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Gravity and handedness of photons

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Abstract

Vacuum fluctuations of quantum fields are altered in presence of a strong gravitational background, with important physical consequences. We argue that a non-trivial spacetime geometry can act as an optically active medium for quantum electromagnetic radiation, in such a way that the state of polarization of radiation changes in time, even in the absence of electromagnetic sources. This is a quantum effect, and is a consequence of an anomaly related to the classical invariance under electric-magnetic duality rotations in Maxwell theory.

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The presence of a gravitational background introduces non-trivial effects in the propagation of electromagnetic radiation, with fundamental implications. The gravitational redshift and the deflection of light rays by massive bodies played a pivotal role in the birth of general relativity [1], and are of great importance in many areas of gravitation, cosmology, and astrophysics. It is also well-known that remarkable new features appear in the physics of quantum fields when they propagate under the influence of gravity. Renowned examples are the spontaneous creation of quanta by the expanding universe [2], the thermal radiation by black holes produced by gravitational collapse [3], and the generation of primordial density perturbations by inflation [4]. In this essay, we discuss a new effect on quantum electromagnetic fields propagating in curved spacetimes.

Our discussion resembles the study of the chiral anomaly for fermions. Recall that the theory of a zero-mass, charged spin-1/2 fermion satisfying the Dirac equation in Minkowski spacetime, $\gamma^\mu \partial_\mu \Psi = 0$, is invariant under chiral transformations: $\Psi \to e^{i\theta\gamma_5}\Psi$. Noether's theorem then implies that the so-called axial-vector current, $j_5^\mu = \bar{\Psi}\gamma^\mu\gamma_5\Psi$, is conserved; $\partial_\mu j_5^\mu = 0$. This classical symmetry is present even if we let fermions interact with a classical electromagnetic field $F_{\mu\nu}$. But it is well-known that quantum fluctuations of the Dirac field lead to an anomaly [5]; the vacuum expectation value of $\partial_\mu j_5^\mu$ does not vanish

$$\langle \partial_\mu j_5^\mu \rangle = -\frac{q^2\hbar}{8\pi^2} F_{\mu\nu}{}^\star F^{\mu\nu} = -\frac{q^2\hbar}{2\pi^2} E_\mu B^\mu \,, \tag{1}$$

where q is the charge of the fermion, and ${}^\star F^{\mu\nu} = 1/(2\sqrt{-g})\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of $F_{\mu\nu}$. In the last equality we have written the Lorentz invariant quantity $F_{\mu\nu}{}^\star F^{\mu\nu}$ in terms of the electric and magnetic parts of $F_{\mu\nu}$. This anomaly has important physical implications, e.g. it provides the mechanism to explain the pion decay to two photons $\pi^0 \to \gamma\gamma$.

With this result in mind, we now analyze a different physical situation: the theory of a source-free electromagnetic field propagating on a classical gravitational background. The corresponding Maxwell action is invariant under duality transformations [6]: $F_{\mu\nu} \to F_{\mu\nu}\cos\theta + {}^\star F_{\mu\nu}\sin\theta$. In terms of the vector potential, the infinitesimal transformation reads $\delta A_\mu = \theta\, Z_\mu$, where Z_μ, when evaluated on-shell, is the potential of the dual field ${}^\star F_{\mu\nu} = \nabla_\mu Z_\nu - \nabla_\nu Z_\mu$. Again, Noether's techniques provide a conserved current j_D^μ, whose expression on-shell is given by [7]

$$j_D^\mu = \frac{1}{2}\left(A_\nu {}^\star F^{\mu\nu} - F^{\mu\nu} Z_\nu\right) \,. \tag{2}$$

Q_D, the spatial integral of the time component of j^μ_D on any hyper-surface, is a conserved quantity of the classical theory in arbitrary spacetime geometries. In Minkowski spacetime, Q_D represents the difference in amplitude between right and left polarized components of the electromagnetic field — this is the V-Stokes parameter. This conservation law says that the state of polarization of light is a constant of motion.

The electromagnetic duality is indeed a remarkable symmetry. It suggested the existence of magnetic charges as a way to retain the symmetry in presence of electromagnetic sources. We argue below, however, that the gravitational interaction spoils the symmetry, even without sources, once quantum fluctuations of the electromagnetic field are taken into account.

