Numerical simulation and field verification of inclined piezocone penetration test in cohesive soils

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NUMERICAL SIMULATION AND FIELD VERIFICATION OF INCLINED PIEZOCONE PENETRATION TEST IN COHESIVE SOILS

A Dissertation

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ABSTRACT

A large strain finite element analysis is performed to analyze the effect of soil anisotropy on the inclined piezocone penetration test in normally consolidated cohesive soils. The piezocone penetration is numerically simulated using the commercial finite element code ABAQUS. The saturated clay is modeled as a two-phase material and the effective stress principle is used to describe its behavior. A frictional contact interface utilizing Mohr-Coulomb's theory was chosen to represent interactions between the surface of the cone and the soil. The Anisotropic Modified Cam Clay Model (AMCCM) by Dafalias (1987) was chosen and implemented into ABAQUS through user subroutine UMAT. The piezocone penetration is numerically simulated by the three-dimensional finite element method using different inclination angles at different initial stress states. A field testing program of inclined cone penetration is also developed and performed in three different locations with varying soil characteristics in Louisiana, using the Continuous Intrusion Miniature Cone Penetration Test System (CIMCPT). The following conclusions are drawn from this study:

1) As compared to the previously conducted calibration chamber tests, the finite element analysis results based on Anisotropic Modified Cam Clay Model (AMCCM) are overall in good agreement with the actual measurements. This indicates that the soil anisotropy plays an important role during piezocone penetrations.

2) Initial stress state strongly affects the tip resistance, sleeve friction and generated excess pore pressures. Coefficient of lateral earth pressure K indicates the degree of initial stress anisotropy. If K=1, no difference is expected between inclined and
vertical penetrations. However, for $K \neq 1$, the tip resistance, sleeve friction and
generated excess pore pressures tend to increase ($K < 1$) or decrease ($K > 1$) when the
orientation of penetration changes gradually from vertical to horizontal. Also, the soil
classification derived from inclined penetration data may require special
consideration.

3) The effect of anisotropic permeability on the tip resistance, sleeve friction and excess
pore pressures during inclined penetrations is negligible for soils with very low
hydraulic conductivities. However, it has significant effect on the dissipation of the
excess pore pressure at the cone tip.
CHAPTER 1
INTRODUCTION

1.1 Background

Recently, there has been increased emphasis on the evaluation of geotechnical parameters using in-situ testing. The electronic cone penetrometer is considered among the preferred devices for in-situ investigation, geomedia characterization and evaluation of soil properties. The cone penetration test (CPT) is robust, simple, fast, relatively economical and provides continuous records with respect to depth. The piezocone penetration test (PCPT) is an extension of CPT and is able to measure the cone tip resistance, sleeve friction and generated pore water pressures simultaneously. These measurements can be effectively utilized for soil stratigraphy and identification and in the evaluation of engineering soil properties and characteristics such as soil classification, strength and deformation characteristics. The estimated geotechnical parameters can be used for direct geotechnical design.

Currently, the use of CPT and PCPT are traditionally limited to vertical penetration. However, the vertical penetration is not always possible due to the existing buildings, structures, facilities, or the lack of access. In certain cases where special geological features of soil bedding exist such as inclined bedding and joints, foliated planes, veins, etc., there is a need to test the soil at an inclined orientation, and measure the properties of sediments along inclined orientations. In addition, due to the alignment of some reinforcing structures (i.e., batter piles, anchors, tie-backs, soil nailing), the stiffness and
strength parameters (i.e., elastic Young's and shear moduli, undrained shear strength) and/or flow characteristics are needed at an inclination plane other than vertical. The inclined piezocone penetration test can effectively be used for profiling the shear strength along the inclined length of anchors, batter piles and other soil inclusions with inclined placement for slope stability, embankments and other purposes.

![Diagram of stress distribution around an inclined cone]

**Figure 1.1 Stress distribution around an inclined cone**

In vertical penetration, the vertical overburden stresses $\sigma_v$, are acting in the direction of penetration, while an axi-symmetric horizontal confining stresses, $\sigma_h$, are acting radially around the cone. This is not the case for inclined penetration and the stresses are not axi-symmetric. The stress distribution around the inclined cone penetrometer is shown in Figure 1.1. The tip resistance is expected to be a function of the angle of inclination, $\alpha$. One can expect that this effect is due mainly to the soil anisotropy.
There are generally two types of anisotropy (Banerjee, et al., 1981). Inherent anisotropy manifests itself in the soil deposits as a result of applied stresses at the time of formation in the form of first-structure on a macroscopic scale or as a fabric orientation on the microscopic scale. Stress or induced anisotropy arises from changes in the effective stress state produced by subsequent loading history. This anisotropy can cause the elastic, strength and compressibility parameters of the soil deposits to vary with direction, and hence cannot be ignored. Experimental results have shown that the yield surface for anisotropically consolidated clay tends to align along the $K_o$ line (e.g., Graham et al., 1983). The pore water pressure as well as the stress-strain response of an anisotropically consolidated clay specimen under undrained shearing were significantly affected by the initial stress ratio (Ladd and Varallyay, 1965; Stipho, 1978). Therefore, to better investigate the effect of inclined penetration, the soil anisotropy has to be taken into account.

Part of this work involved the implementation of an anisotropic soil constitutive law into the finite element model. This is accomplished by developing a user subroutine UMAT and linking it to the general purpose finite element software package ABAQUS/Standard. An analytical model is developed and used to simulate the deep piezocone penetration in saturated cohesive soils by using ABAQUS as well. This numerical model takes into consideration the large deformation of the soil around the cone tip, the nonlinear soil behavior, coupled soil skeleton and pore water interaction, and the soil-piezocone interface friction. The influence of the angle of inclination and soil anisotropy on the cone tip resistance, generated pore pressures as well as stress and strain contours are analyzed.
1.2 Objectives and Scope of Work

The main objectives of this research are:

- Implement the Anisotropic Modified Cam Clay Model (AMCCM: Dafalias, 1987) into the commercial finite element software package ABAQUS through user subroutine UMAT, as shown in appendix.

- Develop an analytical model to simulate the continuous advance of piezocone penetration in cohesive soils, and implement this model into ABAQUS. Verify it by using this analytical model to simulate the piezocone penetration tests conducted at the Louisiana State University Calibration Chamber System (LSU/CALCHAS).

- Apply this model to the three-dimensional simulation of inclined piezocone penetration. Discuss the effect of inclination angle as well as soil anisotropy on the penetration results.

- Conduct a field testing program of inclined cone penetration and compare field results qualitatively with the results obtained from the numerical model.

1.3 Organization of Various Chapters

Immediately following this introduction chapter, Chapter 2 presents an extensive literature review regarding various methods that have been used to interpret piezocone penetration tests. The analytical model and its verification are described in Chapter 3. In Chapter 4, a three-dimensional finite element analysis is performed to simulate the inclined piezocone penetration in cohesive soils. Chapter 5 presents a field testing program of inclined cone penetration conducted at several locations in Louisiana. Finally, Chapter 6 gives the summary and conclusions of this work.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

A rigorous theoretical analysis of cone penetration is extremely difficult. The piezocone penetration test (PCPT) data is affected by many variables, including piezocone design, geometric size, rate of penetration, testing procedures, and soil characteristics (Tumay and Acar, 1985; Kurup et al., 1994; Voyiadjis et al., 1993; Kiousis et al., 1988; Kiousis, 1985). Location and size of the pore pressure element also influence the magnitude of the measured pore pressure (Tumay et al., 1981; Kurup et al., 1994; Voyiadjis et al., 1993) especially in overconsolidated stiff clays, where a large pore pressure gradient develops around the tip (Tumay et al., 1982; Baligh et al., 1981; Mayne et al., 1990). The deep penetration problem also involves large deformations and highly nonlinear material and interface behavior which make it more difficult to be numerically solved.

Over the past four decades, a number of models were proposed to correlate the penetration measurements with different soil properties. The existing approaches can be divided into five categories (Yu et al., 1998): those based on bearing capacity theory (e.g., Meyerhof, 1961), those based on cavity expansion theory (e.g., Torstensson, 1975 & 1977; Vesic, 1972), models based on strain path method (e.g., Baligh, 1985; Levaldoux and Baligh, 1986), numerical models based on finite element analysis (e.g., Kiousis et al., 1988; van den Berg, 1994; Abu-Farsakh et al., 1998 & 2003), and those based on
calibration chamber testing (e.g., Holden, 1971; Tumay and de Lima, 1992). The summary of the basic idea behind these models as well as their capabilities and limitations are discussed below.

2.2 Bearing Capacity Models

One of the first methods for the analysis of cone penetration was to treat it as a bearing capacity problem. Assuming a specific failure mode, a plastic collapse calculation is performed and the resulting vertical pressure is identified as the bearing capacity. A large number of investigators obtained different solutions for the bearing capacity of deep foundations depending on the assumed shape and location of the failure zone. Well-known solutions for axi-symmetric conditions are those published by Meyerhoff (1951, 1961) and Cox et al. (1961). Figure 2.1 shows some of the failure patterns investigated by Vesic (1967).

Yu et al., (1998) reviewed this method and gave the summary of some bearing capacity solutions for cone resistance in both cohesive and cohesionless soils. As he pointed out, the major advantage of this approach is its relative simplicity. It can be easily
accepted by many engineers who are already familiar with bearing capacity calculations. However, this method has two major limitations (Yu et al., 1998). First, in the bearing capacity analysis, the deformations of the soil around the cone are neglected; this means the dependence of the cone resistance on soil compressibility, as observed in laboratory and field testing, cannot be predicted. The other drawback is that the bearing capacity method ignores the influence of the cone penetration process on the initial stress states around the shaft. In particular, the horizontal stress tends to increase around the cone shaft after cone penetration, and the influence of this change on the cone resistance is not considered by bearing capacity analysis. In addition, the bearing capacity model cannot take into account the soil-piezocone interface friction. Therefore, recently, more and more scholars have paid more attention to other kind of approaches.

2.3 Cavity Expansion Models

A method which does account for soil compressibility is the cavity expansion theory. The analogy between cavity expansion and cone penetration was first pointed out by Bishop et al., (1945) after observing that the pressure required to produced a deep hole in an elastic-plastic medium is proportional to that necessary to expand a cavity of the same volume under the same conditions. Two steps need to be followed in order to use a cavity expansion approach to predict cone resistance: (1) Develop theoretical (analytical or numerical) limit pressure solutions for cavity expansion in soils and (2) relate cavity expansion limit pressures to cone resistance (Yu et al., 1998). General solutions associated to cavity expansion in a Mohr-Coulomb material are presented by Vesic (1972), who extended the model by allowing for the possibility that the volumetric strain in the plastic region is not zero. He treated both the case of expanding a spherical cavity
and a cylindrical cavity in an infinite soil mass. Figure 2.2 gives a general explanation of the expansion of a cavity.

Figure 2.2 Expanding a cavity in an infinite soil mass (Vesic, 1972)

Luger (1982) and Carter et al., (1986) presented analytical solutions for limit pressures due to expansion in a non-associated Mohr-Coulomb material. Yu and Houlsby (1991) extended this theory by introducing large strains in the plastic region, whereas the elastic region is treated by a small-strain approach. One of the latest developments is the combination of critical state models and cavity expansion of spherical and cylindrical cavities in soil using a constitutive model based on critical state theory (Been et al., 1991; Collins and Yu, 1996; Cao et al., 2001).

Yu et al., (1998) believe that the cavity expansion approach is more realistic than the bearing capacity theory for the following two reasons: First, both elastic and plastic
deformation of the soil during cone penetration can be taken into account in cavity expansion theory. Second, The cavity expansion approach is able to consider, at least in an approximate manner, both the influence of the cone penetration process on initial stress states and the effect of stress rotations that occur around the cone tip. However, the cavity expansion theory has its own limitations. It is essentially a one-dimensional theory and thus restricts the dependence of field variables (i.e., displacements, strains, stresses, and pore pressures) to the radial coordinates only. Nevertheless, cone penetration is not a one-dimensional problem and hence the field variables depend on the radial and vertical locations. Baligh (1986) also pointed out that one of the inconsistencies in the application of cavity expansion theory to cone penetration problems is that it does not model correctly the strain paths followed by soil elements. Most of the cavity expansion models assume the initial stress condition to be isotropic (i.e., $\sigma_v = \sigma_h$). Unfortunately, this is not the case for almost all field applications.

Torstensson (1975, 1977) proposed a one-dimensional undrained cavity expansion solution (cylindrical or spherical) assuming an isotropic initial stress distribution and an elastic-perfectly plastic soil material. An uncoupled finite difference scheme is used to analyze the pore pressure dissipation and consolidation. The initial excess pore pressure distribution is assumed to develop in the plastic zone, while the initial excess pore pressure in the elastic zone is taken as zero. Distribution of the initial excess pore pressure in the plastic zone at any radius, $r$, is given by Torstensson (1975, 1977):

Cylindrical cavity

$$\Delta u_i = s_u (\ln I_r - 2 \ln \frac{r}{r_o})$$

(2.1)
Spherical Cavity

\[ \Delta u_i = 4s_u \left( \frac{1}{3} \ln I_r - \ln \frac{r}{r_o} \right) \]  

(2.2)

where \( s_u \) is the undrained shear strength, \( I_r \) is the rigidity index which equals \( G/s_u \), and \( G \) is the shear modulus, \( r_o \) is the cone radius.

The excess pore pressure predicted by the cavity expansion model proposed by Torstensson gives lower values than the actual ones. This is expected since the Torstensson model ignores the shear induced excess pore pressure.

2.4 Strain Path Method

A promising approach accounting for large deformations is the strain path method as proposed by Baligh (1985). In this approach, in the first step, soil deformations are considered in terms of a steady flow of soil around a cone. Approximate velocity fields are estimated and differentiated with respect to the spatial coordinates in order to obtain strain rates. Integration of these strain rates along the streamlines defines the strain paths (histories) for individual soil elements around the cone. Finally, a constitutive model is introduced, equilibrium is required and the corresponding stress field around the cone is calculated. The effective stress is determined from the strain path for various soil elements either by using the effective stress approach or the total stress approach. Given the effective stresses, the excess pore pressure can be computed from equilibrium considerations. Based on the original work done by Baligh (1985), an improved understanding of the strain field during penetration in clay is presented by Acar and Tumay (1986).
A combination of a strain path method with the finite element analysis was proposed by Teh and Houlsby (1991) for the analysis of piezocone penetration in cohesive soil. Strain path method analysis provides the initial stress condition, and the large strain finite element analysis provides the equilibrium correction. The full analysis consists of two stages. In the first stage, the undrained penetration of the cone penetrometer into clay is analyzed, with the clay idealized as an incompressible Von Mises material. In the second stage, the consolidation around the static cone is analyzed using uncoupled Terzaghi-Rendulic consolidation theory. The actual geometry of the penetrometer was included explicitly in the analysis instead of using a combination of sources and sinks to approximately simulate the geometry of the penetrometer. Teh and Houlsby (1991) included the influence of rigidity index \( I_r \) on excess pore pressure dissipation and suggested the following modified time factor \( T^* \) for the evaluation of the coefficient of consolidation:

\[
T^* = \frac{c_r t}{r_o^2 \sqrt{I_r}} \tag{2.3}
\]

where \( c_r \) is the radial coefficient of consolidation and \( t \) is the time, \( I_r \) and \( r_o \) are defined the same as in Equation 2.1 & 2.2.

As Yu et al., (1998) pointed out, although promising in theory, the application of the steady state approach to the analysis of cone penetration in soils has not been entirely satisfactory. So far, the use of this method has been largely restricted to undrained clays. Another problem using strain path method is that the calculated stresses do not exactly satisfy the equilibrium equation (Teh and Houlsby, 1991). The discrepancy reflects the
error in the initial flow field. Fortunately, some numerical methods can be adopted to eliminate the inequilibrium of the stresses in an approximate manner.

2.5 Finite Element Method

The cone penetration tests in soils have been analyzed by a large number of researchers using incremental displacement finite element methods. These models can be divided into two groups: small strain analysis and large strain analysis. It is understood that cone penetration is essentially a large deformation problem since the cone penetrates into the soil several times its diameter. De Borst and Vermeer (1984) used an Eulerian approach, in which the soil is modeled as a Von Mises material, which flows through a fixed finite element mesh. A more comprehensive large strain analysis of the cone penetration in clay was performed by van den Berg (1994) using an Arbitrary Lagrangian Eulerian (ALE) formulation. He suggested that a steady state condition is normally reached when the penetration is about three times the cone diameter.

Kiousis et al., (1988) used an elasto-plastic large deformation theory to simulate the cone penetration in cohesive soil. The basic constitutive relation was developed in a spatial reference frame and were subsequently transformed in a Lagrangian coordinates. They did not take into consideration the interface friction between the soil and the penetrometer. The penetration process was simulated by applying incremental nodal vertical displacements of the nodes representing the conical surface of the cone penetrometer until failure was achieved. Undrained condition during penetration was assumed and the excess pore pressure distribution was determined by the following equation:
\[ \dot{\phi} = K \dot{J} \]  

(2.4)

where \( \dot{\phi} \) is the time derivative of the pore water pressure, \( K \) is the undrained bulk modulus of the soil-water system and \( \dot{J} \) is the material time derivative of the Jacobian of deformation. However, this model is not able to exactly predict the spatial excess pore pressures and in-situ states and soil parameters. This is due to many simplified assumptions in the model.

Abu-Farsakh (1997), Voyiadjis and Abu-Farsakh (1997), and Abu-Farsakh, et al. (1998) formulated an elasto-plastic coupled equations using the Updated Lagrangian formulation. These equations are based on the principle of virtual work and the theory of mixtures for inelastic porous media as proposed by Prevost (1980). The Modified Cam Clay Model was used to describe the plastic behavior of the cohesive soils. The continuous piezocone penetration was simulated by applying an incremental, vertical movement of the cone tip boundary. The interface friction between the soil and the penetrometer was also considered. A combination of finite element method and cavity expansion was proposed by Abu-Farsakh et al. (2003). A first stage that allows the piezocone to expand radially from an initial small radius to the piezocone radius at a specified depth is introduced before the numerical simulation of the vertical penetration. Song (1999) incorporated the plastic spin tensor and the Anisotropic Modified Cam Clay Model (Dafalias, 1987) into the finite element model to evaluate the soil consolidation characteristics from piezocone penetrations.

