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Anomalies in the CMB from a cosmic bounce

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We explore a model of the early universe in which the inflationary epoch is preceded by a cosmic bounce, and argue that this scenario provides a common origin to several of the anomalous features that have been observed at large angular scales in the cosmic microwave background (CMB). More concretely, we show that a power suppression, a dipolar asymmetry, and a preference for odd-parity correlations, with amplitude and scale dependence in consonance with observations, are expected from this scenario. The model also alleviates the tension in the lensing amplitude. These signals originate from the indirect effect that non-Gaussian correlations between CMB modes and super-horizon wavelengths induce in the power spectrum. We do not restrict to any specific theory, but rather derive features common to a family bouncing models.

Introduction. Observations have revealed features in the CMB that are in tension with the standard model of cosmology (aka ΛCDM). The signals that have attracted more attention are: (i) Absence of two-point correlations, known as power suppression; (ii) A dipolar or hemispherical asymmetry; (iii) A preference for odd-parity correlations. These anomalies appear only at large angular scales, and are present in data from both the WMAP [1] and Planck [2] satellites. The accumulated evidence makes it difficult to attribute them to residual systematics or foregrounds and, consequently, their interpretation as real features in the CMB is not in dispute. However, each of the observed features deviates from the predictions of the ΛCDM model at modest significances, quantified by means of their p-value [3]. This is the probability of obtaining, from the ΛCDM model, a temperature map with features at least as extreme as the observed ones. The Planck team associates p-values $\lesssim 1\%$ to each anomaly *separately* [2, 4].

These low significances open the door to two interpretations. It is possible that the ΛCDM model is complete, but we observe an atypical portion of the background radiation. Or that we actually observe a typical CMB, but the primordial probability distribution contains new physics at large scales. This is a tantalizing possibility and, as emphasized in [5], it is worth exploring new ideas since, given a theoretical model, new analyses could increase the significance of existing signals.

The goal of this paper is to propose a common origin for the observed anomalies. Our ideas rest on an extension of the so-called non-Gaussian modulation, introduced in [6–8], and further explored in [9, 10], to account for the dipolar asymmetry. The essence of this mechanism is that, if the primordial distribution is not Gaussian, certain features appear with higher probability in individual realizations; i.e., their p-values are larger. Our model respects homogeneity and isotropy at the fundamental level, but predicts that typical realizations look significantly more anisotropic than they would in the absence of non-Gaussianity.

The challenge to materialize this idea has been to find a model with strong enough non-Gaussianity, but yet compatible with Planck's constraints [11]. This extension of the ΛCDM model modifies *only* the standard *ansatz* of an almost-scale invariant and Gaussian primordial spectrum of perturbations. Although there exist several scenarios that predict a bounce [12–24], we will not adhere to any specific theory, but rather focus on generic predictions. We argue that a bounce preceding inflation can induce strong non-Gaussian correlations at scales comparable to, or larger than the horizon. Though we cannot measure these correlations directly, they produce an indirect effect in the CMB that can account for the observed anomalies.

The model. We work in a spatially flat FLRW universe, and model the bounce by a scale factor that behaves as $a(t) = a_B (1 + b\,t^2)^n$ in cosmic time t, where a_B, n and b are constants. The value of the (spacetime) Ricci scalar at the bounce is $R_B = 12\,n\,b$, so this family of bounces are parametrized by n and R_B—the value of a_B is physically irrelevant. Different theories assign different values to R_B and n; e.g. loop quantum cosmology [22, 23] produces $n = 1/6$ and R_B of order one in Planck units. $n = 1/6$ also arises in some higher-derivative scalar-tensor theories [17, 18]. We will consider the ranges $n \in [1/4, 1/7]$, and $R_B \in [10^{-3}, 1]\,\ell_{P\ell}^{-2}$, since they include all interesting cases. It has been proven that, if the matter sector is dominated by a scalar field with an appropriate potential $V(\phi)$, an inflationary phase is an attractor of phase space trajectories after the bounce [25–28]. Hence, the goal of the bounce in our model is not to replace inflation, but to complement it by replacing the big bang singularity and bringing the universe to an inflationary phase.

