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INTRADAY EMPIRICAL ANALYSIS OF ELECTRICITY PRICE BEHAVIOUR

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Abstract. This paper proposes an approach to the intraday analysis of the dynamics of electricity prices. The growth optimal portfolio (GOP) is used as a reference unit in a continuous financial electricity price model. A diversified global portfolio in the form a market capitalisation weighted index approximates the GOP. The GOP, measured in units of electricity, is normalised and then modelled as a time transformed square root process of dimension four. The dynamics of the resulting process is empirically verified. Intraday spot electricity prices from the US and Australian markets are used for this analysis. The empirical findings identify a simple but realistic model for examining the volatile behaviour of electricity prices. The proposed model reflects the historical price evolution reasonably well by using only a few robust and readily observable parameters. The evolution of the transformed time is modelled via a rapidly evolving market activity. A periodic, ergodic process with deterministic volatility is used to model market activity.

1. Introduction

The dynamics of commodity prices plays a central role in valuing corresponding financial contingent claims. Their behaviour also plays a role in the evaluation of capital investments to produce a commodity. However, the main difficulty in implementing and testing commodity price models is that the typical state variables of these models do not seem to be directly observable. Employing a straightforward extension of the techniques developed under the continuous time Black and Scholes [2] and Merton [13] models and their extensions, has enabled researchers and practitioners to approximate the value of financial and real commodity contingent claims.

Schwartz [16] employed three models that accounted for the mean reverting nature of commodity prices in differing ways. Interesting results in modelling commodity prices have also been obtained by Gibson and Schwartz [8], Cortazar and Schwartz [6] and Schwartz and Smith [17], using models similar to the Vasicek [18] interest rate framework. These models engage in examining the longer term dynamics of commodities however they do not provide a robust model for the short term dynamics of commodity prices, particularly the intraday dynamics.

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The highly volatile nature of the behaviour of electricity prices suggests that if a market model can capture the dynamics of this erratic behaviour, then many other, more "well-behaved" commodities can also be modelled in a similar way using this approach. The application of existing commodity valuation models to electricity as a commodity has been shown to be unsatisfactory [5]. A number of alternative methods for modelling electricity have therefore been suggested. For instance, models that incorporate time varying volatility and price jumps have been employed in empirical settings [7]. More generally, modelling the most predictable components of electricity has been conducted [12]. However, each of these approaches ignores some important component in the modelling, such as jumps or stochastic volatility.

Our methodology exploits the notion of the *growth optimal portfolio* (GOP), which maximises expected logarithmic utility. It was originally introduced by Kelly (1956) and later extended and applied in [11], [1], [14], and [4]. Under the standard risk neutral framework, the GOP coincides with the *numeraire portfolio* [11], which converts prices, when expressed in units of the GOP, into martingales under the real world probability measure. In a more general continuous time setting, Platen [14] demonstrates that when prices are denominated by the GOP, they become supermartingales. A diffusion model for the GOP is derived in [15] that is applied in the following analysis. Using a similar framework, [3] empirically constructed and modelled an intraday GOP denominated in USD called the *Market Capitalisation Weighted Index* (MCI), with five minute observation intervals. This index is used as an approximation for the GOP in this paper. Under this framework we construct a market activity model for electricity prices using high-frequency trading data for the US and Australian electricity markets.

Section 2 discusses the data set. Section 3 presents a framework for the GOP denominated in units of intraday electricity prices. Section 4 discusses the empirical analysis of intraday market activity and commodity price behaviour.

2. The Behavioural Characteristics of Electricity

2.1. Electricity Market Data. Electricity prices display a high degree of mean reversion and are subject to significant intraday, day of the week and seasonal patterns. Some are censored from above, but not from below. In the short term, these prices seem to be determined by the level of demand while in the long term, they appear to fluctuate around the cost of production, which emphasises the presence of a mean reversion property. The effect of seasonal variations in electricity prices has been well documented, see [5], [10] and [12].

Local air temperatures heavily influence the demand for electricity which, if excessive, places significant burdens on the electricity generators for the immediate production of electricity. The electricity market is subject to distribution and transmission constraints, such that once fully constrained, the marginal cost of transmission can become practically infinite. In some markets, such as the US and Australia, price caps have been instituted to combat this possible situation. There is also the capacity for slightly negative prices, which occur as a consequence of an inability to freely dispose excess electricity.

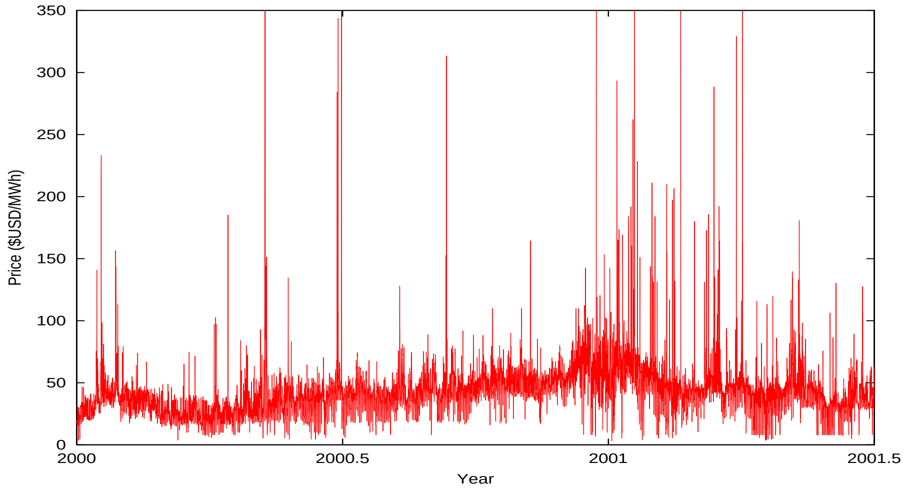

FIGURE 1. NEPOOL spot electricity prices $EP_{USD}(t)$ Jan 2000 - Jul 2001.

The market for electricity is an auction where generators and distributors submit bids encompassing volume and price information. Extremes in temperatures coupled with outages in generation or transmission induce price spikes to occur at random points in time. These price spikes are characteristic for electricity as a traded commodity, which needs to be produced on demand due to limited storage capability.

Finally, it appears that the volatility of observed electricity prices tends to rise more with positive shocks than with negative shocks, a phenomenon referred to as the *inverse leverage effect* [10]. This property will be naturally incorporated in our modelling.

2.2. Data Analysis. The worldwide trend towards deregulating electricity markets is an attempt to promote competition at each stage of the electricity production and supply chain. In the US and Australian wholesale electricity markets, the electricity output from generators is pooled and then scheduled to meet electricity demand. Trading in electricity is conducted in a spot market, which allows instantaneous matching of supply against demand. Generators offer to supply the market with different amounts of energy at particular prices.

Each market boasts its own set of regulations and trading conventions. For instance, in the US, the New England Power Pool (NEPOOL), administered by the independent service operator (ISO) New England, dispatches electricity at an average generation price aggregated across all power providers every hour.

In Australia until 2009, the National Electricity Market Management Company (NEMMCO) was the independent regulator responsible for the dispatch of electricity across four Australian states. The NEMMCO is now known as the Australian Energy Market Operator (AEMO). Dispatch instructions are sent to each generator at five minute intervals to schedule the amount of power to be produced and

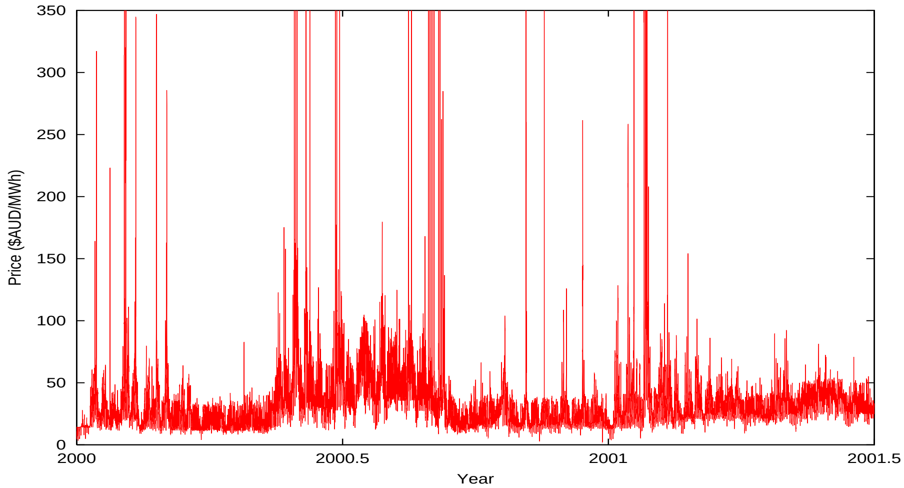

FIGURE 2. NSW spot electricity prices $EP_{AUD}(t)$ Jan 2000 - Jul 2001.

the six dispatch prices recorded during each half hour period are averaged to determine the spot price. Prices are calculated for dispatch intervals in each region in Australia, of which New South Wales (NSW) is the largest. In this analysis, we will refer to US prices as the *NEPOOL electricity price* exclusively expressed in US dollars (USD), and to Australian prices as the *NSW electricity price* exclusively expressed in Australian dollars (AUD).

