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ERRATUM: ABSOLUTE CONTINUITY OF LAWS FOR SEMILINEAR STOCHASTIC EQUATIONS WITH ADDITIVE NOISE (COSA, VOL. 2, NO. 2 (2008) 209–227)

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On page 212 of [1], the proof of Theorem 2.1 is wrong at equation (2.3). Different assumptions are required:

[A1] given $x \in E$, on the time interval $[0,T]$ equation (1.1) has a weak solution $\big((\Omega, \mathbb{F}, \{\mathbb{F}_t\}_{t\in[0,T]}, \mathbb{P}),\ w,\ u\big)$ with initial data x and a. e. path $u \in C([0,T];E)$;

[A2] given any $y \in E$, on any time interval $[t_0,T] \subseteq [0,T]$ equation (1.2) has a unique strong solution z with initial data y and a. e. path $z \in C([t_0,T];E)$.

Here are the corrected statements. For the proofs, see [2].

Theorem 2.1. *Assume* **[A1]**, **[A2]**, *and on the time interval* $[0,T]$ *consider the solutions* u *and* z *of equations (1.1) and (1.2) respectively, with the same initial data* $x \in E$.
If

$$\mathbb{P}\{\int_0^T |G^{-1}F(z(s))|_H^2 ds < \infty\} = 1, \tag{2.1}$$

$$\mathbb{P}\{\int_0^T |G^{-1}F(u(s))|_H^2 ds < \infty\} = 1, \tag{2.1$'$}$$

then
(i) the stochastic process

$$\rho_t(z) = e^{-\int_0^t {}_H\langle G^{-1}F(z(s)), dw(s)\rangle_H - \frac{1}{2}\int_0^t |G^{-1}F(z(s))|_H^2 ds}, \quad 0 \le t \le T,$$

is a positive $\{\mathbb{F}_t\}$*-martingale; in particular,* $\mathbb{E}[\rho_T(z)] = 1$.
(ii) the stochastic process

$$w^*(t) = w(t) + \int_0^t G^{-1}F(z(s))\ ds\ , \qquad t \in [0,T], \tag{2.2}$$

is an H*-cylindrical Wiener process with respect to* $\mathbb{P}^*$*, where the probability measure* $\mathbb{P}^*$ *is defined on* $(\Omega, \mathbb{F}_T)$ *by*

$$d\mathbb{P}^* = \rho_T(z)\ d\mathbb{P}.$$

Theorem 2.3. *Assume* **[A1]**, **[A2]**, *and on the time interval* $[0,T]$ *consider the solutions* u *and* z *of equations (1.1) and (1.2) respectively, with the same initial data* $x \in E$. *If* (2.1) *and* (2.1$'$) *hold, then* $\mathcal{L}^F \sim \mathcal{L}^0$ *and the Radon-Nykodim derivatives are*

$$\frac{d\mathcal{L}^F}{d\mathcal{L}^0}(z) = \mathbb{E}\left[e^{-\int_0^T {}_H\langle G^{-1}F(z(s)),dw(s)\rangle_H - \frac{1}{2}\int_0^T |G^{-1}F(z(s)|_H^2 ds}\Big|\sigma_T(z)\right], \quad \mathbb{P}-a.s. \tag{2.6}$$

$$\frac{d\mathcal{L}^0}{d\mathcal{L}^F}(z) = \mathbb{E}^*[e^{+\int_0^T {}_H\langle G^{-1}F(z(s)),dw^*(s)\rangle_H - \frac{1}{2}\int_0^T |G^{-1}F(z(s))|_H^2 ds}|\sigma_T(z)], \quad \mathbb{P}^* - a.s. \tag{2.7}$$

Moreover, $\mathcal{L}^F$ is unique.

For completeness, we substitute Corollary 2.4 of [1] with the following

Corollary 0.1. *Assume* [**A1**] *and* [**A2**]. *If*

$$|G^{-1}F(v)|_H \leq c\,(1 + |v|_E^p)\,, \quad \forall\, v \in E, \tag{2.8}$$

for some constants $p > 0$ and $c > 0$, then conditions (2.1) *and* (2.1′) *are fulfilled and therefore Theorem 2.3 holds true.*

The main results of Sections 3 and 4 in [1] remain true. More precisely, from previous results the Kuramoto-Sivashinky equation is known to have a unique strong solution (see the discussion after Theorem 3.4 in [1]). Therefore for Section 3, the only change to do is in Theorem 3.4 of [1]: erase in the second line the sentence "there exist a unique weak solution of equation (3.2) on $[0, T]$. Its". In the same way, change Theorem 4.3 in [1]; for this we need to know that the stochastic hyperviscosity-regularized Navier–Stokes equation has a solution. In [3] it has been proved that for any $\alpha \geq \frac{5}{4}$, given $x \in D(A)$ there exists a unique strong solution u such that $u \in C([0, T]; D(A))$ $\mathbb{P}$-a.s. By the way, estimate (4.8) in [1] requires Lemma 4.4 in [3].

References

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