Space-time block coding for multiple transmit antennas over time-selective fading channels

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SPACE-TIME BLOCK CODING FOR MULTIPLE TRANSMIT ANTENNAS OVER TIME-SELECTIVE FADING CHANNELS

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

in

The Department of Electrical and Computer Engineering

by
Ioannis D. Erotokritou
B.S.E.E., Louisiana State University, 2004
May 2006
I would like to dedicate this work to my parents Demetrios and Koulla Erotokritou, who have supported me throughout my academic endeavors, without them this may have never been possible.
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Abstract

The last decade we have witnessed an extraordinary growing interest in the field of wireless communications. Wireless industry has recently turned into a technology known as Multiple-Input Multiple-Output (MIMO) to minimize errors and optimize data speed. This is done by using multiple transmit and receive antennas, as well as appropriate space-time block code techniques.

Due the fact that time-selective fading channels exist, the assumption that the channel is being static over the length of the codeword does not apply in this thesis. The channel state varies over the number of signal intervals, and in such case the orthogonality of the space-time block code will be destroyed leading to irreducible error floor. This thesis deals with orthogonal space-time block coding schemes where time-selective fading channels arise due to Doppler Effect. A realistic channel model was used, where the channel coherency was considered. Modeling the time-selective channels as random processes, a fast and simple decoding algorithm was derived at the receiver. Using MATLAB™ as a simulation tool, we provide simulation results demonstrating the performance for two, three, four, and five transmit antennas over time-selective fading channels. We illustrate that using multiple transmit antennas and space-time coding outstanding performance can be obtained, under the impact of channel variation.
Chapter 1

Introduction and Literature Review

In the last decade, the study of wireless communications with multiple transmit and receive antennas has been conducted expansively in the literature on information theory and communications. It has been known from the information-theoretic results [1], [2], [3], [4], [5] that the application of multiple antennas in wireless systems can significantly improve the channel capacity over the single-antenna systems with the same requirements of power and bandwidth. Based on those results, many communication schemes suitable for data transmission through multiple-antenna wireless channels have been proposed, including Bell Labs Layered Space–Time (BLAST) [1], space–time trellis codes [6], space–time block codes from orthogonal designs [7], and unitary space–time codes [8], [9], among many others.

1.1 Background

It is well-known that the decoding algorithm must have as low as possible complexity in order to find applications in the real-world of wireless communication systems. Base on this criterion, Alamouti [10] developed a simple and effective transmit technique for two transmit antennas which has remarkably low decoding complexity.

Tarokh, Jafarkhani, and Calderbank [7] presented orthogonal designs that can be used as space-time block codes for wireless communications and generalized the Alamouti scheme for more than two transmit antennas. Space–time block coding provides an exceptional link between orthogonal designs and wireless communications. Since this coding scheme achieves full transmit diversity and has a very simple maximum-
likelihood decoding algorithm at the receiver, Alamouti’s space-time block code has been established as a part of the W-CDMA and CDMA-2000 standards.

Orthogonal space-time block codes (O-STBC) achieve high transmit diversity and have a low complexity decoding algorithm at the receiver using any number of transmit and receive antennas (Figure 1.1).

### 1.2 Orthogonal Space-Time Block Codes (O-STBC)

Space–time block coding is a technique used to improve the performance of a wireless transmission system, where the receiver is provided with multiple signals carrying the same information. The concept behind space-time block coding is to transmit multiple copies of the same data through multiple antennas in order to improve the reliability of the data-transfer through the noisy channel. This is shown in Figure 1.2.
In the receiver terminal, space–time coding combines together all the copies of the received signal in an optimal way to extract as much information from each of them as possible.

A complex orthogonal space-time block code for any number of transmits antennas \( n \), is described by a \( p \times n \) transmission matrix \( \Phi \), as shown in Figure 1.3, where \( k, n, \) and \( p \) are positive integers.

![Transmission matrix](image)

**Figure 1.3: Transmission matrix**

A transmission matrix is represented with the following notation \([p, n, k]\), where \( p \) is the length of the block, \( n \) is the number of transmit antennas, and \( k \) the number of information symbols. Every entry is a complex linear combination of the \( k \) complex variables \( z_1, z_2, \ldots, z_k \) and their conjugates \( z_1^*, z_2^*, \ldots, z_k^* \), satisfying the following complex orthonormality condition:

\[
\Phi^H \Phi = \left( |z_1|^2 + |z_2|^2 + \ldots + |z_k|^2 \right) \times I_{n \times n}
\]

(1-1)

where \( \Phi^H \) represents the Hermitian transpose of \( \Phi \), and \( I_{n \times n} \) the \( n \times n \) identity matrix. An orthogonal space-time block code with \( n \) transmit antennas can send \( k \) information symbols, namely \( z_1, z_2, \ldots, z_k \), from signal constellations such as phase-shift keying (PSK) and quadrature amplitude modulation (QAM) in a block of \( p \) channel uses. The rate of the orthogonal code is defined as \( R = \frac{k}{p} \), which reflects the bandwidth efficiency of the code.
The construction of high-rate orthogonal space-time block codes is an essential problem in space-time block coding. The first complex orthogonal space-time block code was proposed by Alamouti [10] as follows:

\[
\mathcal{C} = \begin{pmatrix}
 z_1 & z_2 \\
 -z_2^* & z_1^*
\end{pmatrix}
\]

for two transmit antennas \([p, n, k] = [2, 2, 2]\), which has full rate \( R = \frac{2}{2} = 1 \). Three, four, five, and eight transmit antennas, with different rates were constructed by Liang [12]. Obviously, a complex orthogonal space-time block code with high rate can improve the bandwidth efficiency. Hence, in order to improve the bandwidth efficiency of a complex orthogonal space-time block code for any given number of transmit antennas, we need to apply the space-time block code with the highest rate \( R \) as possible. Orthogonal designs with maximal rates where demonstrated successfully by Liang [12].

### 1.3 Antenna Theory

An antenna is a device used for transmitting and/or receiving electromagnetic waves which are operated in radio frequencies (RF), a range of 10 kHz to 300 GHz. The size and shape of antennas are determined from the frequency of the signal they are designed to receive. An antenna must be tuned to the same frequency band that the radio system to which it is connected operates in, otherwise reception and/or transmission will fail. Therefore, antennas couple electromagnetic energy from the space to other mediums. In the recent years, due to the wireless cellular evolution many antenna technologies were proposed which provide more quality, capacity, and coverage. These types of antenna systems are the sectorized antenna systems, diversity antenna systems and many others. For more information regarding antenna
theory refer to Ref. [13]. However, antennas are operated in a noisy environment where many hostile effects should surpassed or minimize in order the communication to be successful.

### 1.3.1 Multipath Interference Effect

Multipath interference is a phenomenon where two or more waves are transmitted at the same time from a base station and travel through different paths towards the receiving end (Figure 1.4); whereas, before the reception they interfere with each other causing a phase shift.

![Figure 1.4: The effect of multipath on a mobile user](image)

When the waves of multipath signals are out of phase, reduction in signal strength can occur. This phenomenon is known as Rayleigh fading. As shown in Figure 1.5, fade describes the loss of signal strength at the receiver by causing periodic attenuation. In addition, due to the multiple reflections, the same signals could arrive at the receiver end at different times. This effect arises a phenomenon called intersymbol interference, where the receiver cannot sort the incoming information. As a result, the bit error rate increases and distorts the incoming signal.
Figure 1.5: Representation of the Rayleigh fade effect on a user signal (Adapted from Ref. [14])

Another very important phenomenon is the co-channel interference (Figure 1.6), where the same carrier frequency reaches the same mobile receiver from two separate base stations.

