents moneyness, (*T-t)* time to maturity. For details, see section 2.5.
*** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

Period	Dates	Number of Records	*a*	*b*	*c*	*d*	*e*	*f*	Adj. R^2
All	*January 1, 1998 to December 31, 2006*	3061	-0.39	3.68***	1.20*	0.11	0.50**	-3.78***	0.2867
Period 1	*January1, 1998 to August 31, 2000*	1310	-1.21***	1.57***	0.60	1.82***	-0.35**	-1.85***	0.2973
Period 2	*September 1, 2000 to March 6, 2003*	837	0.56	2.87***	-0.29	-1.30*	0.98***	-2.67***	0.2471
Period 3	*March 7, 2003 to December 31, 2006*	914	3.01***	6.46***	0.86	-6.50***	3.62***	-6.14***	0.1554
High Volatility Period	*January 1, 1998 to August 31, 2003*	2297	-0.33	2.83***	0.77	0.21	0.37**	-2.94***	0.2721
Low Volatility Period	*September 1, 2003 to December 31, 2006*	764	4.69***	8.49***	0.85	-9.77***	5.21***	-8.06***	0.1219

Table 2.9: Regression Coefficients for Implied Price Change Volatility for Puts with Two, Three and Four Day Lags

The equation $\sigma = a + b(T\text{-}t) + c(T\text{-}t)^2 + d((F/K)) + e((F/K))^2 + f((F/K))(T\text{-}t) + \varepsilon$ is estimated using implied price change volatility *(σ)* for puts with two (Puts2), three (Puts3) and four (Puts4) day lags. The (*F/K)* represents moneyness, (*T-t)* time to maturity. For details, please see section 2.5.

*** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

Dataset	**Dates**	**Number of Records**	**a**	**b**	**c**	**d**	**e**	**f**	**Adj. R^2**
Puts2	January 1, 1998 to December 31, 2006	410	-0.52*	4.03***	1.61	0.32	0.38*	-4.03***	0.2795
Puts3	January 1, 1998 to December 31, 2006	349	-0.97***	3.05***	3.13	1.16***	0.01	-3.64***	0.3984
Puts4	January 1, 1998 to December 31, 2006	268	-0.75**	1.79**	2.42	0.97*	0.008	-2.40***	0.3917

Table 2.10: Regression Coefficients for Implied Volatility for Calls with One Day Lags

The equation $\sigma = a + b(T\text{-}t) + c(T\text{-}t)^2 + d((F/K)) + e((F/K))^2 + f((F/K))(T\text{-}t) + \varepsilon$ is estimated using implied volatility *(σ)* for calls with one day lags. The (*F/K)* represents moneyness, (*T-t)* time to maturity. For details, see section 2.5.
*** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

Period	Dates	Number of Records	a	b	c	d	e	f	Adj. R^2
All	*January 1, 1998 to December 31, 2006*	2274	11.96***	-8.36***	2.65***	-23.26***	11.49***	7.95***	0.1327
Period 1	*January1, 1998 to August 31, 2000*	910	6.13***	-3.87***	1.58***	-12.10***	6.18***	3.61***	0.0664
Period 2	*September 1, 2000 to March 6, 2003*	699	8.63***	-5.46***	1.73***	-16.92***	8.51***	5.21***	0.0936
Period 3	*March 7, 2003 to December 31, 2006*	665	22.61***	-5.28***	-0.06	-4573***	23.23***	5.56***	0.2254
High Volatility Period	*January 1, 1998 to August 31, 2003*	1722	8.24***	-5.24***	2.02***	-16.17***	8.15***	4.90***	0.0871
Low Volatility Period	*September 1, 2003 to December 31, 2006*	552	16.39***	-4.90***	0.43	-32.93***	16.64***	4.99***	0.0923

Table 2.11: Regression Coefficients for Implied Volatility for Puts with One Day Lags

The equation $\sigma = a + b(T\text{-}t) + c(T\text{-}t)^2 + d((F/K)) + e((F/K))^2 + f((F/K))(T\text{-}t) + \varepsilon$ is estimated using implied volatility *(σ)* for puts with one day lags. The *(F/K)* represents moneyness, *(T-t)* time to maturity. For details, see section 2.5.
*** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

Period	Dates	Number of Records	a	b	c	d	e	f	Adj. R^2
All	January 1, 1998 to December 31, 2006	3061	-0.26	4.03***	3.14***	-0.15	0.63***	-4.54***	0.5434
Period 1	January1, 1998 to August 31, 2000	1310	-0.87***	2.73***	2.89***	1.11***	0.003	-3.35***	*0.7014*
Period 2	September 1, 2000 to March 6, 2003	837	0.74**	2.94***	1.61***	-1.57***	1.09***	-3.16***	0.4928
Period 3	March 7, 2003 to December 31, 2006	914	-0.47	3.48***	2.57***	0.036	0.57**	-3.92***	0.6512
High Volatility Period	January 1, 1998 to August 31, 2003	2297	-0.07	3.51***	2.71***	-0.28	0.60***	-3.95***	*0.5880*
Low Volatility Period	September 1, 2003 to December 31, 2006	764	0.45	4.62***	1.84***	-1.76***	1.45***	-4.87***	0.7052

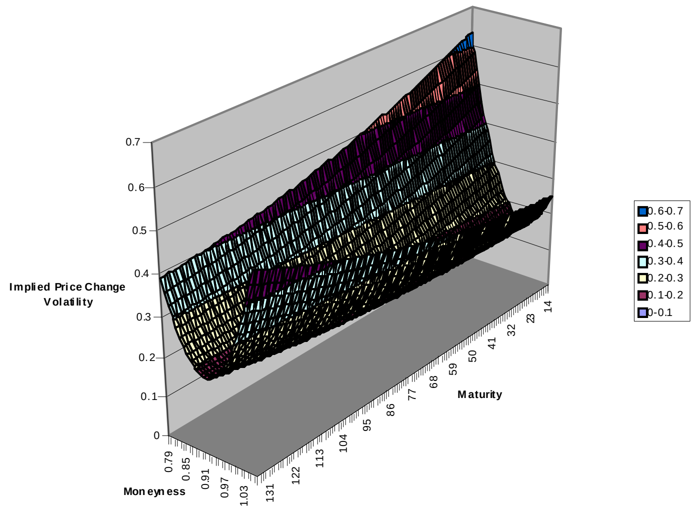

Figure 2.7: Plot of Implied Price Change Volatility versus Maturity and Moneyness for Calls during the Period from January 1998 to December 2006.

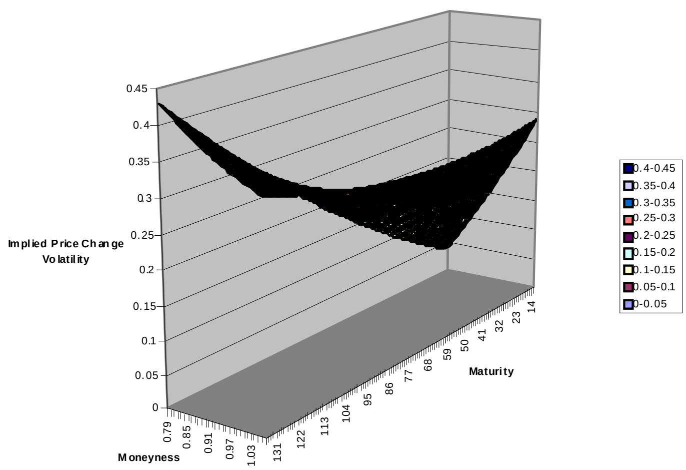

Figure 2.8: Plot of Implied Price Change Volatility versus Maturity and Moneyness for Puts during the Period from January 1998 to December 2006.

2.6 Implications of Differences between Implied Price Change Volatility and Implied Volatility for Hedging

The primary motivation for examining implied price change volatility is to determine if it has some merit over implied volatility in context of hedging. The intuition for the use of implied price change volatility is that it will capture the information implied by price changes rather than price levels in the option and its underlying. On the other hand, a problem with implied price change volatility is that it requires data on consecutive observations on the same contract. In addition, reliable estimates require larger datasets and careful screening of the data. To address this problem, I examine the relation between implied price change volatility(σ_{pc}) and implied volatility (σ_{iv}) using the following equations.

$$\sigma_{pc} = a + b\sigma_{iv} + \varepsilon, \tag{2.12}$$

$$\sigma_{pc} = a + b\sigma_{iv} + c\sigma_{iv}^2 + \varepsilon, \tag{2.13}$$

$$\begin{aligned}\sigma_{pc} &= a + b\sigma_{iv} + c(T-t) + d(T-t)^2 + e(F/K) + \\ &\quad +e(F/K)^2 + g(F/K)(T-t) + \varepsilon,\end{aligned} \tag{2.14}$$

where σ_{pc} is implied price change volatility and σ_{iv} is implied volatility. The coefficients are reported in Table 2.12. The coefficients on implied volatility are strongly significant for both calls and puts. The simpler linear model in Equation (2.12) predicts as well as other two models and is given by

$$\text{Calls} : \hat{\sigma}_{pc} = 0.04495 + 0.68995\hat{\sigma}_{iv} \tag{2.15}$$

for calls while the linear model for puts

$$\text{Puts} : \hat{\sigma}_{pc} = 0.01415 + 1.03284\hat{\sigma}_{iv}. \tag{2.16}$$

Thus, estimates of implied price change volatility for calls (puts) will be smaller (larger) than implied volatility for calls (puts). The relation between the implied price change volatility and implied volatility can be used to create a price change volatility surface similarly as in PBS and used to create hedge ratios. This possibility is investigated in the third essay.

Price change volatility and implied volatility are empirically different. This is important because of hedging implications. I display how errors in the measurement of volatility translate into errors in the hedge ratio in Figure 2.9. Small errors in measurement of volatility can translate to large errors in the hedge ratio, mainly for out-of-the-money and in-the-money call and put options. This effect can be seen by examining $\frac{\partial^2 C}{\partial\sigma\partial F}$in Equation (2.10).

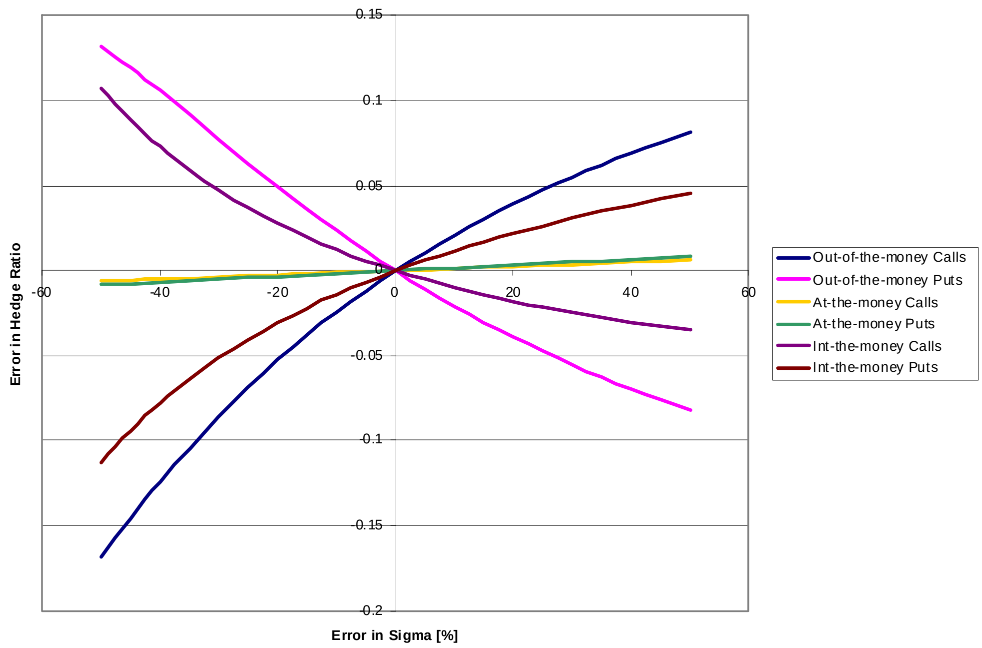

Figure 2.9: Effect of Error in Measurement of Volatility on the Hedge Ratio for Call and Put Options with 30 Days to Maturity

Table 2.12: Relation of Implied Price Change Volatility to Implied Volatility, Maturity and Moneyness for Calls and Puts with One Day Lags

This table shows regression coefficients for following equations: (1) $\sigma_{pc} = a + b\sigma_{iv} + \varepsilon$, (2) $\sigma_{pc} = a + b\sigma_{iv} + c\sigma_{iv}^{\ 2} + \varepsilon$ and (3) $\sigma_{pc} = a + b\sigma_{iv} + c(T\text{-}t) + d(T\text{-}t)^2 + e((F/K)) + f((F/K))^2 + g((F/K))(T\text{-}t) + \varepsilon$, where σ_{pc} represents implied price change volatility and implied volatility, σ_{iv} implied volatility, the (*F/K*) Moneyness and (*T-t*) time to maturity. For details, please see section 2.6. *** Refers to the significance at 99% level, ** to significance at 95% level and * to significance at 90% level.

	Regression	Number of Records	a	b	c	d	e	f	g	Adj. R^2
Calls	**1**	2274	0.04495***	0.68995***						0.2035
	2	2274	-0.01324	1.33903***	-1.66340***					0.2106
	3	2274	4.75202***	0.65734***	0.07192	-0.33019	-9.79925***	5.10185***	-0.03299	0.2107
Puts	**1**	3061	0.01415***	1.03284***						0.4869
	2	3061	0.04962***	0.76122***	0.46481***					0.4882
	3	3061	-0.13311	0.98864***	-0.30349	-1.90497***	0.25513	-0.12018	0.71307**	0.4903

2.7 Conclusion

This study introduces the concept of implied price change volatility, suggesting several areas of investigation. First, why is this an important concept? Second, how does the time series behavior of this measure compare with that of implied volatility and historical volatility. And finally, does implied price change volatility exhibit the same kind of moneyness and maturity biases as those found in implied volatility?

