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Optimal production and delivery scheduling models for a supply chain system of deteriorating items

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OPTIMAL PRODUCTION AND DELIVERY SCHEDULING MODELS
FOR A SUPPLY CHAIN SYSTEM OF DETERIORATING
ITEMS

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Industrial Engineering

in

The Department of Construction Management and Industrial Engineering

By

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TABLE OF CONTENTS

LIST OF TABLES	v
LIST OF FIGURES	vi
ABSTRACT	vii
CHAPTER 1 INTRODUCTION	1
1.1 Literature Review	2
1.1.1 Single-producer single-buyer problem.....	2
1.1.2 Single-producer Multi-buyer problem	3
1.1.3 Inventory problem with deterioration	4
1.1.4 Inventory problem with raw material cost	4
1.2 Drawbacks of Previous Research.....	5
1.3 Research Goals.....	6
1.4 Research Objectives	6
1.5 Applications	7
CHAPTER 2 THE PRODUCER-BUYER MODEL 1	8
2.1 The problem	8
2.2 Assumptions and notations.....	10
2.2.1 Assumptions:.....	10
2.2.2 Notation.....	10
2.3 The Model	13
2.3.1 Buyers' cost.....	13
2.3.2 Producer's cost	14
2.3.3 System Total cost	17
2.4 Solution Procedure	17

2.5 Computational results	19
2.6 Sensitivity analysis.....	22
2.6.1 Effect of θ_b on $TC(T, f)$	22
2.6.2 Effect of θ_{vb} on $TC(T, f)$	24
2.6.3 Effect of A_v and H_v on $TC(T, f)$	25
2.7 Benefits of the model	27
CHAPTER 3 THE PRODUCER-BUYER MODEL 2	28
3.1 The problem	28
3.2 Assumptions and notations.....	31
3.2.1 Assumptions:.....	31
3.2.2 Notation.....	31
3.3 The Model	33
3.3.1 Buyers' cost.....	33
3.3.2 Producer's cost.....	34
3.3.3 System Total cost	36
3.4 Solution Procedure	37
3.5 Computational results	40
3.6 Sensitivity analysis.....	43
3.6.1 Effect of θ_b on $TC_{RM+R}(T, f)$	43
3.6.2 Effect of θ_{vb} on $TC_{RM+R}(T, f)$	44
3.6.3 Effect of A_m and H_m on $TC_{RM+R}(T, f)$	46
3.7 Benefits of the model	48
CHAPTER 4 CONCLUSION.....	49

REFERENCES.....	50
APPENDIX I PROOF OF THE CONCAVITY	55
APPENDIX II THE SPECIFIC SOLVING PROGRAM OF THE MATLAB	57
VITA	58

LIST OF TABLES

Table 1.1 Comparison of characteristics between previous research and this study.....	5
Table 2.1 The optimal solutions of the examples.....	22
Table 2.2 Values of A_v, H_v and $TC(T, f)$ for Figure 2.7.....	25
Table 2.3 Values of A_v / H_v and $TC(T, f)$ for Figure 2.8.....	26
Table 3.1 The optimal solutions for the case with raw material and remanufacturing cost....	42
Table 3.2 The optimal solutions for the case without raw material.....	42
Table 3.3 The optimal solutions for the case without raw material and remanufacturing cost.....	43
Table 3.4 Values of A_m, H_m and $TC_{RM+R}(T, f)$ for Figure 3.7.....	46
Table 3.5 Values of A_m / H_m and $TC_{RM+R}(T, f)$ for Figure 3.8.....	47

LIST OF FIGURES

Figure 2.1 Model 1 System.....	8
Figure 2.2 Buyers' and the producer's inventory levels for Model 1.....	9
Figure 2.3 Algorithm 1's flow chart.....	20
Figure 2.4 Function with respect to f_1 and T	21
Figure 2.5 Effect of θ_b on $TC(T, f)$	23
Figure 2.6 Effect of θ_{vb} on $TC(T, f)$	24
Figure 2.7 Effect of A_v and H_v on $TC(T, f)$	26
Figure 2.8 Effect of A_v / H_v on $TC(T, f)$	27
Figure 3.1 Model 2 System.....	29
Figure 3.2 Buyers' and the producer's inventory levels for Model 2.....	30
Figure 3.3 Algorithm 2's flow chart.....	39
Figure 3.4 Function with respect to f_1 and T	41
Figure 3.5 Effect of θ_b on $TC_{RM+R}(T, f)$	44
Figure 3.6 Effect of θ_{vb} on $TC_{RM+R}(T, f)$	45
Figure 3.7 Effect of A_m and H_m on $TC_{RM+R}(T, f)$	47
Figure 3.8 Effect of A_m / H_m on $TC_{RM+R}(T, f)$	48

ABSTRACT

The market is varying from minute to minute nowadays. Increase cooperation and pursue the optimal interest of the integrated supply chain become a more effective way than act alone in the competition. In this research, an integrated inventory policy between single-producer and multi-buyer is developed and two inventory models are built. The first model extends the research of Lin and Lin (2007) by changing the single-buyer system to the multi-buyers one. Both backorder of buyers and deteriorating items of each party (producer's level, buyers' level, and during transport) are considered herein. The second model is based on the research of Woo *et al.* (2001) and Model 1 by takes raw material cost and remanufacturing proceeds into account additional. In both model, the producer and buyers collaboratively work at minimizing their total operation cost and the problems are solved under an assumption of equal replenishments and production cycles. The algorithms to find the optimal solutions are given, and numerical examples are presented. Sensitivity for systems parameters is also analyzed and all calculations are completed by software Matlab and Maple.

Key Words: Deterioration, raw material, shortage, remanufacturing, inventory, integrated policy, joint cost, minimization.

Running Heads: Single-producer and multiple buyers' inventory

CHAPTER 1 INTRODUCTION

Information networking allows companies to join markets all over the world. It is difficult to accommodate for such competition. Due to the limitation in every respect, action alone is infeasible. In order to increase the sale stability and competitive power between competitors, many companies are trying to increase profits by cooperating with their partners. So, setting up a policy to minimize the joint total cost of the whole supply chain system becomes more and more popular.

The supply chain system is a network that consists of several organizations or units which are linked by products, material, and services flows. It is designed to enhance the cooperation between relevant companies and to maximize the benefit of the whole system. Nowadays, multifarious policies are used in industry. For example, Dell uses “The Toyota Way” to minimize its inventory, which means to minimize the raw material inventories and to get the supplies frequently (some parts are purchase in 1-minute unit) in small amount. As the super carrier of retail trade, Wal-Mart uses the CPFR (Collaborative Planning Forecasting and Replenishment) management model to enhance efficiency. This model can join the companies in the supply chain to forecast the market, manage production and schedule deliveries. By using this model, Wal-Mart’s products price is reduced to 10 percent below most of the competitors (Andraski and Haedicke, 2003).

Because of the great opportunity of cooperation policy in industry applications, much research in various joint models explored the collaboration between producers and buyers. Among these research, inventory level is one of the focuses. For a company, a high inventory level can meet demand of each buyer. However, it means more warehouses, more holding cost, and more deterioration. On the other hand, low inventory level could reduce expenses while companies may have to face a shortage situation which can lead to shortage cost,

reliability loss and low goodwill. For the entire supply chain, the inventory level involves the logistics and cost between partners. So, drawing a balance among all levels of the system and gaining the optimal inventory level could benefit everyone.

In the past, most companies decided the inventory level based only on the demand forecast and storage. The Economic Ordering Quantity (EOQ) was one of the specific models which could minimize the total cost by determine the optimal ordering quantity. However, policies set by the EOQ model are not advantageous to other parties. It often benefits one company while harms its partners. Many problems also emerge when the actual demands do not match the forecast demand or some unexpected incidents occur in the system. Unlike EOQ, the integrated inventory model considers the combined total cost generated in an entire supply chain. Many models on joint inventory management have been built to minimize the total cost of a system. The following section will be devoted to a review of literatures on the related issues while the goal of this study is to build two models to minimize the supply system's cost. The first one is to consider the shortages of buyers and the deteriorated items in all levels. As natural extension to it, the second one will incorporate the raw material cost in building such as system cost function.

1.1 Literature Review

This section reviews literature related to the models of a whole supply chain system. The literature is grouped into categories based on the characteristics of models; both achievements and limitations of different models are analyzed. A summary of the limitations of earlier studies are also listed.

1.1.1 Single-producer single-buyer problem

Due to the limitations of industrial complexity in development and applications, most early models in this area are simple. Goyal (1977) built a single-vender single-buyer joint

economic lot size model with finite demand, which could minimize the total costs for both producer and buyer. Banerjee (1986) developed a joint economic-lot-size model on a lot-for-lot basis for a single-buyer and single-producer system. Both works stand as fundamental models which simulate simple supply chain cooperation. Goyal (1988) improved Banerjee's model (1986) by relaxing the assumption of the lot-for-lot policy—this model can be used in various kinds of supply policy. Golhar and Sarker (1992) first modeled the just-in-time system delivery operations for a producer and buyer's joint inventory system which subsequently followed by several other models along this line to capture more realistic facets of the problem (Sarker and Parija 1994, 1996, Parija and Sarker 1999, Jamal *et al.* 1993, 1997, Sarker *et al.* 1997, 2000). Ramasesh (1990) divided the total ordering cost into the contract order cost and the shipment cost in a lot. Goyal (1995) propounded a model which allows different replenishment size for the buyer. Both of the models reflect more accordance with actual conditions. Lu (1995) developed a single-producer single-buyer model which allowed shipments to be made before a production batch is completed. All shipment sizes are equal in his case. Pan and Yang (2002) built a model with flexible lead time which means the lead time can be shortened by paying some extra cost. Based on this research, they presented another model which includes the quality issue in Yang and Pan (2004). Sajadieh *et al* (2010) presents a joint economic lot-sizing model in which demand is correlated positively with quantities.

1.1.2 Single-producer Multi-buyer problem

In the real world, a producer often supplies to more than one buyer. In that aspect, Woo *et al.* (2001) developed a single-producer, multi-buyer model with common cycle time for all buyers and the producer. Khouja (2003) studied a supply chain network which has multiple firms and each firm have multiple customers under three coordination mechanisms.

Wee and Yang (2004) developed a heuristic solution model for a producer-distributors-retailers inventory system by using the principle of strategic partnership. Zhang *et al.* (2007) extended Woo *et al.* (2001)'s model by allowing buyers with different ordering cycles and each buyer can replenish more than once in one production cycle. Haji *et. al* (2009) introduce a new replenishment policy where the authors claimed that the model is easy to implement and also reduces the impacts of uncertain demand.

1.1.3 Inventory problem with deterioration

In many industries, such as food, cosmetic, and paper, products deteriorate to irreversible conditions. So these unsuitable conditions should be considered in an inventory model to reflect the reality. Ghare (1963) first considered an exponentially decay in an EOQ model. Dave (1981) developed an inventory model in which the number of deteriorating item is relative to time without shortage, this method reflects more to the real condition. Sachan (1984) extended Dave's model by allowing shortage. Goswami (1992) computed deterioration by introducing a deterioration rate associated with time; then the model can explain the deterioration condition more clearly. Lin and Lin (2007) presented a cooperative inventory policy between single-producer and single-buyer, in which deteriorating items and shortage for buyers are considered. Lin *et al.* (2010) built models to optimize the supply system by considering deteriorating items and backorders with a fixed service rate in four scenarios.

1.1.4 Inventory problem with raw material cost

Raw material cost also impact on the entire policy. Wee and Shum (1999) presented a model to discuss this part of cost. Woo *et al.* (2001) developed a single-producer, multi-buyer model with common cycle time and raw material cost to simulate this situation. Zhang *et al.* (2007) relaxed the assumption of common cycle in Woo *et al.* (2001) to a multiple one. Tang *et al.* (2008) built a model to balance the raw material inventory and production demands in iron industry.

1.2 Drawbacks of Previous Research

Much research has been done to optimize supply chain systems. There still are some limitations, such as:

1. **Shortage allowance:** Most research reviewed above assume that shortage is not allowed, especially in single-producer multi-buyer system.
2. **Deterioration allowance:** Most research did not take deteriorated items into account. Some research allowed for deteriorated items, such as Lin and Lin (2007), but they were limited to a single-producer single-buyer system.
3. **Raw material cost:** Most research did not consider raw material cost or it ignored the deterioration of this part.
4. **Remanufacturing proceeds:** Most research neglect the remanufacturing proceeds.

Based on the limitations analyzed above, this study will build a single-producer multi-buyer system with both shortages and deterioration. Table 1.1 shows the characteristics comparison between this study and related research.

Table 1.1 Comparison of characteristics between previous research and this study

Properties considered	Woo <i>et al.</i> (2001)	Zhang <i>et al.</i> (2007)	Lin & Lin (2007)	Lin & Lin (2010)	Model 1	Model 2
Number of venders	1	1	1	1	1	1
Number of buyers	n	n	1	1	n	n
Production rate	constant	constant	constant	constant	constant	Constant
Demand rate	constant	constant	constant	constant	constant	Constant
Shortage	backorder	backorder	backorder	backorder	backorder	backorder
Deterioration	No	No	Yes	Yes	Yes	Yes
Raw material	Yes	Yes	No	No	No	Yes
Remanufacturing	No	No	No	No	No	Yes

1.3 Research Goals

Though several studies have been done in this area, no research explored the system with a single-producer and multiple buyers under both deterioration (items decayed or be destroyed due to unsuitable storage/transportation conditions, overdue/obsolete items are not included here), backordering, raw material cost, and remanufacturing proceeds, which is a common situation in real world. The goal of this research is to investigate the effect of collaboration between one producer and several buyers where all factors listed above exist. The *specific objective* of this study is to build two models of a system which is composed of one producer and several buyers, and to minimize the joint cost of the entire system by optimizing the common cycle time and shortage time. Shortages are allowed for all buyers while it is not allowed for the producer to meet the downstream demand. Deteriorated items for all parties, as well as in transit, are considered. The total cost is composed of ordering cost, setup cost, holding costs, backorder cost and deterioration cost in Model 1 and Model 2 and also considers in the raw material cost and remanufacturing proceeds.

1.4 Research Objectives

The objective of this research is to overcome the limitations of previous research analyzed above. Two models with single-producer multi-buyer problem will be formulated here. The objective is to minimize the total cost in the entire system which includes the setup cost, holding cost, shortage cost, deterioration cost, raw material cost and remanufacturing proceeds. In order to minimize the total cost, the specific objectives of this research are:

1. To develop two cost models for the entire supply system, the first one with shortage and deterioration, the second one with shortage, deterioration, raw material cost and remanufacturing proceeds.
2. To construct the solution procedure.

3. To determine optimal replenishment policies to minimize the system's total cost.

1.5 Applications

The first model is built for the supply systems with single-producer and multi-buyer, which can be treated as a unit. The producer produces the products and delivers them to retailers. Deteriorated items may occur everywhere: the producer's warehouse, shops and during transportation. Shortage and backorders are allowed in shops because sometimes the demand of customer may seasonally vary. Because the inventory level can affect costs each party must decide on the produce/replenishment cycle and shortage level to minimize the total cost and benefits of each.

The second model is an extension of the first one. The producer needs to supply and hold raw materials for production so the cost of this part should also be considered. In the second model, raw material with deterioration cost is taken into account. In addition, parts of deterioration items can be remanufacturing by the producer and reducing the lost in most case, so the proceeds of this part are also considered in Model 2.

