Dynamical investigation of a manned capsule/tether re-entry system

Jeffrey Alan Kornuta
Louisiana State University and Agricultural and Mechanical College

Follow this and additional works at: https://digitalcommons.lsu.edu/gradschool_theses
Part of the Mechanical Engineering Commons

Recommended Citation
Kornuta, Jeffrey Alan, "Dynamical investigation of a manned capsule/tether re-entry system" (2009). LSU Master's Theses. 183. https://digitalcommons.lsu.edu/gradschool_theses/183

This Thesis is brought to you for free and open access by the Graduate School at LSU Digital Commons. It has been accepted for inclusion in LSU Master's Theses by an authorized graduate school editor of LSU Digital Commons. For more information, please contact gradetd@lsu.edu.
DYNAMICAL INVESTIGATION OF A MANNED CAPSULE/TETHER RE-ENTRY SYSTEM

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

in

The Department of Mechanical Engineering

by

Jeffrey Alan Kornuta
B.S., Louisiana State University in Baton Rouge, Louisiana, 2008
May, 2009
Acknowledgements

I would like to thank Excalibur Almaz for their support in funding this research. I would also like to thank Dr. Leroy Chiao for his mentoring and for helping give me the opportunity to work on such an interesting project.

Special thanks goes to my adviser, Dr. Shengmin Guo, for his immense help and guidance on all aspects of this project. His continued support is very much appreciated and has been critical to my successes as a young researcher.

I would also like to recognize Dr. Robert Hoyt for taking the time to help me grasp several key concepts critical to this project. Furthermore, I would like to thank my committee members as well as the entire LSU Mechanical Engineering faculty for their guidance and support throughout my academic career.

Last but certainly not least, I would like to thank my parents, family, and friends for their encouragement and support. I could not have arrived at this point without them.
# Table of Contents

Acknowledgements ii  
List of Tables v  
List of Figures vi  
Abstract x  

1 Introduction 1  
1.1 Uses for Tethers ................................. 1  
1.1.1 Adjoining Objects in Orbit ..................... 1  
1.1.2 Creating Microgravity Environments .............. 2  
1.1.3 Power Generation & Momentum Exchange .......... 2  
1.2 Project Objective .............................. 3  
1.3 Overview of Thesis ............................. 5  

2 Literature Review 6  
2.1 Tether Deployment ............................. 6  
2.2 Tether System Modeling ......................... 9  

3 Modeling of the Capsule/Tether Re-entry System 13  
3.1 Lagrangian Formulation .......................... 13  
3.1.1 Kinetic Energy Derivation ...................... 15  
3.1.2 Potential Energy Derivation .................... 16  
3.1.3 Generalized forces ............................ 17  
3.2 Numerical Implementation ....................... 22  
3.2.1 Validation Case .............................. 23  
3.2.2 Application of Software ...................... 24  

4 Initial Condition Generation 28  
4.1 Massless, Inelastic Tether  ..................... 28  
4.1.1 Derivation of Equations ...................... 29  
4.1.2 Results & Discussion: Inelastic Swing ......... 30  
4.2 Massless, Elastic Tether  ...................... 36  
4.2.1 Derivation of Equations ...................... 37  
4.2.2 Results & Discussion: Elastic Swing .......... 39  

5 Re-entry Dynamics 47  
5.1 Inelastic Tether Re-entry Validation ............. 47  
5.1.1 Literature Validation ........................ 47  
5.1.2 Lumped Mass Validation ...................... 48  
5.2 Tether Length Comparison ....................... 49  
5.2.1 Constant Initial Conditions .................. 49  
5.2.2 Different (Inelastic) Initial Conditions ...... 49  
5.2.3 Different (Elastic) Initial Conditions ......... 54
5.3 Tether Diameter Comparison ........................................... 61
5.4 Capsule Mass Comparison ............................................. 61

6 Capsule Heating ......................................................... 66
6.1 Aerodynamic Heating Theory ........................................ 66
6.2 Capsule Heating Model ............................................... 67
6.3 Heating Results & Analysis .......................................... 68
  6.3.1 Tether Length Comparison ...................................... 69
  6.3.2 Tether Diameter Comparison ................................... 79
  6.3.3 Capsule Mass Comparison ....................................... 79

7 Conclusion .............................................................. 87
7.1 Summary ............................................................... 87
  7.1.1 Results ............................................................. 88
7.2 Recommendations & Future Work ................................. 89

References ................................................................. 91

Appendix A: Atmospheric Data ......................................... 93
  A.1 Mean Density ....................................................... 93
  A.2 Mean Free Path .................................................... 94

Appendix B: Drag Coefficient Data .................................... 95

Appendix C: Re-entry Simulation Code ............................... 96

Appendix D: Initial Condition Generation Code .................... 100

Appendix E: Heat Transfer Code ....................................... 103

Appendix F: TSS Parameters ............................................ 105

Appendix G: TSS Parameters (Krischke) ............................. 106

Vita .............................................................................. 107
List of Tables

6.1 Capsule layer properties. .................................................. 67
F.1 Properties for the station, capsule, and tether. .......................... 105
G.1 Release parameters and capsule/tether properties (Krischke). ......... 106
## List of Figures

1.1 Various phases of tether deployment & tether release. 4

2.1 Diagram of simple TSS. 6

2.2 Tether modeling diagram. 10

3.1 Diagram of the capsule/tether system model under consideration. 14

3.2 System parameters vs. time for the validation case. 25

3.3 Original software workflow for obtaining numerical results. 26

3.4 Improved software workflow for obtaining numerical results. 27

4.1 Capsule-tether system (dumbbell) model immediately before swing. 28

4.2 Station altitude vs. time during inelastic swing. 31

4.3 Station radial velocity vs. time during inelastic swing. 31

4.4 Capsule altitude vs. time during inelastic swing. 32

4.5 Capsule radial velocity vs. time during inelastic swing. 32

4.6 Angular position, \( \alpha \), vs. time during inelastic swing. 33

4.7 Angular velocity, \( \dot{\alpha} \), vs. time during inelastic swing. 33

4.8 Overall capsule velocity vs. time during inelastic swing. 34

4.9 Capsule altitude and velocity vs. tether length at inelastic release. 35

4.10 Overall \( \Delta v \) vs. tether length at inelastic release. 35

4.11 Elastic capsule-tether system (dumbbell) model immediately before swing. 36

4.12 Station altitude vs. time during elastic swing. 39

4.13 Station radial velocity vs. time during elastic swing. 40

4.14 Tether stretch distance, \( x \), vs. time during elastic swing. 40
4.15 Capsule altitude vs. time during elastic swing. .......................... 41
4.16 Capsule radial velocity vs. time during elastic swing. ............... 41
4.17 Angular position, $\alpha$, vs. time during elastic swing. .............. 42
4.18 Angular velocity, $\dot{\alpha}$, vs. time during elastic swing. ........... 43
4.19 Overall capsule velocity vs. time during elastic swing. .............. 43
4.20 Mean stretch distance, $x_{\text{avg}}$ (red) vs. time during swing of 25 km tether. . . . 44
4.21 Capsule altitude and velocity vs. tether length at elastic release. .... 45
4.22 Mean stretch distance, $x_{\text{avg}}$, vs. tether length at elastic release. . . 46
4.23 Overall $\Delta v$ vs. tether length at elastic release. ...................... 46
5.1 Capsule velocity vs. altitude validation (Krischke). ...................... 47
5.2 Capsule velocity vs. altitude validation, number of links. ............. 48
5.3 Capsule altitude vs. time and tether length, 25 km inelastic IC’s. .... 49
5.4 Capsule altitude vs. longitude and tether length, 25 km inelastic IC’s. . 50
5.5 Capsule velocity vs. time and tether length, 25 km inelastic IC’s. .... 50
5.6 Capsule altitude vs. time and tether length, inelastic IC’s. ............. 51
5.7 Capsule velocity vs. time and tether length, inelastic IC’s. ............. 51
5.8 Capsule altitude vs. time for various tether lengths, inelastic IC’s. .... 52
5.9 Capsule altitude vs. longitude for various tether lengths, inelastic IC’s. . . 53
5.10 Capsule velocity vs. time for various tether lengths, inelastic IC’s. .... 53
5.11 Tether angles, $\alpha_i$, vs. time for various tether lengths, inelastic IC’s. . . 55
5.12 Capsule altitude vs. time and tether length, elastic IC’s. .............. 57
5.13 Capsule velocity vs. time and tether length, elastic IC’s. .............. 57
5.14 Capsule altitude vs. time for various tether lengths, elastic IC’s. ....... 58
5.15 Capsule altitude vs. longitude for various tether lengths, elastic IC’s. . . . . 59
5.16 Capsule velocity vs. time for various tether lengths, elastic IC’s. . . . . . . 60
5.17 Capsule altitude vs. time for various tether lengths and diameters, correspond- ing IC’s. ............................................................. 62
5.18 Capsule velocities vs. time for various tether lengths and diameters, correspond- ing IC’s. ............................................................. 63
5.19 Capsule altitude vs. time and capsule mass, same IC’s. ......................... 64
5.20 Capsule altitude vs. longitude and capsule mass, same IC’s. ................... 64
5.21 Capsule velocity vs. time and capsule mass, same IC’s. ......................... 65
6.1 1-D heat transfer model for the capsule. ................................................. 67
6.2 Convective heat flux vs. time and tether length, 25 km inelastic IC’s. . . . . 69
6.3 Convective heat flux vs. altitude and tether length, 25 km inelastic IC’s. . . 70
6.4 Capsule temperature vs. time, tether length, and material depth; inelastic IC’s. ............................................................. 71
6.5 Capsule temperature vs. altitude, tether length, and material depth; inelas- tic IC’s. ............................................................. 72
6.6 Convective heat flux vs. time and tether length, inelastic IC’s. ................... 73
6.7 Temperature (x = 0) vs. time and tether length, inelastic IC’s. ................. 74
6.8 Temperature (x = h₁/4) vs. time and tether length, inelastic IC’s. .............. 75
6.9 Convective heat flux vs. time and tether length, elastic IC’s. ..................... 76
6.10 Temperature (x = 0) vs. time and tether length, elastic IC’s. ................. 77
6.11 Percent decrease in maximum heat flux from tether drag. ...................... 78
6.12 Percent decrease in maximum temperature from tether drag. ................ 78
6.13 Convective heat flux vs. time and (2 mm) tether length, inelastic IC’s. . . . 80
6.14 Capsule temperature vs. time and (2 mm) tether length, inelastic IC’s. . . . 81
6.15 Decrease in maximum heat flux from a 1 mm and 2 mm tether. . . . . . . . . 82
6.16 Decrease in maximum temperature from a 1 mm and 2 mm tether. . . . . . . 82
6.17 Convective heat flux vs. time and capsule mass, inelastic IC’s. . . . . . . . 83
6.18 Capsule temperature vs. time and capsule mass, inelastic IC’s. . . . . . . . 84
6.19 Decrease in maximum heat flux from tether with capsule mass. . . . . . . . 85
6.20 Decrease in surface temperature from tether with capsule mass. . . . . . . . 86
A.1 Mean density, ρ, vs. altitude for Earth’s atmosphere. . . . . . . . . . . . 93
A.2 Mean free path, λ, vs. altitude for Earth’s atmosphere. . . . . . . . . . . 94
Abstract

The use of tethers in space has an exciting promise in future astronautical applications, with the possibility of providing more sophisticated functionality to satellites and spacecraft. Some of these applications include adjoining satellites, creating microgravity environments, generating power, and transferring momentum between spacecraft. The focus of this project is to investigate the possible reduction in convective heat flux and temperatures on a manned capsule as a result of re-entry with an attached momentum exchange tether, including how various tether parameters affect these results.

Using a “bottom-up” approach by modeling the system as a series of lumped masses and rigid rods (links) in conjunction with Lagrange’s equations, software was developed using Mathematica® that is capable of generating the equations of motion for any arbitrary number of links. To solve the resulting equations of motion, a separate dynamic dumbbell station/capsule swing model was developed for both an inelastic and elastic tether to provide a realistic initial condition vector for the re-entry simulation. Mathematica® was also used to solve the equations of motion numerically, allowing for a simple and effective software workflow.

For the analysis, the resulting motion of the capsule was studied in the case of varying tether length, tether diameter, and capsule mass. Accordingly, the resulting heat loads on the capsule were calculated using a simple 1-D multilayer heat transfer model based on the previously-analyzed dynamic cases. For certain cases, the presence of a tether can reduce the convective heat flux by almost 60% and the surface temperature by just over 20% when compared to an equivalent tether-less system.

It is debatable as to whether or not a momentum exchange tether would serve as an effective hypersonic parachute for a manned re-entry mission. Such a conclusion depends on yet unexplored or undetermined parameters (e.g. the fatigue properties of the capsule’s heat shield or the actual weight of the capsule before re-entry). Nevertheless, if designing a mission to use a relatively light capsule and a long/thick tether, this feasibility
study certainly suggests that further investigation into this subset of tether de-orbiting is warranted.
1 Introduction

The concept of implementing tethers for use in space began with Tsiolokovskii in 1895 with his ‘Day-Dreams of Earth and Heaven’ [1]. The idea was simple: artificial gravity could be created in space using a centrifugal force from a spinning tether/counter-weight system. Since then, many authors, particularly science fiction writers, have borrowed the idea of space tethers for use in their stories. Such a well known example would be The Fountains of Paradise by the late Arthur C. Clarke [2]. In Clarke’s elaborate story, a complex orbital tower is suspended in GEO (Geosynchronous Earth Orbit) using a tether anchored to Earth. Stories such as these have inspired many scientists and engineers to study the possibilities of using tethers for a variety of astronautical purposes.

1.1 Uses for Tethers

1.1.1 Adjoining Objects in Orbit

A serious consideration for applying tethers to space-related missions includes simply adjoining objects in orbit [3]. These objects, whether spacecraft or satellites, could benefit in several ways by being connected via a tether. First, spacecraft connected by a tether could easily share electric power or other supplies by a simple transport mechanism. By connecting two (or multiple) spacecraft in this way, complicated and potentially dangerous docking maneuvers could be avoided, thus saving fuel and eliminating the risk of an accident. An additional use for joining objects in space would be to align an array of various sensors or satellites across a certain region of Earth’s orbit. By creating an array of sensors or satellites that span a large distance, a single system could measure or transmit information from several locations simultaneously, including regions which are located very far apart. This concept has been studied in-depth by Chobotov et al. [4].
1.1.2 Creating Microgravity Environments

Another exciting application of space tethers involves creating microgravity environments in space. The most obvious benefit of microgravity would be improving the living conditions aboard space stations for humans through the production of centrifugal force, since even the smallest amount of gravity in these situations can enhance the living experience of an astronaut [3]. Likewise, production of microgravity can aid in the servicing of spacecraft, particularly during refueling [5]. In the presence of zero-gravity, liquids (in this case, propellant) tend to fragment into small droplets from the effects of surface tension; hence, this property proposes a definite problem for refueling operations in space. Thus, through the help of a tether, a centrifugal force could be created that recreates a small fraction of gravity—enough to make a refueling operation possible.

1.1.3 Power Generation & Momentum Exchange

Perhaps the most exciting possibilities of space tethers lie with conducting tethers and momentum exchange between spacecraft. Conducting electrodynamic tether systems are interesting because of the way they interact with the geomagnetic field. If a conducting insulated tether is deployed vertically from a spacecraft and an induced EMF is driven against it with the help of an on-board power supply, then the Ampere forces along the tether could produce small thrusts for a spacecraft [6]. However, if this process is reversed, this conducting tether could operate as a power generator by passing through the Earth’s magnetic field at a non-zero velocity. Although the Ampere forces decelerate the spacecraft-tether system in this case, the energy generated could be produced with an efficiency of 90% or more [6].