That the transformation law for the basic variables A_μ looks complicated—non-local in fact—indicates that the free Maxwell theory is more naturally formulated in terms of other variables, namely self- and anti-self dual fields. We will reformulate the theory in terms of these variables, and in doing so we will see an interesting analogy with the dynamics of fermions and the associated chiral symmetry. Powerful techniques developed for fermions will then become available to study the electromagnetic duality in the quantum theory. We focus first on Minkowski spacetime, and later allow arbitrary gravitational fields.

Define the complex fields $\vec{H}_\pm := \frac{1}{2}[\vec{E} \pm i\vec{B}]$. It is easy to see that they transform under duality by a simple phase, $\vec{H}_\pm \to e^{\mp i\theta}\vec{H}_\pm$. $\vec{H}_+$ and $\vec{H}_-$ are called the self- and anti-self-dual parts of the electromagnetic field. They transform under the $(1,0)$ and $(0,1)$ representations of the Lorentz group, respectively. Define the associated vector potentials $\vec{H}_\pm =: \pm i\,\vec{\nabla}\times\vec{A}_\pm$. The Maxwell equations, in the radiation gauge, read

$$\vec{\nabla}\times\vec{A}_\pm = \pm i\,\partial_t\vec{A}_\pm\ , \quad \vec{\nabla}\cdot\vec{A}_\pm = 0. \tag{3}$$

These equations can be derived either by directly writing Maxwell's equations $dF = d\,{}^\star F = 0$ in terms of $\vec{A}_\pm$, or as Hamilton's equations for the Maxwell Hamiltonian using $2\vec{A}_-$ and $-\vec{H}_+$—or equivalently $2\vec{A}_+$ and $-\vec{H}_-$—as canonically conjugate variables.

The four equations (3) for $\vec{A}_+$ can be written more compactly as $(\alpha^a)^b{}_i\partial_a A^i_+ = 0$, where the components of the $(\alpha^a)^b{}_i$ matrices are easily extracted from (3)—the spacetime indices a,b run from 0 to 3, and the internal index i runs from 1 to 3. The equations for the anti-self-dual potential $\vec{A}_-$ are the complex-conjugates of the equation for $\vec{A}_+$. Both sets of equations can be combined in

$$\beta^a\partial_a\Psi(x) = 0\ , \tag{4}$$

where we have defined

$$\Psi \equiv \begin{pmatrix} A_+^{\ i} \\ A_{-\,i} \end{pmatrix} \ , \qquad \beta^a \equiv i \begin{pmatrix} 0 & (\bar{\alpha}^a)_b^{\ i} \\ -(\alpha^a)^b_{\ i} & 0 \end{pmatrix} \ . \tag{5}$$

(The bar denotes complex conjugation). The β^a-matrices satisfy the following (anti-) commutation properties: $\bar{\beta}^{(a}\beta^{b)} = -\eta^{ab}\,\mathbb{I}$, $\bar{\beta}^{[a}\beta^{b]} = 2\ \text{diag}\big({}^{+}\Sigma^{ab}, {}^{-}\Sigma^{ab}\big)$, where η_{ab} is the Minkowski metric and ${}^{\pm}\Sigma^{ab}$ are the generators of the $(1,0)$ and $(0,1)$ representations of the Lorentz group, respectively. α^a and β^a are the spin-1 analog of the Pauli $\sigma^a = (I, \vec{\sigma})$ and Dirac γ^a matrices, respectively. Duality transformations are now written as

$$\Psi \to e^{i\theta\beta_5}\Psi = \begin{pmatrix} e^{-i\theta}A_+^i \\ e^{i\theta}A_{-\,i} \end{pmatrix} \ , \tag{6}$$

where $\beta_5 \equiv \frac{i}{16}\epsilon_{\mu\nu\sigma\rho}\bar{\beta}^\mu\beta^\nu\bar{\beta}^\sigma\beta^\rho = \text{diag}(-\mathbb{I}_{3\times 3}, \mathbb{I}_{3\times 3})$.

The space of solutions of (4) is spanned by transverse monochromatic waves oscillating with positive and negative frequencies. The relation between self-duality and helicity is as follows. Self-dual monochromatic waves have (negative) positive helicity for (negative) positive frequency modes, while the opposite occurs for anti-self dual solutions. This is analog to the relation between chirality and helicity for fermions.