The importance of the concept arises primarily from it application to hedging issues. Hedging effectiveness depends on having portfolios with offsetting price changes. With respect to options, this means that option price change should be offset by a price change in the synthetic portfolio. Therefore, I develop the concept of volatility implied by price changes. In local time, implied price change volatility should exactly replicate changes in option price.

Implied price change volatility shows time series behavior similar to that of implied volatility and the moving average of historical volatility. The high and low volatility periods coincide for all volatility measures. However, the implied price change volatility is more disperse than either of these measures. Similarly as implied volatilities, the implied price change volatilities estimated from calls are smaller than those estimated from puts. It is consistent with findings of Bollen and Whaley (2004) who attribute this phenomena to different demand curves for calls and puts. This difference, however, is more pronounced for implied price change volatility than for implied volatility. The implied price change volatilities are smaller (larger) than implied volatilities for calls (puts) for all subperiods. The implied volatilities estimated from one, two, three and four day changes are very similar. It suggests that the implied price change volatilities are stable across different lags. Money and maturity biases are also consistent with those found in implied volatilities, although the maturity bias is not so pronounced and calls exhibit more bias than puts.

Finally, estimates of implied price change volatility can be obtained from implied volatility, making it easy to convert a PBS surface of implied volatilities to a PBS surface in price change volatility. This simplifies the use of implied volatilities in hedging applications.

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Chapter 3 Price Changes in the S&P 500 Options and Futures Contracts: A Regression Analysis

3.1 Introduction

In recent years, capital markets exhibited increasing use of derivative instruments. The important factors allowing for this growth in derivative markets have been both hedging requirements of investors as well as advances in the development of valuation models. The original option pricing formula was developed by Fisher Black and Myron Scholes in 1973. Since then new models that allow for additional variables, such as stochastic volatility (Hull and White (1987), Heston (1993), Stein and Stein (1991)), jumps (Merton (1976), Bates (1996)) and discrete time GARCH models (Duan (1995), Heston and Nandi (2000)) were developed. The empirical research testing the ability of these models to correctly price derivatives is extensive. This includes the work, for example, of MacBeth and Merville (1979), Rubinstein (1985), Shastri and Tandon (1986), and Whaley (1986). However, most of the papers tested models using the absolute error or absolute percentage error as the loss function.

The use of price *changes* in evaluation of option pricing models is advantageous for several reasons. First, it mitigates some statistical problems such as heteroskedasticity and autocorrelation of errors that unavoidably burden evaluation of price level (hereafter, price) models. For example, if errors in option price behave like a random walk, price change errors should be independent. In addition, price change models may be also less sensitive to omitted variables, especially when pricing gradients on missing state variables are relatively flat. Further, testing price changes has direct implications for delta hedging. The local price change of the option in response to a change in the underlying is captured by the first partial ("delta") of the option with respect to underlying.

An accurate delta-hedge requires an accurate model for computing delta.

The Black Scholes model and its American counterpart, the binomial model, continues to be the widely used among practitioners. Specifically, to price options on the underlying asset, traders estimate volatilities on wide range of options with different maturities and moneyness and create so called "volatility surface" relating implied volatility to maturity and moneyness. Then, this "volatility surface" is used to price options of desired maturity and moneyness on the same underlying. This is called a Practitioner Black-Scholes Model (PBS). Christoffersen and Jacobs (2004) found that when PBS is frequently recalibrated, it outperforms more complex models such as Heston's stochastic volatility model (1993).

In this study, I examine price changes of the S&P 500 futures options using the American version of the binomial model. I investigate how different measures of volatility influence the performance of the model. In addition to traditional volatility measures such as implied volatility and historical volatility, I use a newly proposed volatility measure that can be referred to as "implied price change volatility." The concept of the implied price change volatility is developed in the Essay 1. The implied price change volatility is the parameter that equates the observed price changes to the model price changes. It is a volatility measure analogous to implied volatility but where the focus is on price changes instead of price levels.

In this essay, I examine the American version of binomial model using a volatility parameter determined by 1) implied price change volatility, 2) implied volatility and 3) the volatility of S&P 500 index. The price change model is tested using parameters estimated out of sample. That is, I estimate parameters in the first period and then use these estimated parameters for the regressions in the second period. For data, I use options on the S&P 500 futures and their underlying. The futures contracts are used because they are directly traded, highly liquid and do not suffer from staleness and more complex arbitrage considerations as does the basket of stocks underlying the S&P 500 spot contract. In addition, the futures contracts reflect the market's assessment of future dividends.

I find high R^2 when I regress observations on the Black's model changes. One reason the R^2 are not even higher is due to the presence of "wrong" signs. Bakshi, Cao and Chen (2000) in their empirical study of price changes in S&P 500 index options found that prices of call (put) options frequently move in the opposite (the same) direction as the prices of underlying. That is, the signs are "wrong." These apparent violations can be due to various reasons such as to stochastic volatility, omitted variables or the segmentation of the market for derivatives and their underlying. In addition, they may be caused by nonsynchronious trading or staleness of the data. Since my dataset is very extensive and S&P 500 futures options are traded very frequently, I examine whether similar violations can be found also in the S&P 500 futures contracts.

The regression setup on this study closely follows a working paper of Hilliard, Hilliard and Schwartz. However, it contains a far larger dataset (January 1998 to December 2006) and use an additional test based on the implied price change volatility developed in Essay 1. This chapter is organized as follows: Section 3.2 develops the regression approach for the price change model while section 3.3 provides a description of data. The regression results are summarized in section 3.4. Section 3.5 compares sign violations in the S&P 500 futures options with the findings of Bakshi, Cao and Chen (2000) in the S&P 500 options and section 3.6 concludes.

3.2 The Regression Approach

Option prices are assumed to be driven by a single factor and the usual no-arbitrage assumptions. The pricing equation is expanded by Ito's formula to give a regression setup where price changes are expressed as function of the underlying and time. The regression is used to test the null that Black's model is an unbiased estimator of price change and to quantify adjustments to the standard hedging ratios. The adjusted hedge ratios are tested in Essay 3.

The Ito expansion for the local change in derivative price, say H, is written

$$dH = H_F dF + H_t dt + \frac{1}{2} H_{FF} dF^2, \tag{3.1}$$

where F is Geometric Brownian motion, $H_F \equiv \frac{\partial H}{\partial F}$, $H_{FF} \equiv \frac{\partial^2 H}{\partial F^2}$ and $H_t \equiv \frac{\partial H}{\partial t}$. For infinitesimal changes in dF, the second order terms are of order dt and are non-random.

For small but non-local changes in F, the expression is approximate in the sense of a Taylor's series expansion. However, there are omitted terms that are $O(\Delta^{\frac{3}{2}})$ and the dH and dF are replaced by non-infinitesmal changes, ΔH and ΔF. For non-local changes, coefficients of partials with respect to the underlying are thus random and hedge ratios depend on first and second (and higher) order terms. A testable version of Equation (3.1) is written in regression form as

$$\Delta H = \alpha + \beta_1 H_F \Delta F + \beta_2 H_t \Delta t + \beta_3 H_{FF} \frac{\Delta F^2}{2} + \varepsilon, \tag{3.2}$$

where ε is the zero mean error term, $\Delta H = H_1 - H_0$, $\Delta F = F_1 - F_0$ and $\Delta t \equiv t_1 - t_0$.[8] Under BS assumptions, ε is zero. However, violation of any of the assumptions leads to non-zero errors and/or problems in empirical tests. The errors are principally those caused by stochastic process misspecification, parameter estimation errors, non-zero transactions costs and non-synchronous transactions. Dennis and Mayhew (2004) note that microstructure induced errors can affect empirical tests.

The null hypothesis that the model produces unbiased estimates of price change implies

$$E[\Delta H | \Delta F, \Delta t] = H_F \Delta F + H_t \Delta t + H_{FF} \frac{\Delta F^2}{2}. \tag{3.3}$$

With respect to the coefficients, the null of unbiased estimates is

$$H_0 : (a, \beta_1, \beta_2, \beta_3) = (0, 1, 1, 1). \tag{3.4}$$

[8] Note that a multivariate form of the Ito expansion can be used to develop price change equations for options depending on multiple state variables.

To develop the test, option pricing models are parameterized at time t and all right hand side gradients (Greeks) H_F, H_{FF}, and H_t are computed. At time $t + \Delta t$, the right hand side components ΔF and ΔF^2 are computed so that the regression equation is completely specified. The dependent variable ΔH is the observed change in option price and the right hand side is the change predicted by the option pricing model given changes in the state variables.

Absent statistical considerations, evaluating a price change model is the same as evaluating a price level model. In price level models, the underlying price is assumed known and model price is compared to the observed price. In price change models, underlying price change is assumed known and model price change is compared to observed price change.

In the sections that follow, regressions are developed for the Black's model and an American version of the binomial model. In addition, following Figlewski (2002), a naive model of option price change versus stock price change is tested. Figlewski tests a *price level* version of the Black-Scholes model using regressions against an informationally passive benchmark. The benchmark used is the exercise value of the option, i.e., $Max\{0, F - K\}$, where F is the price of the underlying and K is strike price. As a minimum threshold, the model should best the informationally passive benchmark in R^2 or other measures of model validity. The same procedure can be used for evaluating models of *price change*. One informationally passive benchmark is a regression of option price change versus price change in the underlying. I expect the binomial model will outperform this informationally passive benchmark (hereafter a naive model).

Because of possible confusion between Δ as a gradient and Δ as a small change, the notation for a small change used hereafter is $d(\cdot)$ instead of $\Delta(\cdot)$,e.g., dF replaces $\Delta F.$ This means, for example, that if F is a random state variable, dF^2 is still random since dF is non-local.

3.3 Data

3.3.1 S&P 500 Futures Options and Futures Contracts

The data consists of the prices and contract specifications of futures options, their underlying and contemporaneous risk-free interest rates. The underlying is the S&P 500 futures contract traded on the Chicago Mercantile (CME). This contract is among the most liquid of all derivative contracts and is also used extensively for hedging well-diversified equity portfolios. Observations were taken from January 1998 to December 2006 from the CME's Time and Sales database. To be a valid observation, an option trade of $0.25 or more must occur within 30 seconds of a futures trade of the same maturity. In fact, the average delay between the option trade and the corresponding futures trade was five seconds.

The risk-free rate is calculated from Libor rates based on the British Bankers Association Data. The Libor rates are converted to continuously compounded yields. The Libor data are monthly with the shortest maturity overnight, one and two weeks. Daily Libor rates are obtained by interpolation.

Descriptive statistics for the option data are summarized in Table 3.1 and 3.2. The complete dataset contains 76,544 call options and 101,010 put options. The regressions require price change data for each contract and the associated Greeks calculated on the day(s) prior to the price change. Therefore, for each strike price, the contract that trades closest to 10 am is selected and Greeks are calculated. Thus, if five strikes prices are recorded, five sets of Greeks are computed. For each strike price, only contracts that trade with 1 or 2 day lags are selected, resulting in datasets with 1 or 2 day lags for calls and puts[9]. For a one day lag, a valid observation consists of a trade on the

[9] Datasets for calls and puts with one day lags contain prices on the same contract (same strike price and expiration) that traded on both Monday and Tuesday, or Tuesday and Wednesday, or Wednesday and Thursday, or Thursday and Friday. Datasets for calls and puts with two day lags contain prices on contracts traded on Monday and Wednesday or Wednesday and Friday.

Table 3.1: Data Descriptions for Calls according to Years

Data used for this paper are American style options on S&P 500 futures traded on the CME from January 1998 to December 2006. Symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where F is a price of the futures contract and K is a strike price of the option. Calls are in the money for $F/K > 1$.

		Dataset									
		All	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Number of Call Options		76544	15049	10876	9406	7663	9549	7686	5570	5766	4979
Number of Strike Prices		186	79	77	80	103	99	77	52	46	58
Average Difference between the Trade of Option and Future		5 seconds	4 seconds	4 seconds	5 seconds	5 seconds	4 seconds	5 seconds	6 seconds	6 seconds	6 seconds
Maturity	**Range**	1-238 days	1-156 days	1-238 days	1-150 days	1-167 days	1-238 days	1-162 days	1-120 days	1-160	1-149 days
	1-30 days	46901	10221	6958	5806	4726	4901	4500	3024	3576	3189
	31-60 days	20138	3242	2533	2261	1876	3377	2208	1855	1456	1330
	61-90 days	7647	1246	1110	1128	756	1088	764	565	626	364
	91-120 days	1801	323	264	207	298	175	209	126	105	94
	121-150 days	50	16	10	4	6	6	4	0	2	2
	151-180 days	5	1	0	0	1	1	1	0	1	0
	181-210 days	0	0	0	0	0	0	0	0	0	0
	211-240 days	2	0	1	0	0	1	0	0	0	0
	241-270 days	0	0	0	0	0	0	0	0	0	0
Moneyness	X1	8720	1081	1347	1788	1269	1799	1192	167	56	21
	X2	31711	6257	4774	4277	3377	3807	2950	2429	1823	2017
	At the money	34519	7394	4495	3221	2892	3564	3295	2889	3849	2920
	X3	1194	256	199	109	102	218	206	56	32	16
	X4	400	61	61	11	23	161	43	29	6	5

Table 3.2: Data Descriptions for Puts according to Years

Data used for this paper are American style options on S&P 500 futures traded on the CME from January 1998 to December 2006. Symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where F is a price of the futures contract and K is a strike price of the option. Puts are in the money for $F/K < 1$.