In addition to conducting tethers, an appealing application of tether systems involves the transfer of energy between orbiting objects through the sharing of momentum. Unlike the burning and release of propellants to position a spacecraft, tether-assisted maneuvers involve pure energy and angular momentum exchange between objects when performed
in the vacuum of space. This method has the potential to produce sizable fuel savings for spacecraft, as suggested by Carroll [7]. This transfer of momentum has several compelling uses for spacecraft operations, such as payload launching and releasing. If a spacecraft wishes to release a payload into upper orbit, a tether could be deployed in order to sling-shot the payload while simultaneously placing the orbiter into a re-entry path. Conversely, a spacecraft could also propel itself into upper orbit without thrusters by “swinging” an object into a re-entry trajectory towards Earth [3]. Both of these methods utilize the pure exchange of energy through angular momentum, which would be extremely appealing for those who wish to perform efficient maneuvers in space that usually require a sufficient amount of fuel.

1.2 Project Objective

The overall objective of this project is to quantify the benefits of using a momentum exchange tether as a hypersonic parachute during the de-orbiting a manned re-entry capsule from a base station. Specifically, the author investigates the possible reduction in aerothermal heat flux and temperatures on the capsule as a result of re-entry with an attached tether, including how various tether parameters affect these results. In this way, not only could the utilization of a TSS (Tethered Space System) for manned re-entry reduce the overall cost of long-term space operations though curtailing the amount of required fuel to perform a standard de-orbit maneuver (in both the capsule and base station), but leaving the tether attached to the capsule after its released from the base station could also provide additional drag during the pre-parachute phase—effectively altering the role of the momentum exchange tether to that of a hypersonic parachute. As such, the presence of a tether during re-entry could reduce mission costs by extending the lifetime of the capsule’s heat shield or by reducing the capsule weight through curtailing the amount of required heat shield material. The results of this preliminary research will be use to determine if further investigation into this topic is warranted.
The various phases of tether deployment and release for a capsule de-orbiting maneuver are as follows [Figure 1.1]:

1. Phase 1: The capsule is pushed/released away from the front of the station (pointing downward) with some initial velocity.

2. Phase 2: Once Earth’s gravity gradient has pulled the capsule to the desired tether length, the deployment mechanism is halted, which causes the capsule to swing toward the local vertical.

3. Phase 3: As soon as the capsule has reached the local relative equilibrium (local vertical), the tether is severed at the deployer end.

4. Phase 4: After severing the tether, the capsule/tether system proceeds into a re-entry trajectory toward Earth while the base station is put into a higher orbit due to momentum conservation.

This project focuses mainly on Phase 4, during the capsule/tether system’s re-entry into the atmosphere. However, Phases 2 and 3 are also investigated partially in order to properly analyze the re-entry scenario in Phase 4.
1.3 Overview of Thesis

- Chapter 1 introduces the topic of space tethers and their many potential applications, including the main objective of the project and an outline of this thesis.

- Chapter 2 describes the literature review performed on the topic of momentum exchange tethers for the purpose of atmospheric re-entry.

- Chapter 3 describes the dynamic modeling of the capsule/tether system during Phase 4, including the numerical implementation of the equations of motion and the application of appropriate software.

- Chapter 4 explains the modeling of the capsule/station system during swing and release (Phases 2 and 3) for the purpose of finding the conditions of the capsule and tether during release. These release conditions are used as initial conditions for the modeling of the re-entry phase in Chapter 3.

- Chapter 5 analyzes the resulting dynamics of the capsule/tether system during re-entry, including the effects of tether length, tether diameter, and capsule mass on the system’s dynamics.

- Chapter 6 introduces the heat transfer model used for the capsule and analyzes the resulting heat loads (convective heat fluxes and temperatures) on the capsule’s leading edge. Following Chapter 5, the effects of tether length, tether diameter, and capsule mass on the both the convective heat flux and capsule’s temperature profile are explored.

- Chapter 7 summarizes the results from the analysis and provides suggestions for future work regarding the application of momentum exchange tethers for use as a hypersonic parachute.
2 Literature Review

2.1 Tether Deployment

With regards to using momentum exchange tethers for de-orbiting a payload, many recently published papers deal with novel methods of tether deployment [8, 9, 10]. This result is not surprising: deployment is considered to be one of the most delicate tether operations [10], and several of the tether missions flown in the past two decades have had issues or problems concerning deployment—including SEDS-1 [11], TSS-1 [12], and MAST [13]. One interesting paper on tether deployment concerns a tether-assisted re-entry of a capsule from the ISS [14]. This paper by Zimmermann is significant because of its similarities to the system described in this paper: it describes the equations of motion for a simple TSS whose purpose is to de-orbit a capsule. The system is described in Figure 2.1, where $\ell$ is the tether length, $\theta$ is the in-plane angle, and $\phi$ is the out-of-plane angle.

![Diagram of simple TSS](Image)

Figure 2.1: Diagram of simple TSS.

The equations of motion for this system are derived by Stuiver from modified Hamil-
tonians [15]:

\[ \ddot{\theta} + \left( \dot{\theta} - \omega \right) \left( \frac{2 \dot{\ell}}{\ell} - 2 \dot{\phi} \tan \phi \right) + 3 \omega^2 \sin \theta \cos \theta = 0 \] (2.1)

\[ \ddot{\phi} + \frac{2 \dot{\ell}}{\ell} \dot{\phi} + \left[ \left( \dot{\theta} - \omega \right)^2 + 3 \omega^2 \cos^2 \theta \right] \sin \phi \cos \phi = 0 \] (2.2)

\[ \ell - \ell \left[ \dot{\phi}^2 + \left( \dot{\theta} - \omega \right)^2 \cos^2 \phi + \omega^2 \left( 3 \cos^2 \phi \cos^2 \theta - 1 \right) \right] + \frac{T}{M} = 0, \] (2.3)

where \( \omega \) is the orbit angular velocity, \( \mu \) is the gravitational parameter, \( T \) is the tether tension, \( F_B \) is the brake force from the deployer’s braking mechanism, and \( T = F_B \) for a massless tether. It is noted that in Eq. 2.1–2.3, the base station is assumed to be much larger than the re-entry capsule. According to Glässel et al. [16], pure in-plane motion will not excite out-of-plane motion, but not vice-versa. Thus, for a massless tether (dumbbell model), the in-plane state equations that may be used for numerical simulations are

\[ x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T \] (2.4)

\[ \dot{x}_1 = x_3 \] (2.5)

\[ \dot{x}_2 = x_4 \] (2.6)

\[ \dot{x}_3 = -\left( x_3 - \omega \right) \frac{2 x_4}{x_2} - 3 \omega^2 \sin x_1 \cos x_1 \] (2.7)

\[ \dot{x}_4 = x_2 \left[ \left( x_3 - \omega \right)^2 + \omega^2 \left( 3 \cos^2 x_1 - 1 \right) \right] - \frac{u}{M}, \] (2.8)

where \( u = F_B \) for \( T \geq 0 \).

Using these equations to model the TSS numerically, Zimmermann investigates optimal control of a tether deployment system specifically for a re-entry capsule by studying two different cost functions:

\[ J_1 = \int_{t_a}^{t_e} F_B^2 \, dt \] (2.9)

\[ J_2 = \int_{t_a}^{t_e} F_B^2 \, dt, \] (2.10)
where \( t_a \) and \( t_c \) are the initial and final time of the deployment phase, \( J_1 \) is a cost function that minimizes the control effort, and \( J_2 \) is a cost function that minimizes tension variation. From the results found in [16], both cost functions are effective for controlling deployment; however, minimizing the tension variation seems to be more desirable for effectively minimizing unwanted oscillations.

Zimmermann et al. separates the tether deployment into three phases:

1. Deployment towards the local vertical, stabilizing the capsule 1-2 km below the station. This phase includes high sensitivity with respect to perturbations since the gravity gradient forces are still low.

2. Deployment towards the final tether length. Active braking is included in both Phases 1 and 2. Phase 2 is considered terminated when \( \dot{\ell} = 0 \).

3. Tether swing-back and severing at the local vertical. Note that \( \dot{\ell}, \ddot{\ell} = 0 \) during this phase (\( \ell = \) constant), and the deployment mechanism is blocked.

As for deployment stability, Bainum and Kumar used the Routh-Hurwitz stability criterion to discover that the system in Figure 2.1 is stable (during Phase 2) as long as the following condition is met [17]:

\[
\left| \frac{\dot{\ell}}{\Omega \ell} \right| < C, \tag{2.11}
\]

where \( \Omega \) is the orbital angular rate of the TSS and \( C \approx 0.75 \).

It is noted that Eq. 2.11 assumes the base station is massive; thus, it does not apply to the TSS in this paper. However, an approach similar to Bainum and Kumar could be taken in order to find the required deployment stability criterion. Hence, given the abundance of research in tether deployment control and the recent deployment success during the YES2 (Young Engineer’s Satellite 2) mission [18], future prospects for a viable tether deployment strategy for a re-entry mission are promising.
2.2 Tether System Modeling

With regard to previous literature on the topic of modeling a re-entering capsule/tether system, the author has only discovered one paper on the topic. This paper, which was written by Krischke et al. about the capsule/tether system used in the 1993 SEDS-1 mission [19], describes the exact scenario studied in this paper: using a long tether attached to a re-entry capsule for use as a hypersonic parachute. Though Krischke claims that the SEDS-1 capsule could see a heat flux reduction up to one order of magnitude with a 20 km long, heat-resistant tether, there are notable dissimilarities between the system in his paper and in this one. Not only is the capsule mass in his analysis several orders of magnitude less than that which will be used in this study, but his work makes no mention of how the tether parameters would affect the system dynamics and the capsule heating. Thus, given these differences, the author will need to perform additional dynamic modeling.

A typical method for modeling a TSS mathematically includes a system with two point masses that are connected by a perfectly flexible thread [20]. Due to the fact that the tether diameter dimension is much smaller than that of both the tether length and end-bodies, the lack of tether bending stiffness is considered a very reasonable assumption. In this model, we have a tether $\overline{AB}$ connecting two end-masses at both $A$ and $B$; an infinitesimal segment, $ds$, located at an arclength position, $s \in (A, B)$, with an internal tension force, $T(s, t)$; and a geocentric reference frame $XYZ$ with a defining radius vector, $R(s, t)$ [Figure 2.2].

In Figure 2.2, $\rho(s)$ is the mass per unit length at $s$, the gravitational acceleration $g = \mu R/R^3$ (where $R = ||R||$), and $F$ is the sum of all other perturbing forces per unit length exerted on the system. Performing a force balance on element $ds$, we have the following:

$$\rho(s) \frac{\partial^2 R}{\partial t^2} ds = T(s + ds, t) - T(s, t) - \rho(s) \frac{\mu R}{R^3} ds + F ds. \quad (2.12)$$
Figure 2.2: Tether modeling diagram.
Dividing both sides by $\rho \, ds$, we have

$$\frac{\partial^2 \mathbf{R}}{\partial t^2} = \frac{1}{\rho} \frac{\partial T}{\partial s} - \frac{\mu \mathbf{R}}{R^3} + \frac{F}{\rho},$$

(2.13)

assuming that $\rho$ is a constant along the tether. Since the tether does not resist bending, its tension force is always directed along the thread direction,

$$T = T \hat{t}, \quad \hat{t} = \frac{\partial \mathbf{R}}{\partial s} \left\| \frac{\partial \mathbf{R}}{\partial s} \right\|^{-1},$$

(2.14)

where $\hat{t}$ is the unit tangent vector. Adopting Hooke’s law as an appropriate description of elasticity, we have

$$T = E(\gamma - 1), \quad \gamma = \left\| \frac{\partial \mathbf{R}}{\partial s} \right\|,$$

(2.15)

where $E$ is the tether’s extension stiffness.

Similarly, we can perform a force balance on each of the end-masses:

$$m_A \frac{\partial^2 \mathbf{R}_A}{\partial t^2} = T_A - m_A \frac{\mu \mathbf{R}_A}{R_A^3} + F_A,$$

$$m_B \frac{\partial^2 \mathbf{R}_B}{\partial t^2} = T_B - m_B \frac{\mu \mathbf{R}_B}{R_B^3} + F_B,$$

(2.16)

where $F_A$ and $F_B$ refer to the overall perturbing forces on each of the end-bodies located at positions $A$ and $B$, respectively. Of course, realizing that $T_A = T(A, t)$ and $T_B = T(B, t)$, we have

$$T_A = E(\gamma_A - 1), \quad \gamma_A = \left\| \frac{\partial \mathbf{R}}{\partial s} \right\|_A,'$$

$$T_B = E(\gamma_B - 1), \quad \gamma_B = \left\| \frac{\partial \mathbf{R}}{\partial s} \right\|_B,$$

(2.17)

which allows Eq. 2.16 to act as “end conditions” for the PDE (Partial Differential Equation) found in Eq. 2.13. Combining Eq. 2.13–2.15, 2.16, and 2.17, we are left with the following
set of governing equations for the system:

$$\frac{\partial^2 R}{\partial t^2} = \frac{1}{\rho} \frac{\partial^2 R}{\partial s^2} \left( E - \left\| \frac{\partial R}{\partial s} \right\|^{-1} \right) - \frac{\mu R}{R^3} + \frac{F}{\rho}$$

$$\frac{\partial^2 R_A}{\partial t^2} = \frac{1}{m_A} \frac{\partial R_A}{\partial s} \left( E - \left\| \frac{\partial R_A}{\partial s} \right\|^{-1} \right) - \frac{\mu R_A}{R^3_A} + \frac{F_A}{m_A}$$

$$\frac{\partial^2 R_B}{\partial t^2} = \frac{1}{m_B} \frac{\partial R_B}{\partial s} \left( E - \left\| \frac{\partial R_B}{\partial s} \right\|^{-1} \right) - \frac{\mu R_B}{R^3_B} + \frac{F_B}{m_B}.$$ (2.18)

Adopting the simplified notation

$$(\Box)' = \frac{\partial}{\partial s}, \quad (\Box)'' = \frac{\partial^2}{\partial s^2}, \quad (\ddot{\Box}) = \frac{\partial^2}{\partial t^2}$$ (2.19)

and applying them to Eq. 2.18, we are left with the following set of coupled PDEs:

$$\ddot{R} = \frac{1}{\rho} R'' \left( E - \left\| R' \right\|^{-1} \right) - \frac{\mu R}{R^3} + \frac{F}{\rho}$$ (2.20)

$$\ddot{R}_A = \frac{1}{m_A} R'_A \left( E - \left\| R'_A \right\|^{-1} \right) - \frac{\mu R_A}{R^3_A} + \frac{F_A}{m_A}$$ (2.21)

$$\ddot{R}_B = \frac{1}{m_B} R'_B \left( E - \left\| R'_B \right\|^{-1} \right) - \frac{\mu R_B}{R^3_B} + \frac{F_B}{m_B}.$$ (2.22)

which are the governing equations for the system.