In this work, finite element method was selected to analyze the cone penetration problem because, among all the above mentioned methods, the approaches based on the
finite element method are perhaps the only ones that permit taking into consideration the various factors influencing the problem in a most integrated and consistent manner (Cividini and Gioda, 1988).

### 2.6 Calibration Chamber Testing

Due to the difficulties of rigorous simulation and analyses of the cone penetration test in the field, large calibration chambers have been used for many years to establish empirical correlations between cone resistance and soil properties. Laboratory-prepared soil specimens have many advantages over field deposits for research and calibration purposes. Many of the uncertainties in the field have been mentioned. The representative flaws of field tests are soil inhomogeneity and uncertainties regarding the magnitude of in-situ stresses and stress history of the deposit. Laboratory calibration tests, on the other hand, have been noted as a solution to eliminate such disadvantages of field tests since homogeneous, reproducible and instrumented soil specimens, subjected to a known stress history can be prepared and tested under controlled boundary conditions (Kurup, 1993; Lim, 1999).

In general, two types of calibration chambers are used: chambers with rigid or flexible walls. The former type chamber imposes a boundary condition of zero lateral strain on the specimens; the latter type allows lateral movement. Early calibration chambers were mostly rigid wall systems (Tcheng, 1966; Melzer, 1968) which could not control lateral movements. This disadvantage was overcome by introduction of an advanced flexible wall calibration chamber by Holden (1971) at Country Roads Board, Australia. It was designed to test specimens 0.76m in diameter and 0.91m in height. The first unique calibration chamber to test clayey soils was developed at Purdue University
to calibrate a miniature pressuremeter, and houses a sample of 203.2mm in diameter and
337mm in height (Huang, 1986). The LSU/CALCHAS (Tumay and de Lima, 1992)
consists of a calibration chamber, a panel of computer-based controls, a data acquisition
and control system, a hydraulics and chucking system, and a penetration depth
measurement system. This chamber allows testing of different sizes of cone
penetrometers under four traditional controlled boundary conditions (BC1 through BC4).
It can house specimens 525mm in diameter and 815mm in height.

Although calibration chamber testing has been widely used to obtain correlations
between cone resistance and soil properties, there are some limitations. Yu (1998)
pointed out that calibration chambers are of limited size and therefore chamber
correlations are not applicable to field situations without the application of correction
factors (Parkin and Lunne, 1982; Ghionna and Jamiolkowski, 1991). In the last ten years,
significant progress has been made in developing theoretical methods for assessing the
effects of chamber size and various boundary conditions on measured cone resistance
(e.g., Yu, 1990; Schnaid and Houlsby, 1991; Salgado, 1993; Salgado, et al., 1997).

2.7 Inclined Cone Penetration Tests

All the methods mentioned above deal with vertical cone penetration tests (VCPT).
These interpretations are based on the assumption that penetration is performed vertically
with an axi-symmetry along the vertical axis. So far no work has been published related
to inclined cone penetration tests (ICPT). The only reported experimental study was
conducted by Broere and van Tol (1998) on the horizontal cone penetration testing. A test
series were performed in a rigid wall calibration chamber, where both vertical and
horizontal cone penetrations were carried out. A simple cavity expansion model was used
to explain the relationship between the vertical and horizontal cone resistances. The model is based on pure elastic analysis and is too simple to describe the complicated penetration process, as realized by the authors.

In vertical penetration, the vertical overburden stresses, $\sigma_v$, are acting in the direction of penetration, while axi-symmetric horizontal confining stresses, $\sigma_h$, are acting radially around the cone. This is not the case for inclined penetration in which the stresses are not axi-symmetric. The stress distribution around an inclined cone penetrometer is shown in Figure 1.1. The tip resistance is expected to be a function of the angle of inclination, $\alpha$. To take this effect into consideration, it is necessary to carry out a three-dimensional analysis for the inclined penetration.
CHAPTER 3
NUMERICAL MODEL AND ITS VERIFICATION

3.1 Introduction

As stated in the last chapter, the finite element method is perhaps one of the methods that permits taking into consideration the various factors influencing the piezocone penetration problem in a most integrated and consistent manner (Cividini and Gioda, 1988). In this chapter, an analytical model to analyze the deep piezocone penetration in saturated cohesive soils is presented and verified with the controlled calibration chamber test results. The commercial software package, ABAQUS/Standard, is used for this study. The Anisotropic Modified Cam Clay Model (AMCCM, Dafalias, 1987) is chosen as the soil constitutive law. This model is implemented through ABAQUS user subroutine UMAT (as shown in Appendix).

3.2 The Finite Element Program ABAQUS/Standard

The general purpose finite element program ABAQUS/Standard (called ABAQUS in this work) was used to perform the numerical simulations of the piezocone penetrations. ABAQUS is a powerful finite element software package that can take into consideration the large deformation of the soil around the cone tip, the coupled behavior of porous media, and the soil-piezocone interface friction. The reader is referred to the ABAQUS Manuals (Hibbitt, Karlsson & Sorensen, Inc., 2002) for further details. A brief introduction to the theoretical formulations appears below.
3.2.1 Large Deformation Formulation

To formulate the incremental equilibrium equations in continuum mechanics, either the Eulerian formulation or the Lagrangian formulation can generally be utilized. Briefly, in a Lagrangian formulation all quantities (i.e., stress and strain) are referred to the coordinates \( X_i \) associated with some reference configuration; for instance, the initially undeformed configuration of the body. On the other hand, in an Eulerian formulation, all quantities are referred to the coordinates \( x_i \) associated with the currently deforming configuration of the body. The difference between total Lagrangian description and
updated Lagrangian description is the reference state utilized in each formulation. The reference state for the Total Lagrangian incremental formulation is the same for all increments and is usually the undeformed, unstressed state of the body. On the other hand, the reference state for the Updated Lagrangian incremental formulation is the previously deformed configuration of the body which is updated at the end of each incremental step.

Consider the motion of a body with respect to a fixed Cartesian coordinate system as shown in Figure 3.1. The body occupies a volume $V_0$, $V_n$, and $V_{n+1}$ at load increments 0, n, and n+1, respectively, corresponding to time $t_0$, $t_n$ and $t_{n+1}$. The 0 configuration denotes the initial undeformed configuration of an arbitrary deformable body. The n configuration represents the previous deformed configuration while n+1 configuration represents the current deformed configuration of the body. The displacement from n to n+1 configuration is represented by

$$u_i = u_{i+1} - u_i$$

(3.1)

The relation between the coordinates at n+1 and n is given by

$$x_i = x_{i+1} = X_i + u_i$$

(3.2)

From Bathe (1990), the principle of virtual work in an Updated Lagrangian reference frame is obtained by the following equation:

$$\int_{V_n} S_{ij} \delta(u_{n+1} E_{ij}) dV_n = \int_{V_{n+1}} F_i \delta u_i dV_{n+1} + \int_{A_{n+1}} T_i \delta u_i dA_{n+1}$$

(3.3)

The integration at the left-hand side of the equation (3.3) is with respect to the configuration at time step $t_n$ while the right-hand side is with respect to time step $t_{n+1}$. $V^n$ is the volume of the element at the n$^{th}$ configuration; $S_{ij}$ is the second Piola-Kirchhoff
stress tensor at time step \( t_{n+1} \) referred to \( n^{th} \) configuration; \( \delta(\mathbf{E}_{ij}^{n}) \) is the increment of Green-Lagrangian strain tensor from \( n^{th} \) to \( (n+1)^{th} \) configuration. The second Piola-Kirchhoff stress tensor \( \mathbf{S}_{ij}^{n+1} \) is related to the Cauchy stress tensor \( \mathbf{\sigma}_{rs}^{n+1} \) through:

\[
\mathbf{S}_{ij}^{n+1} = J^s \left( \frac{\partial X^1_i}{\partial x^r} \frac{\partial X^1_j}{\partial x^s} \mathbf{\sigma}_{rs}^{n+1} \right)
\]  

(3.4)

where \( J^s \) is the corresponding Jacobian for the solids. The Green-Lagrangian strain tensor \( \mathbf{E}_{ij}^{n+1} \) can be decomposed into linear and nonlinear parts:

\[
\mathbf{E}_{ij}^{n+1} = \mathbf{e}_{ij}^{n} + \mathbf{\eta}_{ij}^{n}
\]  

(3.5)

where

\[
\mathbf{e}_{ij}^{n} = \frac{1}{2} \left( \mathbf{u}_{i,j}^{n} + \mathbf{u}_{j,i}^{n} \right)
\]  

(3.6)

\[
\mathbf{\eta}_{ij}^{n} = \frac{1}{2} \left( \mathbf{u}_{k,i}^{n} \mathbf{u}_{k,j}^{n} \right)
\]  

(3.7)

where \( \mathbf{u} \) is the displacement vector. In equation (3.3), \( \mathbf{F} \) is the body force, \( \mathbf{\delta u} \) is the variation of the displacement, and \( \mathbf{T} \) is the surface traction.

The second Piola-Kirchhoff stress tensor during a finite deformation can be expressed using the following relationship:

\[
\mathbf{S}_{ij}^{n+1} = \mathbf{\sigma}_{ij}^{n} + \mathbf{dS}_{ij}^{n}
\]  

(3.8)

In equation (3.8), \( \mathbf{\sigma}_{ij}^{n} \) is the Cauchy stress tensor at \( n^{th} \) configuration and \( \mathbf{dS}_{ij}^{n} \) is the incremental stress tensor from configuration \( n \) to \( n+1 \) as indicated in Figure 3.1. The large deformation soil behavior is assumed to be described by the following constitutive law in an incremental form as
\[ dS_{ij} = D_{ijrs}^* dE_{rs} \]  

(3.9)

where \( D_{ijrs}^* \) is a constitutive relationship suitable for large deformation. Using the procedures suggested by Washizu (1982),

\[ dS_{ij} = d\sigma_{ij}^J + \sigma_{ij} d\varepsilon_{kk} - \sigma_{ik} d\varepsilon_{jk} - \sigma_{jk} d\varepsilon_{ik} \]  

(3.10)

where \( d\sigma_{ij}^J \) is the Janmann stress increment which can be generally assumed to be

\[ d\sigma_{ij}^J = C_{ijkl} d\varepsilon_{ij} \]  

(3.11)

where \( C_{ijkl} \) is any elasto-plastic constitutive law used in small-deformation analysis. Substituting equation (3.5), (3.6), (3.7), (3.10) and (3.11) into equation (3.3) and neglecting the higher-order terms, we obtain the following equation

\[ \int D_{ijkl} d\varepsilon_{kl} \delta(d\varepsilon_{ij})dV^n + \int \sigma_{ij} n \delta(d\eta_{ij})dV^n = R^{n+1} - \int \sigma_{ij} n \delta(d\varepsilon_{ij})dV^n \]  

(3.12)

where \( R^{n+1} \) is the external virtual work at configuration \( n+1 \), which is also the right-hand side of equation (3.3). Equation (3.12) may be written in a form convenient to finite element coding (Bathe, 1990).

### 3.2.2 Analysis of Porous Media

The word soil actually implies a mixture because the voids of soil skeleton are generally filled with water and air or gas. Hence, soil in general can be considered as a multiphase material whose state can be described by the stresses and displacements (or velocities) within each phase. The stresses carried by the soil skeleton are conventionally called "effective stress", and the part carried by water are called "pore water pressures". If the free drainage condition holds, the steady state pore water pressure depends only on the hydraulic conditions and is independent of the soil skeleton response to external loads. In this case, the soil behavior can be described in a single phase way. The same
description can be applied to the soil if no drainage condition prevails. However, in an intermediate case, in which some flow can take place, there is an interaction between the skeleton strains and the pore water flow. The solution of these problems requires that the soil behavior be analyzed by incorporating the effect of the transient flow of the pore-water through the voids and the stress-strain behavior of soils. Therefore, a multiphase continuum formulation is required for porous media. In this research, it is assumed the soil is fully saturated and is treated as a two-phase problem.

The equilibrium equation for porous media can be derived from equation (3.12) by incorporating the principle of effective stress. The porous media here will be approximately modeled by attaching the finite element mesh to the solid phase. The liquid can flow through this mesh. A continuity equation is, therefore, required for the liquid, equating the rate of increase in liquid mass stored at a point to the rate of mass of liquid flowing into the point within the time increment. The liquid flow is assumed to obey Darcy's law. The continuity equation is satisfied approximately in the finite element model by using excess pore pressure as nodal variables, interpolated over the elements. The equation is integrated in time by using the backward Euler approximation. The total derivative of this integrated variational statement of continuity with respect to the nodal variables is required for the Newton iterations used to solve the nonlinear, coupled, equilibrium and continuity equations.

The principle of effective stress at the state corresponding to time $t_n$ can be written as

$$\sigma_{ij}^n = \sigma_{ij} + \delta_{ij}^n u_w$$

(3.13)

where $u_w$ is the pore water pressure at state $t_n$, by expressing
\[ n+1 u_w = n u_w + du_w \]  
(3.14)

and replacing \( n \sigma_i \) in equation (16) with \( n \sigma_{ij} \), we obtain

\[
\int \int \int (D_{ijkl} \delta \varepsilon_{ij}) \delta \sigma_{kl} \, dV + \int n \sigma_{ij} \delta (d\sigma_{ij}) \, dV = R^{n+1} - \int n \sigma_{ij} \delta (d\sigma_{ij}) \, dV \tag{3.15}
\]

It will be assumed that the movement of the fluid through the soil is governed by Darcy's law. If the fluid has an actual velocity \( v_{wi} \), then the superficial velocity of the fluid relative to the skeleton is \( n(v_{wi} - v_{si}) \), where \( n \) is the soil porosity in the neighborhood of \( x_i \) at time \( t \). This superficial velocity is proportional to the hydraulic gradient, i.e.,

\[
n(v_{wi} - v_{si}) = -k_{ij} \frac{\partial h}{\partial x_j} \tag{3.16}
\]

and \( h \) is defined as

\[
h = \frac{u_w}{\gamma_w} + x_k b_k \tag{3.17}
\]

where \( k_{ij} \) is the permeability tensor, \( \gamma_w \) is the unit weight of the fluid, \( b_k \) are the components of a unit vector parallel to the direction of gravity.

Considering a physical element of the soil skeleton with density \( \rho_s \) at time \( t \), conservation of mass leads to the equation

\[
\frac{d}{dt} [\rho_s (1 - n)] + \theta \rho_s (1 - n) = 0 \tag{3.18}
\]

where \( \theta = \frac{\partial v_{si}}{\partial x_i} \) is the rate of volume strain. It is assumed that the soil particles are incompressible, and \( \rho_s \) is constant, so that

\[
\theta = \frac{n}{1 - n} \tag{3.19}
\]
Similarly, considering the conservation of the mass of the fluid, assuming the fluid is incompressible, one obtains
\[
\dot{n} + n\theta = -\frac{\partial}{\partial x_i}[n(v_{wi} - v_{si})]
\] (3.20)

Equations (3.19) and (3.20) can be combined together to obtain the overall volume behavior of the soil; then
\[
\theta = -\frac{\partial}{\partial x_i}[n(v_{wi} - v_{si})]
\] (3.21)

Equation (3.21) can be expressed by the integral formulation as follows:
\[
\int [n(v_{wi} - v_{si})\frac{\partial \delta u_w}{\partial x_i} - \theta \delta u_w]dV = 0
\] (3.22)

Taking into consideration of Darcy's law, equation (3.22) becomes
\[
\int [k_{ij}\frac{\partial h}{\partial x_j}\frac{\partial \delta u_w}{\partial x_i} + \theta \delta u_w]dV = 0
\] (3.23)

Equations (3.15) and (3.23) are the governing equations of the porous media.

3.2.3 The Interface Modeling

It is extremely important to set up the governing equations of the interaction between the lateral surface of the penetrometer and the soil. This purpose can be achieved by using interface elements. The most commonly used interface elements can be grouped into three main classes; namely the zero-thickness interface elements (Goodman et al., 1968), thin-layer interface elements (Desai et al., 1984) and the constraint approach (Katona, 1983). In ABAQUS/Standard, the constraint approach is adopted. The penetrometer is assumed to be a rigid body and the surrounding soil is deformable. The interaction occurs between these two bodies in terms of two surfaces that may interact.
The lateral surface of the penetrometer is called "master surface" which is rigid. The soil surface is called "slave surface" which is deformable. The nodes on the slave surface are constrained not to penetrate into the master surface; however, the nodes of the master surface can, in principle, penetrate into the slave surface. After this contact pair is defined, a family of contact elements is automatically generated. At each integration point, these elements construct series measures of clearance and relative shear sliding. These kinematic measures are then used, together with appropriate Lagrange multiplier techniques, to introduce surface interaction theories. The interaction simulation consists of two components: one normal to the surfaces and one tangential to the surfaces.

Figure 3.2 Contact pressure-clearance relationship (From ABAQUS manual)

The distance separating two surfaces is called the clearance. The contact constraint is applied when the clearance between two surfaces becomes zero. There's no limit in the
contact formulation on the magnitude of contact pressure that can be transmitted between the surfaces. The surfaces separate when the contact pressure between them becomes zero or negative and the constraint is removed. Separated surfaces come into contact when the clearance between them reduces to zero. This contact pressure-clearance relationship is shown in Figure 3.2.

![Figure 3.3 Contact frictional behavior (From ABAQUS manual)](image)

When surfaces are in contact, they usually transmit shear as well as normal forces across the interface. Coulomb friction model will be used to describe the tangential interaction of the surfaces. The product $\mu p$, where $\mu$ is the coefficient of friction and $p$ is the contact pressure between the two surfaces, gives the limiting frictional shear stress for the contacting surfaces. The contacting surfaces will not slip until the shear stress across their interface equals the limiting frictional shear stress, $\mu p$. The solid line in Figure 3.3
summarizes the behavior of the Coulomb friction model: there is zero relative motion (slip) of the surfaces when they are sticking (the shear stresses are less than $\mu p$).

Figure 3.4 Contact algorithm (From ABAQUS manual)

During the piezocone penetration, states of slave nodes can be identified as "open" (a positive clearance) or "closed" (clearance equal to zero). The contact algorithm in ABAQUS is shown in Figure 3.4. ABAQUS examines the state of all contact interactions at the start of each increment to establish whether slave nodes are open or closed. In Figure 3.4, $p$ denotes the contact pressure at a slave node and $h$ denotes the clearance between a slave node and the master surface. If a node is closed, ABAQUS determines whether it is sliding or sticking. ABAQUS applies a constraint for each closed node and removes constraints from any node where the contact state changes from closed to open.
ABAQUS then carries out an iteration and updates the configuration of the model using the calculated corrections.