		Dataset									
		All	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Number of Put Options		101010	20495	14692	12365	10532	11422	9750	7588	6778	7388
Number of Strike Prices		183	92	98	92	98	99	88	67	62	77
Average Difference between the Trade of Option and Future		5 seconds	4 seconds	4 seconds	4 seconds	5 seconds	4 seconds	5 seconds	6 seconds	6 seconds	6 seconds
Maturity	**Range**	1-265 days	1-204 days	1-202 days	1-265 days	1-176 days	1-181 days	1-156 days	1-185 days	1-147 days	1-174 days
	1-30 days	57367	12429	8258	6845	5950	5762	5135	4097	4035	4856
	31-60 days	27801	4952	3918	3352	2619	4043	3034	2331	1800	1752
	61-90 days	12452	2193	2092	1786	1408	1317	1293	947	826	600
	91-120 days	3227	853	408	367	545	272	286	206	115	175
	121-150 days	126	58	22	9	5	20	1	5	2	4
	151-180 days	29	9	3	3	5	7	1	0	0	1
	181-210 days	6	1	1	1	0	1	0	2	0	0
	211-240 days	0	0	0	0	0	0	0	0	0	0
	241-270 days	2	0	0	2	0	0	0	0	0	0
Moneyness	X1	333	47	13	96	116	43	18	0	0	0
	X2	1358	285	119	198	316	254	69	58	16	43
	At the money	29562	5798	3496	3049	2941	3263	2872	2848	2742	2553
	X3	29368	5859	4027	3547	2678	2754	2804	2502	2314	2883
	X4	40389	8506	7037	5475	4481	5108	3987	2180	1706	1909

same contract the following day. In each case, trades closest to 10 am are used since this is typically the period of heaviest trading. Thus, the time lag between observations is typically close to a multiple of 24 hours. This setup mitigates the problem of dependent observations because there is no overlap of data within lag class.

A characteristic of the data is that the majority of both calls and puts are at-the-money or out-of-the-money options. This is consistent with the notion that participants of option markets seek relatively inexpensive insurance or means of low cost speculation. The dataset contains both short and long term options, but short term options (less than 60 days) constitute the bulk of the trades.

3.3.2 Volatilities Used in Regressions

In my regressions, I use several different volatility measures: the average implied price change volatility (AVPCIV), the average implied volatility (AVIV) and the average S&P 500 index volatility (AV500). In addition to volatility averages, I also use the contract implied volatility (IV) estimated on the first day of a price change (please, see the explanation of IV in the following paragraph) and the fitted implied price change volatility (FPCIV)[10].

To ensure that regressions are tested in out-of-the-sample setup, I estimate all volatility parameters in the estimation period, i. e. in the time period from January 1998 to December 1998 (Table 3.3). Then, I use these estimated volatility parameters for calculation of Greeks and in the regressions in the testing period, i.e. in the time period from January 1999 to December 2006 (datasets for calls and puts with one and two day lags used for regressions are described in Table 3.4).

The volatility of the S&P 500 index is estimated from returns on the S&P 500 spot index

[10] FPCIV is calculated from estimated relation of PCIV with IV. This relation is estimated according to the equation 2.12.

Table 3.3: Average Volatilities Used for Calculations of Greeks for Regressions

Averages of different measures of volatilities are estimated from the S&P 500 futures options from January 1998 to December 1998. Datasets for one and two day lags are used to estimate appropriate parameters.

		Number of Records Used for Estimation	Average of Implied Price Change Volatility (AVPCIV)	Average of Implied Volatility (AVIV)
Calls				
	One Day Lags	372	0.1732	0.1958
	Two Day Lags	34	0.1996	0.2084
Puts				
	One Day Lags	531	0.3251	0.3016
	Two Day Lags	60	0.3126	0.2854

in time period from January 1998 to December 1998. The value of AV500 is 0.2449 for the estimation period.

In addition to volatility averages, I also use the contract implied volatility (IV). By contract implied volatility (IV) I mean the implied volatility estimated for a specific contract i.e. a contract with a specific strike price and an expiration. The contract implied volatility (IV) is estimated on the first day of the price change, for example for the price change that occurs from Monday to Tuesday, IV is estimated on Monday.

As a continuation of the first essay, I estimate this relation between PCIV and IV from data from January 1998 to December 1998. The estimated relation is:

$$\sigma_{pc} = a + b\sigma_{iv} + \varepsilon, \tag{3.5}$$

where σ_{pc} is PCIV and σ_{iv} is IV.

The estimated coefficients are shown in Table 3.5. Then I use this estimated relation to calculate FPCIV in the tested period (January 1999 to December 2006). As is mentioned in the first essay, the estimation of PCIV requires a large dataset and lenghtly estimation. Since the relation between PCIV and IV exists, this relation helps to overcome this problem.

Table 3.4: Data Description for Datasets of Calls and Puts Used in Regressions

Datasets for one day lags contain contracts of the same strike price traded on consecutive days (Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday or Thursday and Friday). Similarly, datasets with two day lags contain records traded on day one and day three (Monday and Wednesday or Wednesday and Friday). These datasets were used for estimation of volatility parameters and for regressions.

Data are pit-traded American style options on S&P 500 futures traded on CME from January 1999 to December 2006. The symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where *F* is a price of the futures contract and *K* is a strike price of the option. Calls (puts) are *in- (out-of-) the-money* for $F/K > 1$.

		Calls		Puts	
		1 Day Lags	**2 Day Lags**	**1 Day Lags**	**2 Day Lags**
Number of Calls		8098	813	0	0
Number of Puts		0	0	11008	1137
Number of Strike Prices		161	138	151	132
Maturity	**Range (Days)**	1 – 119 Days	1 – 106 Days	1 – 113 Days	1 – 125 Days
	1-30 days	4585	335	5766	525
	31-60 days	2367	309	3346	358
	61-90 days	1017	156	1697	227
	91-120 days	129	13	199	26
	121-150 days	0	0	0	1
	151-180 days	0	0	0	0
	181-210 days	0	0	0	0
	211-240 days	0	0	0	0
	241-270 days	0	0	0	0
Moneyness	**X1**	1192	156	61	10
	X2	3488	351	131	26
	At-the-money	3253	273	2843	270
	X3	141	25	3094	305
	X4	24	8	4879	526

Table 3.5: Estimation of Fitted Implied Price Change Volatility (FPCIV)

The relation for fitted implied price change volatility (FPCIV) is estimated from January 1998 to December 1998. The estimation model *is* $\sigma_{PC} = \alpha + \beta\, \sigma_{IV} + \varepsilon$, where σ_{PC} is implied price change volatility (PCIV) and σ_{IV} is implied volatility (IV).
*** Refers to the significance at 99% level, ** to the significance at 95% level and * to the significance at 90% level.

		Number of Records Used for Estimation	α	β	Adjusted R²
Calls					
	One Day Lags	372	0.12675***	0.23747***	0.0166
	Two Day Lags	34	-0.00546	0.98387**	0.1288
Puts					
	One Day Lags	531	-0.01629	1.13164***	0.5525
	Two Day Lags	60	-0.02368	1.17835***	0.6016

3.4 Regression Results

3.4.1 Calls

Scatter plots of market versus model predictions are shown in Figures 3.1 through 3.4. These plots show the tight linear relationship quantified by the regressions.

The regression models for calls are defined as :

$$\text{Naive Model:} \quad dC = \alpha + \beta_1(dF) + \varepsilon, \tag{3.6a}$$

$$\text{B1:} \quad dC = \alpha + \beta_1(C_F dF) + \beta_2(\frac{1}{2}C_{FF}dF^2) + \beta_3(C_t dt) + \varepsilon, \tag{3.6b}$$

$$\text{B2:} \quad dC = \alpha + \beta_1(C_F dF + \frac{1}{2}C_{FF}dF^2 + C_t dt) + \varepsilon, \tag{3.6c}$$

where the Greeks are defined as $\Delta \equiv C_F$, $\Gamma \equiv C_{FF}$ and $\theta \equiv C_t$.

Model B1 is expected to exhibit the highest R^2 since the coefficients of the different Greeks are not restricted to equality. Instead, each Greek is allowed to have a historical, but possibly different scalar multiple. Model B2 is a regression test of unbiasedness of the American Binomial model. In addition, these models are tested also with two additional variables - moneyness (M) and maturity (T):

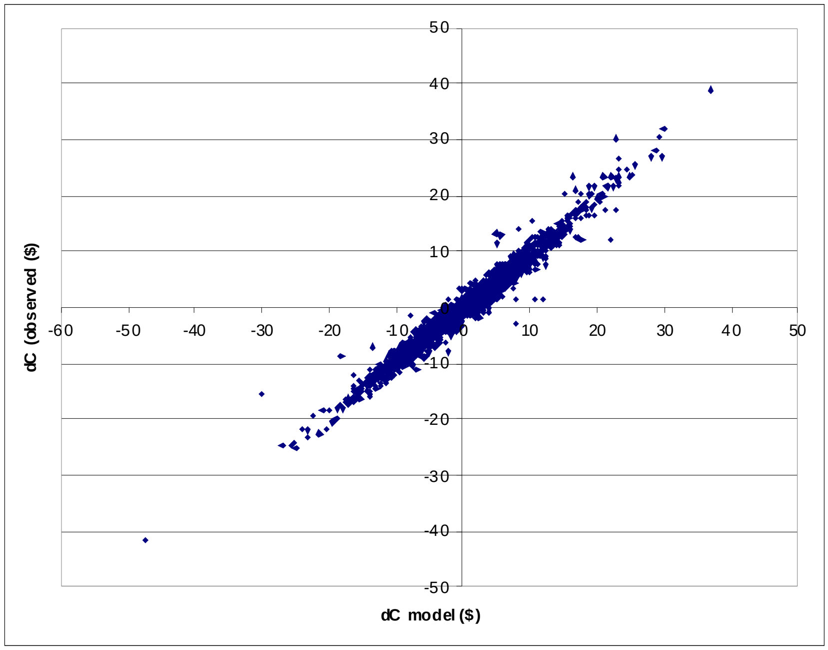

Figure 3.1: American Version of Binomial Model for Calls with One Day Lags Using the Contract Implied Volatility (IV)

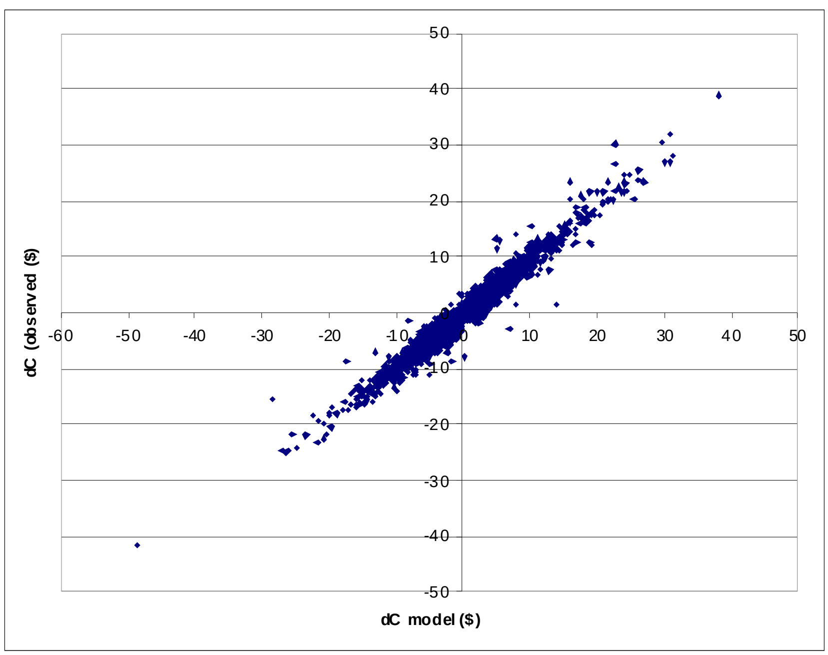

Figure 3.2: American Version of Binomial Model for Calls with One Day Lags Using the Fitted Implied Price Change Volatility (FPCIV)

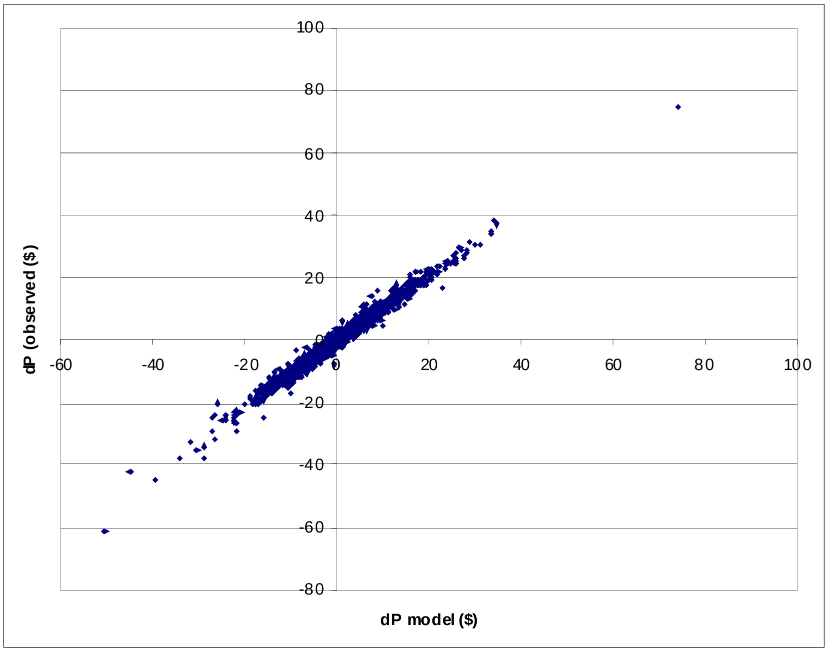

Figure 3.3: American Version of Binomial Model for Puts with One Day Lags Using the Contract Implied Volatility (IV)