Due to the inherent complexity in solving the PDEs in Eq. 2.20–2.22 using a general “top-down” approach, the author has decided to solve the TSS in this paper using a simplified “bottom-up” approach. By reconstructing the system as a series of lumped masses and rigid rods, one may use a more straight-forward method of obtaining the equations of motion (such as Lagrange’s equations) while still capturing a majority of the system’s physics. The author has employed such a method to find the governing system of ODEs (Ordinary Differential Equations) in Chapter 3.
3 Modeling of the Capsule/Tether Re-entry System

3.1 Lagrangian Formulation

The following section describes the derivation of a dynamic capsule/tether system model in the presence of an atmosphere during re-entry using the simplicity of Lagrange’s equations. The capsule is modeled as a point mass, and the tether is modeled as an \( N \) number of point masses adjoined by massless, rigid links in a two-dimensional, equatorial configuration space [21]. The reference frame is given as \( \hat{\mathbf{c}} \) and the inertial frame as \( \hat{\mathbf{e}} \), and the two are related through the relation \( \omega_{c/e} = \dot{\theta} \hat{\mathbf{c}}_3 \). To begin modeling, Lagrange’s equations are presented:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \\
L \equiv T - V,
\]

where each \( Q_i \) represents the forces not derivable from a potential such as aerodynamic drag and other nonconservative forces. Each \( q_i \) represents a variable that describes the corresponding degree of freedom of the system. For this system, there are \( n = N + 2 \) degrees of freedom, which leads the configuration space to be described as

\[
\mathbf{q} \in \mathbb{R}^{n \times 1}, \quad \mathbf{q} = \left[ R \ \theta \ \alpha_1 \ \alpha_2 \ \ldots \ \alpha_N \right]^T.
\]

To find \( L \), the both the kinetic energy, \( T \), and the potential energy, \( V \), must be found. The advantage of using the Lagrangian formulation for finding the equations of motion lies in its simplicity of derivation: in general, since \( T = T(q, \dot{q}) \) and \( V = V(q) \), no accelerations need to be calculated in order to obtain the governing dynamical equations \( (L \neq \ddot{q}) \). Only positions and velocities need to be found. Moreover, the expressions for both \( T \) and \( V \) may be expressed with respect to the local reference frame.
Figure 3.1: Diagram of the capsule/tether system model under consideration.
3.1.1 Kinetic Energy Derivation

The kinetic energy of the system consists of contributions from both the capsule and each of the tether’s point masses [Figure 3.1]. The kinetic energy for the capsule is given as

\[ T_c = \frac{1}{2} m_c (v_c \cdot v_c) = \frac{1}{2} m_c \left( R^2 + R^2 \dot{\theta}^2 \right), \]  

(3.2)

since the the position of the capsule is simply \( R \hat{c}_1 \). The position to each \( i \)-th mass of the tether is given as

\[ \mathbf{R}_i = \left( R + \sum_{n=1}^{i} \ell_n \cos \alpha_n \right) \hat{c}_1 + \left( \sum_{n=1}^{i} \ell_n \sin \alpha_n \right) \hat{c}_2. \]  

(3.3)

Using this definition, the velocity of each tether mass can be found by taking

\[ \mathbf{v}_i = \frac{d\mathbf{R}_i}{dt} + \omega_{c/e} \times \mathbf{R}_i, \]  

(3.4)

which results in

\[ \mathbf{v}_i = \left( \dot{R} - \sum_{n=1}^{i} \ell_n (\dot{\alpha}_n + \dot{\theta}) \sin \alpha_n \right) \hat{c}_1 + \left( R \dot{\theta} + \sum_{n=1}^{i} \ell_n (\dot{\alpha}_n + \dot{\theta}) \cos \alpha_n \right) \hat{c}_2, \]  

(3.5)

where \( \ell_n \) is the same across all links. With the velocities in place, the kinetic energy of the tether is

\[ T_t = \sum_{i=1}^{N} \left[ \frac{1}{2} m_i (\mathbf{v}_i \cdot \mathbf{v}_i) \right]. \]  

(3.6)

Thus, the total kinetic energy is given as

\[ T = T_c + T_t = \frac{1}{2} m_c \left( R^2 + R^2 \dot{\theta}^2 \right) + \frac{1}{2} \sum_{i=1}^{N} m_i \left\{ \left( \dot{R} - \sum_{n=1}^{i} \ell_n (\dot{\alpha}_n + \dot{\theta}) \sin \alpha_n \right)^2 + \left( R \dot{\theta} + \sum_{n=1}^{i} \ell_n (\dot{\alpha}_n + \dot{\theta}) \cos \alpha_n \right)^2 \right\}. \]  

(3.7)
3.1.2 Potential Energy Derivation

For a Newtonian gravitational field,

$$V = -\mu \frac{m}{R},$$  \hspace{1cm} (3.8)

where $\mu$ is the gravitational constant and $R$ is the distance from the center of Earth to the point in question. In this model, higher orders of the geopotential are ignored since the system is assumed to follow an equatorial path. Thus, the potential energy of the capsule is simply

$$V_c = -\mu \frac{m_c}{R}. \hspace{1cm} (3.9)$$

The potential energy of each tether mass is given as

$$V_i = -\mu \frac{m_i}{\|R_i\|}, \hspace{1cm} (3.10)$$

where $\|R_i\|$ is the radial distance from the reference frame (center of earth) to the $i$-th mass.

Using Eq. 3.3:

$$\|R_i\| = \sqrt{R_i \cdot R_i} = \left\{ \left( R + \sum_{n=1}^i \ell_n \cos \alpha_n \right)^2 + \left( \sum_{n=1}^i \ell_n \sin \alpha_n \right)^2 \right\}^{1/2}. \hspace{1cm} (3.11)$$

Thus, knowing $V = V_c + V_t$, we have

$$V = -\mu \frac{m_c}{R} + \sum_{i=1}^N V_i$$

$$= -\mu \frac{m_c}{R} - \mu \sum_{i=1}^N m_i \left\{ \left( R + \sum_{n=1}^i \ell_n \cos \alpha_n \right)^2 + \left( \sum_{n=1}^i \ell_n \sin \alpha_n \right)^2 \right\}^{-1/2}. \hspace{1cm} (3.12)$$
3.1.3 Generalized forces

In Lagrange’s equations, each \( Q_i \) term represents the nonconservative forces exerted on the system. For instance, in this model, the \( Q_i \) terms only embody the aerodynamic drag exerted on both the capsule and the tether. Since Lagrange’s equations result in a set of \( n \) ODEs representing each generalized coordinate, \( q_i \), in the configuration space, each \( Q_i \) term is distinct in that it represents the contribution of all the nonconservative forces toward its respective generalized coordinate. Explained mathematically,

\[
Q_i = \sum_{k=1}^{m} \left( \mathbf{F}_k \cdot \frac{\partial \mathbf{R}_k}{\partial q_i} \right); \quad i = \{1, 2, \ldots, n\}, \tag{3.13}
\]

where \( m \) is the number of nonconservative forces and \( \mathbf{R}_k \) is the corresponding point of application for the \( k \)-th force. It is noted that both \( \mathbf{F}_k \) and \( \mathbf{R}_k \) must be expressed in inertial coordinates.

In general, aerodynamic drag is described as

\[
\mathbf{F}_D = -\frac{1}{2} \rho C_D S \mathbf{v}, \tag{3.14}
\]

where \( C_D \) is the drag coefficient, \( S \) is the frontal area of the body, \( \mathbf{v} \) is the velocity with respect to the atmosphere (wind), and \( \rho \) is the atmospheric density [Appendix A].

Assuming the atmosphere rotates with the planet at rate \( \Omega \), the velocity of the atmosphere (wind) with respect to the capsule, assuming an equatorial orbit, is

\[
\mathbf{v}_{wc} = \dot{\mathbf{R}} \mathbf{\hat{c}}_1 + \mathbf{R} \left( \dot{\theta} - \Omega \right) \mathbf{\hat{c}}_2, \tag{3.15}
\]

since \( \omega_{e/e} = \left( \dot{\theta} - \Omega \right) \mathbf{\hat{c}}_3 \). Thus, the aerodynamic force on the capsule is

\[
\mathbf{F}_c = -\frac{1}{2} \rho_c C_{Dc} S_c \sqrt{\mathbf{v}_{wc} \cdot \mathbf{v}_{wc}} \left[ \dot{\mathbf{R}} \mathbf{\hat{c}}_1 + \mathbf{R} \left( \dot{\theta} - \Omega \right) \mathbf{\hat{c}}_2 \right] = F_{c1} \mathbf{\hat{c}}_1 + F_{c2} \mathbf{\hat{c}}_2. \tag{3.16}
\]

Accordingly, since the capsule is assumed to be a point mass with a spherical body, the
position describing the point of application for the capsule’s drag force is as follows (in inertial coordinates):

\[ \mathbf{R}_c = R \hat{c}_1 = R (\cos \theta \hat{e}_1 + \sin \theta \hat{e}_2). \]  

(3.17)

Using the above information, the generalized forces on the capsule can be identified using Eq. 3.13:

\[ Q_{\mathbf{R},c} = F_c \cdot \frac{\partial \mathbf{R}_c}{\partial \mathbf{R}} = F_{c1} \]  

(3.18)

\[ Q_{\theta,c} = F_c \cdot \frac{\partial \mathbf{R}_c}{\partial \theta} = RF_{c2}, \]  

(3.19)

while \( Q_{\alpha,c} = 0 \) since \( \mathbf{R}_c \notin \alpha_j \).

The generalized forces on the tether must now be calculated. The differential force on the \( i \)-th link at a distance \( \xi \) along the link (separated into normal and tangential components) is

\[ d\mathbf{F}_i = -\frac{1}{2} \rho \mathbf{v}_{\xi \hat{i}} \mathbf{v}_{\xi \hat{i}} (C_{D\xi} \, dS_\perp + C_{Df\xi} \, dS_\parallel), \]  

(3.20)

where \( \mathbf{v}_{\xi \hat{i}} \) is the velocity of the point at distance \( \xi \) along the \( i \)-th link with respect to the atmosphere. Both \( C_{D\xi} \) and \( C_{Df\xi} \) represent the pressure drag coefficient and skin friction drag coefficient, respectively. This velocity is described as

\[ \mathbf{v}_{\xi \hat{i}} = \left( \dot{R} - \dot{\xi} (\ddot{\alpha}_i + \ddot{\theta} - \Omega) \sin \alpha_i - \sum_{n=1}^{i-1} \ell_n (\ddot{\alpha}_n + \ddot{\theta} - \Omega) \sin \alpha_n \right) \hat{c}_1 \]

\[ + \left( R (\ddot{\theta} - \Omega) + \dot{\xi} (\ddot{\alpha}_i + \ddot{\theta} - \Omega) \cos \alpha_i + \sum_{n=1}^{i-1} \ell_n (\ddot{\alpha}_n + \ddot{\theta} - \Omega) \cos \alpha_n \right) \hat{c}_2. \]  

(3.21)

Both \( dS_\perp \) and \( dS_\parallel \) correspond to the differential areas perpendicular and parallel to the
tether link, respectively:

\[ dS_\perp = d_i \, \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2 \right) \frac{\mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2}{v_{w\xi_i}} \, d\xi \]  
\[ (3.22) \]

\[ dS_\parallel = \pi d_i \, \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1 \right) \frac{\mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1}{v_{w\xi_i}} \, d\xi , \]
\[ (3.23) \]

where \( d_i \) is the diameter of the \( i \)-th rod, and \( \hat{\mathbf{b}}_1 \) and \( \hat{\mathbf{b}}_2 \) are unit vectors along and normal to the tether, respectively. Using Eq. 3.22–3.23 and plugging them back into the equation for \( d\mathbf{F}_i \) [Eq. 3.20]:

\[ d\mathbf{F}_i = -\frac{1}{2} \rho_{\xi_i} d_i \left[ C_{D\xi_i} \, \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2 \right) \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2 + \pi C_{Df\xi_i} \, \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1 \right) \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1 \right] \mathbf{v}_{w\xi_i} \, d\xi , \]
\[ (3.24) \]

where

\[ \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2 = \mathbf{v}_{w\xi_i} \cdot (-\sin \alpha_i \hat{\mathbf{c}}_1 + \cos \alpha_i \hat{\mathbf{c}}_2) = D_1 + D_2 \xi, \]
\[ (3.25) \]

\[ \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1 = \mathbf{v}_{w\xi_i} \cdot (\cos \alpha_i \hat{\mathbf{c}}_1 + \sin \alpha_i \hat{\mathbf{c}}_2) = D_3 + D_4 \xi. \]
\[ (3.26) \]

We can assume that the expressions \( \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1 \right) \) and \( \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2 \right) \) do not change along each link, thus simplifying the expressions by setting \( \xi = 0 \):

\[ \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_1 \right) = \text{sgn}(D_3) = \sigma_1 , \]
\[ (3.27) \]

\[ \text{sgn} \left( \mathbf{v}_{w\xi_i} \cdot \hat{\mathbf{b}}_2 \right) = \text{sgn}(D_1) = \sigma_2 . \]
\[ (3.28) \]

Plugging in Eqs. 3.25, 3.26, 3.27, and 3.28 into Eq. 3.24, we have

\[ d\mathbf{F}_i = -\frac{1}{2} \rho_{\xi_i} d_i \left[ \sigma_2 C_{D\xi_i}(D_1 + D_2 \xi) + \pi \sigma_1 C_{Df\xi_i}(D_3 + D_4 \xi) \right] \mathbf{v}_{w\xi_i} \, d\xi . \]
\[ (3.29) \]

Unfortunately, due to the fact that the expressions for \( \rho_{\xi_i} \), \( C_{D\xi_i} \), and \( \mathbf{v}_{w\xi_i} \) are quite complex and depend on \( \xi \), integrating Eq. 3.29 does not result in an analytical expression\(^1\).

\(^1\)Though \( \rho_{\xi_i} \) could reasonably be assumed to be an exponential function, the combination of \( \rho_{\xi_i} \), \( C_{D\xi_i} \), and
However, for a large number of links, it should be reasonable to assume that $F_i$ is constant along each link. Taking $F_i(\xi) = F_i(\ell_i/2)$, we have

$$dF_i = \left[ -\frac{1}{2} \rho_i d_i \{ \sigma_2 C_{DDi}(D_1 + D_2 \xi) + \pi \sigma_1 C_{DFi}(D_3 + D_4 \xi) \} v_{wi}\right]_{\xi=\ell_i/2} d\xi. \quad (3.30)$$

Integrating Eq. 3.30 yields

$$F_i = \int_0^{\ell_i} dF_i \bigg|_{\xi=\ell_i/2} = -\frac{1}{4} \rho_{i,1/2} d_i \ell_i \left[ \sigma_2 C_{Di,1/2} \left( D_1 + D_2 \frac{\ell_i}{2} \right) + \pi \sigma_1 C_{Di,1/2} \left( D_3 + D_4 \frac{\ell_i}{2} \right) \right] v_{wi,1/2} \quad (3.31)$$

where the subscript $(i, 1/2)$ represents the location $\ell/2$ of the $i$-th link. At this point of application, the position vector is

$$R_{i,1/2} = \left( R + \frac{\ell_i}{2} \cos \alpha_i + \sum_{n=1}^{i-1} \ell_i \cos \alpha_n \right) \hat{e}_1 + \left( \frac{\ell_i}{2} \sin \alpha_i + \sum_{n=1}^{i-1} \ell_i \sin \alpha_n \right) \hat{e}_2. \quad (3.32)$$

Of course, we realize that $F_i$ and $R_{i,1/2}$ must be written in inertial coordinates:

$$F_i = F_i \cdot (\cos \theta \hat{e}_1 + \sin \theta \hat{e}_2) \quad (3.33)$$

$$R_{i,1/2} = R_{i,1/2} \cdot (\cos \theta \hat{e}_1 + \sin \theta \hat{e}_2). \quad (3.34)$$

$v_{wi}\xi$ prevents the expression, $dF_i$, from being integrated in a straight-forward manner. $C_{DDi}$ is particularly complex due to its dependence on $\xi$ through the Knudson number,

$$Kn = \frac{\lambda_\infty}{L},$$

where $L$ is the characteristic length of the capsule (the diameter) and $\lambda_\infty$ is the mean free path of the freestream flow [Appendix A,B].
The generalized force expressions can now be found from the following:

\[ Q_{R,t} = \sum_{j=1}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial R} \right) \]  
(3.35)

\[ Q_{\theta,t} = \sum_{j=1}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial \theta} \right) \]  
(3.36)

\[ Q_{\alpha_i,t} = \sum_{j=1}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial \alpha_i} \right). \]  
(3.37)

However, since \( F_j \cdot (\partial R_{j,1/2}/\partial \alpha_i) = 0 \) for \( j < i \), we can simplify Eq. 3.37 with the expression

\[ Q_{\alpha_i,t} = \sum_{j=i}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial \alpha_i} \right), \]  
(3.38)

which leaves \( N \) terms in \( Q_{\alpha_1,t} \), \( N-1 \) terms in \( Q_{\alpha_2,t} \), two terms in \( Q_{\alpha_{N-1},t} \), and one term in \( Q_{\alpha_N,t} \). This completes the generalized force expressions, leaving the following for the system which has \( n \) generalized coordinates and \( N \) tether links:

\[ Q_1 = Q_{R,c} + Q_{R,t} = F_{c1} + \sum_{j=1}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial R} \right) \]

\[ Q_2 = Q_{\theta,c} + Q_{\theta,t} = RF_{c2} + \sum_{j=1}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial \theta} \right) \]

\[ Q_3 = Q_{\alpha_1,t} = \sum_{j=1}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial \alpha_1} \right) \]

\[ Q_4 = Q_{\alpha_2,t} = \sum_{j=2}^{N} \left( F_j \cdot \frac{\partial R_{j,1/2}}{\partial \alpha_2} \right) \]  
(3.39)

\[ Q_{n-1} = Q_{\alpha_{N-1},t} = F_{N-1} \cdot \frac{\partial R_{N-1,1/2}}{\partial \alpha_{N-1}} + F_N \cdot \frac{\partial R_{N,1/2}}{\partial \alpha_N} \]

\[ Q_n = Q_{\alpha_N,t} = F_N \cdot \frac{\partial R_{N,1/2}}{\partial \alpha_N}. \]
3.2 Numerical Implementation

Once Lagrange’s equations are found, we are left with $n$ second-order ODEs. In order to solve for these equations numerically, several steps must first be taken. If

$$\mathbf{q} = \begin{bmatrix} R & \theta & \alpha_1 & \alpha_2 & \cdots & \alpha_N \end{bmatrix}^T,$$

then we need to convert the equations of motion into the form

$$[\mathbf{M}]\ddot{\mathbf{q}} = f(\mathbf{q}, \dot{\mathbf{q}}, t),$$

where $[\mathbf{M}]$ is the system’s mass matrix. But how can we systematically find the elements of $[\mathbf{M}]$? Since the second-order derivatives of the generalized coordinates only arise from the $(d/dt)(\partial T/\partial \dot{q}_i)$ terms of Lagrange’s equations [21], let

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) \equiv \tilde{T}_{2i} + \tilde{T}_{1i},$$

where

$$\tilde{T}_{2i} = \sum_{i=1}^{n} M_{ji}(\mathbf{q}, t) \ddot{q}_i, \quad \tilde{T}_{1i} = \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \tilde{T}_{2j}.$$ (3.43)

In this way, the $M_{ji}$ terms that comprise the mass matrix $[\mathbf{M}]$ may be found.