These models can be used widely in food processing, pharmacy, and other industry areas. For instance, a food processing company provides hard candy to several groceries. Each party has their own warehouse. Hard candy may deteriorate in unfavorable storage conditions. However, a part of them can be remanufactured to retrieve the loss. Transit also may destroy goods. The raw materials, such as sugar and food flavor may also damaged in warehouses. The food processing company can meet all demands of groceries. They make delivery to each grocery once in the beginning of every replenishment cycle. Shortage may appear in each grocery and backorders are allowed.

CHAPTER 2 THE PRODUCER-BUYER MODEL 1

In this chapter, a joint cost model of this two-stage supply chain system will be built. Both deterioration items and back-order situation will be considered. The inventory level of the raw materials and finished products will show the general storage situations. Then, according to following which functions of the producer's and buyers' inventory levels and total cost, a model will be developed.

2.1 The problem

A supply system with a single producer and multiple buyers is considered, where the products may deteriorate at all levels (at producer, in-transit and at buyers). Figure 2.1 illustrates of the system and Figure 2.2 depicts inventory levels of products at both producer and buyers. In order to integrate these units of the system, consider that all parties have the same replenishment/production cycle. The producer's production capability is sufficient to meet all buyers' demands while shortage is allowed for all buyers.

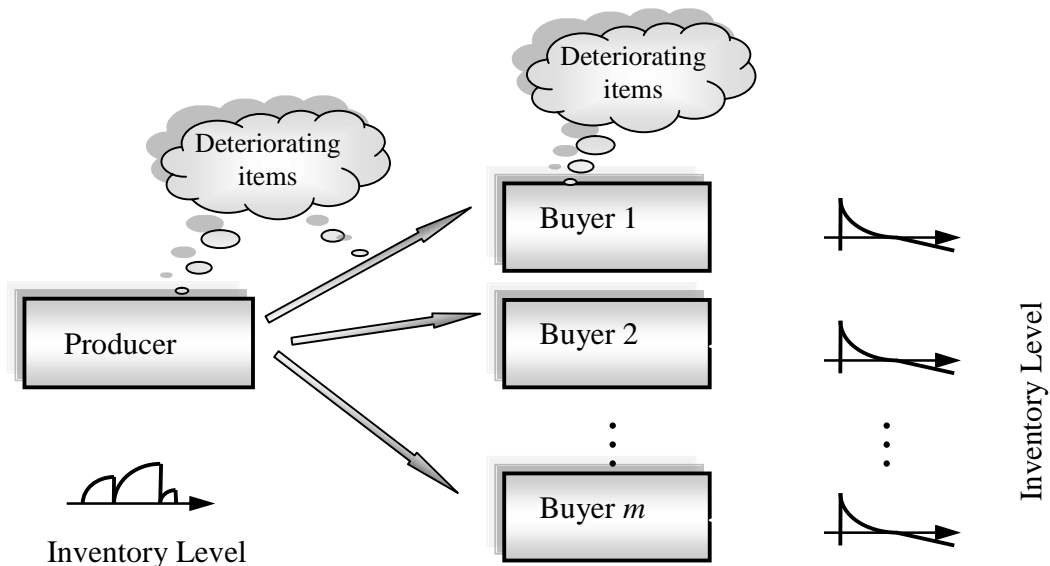


Figure 2.1 Model 1 System

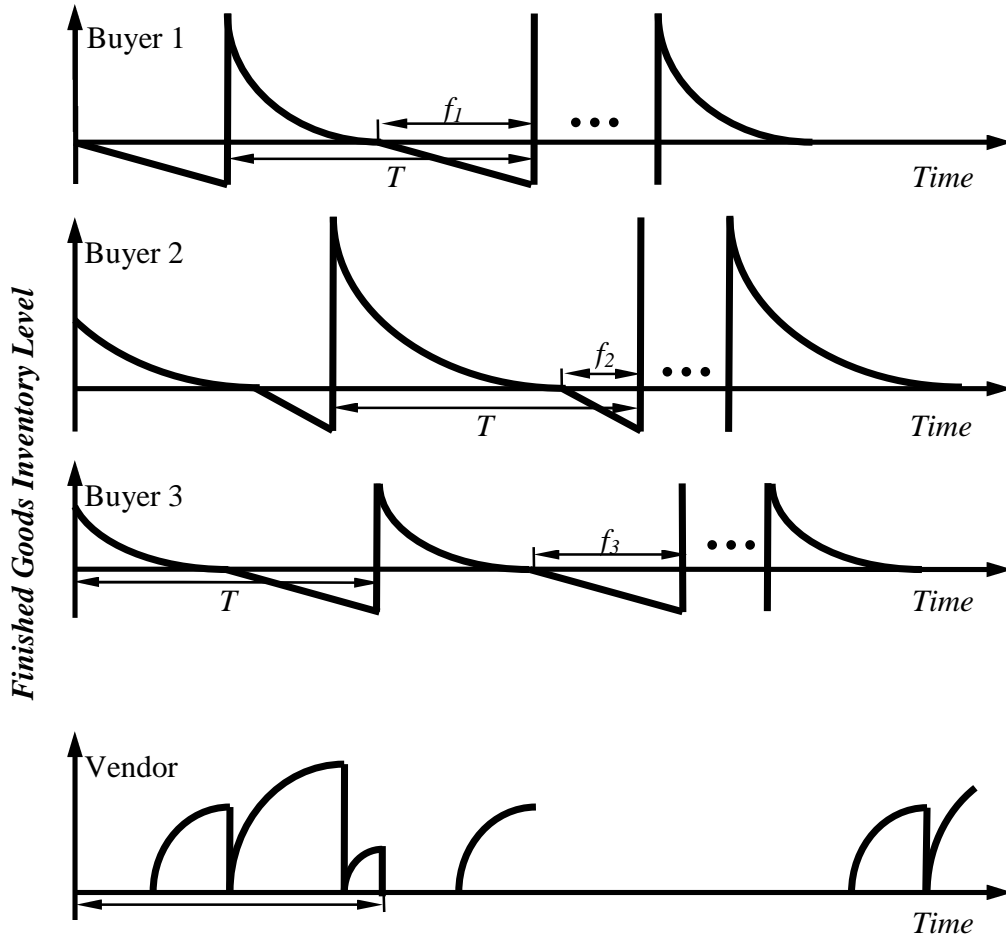


Figure 2.2 Buyers' and the producer's inventory levels for Model 1

Because of the presence of deteriorating items, the inventory level of the producer and all buyers are nonlinear. In each production cycle, the producer produces goods to supply all buyers. Since the production rate, is not less than the total demand rate, production may be allowed to stop for a period in each cycle. When production starts at the beginning of each cycle, there is only a small quantity of deteriorated items so the inventory curve rises more rapidly, and growth slows as the inventory level increases, due to the appearance of many deteriorated items at the later time. The inventory level decreases suddenly when a delivery is made, and becomes zero when all deliveries are made at the end of a cycle.

On the other hand, all buyers receive products at the beginning of each replenishment cycle. The slope of tangent to inventory level increases over time because the number of deteriorated items decreases as the inventory level decreases. During the shortage period, inventory level becomes negative, and continues decreasing in linear fashion because no deterioration exists in this situation.

2.2 Assumptions and notations

There are many uncertainties in an actual supply system, such as machine stoppage, interruptions of shipping, company policy changes, and market fluctuation. In this section, some assumptions to simplify the model and a set of notations to model the problems are stated below.

2.2.1 Assumptions:

The fundamental assumptions used in the study are as follows:

1. Shortages are not allowed for the producer.
2. The buyers' replenishment information is available to the producer.
3. All buyers' replenishment cycles are equal to the producer's production cycle.
4. The ordering, holding and shortage costs are the same for all buyers.

2.2.2 Notation

The fundamental notations used in the study are showed as follows.

(a) Indices:

- | | |
|-----|---------------------------------------|
| m | Total number of buyers, |
| n | Total number of cycles, |
| i | Number of cycle, $i = 1, 2, \dots, n$ |
| j | Number of buyer, $j = 1, 2, \dots, m$ |

(b) *System parameters:*

A_b	Ordering cost by a buyer per cycle, (dollars/order)
A_v	Setup cost of producer per production run or batch, (dollars/batch)
c_b^d	The cost of each deteriorated unit at the buyer's level, (dollars/unit)
c_v^d	The cost of each deteriorated unit at the producer's level, (dollars/unit)
D_j	Demand rate for buyer j , which is a known (units/year),
H_b	Holding cost of buyers (\$/unit/unit-time),
H_v	Holding cost of the producer (\$/unit/unit-time),
P	Production rate for producer (units/year), which is a known and $P \geq \sum_{j=1}^m D_j$,
$\hat{\pi}_b$	Backlogging cost of buyers (\$/unit short/unit-time),
θ_b	Proportion of the on-hand (positive) inventory deteriorated at the buyer's level
θ_v	Proportion of the on-hand inventory deteriorated at the producer's level
θ_{vb}	Proportion of the goods deteriorated in transit from the producer to buyers

(c) *Intermediate variables:*

$I_j^{b+}(t)$	Instantaneous inventory level of products at buyer j at time t , (unit)
$I_j^{b-}(t)$	Instantaneous shortage level of buyer j at time t , (unit)
$I^v(t)$	Instantaneous inventory level of the producer at time t , (unit)
I_j^{b+}	Total quantity carried by buyer j in any cycle, (unit)
I_j^{b-}	Total shortage inventory at the buyer j in any cycle, (unit)
I_j^v	Total inventory carried by the producer for buyer j in any cycle, (unit)

I^v	Total inventory carried by the producer, (unit)
t_i	The end time of cycle i
t_i^s	The beginning time of shortage at buyer's level in cycle i
t_i^p	The beginning time of production at producer's level in cycle i
t_{ij}	The time when the vender finish production for buyer j in cycle i
T_j	Total time for the vender to produce for buyer j in each cycle
T_i	$= \sum_{j=1}^m T_j$, total time for the vender to produce for all buyers in cycle i
W_j^b	Total number of deteriorated units of buyer j in any cycle
W^v	Total number of deteriorated units of the producer for buyer j in any cycle
TC_j^b	Buyer j 's total cost in any cycle
TC^b	All buyers' total cost in any cycle
TC^v	The producer's total cost in any cycle
$TC(T, f)$	The joint cost of the producer and all buyers in any cycle, which is a function of T (common cycle time) and f (a vector of fractions of shortage time to the cycle time for all buyers).

(d) Decision Variables:

f	A vector of $(f_1, f_2, f_3, \dots, f_m)$
f_j	Fraction of shortage time to cycle time for buyer j
T	Common replenishment and production cycle for buyers and producer

2.3 The Model

A mathematical model of the total joint-cost of the entire system is built here. Inventory levels of the producer and the buyers are first modeled to reflect the total cost of the system. Deterioration items are considered for both parties as well as in transit.

2.3.1 Buyers' cost

The cost for each buyer is composed of ordering cost, holding cost, backlog cost, and deterioration cost. For buyer j ($j = 1, 2, \dots, m$), according to Lin and Lin (2006), the buyer's instantaneous inventory level is $I_j^{b+}(t)$ and instantaneous shortage level is $I_j^{b-}(t)$ at time t can be described as

$$\begin{cases} \frac{dI_j^{b+}(t)}{dt} + \theta_b I_j^{b+}(t) = -D_j & (t_{i-1} \leq t \leq t_i^s, i = 1, 2, \dots, n) \\ \frac{dI_j^{b-}(t)}{dt} = -D_j & (t_i^s \leq t \leq t_i, i = 1, 2, \dots, n) \end{cases}, \quad (2.1)$$

where D_j is the demand of buyer j , θ_b is the deteriorating rate of the buyer's inventory. The demand equals to the instantaneous inventory plus deteriorate item without shortage. After the shortage appears, demand equals to the instantaneous inventory. So the solution to this equation is obtained as

$$\begin{cases} I_j^{b+}(t) = e^{-\theta_b t} \int_t^{t_i^s} e^{\theta_b u} D_j du = \frac{D_j}{\theta_b} (e^{\theta_b (t_i^s - t)} - 1) & (t_{i-1} \leq t \leq t_i^s, i = 1, 2, \dots, n) \\ I_j^{b-}(t) = D_j (t_i^s - t) & (t_i^s \leq t \leq t_i, i = 1, 2, \dots, n) \end{cases}, \quad (2.2)$$

where t_i^s is the beginning time of shortage at buyer's level in cycle i .

Let t_i be the end time of cycle i , f_j be the fraction of shortage time to cycle time for buyer j and T be the common replenishment and production cycle for buyers and producer. The total amount of inventory (I_j^{b+}) and the shortage (I_j^{b-}) carried by the buyer j in cycle i are the integral of equation (2.2) as given below

$$\begin{cases} I_j^{b+} = \int_{t_{i-1}}^{t_i^s} I_j^{b+}(t)dt = \frac{D_j}{\theta_b^2} (e^{\theta_b T(1-f_j)} - 1) - \frac{D_j}{\theta_b} T(1-f_j) \\ I_j^{b-} = \int_{t_i^s}^{t_i} I_j^{b-}(t)dt = -\frac{D_j}{2} T^2 f_j^2 \end{cases} \quad (2.3)$$

The number of deteriorated items W_j^b equals to the instantaneous inventory at the beginning of cycle i , $I_j^{b+}(t_{i-1})$ minus the total demand in the cycle, which is given by

$$W_j^b = I_j^{b+}(t_{i-1}) - \int_{t_{i-1}}^{t_i^s} D_j du = \frac{D_j}{\theta_b} (e^{\theta_b T(1-f_j)} - 1) - D_j T(1-f_j). \quad (2.4)$$

Then, the total cost of buyer j , TC_j^b in cycle i is

$$\begin{aligned} TC_j^b = & \frac{1}{T} \{ A_b + H_b \left[\frac{D_j}{\theta_b^2} (e^{\theta_b T(1-f_j)} - 1) - \frac{D_j}{\theta_b} T(1-f_j) \right] + \hat{\pi}_b \frac{D_j}{2} T^2 f_j^2 \\ & + c_b^d \left[\frac{D_j}{\theta_b} (e^{\theta_b T(1-f_j)} - 1) - D_j T(1-f_j) \right] \}, \end{aligned} \quad (2.5)$$

where A_b represents the ordering cost, H_b is the holding cost of buyers, $\hat{\pi}_b$ is the backlogging cost of buyers and c_b^d is the cost of each deteriorated unit at the buyer's level. The total cost

TC^b of all m buyers in cycle i is thus given by

$$\begin{aligned} TC^b = & \frac{1}{T} \{ m A_b + \sum_{j=1}^m H_b \left[\frac{D_j}{\theta_b^2} (e^{\theta_b T(1-f_j)} - 1) - \frac{D_j}{\theta_b} T(1-f_j) \right] + \sum_{j=1}^m \hat{\pi}_b \frac{D_j}{2} T^2 f_j^2 \\ & + \sum_{j=1}^m c_b^d \left[\frac{D_j}{\theta_b} (e^{\theta_b T(1-f_j)} - 1) - D_j T(1-f_j) \right] \}. \quad (j = 1, 2, \dots, m) \end{aligned} \quad (2.6)$$