The generalization to curved spacetimes is straightforward. The Minkowski metric η_{ab} and the ordinary derivative ∂_a are replaced by the curved metric tensor $g_{\mu\nu}$ and the associated covariant derivative ∇_μ, and the curved spacetime α- and the β-matrices are obtained from the flat spacetime ones by using an orthonormal tetrad or vierbein, $(\alpha^\mu)^\nu_{\ i}(x) = e^\mu_a(x)\, e^\nu_b(x)\, (\alpha^a)^b_{\ i}$.

We now explore the duality symmetry in the quantum theory. We will follow the functional-integral strategy used in [9] to establish the chiral anomaly of fermions. The important question is whether the measure of the path integral respects the symmetry of the action. The calculation follows steps similar to the fermionic case, after replacing all the structures associated to the $(1/2, 0)$ and $(0, 1/2)$ representations of the Lorentz group with the $(1, 0)$ and $(0, 1)$ ones, and taking care of additional subtleties arising from the gauge freedom. The reader is referred to [7] for details. In spite of the differences, the result is remarkably similar to the fermionic chiral anomaly. Namely, quantum fluctuations spoil the classical symmetry and the expectation value of the divergence of (2) —for any vacuum state— becomes

$$\langle \nabla_\mu j^\mu_D \rangle = \frac{\hbar}{24\pi^2} R_{\mu\nu\lambda\sigma}{}^\star R^{\mu\nu\lambda\sigma} = \frac{2\hbar}{3\pi^2}\, E_{\mu\nu}B^{\mu\nu}, \tag{7}$$

where in the last equality we have written the Chern-Pontryagin topological density $R_{\mu\nu\lambda\sigma}{}^{\star}R^{\mu\nu\lambda\sigma}$ in terms of the electric and magnetic parts of the Weyl tensor. Compare this expression with (1). As a consequence, the duality charge Q_D is not conserved in general spacetimes, and its change between any two instants is given by

$$\Delta Q_D = \frac{2\hbar}{3\pi^2}\int_{t_1}^{t_2}\int_{\Sigma} d^4x\sqrt{-g}\, E_{\mu\nu}B^{\mu\nu} \ . \tag{8}$$

An analog of this effect arises for fermions in the creation of pairs from the vacuum by strong electric fields. In this situation the presence of a magnetic field would induce a non-zero net chirality on the particles created, as predicted by (1) [8]. Likewise, apart from gravitational tidal forces produced by $E_{\mu\nu}$, frame dragging effects, described by $B_{\mu\nu}$, are necessary to induce net polarization on the created photons.

To illustrate this, consider the process of collapse of a neutron star into a Kerr black hole. In the vicinity of the region where the event horizon will form, the gravitational field is strongly changing and *spontaneous* creation of photons will occur. Our result indicates that the created photons, when measured far from the star, will carry a net polarization given by ΔQ_D. Numerical simulations indicate that for a neutron star of $M = 1.73$ solar masses and angular momentum $J = 0.36M^2$ the collapse produces around 30 photons per second more with one circular polarization than the other. This process has no classical counterpart and is different from the standard, late-time Hawking radiation, which does not contribute to ΔQ_D.[4]Although this number is small—given the short duration of the gravitational collapse—it is significant if we compare it with the ≈ 20 total photons per second, steadily emitted by the formed black hole via Hawking radiation, with net polarization equal zero [10]. It is expected that ΔQ_D becomes significantly larger in more violent processes, as for instance the collision and merger of two black holes as the ones observed by LIGO [11]. But more importantly, the existence of spontaneous creation of photons implies that the *stimulated* counterpart must exist. Therefore, electromagnetic radiation traveling in spacetimes with a non-zero value of (8), such as the ones mentioned above, will experience a change in its net polarization. This may have implications for CMB photons; their propagation through the large scale structure would not only bend their trajectories but also affect their polarization.

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[4] In fact, for an exact Kerr geometry expression (8) yields zero. Therefore, ΔQ_D comes from the (transient) process of collapse, in contrast to the Hawking effect, which is associated with the final, stationary black hole configuration.

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