
Benedetta Ferrario
ERRATUM: ABSOLUTE CONTINUITY OF LAWS FOR SEMILINEAR STOCHASTIC EQUATIONS WITH ADDITIVE NOISE

BENEDETTA FERRARIO

On page 212 of [1], the proof of Theorem 2.1 is wrong at equation (2.3). Different assumptions are required:

[A1] given \( x \in E \), on the time interval \([0, T]\) equation (1.1) has a weak solution \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in [0, T]}, \mathbb{P}), w, u\) with initial data \( x \) and a.e. path \( u \in C([0, T]; E) \);

[A2] given any \( y \in E \), on any time interval \([t_0, T] \subseteq [0, T]\) equation (1.2) has a unique strong solution \( z \) with initial data \( y \) and a.e. path \( z \in C([t_0, T]; E) \).

Here are the corrected statements. For the proofs, see [2].

**Theorem 2.1.** Assume [A1], [A2], and on the time interval \([0, T]\) consider the solutions \( u \) and \( z \) of equations (1.1) and (1.2) respectively, with the same initial data \( x \in E \).

If
\[
\mathbb{P}\{ \int_0^T |G^{-1}F(z(s))|_H^2 ds < \infty \} = 1, \tag{2.1}
\]
\[
\mathbb{P}\{ \int_0^T |G^{-1}F(u(s))|_H^2 ds < \infty \} = 1, \tag{2.1'}
\]
then

(i) the stochastic process
\[
\rho_t(z) = e^{-\frac{1}{2} \int_0^t H(G^{-1}F(z(s)), dw(s))} H - \frac{1}{2} \int_0^t |G^{-1}F(z(s))|_H^2 ds, \quad 0 \leq t \leq T,
\]
is a positive \( \{\mathcal{F}_t\} \)-martingale; in particular, \( \mathbb{E}[\rho_T(z)] = 1 \).

(ii) the stochastic process
\[
w^*(t) = w(t) + \int_0^t G^{-1}F(z(s)) \, ds, \quad t \in [0, T], \tag{2.2}
\]
is an \( H \)-cylindrical Wiener process with respect to \( \mathbb{P}^* \), where the probability measure \( \mathbb{P}^* \) is defined on \((\Omega, \mathcal{F}_T)\) by
\[
d\mathbb{P}^* = \rho_T(z) \, d\mathbb{P}.
\]

**Theorem 2.3.** Assume [A1], [A2], and on the time interval \([0, T]\) consider the solutions \( u \) and \( z \) of equations (1.1) and (1.2) respectively, with the same initial data \( x \in E \). If (2.1) and (2.1') hold, then \( \mathcal{L}^F \sim \mathcal{L}^0 \) and the Radon-Nykodim derivatives are
\[
\frac{d\mathcal{L}^F}{d\mathcal{L}^0}(z) = \mathbb{E}\left[ e^{-\frac{1}{2} \int_0^T H(G^{-1}F(z(s)), dw(s))} H - \frac{1}{2} \int_0^T |G^{-1}F(z(s))|_H^2 ds | \sigma_T(z) \right], \quad \mathbb{P} \text{- a.s.} \tag{2.6}
\]
\[ \frac{d\mathcal{L}^0}{d\mathcal{L}^F}(z) = \mathbb{E}^{\ast}[e^{\frac{1}{2} \int_0^T |H^{-1}F(z(s))|^2 H^{-\frac{1}{2}} \int_0^T |H^{-1}F(z(s))|^2 H^{-\frac{1}{2}} ds} | \sigma_T(z)], \quad \mathbb{P}^\ast \text{ a.s.} \] 

(2.7)

Moreover, \( \mathcal{L}^F \) is unique.

For completeness, we substitute Corollary 2.4 of [1] with the following

**Corollary 0.1.** Assume [A1] and [A2]. If

\[ |G^{-1}F(v)|_H \leq c (1 + |v|_{\mathcal{H}}^p), \quad \forall v \in E, \] 

(2.8)

for some constants \( p > 0 \) and \( c > 0 \), then conditions (2.1) and (2.1') are fulfilled and therefore Theorem 2.3 holds true.

The main results of Sections 3 and 4 in [1] remain true. More precisely, from previous results the Kuramoto-Sivashinsky equation is known to have a unique strong solution (see the discussion after Theorem 3.4 in [1]). Therefore for Section 3, the only change to do is in Theorem 3.4 of [1]: erase in the second line the sentence "there exist a unique weak solution of equation (3.2) on \([0, T)." Its".

In the same way, change Theorem 4.3 in [1]; for this we need to know that the stochastic hyperviscosity-regularized Navier-Stokes equation has a solution. In [3] it has been proved that for any \( \alpha \geq \frac{3}{2} \), given \( x \in D(A) \) there exists a unique strong solution \( u \) such that \( u \in C([0, T]; D(A)) \) \( \mathbb{P}\-a.s. \) By the way, estimate (4.8) in [1] requires Lemma 4.4 in [3].

**References**


Benedetta Ferrario: Dipartimento di Matematica, Università di Pavia, 27100 Pavia, Italy

E-mail address: benedetta.ferrario@unipv.it