Before checking for equilibrium of forces or moments, ABAQUS first checks for changes in the contact conditions at the slave nodes. Any node where the clearance after the iteration becomes negative or zero has changed status from open to closed. Any node where the contact pressure becomes negative has changed status from closed to open. If any contact changes are detected in the current iteration, ABAQUS labels it as a severe discontinuity iteration and no equilibrium checks are carried out. ABAQUS then modifies the contact constrains to reflect the change in contact status after the first iteration and tries a second iteration. ABAQUS repeats the procedure until an iteration is completed with no changes in contact status. This iteration becomes the first equilibrium iteration, and ABAQUS performs the normal equilibrium convergence checks. If the convergence checks fail, ABAQUS performs another iteration. ABAQUS repeats the entire process until convergence is achieved, as summarized in Figure 3.4.

3.3 The Anisotropic Modified Cam Clay Model (AMCCM)

One of the main characteristics of soil is its anisotropy. There are generally two types of anisotropy (Banerjee, et al., 1981). Inherent anisotropy manifests itself in the soil deposits as a result of applied stresses at the time of formulation in the form of first-structure on a macroscopic scale or as a fabric orientation on the microscopic scale. Stress or induced anisotropy arises from changes in the effective stress state produced by subsequent loading history. This anisotropy can cause the stiffness, strength and compressibility parameters of the soil deposits to vary with direction and hence cannot be ignored. Experimental results have shown that the yield surface for anisotropically
consolidated clay tends to align along the $K_0$ line (e.g., Graham et al., 1983). The pore water pressure as well as the stress-strain response of an anisotropically consolidated clay specimen under undrained shearing were significantly affected by the initial stress ratio (Ladd and Varallyay, 1965; Stipho, 1978).

Modified Cam-lay model, which is based on the critical state soil mechanics theory, has been widely used for years. The shape of the yield surface is an ellipse. It is aligned along the p axis and hardens isotropically according to the linear $e-lnp$ relationships following an associated flow rule. This model, however, is only valid for isotropic stress conditions and for normally to lightly overconsolidated clays, involving mainly monotonic loading. There have been several attempts to model anisotropic soil behavior by extending the critical state soil models. An anisotropic model for normally consolidated clay was proposed by Ohta and Hata (1971). This model allows the yield surface to be inclined at the origin of the stress space at the start of shearing. However, it does not rotate further with shearing. Prevost (1978) also presented an anisotropic model that accounted for induced anisotropy, but it is based on the nested surface and it is difficult to memorize the size and location of each surface.

Banerjee and Yousif (1986) proposed an anisotropic model with the yield surface oriented along the $K_0$ line. The yield function was developed from the experimental results. Both isotropic and anisotropic hardening rules were used. The isotropic hardening rule was the same as that of the modified Cam-clay model, whereas the anisotropic/rotational hardening rule was expressed as an empirical function that captures the critical state of soils.
Anandarajah and Dafalias (1985, 1986) introduced induced stress invariants based on the fabric tensor to replace the corresponding stress invariants in the yield function of the modified Cam-clay model. The models were based on bounding surface plasticity. The isotropic, anisotropic and distortional hardening rules were used to describe the evolution of the bounding surface as plastic strain develops.

Dafalias (1987) proposed an anisotropic theory based strictly on the critical state concept. An anisotropic dissipating energy measurement $\alpha$ was introduced in the triaxial space. The theory was generalized into three-dimensional stress space by assuming $\alpha = \sqrt{3\alpha_{ij}\alpha_{ij}}/2$, which expresses the yield function in terms of anisotropic tensor. When $\alpha=0$, the function reduces to the modified Cam-clay model. Crouch and Wolf (1992) further introduced a constant shape parameter to the yield function in order to control the tensile strength. Crouch and Wolf (1995) generalized the model into three-dimensional stress space but no validation was presented. Both models of Crouch assumed a constant shape parameter and did not consider distortional hardening.

The MIT series of soil models (Whittle, 1993; Whittle and Kavvadas, 1994) are extensions of the modified Cam-clay model. The equivalent shear stress is defined so that rotation of yield surface is allowed during subsequent shearing. In order to capture the critical state, a non-associated flow rule was adopted. Thus the key features of the MIT-E1 model include an anisotropic yield surface, kinematic plasticity and the capturing of strain-softening behavior under undrained conditions. Two additional features were incorporated in the MIT-E3 model: small strain nonlinear elasticity using a closed loop hysteric stress-strain formulation, and the bounding surface plasticity to model
overconsolidated clay behavior. However, the plasticity model with non-associated flow
rule does not satisfy Drucker's postulate; thus the resulting solution may not be unique.

Figure 3.5 Anisotropic yield surface in p-q space (Dafalias, 1987)

In this work, the anisotropic modified Cam-clay model developed by Dafalias
(1987) was used. This model is simple compared to other ones, while the most important
features of anisotropic soil behavior can still be captured. An illustration of the yield
surface is shown in Figure 3.5. The yield surface consists of a rotated and distorted
ellipse and the degree of rotation and distortion is determined by the value of $\alpha$. The
normal to $f=0$ at different characteristic points is indicated by a corresponding arrow. The
normals at points C and C', intersections with the critical state lines (CSL), are along the
q-axis. The normals at points O and A are along the p-axis; point A is such that
\[ \eta = \frac{q}{p} = \frac{q_0}{p_0} = \alpha. \] Note that \( p_0 \) is not the intersection of \( f = 0 \) with the \( p \)-axis. Point B represents a typical stress state on \( f = 0 \), with \( \eta = q/p \).

The Anisotropic Modified Cam Clay Model for general stress state is generalized as follows (Dafalias, 1987):

\[ f = p^2 - pp_0 + \frac{3}{2M^2} \left[ (s_{ij} - p\alpha_{ij})(s_{ij} - p\alpha_{ij}) + (p_0 - p)p\alpha_{ij}\alpha_{ij} \right] = 0 \tag{3.24} \]

\[ \alpha_{ij} = \langle \lambda \rangle \frac{1 + e_0}{\lambda - \kappa} \left\{ \begin{array}{c} r \frac{\partial f}{\partial \sigma_{mn}} \left( \frac{c}{p_0} (s_{ij} - xp\alpha_{ij}) \right) \\ \end{array} \right\} \tag{3.25} \]

In equation (3.25), \( <> \) is the Macauley bracket, \( \langle \lambda \rangle \) is the loading index, \( \lambda \) is the compression index in \( e \) vs. \( lnp \) curve, \( \kappa \) is the recompression index in \( e \) vs. \( lnp \) curve, \( e_0 \) is the initial void ratio, \( p \) is the mean effective stress, \( q \) is the deviatoric stress, \( M \) is the slope of the critical state line, \( p_0 \) controls the yield surface size, \( s_{ij} \) is the deviatoric stress tensor, and \( c \) and \( x \) are material constants.

### 3.4 Elastoplastic Constitutive Relation

The elastoplastic stress-strain relation for AMCCM (Dafalias, 1987) has been derived by Voyiadjis and Song (2000). Here only a brief review of the derivation work is given below.

From equation (3.24), one obtains:

\[ \frac{df}{dp} \ dp + \frac{\partial f}{\partial s_{ij}} \ ds_{ij} + \frac{\partial f}{\partial \varepsilon^p} \ d\varepsilon^p + \frac{\partial f}{\partial \alpha_{ij}} \ d\alpha_{ij} = 0 \tag{3.26} \]

Making use of the normality rule (flow rule), (3.27) is obtained as follows:

\[ d\varepsilon^p_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} = d\lambda B_{ij} \tag{3.27} \]
where $B_{ij}$ is defined as follows:

$$B_{ij} = \frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial p} \frac{\partial p}{\partial \sigma_{ij}} + \frac{\partial f}{\partial s_{kl}} \frac{\partial s_{kl}}{\partial \sigma_{ij}}$$  \hspace{1cm} (3.28)

For the case of small elastic strains one makes use of the additive decomposition of the incremental strain:

$$de_{ij}^e = de_{ij} - de_{ij}^p$$  \hspace{1cm} (3.29)

In order to obtain the constitutive relation one uses the following relations:

$$d\sigma_{ij} = C_{ijkl} (de_{kl} - de_{kl}^p) = C_{ijkl}de_{kl} - C_{ijkl}d\lambda B_{kl}$$  \hspace{1cm} (3.30)

where $\varepsilon_{ij}^e$ is the elastic strain, $\varepsilon_{ij}$ is the total strain, $\varepsilon_{ij}^p$ is the plastic strain, and $C_{ijkl}$ is the elastic stiffness tensor. Using equations (3.27), (3.28), (3.29), and (3.30), one obtains

$$d\lambda = \frac{B_{ij}C_{ijkl}de_{kl} + \frac{\partial f}{\partial \alpha_{ij}}d\alpha_{ij}}{B_{ij}C_{ijkl}B_{kl} - \frac{\partial f}{\partial \varepsilon_{ij}^p}B_{ii}}$$  \hspace{1cm} (3.31)

Hence the incremental stress strain relation is obtained as follows:

$$d\sigma_{ij} = \left( C_{ijkl}B_{mn}B_{rs}C_{mns} + C_{ijpq} \frac{\partial f}{\partial \alpha_{mn}} d\alpha_{mn} B_{pq} \frac{1}{3} de_{kl}^j \right) de_{kl}$$  \hspace{1cm} (3.32)

Equation (3.32) is the well-known elastoplastic stiffness equation in the form $[D^{ep}] = [D^e] - [D^p]$, where $[D^{ep}]$ represents the elastoplastic stiffness, $[D^e]$ represents the elastic stiffness and $[D^p]$ represents the plastic reduction.

Since this model is not directly available in ABAQUS, implementation work needs to be done. This can be achieved by using ABAQUS user subroutine UMAT.
3.5 Implementation of AMCCM

UMAT is an interface provided by ABAQUS/Standard that allows a constitutive model to be programmed using FORTRAN. In this work, the Anisotropic Modified Cam Clay Model (AMCCM) is coded and implemented into ABAQUS by the author through the user subroutine UMAT (as shown in Appendix). This user subroutine UMAT is called at each integration point for each iteration of every increment. The following command in the ABAQUS input file is used to call the subroutine UMAT:

* USER MATERIAL, TYPE=MECHANICAL, CONSTANTS = (number of constants)

For AMCCM, there are 6 material constants; namely \( \lambda \) (the compression index in \( e \) vs. \( \ln p \) curve), \( \kappa \) (the recompression index in \( e \) vs. \( \ln p \) curve), \( M \) (the slope of the critical state line), \( \nu \) (Poisson’s ratio), \( c \) and \( x \) (two new constants beyond MCCM).

The main function of UMAT is to update the stresses at the end of the increment. The numerical algorithm for integrating the rate constitutive equations is called a constitutive integration algorithm or a stress update algorithm. An effective stress update algorithm has to enforce consistency at the end of the time step (i.e., \( f_{n+1} = 0 \)) to avoid drift from the yield surface. There are many different approaches to integrating the constitutive equations. A summary of the principal methods is given by Simo and Hughes (1998). Some key issues in the numerical implementation of constitutive models are addressed by Hughes (1984). Here we focus on a class of methods called return mapping algorithms which are robust and accurate and which are widely used in practice (Belytschko et al., 2000).

Return mapping schemes consists of an initial elastic-predictor step, involving an excursion (in stress space) away from the yield surface and a plastic-corrector step which
returns the stress to the updated yield surface. Two ingredients of the method are an integration scheme which transforms the set of constitutive equations into a set of nonlinear algebraic equations and a solution scheme for the nonlinear algebraic equations. Specifically in this work, the fully implicit backward Euler scheme is followed.

In the fully implicit backward Euler method, the increments in plastic strain and internal variables are calculated at the end of the step and the yield condition is enforced at the end of the step. Thus the integration scheme is written as below (Belytschko et al., 2000):

\[
\begin{align*}
\varepsilon_{n+1} &= \varepsilon_n + \Delta \varepsilon \\
\varepsilon_{n+1}^p &= \varepsilon_n^p + \Delta \lambda_{n+1} r_{n+1} \\
q_{n+1} &= q_n + \Delta \lambda_{n+1} \rho_{n+1} \\
\sigma_{n+1} &= C : (\varepsilon_{n+1} - \varepsilon_{n+1}^p) \\
f_{n+1} &= f(\sigma_{n+1}, q_{n+1}) = 0
\end{align*}
\]  

(3.33)
where ε is the strain tensor, Δε is the strain increment tensor, σ is the stress tensor, ∆λ is the increment of plastic flow scalar, r is the plastic flow direction tensor. A set of internal variables is denoted collectively as q whose evolution equation can be specified as
q = Q h(σ, q). C is the elastic modulus (fourth-order tensor), and f is the yield surface.

A geometric interpretation of the algorithm is given in Figure 3.6. From (3.33)_{2}, the plastic strain increment is given by

\[ \Delta \varepsilon_{p}^{n+1} = \varepsilon_{p}^{n+1} - \varepsilon_{p}^{n} = \Delta \lambda_{n+1} r_{n+1} \]  

(3.34)

Substituting this expression into (3.33)_{4} gives

\[ \sigma_{n+1} = C : (\varepsilon_{n+1} - \varepsilon_{n} - \Delta \varepsilon_{p}^{n+1}) = C : (\varepsilon_{n+1} + \Delta \varepsilon - \varepsilon_{n} - \Delta \varepsilon_{p}^{n+1}) \]

\[ = C : (\varepsilon_{n} - \varepsilon_{p}) + C : \Delta \varepsilon - C : \Delta \varepsilon_{p}^{n+1} = (\varepsilon_{n} + C : \Delta \varepsilon) - C : \Delta \varepsilon_{p}^{n+1} \]

(3.35)

where \( \sigma_{n+1}^{trial} = \sigma_{n} + C : \Delta \varepsilon \) is the trial stress of elastic predictor and the quantity \( -\Delta \lambda_{n+1} C : r_{n+1} \) is the plastic corrector which returns or projects the trial stress onto the suitably updated (accounting for hardening) yield surface along the plastic flow direction (Figure 3.6). The elastic-predictor phase is driven by the increment in total strain while the plastic-corrector phase is driven by the increment \( \Delta \lambda_{n+1} \) in the plasticity parameter. Thus, during the elastic-predictor stage, the plastic strain and internal variables remain fixed, and during the plastic-corrector stage, the total strain is fixed.

### 3.6 Verification of the Model

#### 3.6.1 Introduction to the Calibration Chamber Testing

Kurup et al. (1993) ran a series of miniature piezocone penetration tests (MPCPT) on cohesive soil specimens prepared for the Louisiana State University Calibration...
Chamber System (LSU/CALCHAS) (Tumay and de Lima, 1992). The specimens were prepared in two stages: (1) slurry consolidation in a consolidometer from a high-water-content soil slurry and (2) reconsolidation in a calibration chamber to higher stresses that are free from the rigid boundary effects of a slurry consolidometer. Soil slurry was prepared by mixing kaolinite and fine sand with deionized water at a water content of twice the liquid limit. A mixture of 50% kaolinite and 50% fine sand by weight was used to prepare the K-50 specimens. Details of the slurry consolidation process and specimen quality are given by Kurup (1993) and Voyiadjis et al., (1993).

At the end of the first stage of slurry consolidation, the specimen (525mm in diameter and 812mm high) enclosed in the membrane was transferred into the Louisiana State University Calibration Chamber System (LSU/CALCHAS), developed by de Lima (1990) and Tumay and de Lima (1992), where it was subjected to a second stage of consolidation to higher stresses. This two-stage sample-preparation technique has been found to reduce the rigid boundary effects from the slurry consolidometer and is successful in preparation homogeneous soil specimens subjected to a known stress history. This LSU/CALCHAS is a double-walled flexible chamber capable of simulating the four traditional penetration boundary conditions commonly referred to in the literature as:

- BC1: Constant vertical stress and constant lateral stress
- BC2: Zero vertical strain and zero lateral strain
- BC3: Constant vertical stress and zero lateral strain
- BC4: Zero vertical strain and constant lateral stress
The miniature piezocone penetrometer used for the tests has a projected cone area of 1 cm² and a cone apex angle of 60°. The penetrometer has two alternatives for the filter location. The choice is available for the filter located in the lowest 1/4 of the cone at the very tip (U1 configuration), or starting 0.5mm above the base of the cone with 2mm vertical height (U2 configuration). Full details of the test procedure can be found in Kurup et al., (1994). Table 3.1 presents a summary of the stress history of two soil specimens tested in the calibration chamber.

Table 3.1 Summary of stress history of soil specimens (From Kurup et al., 1994)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Final effective stress (kPa)</th>
<th>Lateral stress coefficient K</th>
<th>OCR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>vertical</td>
<td>horizontal</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>207.0</td>
<td>207.0</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>207.0</td>
<td>107.6</td>
<td>0.52</td>
</tr>
</tbody>
</table>

3.6.2 Finite Element Model

The above two calibration chamber tests were simulated using the proposed model to verify its validation. The piezocone penetration analysis is treated as an axi-symmetric boundary value problem with the need of changing the boundary conditions as the piezocone penetrometer advances. The penetrometer is assumed to be infinitely stiff and no tensile stresses are allowed to develop along the centerline boundaries. For numerical purposes, the piezocone penetrometer is assumed to be initially pre-bored to a certain depth with the initial stresses remaining unchanged. The continuous penetration of the piezocone was simulated by applying an incremental vertical displacement at the nodes representing the piezocone boundary. The vertical displacement (penetration) was applied at the rate of 2 cm/s, which is the same rate as used for the calibration chamber tests. The
Mohr-Coulomb frictional model is used to define the sliding potential between the cone surface and the soil. The soil-penetrometer interface friction coefficient is taken to be 0.25, which corresponds to an angle of friction $\delta = 14^o$ between the soil and the piezocone surface.

