8-1-2018

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Received: 16 October 2017 | Accepted: 12 April 2018

DOI: 10.1111/nrm.12172

WILEY Natural Resource Modeling

An integrated approach to analyzing risk in bioeconomic models

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Abstract

We provide a new integrated approach to analyzing risk in a setting where both exogenous and endogenous risks are present. We explicitly model these risks to understand their implications to the precautionary principle. We find that, among other things, the increase of exogenous risk in the presence of endogenous risk has a different impact on precaution than the increase in exogenous risk in the presence of only exogenous risk. While the latter is known to decrease precaution, the former unambiguously increases the precaution in the presence of stock effect.

Recommendations for Resource Managers

- Two kinds of risk are prevalent. Endogenous risk is created by human action. Exogenous risk is created by nature.
- Endogenous risk can be minimized by taking several actions. If there is only endogenous risk, resource managers should take precaution in extracting the resources.
- Managers should take precaution in extracting resources if there are both endogenous and exogenous risk and if there is stock effect. Without stock effect, results are ambiguous.
- If there is only exogenous risk, managers can do nothing. Therefore, managers should allow for the depletion of resources.

KEYWORDS

exogenous risk, robustness principle, steady state analysis

JEL CLASSIFICATIONS

Q53, Q54, C61

1 | INTRODUCTION

We begin our discussion with a rather obvious example. At any moment, humanity may be fully or partially destroyed by a falling meteor or some other disturbance in the universe. Such risks are in general very small but can be considered constant over time, and are purely exogenous from the perspective of the humanity. Humanity also faces the risk of partial destruction to itself by its own hand, for example, in a way explained by Malthus. In particular, in the absence of substantial technological progress, the rising population and subsequent scarcity of resources may lead to a war or some other form of calamity and hence move us toward our own destruction. Such risks are by definition endogenous from the perspective of the humankind. Almost all bioeconomic systems face both endogenous and exogenous risks. In this paper, we provide a framework to analyze situations in which both such risks are present. We use this framework to analyze a system that faces the risk of catastrophe once its state crosses an uncertain threshold and determine whether the precautionary principle holds in such a setting.

The literature on the impact of uncertain threshold, beyond which catastrophe sets in, is extensive. Clarke and Reed (1994) found that exogenous risk decreases precaution, whereas endogenous, avoidable, risk increases it. In a series of papers, Tsur and Zemel (1998; 2004) noted that the ambiguity regarding precaution was due to the reversibility of the catastrophic effect. By focusing on irreversible catastrophic events (Tsur & Zemel, 2004), they found that an increase in exogenous risk increases the degree of precaution, in particular, if the uncertainty is only about the location of the threshold beyond which irreversibility kicks in. On the other hand, Brozovic and Schlenker (2011) found nonmonotonicity in this setting, noting that an increase in risk may increase or decrease precaution. Polasky, De Zeeuw, and Wagener (2011) integrated all these conditions into one general framework. In their work, the planner faces the risk of a regime shift, leading to either a stock effect (i.e., causing the stock to be reduced to zero) or a system dynamics effect (i.e., causing the system dynamics to shift to less desirable dynamics). If the regime shift causes the stock to collapse, and if the probability of such regime shift is exogenous, it will result in increased exploitation. However, if the regime shift is endogenous and if it involves a change in system dynamics, then it will lead to decreased exploitation. Another important result of the paper is that the endogenous probability of regime shift with a stock effect (i.e., regime shift involves reduction of stock to zero) implies that the result will be ambiguous –the case in which many papers of the past have focused.

All these papers either deal with endogenous risk or exogenous risk, but not both risks at the same time. However, as explained in the opening paragraph, systems generally face both types of risks. The endogenous risk is controllable, whereas exogenous risk is not controllable. This paper explicitly uses the mixture of these two types of risk to examine the precautionary principle.