2.3.2 Producer's cost

For the producer, its production capability is sufficient to supply all buyers. Setup cost per production run, holding cost, and deterioration cost are considered into the total cost. Deterioration in both producer's warehouse and shipment process are taken into account. For the producer, the instantaneous inventory level $I^v(t)$ can be described as

$$\begin{cases} \frac{dI^v(t)}{dt} + \theta_v I^v(t) = P & (t_{ij-1} \leq t \leq t_{ij}, i=1,2,\dots,n; j=1,2,\dots,m), \\ I^v(t) = 0 & (t_{i-1} \leq t \leq t_i^p, i=1,2,\dots,n) \end{cases} \quad (2.7)$$

θ_v is the proportion of the deteriorated items at the producer's level and t_i^p is the beginning time of production at producer's level in cycle i . The solution to this equation can be solved as

$$I^v(t) = e^{-\theta_v t} \int_{t_{ij-1}}^t e^{\theta_v u} P du = \frac{P}{\theta_v} (1 - e^{\theta_v(t_{ij-1}-t)}) \quad (t_{ij-1} \leq t \leq t_{ij}). \quad (2.8)$$

Because the products' quantity received by every buyer at the beginning of each cycle equals to the delivery quantity of the producer minus the number of deteriorated item during the transit from the producer to buyers, the inventory level of the producer and the buyer can be shown by the function

$$(1 - \theta_{vb}) \frac{P}{\theta_v} (1 - e^{\theta_v(t_{ij-1}-t_{ij})}) = \frac{D_j}{\theta_b} (e^{\theta_b T(1-f_j)} - 1) + D_j T f_j, \quad (2.9)$$

where θ_{vb} is the proportion of the deteriorated items in transit from the producer to buyers and t_{ij} is the time when the vender finishes production for buyer j in cycle i . Then, the producer's produce time for buyer j in each cycle, T_j can be solved by the above function and presented as

$$T_j = -\frac{1}{\theta_v} \ln \left\{ 1 - \frac{D_j \theta_v}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T(1-f_j)} - 1) + \theta_b T f_j] \right\}, \quad (2.10)$$

and the producer's total produce time in cycle i , T_i equals to

$$T_i = -\sum_{j=1}^m \frac{1}{\theta_v} \ln \left\{ 1 - \frac{D_j \theta_v}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T(1-f_j)} - 1) + \theta_b T f_j] \right\}. \quad (2.11)$$

Now, the amount of inventory carried by the producer for buyer j in cycle i , I_j^v can be obtained by computing the integral of equation(2.8):

$$I_j^v = \frac{P}{\theta_v} [T_j + \frac{1}{\theta_v} (e^{-\theta_v T_j} - 1)], \quad (2.12)$$

and the amount of inventory I^v carried by the producer's in cycle i is

$$I^v = \sum_{j=1}^m [\frac{P}{\theta_v} T_j + \frac{P}{\theta_v^2} (e^{-\theta_v T_j} - 1)]. \quad (2.13)$$

The total number W^v of deteriorated units for the producer in each cycle is equals to the quantity minus the original delivery quantity to all buyers

$$\begin{aligned} W^v &= T_i P - \sum_{j=1}^m I^v(t_{ij}) \\ &= -P \sum_{j=1}^m \frac{1}{\theta_v} \ln \{1 - \frac{D_j \theta_v}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1-f_j)} - 1) + \theta_b T f_j]\} - \sum_{j=1}^m \frac{P}{\theta_v} (1 - e^{-\theta_v T_j}), \end{aligned} \quad (2.14)$$

where $T_i = \sum_{j=1}^m T_{ij}$ is the total time for the vender to produce for all buyers in cycle i . A_v is the setup cost of producer per production run or batch, H_v is the holding cost of the producer and c_v^d is the cost of each deteriorated unit at the producer's level, the total cost of the producer in cycle i , TC^v consists of the setup cost, holding cost and deteriorated item cost, which can be given by

$$TC^v = \frac{1}{T} \{A_v + \text{Error! Objects cannot be created from editing field codes.}$$

$$- c_v^d P \sum_{j=1}^m \frac{1}{\theta_v} \ln \{1 - \frac{D_j \theta_v}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1-f_j)} - 1) + \theta_b T f_j]\}$$

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(2.15)

2.3.3 System Total cost

Then, for shortage time fraction vector $f = (f_1, f_2, f_3, \dots, f_m)$ and cycle time T , the system's joint total cost $TC(T, f)$ in a cycle is the producer's and all buyers' cost, which can be given by

$$\begin{aligned}
 TC(T, f) &= TC^b + TC^v \\
 &= \frac{1}{T} \left\{ mA_b + \sum_{j=1}^m H_b \left[\frac{D_j}{\theta_b^2} (e^{\theta_b T(1-f_j)} - 1) - \frac{D_j}{\theta_b} T(1-f_j) \right] + \sum_{j=1}^m \hat{\pi}_b \frac{D_j}{2} T^2 f_j^2 \right. \\
 &\quad \left. + \sum_{j=1}^m c_b^d \left[\frac{D_j}{\theta_b} (e^{\theta_b T(1-f_j)} - 1) - D_j T(1-f_j) \right] \right\} \\
 &\quad + \frac{1}{T} \left\{ A_v + H_v \sum_{j=1}^m \left[\frac{P}{\theta_v} T_j + \frac{P}{\theta_v^2} (e^{-\theta_v T_j} - 1) \right] \right. \\
 &\quad \left. - c_v^d P \sum_{j=1}^m \frac{1}{\theta_v} \ln \left\{ 1 - \frac{D_j \theta_v}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T(1-f_j)} - 1) + \theta_b T f_j] \right\} \right. \\
 &\quad \left. - c_v^d \sum_{j=1}^m \frac{P}{\theta_v} (1 - e^{-\theta_v T_j}) \right\}. \tag{2.16}
 \end{aligned}$$

2.4 Solution Procedure

To guarantee that there is a minimum total cost, the second-order derivative equations of the total cost function are to be positive. However, in this case, the total cost function $TC(T, f)$ is a higher-order exponential function so it is not convenient to evaluate the second-order derivative in closed-form directly. An indirect approach to check the convexity of $TC(T, f)$ is employed here by evaluating the response surface of the total cost function over a possible range of the given set of parametric values. Software “Matlab” is used to solve the problem and the computation results indicate that the response surface of the total cost function $TC(T, f)$ is convex in T and f_j within a reasonable range. As a fraction, f_j 's range is $[0, 1]$; for T , a reasonable range means an acceptable delivery cycle time for a

common company, usually from 0 to 1 year (this paper choose 0 to 365 days as the range of T in the numerical examples' section). The computational code is given by Appendix I.

Then, the optimal results are found by solving simultaneously $\partial TC(T, f)/\partial T = 0$ and $\partial TC(T, f)/\partial f_j = 0$ for $j = 1, 2, \dots, m$; but for the same reason, it is not easy to evaluate the first-order derivative in closed-form directly. A computer algorithm is written to search the optimal result, which is list below:

Algorithm 1. Calculation of optimal total cost

Step 0: (a) Initialize system parameters $m, j, A_b, A_v, c_b^d, c_v^d, D_j, H_b, H_v, P, \hat{\pi}_b, \theta_b, \theta_v,$

$\theta_{vb}, \Delta TC, \Omega_T$ and Ω_f .

(b) Set the range of $T \in (0, a]$ and $f_j \in [0, 1]$, for $j = 1, \dots, m$.

(c) Set $T = 0$ and $f_j = 0$, initialize ΔT and Δf_j ; for $j = 1, \dots, m$.

(d) Set $T^* = 0, f_j^* = 0, TC(T, f)^* = \infty$.

Step 1. Repeat $T = T + \Delta T$

Repeat $f_j = f_j + \Delta f_j$

Compute $TC(T, f)$.

If $TC(T, f) < TC(T, f)^* = \infty$,

$TC(T, f)^* = TC(T, f)$.

until $f_j = 1$

until $T = a$.

Set $T = T^*, f_j = f_j^*$

Step 2. (a) Compute $\frac{\partial TC(T, f)}{\partial T}$ and $\frac{\partial TC(T, f)}{\partial f_j}$ for $j = 1, 2, \dots, m$.

$$(b) \text{ If } \frac{\partial TC(T, f)}{\partial T} \geq \Delta TC \text{ and } \frac{\partial TC(T, f)}{\partial f_j} \geq \Delta TC$$

Set the rank of $T \in (T - \Delta T, T + \Delta T]$, $f_j \in [f_j - \Delta f_j, f_j + \Delta f_j]$, $\Delta T = \Delta T / \Omega_T$,

$\Delta f_j = \Delta f_j / \Omega_f$, go to Step 1.

Step 3. Output T^* , f_j^* , $TC(T, f)^*$, $\partial TC(T, f) / \partial T$ and $\partial TC(T, f) / \partial f_j$.

Step 4. STOP.

The algorithm 1's flow chart is showed in Figure 2.3 and the computational code is given in Appendix II. In Algorithm 1, all variables' objective areas are searched together. However, it might take too long time to run the computer program when the buyer number m is very large. So a better way is searching small possible ranges for each f_j separately first, then searching the final optimal solution in these areas together. Another feasible way is only choosing some important variables to consider and ignoring others.

2.5 Computational results

In this section, three numerical examples are presented for investigating the performance of the model. A Matlab program given in Appendix II is used to solve these examples.

Example 2.1 A single producer, single buyer system

The system parameters are: $m = 1$, $A_b = 50$, $A_v = 2000$, $H_b = 3$, $H_v = 2$, $c_b^d = 12$, $c_v^d = 10$, $\hat{\pi}_b = 12$, $\theta_b = 0.03$, $\theta_v = 0.01$, $\theta_{vb} = 0.015$, $P = 500$ units/year, $D_1 = 280$ units/year. Set $\Delta TC = \$0.01$, $\Omega_T = 10$ and $\Omega_f = 10$.

The results are listed in Table 2.1 in which the cycle time $T^* = 1.9100$ years and $f_1^* = 0.2262$. The system's total cost $TC(T, f)^*$ reaches the minimum value to \$2,121. Both $\partial TC(T, f) / \partial T$ and $\partial TC(T, f) / \partial f_1$ are computed; both of them are equal which confirms the

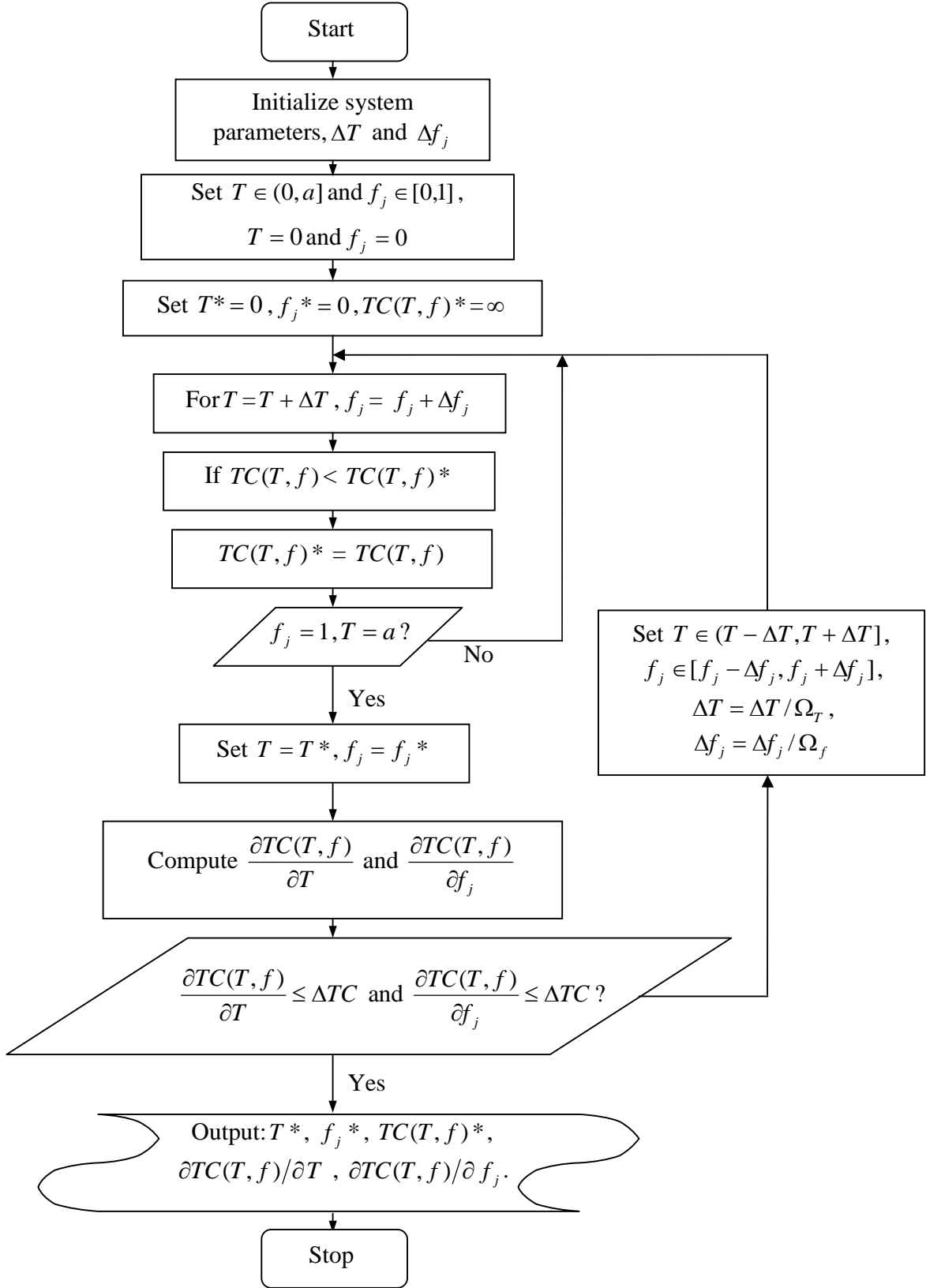


Figure 2.3 Algorithm 1's flow chart

result. In order to see the quality of the total cost function intuitively, a total cost function with respect to f_1 and T are also drawn as shown in Figure 2.4.

Example 2.2 A single producer, two buyers system

The system parameters are: $m = 2$, $A_b = 50$, $A_v = 2000$, $H_b = 3$, $H_v = 2$, $c_b^d = 12$, $c_v^d = 10$, $\hat{\pi}_b = 12$, $\theta_b = 0.03$, $\theta_v = 0.01$, $\theta_{vb} = 0.015$, $P = 500$ units/year, $D_1 = 280$ units/year, $D_2 = 180$ units/year. With the same value of $\Delta TC = \$0.01$ $\Omega_T = 10$ and $\Omega_f = 10$, the following results:

The results are listed in Table 2.1 where the cycle time $T^* = 1.5522$ years, $f_1^* = 0.2248$, and $f_2^* = 0.2238$, and the system's total cost $TC(T, f)^* = \$2,683$. It is also found that $\partial TC(T, f)/\partial f_1 \approx 0$, $\partial TC(T, f)/\partial f_2 \approx 0$ and $\partial TC(T, f)/\partial T \approx 0$, which confirm the accuracy of the results.