A number of different meshes with different degrees of refinement were tried first in order to obtain the appropriate mesh for the analysis of the piezocone penetration problem that satisfies convergence to a unique solution. The finite element mesh as well as the boundary condition is presented in Figure 3.7. One should note that soil specimens 1 and 2 were penetrated at different boundary conditions. Soil specimen 1 was penetrated under boundary condition 1 (BC1), while Soil specimen 2 was penetrated under boundary condition 3 (BC3), as shown respectively in Figure 3.7(a) and 3.7(b). To get a smooth stress state around the corner point (between the cone base and the shaft), the shape of the cone is rounded. Since the eight-node quadrilateral elements with full or reduced integration failed to give good results (de Borst, 1982; van den Berg and Vermeer, 1988), the four-node axi-symmetric pore pressure element with reduced integration (ABAQUS element type CAX4RP) is used in this analysis. The soil properties and initial conditions are summarized in Table 3.2. It should be noted that all the soil properties were determined by laboratory tests except hardening parameter $c$ and $x$. Dafalias (1987) suggested a way to estimate the hardening parameter $x$ through the following equation:

$$x = \frac{3\eta(1 - \frac{K}{\lambda})}{\eta^2 + 3\eta(1 - \frac{K}{\lambda}) - M^2}$$

(3.33)
where $\eta = q/p$. Since the determination of $x$ is related to the initial stress state, specimen 2 is chosen to calculate the $x$ value, as shown in Table 3.2. The hardening parameter $c$ is treated as a fudge factor here.

Figure 3.7(a) Finite element mesh and boundary conditions for soil specimen 1
Figure 3.7(b) Finite element mesh and boundary conditions for soil specimen 2

Table 3.2(a) Input parameters for FEM analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression index $\lambda$</td>
<td>0.11</td>
</tr>
<tr>
<td>Recompression index $\kappa$</td>
<td>0.024</td>
</tr>
<tr>
<td>Permeability $k$</td>
<td>$5 \times 10^{-10}$ m/s</td>
</tr>
<tr>
<td>Poisson's ratio $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Slope of critical line $M$</td>
<td>1.2</td>
</tr>
<tr>
<td>Initial void ratio $e$</td>
<td>1.0</td>
</tr>
<tr>
<td>Unit weight of water $\gamma_w$</td>
<td>9.8 kN/m$^3$</td>
</tr>
<tr>
<td>Hardening parameter $c$</td>
<td>0.05</td>
</tr>
<tr>
<td>Hardening parameter $x$</td>
<td>2.3</td>
</tr>
</tbody>
</table>
Table 3.2(b) Initial stress state for numerical simulation

<table>
<thead>
<tr>
<th>K</th>
<th>(\sigma_v) (kPa)</th>
<th>(\sigma_h) (kPa)</th>
<th>(\alpha_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>207</td>
<td>207</td>
<td>0</td>
</tr>
<tr>
<td>0.52</td>
<td>207</td>
<td>107.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

It we assume during the normally consolidation process of a soil specimen, the yield surface expands in size only without any rotations, we can calculate the initial rotation angle of the yield surface as following (Dafalias, 1987):

\[
\alpha_0 = \frac{\eta}{x}
\]  \hspace{1cm} (3.34)

Obviously, we have \(\alpha_0=0\) for soil specimen 1 and \(\alpha_0=0.3\) for soil specimen 2.

3.6.3 Numerical Predictions versus Experiments

Kiousis (1985) reported a steady state about tip resistance is almost reached when the penetration depth is about 1/3 the diameter of the cone. In this study, the penetration depth of the penetrometer into the soil was chosen to be 6 mm, which is a little larger than the radius of the miniature cone (5.6 mm). If the penetration distance is too long, the soil elements may be distorted too much and fail to yield accurate results. For each soil specimen, the predictions using Anisotropic Modified Cam Clay Model (AMCCM) are compared with the measurements of the calibration chamber tests. The soil properties and other input parameters for the finite element analysis are shown in Table 3.2.

The predicted tip resistance and excess pore pressure (U1 configuration, filter located at the very tip of the cone) profiles and the measured ones for soil specimen 1 are shown in Figure 3.8(a). The predicted tip resistance and excess pore pressure (U1 configuration) profiles and the measured ones for soil specimen 2 are shown in Figure 3.7(b). One should note that the calibration chamber MPCPT tests usually take the
"steady value" as a single measurement, as shown in both figures. These comparisons are also presented in Table 3.3.

As can be seen from Figure 3.8 and Table 3.3, the finite element calculations based on AMCCM are overall in agreement with the experimental results. Very good agreements can be seen between the finite element calculations and the corresponding measured results for soil specimen 2. The predicted tip resistance is about 1.4% less than the measured one, and the predicted tip pore pressure is almost the same as the measured one. For soil specimen 1, the agreements between calculations and experiments are not so good as for soil specimen 2. The predicted tip resistance based on AMCCM is about 19% less than the measured one, but the predicted tip pore pressure is about the same as the measured one. Because the history of soil specimen 2 is anisotropically consolidated while the soil specimen 1 is isotropically consolidated (Table 3.1), it may suggest that the AMCCM is more suitable for the simulation of soil behavior with an anisotropic initial stress state than with an isotropic initial stress state.

Since the finite element predictions based on AMCCM are overall in agreement with the corresponding experimental results, we believe that the proposed model is capable of analyzing the piezocone penetration problems with acceptable accuracy. Consequently, it will be used to perform all the theoretical calculations in the following chapter.
Figure 3.8(a) Measured versus calculated cone tip resistance and excess pore pressure profile for specimen 1

Figure 3.8(b) Measured versus calculated cone tip resistance and excess pore pressure profile for specimen 2
Table 3.3 Comparison between measured and predicted results at steady state condition

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Tip Resistance (MPa)</th>
<th>Tip Pore Pressure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Experiment</td>
</tr>
<tr>
<td>1</td>
<td>0.96</td>
<td>1.19</td>
</tr>
<tr>
<td>2</td>
<td>0.68</td>
<td>0.69</td>
</tr>
</tbody>
</table>
CHAPTER 4

THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS FOR INCLINED PENETRATION

4.1 Introduction

The cone penetration test (CPT) has been used for decades to investigate the properties of soil in situ. Essentially, the test consists of pushing a penetrometer with a standard geometry (cylindrical with a diameter of 35.7 mm and a conical point with an apex angle of 60°) into the soil at a rate of 20 mm/s, while measuring a number of parameters. It has been increasingly used because of its important advantages, such as simplicity, speed and continuous profiling (Salgado, et al., 1997). Originally, the penetrometer was used to measure only the tip resistance, defined as the vertical force acting on the tip of the penetrometer divided by the projected area of the tip (10 cm² for the standard penetrometer). Later, sensors have been incorporated into the cone to measure the friction along a lateral sleeve (150 cm² for the standard penetrometer). Measurements of the generated pore pressures while advancing the penetrometer into the ground and their subsequent dissipations were first made in the early 1970s (Janbu and Senneset, 1974; Wissa et al., 1975; Torstensson, 1975). Subsequent developments in transducer technology during the late 1970s involved the incorporation of piezometric elements to the standard electric cone penetrometers, which made possible simultaneous measurements of pore pressures, tip resistance and sleeve friction (Tumay et al., 1981;
Baligh et al., 1981; Campanella and Robertson, 1981; de Ruiter, 1982; Zuidberg et al., 1982; Smits, 1982).

Currently, measurements are traditionally taken from ground level in the vertical direction to gain information about stratification and soil properties. However, it is not always possible to perform vertical penetration tests due to the existing buildings, structures, facilities or the lack of access. In addition, sometimes, where special geological features of soil bedding exist such as inclined bedding and joints, foliated planes, veins, etc., there is a need to test the soil at an inclined orientation, and measure the properties of sediments along inclined orientations. For example, with the introduction of the mechanized tunnel boring machines (TBM), profiling the soil along the tunnel alignment has becomes a necessity. In this case, horizontal and/or inclined cone penetration tests can provide continuous soil profiling ahead of the TBM and will assist cost-effective operation of the TBM (Kurup et al., 1997).

Although the equipment used to perform a vertical cone penetration test (VCPT) may easily be converted to allow its use in an inclined cone penetration test (ICPT), the measurements obtained cannot be interpreted as easily. In vertical penetration, the vertical overburden stresses, $\sigma_v$, are acting in the direction of penetration, while an axi-symmetric horizontal confining stresses, $\sigma_h$, are acting radially around the cone. This initial stress state around the cone is used implicitly in most theoretical models (e.g, Vesic, 1972; Carter et al., 1986; Salgado, et al., 1997; Abu-Farsakh et al., 1997 & 2003; Yu et al., 2000). However, this is not the case for inclined penetration and the stresses are not axi-symmetric. The stress distribution around an inclined cone penetrometer is shown in Figure 1.1. The measurements (tip resistance, sleeve friction, etc.) are expected to be a
function of the angle of inclination $\alpha$. Here, $\alpha$ is the angle between the vertical direction and the inclined penetration direction.

Since the approaches based on the finite element method are perhaps those that permit taking into consideration the various factors influencing the problem in the most integrated and consistent manner (Cividini and Gioda, 1988), in this chapter, a three-dimensional finite element analysis is performed for the inclined piezocone penetration in saturated cohesive soils using ABAQUS. Readers can refer to Chapter 3 to see how ABAQUS deals with large deformation analysis, porous media analysis and interface friction modeling, as well as their theoretical background. The Anisotropic Modified Cam Clay Model (AMCCM, Dafalias, 1987), which was also introduced in Chapter 3, was implemented into ABAQUS through the user subroutine UMAT to describe the soil behavior under inclined penetration. The piezocone penetration analysis is treated as a boundary problem with the need of changing the boundary conditions as the piezocone penetrometer advances. The penetrometer is assumed to be infinitely stiff. For numerical purposes, the piezocone penetrometer is assumed to be initially pre-bored to a certain depth with the initial stresses remaining unchanged. The continuous penetration of the piezocone is simulated by applying an incremental displacement along the inclined penetration direction at the nodes representing the piezocone boundary. The piezocone was penetrated at the rate of 2 cm/s, which is the same as mostly used for field tests. The Mohr-Coulomb frictional model is used to define the sliding potential between the cone surface and the soil. The soil-penetrometer interface friction coefficient is taken to be 0.25, which corresponds to an angle of friction $\delta=14^\circ$ between the soil and the piezocone surface. The numerical simulations are performed for the same normally consolidated
soil, but with a different initial stress state. The tip resistance, sleeve friction and excess pore pressure profiles with penetration, and developed strain, stress and excess pore pressure fields around the piezocone are presented and discussed.

4.2 Finite Element Mesh

A number of different meshes with different degrees of refinement were tried first in order to obtain the appropriate mesh for the analysis of the piezocone penetration problem that satisfies convergence to a unique solution. The three-dimensional finite element mesh is presented in Figure 4.1. To get a smooth stress state around the corner point (between the cone base and the shaft), the shape of the cone is rounded. Since the eight-node quadrilateral elements with full or reduced integration failed to give good results (de Brost, 1982; van den Berg and Vermeer, 1988), the eight-node trilinear displacement and pore pressure element C3D8RP with reduced integration is used in this analysis. The finite element discretization consists of 6146 elements. All the displacement perpendicular to the outer surfaces is constrained as zero except at the top surface. Drainage is allowed only at the outer boundary sides of the FE mesh. Since the standard cone (diameter=35.7 mm) is most widely used, this cone geometry is simulated. Because of symmetry, only half of the soil mass is needed to perform the analysis, as shown in Figure 4.1(a). The soil mass is 1068 mm × 1068 mm × 534 mm in dimension, and the cone tip is initially located at the center of the symmetric plane, as shown in Figure 4.1(a). This mesh remains the same during the inclined penetration simulation, and only the initial stress tensor components should be changed according to the new space coordinates. Figure 4.2 illustrate how to choose the soil boundaries during inclined
penetration. The soil properties and initial conditions are the same as those in Chapter 3, which are also summarized in Table 4.1.

Table 4.1(a) Input parameters for FEM analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression index $\lambda$</td>
<td>0.11</td>
</tr>
<tr>
<td>Recompression index $\kappa$</td>
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<td>Permeability $k$</td>
<td>$5 \times 10^{-10}$ m/s</td>
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<td>Poisson's ratio $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Slope of critical line $M$</td>
<td>1.2</td>
</tr>
<tr>
<td>Initial void ratio $e$</td>
<td>1.0</td>
</tr>
<tr>
<td>Unit weight of water $\gamma_w$</td>
<td>9.8kN/m$^3$</td>
</tr>
<tr>
<td>Hardening parameter $c$</td>
<td>0.05</td>
</tr>
<tr>
<td>Hardening parameter $x$</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 4.1(b) Initial stress state for numerical simulation

<table>
<thead>
<tr>
<th>K</th>
<th>$\sigma_v$ (kPa)</th>
<th>$\sigma_h$ (kPa)</th>
<th>$\alpha_0$</th>
</tr>
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<tbody>
<tr>
<td>0.4</td>
<td>100</td>
<td>40</td>
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<td>100</td>
<td>140</td>
<td>0.14</td>
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</table>
Figure 4.1(a) Three-dimensional finite element mesh

Figure 4.1(b) The finite element mesh near the cone tip
4.3 Results of the Numerical Simulation

For each different initial stress state (shown in Table 4.1(b)), a vertical penetration ($\alpha=0$), an inclined penetration for $\alpha=30^\circ$, an inclined penetration for $\alpha=60^\circ$ and a horizontal penetration ($\alpha=90^\circ$) are performed. Kiousis (1985) reported a steady state about tip resistance is almost reached when the penetration depth is about 2/3 the radius of the cone. In this study, the penetration depth of the penetrometer into the soil was chosen to be 20 mm, which is a little bit larger than the radius of the penetrometer (17.8 mm). If the penetration distance is too long, the soil elements may be distorted too much and fail to yield accurate results.

4.3.1 Tip Resistance Profiles

The tip resistance for vertical penetration is defined as the vertical force acting on the tip of the penetrometer divided by the projected area of the tip. Similarly, the tip
resistance for inclined cone penetration can be defined as the total force acting on the tip of the penetrometer in the penetration direction divided by the projected area of the tip. This projected area of the tip is related only to the geometry of the penetrometer and hence will not change during the inclined penetrations.

Figure 4.3(a)-(f) shows the tip resistance profiles at each inclination angle for each K value. Here, the K is defined as the coefficient of lateral earth pressure (i.e., $K=\sigma_h/\sigma_v$). For K is equal to 1, the initial stress state is isotropic and no difference is expected for different inclination angles, as shown in Figure 4.3(d). However, for K values not equal to 1, different values of tip resistance are obtained for different inclination angels, as shown in Figure 4.3(a), (b), (c), (e) and (f). The tip resistance approaches the steady state value at about 20 mm penetration depth, and these steady state values for each different K are compared in Figure 4.4.

![Figure 4.3 Tip resistance profiles at different K values for inclined penetrations](image)
(Figure 4.3 continued)
Figure 4.4 Comparison of steady state tip resistance during inclined penetrations for different K values ($\sigma_v=100$ kPa)

As can be seen from both Figure 4.3 and Figure 4.4, the tip resistance is strongly affected by the initial stress state. The higher the initial stresses are, the higher the tip resistance will be. It should be noted that this trend is observed based on a constant initial
vertical stress ($\sigma_v=100$ kPa), in which condition a higher K value means a higher initial stress state. For vertical penetration, the tip resistance increases from 0.36 MPa to 0.72 MPa when K increases from 0.4 to 1.4.

When the initial stress state is isotropic (K=1, i.e., $\sigma_h=\sigma_v$), the tip resistance will be the same regardless of the inclination angle the soil is penetrated at. But for an anisotropic initial stress state (K$\neq$1, i.e., $\sigma_h\neq\sigma_v$), different values of tip resistance are observed, as shown in both Figure 4.3 and Figure 4.4. However, for different K values, the trend of change of the tip resistance varies with the change of inclination angles. For K<1 (i.e., K=0.4, 0.6, 0.8), the tip resistance tends to increase when the inclination angle $\alpha$ increases. For K>1 (i.e., K=1.2, 1.4), the tip resistance tends to decrease when the inclination angle $\alpha$ increases. This may imply that, when penetrating from an initial stress state with a constant vertical stress, for K<1, the tip resistance will gradually increase as the penetration changes gradually from vertical to horizontal direction; for K>1, the tip resistance will gradually decrease when the penetration changes gradually from vertical to horizontal direction.

It is also noted that, the further the initial stress state is away from isotropic, the more the tip resistance varies at different penetration angles. Again, this is based on the penetrations from a constant initial vertical stress. For instance, the horizontal tip resistance is 27%, 15% and 7.7% higher than the corresponding vertical tip resistance for K=0.4, K=0.6 and K=0.8, respectively. The vertical tip resistance is 8.6% and 8.0% higher than the corresponding horizontal tip resistance for K=1.4 and K=1.2, respectively. This also indicates that the difference in tip resistance between inclined
penetration and vertical penetration is not significant for K values close to 1 (both maximum differences are about 8% for K=0.8 and K=1.2).

4.3.2 Sleeve Friction Profiles

The sleeve friction is defined as the total shear force in the penetration direction divided by the sleeve surface area (150 cm$^2$). Figure 4.5(a)–(f) shows the sleeve friction profiles at different inclination angles for different K values. For K equal to 1, the initial stress state is isotropic (i.e., $\sigma_h=\sigma_v$) and no difference is expected for different inclination angles, as shown in Figure 4.5(d). For K values other than 1 (i.e., $\sigma_h\neq\sigma_v$), different values of sleeve friction are recorded for different inclination angels, as shown in Figure 4.5(a), (b), (c), (e) and (f). Obviously, at the penetration depth of 20 mm, a steady state for the sleeve friction still has not been reached. The author believes this happens because of the relatively short penetration depth. However, when the penetration distances are large, severe mesh distortions happen in zones of high strain concentrations around the cone tip, which lead to a severe loss of accuracy and numerical divergence. Therefore, the simulated penetration depth in this study was limited to 20 mm. The standard penetrometer has a sleeve friction area of 150 cm$^2$, which corresponds to a friction sleeve length of 134 mm. Since the penetration distance is only 15% of the friction sleeve length, one expects most of the normal and shear stresses on the sleeve have not been fully mobilized with such a short penetration distance as compared to the friction sleeve length. This is confirmed as depicted in Figures 4.12 to 4.14, where most of the octahedral normal and shear stress bulbs are around the cone tip, and not around the cone shaft. Nevertheless, for preliminary comparison purposes, the sleeve friction values at penetration depth of 20 mm for different K values are presented in Figure 4.6.
Figure 4.5 Sleeve friction profiles at different K values for inclined penetrations

(a) $K=0.4$

(b) $K=0.6$

(c) $K=0.8$

(d) $K=1.0$

Figure 4.5 Sleeve friction profiles at different K values for inclined penetrations
Figure 4.5 continued

(e) $K=1.2$

(f) $K=1.4$

Figure 4.6 Comparison of sleeve friction during inclined penetrations for different $K$ values at penetration depth of 20 mm ($\sigma_v=100$ kPa)
As can be seen from both Figure 4.5 and Figure 4.6, the sleeve friction is strongly affected by the initial stress state. The higher the initial stresses are, the higher the sleeve friction will be. It should be noted that this trend is observed based on a constant initial vertical stress ($\sigma_v=100$ kPa), in which condition a higher K value means a higher initial stress state. For vertical penetration, the sleeve friction increases from 9.1 kPa to 31.9 kPa when K increases from 0.4 to 1.4.