For this study, hourly spot prices have been obtained for the NEPOOL and half-hourly spot electricity prices for NSW were obtained. Although electricity prices have been deregulated in both countries since 1998, there appears to be some structural inhibitors in the prices prior to 2000, resulting in a clearly defined and seemingly artificial level to which prices revert. This hints at the existence of some residual regulatory effects within each market, and therefore we ignore spot electricity price data prior to January 2000.

We denote the spot electricity price at time t by $EP_d(t)$, where we set $d = USD$ when we consider the NEPOOL spot electricity price, which is denominated in USD. Similarly, we denote by EP_{AUD} the NSW spot electricity price at time t, which is denominated in AUD.

We conduct our modelling of electricity prices based on spot price data for the NEPOOL and NSW electricity markets from January 2000 to July 2001, which amounts to 13128 spot electricity prices for the NEPOOL and 26256 spot electricity prices for NSW. We shall ignore slightly negative prices as they are of insignificant magnitude and occur at infrequent and irregular times. Figures 1 and 2 show spot electricity prices $EP_{USD}(t)$ and $EP_{AUD}(t)$ for the eighteen month period for each market. The prices are clearly fluctuating, featuring numerous spikes based upon an underlying time dependent reference level. Some prices during this period reached levels in excess of $5000/MWh. Higher prices appear to be aligned

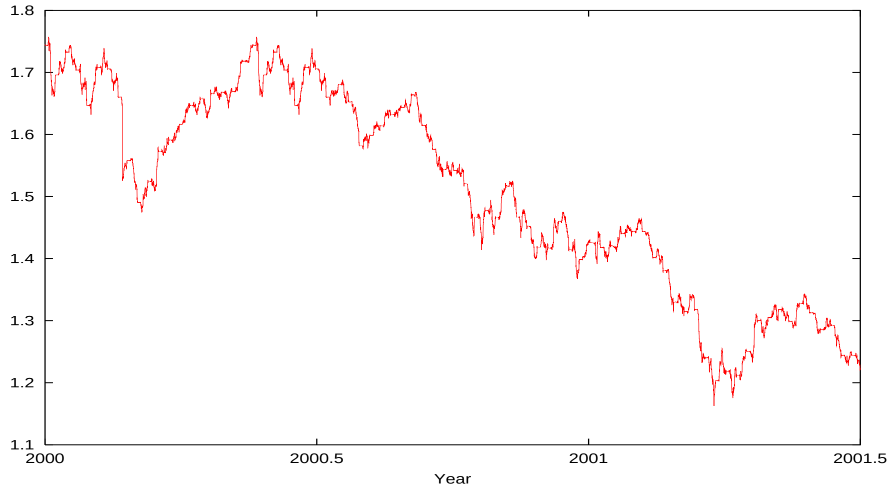

FIGURE 3. Market Capitalisation Weighted Index (MCI) in USD, Jan 2000 - Jul 2001.

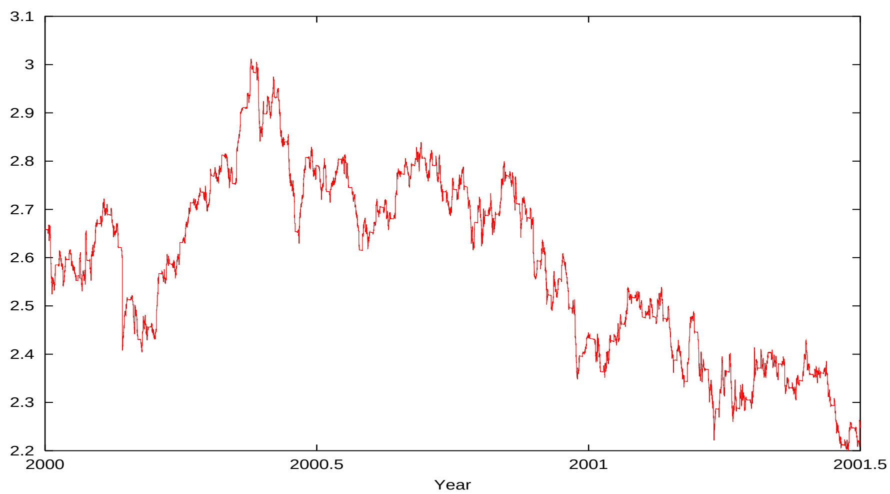

FIGURE 4. Market Capitalisation Weighted Index (MCI) in AUD, Jan 2000 - Jul 2001.

with the extremes in temperatures, primarily during extreme winter and summer conditions.

Following the approach by Platen [14] we use the market capitalisation-weighted world stock index (MCI) as a diversified portfolio of stocks and as a proxy for the *growth optimal portfolio* (GOP). In Figures 3 and 4 we display the intraday MCI denominated in USD and AUD, which is denoted at time t by $S_d^{(MCI)}(t)$ with

$d \in \{USD,\ AUD\}$, as constructed in [3]. The stock markets aggregated in this MCI account for more than 95 percent of the total world market capitalisation. The intraday MCI is formed as a self financing portfolio using 34 local stock market indices taken from almost all financial markets throughout the world, weighted by market capitalisation and rebalanced according to a strict set of rules. This index generally mimics the Morgan Stanley Capital Growth World Index (MSCI) over the period, but it provides observed values at a five minute frequency. In fact, the MCI outperforms the MSCI over the time period used in this analysis.

To be consistent with the observation frequency, for NEPOOL data we arithmetically average twelve consecutive five minute intervals to compute an hourly average for the MCI in USD with values shown in Figure 3. For NSW data, we arithmetically average six consecutive five minute intervals to compute the half hourly average for the MCI in AUD shown in Figure 4. This method of averaging the index data over time is employed since the hourly NEPOOL and half hourly NSW spot prices for electricity are similarly obtained by averaging the dispatch prices over consecutive five minute intervals. These are the highest frequencies at which electricity price data were available for the two markets. The time steps for both the MCI values and the spot electricity prices are synchronised by this method.

We now denominate the MCI in units of the ith spot electricity price, $i \in EP = \{EP_{USD},\ EP_{AUD}\}$, at time $t \in [0, T]$ by setting

$$S^{(MCI)}_{EP_d}(t) = \frac{S^{(MCI)}_d(t)}{EP_d(t)}, \tag{2.1}$$

for $d \in \{USD,\ AUD\}$.

Since the observation intervals for the processes are synchronised, the values for $S^{(MCI)}_i(t),\ i \in EP$, at the observation times $t \in [0, T]$ are easily obtained. It is appropriate to normalise the MCI $S^{(MCI)}_i(t)$ at time t when denominated in units of the ith electricity price. For this, we introduce the ith normalisation function

$$\bar{\alpha}_i(t) = \xi_i e^{\eta_i t}, \tag{2.2}$$

for $i \in EP$ and $t \in [0, T]$, where $\xi_i > 0$ is a *normalisation factor* and $\eta_i > 0$ is the *growth rate* of the ith normalisation function. Thus we introduce the ith *normalised* MCI $Y_i(t)$ at time t for the corresponding electricity price in the form

$$Y_i(t) = \frac{S^{(MCI)}_i(t)}{\bar{\alpha}_i(t)}, \tag{2.3}$$

for $i \in EP$ and $t \in [0, T]$. This representation is a normalised MCI time series which will assist with the formulations through the remainder of the paper.

We choose $\xi_{EP_{USD}} = 28$ and $\xi_{EP_{AUD}} = 63$ as the appropriate normalisation factors. The values that we use for the growth rates are $\eta_{EP_{USD}} = 0.0127$ and $\eta_{EP_{AUD}} = 0.0257$, which follows from a detailed analysis of the behaviour of electricity prices over the long term and will be given in Section 3. However, it transpires that for the relatively short time period of 1.5 years that we consider, the particular value of the growth rate is not really important. The inclusion of the above normalisation function accommodates both the long term growth rate

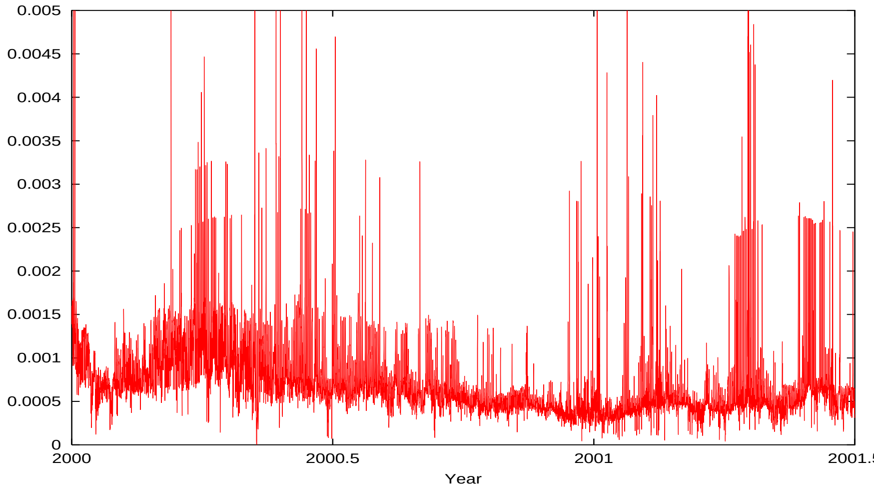

FIGURE 5. Normalised MCI in denomination of NEPOOL electricity prices $Y_{EP_{USD}}(t)$.