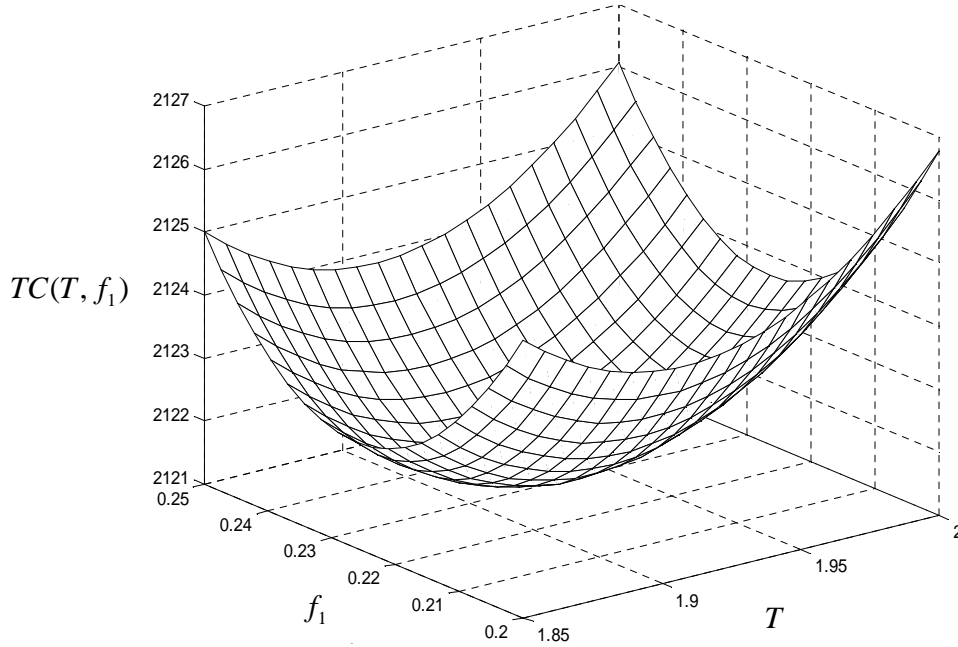


Figure 2.4 Function with respect to f_1 and T

Table 2.1 The optimal solutions of the examples

T^*	f_1^*	f_2^*	f_3^*	$\frac{\partial TC(T, f)}{\partial T}$	$\frac{\partial TC(T, f)}{\partial f_j}$	$TC(T, f)^*$
1.9100	0.2262	-	-	0	0	\$2121
1.5522	0.2248	0.2238	-	0	0	\$2683
1.4282	0.2230	0.2225	0.2226	0	0	\$2993

Example 2.3 A single producer, three buyers system.

The system parameters are: $m = 3$, $A_b = 50$, $A_v = 2000$, $H_b = 3$, $H_v = 2$, $c_b^d = 12$, $c_v^d = 10$, $\hat{\pi}_b = 12$, $\theta_b = 0.03$, $\theta_v = 0.01$, $\theta_{vb} = 0.015$, $P = 1000$ units/year, $D_1 = 280$ units/year, $D_2 = 180$ units/year, $D_3 = 200$ units/year.

Similarly for $\Delta TC = \$0.01$ $\Omega_T = 10$ and $\Omega_f = 10$, calculations for three retailers yields the optimal cycle time $T^* = 1.4282$, years $f_1^* = 0.2230$, $f_2^* = 0.2225$, $f_3^* = 0.2226$, and the system's minimal total cost $TC(T, f_1, f_2, \dots, f_j)^* = \$2,993$. The fact that the values of $\partial TC(T, f)/\partial f_1 \approx 0$, $\partial TC(T, f)/\partial f_2 \approx 0$, $\partial TC(T, f)/\partial f_3 \approx 0$ and $\partial TC(T, f)/\partial T \approx 0$ also conforms the similar accurate results.

2.6 Sensitivity analysis

System parameters are assumed to be static values in this model. However, they are only estimated numbers and might change depending different circumstances. So it is necessary to do a sensitivity analysis of the cost function to show the effects of these changing parameters. Four important parameters θ_b , θ_{vb} , A_v , and H_v are discussed here.

2.6.1 Effect of θ_b on $TC(T, f)$

To observe the impact of inventory deteriorated proportion θ_b (buyer's level), the partial derivative of θ_b is derived as:

$$\begin{aligned}
\frac{\partial TC(T, f)}{\partial \theta_b} = \frac{1}{T} & \left\{ H_b \left[-\frac{2D_1(e^{\theta_b T(1-f_1)} - 1)}{\theta_b^3} + \frac{D_1 T(1-f_1)e^{\theta_b T(1-f_1)}}{\theta_b^2} + \frac{D_1 T(1-f_1)}{\theta_b^2} \right] \right. \\
& + c_b^d \left[-\frac{D_1(e^{\theta_b T(1-f_1)} - 1)}{\theta_b^2} + \frac{D_1 T(1-f_1)e^{\theta_b T(1-f_1)}}{\theta_b} \right] \\
& \left. - \frac{C_v^d P \left[\frac{D_1 \theta_v (e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b^2 (1-\theta_{vb}) P} - \frac{D_1 \theta_v (T(1-f_1)e^{\theta_b T(1-f_1)} + T f_1)}{\theta_b (1-\theta_{vb}) P} \right]}{\theta_v \left[1 - \frac{D_1 \theta_v (e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b (1-\theta_{vb}) P} \right]} \right\}
\end{aligned} \tag{2.17}$$

By equation (2.17) and $\theta_b \in [0,1]$, the effect to the total cost $TC(T, f)$ is shown in

Figure 2.5.

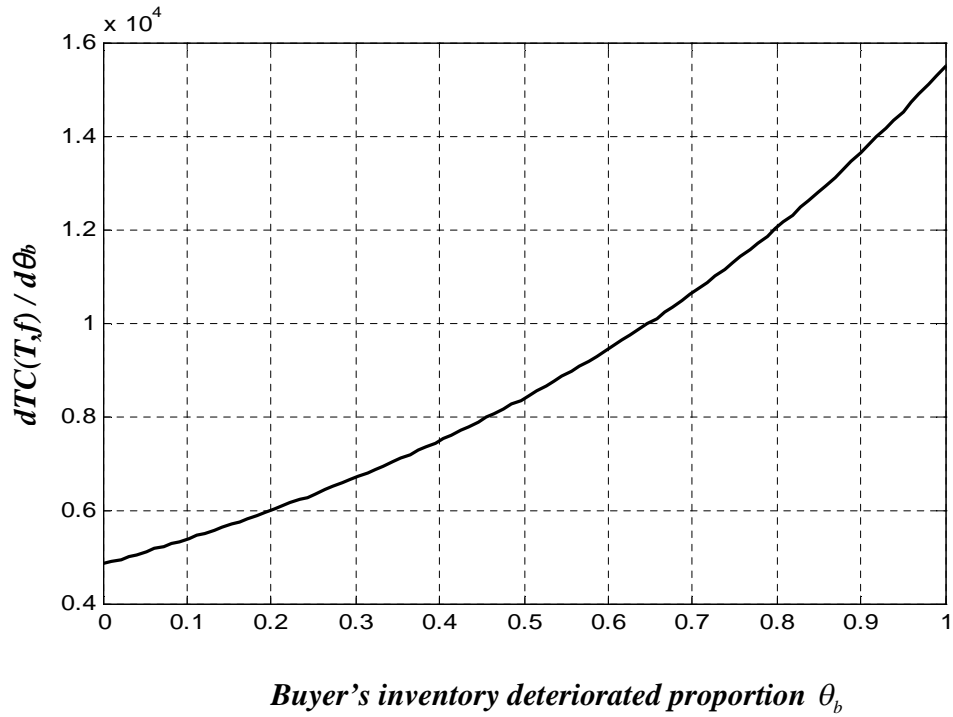


Figure 2.5 Effect of θ_b on $TC(T, f)$

According to Figure 2.5, the term $\partial TC(T, f_1) / \partial \theta_b$ is positive and increases with θ_b and is increasing as deterioration items emerging. So reducing the deterioration in buyer's level can reduce expenses.

2.6.2 Effect of θ_{vb} on $TC(T, f)$

The change of total cost $TC(T, f)$ with respect to the deteriorated proportion in transit

θ_{vb} can be shown as:

$$\frac{\partial TC(T, f)}{\partial \theta_{vb}} = \frac{c_v^d D_1 (e^{\theta_b T (1-f_1)} - 1 + \theta_b T f_1)}{T \theta_b (1 - \theta_{vb})^2 \left[1 - \frac{D_1 \theta_v (e^{\theta_b T (1-f_1)} - 1 + \theta_b T f_1)}{\theta_b (1 - \theta_{vb}) P} \right]}. \quad (2.18)$$

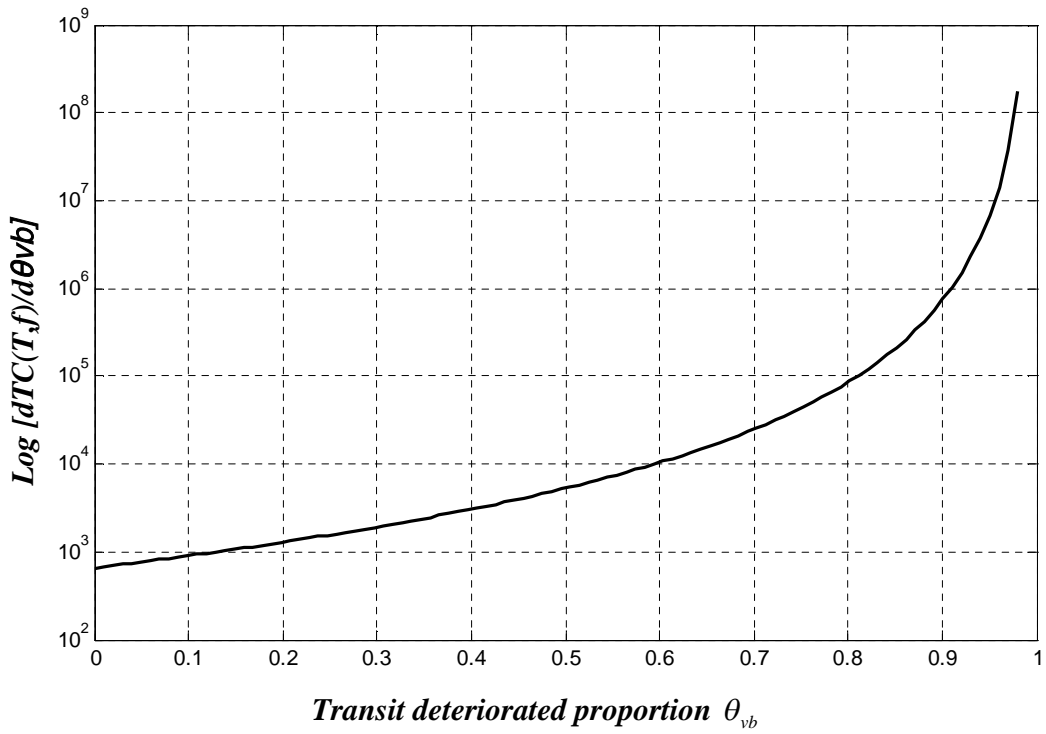


Figure 2.6 Effect of θ_{vb} on $TC(T, f)$

Let θ_{vb} change from 0 to 1 and keep the other parameters unchanged, then the effect on the total cost $TC(T, f)$ is given in Figure 2.6. In this case, $\log \partial TC(T, f_1) / \partial \theta_{vb}$ substitutes for $\partial TC(T, f_1) / \partial \theta_{vb}$ to show the trend more clearly due to the small change. In Figure 2.6, it is seen that $\partial TC(T, f_1) / \partial \theta_{vb}$ is increasing when θ_{vb} is increasing, which means reducing θ_{vb} can also decrease the total cost.

2.6.3 Effect of A_v and H_v on $TC(T, f)$

Set up cost and holding cost are two major costs of the producer, so the effects of them on $TC(T, f)$ are also analyzed.

Table 2.2 Values of A_v , H_v and $TC(T, f)$ for Figure 2.7

$A_v \backslash H_v$	$TC(T, f)$									
	1	2	3	4	5	6	7	8	9	10
10	914	1,074	1,234	1,394	1,554	1,714	1,874	2,034	2,194	2,354
50	935	1,095	1,255	1,415	1,575	1,735	1,895	2,055	2,215	2,375
100	961	1,122	1,282	1,442	1,602	1,762	1,922	2,082	2,242	2,402
150	988	1,148	1,308	1,468	1,628	1,788	1,948	2,108	2,268	2,428
200	1,014	1,174	1,334	1,494	1,654	1,814	1,974	2,134	2,294	2,454
250	1,040	1,200	1,361	1,521	1,681	1,841	2,001	2,161	2,321	2,481
300	1,067	1,227	1,387	1,547	1,707	1,867	2,027	2,187	2,347	2,507
350	1,093	1,253	1,413	1,573	1,733	1,893	2,053	2,213	2,373	2,533
400	1,119	1,279	1,439	1,600	1,760	1,920	2,080	2,240	2,400	2,560
450	1,146	1,306	1,466	1,626	1,786	1,946	2,106	2,266	2,426	2,586
500	1,172	1,332	1,492	1,652	1,812	1,972	2,132	2,292	2,452	2,612
550	1,198	1,358	1,518	1,678	1,838	1,998	2,159	2,319	2,479	2,639
600	1,225	1,385	1,545	1,705	1,865	2,025	2,185	2,345	2,505	2,665
650	1,251	1,411	1,571	1,731	1,891	2,051	2,211	2,371	2,531	2,691
700	1,277	1,437	1,597	1,757	1,917	2,077	2,237	2,397	2,557	2,717
750	1,304	1,464	1,624	1,784	1,944	2,104	2,264	2,424	2,584	2,744
800	1,330	1,490	1,650	1,810	1,970	2,130	2,290	2,450	2,610	2,770
850	1,356	1,516	1,676	1,836	1,996	2,156	2,316	2,476	2,636	2,796
900	1,382	1,543	1,703	1,863	2,023	2,183	2,343	2,503	2,663	2,823
950	1,409	1,569	1,729	1,889	2,049	2,209	2,369	2,529	2,689	2,849
1000	1,435	1,595	1,755	1,915	2,075	2,235	2,395	2,555	2,715	2,875

Table 2.2 lists the values of A_v , H_v , corresponding $TC(T, f)$ and the figure is shown in Figure 2.7. The effect of A_v / H_v on $TC(T, f)$ is given in Figure 2.8(see page 28) while the values used are listed in Table 2.3. As shown above, it is seen that $TC(T, f)$ is rising when A_v / H_v rises. However, the increasing of H_v can more rapidly raise the total cost. This

is due to the large quantity of products which are held by the producer. So controlling the holding cost is significant for the producer.