Suppose the hazard rate of exogenous risk is given by λ_1, a constant, and the hazard rate of endogenous risk is given by $\lambda(s)$, where s is the level of stock. The hazard rate denotes the risk of crossing an unknown threshold level $s^*$when the state variable is s. When state variable s is monotonic, we can induce the distribution for the arrival time of the catastrophe. Specific examples of such induction can be found in literature and is also given below. Suppose τ is the random arrival time of catastrophe, and $\lambda_\tau(s)$ is the hazard rate of such arrival when stock level is s. The probability that catastrophe will occur before the next Δt time period when the stock is s is given by:

The probability that both events will happen in the next Δt time period + the probability that only one of them happen in the next Δt period

$$= (\lambda_1 \Delta t)(\lambda_\tau(s)\Delta t) + (\lambda_1 \Delta t)(\lambda_\tau(s)\Delta t) + (1 - \lambda_1 \Delta t)(\lambda_1(s)\Delta t)$$

$$\simeq (\lambda_1 + (\lambda_\tau(s))\Delta t).$$

Similarly, the probability that no such catastrophe will occur in the next Δt time period is

$$= \text{Probability that none of them will happen in the next period}$$
$$= (1 - \lambda_1 \Delta t)(1 - \lambda_\tau(s)\Delta t)$$
$$\simeq 1 - (\lambda_1 + \lambda_\tau(s)\Delta t).$$

We term this representation of risk, in which both exogenous and endogenous hazard are explicitly specified, composite risk.

2 | MODEL

To better explore this idea, we assume we are looking at forest resources that may face the risk of fire. The exogenous risk of fire requires no explanation, but the nature of endogenous risk probably does. We assume that having more trees increases the forest resources, but also increases the risk of fire. This is a natural assumption in cases when falling leaves provide a natural path for fire to spread. Once a fire occurs in the forest, the growth rate of the forest is impacted, because the carrying capacity of the forest will be reduced. One primary reason for the reduction in carrying capacity is the loss of trees, which will lead to an increase in erosion due to possible floods and other natural causes. This is also a common example given in many papers on the risk of fire, including Reed (1984). Yoder (2004) includes an example of an increasing hazard rate for forest fire as the forest ages.

In this paper, we use the framework employed by Polasky et al. (2011). Say a planner is solving an infinite horizon problem and has to decide the amount of harvest for each period. The parameters of the models are r (representing patience) and p (>0, net price per unit of harvest, considered to be fixed during the planning horizon). The primitive function of the model also includes $\lambda(s)$, a purely endogenous hazard (i.e., the probability of a catastrophe occurring when stock level is $s + \Delta s$, given that it has not occurred yet). The model is represented as:

$$\underset{\{h\}}{\max} \int_0^\infty e^{-rt} ph(t)dt$$

s.t.

$$s(0) = s_0; s(t) \geq 0, h(t) \geq 0 \, \forall \text{t},$$

where $G(s(t)) = (s(t))$ before the catastrophe, and $G(s(t)) = (s(t))$ after the catastrophe. In this paper, catastrophe is defined as the one-time permanent shift in system dynamics, with the growth rate changing from $G_1$to G_2. In the paper by Polasky et al. (2011), $G_i(s) = gs(1 - \frac{s}{K_i})$, $i = 1, 2$, (i.e., the source of the change in system dynamics is the change in carrying capacity).In our example, fire destroys trees, and the lack of trees results in the likelihood of more erosion. Erosion in the forest affects the land area covered by the forest, which implies a reduction in K_i. This, in turn, results in a reduction in the maximum sustainable yield.

There is only one catastrophe that can happen, post-catastrophe, there is no uncertainty in the model. The Hamilton-Jacobi- Bellman equation for the post-catastrophe value function V_2 is given by

$$0 = \max \left\{ ph - rV_2(s) + V_2'(s)(G_2(s) - h) \right\}.$$

Given the linearity of the control function, the solution to this equation is given by the following conditions:

(1) steady state s_2 satisfies $G_2'(s_2) = r$

(2) the value function and its derivative at s_2 are given by[1]

$$V_2' = \begin{cases} \frac{pG_2(s_2)}{r} e^{-\int_s^{s_2} \frac{r}{G_2(\tau)} d\tau} & \text{for } s < s_2 \\ \frac{pG_2(s_2)}{r} & \text{for } s < s_2 \\ e^{-rt_m(s)} \frac{pG_2(s_2)}{r} + \left(1 - e^{-rt_m(s)}\right) \frac{ph_m}{r} & \text{for } s < s_{2'} \end{cases}$$

where $t_m(s)$ is a solution of the function $t_m'(s) = \frac{1}{h_m - G_2(s)}$ and $V_2'(s_2) = p$.