Once the elements of the mass matrix are found, the $n$ second-order differential equations can be reduced to $2n$ first-order ODEs, allowing the system of equations to be solved using standard numerical software packages. Let

$$\mathbf{x} = \begin{bmatrix} q_1 & q_2 & \cdots & q_n & \dot{q}_1 & \dot{q}_2 & \cdots & \dot{q}_n \end{bmatrix}^T$$

$$= \begin{bmatrix} x_1 & x_2 & \cdots & x_{2n-1} & x_{2n} \end{bmatrix}^T,$$

(3.44) (3.45)
which results in the need for a modified mass matrix:

\[
\tilde{M} = \begin{bmatrix}
1_{n \times n} & 0_{n \times n} \\
0_{n \times n} & M
\end{bmatrix},
\]  

(3.46)

where \([1]_{n \times n}\) and \([0]_{n \times n}\) are the identity and zero matrices, respectively. Thus, we are left with the following set of \(2n\) ODEs:

\[
\frac{dx}{dt} = \tilde{M}^{-1}g(x, t),
\]  

(3.47)

where

\[
g(x, t) = \begin{cases}
g_i(x, t) = x_{i+n}, & 1 \leq i \leq n \\
g_i(x, t) = f_{i-n}(x, t), & n < i \leq 2n
\end{cases}.
\]  

(3.48)

Once the system of ODEs from Eq. 3.47 is obtained, standard numerical routines such as Euler’s method or a fourth-order Runge-Kutta method may be implemented to any desired degree of accuracy.

3.2.1 Validation Case

Before any type of simulation is attempted with the tether/capsule system, a validation case is presented that includes the re-entry of a point mass (capsule). In this way, the successful use of Lagrange’s equations to generate the equations of motion for a simple re-entry system may be confirmed. For such a system, the configuration space is described as follows:

\[
q = \begin{bmatrix} R \\ \theta \end{bmatrix} \in \mathbb{R}^{2 \times 1}
\]  

(3.49)

\[
x = \begin{bmatrix} R & \theta & \dot{R} & \dot{\theta} \end{bmatrix}^T.
\]  

(3.50)
Using the approach in the previous section to find the equations of motion, the following expressions are found:\(^2\):

\[
\begin{align*}
mc\left(R - \dot{R}\theta^2 + \mu \frac{1}{R^2}\right) &= -\frac{1}{2} \rho_r e^{(R_{pl} - R)/H} C_D S_c \dot{R} \sqrt{\dot{R}^2 + R^2(\dot{\theta} - \Omega)^2} \\
mc\left(2R\dot{R}\dot{\theta} + R^2\ddot{\theta}\right) &= -\frac{1}{2} \rho_r e^{(R_{pl} - R)/H} C_D S_c R^2 \left(\dot{\theta} - \Omega\right) \sqrt{\dot{R}^2 + R^2(\dot{\theta} - \Omega)^2},
\end{align*}
\]

where \(C_D\) is assumed to be a constant (\(\approx 1\)).

Rearranging the equations in the form \([\mathbf{M}]\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, t)\) then performing a first-order reduction,

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & m_c & 0 \\
0 & 0 & 0 & m_c x_2^2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix}
= 
\begin{bmatrix}
x_3 \\
x_4 \\
f_1(x, t) \\
f_2(x, t)
\end{bmatrix}.
\]

Unfortunately, since \([\tilde{\mathbf{M}}]\) has a very large condition number due to poor scaling from the \(\tilde{M}_{44}\) element, computing its inverse accurately proves challenging from a traditional prospective. Fortunately, MATLAB\textsuperscript{\textregistered} [22] has the ability to solve these types of nonlinear systems of ODEs with poorly-conditioned mass matrices using the ODE solver \texttt{ode115i}().

Setting the mass of the capsule to 4 metric tons, its initial velocity to 7770.5 m/s, its initial altitude to 200 km, and its characteristic area to 25 m\(^2\), the following plots are obtained in Figure 3.2. For this case, the initial velocity is fairly close to the transition point where the capsule will continually orbit around the earth. As a result, the capsule’s angular position extends to approximately 300\(^\circ\) past its initial point—almost a full revolution around Earth.

3.2.2 Application of Software

As mentioned previously, the solution to Eq. 3.53 displayed in Figure 3.2 was solved using MATLAB\textsuperscript{\textregistered}. However, in the general case of a capsule/tether system with an arbitrarily-\(^2\)In this simple case, we will assume the atmospheric density holds an exponential profile with a scale height, \(H\), of 8.5 km and a reference density, \(\rho_r\), of 1.225 kg/m\(^2\).
Figure 3.2: System parameters vs. time for the validation case.
sized configuration space, there must be an automated way of obtaining the system of
equations from Eq. 3.1 and then tailoring them into the form of Eq. 3.47. Otherwise, the
process of solving the equations of motion for many different cases would prove extremely
cumbersome and inefficient.

For this reason, the author has utilized the powerful symbolic manipulation and pro-
gramming capabilities of Mathematica® [23] to generate the equations of motion for the
capsule/tether system given any number of links. In this way, the kinetic energy, potential
energy, and drag forces can be automatically defined for any N number of links, and then
the necessary differentiation may be performed to find the generalized forces and the
resulting equations of motion [Appendix C]. Once these equations of motion are obtained
in Mathematica®, the author utilized the text manipulation capabilities of Python by writ-
ing a script to convert the resulting equations of motion into a form that MATLAB® can
understand. From here, separate function and mass matrix files could be created in order
to solve Eq. 3.47 using the ode115() command [Figure 3.3].

Figure 3.3: Original software workflow for obtaining numerical results.

However, this multi-stepped approach to obtaining numerical results had severe
disadvantages. Not only would conversions from one software package to another
promote human-made errors, but this process still required the user to construct the
MATLAB® equation files and ensure the first-order reduction steps from Eq. 3.43 were
performed correctly. Consequently, the process of performing each simulation required
an exorbitant amount of time, still promoting inefficiency. Though even more time could
have been invested to write a more in-depth Python script to solve some of the problems
presented, the author ultimately rectified this problem by utilizing the built-in numerical
capabilities of Mathematica® to solve the system of ODEs using NDSolve[]—effectively
cutting out the middleman.

This enhanced *modus operandi* greatly simplifies the workflow, saves time, and reduces errors [Figure 3.4]. With the numerical model and enhanced software workflow in place,

![Mathematica](Mathematica)

Figure 3.4: Improved software workflow for obtaining numerical results.

initial conditions are the last remaining piece of the puzzle needed to begin performing re-entry simulations. The next chapter describes the process of developing a swing model for both an inelastic and elastic tether in order to find the desired parameters of the capsule and tether at the release condition for use as re-entry phase initial conditions.
4 Initial Condition Generation

4.1 Massless, Inelastic Tether

As mentioned previously, the model of the capsule/tether system before re-entry includes the time frame immediately after deployment ($\dot{\ell} = 0$, $\alpha \approx 45^\circ$) to the moment right before tether termination ($\alpha = 0$, $\dot{\alpha} = 0$). In this model [Figure 4.1], the tether is assumed to be massless and rigid. Accordingly, running the simulation to find the system parameters at the release condition ($\alpha = 0$) should give good approximate values for use as re-entry phase initial conditions.

![Diagram of Capsule-Tether System](image)

Figure 4.1: Capsule-tether system (dumbbell) model immediately before swing.
4.1.1 Derivation of Equations

Finding the equations of motion for this system is straight-forward: as before, the author utilizes Lagrange’s equations [Eq. 3.1] with

\[ q = \begin{bmatrix} R & \theta & \alpha \end{bmatrix}^T, \quad Q_i = 0. \quad (4.1) \]

Both the station’s and capsule’s position vectors are defined as

\[ \mathbf{r}_s = R \hat{c}_1 \]
\[ \mathbf{r}_c = (R - \ell \cos \alpha) \hat{c}_1 + \ell \sin \alpha \hat{c}_2, \quad (4.2) \]

respectively. Accordingly, the station’s and capsule’s velocities can be obtained:

\[ \mathbf{v}_s = \dot{R} \hat{c}_1 + R \dot{\theta} \hat{c}_2, \]
\[ \mathbf{v}_c = \left[ \dot{R} + \ell (\dot{\alpha} - \dot{\theta}) \sin \alpha \right] \hat{c}_1 + \left[ R \dot{\theta} + \ell (\dot{\alpha} - \dot{\theta}) \cos \alpha \right] \hat{c}_2. \quad (4.3) \]

The kinetic energies of the station and the capsule are

\[ T_s = \frac{1}{2} m_s (\mathbf{v}_s \cdot \mathbf{v}_s) = \frac{1}{2} m_s \left( \dot{R}^2 + R^2 \dot{\theta}^2 \right), \]
\[ T_c = \frac{1}{2} m_c (\mathbf{v}_c \cdot \mathbf{v}_c) = \frac{1}{2} m_c \left\{ \left[ \dot{R} + \ell (\dot{\alpha} - \dot{\theta}) \sin \alpha \right]^2 + \left[ R \dot{\theta} + \ell (\dot{\alpha} - \dot{\theta}) \cos \alpha \right]^2 \right\}, \quad (4.4) \]

where \( m_s \) and \( m_c \) are the masses of the station and capsule, respectively. Thus, the total kinetic energy is

\[ T = T_s + T_c = \frac{1}{2} m_s \left( \dot{R}^2 + R^2 \dot{\theta}^2 \right) + \frac{1}{2} m_c \left\{ \left[ \dot{R} + \ell (\dot{\alpha} - \dot{\theta}) \sin \alpha \right]^2 + \left[ R \dot{\theta} + \ell (\dot{\alpha} - \dot{\theta}) \cos \alpha \right]^2 \right\}. \quad (4.5) \]
The potential energy of both the station and capsule are

\[ \begin{align*}
V_s &= -\mu \frac{m_s}{\| r_s \|} = -\mu \frac{m_s}{R} \quad \text{(4.8)} \\
V_c &= -\mu \frac{m_c}{\| r_c \|} = -\mu m_c \left[ (R - \ell \cos \alpha)^2 + (\ell \sin \alpha)^2 \right]^{-1/2}, \quad \text{(4.9)}
\end{align*} \]

which leaves the following final expression for the potential energy:

\[ V = V_s + V_c = -\mu \frac{m_s}{R} - \mu m_c \left[ (R - \ell \cos \alpha)^2 + (\ell \sin \alpha)^2 \right]^{-1/2}. \quad \text{(4.10)} \]

Once both \( T \) and \( V \) are obtained, we apply the assumption that \( Q_i = 0 \) since drag forces are neglected, and Eq. 3.1 results in the following equations of motion:

\[ \begin{align*}
\mu \frac{m_c (\ell \cos \alpha - R)}{(\ell^2 - 2R\ell \cos \alpha + R^2)^{3/2}} - \mu \frac{m_s}{R^2} + m_s R \dot{\theta}^2 + m_c \dot{\theta} \left( \ell \dot{\alpha} \cos \theta + \dot{\theta} (R - \ell \cos \alpha) \right) - m_c \ddot{R} \\
- m_c \left[ \ell \cos \alpha (\ddot{\alpha} - \dot{\theta}) \right] + R + \ell \sin \alpha (\ddot{\alpha} - \dot{\theta}) = 0 \quad \text{(4.11)}
\end{align*} \]

\[ \begin{align*}
m_c \ell \dot{\alpha}^2 \sin \alpha + 2m_c \ell \dot{R} \dot{\alpha} \cos \alpha - 2m_c R \ddot{R} \dot{\theta} - 2m_s R \ddot{R} \dot{\theta} - 2m_c R \ell \dot{\theta} \dot{\alpha} \sin \alpha + m_c \ell \dot{R} \sin \alpha \\
+ m_c \ell (\ell - R \cos \alpha) \ddot{\alpha} + \left( 2m_c \ell R \cos \alpha - m_c \ell^2 - (m_c + m_s) R \right) \ddot{\theta} = 0 \quad \text{(4.12)}
\end{align*} \]

\[ \begin{align*}
- \mu \frac{m_c \ell \dot{R} \sin \alpha}{(\ell^2 - 2R \ell \cos \alpha + R^2)^{3/2}} - 2m_c \ell \dot{R} \dot{\theta} \cos \alpha + m_c \ell R \dot{\theta}^2 \sin \alpha - m_c \ell \dot{R} \sin \alpha \\
- m_c \ell^2 \ddot{\alpha} + m_c \ell (\ell - R \cos \alpha) \ddot{\theta} = 0. \quad \text{(4.13)}
\end{align*} \]

### 4.1.2 Results & Discussion: Inelastic Swing

The following results are obtained from solving the system of equations for the inelastic tether swing scenario [Eq. 4.11–4.13] using Mathematica®. For this set of equations, we assume the system has just completed the deployment phase with \( \alpha_0 = 45^\circ, \dot{\alpha}_0 = 0, \) and
the C.M. (Center of Mass) at an initial altitude (consequently, a constant altitude from conservation of momentum) of 200 km. Figure 4.2 reflects the station's altitude during a swing phase period of approximately 1800 sec. As the capsule swings downward due to the inherent gravity gradient, the altitude of the station raises in turn, reaching a maximum value then returning toward its initial position. Consequently, the station's radial velocity should be at a zero point during this time of max altitude, which is shown in Figure 4.3.

Figure 4.2: Station altitude vs. time during inelastic swing.

Figure 4.3: Station radial velocity vs. time during inelastic swing.
The altitude and radial velocity of the capsule follow suit. As mentioned previously, Earth’s gravity gradient pulls the capsule toward itself, reaching an extremum before swinging back upward [Figure 4.4]. As a result, the radial velocity of the capsule reaches an extremum at this time [Figure 4.5], which is directly analogous to the radial velocity experienced by the station. As with both the capsule and station, longer tethers result in larger altitude and velocity variations.