Figure 1.6: Illustration of co-channel interference

The signals that missed their intended destination become interference for the rest of the users on the same frequency in the same or adjoin cells.

1.3.2 Doppler Effect

The Doppler Effect is the change in frequency of a wave that is perceived by an observer moving towards or away from the source of the waves. It is well-known that Doppler effects generated by high speed mobility are the major reason for the
reduction of data rates in cellular systems. The Doppler Effect may occur from either
motion of the source (Figure 1.7) or motion of the receiver. This thesis will consider
the latter case, where the receiving end (mobile user) is in motion and the source (base
station) is stationary.

![Figure 1.7: a) Stationary source  b) Moving source](image)

It is important to comprehend that the frequency of the signal that the source emits
does not actually change, but the wavelength ($\lambda$) does; consequently, the perceived
frequency is also affected. When the receiving end moves towards the base station the
receiving frequency becomes higher and when is receding the receiving frequency
becomes lower (see Figure 1.7).

The Doppler shift in frequency depends on the velocity between the source and the
receiver and on the speed of propagation of the signal. Doppler frequency is given by
the formula [15] :

$$
\Delta f \approx \pm f_c \times \frac{v}{c}
$$

(1-3)

where $\Delta f$ is the change in frequency, $f_c$ is carrier frequency, $v$ is the speed
difference between the source and the receiver, and $c$ is the speed of light in vacuum.

For the purpose of this paper the carrier frequency will be taken as 2 GHz as specified
in the Enhanced Data rates for GSM Evolution (EDGE) and the speed of light in
vacuum will be equal with $c = 3 \times 10^8 \text{ m/s}$.
1.4 MIMO - Multiple Input Multiple Output

Wireless communication industry has recently turned to a strategy called Multiple-Input Multiple-Output (MIMO). MIMO is the single most important wireless technology as of today. MIMO is a technology evolution where both ends of the wireless link are equipped with antenna array (Figure 1.8).

![Block diagram of MIMO system](image)

Figure 1.8: Block diagram of MIMO system

This can improve the quality (bit-error rate) and the data rate (bits per sec). Therefore, a superior quality of service (QoS) can be achieved, which revenues the wireless provider. Many space-time block codes for different number of transmit/receive antennas have been developed in order to achieve maximum diversity. MIMO takes advantage of multipath interference effect to increase the user and data capacity; it converts it into a positive feature by using the multiple transmitters and/or receivers to increase throughput and reliability. Usually, multiplexing would cause interference, but MIMO uses the additional pathways to transmit more information and then combines the signal at the receiving end; thus provides robustness against multipath fading. MIMO systems can be designed with the receiver knowing the channel state (coherent case) or not (not-coherent case). For the purposes of this thesis the former case will be consider.
1.4.1 Principles of MIMO Systems

An efficient way to improve data rate and transmission reliability over wireless links is through the use of MIMO systems.

Figure 1.9: Basic spatial multiplexing scheme with 3-Tx antennas. Ai, Bi, and Ci represent symbol constellations (Adapted from Ref. [16])

Figure 1.9 shows an intuitive representation of how MIMO systems operate. A simple bit sequence is decomposed into three independent sequences, which then are transmitted simultaneously through multiple antennas. Bit sequences pass through modulation and mapping process using various symbol constellations. The signals use the same frequency spectrum and they naturally mix together in the wireless channel. At the receiver, after having identified the mixing channel matrix (coherent case), the individual bit streams are separated and estimated. This works in the same way as a linear system of three equations. Therefore, each pair of transmit-receive antennas has a single scalar channel coefficient, hence flat fading channel. A more detail
explanation about the functionality of MIMO systems is explained and illustrated in Ref. [16] (also see later in this thesis for more details).

1.4.2 Transmit Diversity Model

In a given symbol period \((T_s)\), two signals are transmitted at the same time, \(t\), from the two antennas (Figure 1.10). Antenna No.1 transmits the signal \(z_1\) and antenna No.2 transmits the signal \(z_2\). At the next symbol period, \(t + T_s\), signal \(-z_2^*\) is transmitted from antenna No.1, and signal \(z_1^*\) is transmitted from antenna No.2. Where \((*)\) is the conjugate complex symbols. This sequence is shown in Table 1.1.

![Discrete time equivalent model](image)

Table 1.1: Transmission sequence for two transmit antennas

<table>
<thead>
<tr>
<th></th>
<th>Antenna 1</th>
<th>Antenna 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (t)</td>
<td>(z_1)</td>
<td>(z_2)</td>
</tr>
<tr>
<td>time (t + T_s)</td>
<td>(-z_2^*)</td>
<td>(z_1^*)</td>
</tr>
</tbody>
</table>

The orthogonal space-time block code which was decrypted in equation (1-2), using two transmit antennas and one receive antenna, can achieve diversity order of two and
full coding rate. This scheme can be generalized to $n$ transmit antennas and one receive antenna in order to achieve greater diversity order.

Chapter 2 introduces the system model which was used to analyze the performance of various number of transmit antennas over time selective-fading channels.

1.5 Thesis Organization

The remainder of this thesis has been organized as follows: Chapter 2 provides in detail the system model that was used. Chapter 3 provides simulator features and specifications of the modeled system. Chapter 4, 5, 6, and 7 present experimental results for two, three, four, and five transmit antennas respectively base on the proposed scheme. Finally, in Chapter 8, we conclude this thesis with a summary of the results and we point to directions of future work.
Chapter 2

The System Model

A realistic channel model was used in this paper, where the channel coherency was considered. Due the fact that time selective or fast fading channels exist, the assumption that the channel is static all over $pT_s$ does not apply in this thesis (where $T_s$ is the symbol period and $p$ is the length of the block).

However, a time-selective fading channel model was used where the channel variation is considered. Consequently, this channel variation will destroy the orthogonality of the channel matrix and therefore will cause error floor at the high signal-to-noise ratio (SNR) region. The block diagram of the system that was used is shown in Figure 2.1.

Figure 2.1: The channel model for n-Tx antennas
In order, to reduce the above error floor a high rate orthogonal time-space block codes are introduced for the cases of two, three, four, and five transmit antennas [12]. The orthogonal designs that will be presented in this thesis have a special structure.

\[
\Phi = \begin{pmatrix}
z_1 & z_2 & z_3 \\
*z_2 & z_1 & 0 \\
*z_3 & 0 & z_1^* \\
0 & *z_3 & z_2^*
\end{pmatrix}
\]

Figure 2.2: Orthogonal time-space block code for 3-Tx antennas

Each row has either only complex symbols or only conjugate (*) complex symbols, as shown in Figure 2.2. This structure gives the flexibility to manipulate the complex number properties in order to demodulate the received signals.

2.1 Encoding

We assume \( n \) transmit antennas and one receive antenna, where the information symbols \( z_1, z_2, \ldots, z_k \) are transmitted using complex orthogonal time-space block codes [12]. All the complex information symbols are first grouped together by a modulator and then passed through an O-STBC encoder. Then they are transmitted over \( T_x = 1, 2, \ldots, p \) symbol periods.