The power spectrum. Scalar perturbations start their evolution in an adiabatic vacuum in the far past, when all Fourier modes of interest are in an adiabatic regime. Their evolution across the bounce excites some of these modes, in such a way that at the onset of inflation their quantum state differs from the Bunch-Davies vacuum by the presence of both excitations and non-Gaussianity. We have evaluated the power spectrum for different values of n and R_B, and the result can be well approximated by three power-laws:

$$\mathcal{P}_{\mathcal{R}}(\vec{k}) \approx \begin{cases} A_s \left(\frac{k}{k_B}\right)^{n_s-1} & k > k_B \\ A_s \left(\frac{k}{k_B}\right)^{q} & k_I < k \leq k_B \\ A_s \left(\frac{k_I}{k_B}\right)^{q} \left(\frac{k}{k_I}\right)^{2} & k \leq k_I, \end{cases} \quad (1)$$

where $k_B = a_B \sqrt{R_B/6}$ and $k_I = 2\pi a_I \sqrt{R_I/6}$ are the characteristic scales of the problem, set by the spacetime curvature at the bounce and at the onset of inflation, respectively (we use $R_I = 5 \times 10^{-10} \ell_{Pl}^{-2}$ [29]). Equation (1) can be understood as follows. Fourier modes with $k > k_B$ are more ultraviolet than k_B at the time of the bounce, and consequently they are not amplified when they propagate across the bounce. Their spectrum is, therefore, entirely determined by inflation. The choice of potential $V(\phi)$ is encoded in the value of A_s and n_s. On the other hand, modes $k_I < k \leq k_B$ are significantly affected by the bounce, and for them $\mathcal{P}_{\mathcal{R}}(k)$ scales as k^q. Our simulations show that q depends on n, and it takes *negative* values, equal to -2, -1.24, -1.1, -0.7 and -0.5 for n equal to $1/4$, 0.21, $1/5$, $1/6$ and $1/7$, respectively. These values are largely independent of R_B. Therefore, the leading order effect of the bounce is an enhancement of $\mathcal{P}_{\mathcal{R}}(\vec{k})$ at infrared scales. Finally, modes $k < k_I$ are so infrared that they are not affected either by the bounce nor by inflation, and for them $\mathcal{P}_{\mathcal{R}}(k)$ is largely suppressed, with a scale dependence given by k^2. If the bounce is responsible for the anomalies in the CMB, then k_B must be of the order of the pivot scale k_*, which today corresponds to $0.002\,\mathrm{Mpc}^{-1}$. We adopt $k_B = k_*$, which makes the effects from the bounce appear for angular multipoles $\ell \lesssim 30$ in the CMB. This is equivalent to fixing the amount of expansion from the bounce to the end of inflation. Concrete models may come with a justification for such a choice, as it is the case e.g. in loop quantum cosmology [30].

Primordial non-Gaussianity is described by the bispectrum $B_{\Phi}(k_1, k_2, k_3)$ (see e.g. [31]), whose details is conveniently encoded in the function $f_{NL}(k_1, k_2, k_3) \equiv B_{\Phi}(k_1, k_2, k_3)/[P_{\Phi}(k_1)P_{\Phi}(k_2) + 1 \leftrightarrow 3 + 2 \leftrightarrow 3]$, where Φ is the Bardeen potential and $P_{\Phi}(k) = \left(\frac{3}{5}\right)^2 \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}$. An exact calculation of f_{NL} requires knowledge of the gravitational action and the matter content of the universe at the bounce. Our goal is rather to obtain an estimation of its overall form, common to all models. The shape of $f_{NL}(k_1, k_2, k_3)$ can be obtained by using Cauchy integral theorem, and by noticing that its amplitude is dominated by the pole with smaller positive imaginary part in the third-order gravitational action (see section V in [32]). In presence of a bounce, this is the pole introduced by the global minimum of the scale factor $a(\eta)$, that in conformal time is at $\eta_p = i\alpha\sqrt{6/R_B} = i\alpha/k_B$, with $\alpha = \sqrt{\frac{n\pi}{2}} \frac{\Gamma[1-n]}{\Gamma[3/2-n]}$, where $\Gamma[x]$ is the Gamma-function.

This general argument tells us that

$$f_{NL}(k_1, k_2, k_3) \approx \mathfrak{f}_{NL} \times e^{-\alpha(k_1+k_2+k_3)/k_B}, \quad (2)$$

where $\mathfrak{f}_{NL}$ parameterizes our ignorance about its amplitude. We see that the scale dependence of non-Gaussianity is controlled by R_B and n. Concrete bouncing models may add additional features to $f_{NL}(k_1, k_2, k_3)$, such as oscillations or other finer details, but (2) approximates well its overall shape. We have checked this in the concrete scenario explored in [32]. For very infrared wavenumbers k_i, we expect $f_{NL}(k_1, k_2, k_3)$ to become small, for the same reason as the $\mathcal{P}_{\mathcal{R}}(k)$ does. This is not captured by (2), but will be incorporated in our calculation by the effective infrared cut-off that the shape of $\mathcal{P}_{\mathcal{R}}(k)$ introduces. As mentioned above, if k_B is close to k_* then the non-Gaussianity (2) is restricted to the most infrared scales in the CMB, and is large only when super-horizon modes are involved.