As shown in both Figure 4.5 and Figure 4.6, for different K values, the trend of change of the sleeve friction varies with the change of inclination angles. For K<1 (i.e., K=0.4, 0.6, 0.8), the sleeve friction tends to increase when the inclination angle $\alpha$ increases. For K>1 (i.e., K=1.2, 1.4), the sleeve friction tends to decrease when the inclination angle $\alpha$ increases. This may imply that, when penetrating from an initial stress state with a constant vertical stress, for K<1, the sleeve friction will gradually increase when the penetration changes gradually from vertical to horizontal position; for K>1, the
sleeve friction will gradually decrease when the penetration changes gradually from vertical to horizontal position.

It is also noted that the further the initial stress state is away from isotropic, the more the sleeve friction varies at different penetration angles. Again, this is based on the penetrations from a constant initial vertical stress. For instance, at the penetration depth of 20 mm, the horizontal sleeve friction is 82%, 38% and 15% higher than the corresponding vertical sleeve friction for $K=0.4$, $K=0.6$ and $K=0.8$, respectively. The vertical sleeve friction is 23% and 11% higher than the corresponding horizontal sleeve friction for $K=1.4$ and $K=1.2$, respectively. Compared to the tip resistance variation, this may imply that the inclined penetration affects the sleeve friction more than the tip resistance.

4.3.3 Tip Pore Pressure Profiles

One important aspect of the piezocone that has not yet been standardized is the location of the porous element (Tumay, et al., 1981; Rad and Tumay, 1985; Chen and Mayne, 1994). The piezocones used in the current practice have three common locations for the porous elements: (1) at the cone tip (U1 configuration), generally mid-way for 10 & 15 cm² cones and the very tip for mini-cones, (2) 0.5 mm above the base of the cone (U2 configuration), and (3) behind the friction sleeve (U3 configuration), as illustrated in Figure 4.7. The U1 configuration at the cone tip (apex), which is called the tip pore pressure, is utilized in this study.

Figure 4.8(a)~(f) shows the tip pore pressure profiles at each inclination angle for each $K$ value. The tip pore pressure almost reaches the steady state value at about 14 mm
penetration depth, which is less than the steady state distance of tip resistance. These steady state values for each different K values are compared in Figure 4.9.

Figure 4.7 Schematic showing common locations of porous filter element

Figure 4.8 Tip pore pressure profiles at different K values for inclined penetrations

(a) K=0.4  (b) K=0.6

Figure 4.8 Tip pore pressure profiles at different K values for inclined penetrations
(c) \( K = 0.8 \)

(d) \( K = 1.0 \)

(e) \( K = 1.2 \)

(f) \( K = 1.4 \)

(Figure 4.8 continued)
Figure 4.9 Comparison of steady state tip pore pressure during inclined penetrations for different K values ($\sigma_v=100$ kPa)
As can be seen from both Figure 4.8 and Figure 4.9, the tip pore pressure is strongly affected by the initial stress state. The higher the initial stresses are, the higher the tip pore pressure will be. It should be noted that this trend is observed based on a constant initial vertical stress ($\sigma_v=100$ kPa) in which condition a higher K value means a higher initial stress state. For vertical penetration, the tip pore pressure increases from 0.18 MPa to 0.38 MPa when K increases from 0.4 to 1.4.

When the initial stress state is isotropic (K=1), the tip pore pressure is the same no matter at what inclination angle the soil is penetrated. But for an anisotropic initial stress state (K\neq1), different values of tip pore pressure are observed, as shown in both Figure 4.8 and Figure 4.9. However, for different K values, the trend of tip pore pressure changes with the change of inclination angles. For K<1 (K=0.4, 0.6, 0.8), the tip pore pressure tends to increase when the inclination angle $\alpha$ increases. For K>1 (K=1.2, 1.4), the tip pore pressure tends to decrease when the inclination angle $\alpha$ increases. This may imply that, when penetrating from an initial stress state with a constant vertical stress, for K<1, the tip pore pressure will gradually increase when the penetration changes gradually from vertical to horizontal position; for K>1, the tip pore pressure will gradually decrease when the penetration changes gradually from vertical to horizontal position.

It is also observed that, the further the initial stress state is away from isotropic, the more the tip pore pressure varies at different penetration angles. Again, this is based on the penetrations from a constant initial vertical stress. For instance, the horizontal tip pore pressure is 18%, 10% and 7% higher than the corresponding vertical tip pore pressure for K=0.4, K=0.6 and K=0.8, respectively. The vertical tip pore pressure is 6% and 3% higher than the corresponding horizontal tip pore pressure for K=1.4 and K=1.2,
respectively. This might suggest that the difference for tip pore pressures at different penetration angles is not so significant (Most of them are less than 10%). Compared to the variations of tip resistance and sleeve friction, it may be concluded that the inclined penetration affects the sleeve friction most, then the tip resistance, and least the tip pore pressure.

4.3.4 Strain Field

The resulting typical strain contours (one normal strain, and one shear strain) around the piezocone tip at the 20 mm penetration depth for K=0.4, K=1.0 and K=1.4 are presented in Figure 4.10, 4.11 and 4.12, respectively. The rectangular coordinate is used and it rotates with the rotation of the penetrometer during the inclined penetrations, as shown in Figure 4.2. It should be noted that the tensile strains are taken as positive while the compressive strains are taken as negative.

The calculated strains, especially the shear strains close to the tip of the cone, are quite large. This demonstrates the need to use large deformation finite element analysis for the piezocone penetration problems. For axial strains (\(\varepsilon_{33}\)), large compressive strains tend to appear at the very tip of the cone, while large tensile strains develop near the cone base. Here, axial strain means the strain along the inclined penetration direction. Very large (>100%) shear strains (\(\gamma_{13}\)) developed at the very tip of the cone. From Figure 4.10, Figure 4.11 and Figure 4.12 we can see that the large strains are located at a very small region around the cone. The contours are quite similar to each other and the values are quite close. This suggests that the soil deformations around the cone might be entirely controlled by the severe pushing of the cone and both the initial stress state and the inclination angle have minor effects on the soil deformations.
4.3.5 Stress Field

The resulting typical stress contours (the octahedral normal stress and the octahedral shear stress) around the piezocone tip at the 20 mm penetration depth for $K=0.4$, $K=1.0$ and $K=1.4$ are presented in Figures 4.13, 4.14 and 4.15, respectively. The rectangular coordinate is used and it rotates with the rotation of the penetrometer during the inclined penetrations, as shown in Figure 4.2. It should be noted that, the compressive stresses are taken as negative. The octahedral stresses are presented here because they provide a combined effect of the straining to which the soil is subjected to. In addition, they are important in the generation of excess pore pressures. The pore pressure developed during the cone penetration can be expressed in terms of the octahedral normal stress and the octahedral shear stress by the following approximate relationship (Henkel, 1959):

$$\Delta u = \Delta \sigma_{oct} + \alpha \Delta \tau_{oct}$$

(5.1)

Both the maximum octahedral normal stress and octahedral shear stress are concentrated on both the upper and lower portions of the conical surface, as shown in Figures 4.13, 4.14 and 4.15. This is in agreement with the findings of Abu-Farsakh (1997). The values of both octahedral stresses are correlated with the initial stress state. The higher the initial stresses are, the greater the resulting stress field is. Again, this trend is observed based on a constant initial vertical stress ($\sigma_v=100$ kPa) in which condition a higher $K$ value means a higher initial stress state. As can be seen from Figures 4.13 and 4.15, the shapes of the stress bulbs at different inclination angles are not so close to each other as those for strain bulbs (Figure 4.10 and 4.12). This may suggest that different angles of penetrations have stronger effects on the stress contour bulbs than on the strain contour bulbs.
Figure 4.10(a) Axial strain ($\varepsilon_{33}$) for vertical penetration at $K=0.4$

Figure 4.10(b) Axial strain ($\varepsilon_{33}$) for 30° penetration at $K=0.4$
Figure 4.10(c) Axial strain ($\varepsilon_{33}$) for 60° penetration at $K=0.4$

Figure 4.10(d) Axial strain ($\varepsilon_{33}$) for 90° penetration at $K=0.4
Figure 4.10(e) Shear strain ($\gamma_{13}$) for vertical penetration at $K=0.4$

Figure 4.10(f) Shear strain ($\gamma_{13}$) for 30° penetration at $K=0.4$
Figure 4.10(g) Shear strain ($\gamma_{13}$) for 60° penetration at K=0.4

Figure 4.10(h) Shear strain ($\gamma_{13}$) for 90° penetration at K=0.4
Figure 4.11(a) Axial strain ($\varepsilon_{33}$) for penetration at $K=1.0$

Figure 4.11(b) Shear strain ($\gamma_{13}$) for penetration at $K=1.0$
Figure 4.12(a) Axial strain ($\varepsilon_{33}$) for vertical penetration at $K=1.4$

Figure 4.12(b) Axial strain ($\varepsilon_{33}$) for $30^\circ$ penetration at $K=1.4$
Figure 4.12(c) Axial strain ($\varepsilon_{33}$) for 60° penetration at $K=1.4$

Figure 4.12(d) Axial strain ($\varepsilon_{33}$) for 90° penetration at $K=1.4$
Figure 4.12(e) Shear strain ($\gamma_{13}$) for vertical penetration at $K=1.4$

Figure 4.12(f) Shear strain ($\gamma_{13}$) for 30° penetration at $K=1.4$
Figure 4.12(g) Shear strain ($\gamma_{13}$) for 60° penetration at K=1.4

Figure 4.12(h) Shear strain ($\gamma_{13}$) for 90° penetration at K=1.4
Figure 4.13(a) Octahedral normal stress ($\sigma_{\text{oct}}, \text{MPa}$) for vertical penetration at $K=0.4$

Figure 4.13(b) Octahedral normal stress ($\sigma_{\text{oct}}, \text{MPa}$) for $30^\circ$ penetration at $K=0.4$
Figure 4.13(c) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for $60^\circ$ penetration at $K=0.4$

Figure 4.13(d) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for $90^\circ$ penetration at $K=0.4$
Figure 4.13(e) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for vertical penetration at $K=0.4$

Figure 4.13(f) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for $30^\circ$ penetration at $K=0.4$
Figure 4.13(g) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for 60° penetration at K=0.4

Figure 4.13(h) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for 90° penetration at K=0.4
Figure 4.14(a) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for penetration at $K=1.0$

Figure 4.14(b) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for penetration at $K=1.0$
Figure 4.15(a) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for vertical penetration at $K=1.4$

Figure 4.15(b) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for $30^\circ$ penetration at $K=1.4$
Figure 4.15(c) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for 60° penetration at $K=1.4$

Figure 4.15(d) Octahedral normal stress ($\sigma_{\text{oct}}$, MPa) for 90° penetration at $K=1.4$
Figure 4.15(e) Octahedral shear stress ($\tau_{oct}$, MPa) for vertical penetration at $K=1.4$

Figure 4.15(f) Octahedral shear stress ($\tau_{oct}$, MPa) for $30^\circ$ penetration at $K=1.4$
Figure 4.15(g) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for 60° penetration at $K=1.4$

Figure 4.15(h) Octahedral shear stress ($\tau_{\text{oct}}$, MPa) for 90° penetration at $K=1.4$
Figure 4.16(a) Excess pore pressures (MPa) for vertical penetration at K=0.4

Figure 4.16(b) Excess pore pressures (MPa) for 30° penetration at K=0.4
Figure 4.16(c) Excess pore pressures (MPa) for 60° penetration at K=0.4

Figure 4.16(d) Excess pore pressures (MPa) for 90° penetration at K=0.4
Figure 4.17 Excess pore pressures (MPa) for vertical penetration at K=1.0

Figure 4.18(a) Excess pore pressures (MPa) for vertical penetration at K=1.4
Figure 4.18(b) Excess pore pressures (MPa) for 30° penetration at K=1.4

Figure 4.18(c) Excess pore pressures (MPa) for 60° penetration at K=1.4
4.3.6 Spatial Excess Pore Pressure Distribution

The spatial distribution of the excess pore pressure developed around the piezocone penetrometer at a penetration depth of 20 mm for \( K = 0.4 \), \( K = 1.0 \) and \( K = 1.4 \) are presented in Figures 4.16, 4.17 and 4.18, respectively. The maximum concentration of the excess pore pressure is located around the lower one third of the cone, which is in agreement with the numerical findings of Abu-Farsakh (1997). A negative pore pressure is likely to develop behind the cone base as shown in Figures 4.16 to 4.18, which is possibly due to the separation between the cone and the soil just behind the base (Kiousis et al., 1988). This separation is partly due to the geometry of the cone because the soil nodes may slide out of the conical surface during the penetration. Since the geometry of the cone (the corner point at the cone base) has been rounded, this effect is minimized. Another cause of the possible separation lies in the introduction of the piezocone to a pre-bored hole.
while keeping the initial stresses unchanged. As a result, the confining pressure is tremendously underestimated. The separation may not happen if the numerical simulation takes into consideration the increase of the confining pressure around the cone (e.g., Abu-Farsakh et al., 2003). Also, because of the pre-bored hole assumption, the initial condition for the excess pore pressure is assumed to be zero everywhere. This is not the case if the piezocone has penetrated into the soil for a finite distance.

The maximum pore pressure increases with the increase of K value, which means that the higher the initial stresses are, the bigger the excess pore pressure will be. Again, this is based on the penetrations from a constant initial vertical stress. As for the spatial excess pore pressure distribution, it is observed that the further the initial stress state is away from isotropic, the more the tip pore pressure contour varies at different penetration angles. Again, this is based on the penetrations from a constant initial vertical stress. For K=1.4 (Figure 4.18), the contour bulbs are quite similar to each other and the corresponding values are also close; for K=0.4 (Figure 4.16), the changes of contour bulbs and corresponding values with different inclination angles are relatively more significant. This suggests that the inclined penetration might have a minor effect on the pore pressure distribution for an initial stress state close to isotropic.

4.3.7 Effect of Hydraulic Conductivity

So far, only initial stress anisotropy (i.e., $\sigma_h \neq \sigma_v$) has been discussed, and the hydraulic conductivity is assumed to be isotropic (i.e., $k_x = k_y = k_z$). Therefore, it is necessary to investigate the effect of anisotropic hydraulic conductivity on inclined piezocone penetrations. In this work, only the initial stress state of K=0.4 ($\sigma_h = 0.4 \sigma_v$, $\sigma_v = 100$ kPa) is selected since this K value matches those of many normally consolidated
soils in nature. To show the effect of penetration at different inclination angles, a vertical penetration ($\alpha=0$), an inclined penetration for $\alpha=45^\circ$ and a horizontal penetration ($\alpha=90^\circ$) are performed. To show the effect of anisotropy in hydraulic conductivity, three different combinations are selected, namely $k_x=k_y=k_z=k$, $k_x=k_y=5k_z=5k$, and $k_x=k_y=10k_z=10k$, and $k=5\times10^{-10}$ m/s. Here, we refer to a fixed rectangular coordinate with $z$ as the vertical direction. The mesh, boundary conditions are the same as in Figure 4.1 and 4.2 and soil parameters are the same as in Table 4.1 except for the coefficient of hydraulic conductivity. After the penetration depth of 20 mm, the piezocone is stopped and the excess pore pressures are allowed to dissipate.

The tip resistance, sleeve friction and tip pore pressure profiles with different combinations of hydraulic conductivities for different penetration angles are shown in Figure 4.19, 4.20 and 4.21, respectively. It is observed that the effect of different combinations of hydraulic conductivities on the tip resistance, sleeve friction and tip pore pressure profiles can be neglected. One should note that this conclusion is based on penetrations at a very low drainage condition ($k_x=k_y=k_z=k$, or $k_x=k_y=5k_z=5k$, or $k_x=k_y=10k_z=10k$, and $k=5\times10^{-10}$ m/s). However, different hydraulic conductivity conditions significantly affect the excess pore pressure dissipation at the cone tip, as shown in Figure 4.22. The higher the radial permeability coefficient is ($k_x$ and $k_y$), the faster the dissipation will be. For the same conductivity condition, the excess pore pressure at the cone tip tends to dissipate fastest for the horizontal penetration, while slowest for the vertical direction. One should note this is observed for normally consolidated soil with $K=0.4$ and $\sigma_v=100$ kPa.
Figure 4.19 Tip resistance profiles at different inclination angles for different hydraulic conductivities ($k=5\times10^{-10}$ m/s, $K=0.4$ and $\sigma_v=100$ kPa)
Figure 4.20 Sleeve friction profiles at different inclination angles for different hydraulic conductivities ($k=5 \times 10^{-10}$ m/s, $K=0.4$ and $\sigma_v=100$ kPa)
Figure 4.21 Tip pore pressure profiles at different inclination angles for different hydraulic conductivities ($k=5\times10^{-10} \text{ m/s}$, $K=0.4$ and $\sigma_v=100 \text{ kPa}$)

(a) $\alpha=0^\circ$

(b) $\alpha=45^\circ$

(c) $\alpha=90^\circ$
Figure 4.22 Dissipation of excess pore pressure at the cone tip for inclined penetrations 
\( (k=5\times10^{-10} \text{ m/s}, K=0.4 \text{ and } \sigma_v=100 \text{ kPa}) \)
CHAPTER 5
FIELD TESTING OF INCLINED CONE PENETRATION

5.1 Introduction

A computer simulated inclined piezocone penetration test, rather than an actual experiment, is conducted in the last chapter, and the resulting stress, strain and pore pressure fields are discussed. At the present time, extensive experimental data for inclined piezocone penetration is not available for a reliable verification purpose of the finite element analysis presented in the last chapter. Although a lot of theoretical and experimental work has been conducted in regard to vertical cone penetration test (VCPT), so far no work has been published related to inclined cone penetration tests (ICPT). The only related experimental study was conducted by Broere and van Tol (1998) on the horizontal cone penetration testing. A test series was performed in a rigid wall calibration chamber, where both vertical and horizontal cone penetrations were carried out in sand. They found that, for intermediate densities, the mean of horizontal tip resistance is approximately 20% higher than the vertical tip resistance, while the sleeve friction is lower horizontally than at a vertical orientation. This rigid wall calibration chamber study supports that the measurements obtained in an ICPT differ from those obtained in a VCPT.