Table 2.3 Values of A_v/H_v and $TC(T, f)$ for Figure 2.8

A_v	H_v	A_v/H_v	$TC(T, f)$
1	2	0.5	1,069
10	2	5	1,074
50	2	25	1,095
100	2	50	1,121
150	2	75	1,147
200	2	100	1,174
250	2	125	1,200
300	2	150	1,226
350	2	175	1,253
400	2	200	1,279
450	2	225	1,305

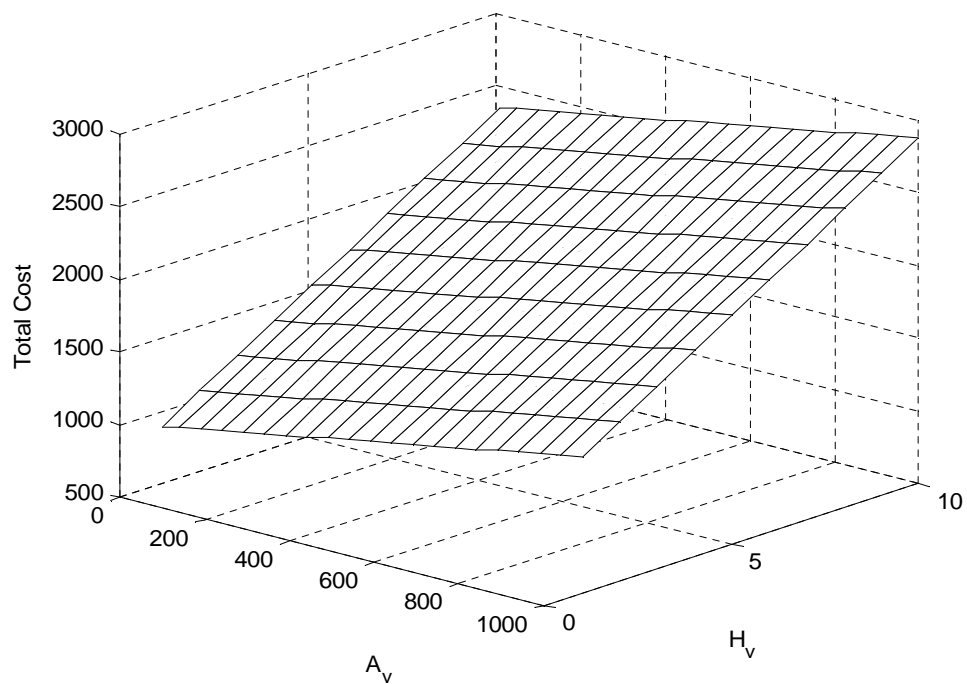


Figure 2.7 Effect of A_v and H_v on $TC(T, f)$

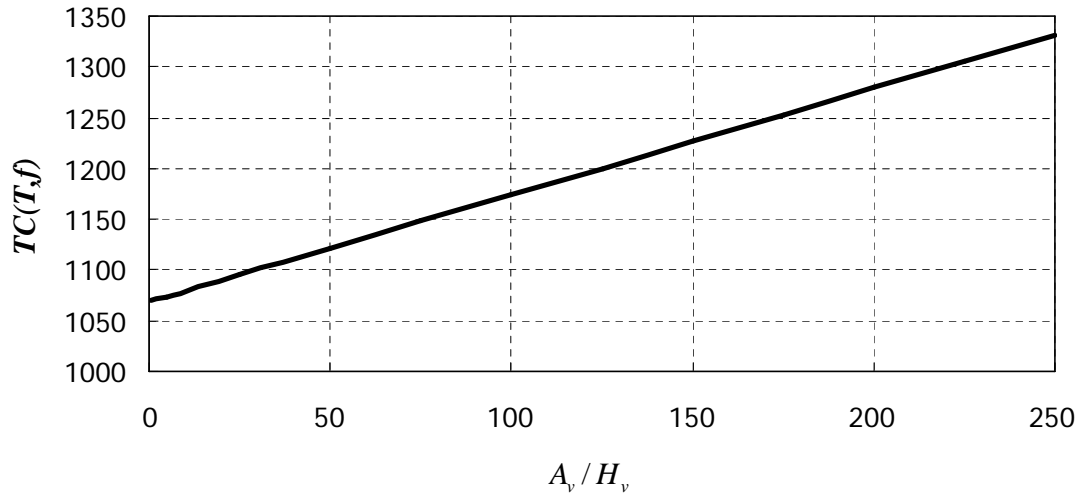


Figure 2.8 Effect of A_v / H_v on $TC(T, f)$

2.7 Benefits of the model

This model can determine the optimal delivery schedule for a supply chain. By applying it, users may build a stable cooperation relationship. The entire supply chain can respond rapidly to the changing market and every user can reduce costs for holding old stocks in the warehouse or for backorders.

CHAPTER 3 THE PRODUCER-BUYER MODEL 2

Model 2 of the entire system's total cost is established in this section. Inventories of the producer and buyers are built in the beginning of the cycle. Raw materials and finished goods costs are expressed separately first and are then jointed to form the joint cost model.

3.1 The problem

A single-producer, multiple-buyer supply system is considered here when the producer pay for both raw materials and finished goods cost while buyers pay for the finished goods cost. Deterioration exist at all levels (at the producer, in-transit and at the buyers) and the remanufacturing process is implemented in producer's level. In order to integrate every partner, consider that all of them have the same replenishment/production cycle. The producer procures raw materials in batches, which can meet n production cycles. The producer's production capability is sufficient to meet all buyers' demands while shortage is allowed at all buyers' levels. Adequate raw materials are supplied to the producer.

The inventory level of the producer and all buyers is nonlinear due to the existence of deteriorating items. The producer get raw material supplement in batches which can meet the requirements of n production cycles. The slope of the tangent to raw material curve increases when the inventory level decreases because the deteriorated raw material number decreases. The inventory level drops to zero when all raw materials are consumed.

Figure 3.1 illustrates the system and Figure 3.2 depicts inventory levels of products at both producer and buyers.

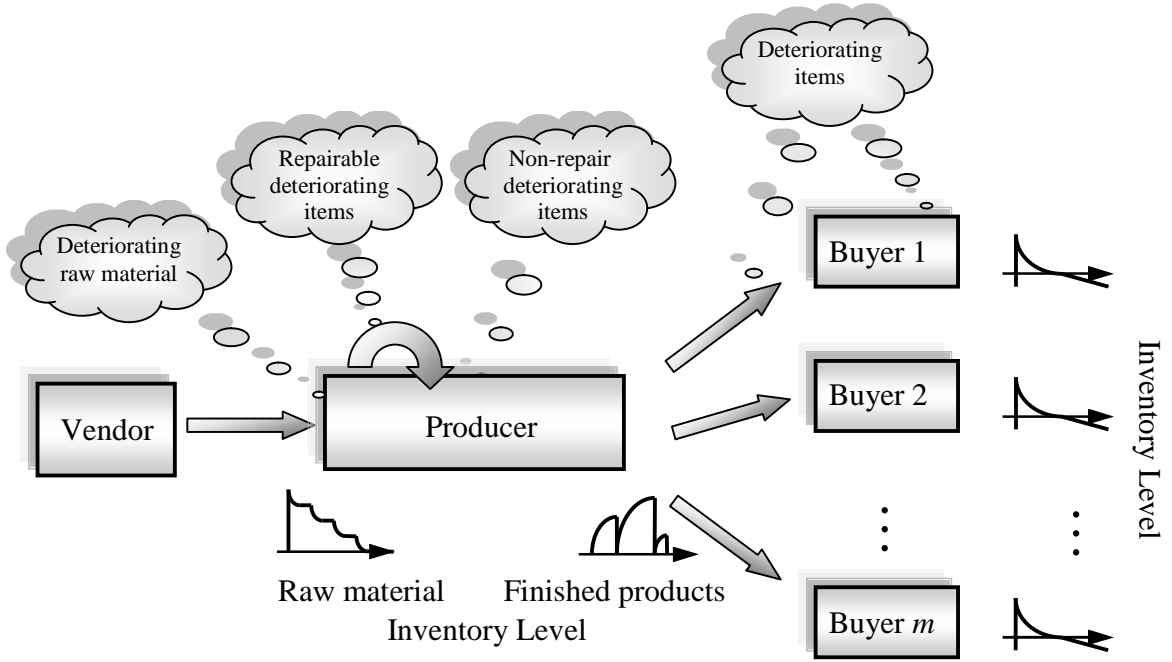


Figure 3.1 Model 2 System

The production rate, P , is not less than the total demand rate $\sum_{j=1}^m D_j$, so production may stop for a period in each cycle. When production starts in each cycle, a general and progressive increase in the number of deteriorated finished goods will occur with the increase inventory level. The curve rises rapidly first and growth slows as the inventory level increases, due to the appearance of more deteriorated items at the later time. The inventory level drops suddenly when a delivery is made, and becomes zero when all deliveries are made.

For all buyers, they receive supplements at the beginning of each replenishment cycle. The slope of tangent to inventory level curve increases as the inventory level decreases because the deteriorated items number decreases. When shortages appear, inventory level becomes negative, and continues decreasing in linear fashion because no deterioration exists in this time.

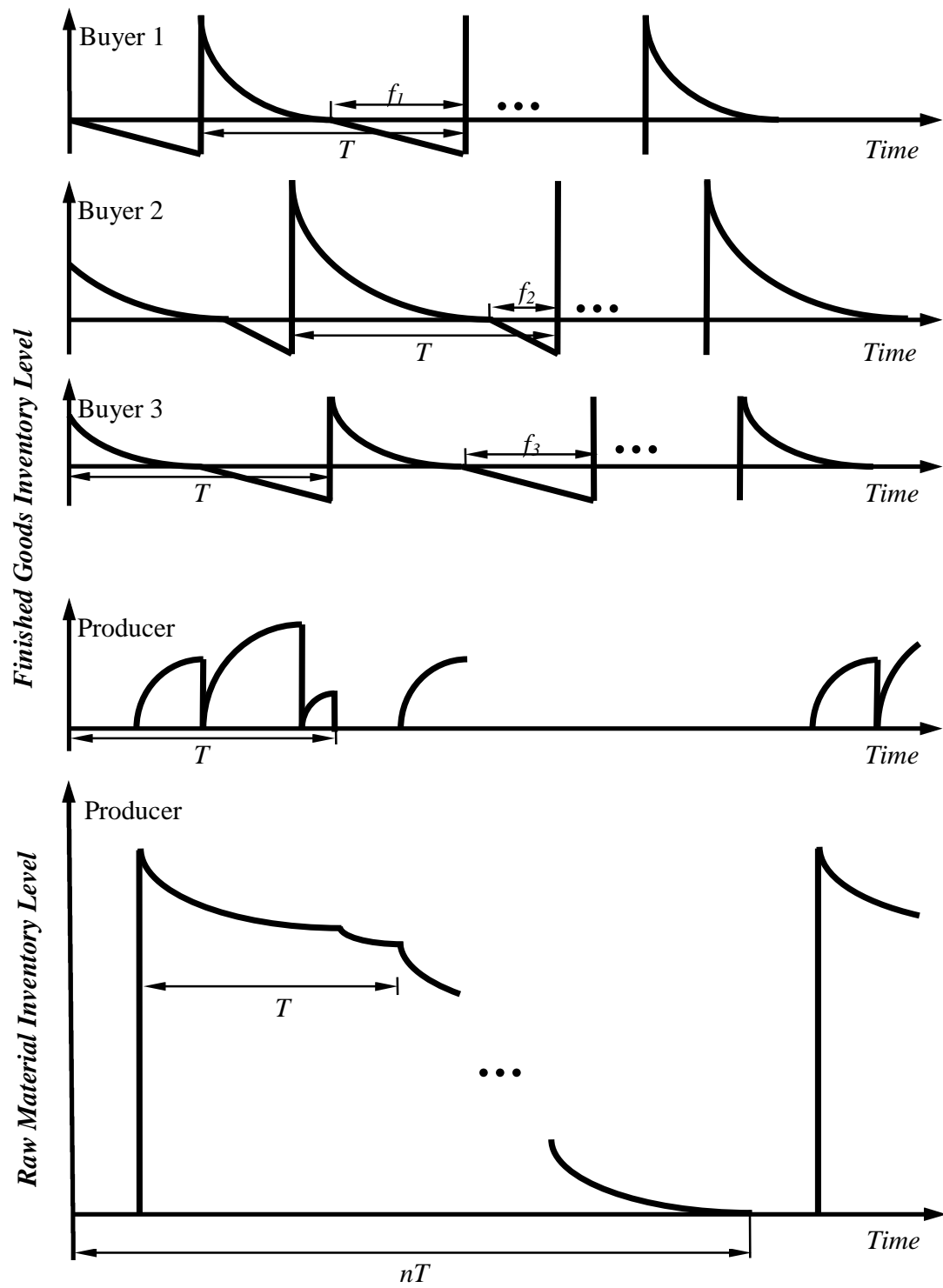


Figure 3.2 Buyers' and the producer's inventory levels for Model 2

3.2 Assumptions and notations

There are many variables in the real world such as an unplanned order, mechanical problem, and policy changes. In this section, some assumptions, regardless of these factors, are considered and notations required to formulate the problem are listed next.

3.2.1 Assumptions:

The assumptions used in this section are as the same as noted in Chapter 2.

3.2.2 Notation

The new notations used in the section are showed as follows; however, notations which are already contained in Chapter 2 are not repeated here.

(a) *System parameters:*

A_{vp} Setup cost of finished goods of producer per production run or batch,
(dollars/batch)

A_{vm} Setup cost of raw materials of producer per production run or batch,
(dollars/batch)

c_{vp}^d The cost of each deteriorated unit of finished goods at the producer's level,
(dollars/unit)

c_{vm}^d The cost of each deteriorated unit of raw materials at the producer's level,
(dollars/unit)

H_{vp} Holding cost of finished goods of the producer (\$/unit/unit-time),

H_{vm} Holding cost of raw materials of the producer (\$/unit/unit-time),

S_{vp} Remanufacturing proceed of finished goods of the producer (\$/unit/unit-time),

M Usage rate of raw materials for producing each finished good,

θ_{vp}	Proportion of the on-hand finished goods inventory deteriorated at the producer's level
θ_{vm}	Proportion of raw materials deteriorated in transit from the producer to buyers
α	The repairable rate of deteriorating items in producer's level

(b) Intermediate variables:

$I^{vp}(t)$	Instantaneous inventory level of the producer at time t , (unit)
I_j^{vp}	Total finished good inventory carried by the producer for buyer j in any cycle, (unit)
I^{vp}	Total finished good inventory carried by the producer, (unit)
I^{vm}	Total raw material inventory carried by the producer in each batch, (unit)
Q_i^{vm}	Ordering quantity of raw material by the producer for cycle i , (unit/order)
Q^{vm}	Total ordering quantity of raw material by the producer in each batch, (unit/batch)
W^{vp}	Total number of deteriorated finished good units of the producer for buyer j in any cycle
W^{vm}	Total number of deteriorated raw material units of the producer for buyer j in any cycle
R^{vp}	The number of remanufacturing items
TC^{vp}	The producer's cost of finished goods in any unit time
TC^{vm}	The producer's cost of raw materials in any unit time
TP^{vp}	The producer's remanufacturing proceeds in any unit time

$TC_0(T, f)$ The joint cost of the producer and all buyers in any cycle (without remanufacturing proceeds and raw material cost), which is a function of T (common cycle time) and f (a vector of fractions of shortage time to the cycle time for all buyers). It is as same as $TC(T, f)$ in Chapter 2.

$TC_R(T, f)$ The joint cost of the producer and all buyers in any cycle (with remanufacturing proceeds but without raw material cost), which is a function of T (common cycle time) and f (a vector of fractions of shortage time to the cycle time for all buyers).

$TC_{RM+R}(T, f)$ The joint cost of the producer and all buyers in any cycle (with both remanufacturing proceeds and raw material cost), which is a function of T (common cycle time) and f (a vector of fractions of shortage time to the cycle time for all buyers).