(3) The optimal harvest rate is given by

$$h = \begin{cases} 0 & \text{for } s < s_2 \\ G_2(s_2) & \text{for } s = s_2, \\ h_m & \text{for } s > s_2 \end{cases}$$

where $h_m \to \infty$.[2]

The problem for the planner before the catastrophe is given by

$$\max E\left\{\int_0^{\tau} e^{-rt} ph(t)dt + e^{-r\tau} V_2(s(\tau))\right\}$$

s.t.

$$\dot{s} = G_1(s(t)) - h(t)$$

$$s(0) = s_0$$

$$s(t) \geq 0,\ h(t) \geq 0\, \forall\, t$$

Here τ is the time of arrival of catastrophe and is random by definition. As explained above, the catastrophe can be the result of either endogenous or exogenous risks or both. We first derive the expression for the hazard function using the arrival time of the catastrophe when the risk is purely endogenous, given that $\lambda(s)$ is known.

Let $\Psi(t)$ define the probability that $\tau \leq t$. Similarly, let $F(s)$ and $f(s)$ be the cumulative distribution and probability distribution corresponding to the hazard rate $\lambda(s)$. It implies that 1-$\Psi(t)$ = prob$(\tau \geq t|\tau > 0)$ = prob$(s^* \geq s_t|s^* \rangle s_0) = \frac{1-F(s_t)}{1-F(s_0)}$. This means the probability distribution function $\psi(t)$ is given by taking the derivative of the expression above:

$\psi(t) = \frac{f(s_t)s}{1-F(s_0)} = \frac{f(s_t)}{1-F(s_0)}(G_1(s) - h)$. Hence the hazard rate associated with time is given by $\lambda_\tau(t) = \frac{\psi(t)}{1-\Psi(t)} = \frac{f(s_t)(G_1(s_t)-h)}{1-F(s_t)} = \lambda(s_t)(G_1(s_t) - h)$. We use this expression for the distribution of random variable τ to calculate the solution. Similarly, $\Psi(t) = \frac{F(s_t)-F(s_0)}{1-F(s_0)}$.[3] This expression allows us to write λ_τ using λ(s), which is assumed to be a primitive (given) function of the model.

In the first period, the planner solves the following problem:

$$\max E_\tau \left[\int_0^\tau e^{-rt} ph(t)dt + e^{-r\tau} V_2(s^c) \right]$$

s.t.

$$\dot{s} = G_1(s(t)) - h(t);$$

$$s(0) = s_0, s(t) \geq 0,\ h(t) \geq 0\, \forall t,$$

where s^c is the level of the state variable immediately after the catastrophe happens. Using Bellman's approach, we get the following expression for the value function

$$w_1(s;t) = \max_h$$

$$\left[\int_t^{t+\Delta t} e^{-rt} ph(x)dx + (1 - (\lambda_1 + \lambda_\tau)\Delta t) w_1(s + \Delta s, t + \Delta t) + (\lambda_1 + \lambda_\tau)\Delta t e^{-r(t+\Delta t)} V_2(s^c) \right]$$

Using a standard dynamic theoretic approach, we now expand $w_1(s + \Delta s;\ \ t + \Delta t)$ around $(s;t)$; set $\lambda_\tau = \lambda$ (s)(G_1(s)-h); and use the expression that $\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_t^{t+\Delta t} e^{-rx} ph(x)dx = e^{-rt} ph(t)$. This gives

$$0 = \max_h e^{-rt} ph(t) - (\lambda_1 + \lambda(G_1 - h))w_1 + w_{1t} + [\lambda_1 + \lambda(G_1 - h)]e^{-rt} V_2 + W_{1s}(G_1 - h)$$

Here, we have suppressed the arguments of w_1, λ, G_1, w_{1s}, w_{1t}.

Using a familiar method of transformation, we convert this into a time autonomous problem. Let $e^{rt} w_1(s;t) = V_1(s)$. Using this format, we replace w_1 (by $e^{-rt}V_1$); w_{1t} (by $-re^{-rt}V_1$), w_{1s} (by $e^{-rt}V_{1s}$) and we get

$$0 = \max_h \left\{ \left[p - V_1' + \lambda V_1 - \lambda V_2 \right] h - \lambda_1 V_1 - \lambda G_1 V_1 - rV_1 + \lambda_1 V_2 + \lambda G_1 V_2 + V_1' G_1 \right\}.$$

We now make a convenient statement:

Lemma 1. *Suppose* the following assumptions are made*:*

(a) Let Ω(s) = V_1(s) − V_2(s). Then $\Omega'(s) > 0$, $\lim_{s\to\infty} \Omega(s) \to 0$. $(b) \lim_{s\to\infty} V_1' > p$ $(c) \lim_{s\to\infty} \lambda(s) \to 0$. $(d) \lim_{s\to\infty} V_1' = 0$. Then there exists a unique root of $f(s) = p - V_1' + \lambda V_1 + \lambda V_2$

Proof. Note that $\lim_{s\to 0} f(s) > 0$. Since $\lim_{s\to\infty} \lambda(s) \to +\infty$, using (d), it is clear that $\lim_{s\to\infty} f(s) > 0$. Continuity of *f*(*s*) then guarantees a root. Furthermore, since $f'(s) = -V_1'' + \lambda'(s)(V_1 - V_2) + \lambda(s)[V_1'(s) - V_2'(s)] > 0$, this function cannot have more than one real root.

The linear nature of our problem implies that the optimal *h* will be given by

$$h = \begin{cases} 0 & if \quad p - V_1' + \lambda V_1 - \lambda V_2 < 0 \\ h^m & if \quad p - V_1' + \lambda V_1 - \lambda V_2 > 0 \\ h^s & if \quad p - V_1' + \lambda V_1 - \lambda V_2 = 0\,. \end{cases}$$

Lemma 1 enables us to assume there exists a state s_1, such that when $s < s_1$ the policy is $h = 0$, and when $s > s_1$, $h = h^m$, where h^m indicates the maximum allowable harvest rate. This assumption implies that $p - V_1'(s) + \lambda(s)V_1(s) - \lambda(s)V_2(s^c) < (>)0$ when $s < (>)s_1$. The policy is a singular when $s = s_1$.

Hence the value function, $V_1(s)$, is given by

$$0 = \lambda_1\left(V_2 - V_1\right) + \lambda\left[G_1V_2 - G_1V_1\right] + V_1'G_1 - rV_1 \quad \text{for } s < s_1 \text{ and} \tag{1}$$

$$0 = \left[p - V_1' + \lambda V_1 - \lambda V_2\right]h^m + \lambda_1(V_2 - V_1) + \lambda(\mathrm{G_1V_2} - \mathrm{G_1V_1}) + V_1'G_1 - rV_1 \quad \text{for } s > s_1 \tag{2}$$

Assuming that the value function is continuously differentiable, we can take

(1) and (2) as two independent equations satisfying both (1) and (2) at $s = s_1$, and solve for $V_1(s_1)$ and $V_1'(s_1)$. The solutions are

$$V_1(s_1) = \frac{\lambda_1V_2 - \lambda G_1V_2 + pG_1 - V_2G_1}{r + \lambda_1} \quad \text{for } s > s_1 \quad \text{and} \tag{3}$$

$$V_1'(s_1) = \frac{(r + \lambda_1 + \lambda G_1)V_1 - (\lambda_1 + \lambda G_1)V_2}{G_1} \tag{4}$$

If we were to know s_1, then (3) and (4) would give an expression for the value function at $s = s_1$. Moreover, differentiating (1) and (2) with respect to s, we get,

When $s < s_1$,

$$\begin{aligned} G_1(s)V_1''(s) = rV_1'(s) - V_1'G_1' - \lambda[G_1V_2' + G_1'V_2 - G_1V_1' - G_1'V_1 - \lambda'[G_1V_2 - G_1V_1] \\ - \lambda_1\left(V_2' - V_1'\right) \end{aligned} \tag{5}$$

When $s > s_1$,

$$\begin{aligned} [G_1(s) - h^m]V_1''(s) = -\left[\lambda'V_1 + \lambda V_1' - \lambda'V_2 - \lambda V_2'\right]h^m + rV_1'(s) - V_1'G_1' \\ - \lambda\left[G_1V_2' + G_1'V_2 - G_1V_1' - G_1'V_1\right] - \lambda'\left[G_1V_2 - G_1V_1\right] - \lambda_1\left[V_2' - V_1'\right] \end{aligned} \tag{6}$$

$$\text{Let } \mathcal{L}_1(s) = \left(\lambda V_1' + \lambda'V_1 - \lambda V_2' - \lambda'V_2\right)$$

and $\mathcal{L}_2(s) = rV_1'(s) - V_1'G_1' - \lambda\left[G_1V_2' + G_1'V_2 - G_1V_1' + G_1'V_1\right] - \lambda'\left[G_1V_2 - G_1V_1\right] - \lambda_1\left(V_2' - V_1'\right)$.