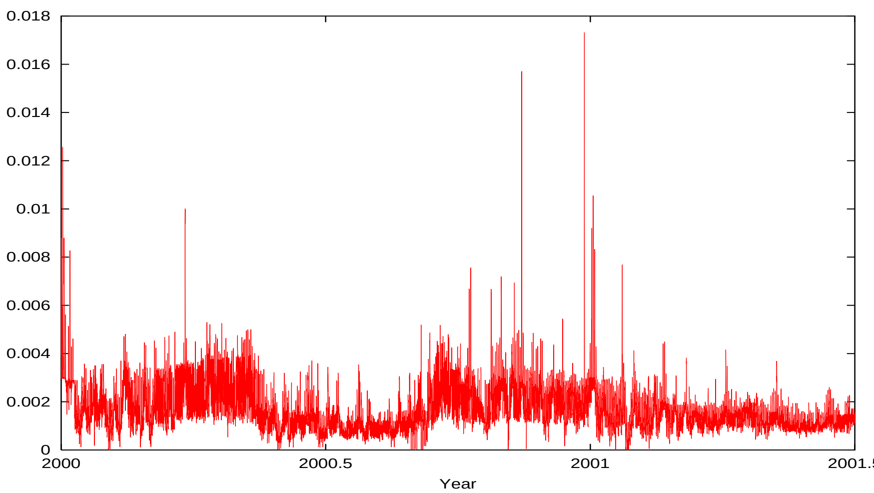

FIGURE 6. Normalised MCI in denomination of NSW electricity prices $Y_{EP_{AUD}}(t)$.

of the MCI in USD or AUD, and the growth implicit in electricity prices related to inflation. The evolution of the normalised MCI $Y_i(t),\ i \in EP$, at time $t \in [0, T]$ is shown in Figure 5 for the denomination in NEPOOL electricity prices, and in Figure 6 for the denomination in NSW electricity prices.

The magnitude of the fluctuations depends on the average level of the normalised MCI $Y_i(t),\ i \in EP$. Periods of high values correspond to periods of large fluctuations and periods of low values correspond to periods of small fluctuations. Taking the square root increases fluctuations when the level of the normalised index is low and decreases fluctuations when the normalised index is high. Therefore, to alter the fluctuations to obtain prices of a similar size, we consider the square root $\sqrt{Y_i(t)}$ of the normalised MCI for $i \in EP$. This allows the fluctuations to be reasonably similar for both high and low levels. The quadratic variation is the sum of the squares of the increments of the process in question when the time discretisation step size vanishes. Its slope provides information about its fluctuations. Figures 7 and 8 illustrate the quadratic variation $\langle\sqrt{Y_i}\rangle_t$ of the square root of the normalised MCI for units of the electricity price for NEPOOL and NSW spot prices, respectively. The graphs in Figures 7 and 8 appear to be relatively smooth, apart from some small periods of major changes in the slope, particularly for the NEPOOL data. They are roughly proportional to a quarter of the time that has elapsed. However, we note that the slope changes in a possibly annually oscillating fashion. The quadratic variation related to NEPOOL prices illustrated in Figure 7, demonstrates the feature of seasonality in a more obvious way than the seasonality evident from the quadratic variation of NSW prices. The New England region in the US experiences far greater extremes in air temperatures than NSW in Australia, which explains the greater degree of seasonality in NEPOOL electricity price fluctuations relative to those of NSW electricity prices.

The slopes of the curves in Figures 7 and 8 are greater during periods of higher average fluctuations, than the slope is during periods with significantly more stable behaviour, and this property exists throughout the data set. Linear regression shows an R^2 value of 0.9584 for the quadratic variation $\langle\sqrt{Y_{EP_{USD}}(t)}\rangle_t$, and an R^2 value of 0.9654 for the quadratic variation $\langle\sqrt{Y_{EP_{AUD}}(t)}\rangle_t$, confirming that it is reasonable to assume linearity, as first approximation. One could make the growth rate η_i time dependent and seasonal, which would allow the curves in Figures 7 and 8 to become almost perfectly straight lines. However, this would need a much longer data set or more detailed modelling to be reasonably founded.

3. A Numeraire Approach

According to [14] the growth optimal portfolio (GOP) can be interpreted as a diversified accumulation index, representative of the best performing long term investment policy. It maximises the expected logarithm of terminal wealth and thus the expected portfolio growth rate. In the very long term, the growth optimal policy almost surely provides higher wealth than alternative strategies [11].

Consider a sequence of market models indexed by a number $d \in \{1, 2, ...\}$. For a given value of d the corresponding financial market model comprises $d + 1$ primary securities. For a given strategy δ, it is convenient to introduce the jth proportion $\pi_\delta^{(j)}$ of the value $S_\delta^{(j)}$ of the corresponding strictly positive portfolio that is invested at time $t \in [0, T]$ in the jth primary security account, $j \in \{0, 1, ..., d\}$ and $d \in \{1, 2, ...\}$. This proportion or fraction is given by the expression

$$\pi_\delta^{(j)}(t) = \delta^{(\delta)}(t)\frac{S_d^{(j)}(t)}{S_d^{(\delta)}(t)}, \tag{3.1}$$

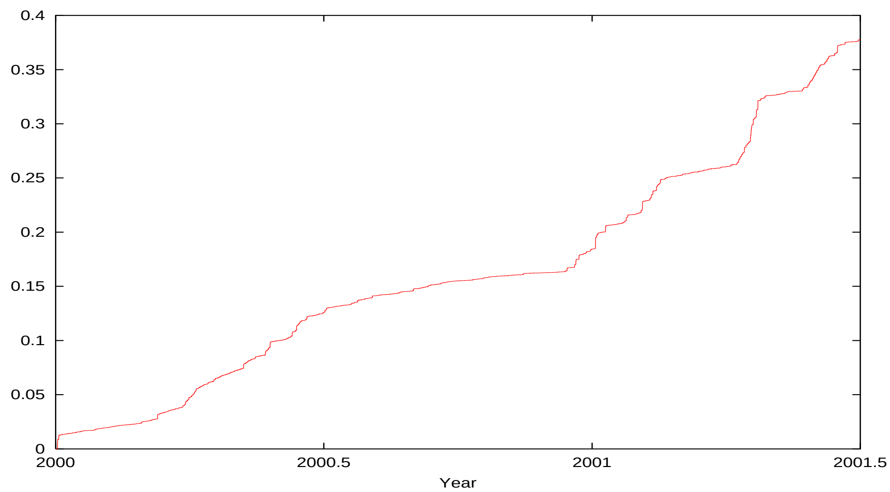

FIGURE 7. Quadratic variation $\langle\sqrt{Y_{EP_{USD}}}\rangle_t$ of the square root of the normalised MCI for NEPOOL electricity prices.

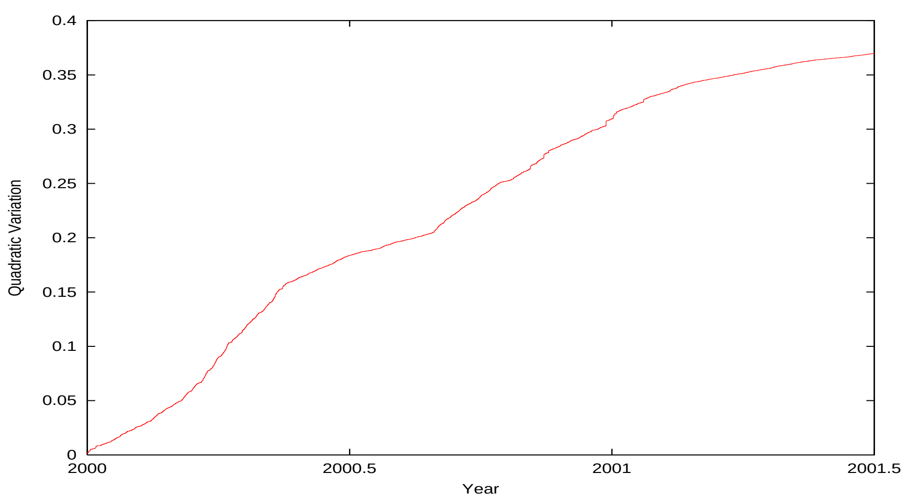

FIGURE 8. Quadratic variation $\langle\sqrt{Y_{EP_{AUD}}}\rangle_t$ of the square root of the normalised MCI for NSW electricity prices.

for $t \in [0,T]$ and $j \in \{0,1,...,d\}$. Note that the proportions always sum to one, that is

$$\sum_{j=0}^{d} \pi_\delta^{(j)}(t) = 1, \tag{3.2}$$

for $t \in [0,T]$.

In order to approximate the GOP using a diversified portfolio we formulate the following definitions.

Definition 3.1. A strictly positive portfolio process $S^{(\delta)}$ is called a *diversified portfolio (DP)* if finite constants $K_1 > 0$, $K_2 > 0$ and $K_3 \in \{1, 2, ...\}$ exist, independent of d, such that

$$|\pi_\delta^{(j)}| \leq \frac{K_1}{d^{\frac{1}{2}+K_2}}, \tag{3.3}$$

almost surely for all $j \in \{0, 1, ..., d\}$, $d \in \{K_3, K_3 + 1, ...\}$ and $t \in [0, T]$.