3.3 The Model

A mathematical model of the total joint-cost of the entire system is built here. Inventory levels of the producer and the buyers for both raw material and finished goods are modeled. The costs of all parties are expressed individually first and then combined.

3.3.1 Buyers' cost

The total cost for each buyer is as the same as function (2.6) given by Chapter 2, which is composed of ordering cost, holding cost, backlog cost, and deterioration cost.

$$\begin{aligned}
 TC^b = & \frac{1}{T} \left\{ mA_b + \sum_{j=1}^m H_b \left[\frac{D_j}{\theta_b^2} (e^{\theta_b T(1-f_j)} - 1) - \frac{D_j}{\theta_b} T(1-f_j) \right] + \sum_{j=1}^m \hat{\pi}_b \frac{D_j}{2} T^2 f_j^2 \right. \\
 & \left. + \sum_{j=1}^m c_b^d \left[\frac{D_j}{\theta_b} (e^{\theta_b T(1-f_j)} - 1) - D_j T(1-f_j) \right] \right\}. \quad (j = 1, 2, \dots, m) \quad (3.1)
 \end{aligned}$$

3.3.2 Producer's cost

The total cost to the producer contains raw material cost, finished goods cost, and remanufacturing proceeds. For raw material, setup cost per batch, holding cost, and deterioration cost are considered. For finished goods, setup cost per production run, holding cost, and deterioration cost are taken into account. Deterioration in both producer's warehouse and shipment process are calculated.

(a) Raw material cost

The usage of raw materials for producing in each production cycle can be described as $\sum_{j=1}^m MD_j T$, where M is the usage rate of raw materials for producing each unit finished good. So, the ordering quantity of raw material for cycle i is

$$Q_i^{vm} = \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i, \quad (3.2)$$

and the amount of raw material ordered by the producer for each batch (n cycle) is

$$Q^{vm} = \sum_{i=1}^n \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i. \quad (3.3)$$

So, the total number, W^{vm} , of deteriorated raw material units for the producer in each cycle equals to the original ordering quantity minus the used quantity:

$$W^{vm} = \sum_{i=1}^n \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i - \sum_{i=1}^n \sum_{j=1}^m MD_j T, \quad (3.4)$$

and the amount of raw material inventory I^{vm} carried by the producer's in cycle i is

$$I^{vm} = \sum_{i=1}^n [Q^{vm} - \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i] \times (1 - \theta_{vm})^{i-1}. \quad (3.5)$$

So, let A_{vm} expresses the setup cost of raw material of each batch, H_{vm} is the holding cost and c_{vm}^d is the deteriorated cost of each raw material unit, the total cost to the producer, in unit

time, TC^{vm} consists of the setup cost, holding cost and deteriorated item cost, which can be given by

$$\begin{aligned}
TC^{vm} = & \frac{1}{nT} \{ A_{vm} + H_{vm} \{ \sum_{i=1}^n [Q^{vm} - \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i] \times (1 - \theta_{vm})^{i-1} \} \\
& + c_{vm}^d [\sum_{i=1}^n \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i - \sum_{i=1}^n \sum_{j=1}^m MD_j T] \}. \quad (i = 1, 2, \dots, n, j = 1, 2, \dots, m)
\end{aligned}
\tag{3.6}$$

(b) Finished good cost

The finished goods cost to the producer is approximately the same as the function in (2.15) (see page 16) in Chapter 2. However, because part of the deterioration items can be remanufactured, so the number of deteriorated finished good units W^{vp} can be shown as

$$\begin{aligned}
W^{vp} = & (1 - \alpha) [T_i P - \sum_{j=1}^m I^v(t_{ij})] \\
= & -P(1 - \alpha) \sum_{j=1}^m \frac{1}{\theta_{vp}} \ln \{ 1 - \frac{D_j \theta_{vp}}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1 - f_j)} - 1) + \theta_b T f_j] \} \\
& - (1 - \alpha) \sum_{j=1}^m \frac{P}{\theta_{vp}} (1 - e^{-\theta_{vp} T_j}),
\end{aligned}
\tag{3.7}$$

and the finished goods total cost to the producer TC^{vp} , which consists of the setup cost, holding cost and deteriorated item cost, can be given by

$$\begin{aligned}
TC^{vp} = & \frac{1}{T} \{ A_{vp} + H_{vp} \sum_{j=1}^m [\frac{P}{\theta_{vp}} T_j + \frac{P}{\theta_{vp}^2} (e^{-\theta_{vp} T_j} - 1)] \\
& - c_{vp}^d P(1 - \alpha) \sum_{j=1}^m \frac{1}{\theta_{vp}} \ln \{ 1 - \frac{D_j \theta_{vp}}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1 - f_j)} - 1) + \theta_b T f_j] \} \\
& - c_{vp}^d (1 - \alpha) \sum_{j=1}^m \frac{P}{\theta_{vp}} (1 - e^{-\theta_{vp} T_j}) \}. \quad (j = 1, 2, \dots, m)
\end{aligned}
\tag{3.8}$$

(c) *Remanufacturing proceeds*

The number of remanufacturing items R^{vp} is equals to the repairable rate of deteriorating items in producer's level α multiplied by the total number of deteriorated finished good units, which is

$$R^{vp} = -P\alpha \sum_{j=1}^m \frac{1}{\theta_{vp}} \ln \left\{ 1 - \frac{D_j \theta_{vp}}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1-f_j)} - 1) + \theta_b T f_j] \right\} - \alpha \sum_{j=1}^m \frac{P}{\theta_{vp}} (1 - e^{-\theta_{vp} T_j}). \quad (3.9)$$

So, the saving of remanufacturing is equals to the remanufacturing proceed of finished goods of the producer, S_{vp} , multiplied by the number of remanufacturing items R^{vp} , which can be given by

$$TP^{vp} = \frac{1}{T} \{ -S_{vp} P \alpha \sum_{j=1}^m \frac{1}{\theta_{vp}} \ln \left\{ 1 - \frac{D_j \theta_{vp}}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1-f_j)} - 1) + \theta_b T f_j] \right\} - S_{vp} \alpha \sum_{j=1}^m \frac{P}{\theta_{vp}} (1 - e^{-\theta_{vp} T_j}) \}. \quad (j=1,2,...,m) \quad (3.10)$$

3.3.3 System Total cost

For the shortage time fraction vector $f = (f_1, f_2, f_3, ..., f_m)$ and cycle time T , the system's joint total cost $TC_{RM+R}(T, f)$, in unit time, contains the producer's and all buyers' cost, which is

$$\begin{aligned} TC_{RM+R}(T, f) = & TC^b + TC^{vm} + TC^{vp} - TP^{vp} \\ = & \frac{1}{T} \{ mA_b + \sum_{j=1}^m H_b \left[\frac{D_j}{\theta_b^2} (e^{\theta_b T (1-f_j)} - 1) - \frac{D_j}{\theta_b} T (1-f_j) \right] + \sum_{j=1}^m \hat{\pi}_b \frac{D_j}{2} T^2 f_j^2 \\ & + \sum_{j=1}^m c_b^d \left[\frac{D_j}{\theta_b} (e^{\theta_b T (1-f_j)} - 1) - D_j T (1-f_j) \right] \} \\ & + \frac{1}{nT} \{ A_{vm} + H_{vm} \left\{ \sum_{i=1}^n [Q^{vm} - \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i] \times (1 - \theta_{vm})^{i-1} \right\} \} \end{aligned}$$

$$\begin{aligned}
& + c_{vm}^d \left[\sum_{i=1}^n \sum_{j=1}^m MD_j T / (1 - \theta_{vm})^i - \sum_{i=1}^n \sum_{j=1}^m MD_j T \right] \} \\
& + \frac{1}{T} \{ A_{vp} + H_{vp} \sum_{j=1}^m \left[\frac{P}{\theta_{vp}} T_j + \frac{P}{\theta_{vp}^2} (e^{-\theta_{vp} T_j} - 1) \right] \} \\
& - c_{vp}^d P (1 - \alpha) \sum_{j=1}^m \frac{1}{\theta_{vp}} \ln \left\{ 1 - \frac{D_j \theta_{vp}}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1 - f_j)} - 1) + \theta_b T f_j] \right\} \\
& - c_{vp}^d (1 - \alpha) \sum_{j=1}^m \frac{P}{\theta_{vp}} (1 - e^{-\theta_{vp} T_j}) \} \\
& + \frac{1}{T} \{ S_{vp} P \alpha \sum_{j=1}^m \frac{1}{\theta_{vp}} \ln \left\{ 1 - \frac{D_j \theta_{vp}}{\theta_b (1 - \theta_{vb}) P} [(e^{\theta_b T (1 - f_j)} - 1) + \theta_b T f_j] \right\} \} \\
& + S_{vp} \alpha \sum_{j=1}^m \frac{P}{\theta_{vp}} (1 - e^{-\theta_{vp} T_j}) \}. \quad (i = 1, 2, \dots, n, j = 1, 2, \dots, m) \quad (3.11)
\end{aligned}$$

3.4 Solution Procedure

Mathematically, in order to prove that there is a minimum total cost, all second-order derivative equations of the total cost function are positive, should be proved. However, the total cost function $TC_{RM+R}(T, f)$ is a higher-order exponential function so it is complicated to show the second-order derivative in closed-form directly. Instead, the response surface over a possible range of the given set of parametric values are evaluated to verify the convexity of $TC_{RM+R}(T, f)$. Software “Matlab” is used to complete the process and the results show that $TC_{RM+R}(T, f)$ is convex in T and f_j within the given range. In this case, f_j are fractions from 0 to 1; a reasonable range for T (an acceptable delivery cycle time for a common user) usually from 0 to 1 year (a range from 0 to 365 days were chosen in the numerical examples’ section here).

The optimal results should be found by calculating the solution of equations $\partial TC_{RM+R}(T, f) / \partial T = 0$ and $\partial TC_{RM+R}(T, f) / \partial f_j = 0$ for $j = 1, 2, \dots, m$; but it is

also difficult to find the first-order derivative in closed-form. So, the computer algorithm listed below is used to search for the optimal result:

Algorithm 2. Calculation of optimal total cost

Step 0: (a) Initialize system parameters $m, n, i, j, A_b, A_{vp}, A_{vm}, c_b^d, c_{vp}^d, c_{vm}^d, H_b, H_{vp}, H_{vm},$

$$S_{vp}, \hat{\pi}_b, P, D_j, M, \theta_b, \theta_{vp}, \theta_{vm}, \theta_{vb}, \alpha, \Delta TC, \Omega_T \text{ and } \Omega_f.$$

(b) Set the range of $T \in (0, a]$ and $f_j \in [0, 1]$, for $j = 1, \dots, m$.

(c) Set $T = 0$ and $f_j = 0$, initialize ΔT and Δf_j ; for $j = 1, \dots, m$.

(d) Set $T^* = 0, f_j^* = 0, TC_{RM+R}(T, f)^* = \infty$.

Step 1. Repeat $T = T + \Delta T$

Repeat $f_j = f_j + \Delta f_j$

Compute $TC_{RM+R}(T, f)^*$.

If $TC(T, f) < TC_{RM+R}(T, f)^* = \infty$,

$$TC_{RM+R}(T, f)^* = TC_{RM+R}(T, f),$$

until $f_j = 1$,

until $T = a$.

Set $T = T^*, f_j = f_j^*$

Step 2. (a) Compute $\frac{\partial TC_{RM+R}(T, f)}{\partial T}$ and $\frac{\partial TC_{RM+R}(T, f)}{\partial f_j}$ for $j = 1, 2, \dots, m$.

(b) If $\frac{\partial TC_{RM+R}(T, f)}{\partial T} \geq \Delta TC$ and $\frac{\partial TC_{RM+R}(T, f)}{\partial f_j} \geq \Delta TC$

Set the rank of $T \in (T - \Delta T, T + \Delta T]$, $f_j \in [f_j - \Delta f_j, f_j + \Delta f_j]$, $\Delta T = \Delta T / \Omega_T$,

$\Delta f_j = \Delta f_j / \Omega_f$, go to Step 1.

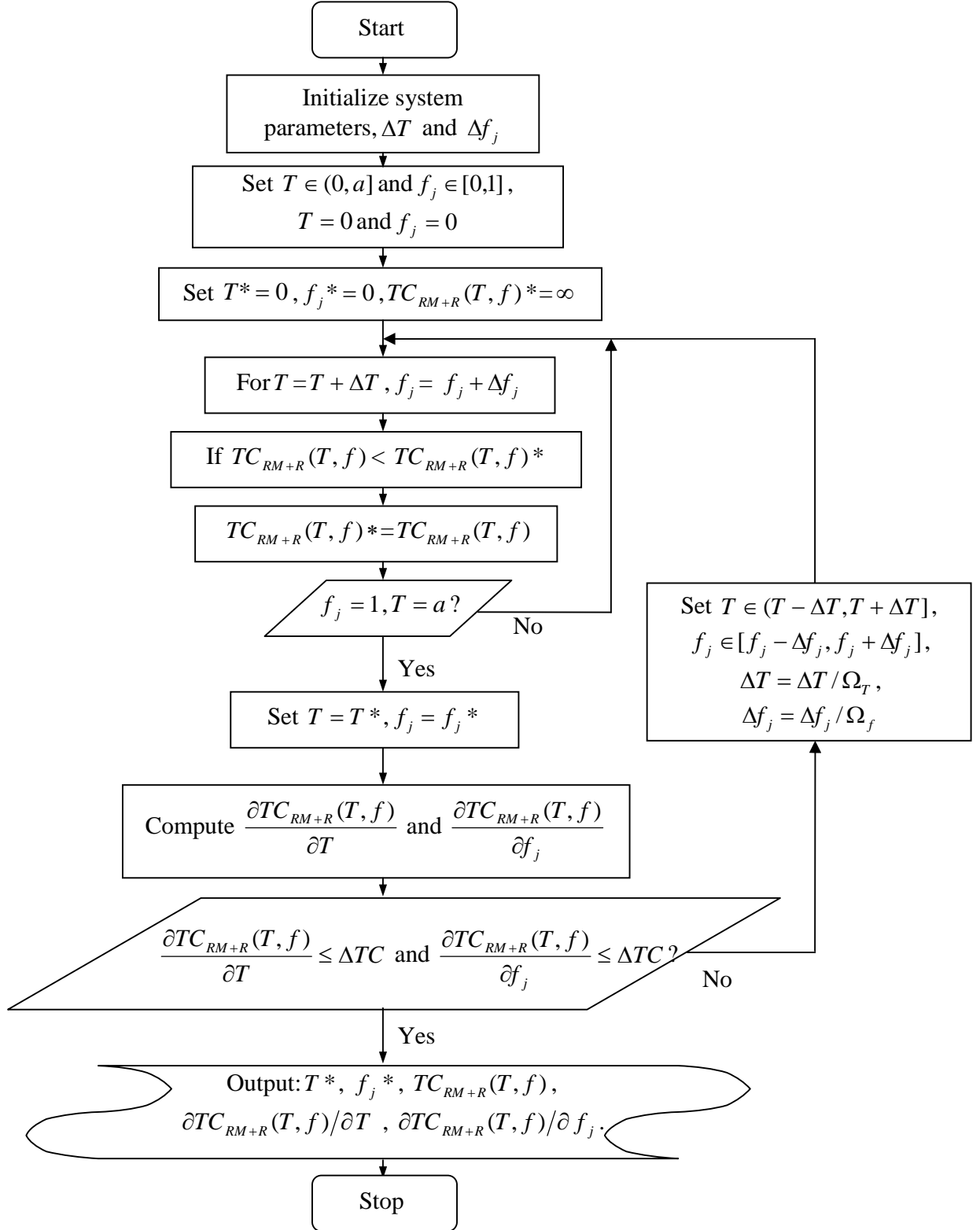


Figure 3.3 Algorithm 2's flow chart

Step 3. Output T^* , f_j^* , $TC_{RM+R}(T, f)^*$, $\partial TC_{RM+R}(T, f)/\partial T$ and $\partial TC_{RM+R}(T, f)/\partial f_j$.