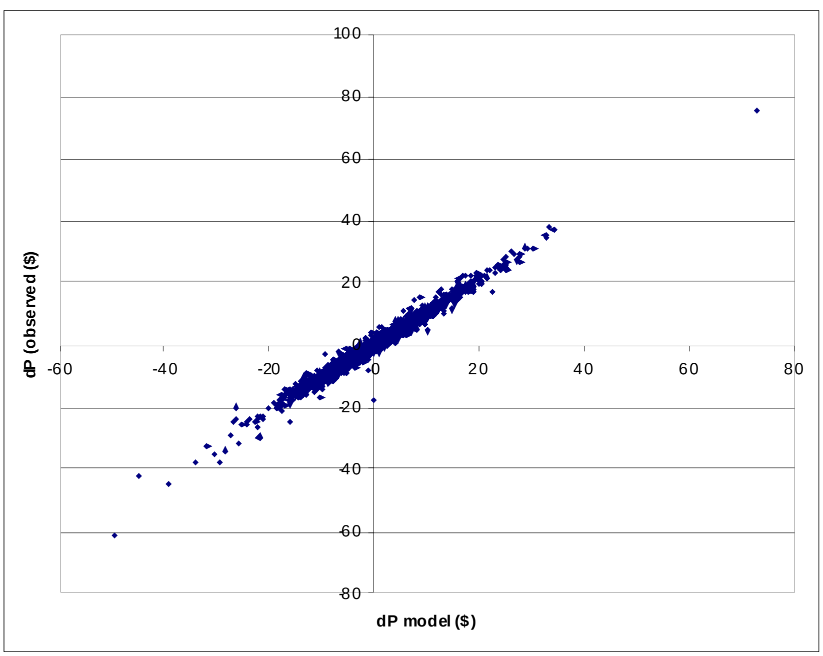

Figure 3.4: American Version of Binomial Model for Puts with One Day Lags Using the Fitted Implied Price Change Volatility (FPCIV)

$$\text{BM1} \quad : \quad dC = \alpha + \beta_1(C_F dF) + \beta_2(\frac{1}{2}C_{FF}dF^2) + \beta_3(C_t dt) + \gamma_1 M + \gamma_2 T + \varepsilon, \tag{3.7a}$$

$$\text{BM2} \quad : \quad dC = \alpha + \beta_1(C_F dF + \frac{1}{2}C_{FF}dF^2 + C_t dt) + \gamma_1 M + \gamma_2 T + \varepsilon. \tag{3.7b}$$

The hypotheses tested are: $\alpha = 0$, $\beta_i = 1$ and $\gamma_i = 0$.

The results for calls with one and two day lags are shown in Tables 3.6 and 3.7[11]. All volatility measures produce high $R^2 s$ ranging from 0.94 to 0.97 compared to the R^2 of the naive model (0.59 for one day lags and 0.56 for two day lags). The highest $R^2 s$ were found when using IV and FPCIV. Slightly higher R^2 were found for IV for one day lags and FPCIV for two day lags.

The α and β_i coefficients are significantly different from zero and one, thus rejecting the unbiasedness null. The β_1 coefficient is smaller than one (ranging from 0.85 to 0.95). It means that the model tends to overestimate price changes in call options. However, the β_1 coefficients are closer to one for AVPCIV (0.94) and FPCIV (0.95) than for other volatility measures. Thus, point estimates are closer to the value of one suggested by the theoretical model. This can be important in developing effective hedges.

Moneyness and maturity effects are strongest for AV500 and AVIV for one day lags. These effects are rather small for the AVPCIV, FPCIV and IV. Moneyness and maturity effects decrease for two day lags.

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11 Black's model and the European binomial models were also tested. Results are almost identical as for American binomial model with differences for β_i coefficients in third decimal places. These results are not shown in tables.

Table 3.6: Regression Coefficients for Calls with One Day Lags

The data consists of S&P 500 futures options traded from January 1999 to December 2006 on the CME. The regressions below follow from an Ito expansion of Black's model and regress local price changes of the call option on option's delta, gamma and theta (Section 3.3.1). The null hypotheses are that the intercept coefficient is $\alpha = 0$, the slope coefficients are $\beta_i = 1$, and the coefficients on moneyness (γ_1) and maturity (γ_2) variables are $\gamma_i = 0$.

*** Refers to the significance at 99% level, ** to the significance at 95% level and * to the significance at 90% level.

Model	**α**	**β_1**	**β_2**	**β_3**	**γ_1**	**γ_2**	**Adjusted R^2**
Panel A. Average of Implied Price Change Volatility (AVPCIV)							
Naïve Model	-0.36***	0.24***					0.5929
B1	-0.11***	0.94***	0.85***	1.02			0.9387
B2	0.17***	0.94***					0.9383
BM1	-0.36	0.94***	0.85***	1.00	0.22	0.33	0.9387
BM2	-0.11	0.94***			-0.12	0.50**	0.9384
Panel B. Average of Implied Volatility (AVIV)							
B1	-0.13***	0.92***	0.89***	0.84***			0.9386
B2	-0.12***	0.91***					0.9385
BM1	0.78**	0.92***	0.89***	0.69***	-1.06***	0.48*	0.9387
BM2	0.43	0.91***			-0.58**	0.10	0.9385
Panel C. Average of S&P 500 Volatility (AV500)							
B1	-0.17***	0.85***	0.93*	0.59***			0.9193
B2	-0.003	0.85***					0.9187
BM1	2.08***	0.86***	0.93*	0.35***	-2.55***	0.83***	0.9199
BM2	1.25***	0.85***			-1.24***	-0.60**	0.9188
Panel D. Contract Implied Volatility (IV)							
B1	-0.17***	0.93***	1.05**	0.95			0.9587
B2	-0.13***	0.93***					0.9584
BM1	0.31	0.93***	1.05**	0.94	-0.50*	-0.11	0.9588
BM2	0.24	0.93***			-0.37	0.20	0.9584
Panel E. Fitted Implied Price Change Volatility (FPCIV)							
B1	-0.080***	0.95***	0.93**	1.24***			0.9497
B2	-0.18***	0.95***					0.9494
BM1	-0.72**	0.95***	0.93**	1.33***	0.71*	-0.21	0.9497
BM2	-0.054	0.95***			0.17	0.39*	0.9494

Table 3.7: Regression Coefficients for Calls with Two Day Lags

The data consists of S&P 500 futures options traded from January 1999 to December 2006 on the CME. The regressions below follow from an Ito expansion of Black's model and regress local price changes of the call option on option's delta, gamma and theta (Section 3.3.1). The null hypotheses are that the intercept coefficient is $\alpha = 0$, the slope coefficients are $\beta_i = 1$, and the coefficients on moneyness (γ_1) and maturity (γ_2) variables are $\gamma_i = 0$.
*** Refers to the significance at 99% level, ** to the significance at 95% level and * to the significance at 90% level.

Model	α	β_1	β_2	β_3	γ_1	γ_2	Adjusted R^2
Panel A. Average of Implied Price Change Volatility (AVPCIV)							
Naïve Model	-0.38***	0.24***					0.5581
B1	-0.088	0.91***	0.73*	0.73**			0.9369
B2	-0.087	0.91***					0.9355
BM1	0.39	0.92***	0.72*	0.65**	-0.63	0.66	0.9368
BM2	-0.080	0.91***			0.0002	-0.069	0.9353
Panel B. Average of Implied Volatility (AVIV)							
B1	-0.089	0.91***	0.73*	0.69***			0.9351
B2	-0.044	0.90***					0.9338
BM1	1.04	0.91***	0.73*	0.57**	-1.33	0.61	0.9350
BM2	0.34	0.90***			-0.36	0.41	0.9336
Panel C. Average of S&P 500 Volatility (AV500)							
B1	-0.10	0.87***	0.73	0.52***			0.9205
B2	0.13**	0.87***					0.9192
BM1	3.19**	0.87***	0.73	0.36*	-3.63**	0.37	0.9209
BM2	1.94	0.87***			-1.68	-1.79	0.9194
Panel D. Contract Implied Volatility (IV)							
Naive	-0.38***	0.24***					0.5581
B1	-0.26***	0.93***	1.10**	0.87*			0.9648
B2	-0.12***	0.93***					0.9637
BM1	1.81	0.93***	1.11**	0.87	-2.02*	-1.22	0.9650
BM2	1.42	0.94***			-1.43	-1.54*	0.9639
Panel E. Fitted Implied Price Change Volatility (FPCIV)							
B1	-0.26***	0.94***	1.10**	0.94			0.9662
B2	-0.16***	0.94***					0.9653
BM1	1.27	0.94***	1.10**	0.96	-1.46	-1.17	0.9663
BM2	1.05	0.94***	-1.12	-1.21			0.9654

3.4.2 Puts

The regression models for puts are analogous to models 3.6a to 3.7b with change in the price of put option dP replacing dC. The results are given in Tables 3.8 and 3.9. As with calls, regression's R^2s are high, ranging from 0.94 to almost 0.98 compared to 0.55 (one day lags) and 0.49 (two day lags) for naive model. The R^2s for regressions using FPCIV and IV are again slightly higher and are in range of 0.97 to 0.98. The α coefficients are significant at the 0.01 level. Also β_1 coefficients are significant at the 0.01 level except for AVPCIV for one day lags and AVPCIV, AVIV and AV500 for two day lags. The β_1 coefficients larger than one mean that the model tends to underestimate puts price changes.

3.5 Sign Violations

To underscore one of the reasons that Black's model (or binomial model) does not fit the data more perfectly, we note that sometimes even the sign of price changes are anomalous. For example, we sometimes observe that a call (put) price will go down (up) following a increase in stock price. This should not happen except for reasons noted earlier, e.g., nonsynchroneous and/or segmented markets, omitted variables, etc.

Bakshi, Cao and Chen (2000), hereafter BCC, use S&P 500 options on the spot to assess sign and magnitude violations. Although the databases are different, our findings on magnitude issues appear to be more consistent with a one-factor no-arbitrage model. Using an hourly interval and the full sample of moneyness levels, BCC regress observed changes in call prices on BSM call price changes and report that $R^2 = 0.51$. Their regression coefficient on the BSM model change (dC_m) is $\beta_1 = 0.51$ and they reject the null hypothesis that $\beta_1 = 1$. In contrast, in this study, the R^2 for regressions of observed daily call price changes on model price changes is 0.94 and $\beta_1 = 0.94$. However, the null hypothesis that $\beta_1 = 1$ is also rejected.

Table 3.8: Regression Coefficients for Puts with One Day Lags

The data consists of S&P 500 futures options traded from January 1999 to December 2006 on the CME. The regressions below follow from an Ito expansion of Black's model and regress local price changes of the put option on option's delta, gamma and theta (Section 3.3.1). The null hypotheses are that the intercept coefficient is $\alpha = 0$, the slope coefficients are $\beta_i = 1$, and the coefficients on moneyness (γ_1) and maturity (γ_2) variables are $\gamma_i = 0$.
*** Refers to the significance at 99% level, ** to the significance at 95% level and * to the significance at 90% level.

Model	**α**	**β_1**	**β_2**	**β_3**	**γ_1**	**γ_2**	**Adjusted R^2**
Panel A. Average of Implied Price Change Volatility (AVPCIV)							
Naïve Model	-0.25***	-0.23***					0.5500
B1	-0.11***	1.01*	1.02	0.45***			0.9478
B2	0.21***	1.00					0.9447
BM1	-0.47***	1.01**	1.02	0.40***	0.29**	0.17	0.9479
BM2	1.32***	1.01			-0.89***	-1.44***	0.9459
Panel B. Average of Implied Volatility (AVIV)							
B1	-0.11***	1.02***	0.98	0.49***			0.9507
B2	0.15***	1.02***					0.9484
BM1	-0.37***	1.02***	0.98	0.44***	0.20*	0.23	0.9507
BM2	1.14***	1.02***			0.81***	-1.11***	0.9492
Panel C. Average of S&P 500 Volatility (AV500)							
B1	-0.11***	1.05***	0.89***	0.62***			0.9485
B2	0.006	1.05***					0.9472
BM1	-0.033	1.05***	0.89***	0.60***	-0.11	0.43**	0.9485
BM2	0.67***	1.05***			-0.58***	-0.28	0.9475
Panel D. Contract Implied Volatility (IV)							
B1	-0.054***	1.05***	1.08**	0.98			0.9733
B2	-0.021***	1.05***					0.9732
BM1	-0.58***	1.05***	1.09**	0.95	0.49***	-0.32**	0.9734
BM2	-0.36***	1.05***			0.36***	-0.53***	0.9733
Panel E. Fitted Implied Price Change Volatility(FPCIV)							
Naïve Model	-0.25***	-0.23***					0.5500
B1	-0.066***	1.04***	1.11***	0.86***			0.9722
B2	0.025***	1.04***					0.9718
BM1	-0.86***	1.04***	1.12***	0.82***	0.74***	-0.37**	0.9724
BM2	-0.40***	1.04***			0.47***	-0.90***	0.9720

Table 3.9: Regression Coefficients for Puts with Two Day Lags

The data consists of S&P 500 futures options traded from January 1999 to December 2006 on the CME. The regressions below follow from an Ito expansion of Black's model and regress local price changes of the put option on option's delta, gamma and theta (Section 3.3.1). The null hypotheses are that the intercept coefficient is $\alpha = 0$, the slope coefficients are $\beta_i = 1$, and the coefficients on moneyness (γ_1) and maturity (γ_2) variables are $\gamma_i = 0$.
*** Refers to the significance at 99% level, ** to the significance at 95% level and * to the significance at 90% level.