Non-Gaussian modulation. Super-horizon perturbations *can* impact the CMB *if* they are correlated with sub-horizon modes. We follow ideas introduced in [6–10] to compute the bias in the statistics of the gravitational potential $\Phi(\vec{k}_1)$ induced by long wavelength modes $\Phi(\vec{q})$. At the lowest non-vanishing perturbative order, we have

$$\langle \Phi_{\vec{k}_1} \Phi_{\vec{k}_2}^{\star} \rangle|_{\Phi_{\vec{q}}} = (2\pi)^3 \delta(\vec{k}_1 - \vec{k}_2) P_{\Phi}(\vec{k}_1)$$
$$+ f_{NL}(\vec{k}_1, -\vec{k}_2) \frac{1}{2} \left(P_{\Phi}(\vec{k}_1) + P_{\Phi}(\vec{k}_2) \right) \Phi_{\vec{q}}, \quad (3)$$

where $\vec{q}$ must take the value $\vec{q} = \vec{k}_1 - \vec{k}_2$. As we can see, $\Phi(\vec{q})$ introduces "non-diagonal" terms, proportional to both the magnitude of $\Phi(\vec{q})$ and the intensity of the correlations, f_{NL}. These terms translate to anisotropic features in the CMB. In a typical patch of the universe, one expects $|\Phi(\vec{q})|$ to be of the same order as $\sqrt{P_{\Phi}(\vec{q})}$. If, on the other hand, one averages over many patches, these contributions vanish; as it must be, since our model respects isotropy at the fundamental level. The non-diagonal terms in (3) induce similar contributions to the CMB temperature covariance matrix

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_{\ell} \delta_{\ell\ell'} \delta_{mm'} + (-1)^{m'} \sum_{LM} A_{\ell\ell'}^{LM} C_{\ell m \ell' -m'}^{LM},$$

where $C_{\ell m \ell' m'}^{LM}$ are Clebsch-Gordan coefficients. We have encoded the non-Gaussian modulation in $A_{\ell\ell'}^{LM}$, known as the Bipolar Spherical Harmonic (BipoSH) coefficients (see [33, 34]). They organize the modulation in an efficient manner: L and M indicate the "shape" of the modulation, while ℓ, ℓ' account for a possible variation of the modulation amplitude at different scales in the CMB. The monopole, $A_{\ell,\ell'}^{00} \propto \delta_{\ell\ell'}$, shifts the value of the spherically symmetric angular spectrum C_{ℓ}, while the dipole $A_{\ell\ell'}^{1M} \propto \delta_{\ell+1,\ell'}$ introduces correlations between multipoles ℓ and $\ell+1$. Our model cannot predict the exact value of the BipoSH coefficients in the sky, as they depend on a

concrete realization of the mode $\Phi(\vec{q})$. But we can compute their mean square values

$$\sqrt{\langle|A_{\ell\ell'}^{LM}|^2\rangle} \approx \left[\frac{1}{2\pi}\int dq\, q^2\, P_\Phi(q)\, |\mathcal{C}_{\ell\ell'}^L(q)|^2\right]^{1/2}$$
$$\times C_{\ell 0 \ell' 0}^{L0}\sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi(2L+1)}}, \qquad (4)$$

where we have defined

$$\mathcal{C}_{\ell\ell'}^L(q) \equiv \frac{2}{\pi}\int dk_1 k_1^2 (i)^{\ell-\ell'}\Delta_\ell(k_1)\Delta_{\ell'}(k_1)P_\phi(k_1)G_L(k_1,q).$$

In this expression, f_{NL} has been expanded using Legendre polynomials $P_L(\mu)$ as $f_{NL}(\vec{k}_1,\vec{q}) = \sum_L G_L(k_1,q)\frac{2L+1}{2}P_L(\mu)$, with $\mu = \vec{k}_1 \cdot \vec{q}$. Thus, the μ-dependence of $f_{NL}(\vec{k}_1,\vec{q})$ translates to the L-dependence of the BipoSH coefficients. To derive (4) we have used that $f_{NL}(\vec{k}_1,\vec{q})$ is larger for $q \ll k_1$.