Figure 4.4: Capsule altitude vs. time during inelastic swing.

Figure 4.5: Capsule radial velocity vs. time during inelastic swing.
Intuitively, we would expect that during these points of altitude extrema we would observe both \( \alpha = 0 \) and an extremum in the angular velocity, \( \dot{\alpha} \). These results are indeed reflected in Figures 4.6 and 4.7. Interestingly, it is observed that the tether length has little to no effect in the variation of both \( \alpha \) and \( \dot{\alpha} \). Of course, at this extremum point of \( \dot{\alpha} \), we also expect the overall capsule velocity to be at an extremum. This result is seen in Figure 4.8, where a longer tether results in a slower release velocity.

Figure 4.6: Angular position, \( \alpha \), vs. time during inelastic swing.

Figure 4.7: Angular velocity, \( \dot{\alpha} \), vs. time during inelastic swing.
So how can this large collection of data be made useful for finding the capsule/tether system’s release parameters? Since the release condition is defined by the point at which the capsule and tether are at the local vertical with respect to Earth ($\alpha = 0$), the author has written a custom program in Mathematica® that calculates the required parameters at this release position over various tether lengths [Appendix D]. The final result provided by this program is an InterpolatingFunction object that returns these desired parameters when a tether length is provided as an input. In this way, for any given tether length, an initial condition vector may be generated on the fly for a capsule/tether re-entry simulation.

Figure 4.9 reflects the capsule’s velocity and altitude at the release point. Of course, as seen from Figures 4.4 and 4.8, an increased tether length results in a lower altitude and also a lower velocity for the capsule at release. It is noted that these parameters vary linearly with tether length for a rigid tether. Similarly, as seen in Figure 4.10, the overall $\Delta v$ (difference between station velocity and capsule velocity) at the release point increases linearly with tether length. Since increasing tether length slows down the capsule relative to the station, this increase in $\Delta v$ makes sense intuitively.
Figure 4.9: Capsule altitude and velocity vs. tether length at inelastic release.

Figure 4.10: Overall $\Delta v$ vs. tether length at inelastic release.
4.2 Massless, Elastic Tether

In this model [Figure 4.11], the tether is assumed to be massless and elastic with

\[ k = \frac{EA}{\ell} = \frac{\pi Ed^2}{4\ell}, \]

where \( E \) is the Young’s modulus of the tether material [Appendix F]. As with the inelastic tether case, we will use this model to find the system parameters at the release condition \((\alpha = 0)\) to obtain approximate values for use as re-entry phase initial conditions.

Figure 4.11: Elastic capsule-tether system (dumbbell) model immediately before swing.
4.2.1 Derivation of Equations

For the elastic tether case, the author utilizes Lagrange’s equations [Eq. 3.1] with

\[ q = \begin{bmatrix} R & \theta & x & \alpha \end{bmatrix}^T, \quad Q_i = 0. \tag{4.15} \]

The station’s and capsule’s position vectors are defined as

\[ r_s = R \hat{c}_1 \tag{4.16} \]
\[ r_c = [R - (\ell + x) \cos \alpha] \hat{c}_1 + [(\ell + x) \sin \alpha] \hat{c}_2, \tag{4.17} \]

respectively. Accordingly, the station’s and capsule’s velocities can be obtained:

\[ v_s = \dot{R} \hat{c}_1 + R \dot{\theta} \hat{c}_2 \tag{4.18} \]
\[ v_c = [\dot{R} - \dot{x} \cos \alpha + (\ell + x) (\dot{\alpha} - \dot{\theta}) \sin \alpha] \hat{c}_1 + [R \dot{\theta} + \dot{x} \sin \alpha + (\ell + x) (\dot{\alpha} - \dot{\theta}) \cos \alpha] \hat{c}_2. \tag{4.19} \]

Consequently, the kinetic energies of the station and the capsule are

\[ T_s = \frac{1}{2} m_s (v_s \cdot v_s) = \frac{1}{2} m_s (\dot{R}^2 + R^2 \dot{\theta}^2), \]
\[ T_c = \frac{1}{2} m_c (v_c \cdot v_c) \]
\[ = \frac{1}{2} m_c \left[ [\dot{R} - \dot{x} \cos \alpha + (\ell + x) (\dot{\alpha} - \dot{\theta}) \sin \alpha]^2 + [R \dot{\theta} + \dot{x} \sin \alpha + (\ell + x) (\dot{\alpha} - \dot{\theta}) \cos \alpha]^2 \right], \tag{4.20} \]

where, again, \( m_s \) and \( m_c \) are the masses of the station and capsule, respectively. Thus, the total kinetic energy is

\[ T = T_s + T_c \]
\[ = \frac{1}{2} m_s (\dot{R}^2 + R^2 \dot{\theta}^2) + \frac{1}{2} m_c \left[ [\dot{R} - \dot{x} \cos \alpha + (\ell + x) (\dot{\alpha} - \dot{\theta}) \sin \alpha]^2 + [R \dot{\theta} + \dot{x} \sin \alpha + (\ell + x) (\dot{\alpha} - \dot{\theta}) \cos \alpha]^2 \right]. \tag{4.21} \]
The potential energy of both the station and capsule are

\[ V_s = -\mu \frac{m_s}{|r_s|} = -\mu \frac{m_s}{R} \]  
\[ V_c = -\mu \frac{m_c}{|r_c|} = -\mu m_c \left[(R - (\ell + x) \cos \alpha)^2 + ((\ell + x) \sin \alpha)^2\right]^{-1/2}, \]  

which leaves the following final expression for the potential energy:

\[ V = V_s + V_c + V_{spring} = -\mu \frac{m_s}{R} - \mu m_c \left[(R - (\ell + x) \cos \alpha)^2 + ((\ell + x) \sin \alpha)^2\right]^{-1/2} + \frac{1}{2} k x^2. \]  

(4.24)

Once both \( T \) and \( V \) are obtained, we apply the assumption that \( Q_i = 0 \) since drag forces are neglected, and Eq. 3.1 results in the following equations of motion:

\[ -m_c \left( (\ell + x) \sin \alpha (\ddot{x} - \dot{\theta}) + (\ell + x) \dot{x} \cos \alpha (\ddot{x} - \dot{\theta}) + \ddot{R} - \ddot{x} \cos \alpha \right) + \dot{x} \sin \alpha \left( \dddot{x} - \dot{\theta} + \ddot{\theta} \ddot{x} \right) + m_c \dot{\theta} \left( (\ell + x) \cos \alpha (\ddot{x} - \dot{\theta}) + R \dot{\theta} + \dot{x} \sin \alpha \right) + \frac{m_c \mu (\ell + x) \cos \alpha - R}{((R - (\ell + x) \cos \alpha)^2 + ((\ell + x) \sin \alpha)^2)^{3/2}} - m_s \ddot{R} + m_s R \dot{\theta}^2 - \frac{m_s \mu}{R^2} = 0 \]  

(4.25)

\[ R \left[ -m_c \left( (\ell + x) \cos \alpha (\dddot{x} - 2 \ddot{\theta}) + \ddot{x} \sin \alpha \right) - 2 m_c (\ell + x) \ddot{x} \sin \alpha \dot{\theta} + m_c (\ell + x) \dot{x}^2 \sin \alpha \right. \]
\[ -2 (m_c + m_s) \dot{R} \dot{\theta} + m_c (\ell + x) \left((\ell + x)(\dddot{x} - 3 \ddot{\theta}) + R \dddot{\theta} + 3 \dot{x} \sin \alpha + 2R \cos \alpha \ddot{\theta} \right) \]
\[ \left. + 2 m_c \dddot{x} \left( \ddot{x} - \ddot{\theta} \right) (\ell - R \cos \alpha + x) - (m_c + m_s) R^2 \dot{\theta} \right] = 0 \]  

(4.26)

\[ \frac{1}{2} \left[ -2 k x + 2 m_c \left( \dddot{x} - \dot{\theta} (\ell - R \cos \alpha + x) + (\ell + x) \dot{x} + \ddot{R} \sin \alpha \right) \right. \]
\[ - \frac{2 m_c \mu (\ell - R \cos \alpha + x)}{((R - (\ell + x) \cos \alpha)^2 + ((\ell + x) \sin \alpha)^2)^{3/2}} - 2 m_c \left(-R \cos \alpha + R \sin \alpha \left( \dot{x} + \dddot{\theta} \right) \right. \]
\[ \left. + R \left( \dddot{x} \cos \alpha \dot{\theta} + \dot{x} \sin \alpha \dddot{\theta} \right) \right] = 0 \]  

(4.27)
\[ m_c(\ell + x) \left[ R \left( \frac{\mu \sin \alpha}{(-2R(\ell + x) \cos \alpha + (\ell + x)^2 + R^2)^{3/2}} - \cos \alpha \dot{\theta} + \sin \alpha \dot{\theta}^2 \right) - l\ddot{\alpha} + l\dot{\theta} \right. \]

\[ \left. - \ddot{R} \sin \alpha - 2\dot{R} \cos \alpha \dot{\theta} - 2\dot{x} \left( \dot{\alpha} - \dot{\theta} \right) - x\ddot{\alpha} + x\dot{\theta} \right] = 0. \quad (4.28) \]

4.2.2 Results & Discussion: Elastic Swing

The following results are obtained from solving the system of equations for the elastic tether swing scenario [Eq. 4.25–4.28] using Mathematica®. For this set of equations, we assume the system has just completed the deployment phase with \( \alpha_0 = 45^\circ, \dot{\alpha}_0 = 0, x_0 = 0, \dot{x}_0 = 0 \), and the C.M. at an initial altitude of 200 km. Similar to the inelastic case, Figure 4.12 reflects the station's altitude during a swing phase period of approximately 1800 sec. As the capsule swings downward, the altitude of the station reaches a maximum value close to that of Figure 4.2. However, it does not seem to be quite as smooth as before.

The radial velocity of the capsule sheds more light onto the station’s motion. From Figure 4.13, we see that the radial velocity of the capsule follows a similar trend as in Figure 4.5; however, there are noticeable oscillations present. These oscillations grow larger with increased tether length and are the result of longitudinal vibrations within the tether. Figure 4.14 reflects the tether’s stretch distance versus time for various tether lengths.
Figure 4.13: Station radial velocity vs. time during elastic swing.

Figure 4.14: Tether stretch distance, x, vs. time during elastic swing.
Figure 4.15: Capsule altitude vs. time during elastic swing.

Figure 4.16: Capsule radial velocity vs. time during elastic swing.
There are two reasons to intuitively expect the tether stretch length to increase with tether length. First, the capsule altitude for long tethers will always be less than that of shorter tethers [Figure 4.15], thus encouraging tether stretching by experiencing an increased gravity force. The second reason stems from the expression for an equivalent spring constant for the tether [Eq. 4.14]: for a given tether material and tether diameter, the increased tether length will ultimately result in a decreased tether stiffness. The effect of longitudinal tether oscillations can also be seen in the radial velocity of the capsule [Figure 4.16]. In addition, they have an effect on both $\alpha$ and $\dot{\alpha}$, though it is noted that this effect is small [Figures 4.17–4.18], as is their dependence on tether length.

![Figure 4.17: Angular position, $\alpha$, vs. time during elastic swing.]

In order to obtain the parameters at the release condition for this elastic swing case, we are presented with an interesting problem. Due to the simplicity in this dynamic model, inherently it does not capture all the physics present in an actual station/capsule momentum transfer (such as the visco-elastic nature of the tether [24], transverse tether motion, etc.). Additionally, an actual system would employ active damping control via braking during the swing to mitigate longitudinal vibration effects [10]. However, in our case we merely wish to obtain realistic initial conditions for the re-entry simulation, so approximating these values (such as $x$) at tether release is satisfactory.
Figure 4.18: Angular velocity, $\dot{\alpha}$, vs. time during elastic swing.

Figure 4.19: Overall capsule velocity vs. time during elastic swing.
In order to perform this estimation, we will approximate the capsule’s radial velocity as zero at release [Figure 4.16], as was the case for the inelastic release [Figure 4.5]. Additionally, we will only acknowledge the tether’s mean extension length, \( x_{avg} \), in order to approximate the capsule’s altitude at the release point. But what is \( x_{avg} \) and how do we find it systematically? Observing Figure 4.14, we see that each extension profile follows a similar mean path when disregarding the sinusoidal oscillations. For our purposes, we will assume this mean path will have the form of a Gaussian distribution:

\[
x_{avg}(t) \approx a \exp \left[ -\frac{(t - b)^2}{2c^2} \right],
\]

where \( a, b, \) and \( c \) are constants that define the function’s distinct shape. Due to the built-in power of Mathematica®, we are able to easily automate the process of finding these constants using the \textit{FindFit[ ]} function. As demonstrated in Figure 4.20 for the case of a 25 km tether, this method works quite well. As with the inelastic tether case, the author has written a custom program that calculates the required parameters at this release position.

![Figure 4.20](image)

Figure 4.20: Mean stretch distance, \( x_{avg} \), (red) vs. time during swing of 25 km tether.
over various tether lengths, resulting in an *InterpolatingFunction* object that will return these parameters when provided a tether length as an input [Appendix D].

Figure 4.21 reflects the capsule’s velocity and altitude at the release point. Following the same trend as the inelastic case, an increased tether length results in a lower altitude and also a lower velocity for the capsule at release. It is noted that these parameters do not differ much from the inelastic case [Figure 4.9] since $x_{\text{avg}}$ never even exceeds 0.5 km [Figure 4.22]. Similarly, the overall $\Delta v$ (difference between station velocity and capsule velocity) at the release point increases linearly with tether length [Figure 4.23] without much deviation from the inelastic case [Figure 4.10].

![Figure 4.21: Capsule altitude and velocity vs. tether length at elastic release.](image_url)
Figure 4.22: Mean stretch distance, $x_{\text{avg}}$, vs. tether length at elastic release.

Figure 4.23: Overall $\Delta v$ vs. tether length at elastic release.
5 Re-entry Dynamics

5.1 Inelastic Tether Re-entry Validation

5.1.1 Literature Validation

Before we proceed with analyzing the results of the model for a re-entry capsule, we will first compare this method to previous documented results for a re-entering mass/tether system. As stated previously, the only known paper to discuss the dynamics of such a system is one written by Krischke et al. [19], who analyzed how a tether might be used as a hypersonic parachute for a small re-entering payload. Using his system parameters [Appendix G], the numerical results of the author’s software is compared to the results found in Krischke’s paper for payload velocity versus altitude [Figure 5.1].

![Figure 5.1: Capsule velocity vs. altitude validation (Krischke).](image)

While the results between the author’s software and Krischke’s software agree qualitatively, we notice that there is certainly some discrepancy between the two. These discrepancies are due to the fact that the author’s method only accounts for a endmass/tether system traveling in a two-dimensional equatorial trajectory, while Krischke’s software, a modified code originally developed at the Harvard-Smithsonian Center for Astrophysics called Master20 [25], takes into account a full three-dimensional trajectory. Accordingly, not only are the drag models different, but his software also includes perturbations such as the $J_2$ zonal harmonic of Earth’s gravitational field to account for Earth’s oblateness. Thus,
it is difficult to make a direct comparison between the results of each piece of software.

Another disparity found in Figure 5.1 lies with the “wobbly” nature of the author’s solution for a system which includes a tether. After some investigation, the author found that this irregularity is due to the fact that the payload’s mass is on the same order as the tether itself. In other words, the trajectory becomes more “wobbly” as the capsule mass approaches the tether mass. It is suspected that including more tether masses (links) in the author’s model would prevent this issue. Fortunately, the mass of the capsule investigated in this paper is about two orders of magnitude heavier than the tether, effectively eliminating this issue.