This means that the proportion $\pi_\delta^{(j)}$ of the value of a DP, which is invested in the jth primary security account, need to decrease slightly faster than $d^{-\frac{1}{2}}$ as $d \to \infty$. This is, for instance, the case if equal proportions are used.

The key pricing features of the financial market have been characterised via the GOP where a model for the dynamics of the GOP has been suggested. In this paper we apply these results to the case of electricity which is a commodity. Instead of referring to the GOP in units of dollars, we express it in units of electricity and analyse its dynamics. More specifically, we interpret the MCI when expressed at time t in units of electricity $S_i^{(MCI)}(t),\ i \in EP$, as being the GOP that is measured in units of electricity. This has consequences for the structure of the dynamics of the value $S_i^{(MCI)}(t)$.

When we express a given portfolio $S^{(\delta)}(t)$ in units of the GOP, then we call the ratio

$$\hat{S}^{(\delta)}(t) = \frac{S^{(\delta)}(t)}{S^{(\delta_*)}(t)}, \tag{3.4}$$

the corresponding *benchmarked portfolio*. By application of the Itô formula and using 3.3 it follows that the benchmarked portfolio $\hat{S}^{(\delta)}(t)$ satisfies the SDE

$$d\hat{S}^{(\delta)}(t) = -\hat{S}^{(\delta)} \sum_{k=1}^{d} \sum_{j=0}^{d} \pi_\delta^{(j)}(t)\sigma^{j,k}(t) dW^k(t) \tag{3.5}$$

with jth *specific volatility*

$$\sigma^{j,k}(t) = b^{j,k}(t) - \theta^k(t) \tag{3.6}$$

for $t \in [0, T]$, $j \in \{0, 1, ..., d\}$ and $k \in \{1, 2, ..., d\}$. This formulation permits specific volatility to be negative under some conditions.

This allows us to introduce the kth *total specific volatility*

$$\hat{\sigma}^k(t) = \sum_{j=0}^{d} |\sigma^{j,k}(t)| \tag{3.7}$$

for $t \in [0, T]$ and $k \in \{1, 2, ..., d\}$.

From 3.5 the GOP value $S_i^{(MCI)}(t)$ defined in 2.1 satisfies the stochastic differential equation (SDE)

$$dS_i^{(MCI)}(t) = S_i^{(MCI)}(t) \left[r_i(t)dt + \theta_i(t)(\theta_i(t)dt + dW_i(t)) \right], \tag{3.8}$$

for $i \in EP$ and $t \in [0, T]$. Here $W_i = \{W_i(t),\ t \in [0, T]\}$ is a standard Wiener process on a given probability space. This representation is standard for contingent claim modelling in financial markets. The volatility $\theta_i(t)$ is the market price for risk with respect to W_i. If the electricity price is interpreted as a currency, then $r_i(t)$ refers to its interest rate. In the case of the given commodity, we call this the *electricity short rate*, which is technically analogous as the short term interest rate is for currencies. In the SDE (3.8) the risk premium is the square of the volatility.

Definition 3.2. A benchmark model is called *regular* if there exist finite constants K_3 and K_4, independent of d, such that

$$E((\hat{\sigma}^k(t))^2) < K_4 \tag{3.9}$$

for all $t \in [0,T]$, $k \in \{1,2,...,d\}$ and $d \in \{K_3, K_3+1,...\}$.

This is a property that can be assumed to be represented by the world stock market consisting of all stocks traded on existing exchanges. The difference between the logarithms of the GOP $S^{(\delta_*)}(t)$ and a given strictly positive portfolio $S^{(\delta)}(t)$ satisfies the SDE

$$d(\log(S^{(\delta_*)}(t)) - \log(S^{(\delta)}(t))) = \frac{1}{2}R^d_\delta(t)dt - \sum_{k=1}^{d}\sum_{j=0}^{d}\pi^{(j)}_\delta(t)\sigma^{j,k}(t)dW^k(t) \tag{3.10}$$

with *tracking rate*

$$R^d_\delta(t) = \sum_{k=1}^{d}(\sum_{j=0}^{d}\pi^{(j)}_\delta(t)\sigma^{j,k}(t))^2 \tag{3.11}$$

for all $t \in [0,T]$. The tracking rate equals the squared diffusion coefficient of the SDE 3.10. It can be interpreted as a measure of the distance between a given portfolio $S^{(\delta)}(t)$ and the GOP $S^{(\delta_*)}(t)$ at time $t \in [0,T]$.

Definition 3.3. For an increasing number d of risky primary security accounts we call a strictly positive portfolio $S^{(\delta)}$ *an approximate GOP* if the corresponding sequence of tracking rates $(R^d_\delta(t))_{d\in\{1,2,...\}}$ vanishes in probability, that is for each $\epsilon > 0$ we have

$$\lim_{d\to\infty} P(R^d_\delta(t)) > \epsilon) = 0 \tag{3.12}$$

for all $t \in [0,T]$.

Under the above assumptions we propose the following limit theorem.

Theorem 3.4. *For a regular benchmark model a diversified portfolio is an approximate GOP.*

Proof. We estimate by using 3.11, 3.3 and 3.9 for a DP $S^{(\delta)}$ in a regular benchmark model its expected tracking rate. That is,

$$e^d_\delta(t) = E(R^d_\delta(t)) \le \sum_{k=1}^{d} E((\sum_{j=0}^{d}|\pi^{(j)(t)}_\delta||\sigma^{j,k}(t)|)^2) \tag{3.13}$$

$$\le \sum_{k=1}^{d}(\frac{(K_1)^2}{d^{1+2K_2}}K_4) \tag{3.14}$$

$$\le (K_1)^2 K_4 d^{-2K_2} \tag{3.15}$$

for $t \in [0,T]$ where $d \in \{K_3, K_3+1, ...\}$. Consequently, since $K_2 > 0$ it follows by the Markov inequality for any given $\epsilon > 0$ that

$$\lim_{d\to\infty} P(R^d_\delta(t)) > \epsilon) \leq \lim_{d\to\infty} \frac{1}{\epsilon} e^d_\delta(t) = 0 \tag{3.16}$$

for all $t \in [0,T]$. □

This proves by Definition 3.3 the Proposition 3.4. This result allows us to conclude that a world stock portfolio that is a DP approximates the GOP and justifies the use of the diversified portfolio to benchmark electricity prices in this analysis as discussed in Section 2.2.

3.1. Discounted GOP. Let us discount the GOP value $S_i^{(MCI)}(t)$ in ith electricity units at time t, see (3.8), by the *electricity price savings account* value

$$S_i^{(0)}(t) = \exp\left\{\int_0^t r_i(s)ds\right\}, \tag{3.17}$$

at time $t \in [0,T]$ for $i \in EP$, where electricity is theoretically accrued at the electricity short rate $r_i(t)$. Let us discount the GOP value $S_i^{(MCI)}(t)$, see (3.8), at time t by the electricity savings account $S_i^{(0)}(t)$. Then the *discounted* GOP

$$\bar{S}_i^{(MCI)}(t) = \frac{S_i^{(MCI)}(t)}{S_i^{(0)}(t)}, \tag{3.18}$$

satisfies by application of the Itô formula, (3.8) and (3.17) the SDE

$$d\bar{S}_i^{(MCI)}(t) = \bar{S}_i^{(MCI)}(t)\theta_i(t)(\theta_i(t)dt + dW_i(t)), \tag{3.19}$$

for $t \in [0,T]$ and $i \in EP$.

By (3.19), using the *discounted GOP drift*

$$\alpha_i(t) = \bar{S}_i^{(MCI)}(t)(\theta_i(t))^2, \tag{3.20}$$

as a parameter process leads to a GOP volatility at time t of the form

$$\theta_i(t) = \sqrt{\frac{\alpha_i(t)}{\bar{S}_i^{(MCI)}(t)}}. \tag{3.21}$$

We consider a rather short time period. Therefore we assume, for simplicity, $r_i(t) = 0$ for $t \in [0,T]$ and $i \in EP$. Similar to the normalisation function $\bar{\alpha}_i(t)$ in (2.2) we model the discounted GOP drift in the form

$$\alpha_i(t) = \bar{\alpha}_i(t)m_i(t), \tag{3.22}$$

for $i \in EP$ and $t \in [0,T]$. Here $m_i = \{m_i(t),\ t \in [0,T]\}$ denotes the nonnegative ith *market activity* process that has an average value of one for $i \in EP$ that will be further specified in Section 3.2. The numeraire-denominated price processes are therefore normalised so that the GOP is aligned with electricity price processes.

To derive a value for the ith net growth rate η_i, several concerns must be addressed. Electricity, as a commodity, is a security that theoretically has a time value. An income can be obtained from lending this commodity since there is value in being able to make the commodity available over certain periods. We avoid particular detailed assumptions regarding the nature of the growth rate or the electricity short rate. These appear as natural quantities in our setup. The notion of a net convenience yield, separated into gross convenience and cost of carry in our setting appears to be unnecessary.