Power suppression, lensing, and parity. A lack of two-point correlations $C(\theta) \equiv \langle\delta T(\hat{n})\delta T(\hat{n}')\rangle$, $\cos\theta \equiv \hat{n}\cdot\hat{n}'$, for $\theta > 60°$ was noticed by COBE and WMAP [35, 36], and confirmed by Planck. The observed value of the estimator $S_{1/2} \equiv \int_{-1}^{1/2}[C(\theta)]^2\,d(\cos\theta)$ [37], which measures the total amount of correlations in $\theta > 60°$, is $S_{1/2}^{\mathrm{obs}} \approx 1500\,\mu K^4$ [2, 4] (see [38] for further details), instead of $S_{1/2} \approx 45000\mu K^4$ predicted by the ΛCDM model. The p-value of $S_{1/2}^{\mathrm{obs}}$ is a fraction of a percent [2, 4, 38]. Our model can account for such a suppression, but with an important subtlety. The monopolar modulation introduced by $A_{\ell\ell}^{00}$ does not change the mean value of $S_{1/2}$, but rather it modifies its variance, increasing the range of typical statistical "excursion", both to *larger and smaller* values away from the mean. In this sense, our model does not predict a power suppression, but rather it increases the probability of observing it. We have computed the value of $\mathfrak{f}_{\mathrm{NL}}$ that makes the probability of observing $S_{1/2} \leq S_{1/2}^{\mathrm{obs}}$ approximately equal to 20% (we have approximated the statistics of $S_{1/2}$ by a Gaussian; corrections are higher order in non-Gaussianity), and show them in Table I. Remarkably, these values of $\mathfrak{f}_{\mathrm{NL}}$ are of the same order found in loop quantum cosmology [32]. Therefore, although the "bare" power spectrum (1) is enhanced with respect to the ΛCDM value at large angles, if the correlations with super-horizon scales are strong, the probability of observing $S_{1/2}^{\mathrm{obs}} \approx 1500\,\mu K^4$ is high, and the observed suppression cannot be considered anomalous.

Next, we can compute other effects that our model *predicts* must come together with a suppression. This is the goal of the rest of the paper. First, we plot in Figures 1 and 2 the details of C_ℓ and $C(\theta)$. To quantify how well our results agree with data, we have carried out a Markov chain Monte Carlo (MCMC) analysis, using TT and low-ℓ EE data [39], by using the CosmoMC software [40]. We

R_B \ n	1/4	0.21	1/5	1/6	1/7
$1\,\ell_{P\ell}^{-2}$	-	959	1334	3326	5031
$10^{-1}\,\ell_{P\ell}^{-2}$	-	1560	2065	4454	6298
$10^{-2}\,\ell_{P\ell}^{-2}$	-	2573	3238	6066	8024
$10^{-3}\,\ell_{P\ell}^{-2}$	-	4372	5234	8518	10530

Table I: Values of the amplitude $\mathfrak{f}_{NL}$ that make the probability of observing $S_{1/2}^{\mathrm{obs}} \leq 1500\,\mu K^4$ approximately equal to 20%. For $n \leq 1/4$, the tilt q of the power spectrum is too negative, and the non-Gaussian modulation cannot produce the observed suppression.
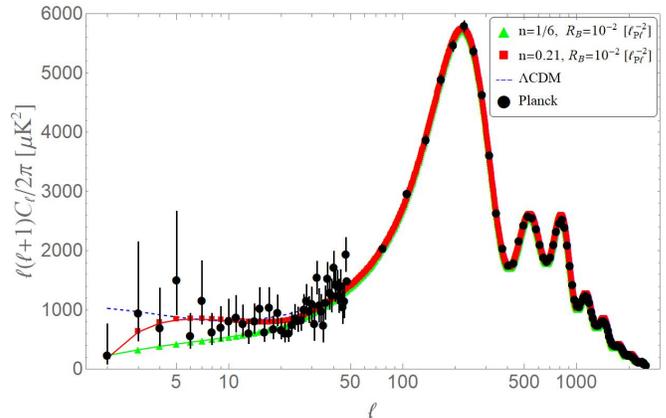

Figure 1: Temperature angular power spectrum for two representative bouncing models, compared with the predictions from the ΛCDM model with the standard ansatz, and data from Planck [2]. Both bouncing models produce $S_{1/2} = 1500\mu K^4$. We have used the best-fit values for A_s and n_s.