5.1.2 Lumped Mass Validation

An additional validation of importance includes checking for major differences in solutions with different numbers of tether masses (links). Figure 5.2 compares the differences in velocity versus altitude for various numbers of tether links. Overall, there is quite a good agreement between the solutions—especially in the 40-60 km region where the maximum heat flux occurs (see Eq. 6.2, Chapter 6). This agreement is encouraging since one of the limitations of the author’s software includes being computationally unable to model more tether links in Mathematica® on 32-bit x86 hardware.
5.2 Tether Length Comparison

5.2.1 Constant Initial Conditions

The following plots [Figures 5.3–5.5] present both the altitude and velocity of the capsule during re-entry for different tether lengths [Appendix F]. These results are all simulated using identical initial conditions taken from an inelastic 25 km tether swing release. In this way, we can study the effect of tether length on the capsule’s dynamics irrespective of the system’s initial conditions.

![Graph showing capsule altitude vs. time and tether length, 25 km inelastic IC’s.]

Figure 5.3: Capsule altitude vs. time and tether length, 25 km inelastic IC’s.

Following intuition, the tether causes the capsule to reach ground level sooner than if the tether was not present [Figure 5.3]. In other words, the added drag from increased tether length results in the slowing down of the capsule sooner, as expected [Figures 5.4 and 5.5]. It is noted that for identical initial conditions, the difference in tether length solutions is smaller than the difference between the solution having no tether and the solutions having a tether.

5.2.2 Different (Inelastic) Initial Conditions

The following section compares tether length solutions with their corresponding initial conditions (IC’s) for an inelastic tether swing release. Figures 5.6 and 5.7 compare the
Figure 5.4: Capsule altitude vs. longitude and tether length, 25 km inelastic IC’s.

Figure 5.5: Capsule velocity vs. time and tether length, 25 km inelastic IC’s.
altitudes and velocities of the different tethered solutions, respectively. As expected, the lower initial altitude and lower overall velocity of the longer tether lengths result in an earlier re-entry and an overall shorter trajectory time.

![Capsule altitude vs. time and tether length, inelastic IC’s.](image)

**Figure 5.6:** Capsule altitude vs. time and tether length, inelastic IC’s.

![Capsule velocity vs. time and tether length, inelastic IC’s.](image)

**Figure 5.7:** Capsule velocity vs. time and tether length, inelastic IC’s.

Figures 5.8 and 5.9 compare altitudes of different tethered solutions with their corresponding untethered solutions, each using the inelastic swing initial conditions of the specified tether length. As expected, in each case the tether reduces the time for de-orbit
Figure 5.8: Capsule altitude vs. time for various tether lengths, inelastic IC’s.
Figure 5.9: Capsule altitude vs. longitude for various tether lengths, inelastic IC’s.
as compared to the untethered solution, even for varying initial conditions. Figure 5.10 compares velocities of different tethered solutions with their corresponding untethered solutions, each using the inelastic swing initial conditions of the specified tether length. Again we see that the capsule velocities of the tethered solutions are reduced sooner as a result of the increased drag from the tether. The motion of the tether during re-entry can be seen in Figure 5.11, which compares the tether angles, $\alpha_i$, of each $i$-th link for different tethered solutions, each using the inelastic swing initial conditions of the specified tether length.

5.2.3 Different (Elastic) Initial Conditions

The following section compares tether length solutions with their corresponding IC’s for an elastic tether swing release. Figures 5.12 and 5.13 compare the altitudes and velocities of the different tethered solutions, respectively. Each of these solutions varies little with respect to the corresponding inelastic IC case. Figures 5.14 and 5.15 compare altitudes of different tethered solutions with their corresponding untethered solutions, each using the elastic swing initial conditions of the specified tether length.

As before when using inelastic IC’s, in each case the tether reduces the time for de-orbit as compared to the untethered solution. Figure 5.16 compares velocities of different tethered solutions with their corresponding untethered solutions, each using the elastic swing initial conditions of the specified tether length. As before, the capsule velocities of the tethered solutions are reduced sooner as a result of the increased drag from the tether. These similarities between solutions using both the elastic and inelastic swing release IC’s are expected given that the release conditions are so close themselves [Figures 4.9 and 4.21].
Figure 5.10: Capsule velocity vs. time for various tether lengths, inelastic IC’s.
Figure 5.11: Tether angles, $\alpha_i$, vs. time for various tether lengths, inelastic IC’s.
Figure 5.12: Capsule altitude vs. time and tether length, elastic IC’s.

Figure 5.13: Capsule velocity vs. time and tether length, elastic IC’s.
Figure 5.14: Capsule altitude vs. time for various tether lengths, elastic IC’s.
Figure 5.15: Capsule altitude vs. longitude for various tether lengths, elastic IC's.
Figure 5.16: Capsule velocity vs. time for various tether lengths, elastic IC’s.
5.3 Tether Diameter Comparison

The following plots [Figures 5.17 and 5.18] compare re-entry solutions with differing tether lengths and diameters, each using the corresponding IC’s for an inelastic tether swing release. Specifically, the solutions are compared for the (realistic) tether diameters of 1 mm and 2 mm. As expected, for each case the thicker tether provides more drag, slowing down the capsule/tether system sooner.

5.4 Capsule Mass Comparison

The following plots [Figures 5.19–5.21] present both the altitude and velocity of the capsule during re-entry for different capsule masses. These results were all simulated using the identical initial conditions of an inelastic 30 km tether swing release. In this way, we can look at the effect of capsule mass on the system’s dynamics irrespective of the initial conditions or the capsule’s geometry. Following intuition, the lighter capsule will slow down sooner due to the increased drag effects.
Figure 5.17: Capsule altitude vs. time for various tether lengths and diameters, corresponding IC’s.
Figure 5.18: Capsule velocities vs. time for various tether lengths and diameters, corresponding IC’s.
Figure 5.19: Capsule altitude vs. time and capsule mass, same IC’s.

Figure 5.20: Capsule altitude vs. longitude and capsule mass, same IC’s.
Figure 5.21: Capsule velocity vs. time and capsule mass, same IC’s.
6 Capsule Heating

6.1 Aerodynamic Heating Theory

As the capsule/tether system is entering the atmosphere at incredible (hypersonic) speeds, a thick boundary layer forms on the capsule surface and results in a strong bow shock that precedes the leading edge. Due to the high temperatures associated with this shock layer, there is a substantial amount of heat transfer into the re-entry vehicle. The convective heat flux from the shock layer to the surface (wall) of the capsule is

\[ q''_w = -\kappa \left( \frac{\partial T}{\partial y} \right)_w, \tag{6.1} \]

where \( \kappa \) is the thermal conductivity, \( T \) is temperature, and \( y \) is the direction normal to the surface. Because the leading edge of the blunt-bodied capsule will experience the brunt of the heat load, the stagnation-point will be our point of interest.

Unfortunately, the solution to Eq. 6.1 requires knowledge of the surrounding flow field, which is difficult to solve since the high-temperature freestream fluid is not in equilibrium, chemically reacting, and possibly ionized. Though the governing equations that describe a compressible, stagnation-point boundary layer may be solved by themselves using a numerical shooting technique (such as Monte Carlo) [26], a much more complex model that accounts for chemical dissociation and other molecular-level physics would also need to be considered. Due to these complexities, a simpler model will be used in order to perform a basic assessment.

A relation used frequently to find the stagnation-point aerodynamic heat flux is one that was developed by Sutton and Graves [27], who numerically solved the boundary layer equations in chemical equilibrium to find the (cold wall) convective heating of a re-entering body. In this model, the stagnation-point convective heat flux is described as

\[ q''_{aero} = K \sqrt{\frac{p}{R_N} v^3}, \tag{6.2} \]
where $\rho$ is the density, $v$ is the freestream velocity, $R_N$ is the nose radius, and $K \approx 1.83 \times 10^{-4} \text{ kg}^{1/2}/\text{m}$ for Earth’s atmosphere. Not only is this convective heating model widely used for re-entry problems [19, 28, 29, 30], but this $R_N^{-1/2}$ dependence describes why re-entry vehicles must have a blunt shape to reduce aerodynamic heating.

### 6.2 Capsule Heating Model

In order to solve for the transient temperature profile at the stagnation-point, we will adopt a method similar to that of Meese and Nørstrud [30], who developed a simplified 1-D heat transfer conduction model for the surface of a reusable re-entry vehicle. This abridged 5-layer model accounts for both convective and radiative effects at the surface while making the assumption that no heat escapes into the capsule interior [Table 6.1, Figure 6.1].

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>$h$ [mm]</th>
<th>$\kappa$ [W/mK]</th>
<th>$\alpha \times 10^6$ [m$^2$/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RCC</td>
<td>152.4</td>
<td>4.3</td>
<td>3.534</td>
</tr>
<tr>
<td>2</td>
<td>RTV-560</td>
<td>2.0</td>
<td>0.424</td>
<td>0.2712</td>
</tr>
<tr>
<td>3</td>
<td>FRCI-12</td>
<td>4.0</td>
<td>0.0505</td>
<td>0.2283</td>
</tr>
<tr>
<td>4</td>
<td>RTV-560</td>
<td>2.0</td>
<td>0.424</td>
<td>0.2712</td>
</tr>
<tr>
<td>5</td>
<td>Al</td>
<td>25.4</td>
<td>167.4</td>
<td>70.6</td>
</tr>
</tbody>
</table>

Table 6.1: Capsule layer properties.

![1-D heat transfer model for the capsule.](image_url)
Taking a control volume around the capsule layers, we have

\[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_x} \frac{\partial T}{\partial t},
\]

(6.3)

where \( \alpha_x \) is a piecewise function of the values in Table 6.1 based on location, \( x \). The boundary conditions for Eq. 6.3 are

\[
\frac{\partial T}{\partial x} (x = 0, t) = -\frac{1}{\kappa_1} \left( q_{\text{aero}}'' - q_{\text{rad}}'' \right) = -\frac{1}{\kappa_1} \left( K \sqrt{\frac{\rho}{R_N}} v^3 - \varepsilon \sigma T^4 \right)
\]

(6.4)

\[
\frac{\partial T}{\partial x} (x = h_{\text{tot}}, t) = 0,
\]

(6.5)

where \( \varepsilon \) is the emissivity (assumed \( \approx 0.85 \)), \( \sigma \) is the Stefan-Boltzmann constant, \( \kappa_1 \) is the thermal conductivity of the first layer, and \( h_{\text{tot}} \) is the total thickness of all five capsule layers. Both Eq. 6.4 and 6.5 hold true \( \forall t \in [0, t_{\text{final}}] \). The single initial condition is

\[
T(x, t = 0) = 294 \text{ K}; \quad \forall x \in [0, h_{\text{tot}}],
\]

(6.6)

which is the temperature of the NASA space shuttle after several Earth orbits (with sunlight) before re-entry [31].

In order to solve Eq. 6.3 together with the conditions in Eq. 6.4–6.6, we will use \texttt{NDSolve[]} in \texttt{Mathematica®} to solve this nonlinear PDE numerically [Appendix E]. With the solution for \( T(x, t) \) in place, temperatures may be found at any desired location at any instance of time. It is noted that the temperatures of the tether will not be found in this model since the tether is assumed to remain intact throughout the entire simulation.

### 6.3 Heating Results & Analysis

The following section will compare various capsule/tether system parameters against their corresponding effects on the heating of the capsule, specifically the aerodynamic
(convective) heat flux and temperatures located at the stagnation region of the capsule [Figure 6.1].

6.3.1 Tether Length Comparison

We will first compare the effects of tether length on the aerodynamic heat flux and temperatures experienced by the capsule, holding all other capsule/tether system parameters constant. The following results are all simulated using identical initial conditions taken from an inelastic 25 km tether swing release. Following intuition and the $v^3$ dependence in the heat flux term [Eq. 6.2], the tether results in a decrease in heat flux experienced by the capsule [Figure 6.2]. In general, an increased tether length results in a decreased heat flux until the point at which the tether length becomes long enough to promote slightly higher peak heating. The small change in heat flux between each tether length also confirms that an analysis of an elastic tether during re-entry is not needed since the maximum elongation of the tether is just over 2% [Appendix F]. The resulting temperatures on the capsule are shown in Figures 6.4 and 6.5 at different depths into the first layer of capsule material. Observing the results, we see that the peak temperatures are experienced at the surface (stagnation-point) of the capsule, while the temperature profiles differ less

Figure 6.2: Convective heat flux vs. time and tether length, 25 km inelastic IC’s.
Figure 6.3: Convective heat flux vs. altitude and tether length, 25 km inelastic IC’s.

Further into the material. As shown in Table 6.1, the first layer consists of RCC (Reinforced Carbon-Carbon) with a depth of $h_1$.

The next set of plots compare the effects of tether length on heat flux and temperatures for an inelastic tether swing release of the specified tether length. From Figure 6.6 we see that increasing tether length does indeed reduce the aerodynamic heat flux on the capsule. However, it is noted that there is a point at which the tether becomes too long due to the fact that the slower release condition will cause a steeper re-entry trajectory. The temperatures of the capsule are shown in Figures 6.7 and 6.8. As expected, not only does the presence of a tether reduce temperatures, but the temperature profile becomes more uniform further into the capsule material.

In the case of an elastic tether swing release, the resulting heat fluxes and temperatures are very similar to that of the inelastic case. Figure 6.9 shows the resulting convective heat flux while Figure 6.10 gives the temperature history at the surface of the capsule ($x = 0$). In each plot, the results mimic their inelastic release equivalent quite closely [Figures 6.6 and 6.7].

A more direct comparison between the tethered and untethered systems for both elastic and inelastic swing releases can be made by considering differences in maximum heat flux and surface temperature. Figure 6.11 reveals the percent decrease in the maximum
Figure 6.4: Capsule temperature vs. time, tether length, and material depth; inelastic IC’s.
Figure 6.5: Capsule temperature vs. altitude, tether length, and material depth; inelastic IC’s.
Figure 6.6: Convective heat flux vs. time and tether length, inelastic IC’s.
Figure 6.7: Temperature ($x = 0$) vs. time and tether length, inelastic IC’s.
Figure 6.8: Temperature \((x = h_1/4)\) vs. time and tether length, inelastic IC’s.
Figure 6.9: Convective heat flux vs. time and tether length, elastic IC’s.
Figure 6.10: Temperature ($x = 0$) vs. time and tether length, elastic IC’s.
convective heat flux experienced by the capsule due to the tether, while Figure 6.12 reflects the percent decrease in maximum surface temperature of the capsule due to the tether.

![Graph of Decrease Aero. Flux](image1)

**Figure 6.11:** Percent decrease in maximum heat flux from tether drag.

![Graph of Decrease Temp.](image2)

**Figure 6.12:** Percent decrease in maximum temperature from tether drag.

Though an increased tether length does indeed reduce aerodynamic heat flux further, this reduction only occurs up until a critical point. At this critical point, as mentioned previously, the slower release velocity caused by the increased tether length will force the system to enter the atmosphere at bit steeper. This steeper trajectory will, in turn, result in higher heat fluxes on the capsule. Overall, the presence of a tether can reduce the maximum heat flux by almost 30% [Figure 6.11], while the resulting decrease in
temperature may be reduced by almost 10% [Figure 6.12].

6.3.2 Tether Diameter Comparison

The tether diameter also affects the heating of the capsule. Figure 6.13 shows the convective heat flux on the capsule for the case of an inelastic swing release, while Figure 6.14 displays the corresponding temperatures at the surface of the capsule. From initial inspection, we see that the resulting heat flux from the 2 mm tether is indeed reduced when compared to the 1 mm case [Figure 6.6]. Accordingly, the capsule surface temperatures are also reduced when compared to the 1 mm case [Figure 6.7].

The 1 mm and 2 mm cases are compared more directly when considering the differences in maximum heat flux and surface temperature. Figure 6.15 reveals the percent decrease in the maximum convective heat flux experienced by the capsule due to the tether, while Figure 6.16 reflects the percent decrease in maximum surface temperature of the capsule due to the tether. Clearly, the 2 mm tether performs better at reducing heat loads by offering up to almost 60% less convective heat flux and 20% less surface temperature than an equivalent case without a tether.