In order to further investigate this effect, a field testing program of inclined cone penetration was undertaken at three different locations in Louisiana with varying soil
characteristics; the corresponding test results are discussed. Because the soil characteristics vary from point to point at each site, the test results may be used only qualitatively since no measurements of stress, strain and pore pressure distributions were made. The theoretical–experimental comparisons are thus restricted to the tip resistance and sleeve friction.

5.2 Continuous Intrusion Miniature Cone Penetration Test System (CIMCPT)

All the field tests were performed by using the Continuous Intrusion Miniature Cone Penetration Test System (CIMCPT, http://www.coe.lsu.edu/facilities/revegits-cimcpt.html) developed and implemented at the Louisiana Transportation Research Center (Tumay and Kurup, 2001). This system is mounted in a four-wheel drive, one ton, all terrain vehicle (Figure 5.1). A novel feature of this new in situ testing vehicle is the chain driven caterpillar-type continuous push device powered by a hydraulic motor to advance the cone penetrometer, which greatly increases productivity and serviceability (Figure 5.2). Hydraulic power is provided by the vehicle's transmission. A pressure compensated flow control valve controls the penetration speed (set for 2 cm/s). The reversible hydraulic motor is capable of continuously inserting and retracting the single, continuous, coiled penetration rod. The penetration rod is a 12.7 mm diameter, 15 m long stainless steel tube. It has a 2 cm² cone penetrometer attached to one end and a connector at the other. The ability to coil and uncoil the thrust rod is one unique feature of this miniature cone system. Coiling eliminates threaded connections and simplifies water proofing. The coiling mechanism also straightens the rod prior to insertion into the soil.
The rods have been proven to withstand more than 300 cycles of coiling and uncoiling. The maximum depth of penetration that can be achieved by the CIMCPT system is 15 m.

Figure 5.1 The CIMCPT vehicle

Figure 5.2 The continuous intrusion system

The miniature cone penetrometer has a projected cone area of 2 cm$^2$, a friction sleeve area of 40 cm$^2$, and a cone apex angle of 60° (Figure 5.3). It is a subtraction type cone (i.e., the tip load cell measures the cone resistance and the sleeve load cell measures
the combined cone resistance and sleeve friction). The tip and sleeve load cells are of the strain gauge type in a Wheatstone full bridge configuration. Both the tip and sleeve load showed zero return, excellent linearity, practically no hysteresis, and high repeatability. The probes are also temperature compensated, thereby reducing drift and increasing accuracy. A displacement transducer that is essentially an optical encoder friction-coupled to the rod measures the penetration depth. The encoder is axially mounted to a wheel, which is located within the cone pushing device that rotates as the cone rod is unwound and pushed into the soil.

Figure 5.3 The 2 cm² miniature friction cone penetrometer

5.3 Description of Field Testing Sites

To investigate the relationship of measurements between the VCPT and ICPT, three sites were selected in Louisiana to conduct inclined penetration tests, namely Baton Rouge site, New Iberia site, and Port Allen site. The index properties at each investigated site are listed in Table 1. In the table, \( w_a \) is water content (%), \( w_L \) is liquid limit (%), \( I_p \) is plasticity index, \( s_u \) is undrained shear strength (kPa) and OCR is overconsolidation ratio.

The Baton Rouge site is located at the intersection of Highland Road and Interstate 10 in Baton Rouge, Louisiana. Beneath a 4-meter thick sandy clay fill lies a thick (>40m) deposit of overconsolidated, desiccated silty clay/clayey silt formed during the
Pleistocene period and deposited in a deltaic environment. The soil is of stiff consistency, low moisture content and fissured with slickensides and occasional sand pockets (Arman and McManis, 1977). Within the depth of 16m, the water content ranges from 23% to 46%, liquid limit ranges from 48% to 70% and plasticity index ranges from 26% to 42%. The OCR varies from 15.6 at a depth of 5.5 m to 5.3 at about 15.9 m.

Table 5.1(a) Summary of soil properties for the investigated sites

<table>
<thead>
<tr>
<th>Site</th>
<th>$w_n$ (%)</th>
<th>$w_L$ (%)</th>
<th>$I_p$ (%)</th>
<th>$s_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baton Rouge site</td>
<td>23-46</td>
<td>48-70</td>
<td>26-42</td>
<td>...</td>
</tr>
<tr>
<td>New Iberia site</td>
<td>23-33</td>
<td>30-35</td>
<td>9-17</td>
<td>38-118</td>
</tr>
<tr>
<td>Port Allen site</td>
<td>31-63</td>
<td>64-115</td>
<td>25-41</td>
<td>18-44</td>
</tr>
</tbody>
</table>

Table 5.1(b) Values of OCR at certain depths for the investigated sites

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>1.5</th>
<th>2.5</th>
<th>3</th>
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The New Iberia site is located 10 miles south of New Iberia at US 90 interchange at Louisiana Highway 88. The soil profile at New Iberia site consists of stiff to medium silty clay soils down to 7.5 m, silty sand and sandy soils from 7.5 m to 12.0 m, interbedded with thin layers of silty clay, silty clay soil from 12.0 m to 13.3 m and sandy soils down to 16.0 m. Within the depth of 16m, the water content ranges from 23% to 33%, liquid limit ranges from 30% to 35% and the plasticity index ranges from 9% to 17%. The OCR varies from 4.3 near the surface to 1.2 at about 7 m.
The Port Allen site is a pavement research facility site located about 2 miles west of the Mississippi River in Port Allen. The soil deposit consists of 3.6 m of medium brown and gray silty clay layer, stiff clay layer from 3.6 m to 5.5 m, soft to medium gray clay from 5.5 m to 6.7m, followed by alternating layers of sandy and silty clay soils from 6.7 to 10.5 m, and sandy layer from 10.5 to 16.0 m. Within the depth of 16m, the water content ranges from 31% to 63%, the liquid limit ranges from 64% to 115%, and the plasticity index ranges from 25% to 41%. The OCR varies from 16.5 at 0.5 m to 2 at 6.5m depth.

Figure 5.4 Probability of soil type (%) based on Zhang and Tumay (1999)
Figure 5.5 Soil types based on Robertson et al. (1986)

Figure 5.4 shows the subsurface soil profiles at these sites based on the computerized probabilistic soil classification technique by Zhang and Tumay (1999) using in situ vertical penetration test results. Figure 5.5 shows the soil profiles based on soil classification method by Robertson et al., (1986). The method based on Zhang and Tumay gives much closer soil classifications to the boring tests when compared to the method based on Robertson's.
Figure 5.6 Installation of CIMCPT on the ground

Figure 5.7 Experiment setup for inclined penetration

Modifications are necessary in order to use CIMCPT system to perform inclined cone penetration tests since the system is originally designed for vertical penetration with the continuous intrusion mechanism affixed on the specific vehicle. A lot of work was
involved since the heavy continuous intrusion system has to be removed from the vehicle and set up on the ground (Figure 5.6). Two anchors were placed into the ground to help provide reaction force. A jack was used to lift one end of the steel frame to achieve certain inclined degrees for inclined penetrations (Figure 5.7).

One vertical and six inclined penetration tests were conducted at the Baton Rouge site; the test layout is shown in Figure 5.8(a). Three vertical and six inclined tests were conducted at the New Iberia and Port Allen sites; the test layout is shown in Figure 5.8(b). The inclination angles are 15°, 30° and 45° from vertical direction respectively at all the three sites.
Figure 5.9(a) Tip resistance profiles at Baton Rouge site

Figure 5.9(b) Sleeve friction profiles at Baton Rouge site
Figure 5.9(c) Friction ratio profiles at Baton Rouge site

Figure 5.10(a) Tip resistance profiles at New Iberia site
Figure 5.10(b) Sleeve friction profiles at New Iberia site

Figure 5.10(c) Friction ratio profiles at New Iberia site
Figure 5.11(a) Tip resistance profiles at Port Allen site

Figure 5.11(b) Sleeve friction profiles at Port Allen site
5.4 Results and Discussions

The change of tip resistance, sleeve friction and friction ratio with depth at different sites are shown in Figures 5.9, 5.10 and 5.11, respectively. For the inclined penetration, "depth" implies the projected distance along the vertical direction, not along the inclined direction. It is observed that both the tip resistance and sleeve friction differ from each other at certain depths for different penetration angles at all locations. The author believes different penetration angles contributed to the differences in penetration measurements; however, it is premature to assume that the differences are due only to the inclination angles. In order to compare the values of tip resistance and sleeve friction due to inclination angles, certain intersection points by vertical penetration and inclined ones were selected, as shown in Figure 5.8, and the corresponding data shown in Table 5.2 to Table 5.7, respectively.
Table 5.2 Comparison of tip resistance (MPa) at Baton Rouge site

<table>
<thead>
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<th>Point ID</th>
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<th>I1 (15^\circ)</th>
<th>I2 (30^\circ)</th>
<th>I3 (45^\circ)</th>
<th>I4 (15^\circ)</th>
<th>I5 (30^\circ)</th>
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Table 5.3 Comparison of sleeve friction (MPa) at Baton Rouge site

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Table 5.4 Comparison of tip resistance (MPa) at New Iberia site

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Table 5.5 Comparison of sleeve friction (MPa) at New Iberia site

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Table 5.6 Comparison of tip resistance (MPa) at Port Allen site

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5.4.1 Tip Resistance

At a first glance of Table 5.2 to Table 5.7, it is very hard to tell whether the tip resistance of a vertical penetration is higher or lower compared to an inclined one, since different conclusions may be drawn at different points (depths). From the results of finite element analysis presented in the last chapter, for normally consolidated soils, when $K<1$ ($K$ is the coefficient of lateral earth pressure at rest.), the tip resistance will gradually increase when the penetration changes gradually from vertical to horizontal position; for $K>1$, the tip resistance will gradually decrease when the penetration changes gradually from vertical to horizontal position. Since the $K$ values are involved, the possible $K$ values for different depths at different locations have to be discussed first.

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<th>V2</th>
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<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>F</td>
<td>2.286</td>
<td>&gt;1</td>
<td>0.018</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>G</td>
<td>2.844</td>
<td>&lt;1</td>
<td>0.022</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>0.017</td>
</tr>
<tr>
<td>H</td>
<td>3.959</td>
<td>&lt;1</td>
<td>0.041</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>I</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>J</td>
<td>1.320</td>
<td>&gt;1</td>
<td>0.020</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>0.013</td>
</tr>
<tr>
<td>K</td>
<td>2.286</td>
<td>&gt;1</td>
<td>0.013</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.026</td>
</tr>
<tr>
<td>L</td>
<td>2.844</td>
<td>&lt;1</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>M</td>
<td>3.959</td>
<td>&lt;1</td>
<td>0.047</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.032</td>
</tr>
</tbody>
</table>
For normally consolidated soil, the relationship of $K$ and the effective stress friction angle $\phi'$ proposed by Jaky (1944) is widely accepted:

$$K = 1 - \sin \phi'$$

(5.1)

From this equation we know $K < 1$ for normally consolidated soil. As for overconsolidated soil, the situation becomes more complicated. Mayne (1982) suggested an equation to estimate $K$ for overconsolidated soils as follows:

$$K = (1 - \sin \phi') \text{OCR} \sin \phi'$$

(5.2)

As both boring tests and vertical penetration tests (Figure 5.4) indicate, most of the soils at the investigated depths at all the testing sites are sandy/silty clay or silts. Consequently, the effective internal friction angle $\phi'$ normally lies between 15° to 30°. Table 5.8 shows the possible values of $K$ estimated by Eq. (5.2) for a different combination of OCR and possible $\phi'$. From Table 5.8, we might draw the following conclusion: for normally and slightly overconsolidated soil ($1 < \text{OCR} < 4$), $K$ is most likely less than 1; for heavily overconsolidated soil ($\text{OCR} > 4$), $K$ is most likely greater than 1.

Table 5.8 Possible $K$ values for different OCR and $\phi'$

<table>
<thead>
<tr>
<th>$\phi'$</th>
<th>OCR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td></td>
<td>0.74</td>
<td>0.89</td>
<td>0.98</td>
<td>1.06</td>
<td>1.12</td>
<td>1.18</td>
<td>1.23</td>
<td>1.27</td>
</tr>
<tr>
<td>30°</td>
<td></td>
<td>0.50</td>
<td>0.71</td>
<td>0.87</td>
<td>1.00</td>
<td>1.12</td>
<td>1.22</td>
<td>1.32</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Considering the OCR values (Table 5.1(b)) and the relationship between $K$ and OCR (Table 5.8), the estimated $K$ values at the intersection points (Figure 5.8) are shown in Table 5.2, Table 5.4 and Table 5.6. In total, there are 26 comparisons of tip resistance between vertical and inclined penetration for $K > 1$. Among them, 18 vertical tip
resistances are higher than the corresponding inclined ones, which means 69% of the measurements of tip resistance at K>1 condition favors our expectation. In total, there are 12 comparisons of tip resistance between vertical and inclined penetration for K<1. Among them, 4 vertical tip resistances are lower than the corresponding inclined ones, which means only 33% of the measurements of tip resistance at K<1 condition favor our expectation.

Table 5.9 Summary of OCR and K values for the selected soil layers

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>site</th>
<th>Depth</th>
<th>Soil Type</th>
<th>OCR</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Baton Rouge</td>
<td>1m–4m</td>
<td>sandy clay</td>
<td>&gt;4</td>
<td>&gt;1</td>
</tr>
<tr>
<td>2</td>
<td>New Iberia</td>
<td>3m–5m</td>
<td>silty clay</td>
<td>&lt;4</td>
<td>&lt;1</td>
</tr>
<tr>
<td>3</td>
<td>Port Allen</td>
<td>1m–2.5m</td>
<td>silty clay</td>
<td>&gt;4</td>
<td>&gt;1</td>
</tr>
<tr>
<td>4</td>
<td>Port Allen</td>
<td>4m–6m</td>
<td>clay</td>
<td>&lt;4</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>

In order to further demonstrate the difference of measurements between vertical and inclined penetrations, four layers of soils are selected considering both site descriptions and vertical penetration test results (Figure 5.4). At Baton Rouge site, heavily overconsolidated (OCR>10) sandy clay lies between a depth of 1m to 4m and the K values are higher than 1. We name it Layer 1. At New Iberia site, slightly overconsolidated (OCR<4) silty clay lies between a depth of 3m to 5m and the K values are less than 1. We name it Layer 2. At Port Allen site, heavily overconsolidated (OCR>4) silty clay lies between a depth of 1m to 2.5m and the K values are higher than 1. We name it Layer 3. Also at Port Allen site, slightly overconsolidated (OCR<4) clay lies between a depth of 4m to 6m and the K values are less than 1. We name it Layer 4. Thus we have 4 layers selected, in which Layer 1 and Layer 3 are heavily
overconsolidated (K>1), while Layer 2 and Layer 4 are slightly overconsolidated (K<1). Table 5.9 shows the summary of information for the above selected layers.

Both Layer 1 (Figure 5.12(a)) and Layer 3 (Figure 5.12(c)) are for heavily overconsolidated clay and the estimated K values are higher than 1. As concluded in the last chapter, for K>1, it is expected that the tip resistance will decrease when the inclination angle $\alpha$ increases. Actually both figures are very close to confirming the expectations except one of the 45°-inclined penetration (I6). For K<1, it is expected that the tip resistance will increase when the inclination angle $\alpha$ increases. But unfortunately, this is hardly seen for both Layer 2 (Figure 5.12(b)) and Layer 4 (Figure 5.12(d)). One thing that should be pointed out is that, both Layer 2 and layer 4 are lightly overconsolidated clay (<1OCR<4), and the K values may be very close to 1 according to Eq. (5.2) and Table 5.8. In this case, the initial stress state is close to isotropic and might contribute very little to the measurements of inclined penetrations and thus other factors (e.g., soil property variations) may contribute more to the difference.

![Figure 5.12(a) Averaged tip resistance for Layer 1 at Baton Rouge site (K>1)](image-url)
Figure 5.12(b) Averaged tip resistance for Layer 2 at New Iberia site (K<1)

Figure 5.12(c) Averaged tip resistance for Layer 3 at Port Allen site (K>1)
As discussed above, these field testing data tend to support the numerical findings of tip resistance change during inclined penetrations. In fact, this effect might also be explained based on the following simple cavity expansion model other than the three-dimensional finite element analysis. Cavity expansion model, first proposed by Vesic (1972), later was adopted and extended by many people (i.e., Luger, 1982; Carter et al., 1986; Yu and Houlsby, 1991; Salgado et al., 1997; Cao et al., 2001). The cavity expansion theory treats the cone penetration as a one-dimensional problem and the initial in situ stresses are assumed to be isotropic, i.e., the vertical stress $\sigma_v$ and horizontal stress $\sigma_h$ are equal. However, this assumption has to be abandoned for the inclined penetration; otherwise the results will be identical for penetrations at different inclination angles.

Broere and van Tol (1998) proposed a cavity expansion model to interpret the different measurements between horizontal and vertical tip resistance. Using the assumption that plane strain conditions hold, the cavity expansion can be entirely described as the expansion of a circular cavity in a plane perpendicular to the penetration.
direction. For VCPT, the initial stress state in this plane is a uniform radial stress ($\sigma_h$), but for ICPT, the confining stress is non-uniformly distributed, as shown in Figure 1.1. Assuming the soil is purely elastic obeying Hook’s law, based on the total work done in expanding the cavity, the following relationship between vertical and horizontal tip resistance is obtained (Broere and van Tol, 1998):

$$\frac{q_{cH}}{q_{cV}} = \frac{1+K}{2K}$$  \hspace{1cm} (5.3)

where $q_{cH}$ is the horizontal tip resistance, $q_{cV}$ is the vertical tip resistance, and $K$ is the coefficient of lateral earth pressure. In fact, this work can be easily extended to the relationship of tip resistance between vertical and inclined penetrations.