Step 4. STOP.

The flow chart of Algorithm 2 is showed in Figure 3.3. Basically, all variables' objective areas should be searched together. However, this method might require considerable computer time when the buyer number, m , is large. Instead, searching each f_j separately, first to narrow the optimal solution range, and then searching all variables together is a faster way. Another practicable way is to consider the significant variables and ignoring others.

3.5 Computational results

Three numerical examples are presented for investigating the performance of the model here.

Example 3.1 A single producer, single buyer system

The system parameters are: $m = 1$, $n = 10$, $A_b = 100$, $A_{vp} = 1000$, $A_{vm} = 500$, $H_b = 3$, $H_{vp} = 2$, $H_{vm} = 1$, $c_b^d = 12$, $c_{vp}^d = 10$, $c_{vm}^d = 3$, $\hat{\pi}_b = 12$, $S_{vp} = 150$, $M = 2$, $\alpha = 0.8$, $\theta_b = 0.02$, $\theta_{vp} = 0.005$, $\theta_{vm} = 0.015$, $\theta_{vb} = 0.01$, $P = 300$ units/month, $D_1 = 200$ units/month. Set $\Delta TC = \$0.01$, $\Omega_T = 10$ and $\Omega_f = 10$.

Results are listed in Table 3.1, the cycle time $T^* = 1.7875$ month and $f_1^* = 0.2167$. The minimum system's total cost $TC_{RM+R}(T, f)^*$ is \$20,117. $\partial TC_{RM+R}(T, f)/\partial T$ and $\partial TC_{RM+R}(T, f)/\partial f_1$ are calculated and both are equal to 0 which confirms the result. To see the quality of the total cost function intuitively, the total cost function figure respect to f_1 and T as shown in Figure 3.4.

Example 3.2 A single producer, two buyers system

The system parameters are: $m = 2$, $n = 10$, $A_b = 100$,
 $A_{vp} = 1000$, $A_{vm} = 500$, $H_b = 3$, $H_{vp} = 2$, $H_{vm} = 1$, $c_b^d = 12$, $c_{vp}^d = 10$, $c_{vm}^d = 3$, $\hat{\pi}_b = 12$,
 $S_{vp} = 150$, $M = 2$, $\alpha = 0.8$, $\theta_b = 0.02$, $\theta_{vp} = 0.005$, $\theta_{vm} = 0.015$, $\theta_{vb} = 0.01$,
 $P = 400$ units/month, $D_1 = 200$ units/month, $D_2 = 120$ units/month. Set $\Delta TC = \$0.01$ $\Omega_T = 10$ and $\Omega_f = 10$.

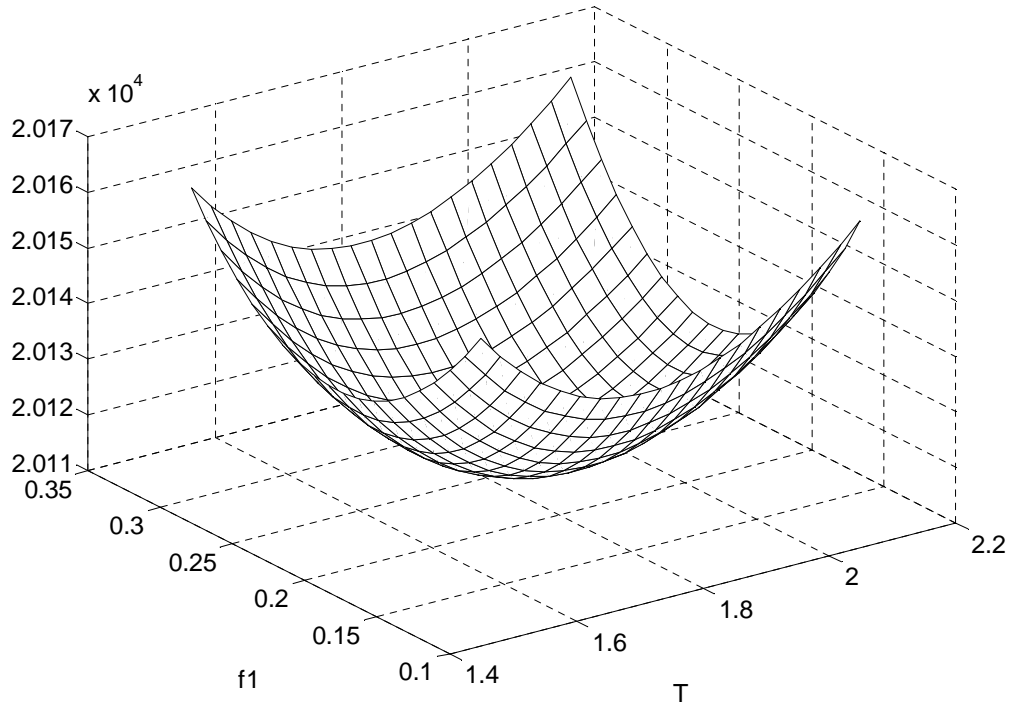


Figure 3.4 Function with respect to f_1 and T

The results are also listed in Table 3.1 where the cycle time $T^* = 1.5567$ month, $f_1^* = 0.2157$, and $f_2^* = 0.2153$, and the system's total cost $TC(T, f)^* = \$31,740$. $\partial TC_{RM+R}(T, f) / \partial f_1$, $\partial TC_{RM+R}(T, f) / \partial f_2$, and $\partial TC_{RM+R}(T, f) / \partial T$ are confirmed equal to 0.

Example 3.3 A single producer, three buyers system.

The system parameters are: $m = 3$, $n = 10$, $A_b = 100$,
 $A_{vp} = 1000$, $A_{vm} = 500$, $H_b = 3$, $H_{vp} = 2$, $H_{vm} = 1$, $c_b^d = 12$, $c_{vp}^d = 10$, $c_{vm}^d = 3$, $\hat{\pi}_b = 12$,
 $S_{vp} = 150$, $M = 2$, $\alpha = 0.8$, $\theta_b = 0.02$, $\theta_{vp} = 0.005$, $\theta_{vm} = 0.015$, $\theta_{vb} = 0.01$,
 $P = 800$ units/month, $D_1 = 200$ units/month, $D_2 = 120$ units/month, $D_3 = 400$ units/month.

Set $\Delta TC = \$0.01$ $\Omega_T = 10$ and $\Omega_f = 10$, optimal cycle time for three retailers are
 $T^* = 1.1182$ month, $f_1^* = 0.2144$, $f_2^* = 0.2143$, $f_3^* = 0.2147$ gg, and the system's minimal
total cost $TC(T, f)^* = \$70,279$. $\partial TC_{RM+R}(T, f) / \partial f_1 \approx 0$, $\partial TC_{RM+R}(T, f) / \partial f_2 \approx 0$,
 $\partial TC_{RM+R}(T, f) / \partial f_3 \approx 0$, and $\partial TC_{RM+R}(T, f) / \partial T \approx 0$, which conform to the accurate results.

Table 3.1 The optimal solutions for the case with raw material and remanufacturing cost

T^*	f_1^*	f_2^*	f_3^*	$\frac{\partial TC(T, f)}{\partial T}$	$\frac{\partial TC(T, f)}{\partial f_j}$	$TC_{RM+R}(T, f)^*$
1.7875	0.2167	-	-	0	0	\$20,117
1.5567	0.2157	0.2153	-	0	0	\$31,740
1.1182	0.2144	0.2143	0.2147	0	0	\$70,279

In order to show the effects of raw materials and remanufacturing, the optimal results in another two situations are provided. The first one is considers remanufacturing proceeds but ignores raw materials cost, which result is list in Table 3.2. The second one is considering the total cost without raw materials cost and remanufacturing proceeds, and the result is showed in Table 3.3. Both situations are based on the same parameters as examples given above.

Table 3.2 The optimal solutions for the case without raw material

T^*	f_1^*	f_2^*	f_3^*	$\frac{\partial TC(T, f)}{\partial T}$	$\frac{\partial TC(T, f)}{\partial f_j}$	$TC_R(T, f)^*$
1.7486	0.2166	-	-	0	0	\$1,252
1.5256	0.2158	0.2153	-	0	0	\$1,566
1.0854	0.2145	0.2143	0.2147	0	0	\$2,388

Table 3.3 The optimal solutions for the case without raw material and remanufacturing cost

T^*	f_1^*	f_2^*	f_3^*	$\frac{\partial TC(T, f)}{\partial T}$	$\frac{\partial TC(T, f)}{\partial f_j}$	$TC_0(T, f)^*$
1.6470	0.2172	-	-	0	0	\$1,326
1.4620	0.2161	0.2154	-	0	0	\$1,633
1.0339	0.2145	0.2143	0.2151	0	0	\$2,505

By comparing the results of Tables 3.1 and 3.3, it is noticed that raw material constitute a high proportion of total cost. So, the inventory of raw material is reduced by getting the supply more frequently and controlling the holding cost of raw material as low as possible is an efficient way to reduce the total cost of inventory system. By comparing the results of Tables 3.2 and 3.3, it appears the total cost can be reduced by considering the remanufacturing process as well.

3.6 Sensitivity analysis

In model 2, system parameters are assumed to be static. However, as the estimates change in different situations, it is necessary to study the sensitivity to know the effect of these changing parameters for a given optimum solution. Because the inventory deteriorated proportions have a strong influence on the total cost, two important parameters θ_b and θ_{vb} will be discussed.

3.6.1 Effect of θ_b on $TC_{RM+R}(T, f)$

In order to observe the impact of inventory deteriorated proportion θ_b (buyer's level), the partial derivative of θ_b is:

$$\begin{aligned} \frac{\partial TC_{RM+R}(T, f)}{\partial \theta_b} = & \frac{1}{T} \left\{ H_b \left[-\frac{2D_1(e^{\theta_b T(1-f_1)} - 1)}{\theta_b^3} + \frac{D_1 T(1-f_1)e^{\theta_b T(1-f_1)}}{\theta_b^2} + \frac{D_1 T(1-f_1)}{\theta_b^2} \right] \right. \\ & \left. + c_b^d \left[-\frac{D_1(e^{\theta_b T(1-f_1)} - 1)}{\theta_b^2} + \frac{D_1 T(1-f_1)e^{\theta_b T(1-f_1)}}{\theta_b} \right] \right\} \end{aligned}$$

$$+ \frac{[S_{vp}\alpha - c_{vp}^d(1-\alpha)]P \left[\frac{D_1\theta_{vp}(e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b^2(1-\theta_{vb})P} - \frac{D_1\theta_{vp}T(1-f_1)(e^{\theta_b T(1-f_1)} + T f_1)}{\theta_b(1-\theta_{vb})P} \right]}{\theta_{vp} \left[1 - \frac{D_1\theta_{vp}(e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b(1-\theta_{vb})P} \right]}. \quad (3.12)$$

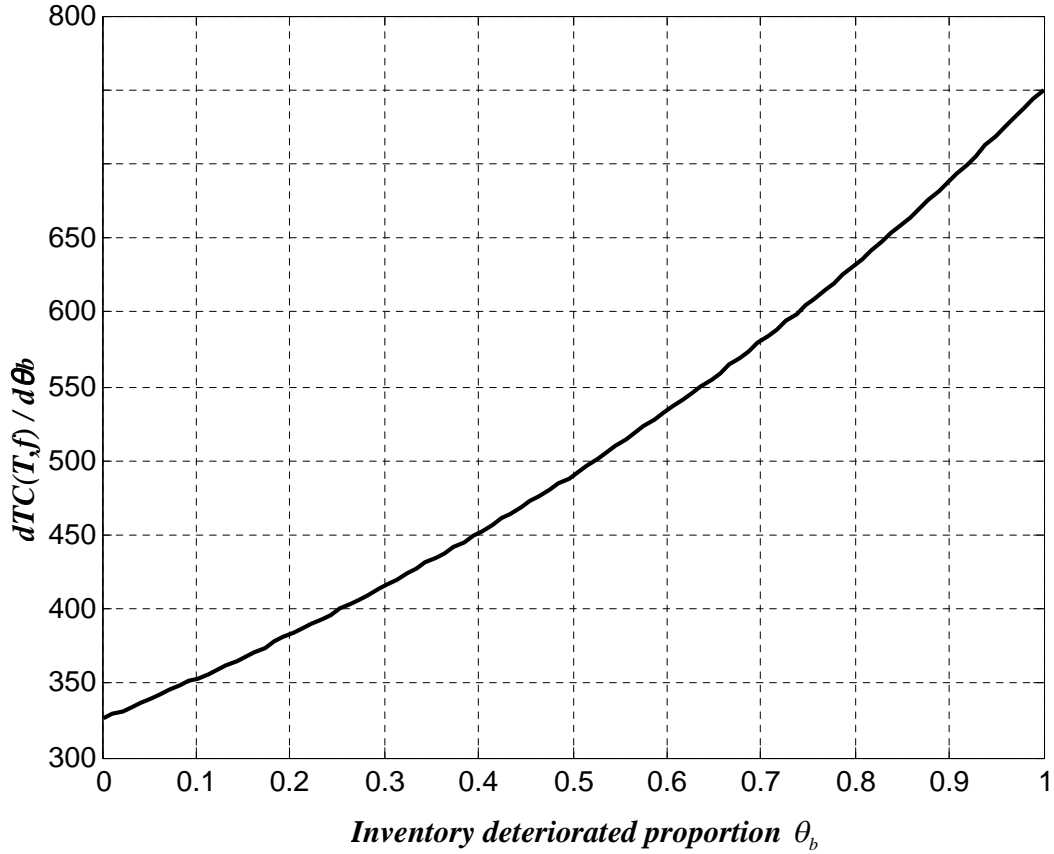


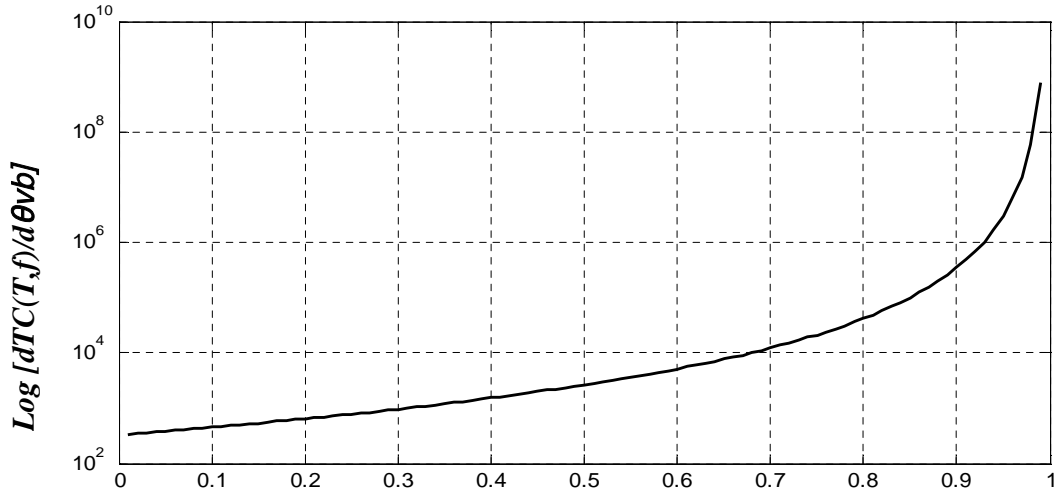
Figure 3.5 Effect of θ_b on $TC_{RM+R}(T, f)$

By equation (3.12) for $\theta_b \in [0,1]$, the effect on the total cost $TC_{RM+R}(T, f)$ is shown in Figure 3.5. According to Figure 3.5, the term $\partial TC_{RM+R}(T, f) / \partial \theta_b$ is positive and increases with θ_b . So, reducing the deterioration in buyer's level can reduce expenses.