3.2. Market Activity. Based on the market activity $m_i(t)$, the ith *market activity time* $\psi_i = \{\psi_i(t),\ t \in [0,T]\}$ can be defined as

$$\psi_i(t) = \int_0^t m_i(s)ds, \tag{3.23}$$

for $t \in [0,T]$ and $i \in EP$. In our case $t = 0$ years corresponds to the starting date of our sample 01/01/2000 00:00:00 for Greenwich Mean Time (GMT). Furthermore, we ensure that on average the market activity time scale elapses approximately as fast as physical time. Approximately $\psi_i(T) \approx 1.5$ will turn out to be equivalent to 30/06/2001 00:00:00 GMT, the terminal date of both sets of data available. To be precise, we assume that asymptotically we have in our model the property that

$$\lim_{T\to\infty} \frac{1}{T} E\left(\psi_i(T)\right) = 1. \tag{3.24}$$

The normalised MCI $Y_i = \{Y_{i,\psi},\ \psi \in [t, \psi_i(T)]\}$ can be conveniently expressed in market activity time and is then obtained as

$$Y_{i,\psi_i(t)} = Y_i(t) = \frac{\bar{S}_i^{(MCI)}(t) m_i(t)}{\alpha_i(t)} = \frac{\bar{S}_i^{(MCI)}(t)}{\bar{\alpha}(t)}, \tag{3.25}$$

for $i \in EP$. Let us introduce the ith market activity in ith activity time, that is

$$m_{i,\psi_i(t)} = m_i(t), \tag{3.26}$$

for $t \in [0, \psi_i(T)]$ and $i \in EP$. It is has been shown via the Itô formula, and using (3.19) and (3.22) in [15], that (3.25) in ith market activity time, satisfies the SDE

$$dY_{i,\psi} = \eta_i\left(\frac{1}{\eta_i} - \frac{Y_{i,\psi}}{m_{i,\psi}}\right)d\psi + \sqrt{Y_{i,\psi}}\, dW_{i,\psi}, \tag{3.27}$$

for $i \in EP$ and $\psi \in [0, \psi_i(T)]$, where

$$dW_{i,\psi_i(t)} = \sqrt{m_i(t)} dW_i(t), \tag{3.28}$$

for $t \in [0,T]$. In the case of $m_{i,\psi} = 1$, the normalised MCI in market activity time, given in (3.25), is a *square root process of dimension four*. In this case, the solution of (3.27) has a long term mean of $\frac{1}{\eta_i}$ and a speed of adjustment parameter η_i. The only parameter that is then relevant in (3.27) is the growth rate η_i, which turns out to be the key growth parameter for the price of electricity.

If we consider the square root of the normalised MCI, then by (3.27) and application of the Itô formula, it evolves according to the SDE

$$d(\sqrt{Y_{i,\psi}}) = \left(\frac{3}{8\sqrt{Y_{i,\psi}}} - \frac{\eta_i}{2} \frac{\sqrt{Y_{i,\psi_i}}}{m_{i,\psi}}\right) d\psi + \frac{1}{2} dW_{i,\psi}, \tag{3.29}$$

for $i \in EP$ and $\psi \in [0, \psi_i(T)]$. It is crucial to note that the diffusion coefficient in (3.29) is constant. Therefore, we obtain in market activity time the quadratic variation of $\sqrt{Y_i}$ in the form

$$\langle\sqrt{Y_i}\rangle_\psi = \frac{\psi}{4}, \tag{3.30}$$

for $i \in EP$ and $\psi \in [0, \psi_i(T)]$. Relation (3.30) holds under general circumstances since no major restrictive assumptions on the actual dynamics of $\bar{S}_i^{(MCI)}$ have been made.

From the market activity time ψ_i given in (3.23) and the quadratic variation of $\sqrt{Y_i}$ in (3.30), we formulate the following result.

Corollary 3.5. *The ith market activity can be calculated as the time derivative*

$$m_i(t) = \frac{d\psi_i(t)}{dt} = 4\frac{d\langle\sqrt{Y_i}\rangle_t}{dt}, \tag{3.31}$$

for $i \in EP$ and $t \in [0, T]$. This shows that the discounted GOP drift is equal to four times the slope of the quadratic variation of the square root of the discounted GOP.

This implies that market activity is directly observable as a time derivative. One needs only to measure the slope of the quadratic variation of the square root of the normalised MCI.

For the intraday observed normalised MCI $Y_i^{(MCI)}(t)$, in units of NEPOOL and NSW electricity, the quadratic variation of its square root has been shown in Figures 7 and 8, respectively. The slope of these graphs at a given time provides by (3.31), a quarter of the corresponding market activity. This allows us to calculate the market activity directly. We simply calculate the numerical derivative corresponding to (3.31) using hourly time steps for the NEPOOL electricity prices and half hourly time steps for NSW electricity prices, and multiply this by four. In Figure 9 we plot the resulting market activity for the NEPOOL series. The market activity fluctuates over a wide range. Therefore, we show in Figure 10 the logarithm $\log(m_i(t))$ of this derivative over a few weeks in April/May 2000 for the NEPOOL series. The NSW electricity series results are similar. It appears that the observed market activity processes show some seasonal patterns and display the characteristic of reverting quickly back to a reference level. It also distinctly shows periods of market inactivity, which typically occur at night when minimal variation in electricity usage is usually experienced.

The quadratic variation of the logarithm of market activity $\langle\log(m_i)\rangle_t$, calculated as a rolling time average over the scenarios, is shown in Figures 11 and 12, which demonstrates that there is minimal seasonal pattern emerging from market activity over the long term. Note that the average slope of the quadratic variation in both Figure 11 and Figure 12 is close to one.

We show in Figures 13 and 14 the quadratic variation of the logarithm of market activity examined at a higher resolution for both the NEPOOL and NSW electricity markets. There is, of course, a distinct *intraday seasonal pattern* which will be discussed in Section 4 below. Figures 13 and 14 illustrate that nights are characterised by a plateau, which means low activity volatility. Notably, despite the seasonal pattern, the graphs in Figures 13 and 14 appear to be roughly linear for the periods when the market is actively trading. The deseasonalisation of the

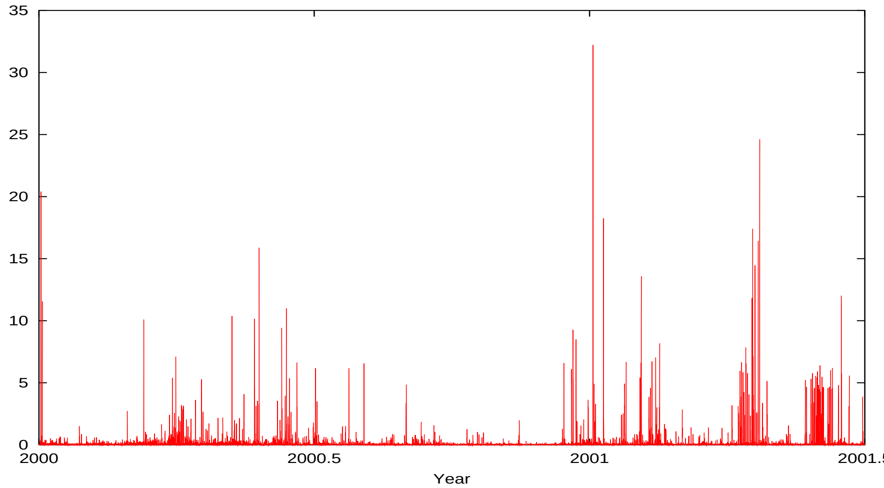

FIGURE 9. Market activity $m_{EP_{USD}}(t)$ for NEPOOL electricity prices.

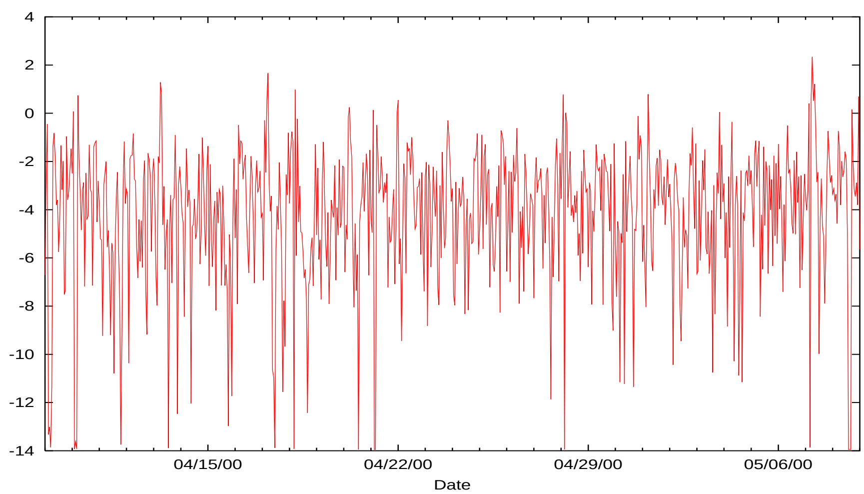

FIGURE 10. Logarithm $\log(m_{EP_{USD}}(t))$ of NEPOOL market activity for April - May 2000.

market activity $m_i(t)$ requires a two stage procedure. The first step deseasonalises the average of the market activity and the second its volatility. This will be outlined below.