6.3.3 Capsule Mass Comparison

The effect of capsule mass on heat loads is investigated in Figures 6.17–6.20 given a 30 km tether with an inelastic swing release. Figure 6.17 shows the convective heat flux on the capsule for various capsule masses, while Figure 6.18 displays the corresponding temperatures at the surface of the capsule. From inspection, we see that the resulting heat flux certainly decreases as the capsule mass decreases. Accordingly, the capsule surface temperatures are also reduced when the capsule mass is decreased. Figure 6.19 reveals the percent decrease in the maximum convective heat flux experienced by the capsule due to the tether, while Figure 6.20 reflects the percent decrease in maximum surface temperature of the capsule due to the tether. These results suggest that significant
Figure 6.13: Convective heat flux vs. time and (2 mm) tether length, inelastic IC’s.
Figure 6.14: Capsule temperature vs. time and (2 mm) tether length, inelastic IC’s.
Figure 6.15: Decrease in maximum heat flux from a 1 mm and 2 mm tether.

Figure 6.16: Decrease in maximum temperature from a 1 mm and 2 mm tether.
Figure 6.17: Convective heat flux vs. time and capsule mass, inelastic IC’s.
Figure 6.18: Capsule temperature vs. time and capsule mass, inelastic IC’s.
reductions can be made in the re-entry heat loads by reducing the capsule mass.

Figure 6.19: Decrease in maximum heat flux from tether with capsule mass.
Figure 6.20: Decrease in surface temperature from tether with capsule mass.
7 Conclusion

7.1 Summary

The use of tethers in space has an exciting promise in future astronautical applications, with the strong possibility of providing more sophisticated functionality to satellites and spacecraft. Some of these applications include, but are not limited to, adjoining satellites, creating microgravity environments, generating power, and transferring momentum between spacecraft. The latter of these was the focus of this project: specifically, the author investigated the possible reduction in convective (aerothermal) heat flux and temperatures on a manned capsule as a result of re-entry with an attached momentum exchange tether, including how various tether parameters affected these results.

After performing a thorough literature review, the author determined that it was necessary to develop software capable of simulating the re-entering capsule/tether system. Using a “bottom-up” approach by modeling the system as a series of lumped masses and rigid rods (links) through the simplicity of Lagrange’s equations, software was developed using Mathematica® that was capable of generating the equations of motion for any arbitrary number of links. In order to solve the resulting equations of motion, a separate dynamic dumbbell station/capsule swing model was developed for both an inelastic and elastic tether to provide a realistic initial condition vector for the re-entry phase. With the initial conditions in hand, the author used the built-in numerical capabilities of Mathematica® to solve the equations of motion numerically, allowing for a simple and effective software workflow.

For the analysis, the resulting motion of the capsule was studied in the case of varying tether length, tether diameter, and capsule mass. Accordingly, using these solutions for the system’s dynamics, the resulting heat loads on the capsule were calculated using a simplified 1-D multilayer heat transfer model [30]. Thus, using this model, the incoming convective heat flux and temperature profile of the capsule material were found and analyzed for each of the dynamic cases.
7.1.1 Results

The following results are provided given a baseline tether material (Zylon®), tether diameter (1 mm), and capsule mass (4800 kg) [Appendix F].

1. The presence of a tether indeed produces more drag at high altitudes, resulting in an earlier re-entry and a shorter re-entry time [Figures 5.4 and 5.5].

2. Increased tether length produces more drag, causing re-entry to occur earlier and decreasing re-entry time [Figures 5.6 and 5.7].

3. Increased tether diameter (2 mm from 1 mm) also results in more drag, decreasing the time of re-entry [Figures 5.17 and 5.18].

4. Decreased capsule mass allows the capsule to slow down sooner due to the increased effect of drag (from the reduced capsule momentum) [Figures 5.19 and 5.21].

5. Increased tether length results in reduced aerodynamic heat fluxes on the capsule up until the point where the slower release velocity actually produces higher fluxes from a steeper re-entry trajectory. For a standard 1 mm tether diameter, the presence of a tether can decrease the convective heat flux on the capsule by almost 30% while decreasing the surface temperature by almost 10% [Figures 6.11 and 6.12].

6. Increased tether diameter (2 mm from 1 mm) also decreases aerodynamic heat fluxes and surface temperatures. For this case, the convective heat flux into the capsule can be reduced by almost 60% and the surface temperature by just over 20% when compared to an equivalent system without a tether [Figures 6.15 and 6.16].

7. Decreased capsule mass results in reduced capsule heat loads. For a 30 km long, 1 mm tether, the presence of a tether can reduce the aerodynamic heat flux by over 20% while reducing the capsule surface temperature by almost 8% [Figures 6.19 and 6.20].
With these reductions in heat loads on the capsule, it is obvious that a de-orbit maneuver using a momentum exchange tether indeed has serendipitous results. However, it is debatable as to whether or not a momentum exchange tether would serve as an effective hypersonic parachute for a manned re-entry mission: such a conclusion depends on yet unexplored or undetermined parameters such as the fatigue properties of the capsule’s heat shield or the actual weight of the capsule before re-entry. Nevertheless, if designing a mission to use a relatively light capsule and a long/thick tether, this feasibility study certainly suggests that further investigation into this subset of tether de-orbiting is warranted.

7.2 Recommendations & Future Work

1. An improved 3-D system model that includes capsule attitude dynamics, capsule lift, and Earth’s oblateness should be developed in order to provide a more accurate assessment of both the heat loads and trajectory information.

2. A thorough evaluation/reassessment of the software algorithm and code structure should be made in order to increase memory efficiency and to allow for a higher (and more accurate) tether resolution to be computed. Additionally, a rewrite of the improved algorithm in FORTRAN or C could also increase computational speed dramatically.

3. Further study into the heat interaction between the hypersonic flow field and capsule surface should be considered in an updated heat transfer model that includes both 3-D capsule attitude dynamics and a more accurate materials selection within the heat shield.

4. The drag coefficient information [Appendix B] should be updated to account for capsule attitude/orientation during re-entry in order to simulate a more accurate flight path.
5. For a more realistic assessment of tether performance, a 2-D/3-D heat transfer model should be constructed for the tether (in conjunction with an appropriate tether failure criterion) in order to uncover when the tether would fail due to convective heating during re-entry.
References


Appendix A: Atmospheric Data

The following data has been pulled from Mathematica®, which is interfaced directly to Wolfram’s database of curated scientific information [23].

A.1 Mean Density

Figure A.1: Mean density, \( \rho \), vs. altitude for Earth’s atmosphere.
A.2 Mean Free Path

Figure A.2: Mean free path, $\lambda$, vs. altitude for Earth’s atmosphere.
Appendix B: Drag Coefficient Data

Following Gallais [29], a drag coefficient approximation (which is taken to be the same for both the capsule, a sphere, and the tether, a cylinder) for a re-entry vehicle may be approximated by

\[ C_D = C_c + \Phi(\text{Kn}_\infty)(C_m - C_c), \]

where \( C_c \) is the drag coefficient at continuum conditions (~1), \( C_m \) is the drag coefficient at molecular conditions (~2), and \( \Phi \) is a bridging function:

\[ \Phi(\text{Kn}_\infty) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\sqrt{\pi} \Delta\text{Kn} \ln \left( \frac{\text{Kn}_\infty}{\text{Kn}_{mi}} \right)}{\Delta\text{Kn}} \right) \right]. \]

In Eq. B.2, \( \Delta\text{Kn} = \ln \text{Kn}_m - \ln \text{Kn}_c \) is the logarithmic width of the intermediate zone, \( \text{Kn}_m \approx 10, \text{Kn}_c \approx 10^{-2} \), and \( \text{Kn}_{mi} \) is the middle value of the intermediate zone (~0.5) [29].
Appendix C: Re-entry Simulation Code

(* ***** Define Global Parameters ***** *)

(* Parameters for Excalibur Almaz System *)
Num = 4; (* Number of links *)
a0 = 45 (π/180); (* Beginning angle of release *)
ρz = 1.56; (* Density of in g/cm^3 *)
d = 0.001; (* Diameter = 1 mm *)
elasticity = 280 × 10^-9; (* Young's modulus = 280 GPa *)
ms = 15000; (* Camelot station mass = 15 metric tons *)
mc = 4800; (* Almaz capsule mass = 4.8 metric tons *)
capRadius = 1.1; (* Capsule diameter = 2.2 meters *)
alto = 200000; (* Initial station/capsule altitude is 200 km *)
Rpl = 6.37 10^6; (* Radius of Earth *)

(* Parameters for Krischeck *)
TotLength = 20000; (* Tether length in km *)
Num = 4; (* Number of links *)
a0 = 45 (π/180); (* Beginning angle of release *)
ρz = 0.97; (* Density of Spectra = 0.97 g/cm^3 *)
d = 0.001; (* Diameter=1 mm *)
mc = 23; (* Endmass = 23 kg *)
capRadius = 0.2; (* Endmass diameter = 0.4 meters *)
alto = 680000; (* Initial station/capsule altitude is 680 km *)
v0 = 7276; (* Initial capsule velocity = 7276 m/s *)
Omega = Cos[40°] [7.27 10^-5]; (* earth's atmospheric rotation velocity *)
Rpl = 6.37 10^6; (* Radius of Earth *)

(* ***** Main Re-entry Simulation Software ***** *)

(* Load IC info. *)
rInfoCase1 = Get["~/Dropbox/MSc/Calculation/rInfoCase1.dat"]; (* rInfoCase2.dat for elastic ICs *)
(* Set tether lengths *)
TotLengths = 10000 {20, 25, 30, 35, 40, 45, 50};

(* Start main loop over tether lengths *)
For[j = 1, j ≤ Length[TotLengths], ++j,
    TotLength = TotLengths[[j]];
    (* Set values & pull density/mean free path data *)
    mass = ρz π / 4 (100 d) 2 (100 TotLength) / 1000; (* Tether mass in kg *)
    values = {m → mass / Num, μ → 3.986 × 10^14, Sc → π capRadius^2},
    Ω → 7.27 10^-5, CDft → 0.01, l → TotLength / Num, Rpl → 6.37 10^6};
    ρ = Get["~/Dropbox/MSc/Calculation/density.dat"]; (* Set Knudson # & drag coefficient data *)
    Knc[alt_] := λ[alt] / (2 capRadius);
    Knt[alt_] := λ[alt] / d;
    CD[alt_] := 1 + (1 + Erf[Sqrt[π] / (Log[10] - Log[.01]) Log[Knc[alt] / .5]]) (2 - 1);
    CDt[alt_] := 1 + (1 + Erf[Sqrt[π] / (Log[10] - Log[.01]) Log[Knt[alt] / .5]]) (2 - 1);
(* Set kinetic & potential energies *)

\[ v_{i[n]} := \left\{ R'[t] - l \sin[\alpha_i[t]] (\theta'[t] + \alpha_i'[t]) - \sum_{n=1}^{i-1} (l \sin[\alpha_n[t]] (\alpha_n'[t] + \theta'[t])), \right. \\
R[t] \theta'[t] + l \cos[\alpha_i[t]] (\theta'[t] + \alpha_i'[t]) + \sum_{n=1}^{i-1} (l \cos[\alpha_n[t]] (\alpha_n'[t] + \theta'[t])) \}; \]

\[ v_{2i[n]} := v_{i[n]} \cdot v_{i[n]} \] // Simplify;

(* Kinetic energy *)

\[ T = \frac{1}{2} \frac{mc (R'[t]^2 + R[t]^2 \theta'[t]^2) + \sum_{i=1}^{\text{num}} \left( \frac{1}{2} m v_{2i[n]} \right)}{R[t]}; \]

\[ V_c = -\mu \frac{mc}{R[t]}; \]

\[ R_{i[n]} := \left\{ R[t] + l \cos[\alpha_i[t]] + \sum_{n=1}^{i-1} l \cos[\alpha_n[t]], l \sin[\alpha_i[t]] + \sum_{n=1}^{i-1} l \sin[\alpha_n[t]] \right\}; \]

\[ R_{absi[n]} := \sqrt{R_{i[n]} \cdot R_{i[n]}}; \]

(* Potential energy *)

\[ V = V_c + \sum_{i=1}^{\text{num}} \left( -\mu \frac{m}{R_{absi[i]}} \right); \]

(* Calculate forces on capsule *)

\[ \text{altCapsule} = (R[t] - R_{ipl}); \]

\[ Q_x = -\frac{1}{2} \text{CD[altCapsule]} \text{Sc} \rho[\text{altCapsule}] \sqrt{(R'[t])^2 + (R[t])^2 (-\Omega + \theta'[t])^2}; \]

\[ Q_y = -\frac{1}{2} \text{(R[t])^2 CD[altCapsule]} \text{Sc} \rho[\text{altCapsule}] (-\Omega + \theta'[t]) \sqrt{(R'[t])^2 + (R[t])^2 (-\Omega + \theta'[t])^2}; \]

(* Set some rotation coordinates stuff *)

\[ \text{eRotate} = \text{RotationTransform}[\theta(t)]; \]

\[ \text{bRotate[i[n]] := RotationTransform[-\alpha_i[t]]}; \]

\[ \text{b2[i[n]] := (-Sin[\alpha_i[t]], Cos[\alpha_i[t]])}; \]

\[ \text{b1[i[n]] := (Cos[\alpha_i[t]], Sin[\alpha_i[t]])}; \]

(* Set position vector for force in inertial coordinates *)

\[ r_{i[2]} := \left\{ R[t] + l/2 \cos[\alpha_i[t]] + \sum_{n=1}^{i-1} l \cos[\alpha_n[t]], l/2 \sin[\alpha_i[t]] + \sum_{n=1}^{i-1} l \sin[\alpha_n[t]] \right\} \] // eRotate;

(* Define velocity vector of tether link midpoint with respect to the atmosphere *)

\[ \text{vw[i[n]] := \{ R'[t] - l/2 (\alpha_i'[t] + \theta'[t] - \Omega) Sin[\alpha_i[t]] - \right. \]

\[ \sum_{n=1}^{i-1} l (\alpha_n'[t] + \theta'[t] - \Omega) Sin[\alpha_n[t]], R[t] (\theta'[t] - \Omega) + \]

\[ \left. \right\} \]
\[ \frac{1}{2} \left( a'_l [t] + \theta' [t] - \Omega \right) \cos[a_l [t]] + \sum_{n=1}^{i-1} \left( a_n ' [t] + \theta' [t] - \Omega \right) \cos[a_n [t]] \]
(* Generate array of equations, including forces *)
GenEquations[T_, V_, q_, Forces_] :=
Module[{n, j, eqns},
  n = Length[q];
  eqns = Array[0, n];
  For[j = 1, j <= n, j++,
    eqns[[j]] = D[D[T, D[q[[j]], t]], t] - D[T, q[[j]]] + D[V, q[[j]]] = Forces[[j]];
  ];
  Return[eqns];
];

(* Generate equations *)
eqns = GenEquations[T, V, q, Forces];

(* Clear more stuff *)
ClearAll[Forces, T, V];

(* Solve for/generate initial conditions *)
Module[{n, m, j, templength},
  n = Num;
  templength = 25000;
  x0 = Array[0, 2 Length[q]];
  x0[[1 ;; 4]] = {R[0] = Rpl + rInfoCase[templength][[3]],
    R'[0] = 0, \[Theta][0] = rInfoCase[templength][[4]], \[Theta]'[0] =
    rInfoCase[templength][[1]]/(Rpl + rInfoCase[templength][[3]])} /. values;
  For[j = 1, j <= n, j++,
    m = 2 (2 + j);
    x0[[m - 1 ;; m]] = {\[alpha][0] = 0, \[alpha]'[0] = -1 (rInfoCase[templength][[2]])} /. values;
  ];
]

(* Solve equations numerically until ground level is reached *)
eqns = Join[eqns, x0] /. values;
NDSolve[eqns, qnt, {t, 0, 4000}, MaxSteps -> \[Infinity],
  Method -> ("EventLocator", "Event" -> Evaluate[R[t] - Rpl /. values],
    "EventAction" -> Throw[Print["rInfoCase", parts]; // Timing >>
    
  ]
];
Appendix D: Initial Condition Generation Code

(* ***** Initial Condition (Swing Simulation) Software ***** *)
(* *** Inelastic Swing Simulation *** *)