3.6.2 Effect of θ_{vb} on $TC_{RM+R}(T, f)$

The change of rate and direction of total cost $TC_{RM+R}(T, f)$, with respect to the proportion of finished goods deteriorated in transit from the producer to buyers, is shown as:

$$\begin{aligned}
\frac{\partial TC_{RM+R}(T, f)}{\partial \theta_{vb}} &= \frac{1}{T} \{ H_{vp} \{ \frac{D_1(e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_{vp} \theta_b (1 - \theta_{vb})^2 [1 - \frac{D_1 \theta_{vp} (e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b (1 - \theta_{vb}) P}]} \\
&\quad - \frac{D_1(e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b \theta_{vp} (1 - \theta_{vp})^2} \} - \frac{[S_{vp} \alpha - c_{vp}^d (1 - \alpha)] D_1(e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b (1 - \theta_{vb})^2 [1 - \frac{D_1 \theta_{vp} (e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b (1 - \theta_{vb}) P}]} \\
&\quad + \frac{[S_{vp} \alpha - c_{vp}^d (1 - \alpha)] D_1(e^{\theta_b T(1-f_1)} - 1 + \theta_b T f_1)}{\theta_b (1 - \theta_{vb})^2} \} . \quad (3.13)
\end{aligned}$$



Inventory deteriorated proportion θ_{vb}

Figure 3.6 Effect of θ_{vb} on $TC_{RM+R}(T, f)$

Increasing the value incrementally of θ_{vb} from 0 to 1 and keeping the other parametric values unchanged, the effect to the total cost $TC_{RM+R}(T, f)$ is given in Figure 3.6. Because the change is small, $\log \partial TC_{RM+R}(T, f_1) / \partial \theta_{vb}$ can replace $\partial TC_{RM+R}(T, f_1) / \partial \theta_{vb}$ is used to illustrate the trend more clearly. In Figure 3.6, it is seen that $\partial TC_{RM+R}(T, f_1) / \partial \theta_{vb}$ is increases when θ_{vb} increases, which indicates reduce θ_{vb} can also decrease the total cost.

3.6.3 Effect of A_m and H_m on $TC_{RM+R}(T, f)$

Because the raw material cost is a major portion of the total cost $TC_{RM+R}(T, f)$, the effects of setup/ordering cost A_m and holding cost H_m of raw materials are also analyzed.

Table 3.4 Values of A_m , H_m and $TC_{RM+R}(T, f)$ for Figure 3.7

$\begin{matrix} H_m \\ A_m \end{matrix}$	$TC_{RM+R}(T, f)$									
	1	2	3	4	5	6	7	8	9	10
10	20,090	38,402	56,714	75,027	93,339	111,652	129,964	148,276	166,589	184,901
35	20,091	38,403	56,716	75,028	93,341	111,653	129,965	148,278	166,590	184,903
60	20,092	38,405	56,717	75,030	93,342	111,655	129,967	148,279	166,592	184,904
85	20,094	38,406	56,719	75,031	93,344	111,656	129,968	148,281	166,593	184,906
110	20,095	38,408	56,720	75,033	93,345	111,657	129,970	148,282	166,595	184,907
135	20,097	38,409	56,722	75,034	93,346	111,659	129,971	148,284	166,596	184,908
160	20,098	38,411	56,723	75,035	93,348	111,660	129,973	148,285	166,597	184,910
185	20,100	38,412	56,724	75,037	93,349	111,662	129,974	148,286	166,599	184,911
210	20,101	38,413	56,726	75,038	93,351	111,663	129,975	148,288	166,600	184,913
235	20,102	38,415	56,727	75,040	93,352	111,664	129,977	148,289	166,602	184,914
260	20,104	38,416	56,729	75,041	93,353	111,666	129,978	148,291	166,603	184,915
285	20,105	38,418	56,730	75,042	93,355	111,667	129,980	148,292	166,605	184,917
310	20,107	38,419	56,731	75,044	93,356	111,669	129,981	148,294	166,606	184,918
335	20,108	38,420	56,733	75,045	93,358	111,670	129,983	148,295	166,607	184,920
360	20,109	38,422	56,734	75,047	93,359	111,672	129,984	148,296	166,609	184,921
385	20,111	38,423	56,736	75,048	93,361	111,673	129,985	148,298	166,610	184,923
410	20,112	38,425	56,737	75,050	93,362	111,674	129,987	148,299	166,612	184,924
435	20,114	38,426	56,739	75,051	93,363	111,676	129,988	148,301	166,613	184,925
460	20,115	38,428	56,740	75,052	93,365	111,677	129,990	148,302	166,614	184,927
485	20,117	38,429	56,741	75,054	93,366	111,679	129,991	148,303	166,616	184,928
500	20,117	38,430	56,742	75,055	93,367	111,679	129,992	148,304	166,617	184,929

Table 3.4 lists the values of A_m , H_m , corresponding $TC_{RM+R}(T, f)$ and the figure is shown in Figure 3.7. The effect of A_m / H_m on $TC_{RM+R}(T, f)$ is shown in Figure 3.8 while the values used are listed in Table 3.5.

Table 3.5 Values of A_m / H_m and $TC_{RM+R}(T, f)$ for Figure 3.8

A_m	H_m	A_m / H_m	$TC_{RM+R}(T, f)$
1	1	1	20,089
5	1	5	20,089
10	1	10	20,091
35	1	35	20,092
60	1	60	20,094
85	1	85	20,095
110	1	110	20,097
135	1	135	20,098
160	1	160	20,100
185	1	185	20,091
200	1	200	20,100

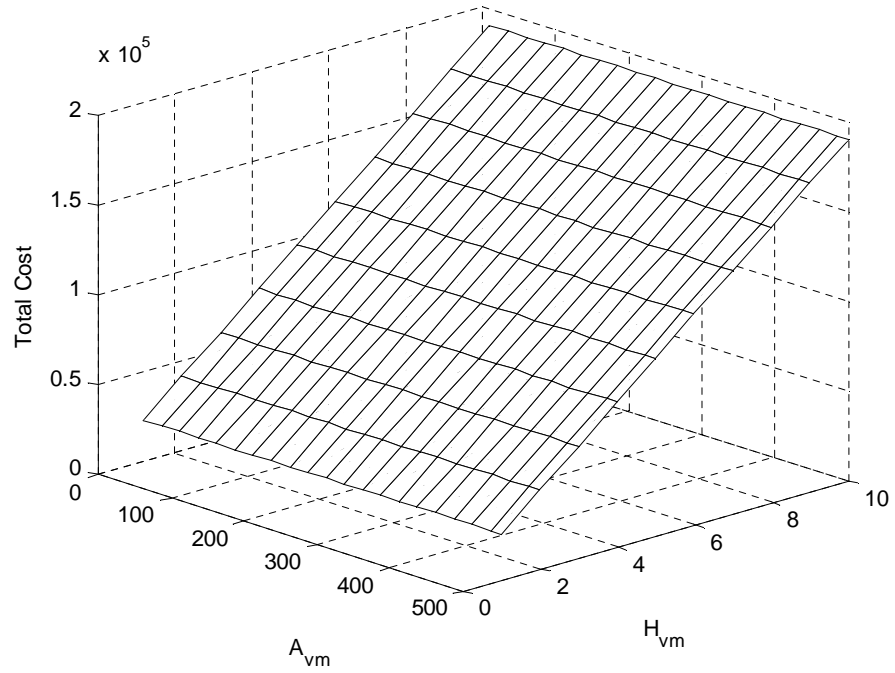


Figure 3.7 Effect of A_m and H_m on $TC_{RM+R}(T, f)$

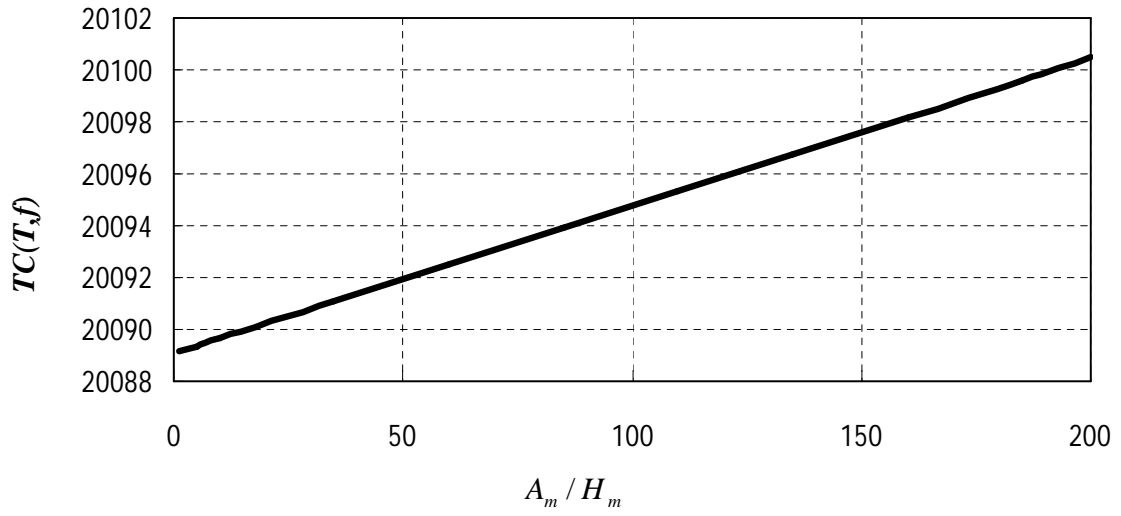


Figure 3.8 Effect of A_m / H_m on $TC_{RM+R}(T, f)$

As in the figures given above, it is appears that $TC_{RM+R}(T, f)$ increases when A_m or H_m rise. However, the increasing of H_m can more rapidly increase the total cost. This is due to the large quantity of raw materials held by the producer. So, reducing the holding cost of raw material is another way to control the total cost.

3.7 Benefits of the model

By applying model 2, users joined by a supply chain may build a stable cooperative relationship and increase market share. They can also respond rapidly to the changing market. Thus, every user could reduce costs of holding outdated stocks.

CHAPTER 4 CONCLUSION

Much research has been done in the area to optimize the total cost of the entire supply system by controlling the inventory levels of all companies linked by a supply chain. However, most of them either did not allow shortages or deterioration, and thus, ignored raw material costs, especially in multi-buyer systems; but in the real world, the latter situations exists.

This research presented two inventory models for single-producer multi-buyer system, which considered shortages, deterioration of items, raw material costs, and remanufacturing proceeds. The solution algorithm, numerical analysis and the sensitivity analysis are presented to show details. By applying the models, companies joined by a supply chain may build a stable cooperation relationship and gain market share. The models can respond rapidly to the changing market and the users could reduce costs of holding outdated stocks in the warehouse.

These two models will overcome the limitations found in the existing literature. Therefore, the outcome of this research will be beneficial to users that deal with the inventory problems similar to the problems in the models.

However, there are still some limitations in this research. For example, the research assumes that all buyers' replenishment cycles are equal. This is infrequent in the real world. Further, research can be done by applying different replenishment cycles of each buyer or extend to a multi-producer system.

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APPENDIX I PROOF OF THE CONCAVITY

The second-order derivative equations of the total cost function are from the “Maple” software, which are list before:

To prove that their values are positive, all values of system parameters are inputted first and then compute the functions by the “Matlab” software. The result shows that all values are positive in the reasonable area. So, the total cost function is convex in this area.

The main specific program of Matlab is list below:

```
clear
clc
global the_b B0 B_11 B_21 B_31 B_41 B_51 B_61 B_12 B_22 B_32 B_42 B_52 B_62
ii = 0;
m = 2;
A_b = 50; A_v = 2000;
H_b = 3; H_v = 2;
c_b = 12; c_v = 10;
pi_b = 12;
the_b = 0.03;
the_v = 0.01;
the_vb = 0.015;
P = 500;
D1 = 280;
D2 = 180;

B0 = m*A_b+A_v;
B_11 = pi_b*D1/2;
B_21 = H_b*D1/the_b^2+c_b*D1/the_b;
B_31 = H_b*D1/the_b+c_b*D1;
B_41 = c_v*P/the_v+H_v*P/the_v^2;
B_51 = D1*the_v/the_b/(1-the_vb)/P;
B_61 = (c_v+H_v/the_v)*D1/the_b/(1-the_vb);

B_12 = pi_b*D2/2;
B_22 = H_b*D2/the_b^2+c_b*D2/the_b;
B_32 = H_b*D2/the_b+c_b*D2;
B_42 = c_v*P/the_v+H_v*P/the_v^2;
B_52 = D2*the_v/the_b/(1-the_vb)/P;
B_62 = (c_v+H_v/the_v)*D2/the_b/(1-the_vb);

for T = 1:0.1:5
    for f1 = 0:0.01:1
        for f2 = 0:0.01:1
```

```

v_dF_dT_2 = dF_dT_2(T,f1,f2);
v_dF_df1_2 = dF_df1_2(T,f1,f2);
v_dF_df2_2 = dF_df2_2(T,f1,f2);
if v_dF_dT_2<0 || v_dF_df1_2<0 || v_dF_df2_2<0
    display('less than zero happen')
end
end
end
ii = ii+1;
ii
end
display('all the second derivative is larger than zero')
v_dF_dT, v_dF_df1, v_dF_df2

```


APPENDIX II THE SPECIFIC SOLVING PROGRAM OF THE MATLAB

The main Matlab program is list below:

```
Z_min = 1e6;
for T = 1.5522336469156:0.0001:1.5522336469158
    for f1 = 0.224816081:0.000001:0.224816083
        for f2 = 0.22376110:0.000000001:0.22396112
            Z = simplified_func(T,f1,f2);
            if Z<Z_min
                Z_min = Z;
                T_min = T;
                f1_min = f1;
                f2_min = f2;
            end
        end
    end
    ii = ii+1;
end
format long
Z_min,T_min,f1_min,f2_min

v_dF_dT = dF_dT(T_min,f1_min,f2_min)
v_dF_df1 = dF_df1(T_min,f1_min,f2_min)
v_dF_df2 = dF_df2(T_min,f1_min,f2_min)
```

VITA

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