![Actual stress state and equivalent stress state around the cone during inclined penetration](image)

**Figure 5.13** Actual stress state and equivalent stress state around the cone during inclined penetration

As shown in Figure 5.13, one must find an "equivalent confining stress" to take into account the non-uniformly distributed initial stress state in the plane perpendicular to the inclined penetration. Here, “equivalent” refers to the total work done in expanding the cavity. For a penetration at an inclined angle $\alpha$ (the angle between the penetration
direction and the vertical direction, as shown in Figure 1.1), when the shear stresses are ignored, one of the principal stresses in this plane is $\sigma_h$, and the other principal stress in this plane, denoted by $\sigma_\alpha$, is a function of inclination angle $\alpha$ as below (Figure 5.13):

$$\sigma_\alpha = \frac{\sigma_v + \sigma_h - \sigma_v - \sigma_h}{2} \cos 2\alpha$$ \hspace{1cm}(5.4)

where $\sigma_v$ and $\sigma_h$ are the initial vertical stress and horizontal stress, respectively. In this chapter, according to conventional geomechanics sign notation, compressive stresses are considered to be positive. Therefore, the radial stress in the plane perpendicular to the penetration direction is

$$\sigma_\theta = \frac{\sigma_u + \sigma_h}{2} \cos 2\theta + \sigma_h$$ \hspace{1cm}(5.5)

It is assumed that the displacement at any point in this plane is still radial during the circular expansion from zero radius to a finite radius. If we define that the work required for expanding such a cavity in an actual initial stress state equals that of expanding the same cavity in a uniformly distributed stress state (equivalent confining stress), we have

$$W = \int_0^a \int_0^{2\pi} \sigma_\theta r d\theta dr = \int_0^a \int_0^{2\pi} \sigma_e r d\theta dr$$ \hspace{1cm}(5.6)

where $a$ is the final radius of the cavity. From Equation 5.5 and 5.6, we get

$$\sigma_e = \frac{\sigma_u + \sigma_h}{2}$$ \hspace{1cm}(5.7)

This is the expression of the equivalent confining stress for the plane perpendicular to the direction of the inclined penetration. This way, we can use cavity expansion theory to take into consideration the effect of the initially non-uniformly distributed stresses in the plane perpendicular to the inclined penetration.
Assuming the tip resistance is proportional to this equivalent confining stress, then the relationship between the inclined tip resistance $q_{cI}$ and vertical tip resistance $q_{cV}$ is:

$$\frac{q_{cI}}{q_{cV}} = \frac{\sigma_a + \sigma_h}{\sigma_h + \sigma_h} = \frac{1 + \sigma_a}{2\sigma_h}$$  \hspace{1cm} (5.8)

This equation coincides with Equation 5.3 for the horizontal penetration. If $K<1$, we have $\sigma_a > \sigma_h$, so $q_{cI} > q_{cV}$; if $K>1$, we have $\sigma_a < \sigma_h$, so $q_{cI} < q_{cV}$. Therefore, Equation 5.8 has the same prediction for the change of tip resistance during inclined penetrations as compared to the finite element findings of the last chapter, and they are both in agreement with the calibration chamber findings (Broere and van Tol, 1998). That is, for coefficient of lateral earth pressure $K<1$ ($\sigma_h < \sigma_v$), we expect tip resistance reaches the minimum value at the vertical position, then increases as the inclination angle $\alpha$ increases until it reaches the maximum value at the horizontal position. For $K>1$ ($\sigma_h > \sigma_v$), we expect the tip resistance reaches the maximum value at vertical position, then decreases as the inclination angle $\alpha$ increases until it reaches the minimum value at the horizontal position.

### 5.4.2 Sleeve Friction

As concluded in the last chapter, for $K<1$, the sleeve friction tends to increase when the inclination angle $\alpha$ increases, while the sleeve friction decreases when the inclination angle $\alpha$ increases for $K>1$. However, Broere and van Tol (1998) reported that the sleeve friction of horizontal penetration is lower than that of vertical penetration for normally consolidated sand after the calibration chamber testing, but no interpretation was given in
regard to this difference. Unfortunately, this field testing data can neither support nor deny the predictions since they are so scattered, as discussed below.

In total, there are 25 comparisons of sleeve friction between vertical and inclined penetration for $K>1$. Among them, 12 vertical sleeve frictions are higher than the corresponding inclined ones, which means 48% measurements of sleeve friction at $K>1$ condition favor our conclusion. In total, there are 12 comparisons of sleeve frictions between vertical and inclined penetration for $K<1$. Among them, 9 vertical sleeve frictions are lower than the corresponding inclined ones, which means 75% measurements of sleeve frictions at $K<1$ condition favor our conclusion.

The same 4 layers of soils are selected to further demonstrate the difference of sleeve friction measurements (Figure 5.14). Both Layer 1 (Figure 5.14(a)) and Layer 3 (Figure 5.14(c)) are for heavily overconsolidated clay and the estimated $K$ values are higher than 1. As stated earlier, for $K>1$, it is expected that the sleeve friction will decrease when the inclination angle $\alpha$ increases. In fact, Layer 1 (Figure 5.14(a)) shows different trends of change in sleeve friction as the inclination angle increases, depending on which set of tests are chosen. Layer 3 (Figure 5.14(c)) somewhat shows the opposite trends of change in sleeve friction as the inclination angle increases, except that both the 45° measurements drop suddenly, which favors the expectations. Both Layer 2 (Figure 5.14(b)) and Layer 4 (Figure 5.14(d)) are for slightly overconsolidated clay and the estimated $K$ values are lower than 1. As stated earlier, for $K<1$, it is expected that the sleeve friction will increase when the inclination angle $\alpha$ increases. Both Layer 2 (Figure 5.14(b)) and Layer 4 (Figure 5.14(d)) support our expectations as the inclination angle increases, except for the 45° inclined penetration.
In fact, the different measurements of sleeve friction during inclined penetrations may also be explained by the following simple equation:

\[ f_{cl} = c_a + \mu \sigma_{nl} \]  \hspace{1cm} (5.9)

\( f_{cl} \) is the sleeve friction at a certain inclined penetration, \( c_a \) is the adhesion between the soil and the penetrometer, \( \mu \) is the friction coefficient, and \( \sigma_{nl} \) is the normal stress acting on the sleeve area. Since the normal stress is not necessarily uniformly distributed in the plane perpendicular to the inclined penetration direction, the equivalent confining stress (Figure 5.13 and Equation 5.7) may be introduced instead. Then Equation 5.9 can be rewritten as:

\[ f_{cl} = c_a + \mu \sigma_e \]  \hspace{1cm} (5.10)

If we assume \( c_a \) and \( \mu \) remain the same for different inclination angles, then the sleeve friction is determined solely by the magnitude of equivalent confining stress \( \sigma_e \). Thus we anticipate that the sleeve friction tends to change in the same direction as the tip resistance changes during the inclined penetrations. That is, for \( K<1 \), the sleeve friction tends to increase when the inclination angle \( \alpha \) increases, while the sleeve friction decreases when the inclination angle \( \alpha \) increases for \( K>1 \). Equation 5.10 has the same prediction for the change of sleeve friction during inclined penetrations as compared to the finite element findings of last chapter, but they are not in agreement with the calibration chamber findings (Broere and van Tol, 1998).

5.4.3 Friction Ratio and Soil Classification

As discussed above, since both tip resistance and sleeve friction tend to change in the same direction as the inclination angle \( \alpha \) increases, it is hard to tell whether the friction ratio will increase or decrease with the change in inclination angles. As a result,
the soil classifications become unclear based on inclined penetration results. However, it is still believed the soil classifications differ from each other for inclined penetrations. To show this effect, at each site a soil classification based on a vertical penetration is compared with that of a 45° inclined penetration, using Zhang and Tumay's classification method (1999), as shown in Figure 5.15(a), (b) and (c). Also, the same four soil layers are selected and the averaged percentages for each type of soil are shown in Table 5.10.

From Table 5.10, it seems the friction ratio tends to increase for both $K>1$ and $K<1$ conditions. As a result, the perceived percentage of fines (clay) may increase while the percentage of coarse material sand may drop when the inclination angle increases. This is observed in both Figure 5.15 and Table 5.10. If this effect holds true for penetrations in other soils, too, the soil classification charts based on friction ratio should be modified to take into account the inclined penetrations. However, more extensive data for various soils are needed in order to further analyze this effect.

![Figure 5.14(a) Averaged sleeve friction for Layer 1 at Baton Rouge site ($K>1$)](image-url)
Figure 5.14(b) Averaged sleeve friction for Layer 2 at New Iberia site (K<1)

Figure 5.14(c) Averaged sleeve friction for Layer 3 at Port Allen site (K>1)
Figure 5.14(d) Averaged sleeve friction for Layer 4 at Port Allen site (K<1)

Table 5.10 Averaged soil type percentage (%) for the selected soil layers

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>K</th>
<th>V1</th>
<th>I6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>clay</td>
<td>silt</td>
</tr>
<tr>
<td>1</td>
<td>&gt;1</td>
<td>38</td>
<td>49</td>
</tr>
<tr>
<td>2</td>
<td>&lt;1</td>
<td>15</td>
<td>57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layer No.</th>
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<th>V3</th>
<th>I3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>clay</td>
<td>silt</td>
</tr>
<tr>
<td>3</td>
<td>&gt;1</td>
<td>81</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>&lt;1</td>
<td>63</td>
<td>33</td>
</tr>
</tbody>
</table>
Figure 5.15(a) Probability of soil type (%) at Baton Rouge site based on Zhang and Tumay (1999)
Figure 5.15(b) Probability of soil type (%) at New Iberia site based on Zhang and Tumay (1999)
Figure 5.15(c) Probability of soil type (%) at Port Allen site based on Zhang and Tumay (1999)
CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 Summary

Vertical penetration can be treated as an axi-symmetric problem since the vertical overburden stresses are acting in the direction of penetration while the axi-symmetric horizontal confining stresses are acting radially around the cone. However, this is not the case for inclined penetration and the stresses are not axi-symmetric. Therefore, it is necessary to carry out a three-dimensional finite element analysis. In this work, a large strain finite element analysis using the commercial finite element code ABAQUS, is used to analyze the effect of soil anisotropy on the inclined piezocone penetration test in saturated cohesive soils. The piezocone penetration analysis is treated as a boundary problem with the need of changing the boundary conditions as the piezocone penetrometer advances. The penetrometer is assumed to be infinitely stiff. For numerical purposes, the piezocone penetrometer is assumed to be initially pre-bored to a certain inclined depth with the initial stresses remaining unchanged. The continuous penetration of the piezocone is simulated by applying incremental displacements along the penetration direction at the nodes representing the piezocone boundary. The penetration rate was 2 cm/s, which is the same as is mostly used for field tests. The saturated clay is modeled as a multi-phase material and the effective stress principle is used to describe its behavior. The Mohr-Coulomb frictional model was used to define the sliding potential between the cone surface and the surrounding soil. The soil-penetrometer interface
friction coefficient is assumed to be 0.25, which corresponds to an angle of friction $\delta=14^\circ$ between the soil and the piezocone surface. The numerical simulations are performed for the same normally consolidated soil, but with a different initial stress state. The tip resistance, sleeve friction and excess pore pressure profiles with penetration, and the developed strain, stress and excess pore pressure fields around the piezocone, as well as the excess pore pressure dissipation at the cone tip are presented and discussed.

To better catch the anisotropic soil behavior during penetration, the Anisotropic Modified Cam Clay Model (AMCCM) proposed by Dafalias (1987) was chosen and implemented into ABAQUS through user subroutine UMAT. For verification purposes, previous conducted calibration chamber tests (Kurup, et al., 1994) were simulated using the above analytical model and the results are compared with the actual measured values. It shows that the results calculated using AMCCM are overall in good agreement with the experimental measurements. Therefore, the subsequent predictions of three-dimensional finite element analysis for inclined piezocone penetrations are based on a solid foundation.

At the present time, extensive experimental data for inclined piezocone penetration is not available for a reliable verification purpose of the presented finite element analysis. As a result, a field testing program of inclined cone penetration was developed and performed in three different locations with varying soil characteristics in Louisiana, using the Continuous Intrusion Miniature Cone Penetration Test System (CIMCPT, http://www.coe.lsu.edu/facilities/revegits-cimcpt.html). The inclination angles were selected to be 15°, 30° and 45° from the vertical direction, respectively, in all three sites. However, since the soil characteristics vary from point to point at each site, and no
measurements of stress, strain and pore pressure distributions were made, the testing results can be evaluated only qualitatively rather than quantitatively.

6.2 Conclusions

In this work, a large strain finite element analysis was performed to analyze the effect of soil anisotropy on the inclined piezocone penetration test in cohesive soils. The piezocone penetration is numerically simulated based on large strain formulations using commercial finite element code ABAQUS. The saturated clay is modeled as a multi-phase material and the effective stress principle is used to describe its behavior. A frictional contact interface utilizing Mohr-Coulomb's theory was chosen to represent interactions between the surface of the cone and the soil. The Anisotropic Modified Cam Clay Model (AMCCM) proposed by Dafalias (1987) was chosen and implemented into ABAQUS through user subroutine UMAT. For verification purposes, a field testing program of inclined cone penetration was developed and performed in three different locations with varying soil characteristics in Louisiana. Based on this study, a number of important conclusions can be drawn:

1) As compared to the previously conducted calibration chamber tests, the finite element analysis results based on Anisotropic Modified Cam Clay Model (AMCCM) are overall in good agreement with the actual measurements. It indicated that the soil anisotropy plays an important role during piezocone penetrations. Therefore, a soil model which can take into account the anisotropic hardening, such as the AMCCM, is necessary for the simulation of soil behavior during piezocone penetrations.

2) Initial stress state strongly affects the tip resistance, sleeve friction and generated excess pore pressures. The higher the initial stresses are, the higher the tip resistance,
sleeve friction and excess pore pressures, as well as the resulting stresses and strains. The coefficient of lateral earth pressure (K) indicates the degree of initial stress anisotropy. If K=1, then there’s no difference expected between inclined and vertical penetrations.

3) A steady state for the tip resistance was almost reached at a penetration depth of 20 mm from a pre-bored hole. For different K values, the trend of tip resistance changes with the change of inclination angles. For K<1, the tip resistance will gradually increase as the penetration changes gradually from vertical to horizontal position; for K>1, the tip resistance will gradually decrease as the penetration changes gradually from vertical to horizontal position. The relationship of tip resistance changing with the inclination angle is almost linear. The further the initial stress state is away from isotropic, the more significant the tip resistance variations at different penetration angles become. The horizontal tip resistance is about 27% and 15% higher than the corresponding vertical tip resistance for K=0.4 and K=0.6, respectively.

4) At the penetration depth of 20 mm, a steady state for the sleeve friction still has not been reached. The author believes this happens because of the relatively short penetration depth. However, when the penetration distances are large, severe mesh distortions happen in zones of high strain concentrations around the cone tip, which lead to a severe loss of accuracy and numerical divergence. Therefore, the simulated penetration depth in this study was limited to 20 mm. The standard penetrometer has a sleeve friction area of 150 cm², which corresponds to a friction sleeve length of 134 mm. Since the penetration distance is only 15% of the friction sleeve length, one expects most of the normal and shear stresses on the sleeve have not been fully
mobilized with such a short penetration distance as compared to the friction sleeve length. Nevertheless, for preliminary comparison purposes, the sleeve friction values at penetration depth of 20 mm for different K values are observed and the following conclusions can be made. For K<1, the sleeve friction will gradually increase as the penetration changes gradually from vertical to horizontal position; for K>1, the sleeve friction will gradually decrease as the penetration changes gradually from vertical to horizontal position. The further the initial stress state is away from isotropic, the more significant the sleeve friction varies at different penetration angles. The horizontal sleeve friction is 82% and 38% higher than the corresponding vertical sleeve friction for K=0.4 and K=0.6, respectively at the penetration depth of 20 mm.

5) A steady state for the tip pore pressure (U1 configuration) was reached at a penetration depth of 14mm from a pre-bored hole. For different K values, it changes in the same way as that of the tip resistance and sleeve friction during inclined penetrations. However, it is not as significant as that of the tip resistance or sleeve friction. The maximum difference was about 18% at K=0.4 condition. The further the initial stress state is away from isotropic, the more the tip pore pressure contour varies at different penetration angles. The inclined penetration seems to have minor effect on the pore pressure distribution for an initial stress state close to isotropic.

6) For a piezocone penetration at a certain inclined angle, different hydraulic conductivities (i.e., k_x=k_y≠k_z) have minor effect on the tip resistance, sleeve friction and tip pore pressure profiles (U1 configuration), provided that the hydraulic conductivity in the soil is very low (k=5x10^{-10} m/s). However, different hydraulic conductivities significantly affect the excess pore pressure dissipation at the cone tip.
The higher the radial coefficient of hydraulic conductivity is \((k_x \text{ and } k_y)\), the faster the dissipation will be. For the same conductivity condition, the excess pore pressure at the cone tip tends to dissipate faster for the horizontal penetration than for the vertical penetration.

7) The predicted strains are large. Very large (>100%) shear strains \((\gamma_{13})\) developed at the very tip of the cone. This demonstrates the need for using a large deformation finite element analysis for the piezocone penetration problems. However, the strain contours are close to each other no matter what the \(K\) value or penetration angle is, except for \(K=0.4\). This may imply that, if the initial stress anisotropy is below a certain degree, its effect on the soil deformations during piezocone penetrations may be negligible.

8) Field testing data confirms the numerical findings of the change of tip resistance during inclined penetrations for heavily overconsolidated soils. That is, the tip resistance will decrease when the inclination angle \(\alpha\) increases for \(K>1\). However, the field testing data for \(K<1\) is too scattered to see a clear trend. A simplified cavity expansion model was given to attribute the different tip resistance measurements to the different initial stress state in the plane perpendicular to the inclined orientation, which is in agreement with the finite element findings.