4. Model for Market Activity

4.1. Activity Volatility. Figures 11 and 12 suggest that due to the approximate linearity of the quadratic variation $\langle \log(m_i) \rangle_t$, market activity is likely to have

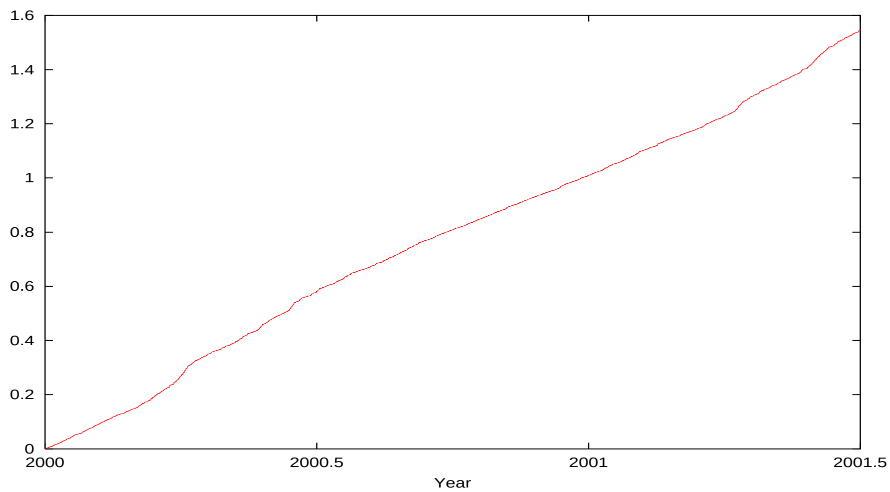

FIGURE 11. Quadratic variation of the logarithm of market activity for NEPOOL $\langle \log(m_{EP_{USD}}) \rangle_t$.

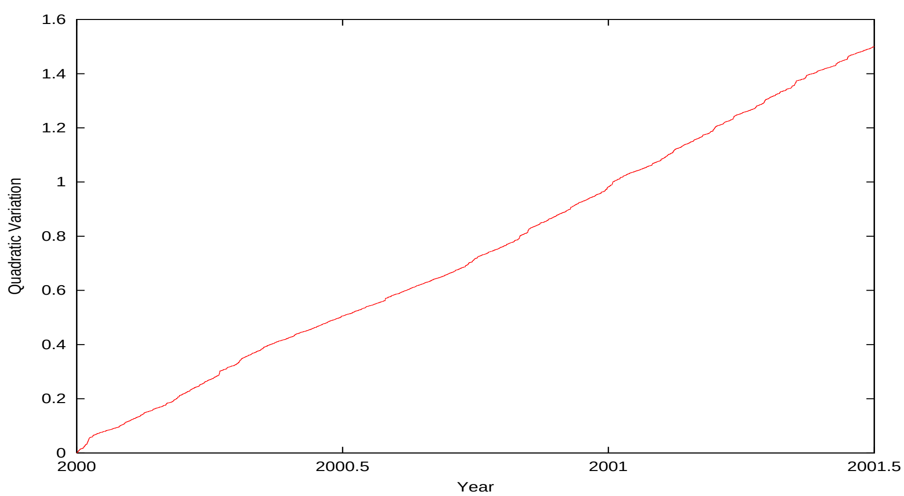

FIGURE 12. Quadratic variation of the logarithm of market activity for NSW $\langle \log(m_{EP_{AUD}}) \rangle_t$.

multiplicative noise. In this paper, we propose a linear mean reverting model for the market activity $m_i(t)$ of electricity that accounts for this feature. It is given by the SDE

$$dm_i(t) = \kappa_i \beta_i^2(t)(\bar{m}_i(t) - m_i(t))dt + \beta_i(t) m_i(t) d\bar{W}_i(t), \tag{4.1}$$

for $i \in EP$ and $t \in [0, T]$, with deterministic *speed of adjustment parameter* $\kappa_i > 0$, *reference market activity* $\bar{m}_i(t) \geq 0$ and *activity volatility* $\beta_i(t) > 0$. Here,

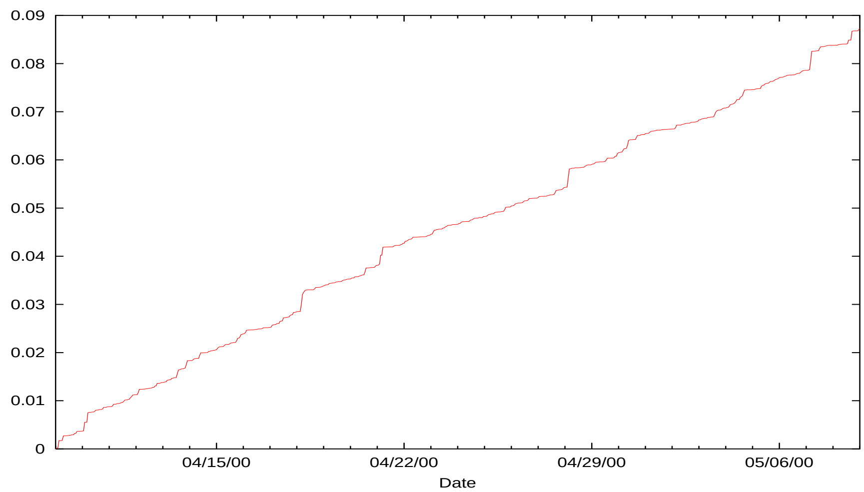

FIGURE 13. Quadratic variation of the logarithm of market activity for NEPOOL $\langle \log(m_{EP_{USD}}) \rangle_t$, April - May 2000.

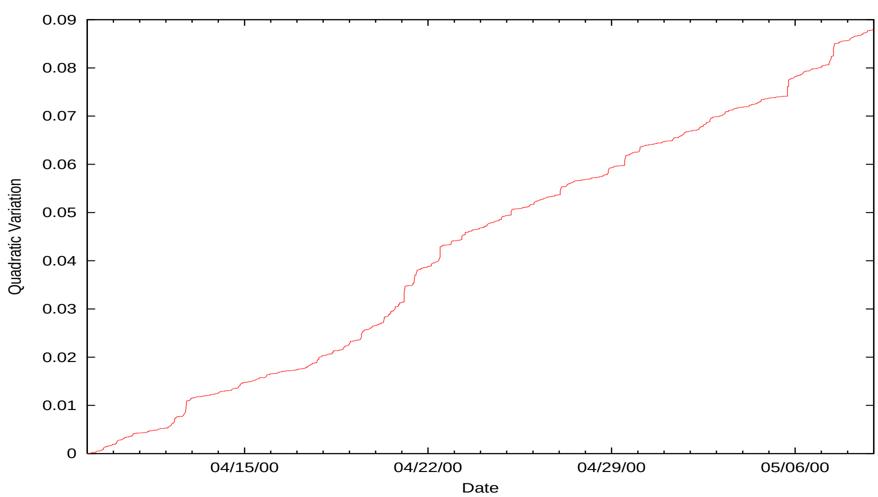

FIGURE 14. Quadratic variation of the logarithm of market activity for NSW $\langle \log(m_{EP_{AUD}}) \rangle_t$, April - May 2000.

$\bar{W} = \{\bar{W}_i(t),\ t \in [0,T]\}$ for $i \in EP$ is an independent standard Wiener process. The reference market activity $\bar{m}_i(t)$ and the activity volatility $\beta_i(t)$ are assumed to exhibit some deterministic seasonal pattern.

Let us also introduce the *expected market activity* $\hat{m}_i(t)$ at time $t \in [0,T]$ as the expectation

$$\hat{m}_i(t) = E(m_i(t)) \tag{4.2}$$

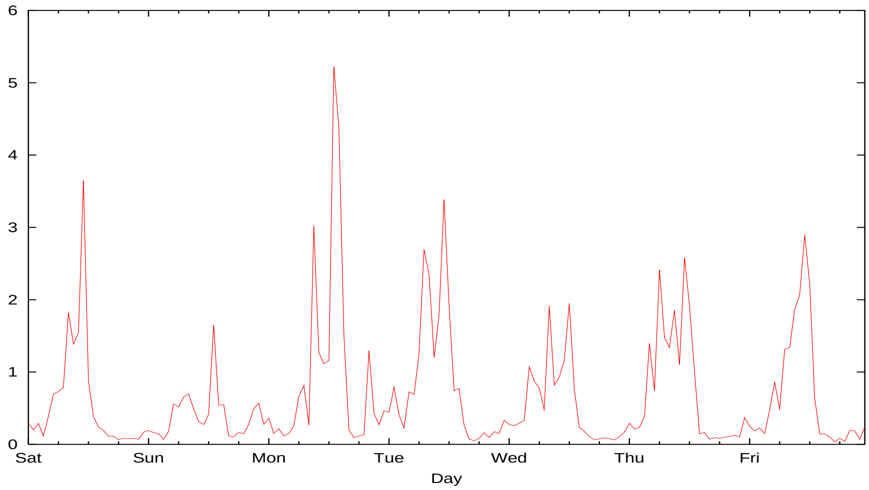

FIGURE 15. Weekly seasonal pattern of expected market activity $\hat{m}_{EP_{USD}}(t)$ for NEPOOL.

for $t \in [0, T]$ and $i \in EP$. To deseasonalise $m_i(t)$, we estimate the expected market activity $\hat{m}_i(t)$ by simply using the Law of Large Numbers for each observation time of the week during the full observation period for NEPOOL and NSW data, respectively. Figures 15 and 16 illustrate the observed weekly pattern in the average market activity $\hat{m}_i(t)$ for $i \in EP$. It indicates that, on average, market activity reaches two distinct peaks during weekdays and one peak on weekends for both NEPOOL and NSW electricity markets. This pattern is very similar over all seasons of the year. We also see that nights display almost no market activity, as would be expected.