The receive signal \( r_i \) at time \( i \) can be estimated by:

\[
r_i = \sum_{j=1}^{n} h_{ji} \times c_{ij} + n_i \quad \text{where} \quad i = 1, 2, 3, \ldots, p
\]  

(2-1)

where \( n_i \) is a complex additive white Gaussian noise (AWGN) with zero mean and variance \( \frac{\sigma_i^2}{2} \), \( c_{ij} \) equals with one of the following complex information symbols

\([\pm z_1, \pm z_2, \ldots, \pm z_k, \pm z_1^*, \pm z_2^*, \ldots, \pm z_k^*] \) from O-STBC matrix (see Figure 1.3), and
$h_{ji}$ denotes the time selective channel from the $j^{th}$ transmit antenna to the receive antenna.

One of the best known models that have been used for a time variant flat-fading channel is the Jake’s model [17], which is the following:

$$h_{ji} = \alpha_{(i-1)} \times h_{j(i-1)} + w_{ji} \quad \text{where } j = 1, 2, \ldots, n$$

$$i = 1, 2, \ldots, p$$

(2-2)

where $w_{ji}$ is noise which has complex Gaussian zero-mean with complex variance $\frac{\sigma_i^2}{2}$ per dimension, and it is statically independent of $h_{j(i-1)}$. The coefficient $\alpha$ can be estimated by:

$$a_i = J_0(2\pi \times i \times f_d \times T_s)$$

(2-3)

where $f_d$ is the Doppler frequency, $T_s$ is the information symbol duration, and $J_0(.)$ is the $0^{th}$ order Bessel function of the first kind (Figure 2.3).

Figure 2.3: Bessel Function of the first kind (Adapted from Ref. [18])
We need to give special attention to \( w_{ji} \) where is another i.i.d complex Gaussian random variable having zero mean and variance \( \sigma_i^2 \) and is statistically independent of \( h_{j(i-1)} \). The variance of \( w_{ji} \) can be calculated from the following:

\[
\sigma_i^2 = 1 - \left| \alpha_{i(i-1)} \right|^2
\]

(2-4)

where it depends on the variable \( \alpha \). The value of \( \alpha \) depends on the terminal speed.

In order to find the Doppler frequency, \( f_d \), we have to use the following formula:

\[
f_d = f_c \times \left( \frac{V}{c} \right)
\]

(2-5)

where \( f_c \) corresponds to the carrier frequency, \( V \) is the speed of the mobile terminal, and \( c \) is the speed of light in vacuum.

The information symbol duration \( T_s \) can be found by:

\[
T_s = \frac{SF}{\text{ChipRate}}
\]

(2-6)

where \( SF \) is the spreading factor. According to Universal Mobile Telecommunications System (UMTS), \( \text{ChipRate} = 3.84 \times 10^6 \frac{\text{chips}}{\text{sec}} \) and \( SF = 128 \).

Therefore, as the length of the codeword increases the more vulnerable is the system to channel variation. As shown in Figure 2.4, a realistic model was used, where the channel is static only over a symbol period \( (T_s) \) instead of being static over \( pT_s \).

![Figure 2.4 The time-selective codeword](image-url)
2.2 Decoding

As mentioned above, the information was encoded based on the following model:

\[
Y_{px1} = H_{pxk} \times S_{kx1} + W_{px1}
\]  

(2-7)

where \( Y \) is the received signal, \( H \) the channel, \( S \) the information, and \( W \) the noise. All the variables are in a matrix form.

Given that the receiver knows the channel state information (coherent case), a simple decoding algorithm was used:

\[
\tilde{S}_{kx1} = d \times [H^H_{kxp} \times Y_{px1}]
\]  

(2-8)

where \( H^H \) denotes conjugate transpose (Hermitian) and the constant \( d \) is given by:

\[
d = \frac{1}{\sum_{k} |h_{k1}|^2 + |h_{k2}|^2 + \cdots + |h_{kp}|^2}
\]  

(2-9)
Chapter 3

Simulation Methodology

MATLAB™ Release 14, V 7.0 was used as a simulator tool in order to perform the simulation experiments.

3.1 Simulator

MATLAB™ is a software package for high-performance in technical computing, integrating programming, visualization, and computation in a very user-friendly environment. Best of all, it also provides extensibility and flexibility with its own high-level programming language. Common uses of MATLAB™ involve [19]:

(a) Mathematics (Arrays and matrices, linear algebra, etc.)
(b) Programming development (Function, data structures, etc.)
(c) Modeling and simulation (Signal Processing etc.)
(d) Data analysis (statistics etc.)
(e) Visualization (graphics, animation etc.)

All the simulations were performed by automation programs which were created by me. The whole simulation program was divided in sub-functions which were built in M-files form, *.m extension. More details about the structure of the software are shown in Figure 3.1.

3.2 Components of Simulation

The following components were modeled, through the simulations:

1. Complex information symbols signals
2. Flat-fading channels (Jake’s model [17])
Figure 3.1: Flowchart of the m-file functions of the simulation software
3. Doppler effect
4. White Gaussian Noise
5. Terminal speed

3.3 Constellations

The complex information symbols \( \{ \pm z_1, \pm z_2, \ldots, \pm z_k, \pm z_1^*, \pm z_2^*, \ldots, \pm z_k^* \} \) are transmitted using complex orthogonal time-space block codes. The information symbols are randomly selected from 4-PSK or/and 8-PSK constellations accordingly in order to achieve transmission of two bits per channel use (Figure 3.2).

![4-PSK constellation](image1)

![8-PSK constellation](image2)

Figure 3.2: a) 4-PSK constellation b) 8-PSK constellation (Phase Shift Key)

The representations of the information symbols are randomly selected from the above constellations and the bit mapping of the signals are built in an array form as shown below:

```plaintext
1> PSK4_constellation = [ 2> 1 i -1 -i 3> ]; 4> 5> PSK4_bitmapping = [ 6> 00 01 11 10 7> ]; 8> 9> %------------------------------------------------------------------------------------
```
The following seven vehicle speeds 0, 25, 50, 75, 150, 200, and 250 $\frac{km}{h}$ were used to calculate the Doppler frequencies and their corresponding constants $f_d \times T_s$.

$T_s$ is the symbol period and equals with $\frac{128}{3.84 \times 10^6}$ s and $f_d$ is the Doppler frequency shift and can be calculated using the equations 1-3, 2-3, and 2-5 (see Table 3.1).

<table>
<thead>
<tr>
<th>Vehicle Speed ($\frac{km}{h}$)</th>
<th>$f_d \times T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$0.001543$</td>
</tr>
<tr>
<td>3</td>
<td>$0.003086$</td>
</tr>
<tr>
<td>4</td>
<td>$0.004630$</td>
</tr>
<tr>
<td>5</td>
<td>$0.009300$</td>
</tr>
<tr>
<td>6</td>
<td>$0.012300$</td>
</tr>
<tr>
<td>7</td>
<td>$0.015400$</td>
</tr>
</tbody>
</table>

It’s obvious, that the value of $a_i$ depends on the applications. Recalling the formula of $a_i$

$$a_i = J_0(2\pi \times i \times f_d \times T_s)$$

(3-1)
we can realize that the terminal speed is the major factor that determines the value of 
\( a_i \). Using the above formula it can be shown that for speeds less than \( 150 \frac{km}{h} \) 
\((V < 150 \frac{km}{h})\), \( \alpha_i \) is greater than 0.9991 (\( a_i > 0.9991 \)) and assuming that \( i \langle 4 \).
Consequently, as the number of transmit antennas increases the length of the block increases as well. In addition, as the terminal speed increases more channel variation occurs. The consequences of the above factors result to an irreducible error floor in the BER curves at the high signal-to-noise ratio region.
Considering the above restrictions and based on realistic scenarios the range of the above speeds \((0 \text{– } 250 \frac{km}{h})\) was chosen among many others in order to determine the performance of different multiple transmit antennas. For instance, high speeds trains in Europe can easily reach 250 kilometers per hour. On the other hand, indoor networks can be built where the receiver is stationary \((0 \frac{km}{h})\).