Model	α	β_1	β_2	β_3	γ_1	γ_2	Adjusted R^2
Panel A. Average of Implied Price Change Volatility (AVPCIV)							
Naïve Model	-0.72***	-0.23***					0.4870
B1	-0.12*	1.03	0.73	0.64***			0.9496
B2	0.016	1.03*					0.9475
BM1	0.30	1.03	0.73	0.69**	-0.36	0.090	0.9496
BM2	0.87***	1.03*			-0.71**	-0.68	0.9477
Panel B. Average of Implied Volatility (AVIV)							
B1	-0.12*	1.02	0.75	0.53***			0.9514
B2	0.19***	1.02					0.9483
BM1	-0.10	1.02	0.75	0.54***	0.019	-0.28	0.9513
BM2	1.40***	1.02			-0.93***	1.72***	0.9492
Panel C. Average of S&P 500 Volatility (AV500)							
B1	-0.12*	1.02	0.76	0.47***			0.9489
B2	0.30***	1.01					0.9449
BM1	-0.35	1.02	0.77	0.46***	0.25	-0.44	0.9489
BM2	1.72***	1.02			-1.05***	-2.40***	0.9463
Panel D. Contract Implied Volatility (IV)							
B1	0.006	1.06***	1.19**	1.31**			0.9784
B2	-0.086***	1.06***					0.9780
BM1	-0.46	1.06***	1.21***	1.33**	0.56**	-1.29***	0.9786
BM2	-0.80***	1.06***			0.70***	-0.58	0.9781
Panel E. Fitted Implied Price Change Volatility (FPCIV)							
B1	-0.022	1.06***	1.20**	1.11			0.9758
B2	0.0009	1.05***					0.9755
BM1	-1.16***	1.06***	1.23***	1.11	1.17***	-1.42***	0.9763
BM2	-0.95***	1.05***			0.99***	-1.25*	0.9758

The violations where observed call price, dC_o and futures prices, dF, move in opposite directions are referred to in BCC as Type I violations. Table 3.10, panel A documents the percentage of violations that I find as a function of moneyness and maturity. The results are quite similar to those of BCC. For one day lags (approximately 10 am until 10 am the next day) there are 9.80% Type I violations. The comparable BCC number is 9.1% using a cash index and 7.2% using a futures proxy for the cash price. The violation percentages decrease with increasing moneyness. This holds true for all maturity classes. It is consistent with the finding of BCC when they use futures prices for the spot price proxy. In addition, Type I violations decrease with increasing option maturity for all moneyness levels. These finding are consistent with the notion that there are fewer violations in higher priced calls while lower priced calls contain more "noise," perhaps partially explained by transaction costs or the higher relative importance of omitted variables.

Table 3.10, panel B records violations for puts. Overall, the results are similar to the results for puts. Similarly as for calls, there is far fewer violation for puts *at-* and *in-the-money*.

The results in this study are similar to those of BCC with respect to violation percentages. However, the regression results for the price change model differ considerable in the R^2s (0.94 versus 0.51) and $\hat{\beta}_1$ being closer to the null ($\hat{\beta} = 0.94$ versus $\hat{\beta} = 0.51$). How can these differences be explained? As noted above, BCC study the S&P 500 spot option and observe price changes in hourly intervals. In this study, I use the S&P 500 futures option and observe price changes over 24 hour intervals. The BCC study used data from March 1994 through August 1994 while my study uses data from January 1998 to December 2006. Still, these factors would not seem to account for the observed differences.

Table 3.10: Price Change Violation Rates

The violations are observations where observed change in the price of call (put) dC_o (dP_o) moves in the opposite (the same) directions with future price, dF. The time interval is approximately 24 hours. Data used are options on the S&P 500 futures from January 1998 to December 2006 that traded with one day lag. Dataset for calls contains 9562 consecutive observations; dataset for puts contains 13253 records. Violation rates are expressed in percents, the actual numbers of violations in each category are shown in parenthesis.

			Maturity		
Moneyness	**All**	**1 – 30 Days**	**31 – 60 Days**	**61 – 90 Days**	**91 – 120 Days**
Panel A: Calls $dC_o*dF<0$					
All	9.80%	6.35%	2.47%	0.90%	0.08%
	(937)	(607)	(236)	(86)	(8)
F/K ≤ 0.925	1.81%	0.60%	0.81%	0.35%	0.06%
	(173)	(57)	(77)	(33)	(6)
0.925 < F/K ≤ 0.975	4.85%	3.21%	1.23%	0.39%	0.02%
	(464)	(307)	(118)	(37)	(2)
0.975 < F/K ≤ 1.025	3.10%	2.52%	0.42%	0.16%	0%
	(296)	(241)	(40)	(15)	(0)
1.025 < F/K ≤ 1.075	0.04%	0.02%	0.01%	0.01%	0%
	(4)	(2)	(1)	(1)	(0)
F/K ≥ 1.075	0%	0%	0%	0%	0%
	(0)	(0)	(0)	(0)	(0)
Panel B: Puts $dP_o*dF>0$					
All	9.39%	5.49%	2.41%	1.30%	0.19%
	(1245)	(728)	(320)	(172)	(25)
F/K ≤ 0.925	0%	0%	0%	0%	0%
	(0)	(0)	(0)	(0)	(0)
0.925 < F/K ≤ 0.975	0.05%	0.05%	0%	0%	0%
	(6)	(6)	(0)	(0)	(0)
0.975 < F/K ≤ 1.025	1.75%	1.34%	0.29%	0.11%	0.01%
	(232)	(177)	(39)	(15)	(1)
1.025 < F/K ≤ 1.075	2.47%	1.83%	0.45%	0.18%	0.02%
	(327)	(242)	(59)	(24)	(2)
F/K ≥ 1.075	5.13%	2.29%	1.68%	1.00%	0.17%
	(680)	(303)	(222)	(133)	(22)

Dennis and Mayhew (2003) investigate microstructure errors in observed options prices and conclude that noise in observations can have a significant effect on model predictions. For example, using a Black-Scholes model, they derive an expression for the probability that an observed call price and the underlying stock price will move in opposite directions. They note that the probability depends on the option tick size and the distribution of true stock price changes. And, importantly, it depends on the probability that the true option price will change by less than one tick.

In my study, I conjecture that most of the variation can be due to differences in the way options and their underlying are matched. BCC use the Berkeley Options data base. In this database, bid-ask option prices are matched with the last underlying price on the spot. In alternative tests, BCC use the lead month futures price as the underlying to mitigate stale prices in the spot index. Their samples are taken at predetermined time points and matched pairs consist of existing mid-point bid-ask quotes and the last price on a spot (futures) transaction. In contrast, this study uses the CME Time and Sales Database to match transaction prices in the futures option and its underlying. The price change observation is taken at a period of peak liquidity and consists of the closest match near 10 am and ends with the closest match near 10 am on the following day. The average displacement in real transaction time between matched pairs averages five seconds and does not exceed 30 seconds. Although this data is virtually synchronous, the transaction prices are nonetheless subject to the criticism of bid-ask bounce.

3.6 Conclusion

There is sufficient evidence to reject the American version of the binomial model of price change for S&P futures options. The model overestimates short-term call price changes and call delta. It underestimates put price changes and put delta. That said, the American version of binomial model still produced R^2s of more than 95%. The model is tested using five different measures of volatility - the average implied price change volatility (AVPCIV), the average implied volatility (AVIV), average of S&P 500 index volatility (AV500), contract implied volatility (IV) and fitted implied price change volatility (FPCIV). The fitted implied price change volatility (FPCIV) and contract implied volatility (IV) produce slightly higher R^2s than other two volatility measures.

In the more extreme case, futures option change may have "wrong signs." Using futures options and a different time frame, my results largely confirm the findings of Bakshi, Cao and Chen (2000) in documenting apparent violations of rational option prices. For example, in this study, daily call price changes and changes in the underlying move in the opposite direction approximately 9.8 percent of the time.

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Chapter 4 Hedging Price Changes in the S&P 500 Options and Futures Contracts: The Effect of Different Measures of Implied Volatility

4.1 Introduction

Market participants as well as businesses face uncertainties arising from the moves of the markets, changes in interest rates, changes in currency values or prices of commodities. Financial derivatives are instruments that can be used to manage these types of risk. The use of financial derivatives increases every year. Derivatives based on specific stocks or indices are traded on CBOE, a large portion of derivative securities, however, are negotiated privately. According to the International Swaps and Derivatives Association's midyear report, the notional amount of privately negotiated derivatives outstanding was $531.2 trillion as of June 30, 2008.

The focus of corporate risk management is to decrease the risk arising from the volatility of cash flows and reducing the probability of financial distress (Stulz (1996))[12]. There are several benefits associated with lower volatilities in cash flow. First, the firm with lower cash flow volatility generally has larger debt capacity and can therefore utilize the tax benefit of debt. Second, lower volatility of cash flows may reduce expected tax liabilities (Smith and Stulz (1985)) since the total tax is convex to revenue. The empirical evidence on these issues is sparse. According to study of Guay and Kothari (2003), the use derivatives to hedge firms' risk exposure is rather modest relative to their cash flows or market value sensitivities. The findings of Graham and Rogers (2002) are consistent with the notion that firms hedge in order to increase their debt capacity and tax benefits of debt. However, they do not find evidence of corporate hedging for

[12] Corporate use of derivatives can be also aimed to increase firms' risk. The argument behind the use of derivatives for the purpose of seeking additional risk is based on the agency problem between the shareholder and debtholder (Jensen and Meckling (1976), Myers (1977)). The shareholders have incentives to increase the risk of the firm to transfer wealth from the debtholders.

purpose of reducing expected tax liabilities.

Firms hedge to reduce risk. The basic approach is to create a financial position in risk factors that are negatively correlated with the core value of the firm. And ideally the financial position would offset changes in core value. For example, a firm may hedge changes in their cash flows due to changes in interest rates or in currency values. The hedge can be created using different instruments, such as futures, swaps or options.

The most common technique for hedging with options consists of matching one or more partial derivatives in the core and hedging positions. The set of first and second partials are referred to as the Greeks. If H is the derivative and x the underlying, $delta = \frac{\partial H}{\partial x}$, $gamma = \frac{\partial^2 H}{\partial x^2}$ and $vega = \frac{\partial H}{\partial \sigma}$. The result of a delta hedging is a portfolio that is insensitive to small changes in the underlying asset. The delta-gamma hedge incorporates the second partial (curvature) in the hedging scheme, while the delta-vega strategy additionally takes into the account changes in volatility. While the delta-gamma and delta-vega hedging strategies improve performance, their advantage is partially offset by the requirement for an additional traded option in the hedging portfolio.

More recent developments have brought about countless new pricing models, each of them relaxing some of the assumptions of the Black-Scholes model. Examples of most well known models include Merton's stochastic interest rate model (1973), the stochastic volatility models of Heston (1993) and Hull and White (1987), Merton (1976) and Bates' jump diffusion models (1991), the stochastic volatility and stochastic interest rates models of Amin and Ng (1993) and Bakshi and Chen (1997a, b) and the stochastic volatility jump diffusion model of Bates (1996).

The downside of these more complex models is that they require estimation of additional parameters. Errors in the estimated parameters then translate into the pricing errors. The basic question is whether these model can *ex ante* outperform the Black-Scholes model. That is, does the complexity of these models negate any theoretical advantage in a real world setting. Bakshi, Cao and Chen (1997) conducted a comprehensive empirical study of competing option pricing models. They compared the hedging performance of the Black-Scholes model with more sophisticated models including stochastic volatility, stochastic interest rates and random jumps. Their results on S&P 500 index options document that stochastic volatility model outperforms other models in hedging applications.

Despite these findings, traders on Wall street have resisted more complex models and rather tend to use a modified version of Black-Scholes model referred to as the "Practitioner Black-Scholes" (PBS) model. The PBS model takes into account non-constant volatilities (as assumed in the Black-Scholes model) by allowing them to differ across maturities and moneyness. This is consistent with the volatility smile, as noted in Rubinstein (1985, 1994)). The PBS model also allow volatilities to be larger for puts than for calls (Bollen and Whaley (2004)).

The basic output of the PBS model is the "volatility surface." The implied volatility surface is computed from option contracts with a wide array of moneyness and maturity specifications. A continuous surface is then constructed from the grid of points by interpolation or splines. The resulting volatility surface can be used to price options of any desired moneyness and maturity. Berkowitz (working paper) offers a theoretical justification for the use of PBS. He claims that the PBS model is a reduced form approximation to an unknown structural model. When recalibrated with sufficient frequency, the PBS model yields good results. Christoffersen and Jacobs (2004) in their empirical study document that when the estimation and evaluation loss functions are correctly defined, the PBS model actually outperforms more sophisticated models, such as Heston’s stochastic volatility model.