4.2. Activity Volatility Time. The deterministic seasonal ith activity volatility $\beta_i(t)$ allows us to introduce the ith *activity volatility time* $\tau_i = \{\tau_i(t),\, t \in [0, T]\}$ as

$$\tau_i(t) = \langle \log(m_i) \rangle_t = \int_0^t (\beta_i(u))^2 du \tag{4.3}$$

for $t \in [0, T]$ and $i \in EP$ and. Again, as with market activity time, the activity volatility time (4.3) requires initialisation, which we have simply set $\tau_i(0) = 0$.

Figures 17 and 18 show the average weekly activity volatility series. Public holidays were counted as Sundays since the behaviour of electricity prices on public holidays closely resembles, on average, the behaviour of electricity prices on Sundays. This shows that the average activity volatility for weekdays is different from the average activity volatility on weekends. It seems that electricity demand and prices appear, in general, to be more predictable on weekdays than on weekends, as reported for the case of Australia in [5]. This feature results in a more stable activity volatility pattern emerging during the week, which is illustrated in Figures 17 and 18.

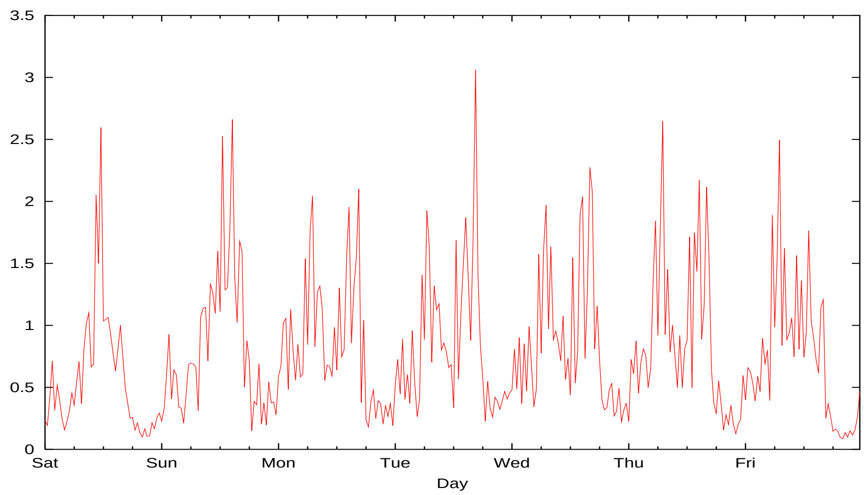

FIGURE 16. Weekly seasonal pattern of expected market activity $\hat{m}_{EP_{AUD}}(t)$ for NSW.

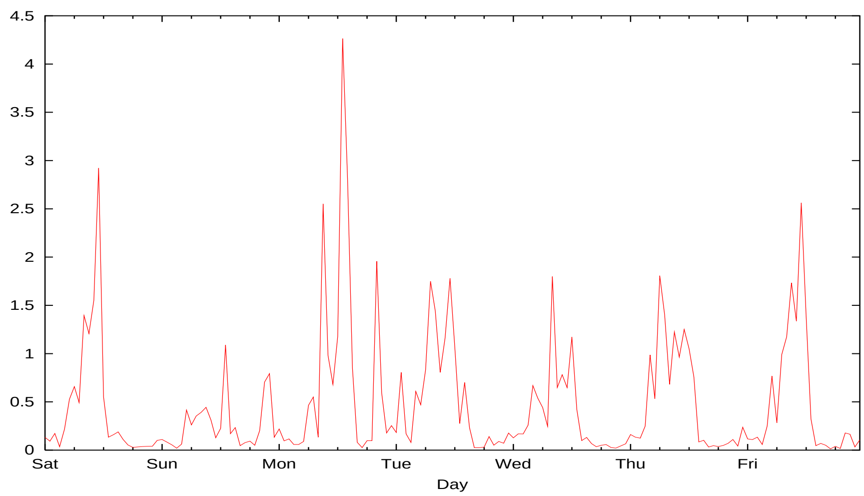

FIGURE 17. Average weekly pattern of activity volatility $\beta_{USD}(t)$ for NEPOOL.

To extract the reference market activity $\bar{m}_i(t)$ it is useful to consider the market activity in activity volatility time $\tau_i(t)$. When we denote the market activity, reference market activity and average market activity in activity volatility time, by $m_{i,\tau_i(t)} = m_i(t)$, $\bar{m}_{i,\tau_i(t)} = \bar{m}_i(t)$ and $\hat{m}_{i,\tau_i(t)} = \hat{m}_i(t)$ respectively, we obtain by (4.1) and (4.3) the SDE

$$dm_{i,\tau} = \kappa_i(\bar{m}_{i,\tau} - m_{i,\tau})d\tau + m_{i,\tau}d\bar{W}_{i,\tau}, \tag{4.4}$$

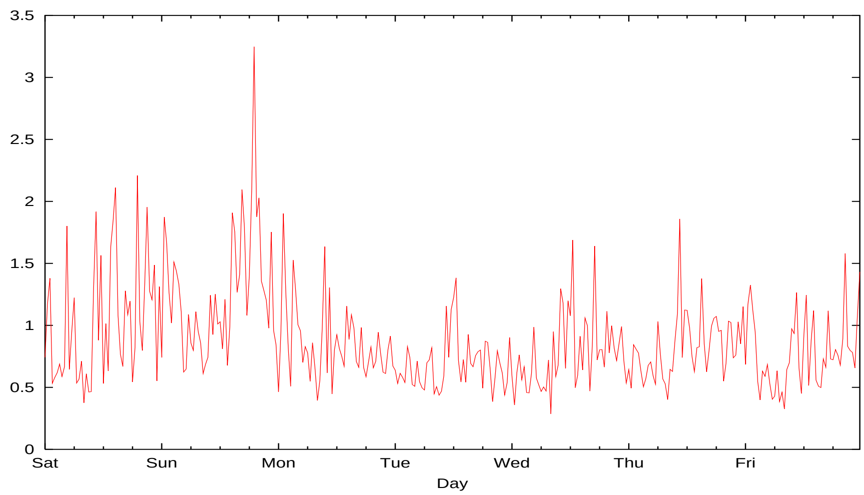

FIGURE 18. Average weekly pattern of activity volatility $\beta_{AUD}(t)$ for NSW.

which progresses in units of $\tau \in [0, \tau_i(T)]$, where

$$d\bar{W}_{i,\tau_i(t)} = \beta_i(t) d\bar{W}_i(t), \tag{4.5}$$

for $t \in [0, T]$ and $i \in EP$.

By taking expectations on both sides of the SDE (4.4) we obtain by (4.2) the ordinary differential equation

$$d\hat{m}_{i,\tau} = \kappa_i(\bar{m}_{i,\tau} - \hat{m}_{i,\tau}) d\tau, \tag{4.6}$$

for $t \in [0, \tau_i(T)]$ and thus the reference level for the market activity in the form

$$\bar{m}_{i,\tau} = \frac{1}{\kappa_i} \frac{d\hat{m}_{i,\tau}}{d\tau} + \hat{m}_{i,\tau}, \tag{4.7}$$

for $\tau \in [0, \tau_i(T)]$ and $i \in EP$. Below we will estimate the speed of adjustment parameter κ_i for $i \in EP$. By (4.7) this gives, together with the average market activity estimated in Figures 15 and 16, an estimate for the reference level $\bar{m}_i(t) = \bar{m}_{i,\tau_i(t)}$. It is clear from (4.7) that we require the derivative of the expected market activity with respect to activity volatility time. Provided that expected market activity is relatively smooth, the reference market activity can be calculated.

From (4.4) and (4.6) and by application of the Itô formula, the logarithm of ith market activity $\log(m_{i,\tau})$ in ith activity volatility time satisfies the SDE

$$d\log(m_{i,\tau}) = \left(\frac{1}{m_{i,\tau}} \frac{d\hat{m}_{i,\tau}}{d\tau} + \frac{\kappa_i \hat{m}_{i,\tau}}{m_{i,\tau}} - \kappa_i - \frac{1}{2} \right) d\tau + d\bar{W}_{i,\tau}, \tag{4.8}$$

for $\tau \in [0, \tau_i(T)]$ and $i \in EP$. We therefore have a nonlinear drift term and a constant diffusion term for the SDE of the logarithm of market activity $\log(m_{i,\tau})$ in activity volatility time.