9) The field test data for sleeve friction measurements are also scattered. However, they support the numerical findings to a certain extent, especially for \(K<1\) situations. A similar explanation was given to attribute the different sleeve friction measurements to the different initial stress state in the plane perpendicular to the inclined orientation. This explanation is also in agreement with the three-dimensional finite
element findings. As a result, the CPT-based soil classifications based on inclined penetrations differ from those based on vertical penetrations.

6.3 Limitations and Recommendations for Future Research

This work presents a preliminary study toward the understanding and simulation of the inclined piezocone penetrations in saturated cohesive soils. Due to the problem complexity and limited time, the numerical model has some limitations which can be improved in future work. Limitations of this work and recommendations for future research can be summarized as follows:

1) The Anisotropic Modified Cam Clay Model (AMCCM) is used in this research. The Modified Cam Clay Model (MCCM) is not suitable for application to heavily overconsolidated clays since it tends to overestimate the strength of the soil in the strain-softening region (Chen and Mizuno, 1990). Similarly, the Anisotropic Modified Cam Clay Model (AMCCM) is not recommended for heavily overconsolidated soils since it is an extension of MCCM. As a result, this work focused only on numerical simulations of inclined piezocone penetrations in normally consolidated soils.

2) The Anisotropic Modified Cam Clay Model (AMCCM) can simulate the anisotropic soil behavior during inclined piezocone penetrations. However, it cannot take into consideration the possible soil substructure changes (e.g., rotation or realignment of soil grains). Therefore, a soil model which can incorporate micro-mechanical behavior of soils is recommended. In addition, the penetration rate of 20 mm/s cause strains to be induced at a very high rate. Therefore, for a more rigorous approach, the rate dependency in the soil model should be involved.
3) In this work, the soil is assumed to be fully saturated and is treated as a two-phase problem. For the application of this study to the more versatile soil conditions, such as unsaturated conditions, an extension of the theory of mixtures for three-phase materials (air, water and solid) is recommended.

4) In this work, the piezocone penetrometer is assumed to be initially pre-bored to a certain depth with the initial stresses remaining unchanged. As a result, the developed soil stresses around the cone could be underestimated. However, tremendous computational errors occurred due to large rotations of involved elements if the penetration started from the top surface of the soil. A valid simulation of this transient state is desirable.

5) Although both the tip resistance and tip pore pressure (U1 configuration) reach to a steady state within the simulated penetration depth, the sleeve friction does not. The author believes this happens because of the relatively short penetration distance. In addition, to fully consider the pore pressure interaction along the cone shaft, a longer penetration distance needs to be simulated, too. However, when the penetration distances are large, severe mesh distortions happen in zones of high strain concentrations around the cone tip, which lead to a severe loss of accuracy and numerical divergence. Therefore, the simulated penetration depth in this study was limited to 20 mm. For a larger penetration distance, either a remeshing technique or Arbitrary Lagrange-Eulerean formulation is recommended for future research.
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APPENDIX: UMAT SUBROUTINE

SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,STRAN,DSTRAN,
2 TIME,DTIME,TEMP,DTMP,PREDEF,DPREDEF,MATERL,NDI,NSHR,NTENS,
3 NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,CELENT,
4 DFGRD0,DFGRD1,NOEL,NPT,KSLAY,KSTEP,KINC)
C
INCLUD 'ABA_PARAM.INC'
C
CHARACTER*80 SOIL
DIMENSION STRESS(NTENS),STATEV(NSTATV),
1 DDSDDDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
2 STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPREDEF(1),
3 PROPS(NPROPS),COORDS(3),DROT(3,3),
4 DFGRD0(3,3),DFGRD1(3,3)
C
DIMENSION EELAS(6),EPLAS(6),SALPHA(6),BB21(3),BB31(3),S(6),A(6),
1 B(6),DUM(6),DELSAL(6),POSTS(6),DELEPS(3,3),DELEPI(3,3),DELEPV(6),
2 EELADSE(6,6),STRE(6),STRA(6),DSTRA(6),PRES(6),BASAL(6),
3 EELAS0(6),EPLAS0(6),SALPH0(6)
C
C -----------------------------------------------------------
C UMAT FOR 3D ANISOTROPIC MODIFIED CAM CLAY MODEL (DAFALIAS, 1987)
C MODIFIED FROM DR SONG'S PREVIOUS WORK BY LEI WEI IN SUMMER 2003
C FULLY BACKWARD EULER INTEGRATION ALGORITHM USED FOR STRESS
UPDATE
C -----------------------------------------------------------
C PROPS(1) - M !SLOPE OF CRITICAL STATE LINE IN STRESS SPACE
C PROPS(2) - NU !PISON'S RATIO
C PROPS(3) - KAPA !SLOPE OF UNLOADING RELOADING LINE IN V-LNP' PLANE
C PROPS(4) - LENBUDA !SLOPE OF NORMAL COMPRESSION LINE IN V-LNP'
PLAN
C PROPS(5) - C !BACK STRESS PARAMETER
C PROPS(6) - X !BACK STRESS PARAMETER
C -----------------------------------------------------------
C STATEV(1-6) - ELASTIC STRAIN TENSOR IN VECTOR FORM
C STATEV(7-12) - PLASTIC STRAIN TENSOR IN VECTOR FORM
C STATEV(13-18) - SALPHA TENSOR IN VECTOR FORM
C STATEV(19-24) - INCREMENT OF SALPHA TENSOR IN VECTOR FORM
C STATEV(25) - VOID RATIO
C STATEV(26) - YIELD SURFACE SIZE INDICATOR (P0)
C STATEV(27) - ALPHA
C
C MAKE COMPREESIVE STRESS AND STRAIN POSITIVE
C
DO 5 K1=1,6

147
STRE(K1)=-STRESS(K1)
STRA(K1)=-STRAIN(K1)
DSTRA(K1)=-DSTRAIN(K1)
5 CONTINUE
C
C STATE VARIABLES
C
PREVOI=STATEV(25)
PC=STATEV(26)
ALPHA=STATEV(27)
DO 170 K1=13,18
   SALPHA(K1-12)=STATEV(K1)
170 CONTINUE
C
DO 200 K1=19,24
   DELSAL(K1-18)=STATEV(K1)
200 CONTINUE
C
C ELASTIC PROPERTIES
C
ENU=PROPS(2)
PROPM=PROPS(1)
PREP=(STRE(1)+STRE(2)+STRE(3))/3.0D0
DO 10 K1=1,3
   PRES(K1)=STRE(K1)-PREP
10 CONTINUE
DO 20 K1=4,6
   PRES(K1)=STRE(K1)
20 CONTINUE
Q1=0.0D0
DO 30 K1=1,3
   Q1=Q1+S(K1)*S(K1)
30 CONTINUE
Q2=0.0D0
DO 40 K1=4,6
   Q2=Q2+2.0D0*S(K1)*S(K1)
40 CONTINUE
PREQ=SQRT(1.5D0*(Q1+Q2))
PREETA=PREQ/PREP
BK=(1.0D0+PREVOI)*PREP/PROPS(3)
G=BK*1.5D0*(1.0D0-2.0D0*ENU)/(1.0D0+ENU)
C
C ELASTIC STIFFNESS
C
DO 60 K1=1,6
   DO 50 K2=1,6
      DDSDDE(K2,K1)=0.0
50 CONTINUE
60 CONTINUE
DO 80 K1=1,3
DO 70 K2 = 1, 3
   DDSDDE(K2, K1) = (3.0D0 * BK - 2.0D0 * G) / 3.0D0
70   CONTINUE
DDSDDE(K1, K1) = (3.0D0 * BK + 4.0D0 * G) / 3.0D0
80   CONTINUE
   DO 90 K1 = 4, 6
      DDSDDE(K1, K1) = G
90   CONTINUE
C
C CALCULATE STRESS FROM ELASTIC STRAINS
C
   DO 110 K1 = 1, 6
      DO 100 K2 = 1, 6
         STRE(K2) = STRE(K2) + DDSDDE(K2, K1) * DSTRA(K1)
100    CONTINUE
110   CONTINUE
C
C RECOVER ELASTIC AND PLASTIC STRAINS
C
   DO 120 K1 = 1, 6
      EELAS(K1) = STATEV(K1) + DSTRA(K1)
      EPLAS(K1) = STATEV(K1 + 6)
120   CONTINUE
C
C UPDATE STRESSES BASED ON TRIAL VALUES
C
      P = (STRE(1) + STRE(2) + STRE(3)) / 3.0D0
   DO 130 K1 = 1, 3
      S(K1) = STRE(K1) - P
130    CONTINUE
   DO 140 K1 = 4, 6
      S(K1) = STRE(K1)
140    CONTINUE
Q1 = 0.0D0
   DO 150 K1 = 1, 3
      Q1 = Q1 + S(K1) * S(K1)
150    CONTINUE
   DO 160 K1 = 4, 6
      Q2 = Q2 + 2.0D0 * S(K1) * S(K1)
160    CONTINUE
Q = SQRT(1.5D0 * (Q1 + Q2))
ETA = Q / P
C
C UPDATE VOID RATIO
C
      EVOID = PREVOI - PROPS(3) * LOG(P / PREP)
C
C JUDGE IF YIELDING OCCURS
C
PY = P*(PROPM*PROPM + ETA*ETA - 2.0D0*ALPHA*ETA)/(PROPM*PROPM - $ALPHA*ALPHA)

IF (PY .LT. PC) nflag=1
IF (PY .LT. PC) GO TO 88

C CALCIULATE Bij

BB11 = (2.0D0*P-PC)/3.0D0
SALPH = S(1)*SALPHA(1) + S(2)*SALPHA(2) + S(3)*SALPHA(3) + $2.0D0*S(4)*SALPHA(4) + 2.0D0*S(5)*SALPHA(5) + $2.0D0*S(6)*SALPHA(6)
BB12 = (PC*ALPHA*ALPHA/3.0D0 - SALPH)/(PROPM*PROPM)
BB21(1) = 3.0D0*(2.0D0*S(1)/3.0D0 - S(2)/3.0D0 - S(3)/3.0D0)/$(PROPM*PROPM)
BB21(2) = 3.0D0*(2.0D0*S(2)/3.0D0 - S(1)/3.0D0 - S(3)/3.0D0)/$(PROPM*PROPM)
BB21(3) = 3.0D0*(2.0D0*S(3)/3.0D0 - S(2)/3.0D0 - S(1)/3.0D0)/$(PROPM*PROPM)
BB31(1) = 3.0D0*P*(2.0D0*SALPHA(1)/3.0D0 - SALPHA(2)/3.0D0 - $SALPHA(3)/3.0D0)/(PROPM*PROPM)
BB31(2) = 3.0D0*P*(2.0D0*SALPHA(2)/3.0D0 - SALPHA(1)/3.0D0 - $SALPHA(3)/3.0D0)/(PROPM*PROPM)
BB31(3) = 3.0D0*P*(2.0D0*SALPHA(3)/3.0D0 - SALPHA(2)/3.0D0 - $SALPHA(1)/3.0D0)/(PROPM*PROPM)

DO 210 K1=1,3
A(K1) = BB11 + BB12 + BB21(K1) - BB31(K1)
210 CONTINUE
A(4) = 3.0D0*(S(4)-P*SALPHA(4))/(PROPM*PROPM)
A(5) = 3.0D0*(S(5)-P*SALPHA(5))/(PROPM*PROPM)
A(6) = 3.0D0*(S(6)-P*SALPHA(6))/(PROPM*PROPM)
BII = A(1) + A(2) + A(3)

C CALCIULATE Cijkl*Bkl

DO 230 K1=1,3
B(K1) = 0.0D0
DO 220 K2=1,3
B(K1) = B(K1) + DDSDDE(K1,K2)*A(K2)
220 CONTINUE
DO 240 K1=4,6
B(K1) = 2.0D0*DDSDDE(K1,K1)*A(K1)
240 CONTINUE

C CALCIULATE dF/dpc*pc(ba)
XI = (1.0D0+PREVOI)/(PROPS(4)-PROPS(3))
AA1 = XI*P*PC*(-1.0D0+(ALPHA*ALPHA/(PROPM*PROPM)))
AA = AA1*BII
C CALCULATE He=Bij*Cijkl*Bkl
C
AB1=0.0D0
DO 250 K1=1,3
   AB1=AB1+A(K1)*B(K1)
250 CONTINUE
AB2=0.0D0
DO 260 K1=4,6
   AB2=AB2+2.0D0*A(K1)*B(K1)
260 CONTINUE
AB=AB1+AB2
C
C CALCULATE Bij*Cijkl*dEPSILONkl
C
AC=0.0D0
DO 270 K1=1,6
   AC=AC+DSTRA(K1)*B(K1)
270 CONTINUE
C
C CALCULATE dF/dALPHAij
C
DO 290 K1=1,6
   DUM(K1)=3.0D0*(P*PC*SALPHA(K1)-P*S(K1))/(PROPM*PROPM)
290 CONTINUE
C
C CALCULATE ALPHAij(ba)
C
BASAL(K1)=ABS(BII)*XI*PROPS(5)*(S(K1)-PROPS(6)*$P*SALPHA(K1))/PC
C
C CALCULATE (dF/dALPHAij)*ALPHAij(ba)
C
AD1=0.0D0
DO 300 K1=1,3
   AD1=AD1+DUM(K1)*BASAL(K1)
300 CONTINUE
AD2=0.0D0
DO 310 K1=4,6
   AD2=AD2+2.0D0*DUM(K1)*BASAL(K1)
310 CONTINUE
AD=AD1+AD2
C
C CALCULATE SCALAR OF FLOW RULE
C
DELBUD=AC/(AB-AA-AD)
C
C UPDATE STRAINS
C
DO 320 K1=1,3
   EPLAS0(K1)=EPLAS(K1)+DELBUD*A(K1)
EELAS0(K1) = EELAS(K1) - DELBUD*A(K1)
320 CONTINUE
   DO 330 K1 = 4, 6
      EPLAS0(K1) = EPLAS(K1) + 2.0D0*DELBUD*A(K1)
      EELAS0(K1) = EELAS(K1) - 2.0D0*DELBUD*A(K1)
   330 CONTINUE

C
C    UPDATE STRESSES
C
   DO 340 K1 = 1, 6
      STRE(K1) = STRE(K1) - DELBUD*B(K1)
   340 CONTINUE
   POSTP = (STRE(1) + STRE(2) + STRE(3))/3.0D0
   DO 350 K1 = 1, 3
      POSTS(K1) = STRE(K1) - POSTP
   350 CONTINUE
   DO 36 K1 = 4, 6
      POSTS(K1) = STRE(K1)
   36 CONTINUE
   Q1 = 0.0D0
   DO 370 K1 = 1, 3
      Q1 = Q1 + POSTS(K1)*POSTS(K1)
   370 CONTINUE
   Q2 = 0.0D0
   DO 38 K1 = 4, 6
      Q2 = Q2 + 2.0D0*POSTS(K1)*POSTS(K1)
   38 CONTINUE
   POSTQ = SQRT(1.5D0*(Q1 + Q2))
   POSTET = POSTQ/POSTP

C
C    UPDATE BACK STRESS SALPHAIj
C
   DO 390 K1 = 1, 6
      DELSA(K1) = DELBUD*BASAL(K1)
   390 CONTINUE
   DO 400 K1 = 1, 6
      SALPH0(K1) = SALPHA(K1) + DELSA(K1)
   400 CONTINUE
   CALL ROTSIG(SALPH0, DROT, SALPHA, 1, 3, 3)
   SALPH1 = 0.0D0
   DO 410 K1 = 1, 3
      SALPH1 = SALPH1 + SALPHA(K1)*SALPHA(K1)
   410 CONTINUE
   SALPH2 = 0.0D0
   DO 440 K1 = 4, 6
      SALPH2 = SALPH2 + 2.0D0*SALPHA(K1)*SALPHA(K1)
   440 CONTINUE
   ALPHA = SQRT(1.5D0*(SALPH1 + SALPH2))
   PC0 = PC
C UPDATE P0
C PY=PC+DELBUD*XI*PC*BII
   IF (PY .LT. PC) THEN
     do 360 k1=1,6
       stre(k1)=pc*stre(k1)/py
     continue
     POSTP=(STRE(1)+STRE(2)+STRE(3))/3.0D0
     DO 380 K1=1,3
       POSTS(K1)=STRE(K1)-POSTP
     380 CONTINUE
     POSTS(4)=STRE(4)
     POSTS(5)=STRE(5)
     POSTS(6)=STRE(6)
     Q1=0.0D0
     DO 420 K1=1,3
       Q1=Q1+POSTS(K1)*POSTS(K1)
     420 CONTINUE
     Q2=0.0D0
     DO 430 K1=4,6
       Q2=Q2+2.0D0*POSTS(K1)*POSTS(K1)
     430 CONTINUE
     POSTQ=SQRT(1.5D0*(Q1+Q2))
     POSTET=POSTQ/POSTP
   else
     pc=py
   ENDIF
C UPDATE VOID RATIO
C EVOID=PREVOI-PROPS(3)*LOG(POSTP/PREP)-(PROPS(4)-PROPS(3))*$LOG(PC/PC0)
C eta=postet
nflag=2
C MAKE COMPRESSIVE STRESS AND STRAIN NEGATIVE
C 88    DO 630 K1=1,6
       STRESS(K1)=-STRE(K1)
       STRAN(K1)=-STRA(K1)
       DSTRAN(K1)=-DSTRA(K1)
     630 CONTINUE
C UPDATE STATE VARIABLES
C DO 640 K1=1,6
     STATEV(K1)=EELAS(K1)
STATEV(K1+6)=EPLAS(K1)
STATEV(K1+12)=SALPHA(K1)
STATEV(K1+18)=DELSAL(K1)

CONTINUE
STATEV(25)=EVOID
STATEV(26)=PC
STATEV(27)=ALPHA

C
RETURN
END
VITA

Lei Wei was born on January 22, 1975, in Wuhan, Hubei Province, People’s Republic of China. He received his Bachelor of Science degree in civil engineering (geotechnical engineering) in 1997 from Tongji University, Shanghai, China. In May 2000, he obtained his Master of Science degree in civil engineering (geotechnical engineering) from the same department and the same university. Then he was admitted to Louisiana State University in the United States of America and joined the graduate program in the department of Civil and Environmental Engineering in August 2000, specializing in geotechnical engineering. He is currently a doctoral candidate in the department of Civil and Environmental Engineering at Louisiana State University. He will receive the degree of Doctor of Philosophy in the spring 2004 commencement.