Needs["VariationalMethods"];
(* Specify positions and velocities of station and capsule *)
alt = alt0;
solni = (R[t] - (l) Cos[a[t]]), (l) Sin[a[t]]);
vs = [R[t], R[t] θ'[t]];
vc = [R'[t] + (l) (a'[t] - θ'[t]) Sin[a[t]], R[t] θ'[t] + (l) (a'[t] - θ'[t]) Cos[a[t]]];

(* Specify kinetic and potential energies *)
1
T = l ms (vs.vsi) + mc (vc.vc);
2
V = -µ
R[t] - µ
mc
√rc.rc

(* Specify values to use, as well as initial conditions *)
valuesi = {µ -> 3.986 × 10^-14, Rpl -> 6.37 × 10^6};
mc
dcm = l temp;
ms + mc

ltemp = R0 = √((rcm + dcm l Cos[a0])^2 + (dcm l Sin[a0])^2);
x0i = [R[0] = r0, R'[0] = 0, θ[0] = 0,
θ'[0] = Sqrt[µ/rcm] / rcm, a[0] = a0, a'[0] = 0]. valuesi;
(* Define the equations of motion, damping forces, custom spring coefficient *)
eqnsi = EulerEquations[T - V, (R[t], θ[t], a[t]), t];

(* Kfun[x_] := Piecewise[((k, x>0), (100 k, x<0))];
eqnsi := eqnsi /. K->Kfun[x[t]]; *)

(* Define an array of both tether lengths
and corresponding spring constant values *)
ltemp = Table[n, {n, 5000, 50000, 5000}];

(* Solve the system of ODEs numerically *)
solni = Array[NDSolve[{eqnsi, x0i} / valuesi /. l -> ltemp[[n]],
{R, θ, a}, {t, 0, 1800}], {n, 1, Length[ltemp]}];

(* Get all data at release point *)
time = 900;
releaseInfo = Array[Null, Length[solni]];
releaseInfo2 = Array[Null, Length[solni]];
times = Array[Null, Length[solni]];
Module[{vctemp, Rcd, vtemp},
For[i = 1, i < Length[solni], i++]

temp = FindRoot[Evaluate[a[t] / solni[[i]]], {t, 800, 1100}];
(*time = t /. temp;*)
times[[i]] = temp;
(* Find velocities and positions *)
vctemp = vc[[2]] / ltemp[i] / solni[[i]] / temp;
vctemp = vctemp[[1]];
Rcd = vc[[1]] / ltemp[i] / solni[[i]] / temp;
Rcd = Rcd[[1]];
RCo = rc[[1]] / ltemp[i] / solni[[i]] / temp;
\begin{verbatim}
Rc0 = (Rc0[1] - Rpl) /. valuesi;
alphad = \alpha'[t] /. solni[i] /. temp; alphad = alphad[1];
\theta temp = \theta[t] /. solni[i] /. temp; \theta temp = \theta temp[1];

Rd = vs[[1]] /. solni[i] /. temp; Rd = Rd[1];
vstemp = vs[[2]] /. solni[i] /. temp; vstemp = vstemp[[1]];
(* releaseInfo={cl vel., t.length, alt., \alpha', c2 vel., \theta,delta v} *)
releaseInfo[i] =
{vtemp, ltemp[i] / 1000, Rc0 / 1000, alphad, Rd, \theta temp, vtemp - vctemp};

(* releaseInfo2={cl vel., t.length, c2 vel.} *)
releaseInfo2[[i]] = {vtemp, ltemp[i] / 1000, Rd};
]

(* Generate interpolating function *)
f = (1000 releaseInfo[[t, 2]],
{releaseInfo[[t, 1]], releaseInfo[[t, 4]], 1000 releaseInfo[[t, 3]],
releaseInfo[[t, 6]], releaseInfo[[t, 7]]}) & @ Range[1, Length[ltemp]]; rInfoCasel = Interpolation[f] >> "/Dropbox/MSc/Calculation/rInfoCasel.dat";

(* *** Elastic Swing Simulation *** *)

Needs["VariationalMethods"];
altn = alt0;
(* Specify positions and velocities of station and capsule *)
rcm = 6.37 \times 10^{-6} + alt0;
rc = (R[t] - (1 + x[t]) \cos[a[t]], (1 + x[t]) \sin[a[t]]);
vs = (R[t], R[t] \theta'[t]);
vc = \{R'[t] - x'[t] \cos[a[t]] + (1 + x[t]) (a'[t] - \theta'[t]) \sin[a[t]],
R[t] \theta'[t] + x'[t] \sin[a[t]] + (1 + x[t]) (a'[t] - \theta'[t]) \cos[a[t]]\};

(* Specify kinetic and potential energies *)
1
\[ T = \frac{1}{2} ms (vs.vs) + \frac{mc}{2} (vc.vc); \]
1
\[ V = \frac{\mu}{R[t]} - \frac{\mu mc}{\sqrt{rc.rc}} + \frac{1}{2} k x[t]^2; \]

(* Specify Rayleigh dissipation function *)
RDF = \frac{1}{2}
\[ c x'[t]^2; \]

(* Specify values to use, as well as initial conditions *)
valuesi = \{\mu \rightarrow 3.986 \times 10^{-14}, Rpl \rightarrow 6.37 \times 10^{-6}, c \rightarrow 0.000\};
dcm = \frac{mc}{ms + mc};
r0 = \sqrt{\frac{(rcm + dcm \cos[a0])^2 + (dcm \sin[a0])^2}{rcm} + \frac{1}{2} mc};
x0i = \{R[0] = r0, R'[0] = 0, \theta[0] = 0, \theta'[0] = \text{Sqrt}[\mu / rcm] / rcm,
\theta[0] = 0, x[0] = 0, a[0] = a0, a'[0] = 0\} /. valuesi;
(* Define the equations of motion, damping forces, custom spring coefficient *)
eqnsi = EulerEquations[T - V, (R[t], \theta[t], x[t], a[t]), t];
qd = \{R'[t], \theta'[t], x'[t], a'[t]\};
QD = -D[RDF, t] & @ qd;
For[i = 1, i \leq \text{Length}[qd], i++,
eqnsi[[i, 2]] = QD[[i]];
]
\end{verbatim}
(* Define an array of both tether lengths and corresponding spring constant values *)
ltemp = Table[n, {n, 5000, 50000, 5000}];
kvalues = (elasticity \pi d^2) / (4 \#) & /@ ltemp;
For[i = 1, i <= Length[ltemp], i++,
kvalues[[i]] = {ltemp[[i]], kvalues[[i]]};
]

(* Solve the system of ODEs numerically *)
solni = Table[NDSolve[{eqnsi, x0i} /. valuesi /. l -> kvalues[[n, 1]] /. k -> kvalues[[n, 2]],
{R, \theta, x, v}, {t, 0, 1800}, {n, 1, Length[ltemp]}];

(* Get all data at release point *)
time = 900;
xfun = a Exp[-(t - b)^2] / 2 c^2;
releaseInfo = Array[Null, Length[solni]]; releaseInfo2 = Array[Null, Length[solni]];
Module[{vtemp, xvalues, fit, xavg, R0, temp, alphad, ttemp, Rd, vtemp, Rd},
For[i = 1, i <= Length[solni], i++,
temp = FindRoot[Evaluate[a[t] /. solni[[i]]], {t, 800, 1100}];

(* Get average extension length *)
xvalues = Table[Flatten[{t, Evaluate[x[t] /. solni[[i]]]}], {t, 0, 1800, 1}];
fit = FindFit[xvalues, xfun, {a, b, c}, t];
xavg = xfun / fit /. temp;

(* Find velocities and positions *)
vtemp = vc[[2]] / . x'[t] -> 0 / . x[t] -> xavg / . l -> ltemp[[i]] / . solni[[i]] / . temp;
vtemp = vtemp /@ ltemp[[1]];
Rcd = vc[[1]] / . x'[t] -> 0 / . x[t] -> xavg / . l -> ltemp[[i]] / . solni[[i]] / . temp;
Rcd = Rcd /@ ltemp[[1]];
Rc0 = rc[[1]] / . x[t] -> xavg / . l -> ltemp[[i]] / . solni[[i]] / . temp;
Rc0 = (Rc0 /@ ltemp[[1]] - Rd) / valuesi;
alphad = a'[t] / . solni[[i]] / . temp; alphad = alphad /@ ltemp[[1]];
ttemp = \theta[t] / . solni[[i]] / . temp; ttemp = ttemp /@ ltemp[[1]];

vtemp = vs[[2]] / . solni[[i]] / . temp; vtemp = vtemp /@ ltemp[[1]];
Rd = vs[[1]] / . solni[[i]] / . temp; Rd = Rd /@ ltemp[[1]];

(* releaseInfo = (c1 vel., t.length, alt., a', c2 vel., \theta, avg. sp. ext.) *)
releaseInfo[i] = 
{vtemp, ltemp[[i]] / 1000, Rc0 / 1000, alphad, Rcd, ttemp, xavg, vtemp - vtemp};

(* releaseInfo2 = (c1 vel., t.length, c2 vel.) *)
releaseInfo2[[i]] = {vtemp, ltemp[[i]] / 1000, Rd};
]

(* Generate interpolating function *)
f = {10000 releaseInfo[[\#, 2]],
{releaseInfo[[\#, 1]], releaseInfo[[\#, 4]], 1000 releaseInfo[[\#, 3]],
releaseInfo[[\#, 6]], releaseInfo[[\#, 7]]} & /@ Range[1, Length[ltemp]];
rInfoCase2 = Interpolation[f] >> "/Dropbox/MSc/Calculation/rInfoCase2.dat";
Appendix E: Heat Transfer Code

(* ***** Heat Load and Temperature Profile Software ***** *)

(* Generate temperature solutions for with and without tether *)
Needs["DifferentialEquations'InterpolatingFunctionAnatomy"];
values = {m -> mass / Num, μ -> 3.986 10^14, Sc -> π capRadius^2, Ω -> 7.27 10^-5, CDt = 0.01,
        1 -> TotLength / Num, Rpl -> 6.37 10^6, a -> 0.95, k -> 1.82 10^-4, t1 -> 0.1524,
        t2 -> 0.002, t3 -> 0.004, t4 -> 0.002, t5 -> 0.0254, e -> 0.85, σ -> 5.67 10^-8};
ρ = Get["~/Dropbox/MSc/Calculation/density.dat"];

(* Define the freestream velocity and heating load (flux *)
vwc = (R'[t], Ω (θ'[t] - Ω));
vtot = \sqrt{vwc.vwc};
qflux[alt_] := k \sqrt{ρ[alt]} / capRadius \ vtot^3 / . values;

(* Set tether lengths to analyze *)
TotelLengths = 1000 {15, 20, 25, 30, 35, 40, 45, 50};

(* Set thermal diffusivity and thermal conductivity piecewise functions *)
afun[x_] := Piecewise[{{3.531 10^-6, 0 \leq x \leq t1}, {0.2712 10^-6, t1 < x \leq t2},
        {0.2283 10^-6, t2 < x \leq t3}, {0.2712 10^-6, t3 < x \leq t4}}, 70.6 10^-6] / . values;
ktfun[x_] := Piecewise[{{4.3, 0 \leq x \leq t1}, {0.424, t1 < x \leq t2},
        {0.0505, t2 < x \leq t3}, {0.424, t3 < x \leq t4}}, 167.4] / . values;

(* Loop over all defined tether lengths *)
For[j = 1, j <= Length[TotelLengths], ++j,
    TotelLength = TotelLengths[[j]];
    (* Pull simulated dynamics solutions *)
sol = Get["~/Dropbox/MSc/Calculation/solutions/solution_4links_" <>
          ToString[TotelLength / 1000] <> "_km.dat"];
solNone = Get["~/Dropbox/MSc/Calculation/solutions/solution_nolinks_" <>
          ToString[TotelLength / 1000] <> "_km.dat"];

(* Define end of time range for diffy-Q's *)
time = InterpolatingFunctionDomain[R /. sol[[2]][[1]]][[1, 2]] / Floor;
timeNone = InterpolatingFunctionDomain[R /. solNone[[2]][[1]]][[1, 2]] / Floor;

(* Set the altitude function *)
alt = Evaluate[(R[t] - Rpl) / . sol[[2, 1]]];
altNone = Evaluate[(R[t] - Rpl) / . solNone[[2, 1]]];

(* Define heat transfer parameters *)
qfun[t_] := Evaluate[qflux[alt] / . values /. sol[[2]][[1]]];
qfunNone[t_] := Evaluate[qflux[altNone] / . values /. solNone[[2]][[1]]];

(* Define PDE and initial conditions *)
eqnsTemp = {D[T[x, t], x, x] == 1 / afun[x] D[T[x, t], t],
        T0 = Derivative[1, 0][T][0, t] == 1 / (kfun[0] (qfun[t] - e σ (T)[0, t]^4)),
        Derivative[1, 0][T][t1 + t2 + t3 + t4 + t5, 0] == T[0, 0] = 294];
 Derivative[1, 0][T][0, t] == \(-\frac{1}{kfun[0]} \) (qfunNone[t] - \( e \circ (T[0, t])^4 \)),

Derivative[1, 0][T][t1 + t2 + t3 + t4 + t5, t] = 0, T[x, 0] = 294

(* Set up equations to solve *)

eqns = Join[eqnsTemp, T0] /. values;

(* Solve equations numerically *)

solnTemp = NDSolve[eqns, T, {x, 0, t1 + t2 + t3 + t4 + t5 /. values}, {t, 0, time}];

solnTempNone = NDSolve[eqnsNone, T, {x, 0, t1 + t2 + t3 + t4 + t5 /. values}, {t, 0, timeNone}];

ClearAll[sol, solNone, time, timeNone, qfun, qfunNone, alt, altNone, eqnsTemp, T0, T0None, eqns, eqnsNone, solnTemp, solnTempNone];
Appendix F: TSS Parameters

Table F.1: Properties for the station, capsule, and tether.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Orbit Altitude</td>
<td>200 km</td>
</tr>
<tr>
<td>Capsule Mass</td>
<td>4,800 kg</td>
</tr>
<tr>
<td>Capsule Diameter</td>
<td>2.2 meters</td>
</tr>
<tr>
<td>Station Mass</td>
<td>15,000 kg</td>
</tr>
<tr>
<td>Tether Material</td>
<td>Zylon® [32]</td>
</tr>
<tr>
<td>Tether Density</td>
<td>1.56 g/cm³</td>
</tr>
<tr>
<td>Tether Elasticity</td>
<td>280 GPa</td>
</tr>
<tr>
<td>Tether Rupture Elongation</td>
<td>2.5%</td>
</tr>
<tr>
<td>Tether Diameter</td>
<td>1 mm</td>
</tr>
</tbody>
</table>
Appendix G: TSS Parameters (Krischke)

Table G.1: Release parameters and capsule/tether properties (Krischke).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Release Altitude</td>
<td>680 km</td>
</tr>
<tr>
<td>Release Velocity</td>
<td>7,276 m/s</td>
</tr>
<tr>
<td>Capsule Mass</td>
<td>23 kg</td>
</tr>
<tr>
<td>Capsule Diameter</td>
<td>0.4 meters</td>
</tr>
<tr>
<td>Tether Material</td>
<td>Spectra® [33]</td>
</tr>
<tr>
<td>Tether Density</td>
<td>0.97 g/cm³</td>
</tr>
<tr>
<td>Tether Diameter</td>
<td>1 mm</td>
</tr>
</tbody>
</table>
Vita

Jeff Kornuta is a Baton Rouge native who grew up on a steady diet of football and crawfish. After graduating from Parkview Baptist High School in 2004, he pursued his Bachelor of Science in Mechanical Engineering at Louisiana State University, where he fell in love with both engineering and the Tigers. Graduating from LSU in 2008 with College Honors, Jeff continued his academic studies toward a Masters of Science in Mechanical Engineering at the University. He plans to continue his pursuit of knowledge toward a PhD in the fall of 2009.