### 3.5 Simulation Parameters

All the simulation parameters that have been used in this thesis are based on European Telecommunication Standards [20].

- **Carrier Frequency:** 2 GHz
- **Transmission Rate:** 144 kbits/sec
- **Terminal Speeds:** \(0 \text{– } 250 \frac{km}{h}\)
- **Channel Realizations:** \(5 \times 10^6\)
- **Signal to Noise Ratio (SNR):** \(0 \text{– } 36 \text{dB}\)
- Bit Error Rate (BER): $0 - 10^{-6}$
- Chip Rate: $3.84 \times 10^6 \frac{\text{chips}}{\text{sec}}$
- Spreading Factor (SF)=128
Chapter 4
Two Transmit Antennas

This chapter presents performance results of two transmit antennas and one receive antenna over the impact of time-varying channels base on the terminal speeds.

4.1 Channel Matrix Calculations

The remarkable Alamouti-coded symbol matrix was used for two transmit antennas, for the time intervals \( t, t + 1 \) [10].

\[
\mathcal{O}_z = \begin{bmatrix}
    z_1 & z_2 \\
    -z^*_2 & z^*_1
\end{bmatrix}
\] (4-1)

Alamouti scheme has low complexity and it can achieve full rate \( R = \frac{2}{2} = 1 \) ([\( p, n, k \)]=[2, 2, 2]).

The complex modulation symbols \( z_k \) are arranged based on a transmit matrix \( \mathcal{O}_z \). It is important to mention that the power of each symbol is normalized, \( E(\lvert z_k \rvert^2) = 1 \). Let \( h_1 \) and \( h_2 \) be the channels from the two transmit antennas to the receive antenna, respectively. In all orthogonal space-time block schemes, a crucial assumption was considered, where \( h_1 \) and \( h_2 \) are constants over two consecutive symbol periods. This thesis, will not consider this assumption, rather the channel state will vary from symbol to symbol, as shown in Figure 4.1.

\[
h_i(t) \neq h_i \cdot (t + 1) \quad \text{where} \quad i = 1, 2
\] (4-2)
Figure 4.1: Transmission model for 2-Tx antennas

At times:

\[ t = 1 \quad y_1 = h_{11} \cdot z_1 + h_{21} \cdot z_2 + w_1 \]  
\[ t = 2 \quad y_2 = h_{22} \cdot z_1^* - h_{12} \cdot z_2^* + w_2 \Rightarrow y_2^* = h_{22}^* \cdot z_1 - h_{12}^* \cdot z_2 + w_2^* \]

From equations 4-3 and 4-4 the receive signal in matrix form is:

\[
\begin{bmatrix}
  y_1 \\
  y_2^*
\end{bmatrix} = \begin{bmatrix}
  h_{11} & h_{21} \\
  h_{22}^* & -h_{12}^*
\end{bmatrix} \cdot \begin{bmatrix}
  z_1 \\
  z_2^*
\end{bmatrix} + \begin{bmatrix}
  w_1 \\
  w_2^*
\end{bmatrix}
\]

where the dimensions of matrix/vectors are: \( Y_{2x1} = H_{2x2} \times S_{2x1} + W_{2x1} \).

As you can see from the above calculations the receive signal is given by:

\[ \tilde{y} = \tilde{H} \cdot s + w \]  

where \( s \) is the signal vector, \( s = [z_1 \quad z_2]^T \), \( w \) is the noise vector, \( w = [w_1 \quad w_2^*]^T \),

and \( \tilde{H} \) is the new mortified coded channel matrix, \( \tilde{H} = \begin{bmatrix}
  h_{11} & h_{21}^* \\
  h_{22}^* & -h_{12}^*
\end{bmatrix} \).

To detect the original information symbols, we take advantage the orthogonal structure of \( \tilde{H} \), so the retrieve symbols can be found by:

\[ \tilde{s} = \tilde{H}^H \cdot \tilde{y} = \frac{\left| h_{11} \right|^2 + \left| h_{12} \right|^2 + \left| h_{21} \right|^2 + \left| h_{22} \right|^2}{2} \cdot s + \tilde{w} \]

4.2 Simulation Results

The following graphs illustrate simulation results in terms of bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for two transmit antennas and one
receive antenna based on seven different vehicle speeds 0, 25, 50, 75, 150, 200, and 
250 $\frac{km}{h}$, for transmission of 2 bits per channel use. The transmission using 
Alamouti’s scheme [10] for two transmit antennas employs the 4-PSK constellation.

Figure 4.2: Bit, symbol, and block error probability versus signal-to-noise 
ratio (SNR) for 2-Tx antennas base on vehicle speed 0, 25, 50, 75, 150, 200, 
and 250 km/h, respectively. (Figure Continue)
(e) 150 km/h
(f) 200 km/h
(g) 250 km/h
4.3 Performance Evaluation

Through analysis and simulations, we present the performance of two transmit antennas and one receive antenna over time-varying fading channels. For comparison and readability purposes we plot all the BER curves for all the aforementioned speeds, as shown in Figure 4.3.

Assuming that the receiver has perfect knowledge of the channel, a very low complexity decoding algorithm is proposed. At low vehicle speeds or slow fading scenarios ($V \leq 75 \frac{km}{h}$), our simulations indicated that the bit error probability has very low variation below the bit error rate of $10^{-5}$. However, above $10^{-5}$ the bit error remains the same for vehicle speeds below $75 \frac{km}{h}$. Consequently, the scheme
performance of the proposed decoding algorithm is excellent and shows no error floor at all at low vehicle speeds and under the presence of channel variation. On the other hand, when the vehicle speed is equal to or higher than $150 \frac{km}{h}$ the receiver exhibits a progressively severe irreducible error floor in the high SNR region. At low vehicle speeds, the assumption that the channel remains static over the length of the codeword is reasonable; whereas, our simulations show that channel variation at higher speeds destroys the orthogonality of transmission matrix leading to irreducible error floor in the high SNR region. As shown in the graph, the higher signal-to-noise ratio the lower the number of errors. Therefore, the desired low error bit probability comes with a costly price of high SNR. This undesirable price will be minimized by the addition of more transmit antennas, which will be discussed in the following chapters.
Chapter 5

Three Transmit Antennas

This chapter exhibits performance results for three transmit antennas and one receive antenna over time-selective fading channels, according to user vehicle speeds.