In this study, I extend the PBS tradition of "simpler is better" by examining how different volatility measures influence out-of-sample hedging performance. I employ Black's model and evaluate hedging effectiveness using historical volatility of the S&P 500 index, implied volatility, and a newly proposed volatility measure, the implied price change volatility. Implied price change volatility is implied from price *changes* rather than price *levels*. It is calculated by equating changes implied by a binomial tree version of Black's model with observed price changes. I also examine whether the hedging performance improves with daily upgrading of implied volatility and implied price change volatility.

The purpose of the study is to evaluate the performance of delta hedges, delta-gamma hedges and delta-vega hedges on the S&P500 Futures Option contract with a particular focus on the use of implied price change volatility. Fundamentally, hedging depends on matching price changes so the intuition is that price change volatility should produce a superior hedge. The calculation of price change volatility is complicated by the fact that it must be estimated from contracts that trade on consecutive days. Therefore this method notably decreases the size of the dataset. The applicability of the implied price change volatility is therefore restricted due to data issue. But one can overcome such restriction to an extent by using the fitted value of implied price change volatility from regression 2.12. To evaluate performance of implied price change volatility, I create hedges using both directly estimated implied price change volatility and fitted implied price change volatility.

My findings indicate that the hedging performance of Black's model improves with daily updating of volatility measures. And the best performance for delta hedges are those computed when implied price change volatility is used. In fact, the delta hedge computed using implied price change volatility produced average relative hedging errors for calls smaller than delta-gamma and delta-vega hedges. It appears that implied price change volatility may more than compensate for using an additional traded option when hedging calls. Thus, it appears that implied price change volatility is arguably the preferred volatility measure for hedging applications. Creating hedges using greeks adjusted by regression coefficients do not consistently improve hedging performance.

Results for puts are similar to those for calls. However, the benefits of implied price change volatility are not so compelling. Delta hedges based on the implied price change volatility produce highest R^2s but have higher average relative errors than hedges using implied volatility. Still, hedges created using daily updated implied volatilities and implied price change volatilities outperform those based on volatility averages.

The rest of the essay is organized as follows. Section 4.2 describes the data and different volatility measures. The hedging setup is described in section 4.3. Section 4.4 discusses the results and section 4.5 concludes.

4.2 Data

The data for this study consists of options on the S&P 500 futures and their underlying from January 1998 to December 2006. Data were obtained from CME's Time and Sales database. The S&P 500 futures contract closely track the price movements of the S&P 500 index. They are traded on the Chicago Mercantile (CME). They are among the most liquid of all derivative contracts with an average of more than one million contracts traded per day in 2006. These contracts are used extensively for hedging well-diversified equity portfolios.

Options were matched with underlying futures such that the option and the futures of the same maturity trade within 30 seconds. Since the volume of trades on the S&P 500 futures is very large, the average delay between the option trade and the corresponding futures trade was five seconds. Only options with price $0.25 and higher were considered. The original dataset for calls and puts is described in Tables 4.1 and 4.2. The complete dataset contains 76,544 call options and 101,010 put options. The majority of call and put options are out-of-the-money or at-the-money. These statistics are consistent with view that options are used as a means of relatively inexpensive insurance or low cost speculation. The dataset contains both short and long dated options. However, short term observations predominate.

The risk-free rate is calculated from Libor rates based on the British Bankers Association Data. The Libor data are monthly with the shortest maturities being overnight, one and two weeks. Daily Libor rates are obtained by interpolation and then converted to continuously compounded yields.

The original datasets were used to identify contracts (options with the same strike price and expiration) that traded on consecutive days. Observations consist of prices and price changes on contracts that traded on Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, and Thursday and Friday. For each day, contracts that traded closest to 10 AM were selected. This is generally the period of highest liquidity. Thus, the time delay between the trades on the option was very close to 24 hours.

4.2.1 The Volatilities

I compare the hedging performance of several different volatility measures: implied volatility, implied price change volatility and historical volatility. I look at both out-of-sample average of these measures and at daily updating. To simplify the orientation in the text, hereafter I will use the following abbreviations for different volatility measures: IV for implied volatility, PCIV for implied price change volatility, AV500 for the average volatility of the S&P 500 index, AVIV for the average implied volatility, AVPCIV for the average implied price change volatility and FPCIV

Table 4.1: Data Description for Calls according to Years

Data used for this study are American style options on S&P 500 futures traded on the CME from January 1998 to December 2006. Symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where F is a price of the futures contract and K is a strike price of the option. Calls are in the money for $F/K > 1$.

		Dataset									
		All	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Number of Call Options		76544	15049	10876	9406	7663	9549	7686	5570	5766	4979
Number of Strike Prices		186	79	77	80	103	99	77	52	46	58
Average Difference between the Trade of Option and Future		5 seconds	4 seconds	4 seconds	5 seconds	5 seconds	4 seconds	5 seconds	6 seconds	6 seconds	6 seconds
Maturity	**Range**	1-238 days	1-156 days	1-238 days	1-150 days	1-167 days	1-238 days	1-162 days	1-120 days	1-160	1-149 days
	1-30 days	46901	10221	6958	5806	4726	4901	4500	3024	3576	3189
	31-60 days	20138	3242	2533	2261	1876	3377	2208	1855	1456	1330
	61-90 days	7647	1246	1110	1128	756	1088	764	565	626	364
	91-120 days	1801	323	264	207	298	175	209	126	105	94
	121-150 days	50	16	10	4	6	6	4	0	2	2
	151-180 days	5	1	0	0	1	1	1	0	1	0
	181-210 days	0	0	0	0	0	0	0	0	0	0
	211-240 days	2	0	1	0	0	1	0	0	0	0
	241-270 days	0	0	0	0	0	0	0	0	0	0
Moneyness	X1	8720	1081	1347	1788	1269	1799	1192	167	56	21
	X2	31711	6257	4774	4277	3377	3807	2950	2429	1823	2017
	At the money	34519	7394	4495	3221	2892	3564	3295	2889	3849	2920
	X3	1194	256	199	109	102	218	206	56	32	16
	X4	400	61	61	11	23	161	43	29	6	5

Table 4.2: Data Description for Puts according to Years

Data used for this study are American style options on S&P 500 futures traded on the CME from January 1998 to December 2006. Symbol *X1* in Moneyness stands for $F/K \leq 0.925$; *X2* for $0.925 < F/K \leq 0.975$, *X3* for $1.025 < F/K \leq 1.075$ and *X4* for $F/K \geq 1.075$, where F is a price of the futures contract and K is a strike price of the option. Puts are in the money for $F/K < 1$.

		Dataset									
		All	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Number of Put Options		101010	20495	14692	12365	10532	11422	9750	7588	6778	7388
Number of Strike Prices		183	92	98	92	98	99	88	67	62	77
Average Difference between the Trade of Option and Future		5 seconds	4 seconds	4 seconds	4 seconds	5 seconds	4 seconds	5 seconds	6 seconds	6 seconds	6 seconds
Maturity	**Range**	1-265 days	1-204 days	1-202 days	1-265 days	1-176 days	1-181 days	1-156 days	1-185 days	1-147 days	1-174 days
	1-30 days	57367	12429	8258	6845	5950	5762	5135	4097	4035	4856
	31-60 days	27801	4952	3918	3352	2619	4043	3034	2331	1800	1752
	61-90 days	12452	2193	2092	1786	1408	1317	1293	947	826	600
	91-120 days	3227	853	408	367	545	272	286	206	115	175
	121-150 days	126	58	22	9	5	20	1	5	2	4
	151-180 days	29	9	3	3	5	7	1	0	0	1
	181-210 days	6	1	1	1	0	1	0	2	0	0
	211-240 days	0	0	0	0	0	0	0	0	0	0
	241-270 days	2	0	0	2	0	0	0	0	0	0
Moneyness	X1	333	47	13	96	116	43	18	0	0	0
	X2	1358	285	119	198	316	254	69	58	16	43
	At the money	29562	5798	3496	3049	2941	3263	2872	2848	2742	2553
	X3	29368	5859	4027	3547	2678	2754	2804	2502	2314	2883
	X4	40389	8506	7037	5475	4481	5108	3987	2180	1706	1909

for fitted implied price change volatility. By fitted implied price change volatility I mean the price change volatility calculated from the estimated relation with implied volatility[13].

As a continuation of Essay 2, I also test the hedging performance of these volatility measures by using greeks adjusted by regression coefficients (Equation 3.6b). To ensure out-of-sample testing, the sample period is divided to two parts: the estimation period from January 1998 to December 1998 and the testing period from January 1999 to December 2006 (description statistics for data used for regression in the testing period is shown in Table 4.3).

4.2.1.1 Volatility Averages

I estimate AVIV, AVPCIV and AV500 during the estimation period and use these averages to set up hedges in the tested period. Estimated volatility averages are shown in Table 4.4. The AVPCIV for calls is 0.1732 while the AVIV is 0.1958. The magnitudes are larger for puts with AVPCIV being 0.3251 and AVIV being 0.3016. The AV500 for the out-of-sample period is 0.2449.

4.2.1.2 Volatility Updating

Consistent with PBS, I also update volatilities daily. This means I estimate the IV and PCIV for a specific contract. When using IV to compute the hedge, the volatility is estimated when the hedge is initiated and hedge results are calculated on the following day. For example, if the one day hedge is initiated on Monday, contract IV is estimated on Monday and hedge results are calculated on Tuesday. For FPCIV, I estimate the relation between the PCIV and contract IV during the estimation period (Table 4.5). The adjusted R^2 is 0.5525 for puts and 0.0166 for calls. Then I use this relation to calculate the FPCIV when the hedge is initiated.

I also update PCIV directly. A valid observation requires that a contract trade on three consecutive days. For example, PCIV would be estimated on Monday and Tuesday. The hedge is

[13] The relation of implied price change volatility with implied volatility is estimated according to the equation 2.12.

Table 4.3: Data Description of Datasets Used for Regressions

Datasets contain contracts of the same strike price and expiration traded on consecutive days (Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday or Thursday and Friday). Data are pit-traded American style options on S&P 500 futures traded on CME from January 1999 to December 2006. The symbol *X1* in Moneyness stands for $F/K < 0.925$; *X2* for $0.925 < F/K < 0.975$, *X3* for $1.025 < F/K < 1.075$ and *X4* for $F/K > 1.075$, where F is a price of the futures contract and K is a strike price of the option. Calls (puts) are *in- (out-of-) the-money* for $F/K > 1$.

		Calls	Puts
Number of Calls		8098	0
Number of Puts		0	11008
Number of Strike Prices		161	151
Maturity	**Range (Days)**	1 – 113 Days	1 – 113 Days
	1-30 days	5766	5766
	31-60 days	2367	3346
	61-90 days	1017	1697
	91-120 days	129	199
	121-150 days	0	0
	151-180 days	0	0
	181-210 days	0	0
	211-240 days	0	0
	241-270 days	0	0
Moneyness	**X1**	1192	61
	X2	3488	131
	At-the-money	3253	2843
	X3	141	3094
	X4	24	4879

Table 4.4: Average Volatilities Used for Calculations of Greeks for Regressions

Averages of different measures of volatilities are estimated from the S&P 500 futures options from January 1998 to December 1998. Datasets for one and two day lags are used to estimate appropriate parameters. Historical volatility (AV500) for this period is 0.2449.

	Number of Records Used for Estimation	Average of Implied Price Change Volatility (AVPCIV)	Average of Implied Volatility (AVIV)
Calls			
One Day Lags	372	0.1732	0.1958
Puts			
One Day Lags	531	0.3251	0.3016

Table 4.5: The Relation between Implied Price Change Volatility and Implied Volatility

The relation for fitted implied price change volatility (FPCIV) is estimated from January 1998 to December 1998. The estimation model *is* $\sigma_{PC} = \alpha + \beta\ \sigma_{PCIV} + \varepsilon$, where σ_{PC} is implied price change volatility and σ_{IV} is implied volatility.
*** Refers to the significance at 99% level, ** to the significance at 95% level and * to the significance at 90% level.

		Number of Records Used for Estimation	α	β	Adjusted R^2
Calls					
	One Day Lags	372	0.12675***	0.23747***	0.0166
Puts					
	One Day Lags	531	-0.01629	1.13164***	0.5525

initiated on Tuesday and the out-of-sample observation is the hedge error observed on Wednesday. Identifying contracts that trade on three consecutive days leads to smaller dataset. Therefore, hedges set up with daily updating of PCIV are executed on a smaller dataset (924 observations).

4.3 Hedging Price Changes

Risk-free returns require that asset price changes, net of deterministic drift, be matched by an equivalent negative price change in the hedging portfolio. The high R^2s observed in regressions of option price changes on price changes of the underlying asset (Essay 2,Equations 3.7a and 3.7b) suggest that it should be possible to effectively hedge option prices in a dynamic hedging environment.

Suppose that a position in an over-the-counter option is to be hedged by the underlying asset. The typical strategy consists of establishing a short synthetic portfolio with a delta that matches that of the option. Under Black's assumptions, this strategy gives risk-free yields when the position is dynamically adjusted at each instant to maintain the match. Continuous hedging is not possible because of institutional features and is not economical in any case because of high transactions costs.

In fact, there are several possible sources of hedging error. These errors are the result of discrete schemes, the omission of additional state variables and stochastic process risk. Because hedges are adjusted at discrete time increments, second order changes in the price of the underlying contribute to both drift and volatility. This effect is sometimes mitigated by including a second option that is exchange traded in the hedging portfolio. This option has an exercise price that differs from that of the option being hedged. Positions in the synthetic portfolio are determined so that both the delta and gamma of the synthetic portfolio match that of the option to be hedged. This strategy is referred to as delta-gamma hedging.