have found that, although all bounces considered here account for $S_{1/2}^{\mathrm{obs}}$ (except $n = 1/4$), not all fit the details of the data equally well. Bounces for which the tilt q of the power spectrum is more negative, do better. For instance, for $n = 0.21$ we have $q = -1.24$, and this value results in a significant improvement in χ^2 of $\Delta\chi^2 = -6.4$, relative to ΛCDM. Hence, this model not only reproduces the overall suppression, but it also fits the details of C_ℓ better. Values of q closer to zero, such as $q = -0.7$ or lower, are not favored from the point of view of χ^2. In this likelihood analysis we do not consider n, R_B and $\mathfrak{f}_{\mathrm{NL}}$ as free parameters; they rather must come out as predictions from individual theories.

We find that the suppression is accompanied by two additional effects in the C_ℓ's. On the one hand, power suppression induces a change in the lensing parameter A_L, making it closer to one than in the ΛCDM model. The relation between power suppression and the value of A_L has been recently pointed out in [41], and we confirm it in our model. More precisely, when we include A_L as a free parameter in our MCMC analysis, we obtain that the mean and standard deviation of the marginalized distribution of A_L is $A_L = 1.179 \pm 0.092$, for $n = 0.21$,
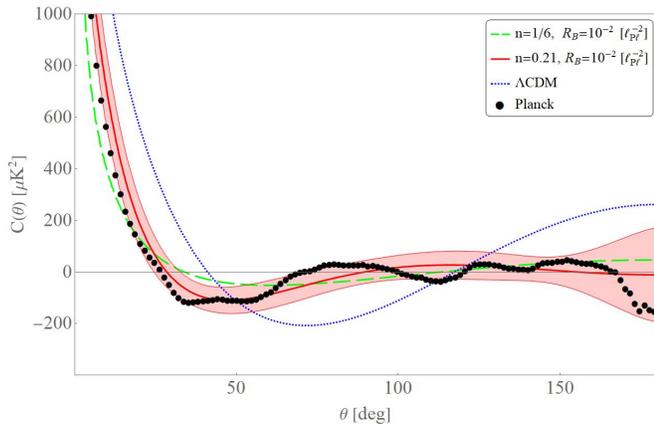
Figure 2: Angular two-point correlations $C(\theta)$. The shadowed region is the cosmic variance of the curve $n = 0.21$, $R_B = 10^{-2}\ell_{P\ell}^{-2}$, and it shows great agreement with data. The same happens for $n = 1/6$. In contrast, data is clearly out of the cosmic variance region (not shown; see Fig. 2 in [2]) of the ΛCDM curve for $\theta \sim 75°$, and $\theta > 170°$.
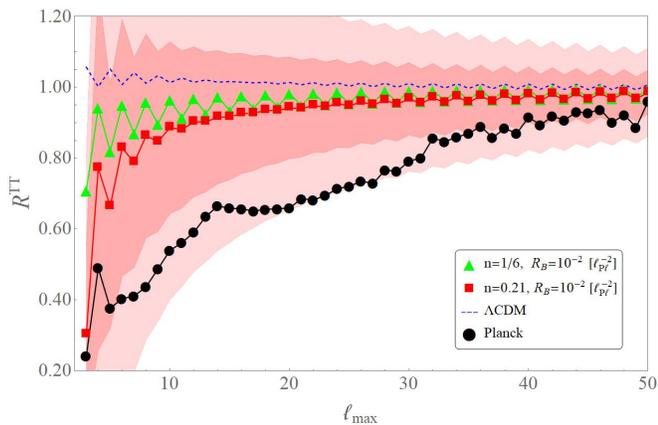

Figure 3: R^{TT} versus ℓ_{max}.

$R_B = 10^{-2}\ell_{P\ell}$. Other values of n and R_B produce similar results. This is to be compared with the ΛCDM value, $A_L = 1.243 \pm 0.096$. Therefore, our model alleviates the tension pointed out in [42] regarding the value of A_L, in the sense that $A_L = 1$ becomes well inside the 2-σ region—without introducing spatial curvature, and hence avoiding the possible "crisis in cosmology" advocated in [42].