5.1 Channel Matrix Calculations

Using the Adams-Lax-Philips [21] construction matrix techniques, which are clearly demonstrated in Ref. [12], the following orthogonal space-time block code was chosen for the simulations of the three antennas.

\[
\varphi_z = \begin{pmatrix}
    z_1 & z_2 & z_3 \\
    -z_2^* & z_1^* & 0 \\
    -z_3^* & 0 & z_1^* \\
    0 & -z_3^* & z_2^*
\end{pmatrix}
\]  

(5-1)

This O-STBC has rate of \( R = \frac{k}{p} = \frac{3}{4} \) ([p, n, k] = [4, 3, 3]) and it has a special structure where each row has either only complex symbols or only conjugate (*) complex symbols. This structure gives the flexibility to use the complex number properties in order to demodulate the receive signals. The complex modulation symbols \([z_1, z_2, z_3, z_1^*, z_2^*, z_3^*]\) are transmitted and arranged according to the transmission matrix \( \varphi_z \). During the time intervals, \( t, t+1, t+2, \) and \( t+3 \) the channel state varies over the length of the codeword. Let \( h_{1i}, h_{2i}, h_{3i} \) be the channel for the transmit antennas one, two, and three respectively, where \( i = 1, 2, 3, 4 \), as shows in Figure 5.1.
At times:

\[ t = 1 \quad y_1 = h_{11} \cdot z_1 + h_{21} \cdot z_2 + h_{31} \cdot z_3 + w_1 \]  \quad (5-2)  

\[ t = 2 \quad y_2 = h_{22} \cdot z_1^* + h_{12} - z_2^* + h_{32} \cdot 0 + w_2 \Rightarrow y_2^* = h_{22}^* \cdot z_1 + h_{12}^* \cdot z_2 + 0 + w_2^* \]  \quad (5-3)  

\[ t = 3 \quad y_3 = h_{23} \cdot 0 - h_{13} \cdot z_3^* + h_{33} \cdot z_1^* + w_3 \Rightarrow y_3^* = h_{33}^* \cdot z_1 - h_{13}^* \cdot z_3 + 0 + w_3^* \]  \quad (5-4)  

\[ t = 4 \quad y_4 = h_{14} \cdot 0 - h_{24} \cdot z_3^* + h_{34} \cdot z_2^* + w_4 \Rightarrow y_4^* = h_{34}^* \cdot z_2 - h_{24}^* \cdot z_3 + 0 + w_4^* \]  \quad (5-5)  

From equations 5-2, 5-3, 5-4, and 5-5 the receive signal in matrix form is:

\[
\begin{bmatrix}
  y_1 \\
  y_2^* \\
  y_3^* \\
  y_4^*
\end{bmatrix} =
\begin{bmatrix}
  h_{11} & h_{21} & h_{31} \\
  h_{22}^* & -h_{12} & 0 \\
  h_{33}^* & 0 & -h_{13}^* \\
  0 & h_{34}^* & -h_{24}^*
\end{bmatrix}
\begin{bmatrix}
  z_1 \\
  z_2^* \\
  z_3^* \\
  z_4^*
\end{bmatrix} +
\begin{bmatrix}
  w_1 \\
  w_2^* \\
  w_3^* \\
  w_4^*
\end{bmatrix}
\]  \quad (5-6)  

where the dimensions of matrix/vectors are: \( Y_{4 \times 1} = H_{4 \times 3} \times S_{3 \times 1} + W_{4 \times 1} \).

As you can see from the above calculations the receive signal is given by:

\[
\tilde{y} = \tilde{H} \cdot s + w
\]  \quad (5-7)  

where \( s \) is the signal vector, \( s = [z_1 \ z_2 \ z_3]^T \), \( w \) is the noise vector, \( w = [w_1 \ w_2^* \ w_3^* \ w_4^*]^T \), and \( \tilde{H} \) is the new mortified orthogonal coded channel matrix.
\[
\phi_h = \begin{bmatrix}
  h_{11} & h_{21} & h_{31} \\
  h_{22}^* & -h_{12}^* & 0 \\
  h_{33}^* & 0 & -h_{13}^* \\
  0 & h_{34}^* & -h_{24}^*
\end{bmatrix}
\] (5-8)

The detection procedure is the same as it is described in chapter 2 and chapter 4.

## 5.2 Simulation Results

The following graphs illustrate simulation results in terms of bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for three transmit antennas and one receive antenna based on five different vehicle speeds 0, 25, 50, 75, and 150 km/h.

The transmission matrix which I mentioned earlier was used, while 4-PSK and 8-PSK constellations were considered in order to achieve transmission of 2 bits per channel use.

![Simulation Results Graphs](image)

(a) 0 km/h  
(b) 25 km/h

Figure 5.2: Bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for 3-Tx antennas base on vehicle speed 0, 25, 50, 75, and 150 km/h, respectively. (Figure Continue)
5.3 Performance Evaluation

Based on analysis and simulations, we present the BER performance of the proposed decoder for three transmit antennas and one receive antenna over time-selective fading channels. For comparison and readability purposes we plot all the BER curves for all the above mentioned speeds, as shown in Figure 5.3.
Assuming that the receiver has perfect knowledge of the channel, a very low complexity decoding algorithm is proposed. Theoretical and simulation results show that the proposed decoder performs reasonably well when the channel varies significantly from one signaling interval to another. At low vehicle speeds ($V < 75 \frac{km}{h}$), our simulations indicated that the bit error probability has very low variation below the bit error rate of $10^{-5}$. At speeds 0, 25, and $50 \frac{km}{h}$ our simulations shows no error floor at all in the high SNR region and SNR values are minimized by 5-dB compare with the BER performance of two-transmit antennas. However, when the vehicle speed is increased above $75 \frac{km}{h}$ an irreducible error floor appears in the high SNR region. Moreover, above the speed of $150 \frac{km}{h}$ this error floor is increased rapidly, whereas the communication between the terminals cannot perform
successfully. As shown in Figure 5.3, using three transmit antennas we achieved the same bit error probability with lower SNR values, compared with the two transmit antenna scheme (Figure 4.3). Overall, three antennas can perform better in some applications but on the other hand, hostile error floor effects must be considered at high vehicle speeds.

In the next chapter, we will consider a scheme where four transmit antennas and one receive antenna take place.
Chapter 6

Four Transmit Antennas

This chapter presents performance results for four transmit antennas and one receive antenna over time-varying channels based on the terminal speeds. In the section 6.2 simulation results were built with 1.5 bits per channel use, where in the section 6.3 2 bits per channel use was considered.

6.1 Channel Matrix Calculations

The space-time block code that was used for the simulation of four transmit antennas and one receive antenna for 2 bits per channel use, is shown in Figure 6.3.

At times: (see Figure 6.1)

\[
\begin{align*}
  t = 1 & \quad y_1 = h_{11} \cdot z_1 + h_{21} \cdot 0 + h_{31} \cdot z_2 + h_{41} \cdot z_3 + w_1 \\
  t = 2 & \quad y_2 = h_{12} \cdot 0 + h_{22} \cdot z_1 + h_{32} \cdot z_4 + h_{42} \cdot z_5 + w_2 \\
  t = 3 & \quad y_3^* = -h_{13}^* \cdot z_2 - h_{23}^* \cdot z_4 + h_{33}^* \cdot z_1 + h_{43}^* \cdot 0 + w_3^* \\
  t = 4 & \quad y_4^* = -h_{14}^* \cdot z_3 - h_{24}^* \cdot z_5 + h_{34}^* \cdot z_1 + h_{44}^* \cdot 0 + w_4^* \\
  t = 5 & \quad y_5 = -h_{15} \cdot z_4 + h_{25} \cdot z_2 + h_{35} \cdot 0 + h_{45} \cdot z_6 + w_5
\end{align*}
\]
\[ t = 6 \quad y_6^* = h_{16} \cdot 0 - h_{26}^* \cdot z_6 - h_{36}^* \cdot z_3 + h_{46}^* \cdot z_2 + w_6^* \]  
\[ t = 7 \quad y_7 = -h_{17} \cdot z_5 + h_{27}^* \cdot z_3 - h_{37}^* \cdot z_6 + h_{47} \cdot 0 + w_7 \]  
\[ t = 8 \quad y_8^* = h_{18}^* \cdot z_6 - h_{28} \cdot 0 - h_{38}^* \cdot z_5 + h_{48}^* \cdot z_4 + w_8^* \]  
\[ (6-6) \]
\[ (6-7) \]
\[ (6-8) \]