Another significant source of error in an equity hedge is the omission of a second state variable such as stochastic volatility. This error can be addressed by adding an exchange traded options that is chosen to match vega[14]. This strategy is referred to as delta-vega hedging.

Adding two exchange traded options to the synthetic portfolio to account for discreteness and stochastic volatility is referred to as delta-gamma-vega hedging. The setup for the synthetic hedging portfolio (V) is as follows:

$$V = \alpha_1 f + \alpha_2 X + \alpha_3 Y + B \tag{4.8}$$

$$dV = \alpha_1 dF + \alpha_2 dX + \alpha_3 dY + rB, \tag{4.9}$$

where f is the value of the underlying futures contract, F is the futures price, r is the risk free rate, X and Y are exchange traded options and B is a default-free bond. Matching delta, gamma and vega implies

$$\begin{pmatrix} 1 & X_F & Y_F & 0 \\ 0 & X_{FF} & Y_{FF} & 0 \\ 0 & X_\sigma & Y_\sigma & 0 \\ 0 & X & Y & B \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{pmatrix} = \begin{pmatrix} -C_F \\ -C_{FF} \\ -C_\sigma \\ -C \end{pmatrix}, \tag{4.10}$$

where C is the option to be hedged. The first row matches delta, the second row gamma, the third row vega and the fourth row establishes the zero-financing condition. Solutions are as

[14] Vega is the rate of change in the price of an option with respect to volatility

follows:

$$\begin{array}{lllll}
\hline
 & \Delta & \Delta\Gamma & \Delta\nu & \Delta\Gamma\nu \\
\hline
\alpha_1 & -C_F & -C_F + \frac{X_F}{X_{FF}}C_{FF} & -C_F + \frac{X_F}{X_\sigma}C_\sigma & -C_F + \frac{A}{D}C_{FF} + \frac{B}{D}C_\sigma \\
\alpha_2 & 0 & -\frac{C_{FF}}{X_{FF}} & 0 & \frac{Y_\sigma}{D}C_{FF} - \frac{Y_{FF}}{D}C_\sigma \\
\alpha_3 & 0 & 0 & -\frac{C_\sigma}{X_\sigma} & -\frac{X_\sigma}{D}C_{FF} + \frac{X_{FF}}{D}C_\sigma \\
B & -C & -C-\alpha_2 X & -C-\alpha_3 X & -C-\alpha_2 X - \alpha_3 Y \\
\hline
\end{array} \tag{4.11}$$

where $A = -X_F Y_\sigma + X_\sigma Y_F$, $B = X_F Y_{FF} - X_{FF} Y_F$ and $D = -X_{FF} Y_\sigma + Y_{FF} X_\sigma$.

4.3.1 Test Setup

I examine the delta hedge, the delta-gamma hedge and the delta-vega hedge using a variety of volatility inputs. In addition to different volatility inputs, hedges using greeks adjusted with regression coefficients and regression coefficients accounting for moneyness and maturity effects are also examined (See Essay 2).

The delta-gamma-vega hedge requires that the underlying and two exchange traded options trade on consecutive days. The dataset and filters used did not produce a sufficient number of observations to consider a reliable test of the delta-gamma-vega hedge.

4.3.1.1 Error Metrics

The hedging performance is evaluated based on several error metrics standard in the literature. They are:

$$\text{Average Error} \quad : \quad \overline{e} = \frac{\Sigma e}{n} \tag{4.12}$$

$$\text{Error volatility} \quad : \quad \sigma_{error} = \frac{\Sigma(e-\overline{e})^2}{n} \tag{4.13}$$

$$\text{Average Absolute Error} \quad : \quad |\overline{e}| = \frac{\Sigma\,|e|}{n} \tag{4.14}$$

$$\text{Average Relative Error} \quad : \quad \overline{e}_{rel} = \frac{|\overline{e}|}{\overline{C}}.100 \tag{4.15}$$

$$\text{Relative Volatility} \quad : \quad R^2 = 1 - \frac{var(e)}{var(dC)}, \tag{4.16}$$

where $\overline{C}$ is the average price of option hedged

4.3.1.2 Delta Hedge

The delta hedge consists of the underlying asset and a risk free bond. The hedging portfolio is formed on day one using parameters available or estimated on day one (when the hedge is initiated). Then, then the hedge is liquidated on day two. The price change in the hedging portfolio $dV = -C_F dF + rB$[15] is recorded and the hedging error is calculated as $e = dC + dV$, where dC is the observed change in the price of the hedged option.

4.3.1.3 Delta-Gamma Hedge

The delta-gamma hedge consists of the underlying, risk free bond and an additional traded option. I denote the option to be hedged as the target option. The second traded option that is used for hedging is denoted at the instrumental option . The hedging requires that each target option $C_1(S, K, T)$ be paired with an instrumental option $C_2(S, K^*, T)$. I find an instrumental option such that the value of hedging portfolio must be computed at approximately the same time as the option to be hedged (the difference between the trade of the instrumental and target option cannot exceed 30 minutes). I choose the instrumental option to be an option with the same maturity T, but different strike price K^*. When more than one option is available, I choose an option that trades closer to at-the-money.

The hedging portfolio is created on day one using parameters from day one and the hedge is liquidated on day two. The price change of the hedging portfolio is

$$dV = (-C_F + \frac{X_F}{X_{FF}} C_{FF})dF - \frac{C_{FF}}{X_{FF}} dX + r(-C + \frac{C_{FF}}{X_{FF}} X).^{16} \tag{4.17}$$

The hedging error is calculated as

$$e = dC + dV. \tag{4.18}$$

15 For hedges using greeks adjusted regression coefficients, the price change of the hedging portfolio is $dV = -\beta_1 C_F dF + rB$, where β_1 is estimated in the testing period according to the equation 3.6b.

4.3.1.4 Delta-Vega Hedge

The delta-gamma hedge consists of the underlying, a risk free bond and one instrumental option. The approach is analogous to the delta-gamma hedge. The change in the value of hedged portfolio is

$$dV = (-C_F + \frac{X_F}{X_\sigma}C_\sigma)dF - \frac{C_\sigma}{X_\sigma}dY + r(-C + \frac{C_\sigma}{X_\sigma}X). \tag{4.19}$$

The same error metrics are calculated as for delta and delta-gamma hedges.

4.4 Results

Tables 4.6 through 4.8 give results for calls for the delta hedge, the delta-gamma hedge and the delta-vega hedge. The corresponding results for puts are given in Tables 4.9 through 4.11.

4.4.1 Calls

In the tables that follow, I focus primarily on the average relative error metric (Equation 4.15) and R^2(Equation 4.16). The average relative metric is perhaps most commonly used because it is more easily interpreted since it is in percentages.

4.4.1.1 Delta Hedges

Results for delta hedge for calls are given in Table 4.6. *Ex ante*, one would expect that daily updated volatility and deltas adjusted by regression coefficients would provide the best hedging results. Results are consistent with this notion. Generally, hedges using volatility averages perform worse than hedges using daily updated volatilities. The worst performance of all volatility measures are hedges based on AV500 with an average relative error of 10.8% (Panel C). The hedges using daily updated IV and FPCIV perform better (average relative errors are 7.9% and 8.3%, Panels D and E, respectively).

Delta hedges using PCIV directly far outperform other volatility measures (Panel F). The average relative error is 5.54% and $R^2 = 0.95$ (compared to 0.89 for IV). The difference between

Table 4.6: Delta Hedge for One Day Price Changes of Call Options

Data used for the testing of the delta hedge were American style call options on S&P 500 futures traded on CME from January 1999 to December 2006. The hedging portfolio $V = \alpha_I f + B$ was created and the one day price change in the hedging portfolio was calculated as $dV = -C_F dF + + rB$. The V refers to the price of the hedging portfolio, C_F to the Gamma of the option, F to the futures price, r to the risk free rate and B to the risk free bond. Detail description of the Delta hedge is in the section 4.3.1.2.

Model	Average Error	Error Volatility	Average Absolute Error	Average Relative Error [%]	Relative Volatility (R^2)
Panel A. Average of Implied Price Change Volatility (AVPCIV)					
Without Adjustment	-0.5192	2.2017	1.066	8.7215	0.8784
Adjusted by Regression Coefficients	-0.5327	2.1615	1.056	8.6393	0.8806
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.5327	2.1616	1.0564	8.6394	0.8806
Panel B. Average of Implied Volatility (AVIV)					
Without Adjustment	-0.5369	2.2606	1.1154	9.1219	0.8751
Adjusted by Regression Coefficients	-0.5554	2.1538	1.0822	8.8502	0.8810
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.5550	2.1531	1.0824	8.8517	0.8811
Panel C. Average of S&P 500 Index Volatility (AV500)					
Without Adjustment	-0.5726	2.8735	1.3168	10.7690	0.8413
Adjusted by Regression Coefficients	-0.5995	2.4667	1.1990	9.8054	0.8637
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.5988	2.4648	1.2001	9.8148	0.8638
Panel D. Contract Implied Volatility (IV)					
Without Adjustment	-0.5438	1.9281	0.9636	7.8800	0.8935
Adjusted by Regression Coefficients	-0.5591	1.8668	0.9425	7.7077	0.8969
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.5590	1.8664	0.9425	7.7078	0.8969
Panel E. Fitted Implied Price Change Volatility (FPCIV)					
Without Adjustment	-0.5223	2.0365	1.0144	8.2958	0.8875
Adjusted by Regression Coefficients	-0.5341	2.0095	1.0067	8.2331	0.8890
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.5344	2.0100	1.0068	8.2335	0.8890
Panel F. Implied Price Change Volatility					
Without Adjustment	-0.1343	0.7656	0.5744	5.5378	0.9489

Table 4.7: Delta-Gamma Hedge for One Day Price Changes of Call Options

Data used for the testing of the delta hedge were American style call options on S&P 500 futures traded on CME from January 1999 to December 2006. The hedging portfolio was created on day one using parameters of day one according to section two and liquidated on day two. Detail description of the Delta-Gamma hedge is in the section 4.3.1.3.

Model	**Average Error**	**Error Volatility**	**Average Absolute Error**	**Average Relative Error [%]**	**Relative Volatility (R^2)**
Panel A. Average of Implied Price Change Volatility					
Without Adjustment	-0.1625	2.3418	0.9352	7.3948	0.9811
Adjusted by Regression Coefficients	-0.1656	2.3256	0.9358	7.3999	0.9812
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.1656	2.3255	0.9358	7.3999	0.9812
Panel B. Average of Implied Volatility					
Without Adjustment	-0.1732	1.8365	0.9020	7.1329	0.9852
Adjusted by Regression Coefficients	-0.1767	1.8479	0.9105	7.2000	0.9851
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.1766	1.8475	0.9103	7.1984	0.9851
Panel C. Average of S&P 500 Index Volatility					
Without Adjustment	-0.1799	1.7711	0.9044	7.1516	0.9857
Adjusted by Regression Coefficients	-0.1832	1.8640	0.9304	7.3572	0.9849
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.1831	1.8610	0.9296	7.3512	0.9850
Panel D. Contract Implied Volatility					
Without Adjustment	-0.2017	1.5967	0.8375	6.6225	0.9871
Adjusted by Regression Coefficients	-0.2027	1.6210	0.8471	6.6982	0.9869
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.2027	1.6207	0.8470	6.6973	0.9869
Panel E. Implied Price Change Volatility Calculated from the Relation with Contract Implied Volatility					
Without Adjustment	-0.1830	1.7461	0.8754	6.9225	0.9859
Adjusted by Regression Coefficients	-0.1852	1.7399	0.8780	6.9424	0.9859
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.1852	1.7398	0.8780	6.9429	0.9859

Table 4.8: Delta-Vega Hedge for One Day Price Changes of Call Options

Data used for the testing of the delta hedge were American style call options on S&P 500 futures traded on CME from January 1999 to December 2006. The hedging portfolio was created on day one using parameters of day one according to section two and liquidated on day two. Detail description of the Delta-Vega hedge is in the section 4.3.1.4.

Model	**Average Error**	**Error Volatility**	**Average Absolute Error**	**Average Relative Error [%]**	**Relative Volatility (R^2)**
Panel A. Average of Implied Price Change Volatility					
Without Adjustment	-0.1645	2.3412	0.9345	7.3897	0.9811
Panel B. Average of Implied Volatility					
Without Adjustment	-0.1755	1.8348	0.9019	7.1320	0.9852
Panel C. Average of S&P 500 Index Volatility					
Without Adjustment	-0.1823	1.7677	0.9041	7.1495	0.9857
Panel D. Contract Implied Volatility					
Without Adjustment	-0.1928	1.6437	0.8477	6.7034	0.9867
Panel E. Implied Price Change Volatility Calculated from the Relation with Contract Implied Volatility					
Without Adjustment	-0.1845	1.7701	0.8797	6.9564	0.9857
Panel F. Implied Price Change Volatility					
Without Adjustment	-0.4410	0.7728	0.7336	3.3350	0.9827

the performance of PCIV estimated directly and FPCIV is not surprising due to the low R^2 (0.0166).

4.4.1.2 Delta Hedges Adjusted by Regression Coefficients

Hedging performance improved only slightly for hedges using delta's adjusted by regression coefficients. The improvement was the most pronounced for hedges based on the AV500 (average relative error decreased from 10.8% to 9.8%) There was no improvement using greeks adjusted by regression coefficients taking when moneyness and maturity controls were added.