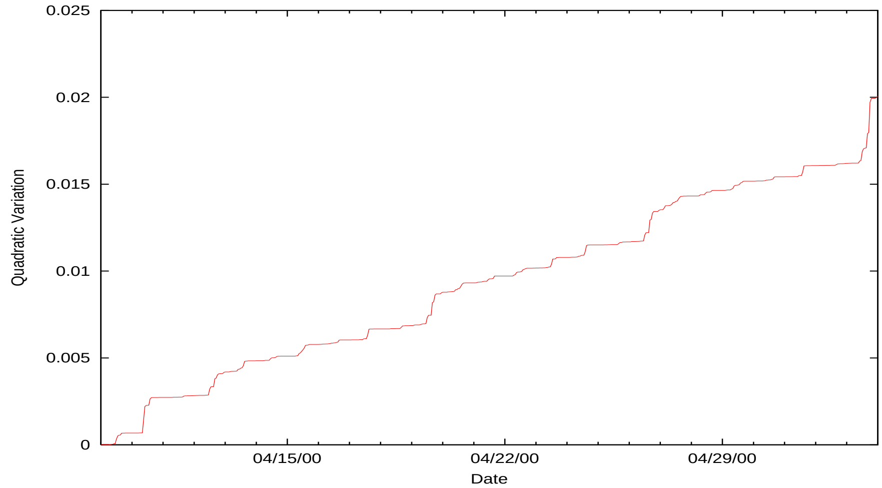

FIGURE 19. Quadratic variation of the logarithm of market activity $\langle \log(m_{EP_{USD}}) \rangle_\tau$ for NEPOOL in activity volatility time.

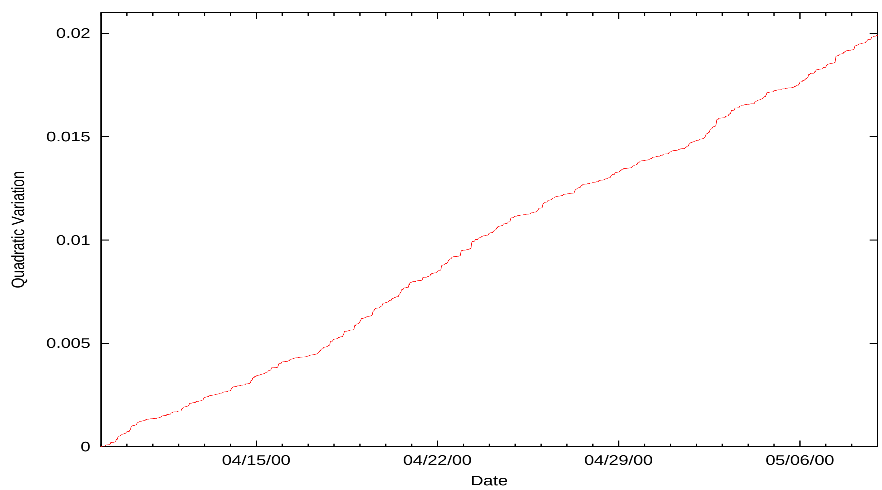

FIGURE 20. Quadratic variation of the logarithm of market activity $\langle \log(m_{EP_{AUD}}) \rangle_\tau$ for NSW in activity volatility time.

The quadratic variation of the logarithm of market activity for the NEPOOL and NSW electricity markets is shown in Figures 19 and 20 in the respective activity volatility time. The higher frequency of NSW data compared with the NEPOOL data translates into a smoother quadratic variation curve as is evident in these figures. The relative linearity of these curves supports the presence of multiplicative noise, which is assumed in the SDE (4.1) for market activity. Theoretically, this leads by (4.8) to $\langle \log(m_i) \rangle_\tau = \tau$. In this paper we interpret the average weekly activity volatility, shown in Figures 17 and 18, as activity volatility $\beta_i(t)$ for each week of the period considered.

4.3. Estimation of Speed of Adjustment. The only parameter in (4.1) remaining to be estimated is the speed of adjustment parameter $\kappa_i,\ i \in EP$, that controls the average strength of mean reversion. Figure 21 shows the histogram of the logarithm $l_{i,\tau} = \log(m_{i,\tau})$ of the market activity for the NEPOOL electricity market, $i \in EP$. As shown, a concentration of negative spikes at $\log(m_{EP_{USD},\tau}) \approx -14$ exists in Figure 21 respectively, due to the opening and closing effects that occur around the relatively benign activity level typical for each night. As in [3], we shall exclude this distortion from our analysis by forming a restricted log-likelihood function, where only the values for $\log(m_{EP_{USD},\tau}) > -10$ and $\log(m_{EP_{AUD},\tau}) > -15$ for each histogram, respectively, are considered.

We require a robust estimation technique to estimate κ_i. Under the simplifying assumption that $\bar{m}_{i,\tau} = 1,\ i \in EP$, the market activity process can be shown to have as a stationary density, an inverse gamma density with a scale parameter of 1. The stationary transition density $\bar{p}(l;\kappa_i)$ of the logarithm of market activity in activity volatility time can therefore be written as

$$\bar{p}(l;\kappa_i) = \frac{(\kappa_i)^{\kappa_i}}{\Gamma(\kappa_i)} \exp\left\{-\kappa_i e^{-l}\right\} e^{-l(\kappa_i - 1)}, \tag{4.9}$$

for $i \in EP$.

A maximum likelihood technique with the above restriction was applied. A plot of the estimated probability density function of $\log(m_{i,\tau})$, based on the resulting maximum likelihood estimate of κ_i, is shown for the US data in Figures 21. The Australian results are largely similar. We estimated $\kappa_{EP_{USD}}$ to be about $\hat{\kappa}_{EP_{USD}} = 11.4$ with a 99% confidence interval of $(10.8, 14.5)$, and $\kappa_{EP_{AUD}}$ to be about $\hat{\kappa}_{EP_{AUD}} = 25.2$ with a 99% confidence interval of $(23.8, 28.5)$. This translates into an expected reversion to the mean with a half life time following a shock of about 22 days for NEPOOL electricity prices and about 10 days for NSW electricity prices.

Maximum likelihood estimates for a two parameter $G(l;\kappa_i,\bar{m}_i)$ inverse gamma density function were also obtained. For both the NEPOOL and NSW data, the reference level parameter $\bar{m}_i$ yielded a value of approximately 1, with estimates for the shape parameter κ_i converging close to the above values obtained using a single parameter estimate for the likelihood function of the inverse gamma density, $i \in EP$. This further strengthens the argument for the applicability of the inverse gamma density function as an appropriate fit for the stationary density of the logarithm of market activity, from which a reliable estimate of the speed of adjustment parameter can be obtained.

In Section 3 it was shown that the normalised MCI, when observed in market activity time, resembles a square root process of dimension four. By using market activity time we know that the quadratic variation of its square root should be linear with a slope close to 0.25, see (3.30). This relationship is confirmed by performing a simple linear regression of the quadratic variation of the square root

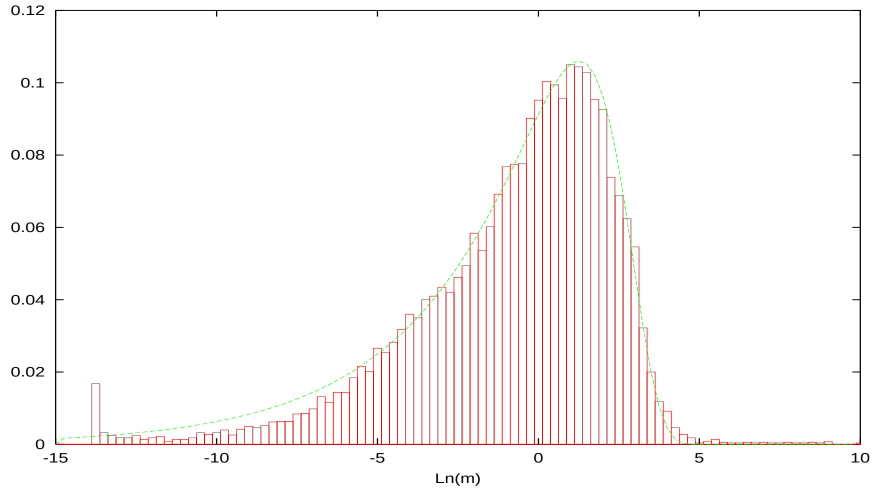

FIGURE 21. Histogram and estimated probability density function of $\log(m_{EP_{USD}}(t))$.

of the normalised MCI against market activity time. The slope coefficient for the NEPOOL electricity market is 0.2531 with corresponding $R^2 = 0.9584$, and for the NSW electricity market, the slope coefficient is 0.2517 with corresponding $R^2 = 0.9985$.

This makes the above derived and calibrated model a largely accurate *intraday* description of electricity prices. Additionally, the corresponding market activity process in activity volatility time is largely shown to have the hypothesised dynamics, when the distorting effects that occur at the close and opening of trading are omitted. On the basis of the above detailed modelling and calibration, it is possible to price electricity derivatives which, however, is beyond the scope of this paper.

5. Conclusion

We have examined the behaviour of an intraday world stock index when expressed in units of electricity prices for two distinct markets. We were able to construct a model for electricity prices that allows for both seasonalities and long term growth. The behaviour of the market activity of electricity prices was inferred through this analysis. A simple way of calculating market activity was subsequently demonstrated. The model for market activity yielded robust results for high frequency electricity price data. This suggests that similar models can possibly be employed for other types of commodities. Market activity is shown to contain seasonal patterns in both the drift and the diffusion term within a coherent format. We showed that the market activity can be modelled as a strongly mean reverting process. Furthermore, we confirmed that the normalised index closely follows a square root process of dimension four in market activity time. Using the techniques outlined in this paper, a method of obtaining derivative prices for electricity will be described in forthcoming work.

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