\[
\begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3^* \\
    y_4^* \\
    y_5 \\
    y_6^* \\
    y_7 \\
    y_8^*
\end{bmatrix} =
\begin{bmatrix}
    h_{11} & h_{31} & h_{41} & 0 & 0 & 0 \\
    h_{22} & 0 & 0 & h_{32} & h_{42} & 0 \\
    h_{33}^* & -h_{13}^* & 0 & -h_{23}^* & 0 & 0 \\
    h_{44}^* & 0 & -h_{14}^* & 0 & -h_{24}^* & 0 \\
    0 & h_{25} & 0 & -h_{15} & 0 & h_{45} \\
    0 & h_{46}^* & -h_{36}^* & 0 & 0 & -h_{26}^* \\
    0 & 0 & h_{27} & 0 & -h_{17} & -h_{37} \\
    0 & 0 & 0 & h_{48}^* & -h_{38}^* & h_{18}^*
\end{bmatrix}
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4 \\
    z_5 \\
    z_6 \\
    z_7 \\
    z_8^*
\end{bmatrix} +
\begin{bmatrix}
    w_1 \\
    w_2 \\
    w_3 \\
    w_4 \\
    w_5 \\
    w_6 \\
    w_7 \\
    w_8^*
\end{bmatrix}
\]  
\[ (6-9) \]

where the dimensions of matrix/vectors are: \( Y_{8\times 1} = H_{8\times 6} \times S_{6\times 1} + W_{8\times 1} \).

And again the receive signal is given by:

\[ \tilde{y} = \tilde{H} \cdot s + w \]  
\[ (6-10) \]

where \( s \) is the signal vector, \( s = [z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6]^T \), \( w \) is the noise vector, \( w = [w_1 \ w_2 \ w_3 \ w_4 \ w_5 \ w_6 \ w_7 \ w_8^*]^T \), and \( \tilde{H} \) is the new orthogonal mortified coded channel matrix (equation 6-11).

\[ \phi^h =
\begin{bmatrix}
    h_{11} & h_{31} & h_{41} & 0 & 0 & 0 \\
    h_{22} & 0 & 0 & h_{32} & h_{42} & 0 \\
    h_{33}^* & -h_{13}^* & 0 & -h_{23}^* & 0 & 0 \\
    h_{44}^* & 0 & -h_{14}^* & 0 & -h_{24}^* & 0 \\
    0 & h_{25} & 0 & -h_{15} & 0 & h_{45} \\
    0 & h_{46}^* & -h_{36}^* & 0 & 0 & -h_{26}^* \\
    0 & 0 & h_{27} & 0 & -h_{17} & -h_{37} \\
    0 & 0 & 0 & h_{48}^* & -h_{38}^* & h_{18}^*
\end{bmatrix}
\]  
\[ (6-11) \]
To detect the original information symbols, we take advantage the orthogonal structure of $\tilde{H}$, so the retrieve symbols can be found by:

$$\tilde{s} = \tilde{H}^H \cdot \tilde{y}$$

(6-12)

6.2 Comparison of Performance of Two O-STBC for Four Transmit Antennas

Orthogonal designs have been used as space–time block codes for wireless communications with multiple transmit antennas ($n$). This section of the chapter presents a comparison of performance of two orthogonal space-time block codes with different rates, $R_1 = \frac{4}{8}$ and $R_2 = \frac{6}{8}$, for four transmit antennas over time-selective fading channels. It is shown that under time-selectiveness and once the vehicle speed rises above a certain value, the code with rate of $\frac{6}{8}$ is much more efficient than the code with rate $\frac{4}{8}$ [12, 22].

6.2.1 Orthogonal Designs

The aim of this section is to introduce a new orthogonal space-time code design, which minimizes the floor error that arises due to the terminal speed. The orthogonal designs that will be presented in this chapter have a special structure. Each row has either only complex symbols or only conjugate ($\ast$) complex symbols (Figure 6.2 and Figure 6.3). This structure gives the flexibility to manipulate the complex number properties in order to demodulate the receive signals. The new orthogonal design has many advantages over the conventional code [22] that has been used so far.
6.2.2 The Conventional O-STBC

This space-time block code, (Figure 6.2) can find application for 4 transmit antennas \((n)\) to send 4 information symbols \((k)\) in a block of 8 channel uses \((p)\). The rate of this orthogonal code is therefore, \(R = \frac{k}{p} = \frac{4}{8} = \frac{1}{2} [22].\)

\[
\varphi = \begin{pmatrix}
Z_1 & Z_2 & Z_3 & Z_4 \\
-Z_2 & Z_1 & -Z_4 & Z_3 \\
-Z_3 & Z_4 & Z_1 & -Z_2 \\
-Z_4 & -Z_3 & Z_2 & Z_1 \\
Z_1^* & Z_2^* & Z_3^* & Z_4^* \\
-Z_2^* & Z_1^* & -Z_4^* & Z_3^* \\
-Z_3^* & Z_4^* & Z_1^* & -Z_2^* \\
-Z_4^* & -Z_3^* & Z_2^* & Z_1^*
\end{pmatrix}
\]

Figure 6.2: The conventional code \([p, n, k] = [8, 4, 4]\)

6.2.3 The New High-Rate O-STBC

The new design of transmission matrix for 4 transmit antennas of size 8×4 is given in Figure 6.3. This matrix is clearly an orthogonal space-time block code and sends \(k = 6\) information symbols in a block of \(p = 8\) channel uses. The rate of this orthogonal code is therefore, \(R = \frac{k}{p} = \frac{6}{8} = \frac{3}{4} [12].\)

\[
\varphi = \begin{pmatrix}
Z_1 & 0 & Z_2 & Z_3 \\
0 & Z_1 & Z_4 & Z_5 \\
-Z_2^* & -Z_4^* & Z_1^* & 0 \\
-Z_3^* & -Z_5^* & 0 & Z_1^* \\
-Z_4 & Z_2 & 0 & Z_6 \\
0 & -Z_6^* & -Z_3^* & Z_2^* \\
-Z_5 & Z_3 & -Z_6 & 0 \\
Z_6^* & 0 & -Z_3^* & Z_4^*
\end{pmatrix}
\]

Figure 6.3: The new High-Rate O-STBC \([p, n, k] = [8, 4, 6]\)
6.2.4 Comparison of the Orthogonal Designs

Clearly, the new orthogonal design has a greater rate than the conventional code. Consequently, the new high rate orthogonal design can achieve bigger diversity gain by transmitting additional two more information symbols. Another big advantage of the new orthogonal design is that in eight symbol periods ($T_s$) transmits zero (nothing), which saves power consumption to the transmitter. The simulations show that the new orthogonal design can efficiently reduce the error floor at the high signal-to-noise ratio (SNR) region. In addition, it provides better performance in high vehicle speed values.