4.4.1.3 Delta-Gamma and Delta-Vega Hedges

Including an additional option in the hedging portfolio improved the hedging performance of all volatility measures (Tables 4.7 and 4.8). The R^2s are 0.98 for both delta-gamma and delta-vega hedges. For the delta-gamma hedge, the lowest average relative error were produced using IV (6.62%) and FPCIV (9.92%). Regression adjusted deltas and gammas do not yield any improvement in the hedging performance. The results for delta-vega hedges are similar. The best performance was observed for hedges with IV (average relative error $= 6.7\%$) and FPCIV (average relative error 7%).

4.4.2 Puts

Theoretically, the performance of put hedges should not differ from that of call hedges. The underlying process is assumed to be geometric Brownian motion and the pricing models are built on the same assumptions. However, point estimates of implied put volatilities are about 50% larger than implied call volatilities. Therefore, expectations are that the performance of put hedges are likely to be notably different.

4.4.2.1 Delta Hedges and Regression Adjusted Hedges

Results for delta hedges for puts are given in Table 4.9. Delta hedges for puts generally have lower average relative errors than calls. The best performing put hedge has relative error 7.3%

Table 4.9: Delta Hedge for One Day Price Changes of Put Options

Data used for the testing of the delta hedge were American style put options on S&P 500 futures traded on CME from January 1999 to December 2006. The hedging portfolio $V = \alpha_I f + B$ was created and the one day price change in the hedging portfolio was calculated as $dV = -C_F dF + + rB$. The V refers to the price of the hedging portfolio, C_F to the Gamma of the option, F to the futures price, r to the risk free rate and B to the risk free bond. Detail description of the Delta hedge is in the section 4.3.1.2.

Model	**Average Error**	**Error Volatility**	**Average Absolute Error**	**Average Relative Error [%]**	**Relative Volatility (R^2)**
Panel A. Average of Implied Price Change Volatility					
Without Adjustment	-0.6331	4.0752	1.0752	8.7157	0.8014
Adjusted by Regression Coefficients	-0.6318	4.0870	1.0791	8.7471	0.8008
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.6317	4.0878	1.0793	8.7491	0.8008
Panel B. Average of Implied Volatility					
Without Adjustment	-0.6283	4.0308	1.0551	8.5530	0.8036
Adjusted by Regression Coefficients	-0.6254	4.0517	1.0609	8.5999	0.8026
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.6253	4.0522	1.0610	8.6009	0.8025
Panel C. Average of S&P 500 Index Volatility					
Without Adjustment	-0.6155	4.1220	1.0571	8.5686	0.7991
Adjusted by Regression Coefficients	-0.6082	4.1457	1.0576	8.5731	0.7980
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.6083	4.1453	1.0576	8.5727	0.7980
Panel D. Contract Implied Volatility					
Without Adjustment	-0.6060	3.6905	0.9019	7.3110	0.8202
Adjusted by Regression Coefficients	-0.5986	3.7137	0.8975	7.2752	0.8190
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.5984	3.7153	0.8976	7.2762	0.8190
Panel E. Implied Price Change Volatility Calculated from the Relation with Contract Implied Volatility					
Without Adjustment	-0.6102	3.6759	0.9096	7.3735	0.8209
Adjusted by Regression Coefficients	-0.6046	3.7004	0.9126	7.3973	0.8197
Adjusted by Regression Coefficients, Moneyness and Maturity	-0.6044	3.7027	0.9129	7.4002	0.8196
Panel F. Implied Price Change Volatility					
Without Adjustment	-0.6120	1.4330	0.9344	9.3589	0.9088

(Panel D), versus 7.8% for the best performing call hedge. However, the best performing put hedges have lower R^2s (0.82 versus 0.89). Delta hedges set up using daily updated volatility parameters outperform hedges using average volatilities. The best performing hedges have average relative error 7.3% (IV, Panel D) and average relative error 7.4% (FPCIV, Panel E). The use of delta's adjusted by regression coefficient's does not improve the performance of these hedges. Delta hedges based on directly estimated PCIV (Panel E) produce the highest R^2s (0.91) but have higher average relative errors (9.4%).

4.4.2.2 Delta-Gamma and Delta-Vega Hedges

Delta-gamma (Table 4.10) and delta-vega (Table 4.11) hedges for puts are less effective than the corresponding hedges for calls. The best results are achieved by hedges based on daily updated IV and FPCIV with average relative errors of 8.8% and R^2s of 0.97 (Table 7, panels D and E).

4.5 Conclusion

This study compares the performance of hedges based on different volatility measures in out-of-sample hedging applications. The volatility measures used are historical volatility (AV500) and averages of implied volatility (AVIV) and implied price change volatility (AVPCIV). In addition to these measures, hedges are also form using daily updated implied volatility (IV), fitted implied volatility (FPCIV) and directly estimated implied price change volatility (PCIV).

Data used for this study are options on the S&P 500 futures contract from January 1998 to December 2006. The data are divided to two periods: an estimation period (January 1998 to December 1998) and out-of-sample testing period (January 1999 to December 2006). Volatility averages and the relation for calculation of fitted implied price change volatility (FPCIV) are first estimated. Then the hedging performance using each volatility measure is evaluated in the out-of-sample testing period. The hedges are set up on day one using greeks calculated based on

Table 4.10: Delta-Gamma Hedge for One Day Price Changes of Put Options

Data used for the testing of the delta hedge were American style put options on S&P 500 futures traded on CME from January 1999 to December 2006. The hedging portfolio was created on day one using parameters of day one according to section two and liquidated on day two. Detail description of the Delta-Gamma hedge is in the section 4.3.1.3.

Model	**Average Error**	**Error Volatility**	**Average Absolute Error**	**Average Relative Error [%]**	**Relative Volatility (R^2)**
Panel A. Average of Implied Price Change Volatility					
Without Adjustment	0.1623	2.8371	0.7955	10.2838	0.9565
Adjusted by Regression Coefficients	0.1634	2.7689	0.7944	10.2690	0.9575
Adjusted by Regression Coefficients, Moneyness and Maturity	0.1634	2.7650	0.7943	10.2681	0.9576
Panel B. Average of Implied Volatility					
Without Adjustment	0.1407	3.4497	0.8064	10.4239	0.9470
Adjusted by Regression Coefficients	0.1442	3.1996	0.8032	10.3827	0.9509
Adjusted by Regression Coefficients, Moneyness and Maturity	0.1443	3.1954	0.8031	10.3820	0.9510
Panel C. Average of S&P 500 Index Volatility					
Without Adjustment	0.0402	11.0020	0.9045	11.6927	0.8311
Adjusted by Regression Coefficients	0.0655	7.3411	0.8852	11.4433	0.8873
Adjusted by Regression Coefficients, Moneyness and Maturity	0.0653	7.3588	0.8853	11.4447	0.8870
Panel D. Contract Implied Volatility					
Without Adjustment	0.1273	1.9183	0.6809	8.8016	0.9706
Adjusted by Regression Coefficients	0.1276	1.9235	0.6829	8.8274	0.9705
Adjusted by Regression Coefficients, Moneyness and Maturity	0.1276	1.9238	0.6830	8.8284	0.9705
Panel E. Implied Price Change Volatility Calculated from the Relation with Contract Implied Volatility					
Without Adjustment	0.1371	1.8098	0.6811	8.8051	0.9722
Adjusted by Regression Coefficients	0.1371	1.8091	0.6805	8.7964	0.9722
Adjusted by Regression Coefficients, Moneyness and Maturity	0.1371	1.8092	0.6805	8.7961	0.9722

Table 4.11: Delta-Vega Hedge for One Day Price Changes of Put Options

Data used for the testing of the delta hedge were American style put options on S&P 500 futures traded on CME from January 1999 to December 2006. The hedging portfolio was created on day one using parameters of day one according to section two and liquidated on day two. Detail description of the Delta-Vega hedge is in the section 4.3.1.4.

Model	**Average Error**	**Error Volatility**	**Average Absolute Error**	**Average Relative Error [%]**	**Relative Volatility (R^2)**
Panel A. Average of Implied Price Change Volatility					
Without Adjustment	0.1665	2.8707	0.7984	10.3207	0.9559
Panel B. Average of Implied Volatility					
Without Adjustment	0.1444	3.5393	0.8106	10.4781	0.9457
Panel C. Average of S&P 500 Index Volatility					
Without Adjustment	0.0274	13.9582	0.9232	11.9334	0.7857
Panel D. Contract Implied Volatility					
Without Adjustment	0.1701	2.1049	0.7178	9.2789	0.9677
Panel E. Implied Price Change Volatility Calculated from the Relation with Contract Implied Volatility					
Without Adjustment	0.1827	2.0182	0.7246	9.3671	0.9690

parameters estimated on day one. The hedges are liquidated on day two and hedging errors are quantified using standard metrics.

As expected, delta-gamma and delta-vega hedges lead to lower average relative errors and higher R^2s than delta hedges for calls. The hedges using daily updated volatilities (IV, FPCIV, PCIV) consistently outperform average volatilities (AV500, AVIV, AVPCIV). The best performance for delta hedges are based on directly estimated implied price change volatilities (PCIV). Their average relative errors are even smaller than those of delta-gamma and delta-vega hedges for calls. This finding suggests that using implied volatilities estimated from price changes rather than price levels may result in more effective hedges for call portfolios.

Results on puts are similar but the benefits of the use of implied price change volatility (PCIV) are more ambiguous. Implied price change volatility (PCIV) produces hedges with the highest R^2s but their average relative errors are higher than hedges based on other volatility measures.

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Chapter 5 Summary and Conclusions

In my dissertation, I develop the concept of implied price changes volatility. Implied price change volatility equates the observed price change in the option to the model price change. Implied price change volatility is analogous to implied volatility but it is estimated from option price changes instead of price levels. The importance of this concept arises from its direct use in hedging applications. The careful choice of a volatility measure is important in creating effective hedges.

Data used for this study are pit traded transactions data on American options on the S&P 500 futures and their underlying from 1998 to 2006. My findings indicate that the implied price change volatility shows similar time series behavior as implied volatility and moving average of historical volatility. The implied price change volatility, however, is more disperse than other volatility measures. As with implied volatility, the implied price change volatility estimated from call prices is smaller than that estimated from put prices. It may be attributed to different demand curves for call and options. Investors seeking protection of their portfolios may bid up prices of put options which results in higher volatilities estimated from put options (Bollen and Whaley (2004)). I find that the difference between implied price change volatilities estimated from put and call options is larger than this difference for implied volatility. Moneyness and maturity effects of implied price change volatility are similar to those found in implied volatility. These effects are more pronounced for calls than for puts.

One of the binding conditions for the use of implied price change volatility is the requirement for large datasets. To estimate this volatility parameter, one needs to gather information on consecutive daily trades on the same contract (i.e., contracts with the same strike price and expiration). For example, a valid observation would be near synchronous transactions on an option and underlying on the same contract on both Monday and Tuesday. This may be problem for less liquid options or shorter datasets. Therefore, to estimate implied price change volatility indirectly, I investigate whether there is a significant relation between implied price change volatility and implied volatility. I estimate and use this relation in regression and hedging applications.

In the second part of the study, I examine the performance of different volatility measures in regression setup. I use Ito's expansion of the Black's model and investigate how different volatility measures are able to explain changes in option prices. Specifically, I compare the performance of the average implied price change volatility, the average of implied volatility and the average of historical volatility. In addition, I also examine the performance of daily updated implied price change volatility and fitted implied price change volatility calculated from its relation with implied volatility. I find that Black's model overestimates changes in S&P 500 futures call options and underestimates changes in put options. However, regressions based on all volatility measures achieve high R^2s of more than 95%. The highest R^2s are the result of regressions based on daily updated implied volatilities and fitted implied price change volatilities.

In the third part of my dissertation, I examine how different volatility measures influence out-of-sample hedging performance. I create delta, delta-gamma and delta-vega hedges. A delta hedge can be created from the underlying and a risk free bond. A delta-gamma or delta-vega hedges requires an additional traded option in the hedging portfolio. Other things equal, the simpler delta hedge is preferred for statistical reasons since the data set is larger.

For creating hedging portfolios, I use the previously mentioned volatility measures and also directly estimated implied price change volatilities. My findings indicate that hedging performance improves with daily updating of volatilities, i.e., the best performing hedges are based on daily updated implied volatility and daily updated implied price change volatility. For example, I use the Monday to Tuesday implied price change estimate as input to the Tuesday to Wednesday hedge. Using these directly estimated implied price change volatilities greatly improves the hedging performance of delta hedges. In fact, the delta hedge based on daily updated implied price change volatility is comparable to delta-gamma or delta-vega hedges for calls. This finding suggests that the volatility information extracted from price changes contains additional information that is beneficial for predicting price changes in call options.

Appendix A Abbreviations and Definitions of Volatility Measures

Implied volatility (IV): Volatility implied from an option price using Black-Scholes model.

Price change implied volatility (PCIV): Volatility implied from a change in the option price using the American version of binomial model.

Fitted implied price change volatility (FPCIV): Implied price change volatility calculated from the estimated relation with implied volatility (Equation 2.12).

Historical volatility of the S&P 500 index (AV500)

Average of implied volatility (AVIV)

Average of implied price change volatility (AVPCIV)

Vita

Jitka Hilliard was born in Prague, Czech Republic. In the Czech Republic she earned a Master of Science degree in biochemistry at the Charles University and an Engineer and a Doctor of Philosophy degrees at the Institute of Chemical Technology. She obtained a Master of Science degree in business administration at Louisiana State University in 2006.

Her research interests includes option pricing, investment and corporate finance. She coauthored and presented papers at the national meeting of the Financial Management Association at Salt Lake City in 2006 and at the international meeting of the Financial Management Association in Prague in 2008.

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