On the other hand, we observe that the power suppression also produces a preference for odd-parity multipoles, as measured by $R^{TT}(\ell_{max}) = D_+(\ell_{max})/D_-(\ell_{max})$, where $D_{+,-}(\ell_{max})$ is the average value of $\ell(\ell+1)C_\ell/2\pi$ in even $(+)$ or odd $(-)$ multipoles, up to ℓ_{max} [4]. In Fig. 3 we show R^{TT} versus ℓ_{max}, and the 1σ and 2σ cosmic variance region for $n = 0.21$, $R_B = 10^{-2}\ell_{P\ell}$, (see also Fig. 25 in [2]). In contrast to ΛCDM, our model produces a clear preference for odd multipoles (i.e. $R^{TT}(\ell_{max}) < 1$).

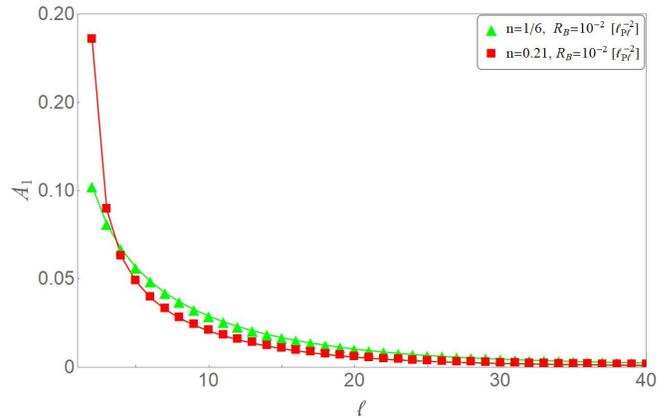We have also checked that, although the values of $\mathfrak{f}_{NL}$

Figure 4: Amplitude of the dipolar modulation, as quantified by $A_1(\ell)$. The power suppression discussed above contributes to increase the amplitude of $A_1(\ell)$ at low ℓ's. In ΛCDM, $A_1(\ell) = 0$.

in Table I are significantly larger than one, the perturbative expansion remains under control, due to the smallness of $\mathcal{P}_\mathcal{R} \ll 1$.

A dipolar modulation in the CMB has been consistently observed in data from WMAP [43] and Planck [2, 4, 44]. In terms of the BipoSH coefficients, this signal can be explained from a non-zero value of $A_{\ell\,\ell+1}^{1M}$. Planck's observations have been reported in terms of

$$A_1(\ell) \equiv \frac{3}{2}\sqrt{\frac{1}{3\pi}\sum_M |A_{\ell,\ell+1}^{1M} \times \mathcal{G}_\ell^{-1}|^2}\,, \quad (5)$$

where $\mathcal{G}_\ell \equiv (C_\ell + C_{\ell+1})\sqrt{\frac{(2\ell+1)(2\ell+2)}{12\pi}}\,C_{\ell,0,\ell+1,0}^{10}$ is the so-called form factor. The signal has been reconstructed in bins of width $\Delta\ell = 64$, up to $\ell_{max} = 512$, and A_1 deviates significantly ($\sim 3\sigma$) from what is expected from ΛCDM only in the first bin, were $A_1^{obs} = 0.068 \pm 0.023$ [4].

Figure 4 shows our result for $A_1(\ell)$, again for the same representative values of n and R_B. We obtain that A_1 is large only for low multipoles $\ell \lesssim 30$. Although details vary slightly among different models, the averaged value of the amplitude for $\ell \lesssim 30$ is also in consonance with observations.

The Planck satellite has also looked for a quadrupolar modulation, and found that the results are compatible with what is expected from the ΛCDM model [29]. We have computed the amplitudes of $A_{\ell\ell}^{LM}$ for $L > 1$ in our model, and checked that they all satisfy Planck's constraints (for details, see [45]).

Discussion. The anomalies in the CMB include strong deviations from scale invariance, isotropy and parity. It is precisely this heterogeneous character that has made the search for a common origin a challenging task. We have argued that an extension of the ΛCDM model, where a cosmic bounce precedes the inflationary era, can collec-

tively account for these signals, as the result of the modulations that very long wavelength perturbations imprint on CMB scales.

We have not adhered to any concrete bouncing theory, but rather introduced a series of approximations to estimate the effects of a generic bounce. The values of the parameters needed to account for the anomalies are in consonance with those coming out from concrete theories. In fact, we have complemented our analysis with calculations using the bounce predicted by loop quantum cosmology, and have checked that our approximations are well justified. Further detailed calculations will appear in a companion publication.

We conclude that it may be premature to disregard the large scale anomalies as mere flukes of the ΛCDM model, and advocate the fascinating possibility that they are imprints of pre-inflationary physics, which carry information about that extreme epoch. Future work will focus on extending our predictions to tensor modes, in order to construct additional ways to test our ideas.

Acknowledgements

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