6.2.5 Simulation Results for 1.5 Bits per Channel Use

The following section provides simulation results for the performance of the above orthogonal space-time block codes. The new orthogonal design code with rate $\frac{3}{4}$ is compared with the conventional code, which has rate $\frac{1}{2}$. The transmission model that was used is described in detail in chapter 2 and is similar with the transmission model for wireless communication systems with multiple antennas as described in Ref. [23]. A simple decoding algorithm under the assumption that the receiver knows the channel state information is also described in chapter 2. The receiver estimates the transmitted bits by using the signals of the receive antennas (coherent case). Figure 6.5 and Figure 6.6 show bit error rates (BER), for transmission of 1.5 bits per channel use for four transmit antennas and one receive antenna, with rates $\frac{4}{8}$ and $\frac{6}{8}$, respectively. In order to achieve a transmission with 1.5 bits per channel use, 8-PSK (Phase Shift Key) constellation for the conventional orthogonal design and 4-PSK
constellation for the new orthogonal design were used. Channel matrix $H$ was generated using the equation 2-2, Jake’s model [17]. Each graph in Figure 6.4 presents the performance results for terminal speeds 0, 25, 50, and $75 \frac{km}{h}$. The bit error rate at each SNR $(E_b/N_0)$ point is averaged over $5 \times 10^6$ channel realizations.

Figure 6.4: The BER vs. SNR performance between the two orthogonal designs for terminal speeds 0, 25, 50, and $75 \frac{km}{h}$ respectively.
Figure 6.5: The BER performance of the O-STBC with rate $\frac{4}{8}$

Figure 6.6: The BER performance of the O-STBC with rate $\frac{6}{8}$
6.2.6 Conclusions

The section 6.2 has shown simulation results for the performance of the new orthogonal space-time block code. It is clearly shown that the new orthogonal design (Figure 6.6) has better performance compared with the conventional orthogonal design (Figure 6.5). It reduces efficiently the error floor at the high signal-to-noise ratio region, especially when the terminal speed is $50 \frac{km}{h}$ (Figure 6.4c).

6.3 Simulation Results for 2 Bits per Channel Use

The graphs in Figure 6.7 illustrate simulation results in terms of bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for four transmit antennas and one receive antenna base on four different vehicle speeds 0, 25, 50, and $75 \frac{km}{h}$, for transmission of 2 bits per channel use. The orthogonal transmission matrix that was used is the one that was demonstrated in Figure 6.3. It’s important to mention, that in order to achieve a transmission rate of 2 bits per channel use, 4-PSK and 8-PSK constellations where used. Therefore, two symbols where selected from 4-PSK constellation and four symbols from 8-PSK constellation since the orthogonal design has rate of $R = \frac{k}{p} = \frac{6}{8}$.

6.4 Performance Evaluation

The analysis and simulations, for four transmit antennas and one receive antenna have shown that for vehicle speeds above $25 \frac{km}{h}$ a significant amount of error floor appears at high regions of SNR, as shown in Figure 6.8.
Figure 6.7: Bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for 4-Tx antennas base on vehicle speeds 0, 25, 50, and 75 km/h, respectively.
Figure 6.8: BER comparisons between different values of speeds, for 4-Tx antennas

On the other hand, the scheme performance below $25 \frac{km}{h}$ is excellent and shows no error floor at all, under the presence of channel variation. Consequently, when the terminals speeds are kept below $25 \frac{km}{h}$, a very good performance appears where the bit error probability is very low at low signal-to-noise ratio areas. The performance of this scheme has shown that it can find real applications, which are discussed in detail in chapter 8.
Chapter 7

Five Transmit Antennas

This chapter demonstrates performance results for five transmit antennas and one receive antenna over time-selective fading channels, according on user vehicle speeds.

7.1 Channel Matrix Calculations

An outstanding orthogonal matrix construction procedure was demonstrated in Ref. [12], where using the proposed construction matrix technique, the following orthogonal space-time block code was obtained.

\[
\Phi_z = \begin{pmatrix}
z_1 & 0 & z_2 & z_3 & z_4 \\
0 & z_1 & z_5 & z_6 & z_7 \\
-z_2^* & -z_5^* & z_1^* & 0 & 0 \\
-z_3^* & -z_6^* & 0 & z_1^* & 0 \\
-z_4^* & -z_7^* & 0 & 0 & z_1^* \\
-z_5 & z_2 & 0 & z_8 & z_9 \\
0 & -z_8^* & -z_3^* & z_2^* & 0 \\
0 & -z_9^* & -z_4^* & 0 & z_2^* \\
-z_6 & z_3 & -z_8 & 0 & z_{10} \\
0 & -z_{10}^* & 0 & -z_4^* & z_3^* \\
-z_7 & z_4 & -z_9 & -z_{10} & 0 \\
z_8^* & 0 & -z_6^* & z_5^* & 0 \\
z_9^* & 0 & -z_7^* & 0 & z_5^* \\
z_{10}^* & 0 & 0 & -z_7^* & z_6^* \\
0 & 0 & z_{10}^* & -z_9^* & z_8^*
\end{pmatrix}_{15 \times 5}
\] (7-1)
The above orthogonal space-time block channel matrix has rate of \( R = \frac{k}{p} = \frac{10}{15} \) \([p, n, k]=[15, 5, 10]\) and it is used for five transmit antennas. The complex modulation symbols are transmitted and arranged according to the above transmission matrix \( \varphi_z \) in such a way that each row has either only complex symbols or only conjugate (\( \ast \)) complex symbols. During the fifteen time intervals, channel state varies over the length of the codeword. Let \( h_{i1}, h_{i2}, h_{i3}, h_{i4}, h_{i5} \) be the channel for the transmit antennas one, two, three, four, and five, respectively, where \( i = 1, 2, \cdots, 15 \).

The corresponding orthogonal channel matrix is shown below:

\[
\varphi_h = \begin{bmatrix}
  h_{11} & h_{31} & h_{41} & h_{51} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  h_{22} & 0 & 0 & 0 & h_{32} & h_{42} & h_{52} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  h_{33}^* & -h_{13}^* & 0 & 0 & -h_{23}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  h_{44}^* & 0 & -h_{14}^* & 0 & 0 & -h_{24}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  h_{55}^* & 0 & 0 & -h_{15}^* & 0 & 0 & -h_{25}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & h_{26} & 0 & 0 & -h_{16} & 0 & 0 & h_{46}^* & h_{56}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & h_{37}^* & -h_{37}^* & 0 & 0 & 0 & 0 & 0 & -h_{27}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & h_{38}^* & 0 & -h_{36}^* & 0 & 0 & 0 & 0 & 0 & 0 & -h_{28}^* & 0 & 0 & 0 & 0 \\
  0 & 0 & h_{29} & 0 & 0 & -h_{19} & 0 & 0 & -h_{39} & 0 & h_{59}^* & 0 & 0 & 0 & 0 \\
  0 & 0 & h_{310}^* & -h_{410}^* & 0 & 0 & 0 & 0 & 0 & 0 & -h_{210}^* & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & h_{211} & 0 & 0 & -h_{111} & 0 & -h_{311} & -h_{411} & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & h_{312}^* & -h_{312}^* & 0 & h_{112}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & h_{513} & 0 & -h_{313} & 0 & h_{113}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & h_{514}^* & -h_{414}^* & 0 & 0 & 0 & h_{114}^* & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{515}^* & -h_{415}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Using the same procedure as decrypted in chapters 4, 5, and 6 the receive signal on a matrix form is constructed below.

\[
\tilde{y} = \tilde{H} \cdot s + w
\]  

(7-3)

\[
\begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3 \\
    y_4 \\
    y_5 \\
    y_6 \\
    y_7 \\
    y_8 \\
    y_9 \\
    y_{10} \\
    y_{11} \\
    y_{12} \\
    y_{13} \\
    y_{14} \\
    y_{15}
\end{bmatrix} =
\begin{bmatrix}
    h_{11} & h_{31} & h_{41} & h_{51} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    h_{32} & 0 & 0 & 0 & h_{32} & h_{42} & h_{52} & 0 & 0 & 0 & 0 \\
    h_{33} & -h_{13} & 0 & 0 & -h_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\
    h_{44} & 0 & -h_{14} & 0 & 0 & -h_{24} & 0 & 0 & 0 & 0 & 0 \\
    h_{35} & 0 & 0 & -h_{15} & 0 & 0 & -h_{25} & 0 & 0 & 0 & 0 \\
    0 & h_{26} & 0 & 0 & 0 & 0 & 0 & -h_{16} & 0 & 0 & 0 \\
    0 & 0 & -h_{27} & 0 & 0 & 0 & 0 & 0 & -h_{37} & 0 & 0 \\
    0 & 0 & 0 & h_{58} & 0 & 0 & 0 & 0 & 0 & -h_{38} & 0 \\
    0 & 0 & 0 & 0 & h_{29} & 0 & -h_{39} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & h_{210} & -h_{310} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & h_{211} & 0 & 0 & -h_{311} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & h_{412} & -h_{312} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & h_{513} & 0 & -h_{313} & 0 & h_{413} & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{414} & -h_{314} & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{515} & -h_{315} & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{515} & -h_{315}
\end{bmatrix}
\begin{bmatrix}
    w_1 \\
    w_2 \\
    w_3 \\
    w_4 \\
    w_5 \\
    w_6 \\
    w_7 \\
    w_8 \\
    w_9 \\
    w_{10} \\
    w_{11} \\
    w_{12} \\
    w_{13} \\
    w_{14} \\
    w_{15}
\end{bmatrix}
\]

The dimensions of matrix/vectors are: 
\[ Y_{15x1} = H_{15x10} \times S_{10x1} + W_{15x1} \cdot w \]

Where \( s \) is the signal vector, 
\[ s = [z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6 \ z_7 \ z_8 \ z_9 \ z_{10}]^T \]

\( w \) is the noise vector,
\[ w = [w_1 \ w_2 \ w_3 \ w_4 \ w_5 \ w_6 \ w_7 \ w_8 \ w_9 \ w_{10} \ w_{11} \ w_{12} \ w_{13} \ w_{14} \ w_{15}]^T \]

and \( \tilde{H} \) is the new mortified coded channel matrix as shown in equation 7-2.

### 7.2 Simulation Results

The graphs in Figure 7.1 illustrate simulation results in terms of bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for five transmit antennas and one receive antenna for four different vehicle speeds 0, 25, 50, and 75 \( \frac{km}{h} \), for transmission of 2 bits per channel use. 8-PSK constellation was considered.
Figure 7.1: Bit, symbol, and block error probability versus signal-to-noise ratio (SNR) for 5-Tx antennas for speeds 0, 25, 50, and 75 km/h, respectively.
7.3 Performance Evaluation

Clearly, simulation results demonstrated that the performance of five transmit antennas and one receive antenna over time-varying fading channels while the terminal user has a certain amount of speed, does not perform successfully. Comparison of the simulation results are shown in Figure 7.2.

Our simulations indicated progressively severe irreducible error floor in all SNR regions. The failure of this scheme is due to the length of the block. Each codeword has fifteen dependant channel states which at the end, they totally destroy the orthogonality of the transmission matrix. Due to this phenomenon, error control coding techniques are applied in order to reduce the bit error probability. On the other hand, when the receiver is stationary the scheme performance of the proposed decoding algorithm is excellent, as indicated in Figure 7.2. It achieves low values of probability.
errors with low amount of signal-to-noise ratio. This scheme can successfully find many applications where the receiver terminal is stationary, such as indoor wireless networks.
Chapter 8

Conclusions

Through our work, we investigated and demonstrated that significant gains can be achieved by increasing the number of transmit antennas. We provided a special family of complex orthogonal space-time codes for transmission using multiple transmit antennas. The encoding and decoding of these codes have low complexity. Earlier works have ignored the variation of the channel state over the length of the codeword. This thesis presented performance results where the channel state varies from symbol to symbol. For comparison and readability purposes we plot all the corresponding BER curves for each number of transmit antennas, according to terminal speed, as shown in the Figure 8.1.

Figure 8.1(a), shows bit error probability curves for different amount of transmit antennas, where the receiver is stationary. It can been seen from the Figure 8.1(a), that at bit error rates equal to $10^{-6}$, the scheme with five transmit antennas achieves more than 2-dB gain over the scheme with four transmit antennas. Moreover, the performance of four transmit antennas achieve 5-dB gain over the three transmit antennas and lastly the three transmit antennas more than 5-dB gain over the two transmit antennas. Previous works confirm our results [11, 22, 24]. A possible application of the scheme is to provide diversity improvement in all stationary receive units in a wireless system, such as an indoor wireless network.

Figure 8.1(b), shows bit error probability curves for different amount of transmit antennas, where the receiver have speed of $25 \frac{km}{h}$. Clearly, the following simulations demonstrate that significant gains can be achieved by increasing the number of transmit antennas.
Figure 8.1: Bit error probability versus SNR for different number of transmit antennas
It’s essential to observe that the two, three, and four transmit antennas schemes shows no error floor at any value of SNR. However, when it comes for the scheme of five transmit antennas under the speed of $25 \frac{km}{h}$, an error floor appears at high SNR regions. This happens due the channel variations between the transmitted symbols and due to the long length of the codeword. In addition, Figure 8.1(c), shows the performance results of the abovementioned schemes under the speed of $50 \frac{km}{h}$, where the three transmit antennas can achieve better diversity gain (3-dB) than two transmit antennas. However, four and five transmit antennas schemes appear to suffer from error floor. In Figure 8.1(d), shows that all the schemes except from the two transmit antenna scheme suffer from error floor at the high SNR regions under the speed of $75 \frac{km}{h}$. Lastly, the performance results of two and three transmits antennas schemes under the vehicle speed of $150 \frac{km}{h}$ are demonstrated in Figure 8.1(e).

Future studies can consider combination of multiple transmit antennas and error control coding techniques in order to suppress the above error floors.
Bibliography


Vita

Ioannis D. Erotokritou was born in Nicosia, Cyprus, on August 3rd, 1979. In the months after completing his high school education from Acropolis Lyceum in July of 1997, he enrolled as a student in the Higher Technical Institute in Cyprus, with focus in electrical engineering. After completing his first degree in electrical engineering, in June 2000, he joined the Greek Army and the Cyprus National Guard where he served as an officer during his 26 month term. After he was discharged from the army in 2002, he enrolled in the Department of Electrical and Computer Engineering at Louisiana State University where he graduated with a first class in spring 2004 with a Bachelor of Science in Electrical Engineering. Upon graduation, Ioannis was accepted as a graduate student in the same department and he is expected to get his Master of Science in Electrical Engineering with specialization in telecommunications and networking in spring 2006.