Lemma 2. *If* $\lambda'(s) > 0$, $\mathcal{L}_1(s) > 0 \forall s$, *then the necessary condition for* $\mathcal{L}_1(s) \le 0$ is $\lambda'(s) < 0$.

Proof. It should be obvious, noting that $\mathcal{L}_1(s) = \frac{d}{ds}[\lambda(V_1 - V_2)] = \frac{d}{ds}[\lambda\Omega] = \lambda\Omega' + \lambda'\Omega$. We know $\lambda, \Omega,$ and Ω' are strictly positive. Hence unless, $\lambda' < 0$, $\mathcal{L}_1(s) > 0$. Note, however, that $\lambda' < 0$ is necessary but not sufficient condition for $\mathcal{L}_1(s) \le 0$. By observing (5) and (6), it is clear that the concavity of V_1 would imply (i) The right hand side of (5) is negative at $s \le s_1$, and (ii) the right hand side of (6) is positive at $s \ge s_1$.[4] Furthermore, notice that from Lemma 2,

$\mathcal{L}_1(s) > 0$; because $\lambda'(s) > 0$ [by assumption], which trivially implies that $\lambda'(s) > -\lambda\frac{V_1'-V_2'}{V_1-V_2}$. Taking the limit from the left, we get $\lim_{s\to s_1^-} V_1'' = \lim_{s\to s_1^-} \frac{\mathcal{L}_2(s)}{G_1(s)}$, and taking limit from the right, we get $\lim_{s\to s_1^+} V_1'' = \lim_{s\to s_1^+} \frac{\mathcal{L}_2(s)-\mathcal{L}_1(s)h^m}{G_1 h^m}$. Assuming that the limit exists at $s = s_1$, and allowing for $h^m \to \infty$, we get

$$G_1(s_1) = \frac{\mathcal{L}_2(s_1)}{\mathcal{L}_1(s_1)}. \tag{7}$$

This expression is valid, since by assumption $\mathcal{L}_1(s_1) > 0$. Now replacing $\mathcal{L}_2$ and $\mathcal{L}_1$, by their definition, we get

$$G_1 = \frac{rV_1'(s) - V_1'G_1' - \lambda\left[G_1V_2' + G_1'V_2 - G_1V_1' + G_1'V_1\right] - \lambda'\left[G_1V_2 - G_1V_1\right] - \lambda_1\left(V_2' - V_1'\right)}{\lambda V_1' + \lambda' V_1 - \lambda V_2' - \lambda' V_2}. \tag{8}$$

Upon further simplification, we get

$$[G_1' - r - \lambda_1]V_1' = \lambda G_1'V_1 - \lambda G_1'V_2 - \lambda_1 V_2'. \tag{9}$$

From (8), we know that in the limiting case when $\lambda(s)$ and λ_1 are identically zero, $G_1'(s_1) = r$, which is the standard solution in the absence of risk.

We also note that all functions in (9) are known as the values of V_1, V_2, V_1', and V_2' have all been calculated previously.

Equation (9) provides a framework to compare the steady state solutions under endogenous risk. First, notice that (9) can be written as

$$G_1' = \frac{rV_1' + \lambda_1\left(V_1' - V_2'\right)}{V_1' - \lambda\left(V_1 - V_2\right)}. \tag{10}$$

Our discussion of the precautionary principle for endogenous risk rests crucially on (10).

(i) First, it is clear that in the absence of exogenous risk, the impact of endogenous risk is to increase the value of G_1', and hence decrease the level of steady state that the planner would be in if there were no such risk. Since $V_1 - V_2 > 0$, (since the dynamics is assumed to be changing for worse, by definition V_2 should be less than V_1), it is clear that the impact of endogenous risk (or its increase) is unambiguously in favor of precaution. (To avoid the negative value of the steady state, we assume the denominator is not negative). In the forest management problem, it is implied that the risk of catastrophic wildfire causes decision makers to maintain a less steady state stock of forest, if they believe more trees will increase the hazard of wildfire. Precaution (or the propensity to stay farther away from the unknown threshold point triggering catastrophe) is an unambiguous result.

(ii) When the system faces the risk of extinction, i.e., $V_2 = V_2' = 0$, a situation much studied in the literature, mainly under the topic of irreversible catastrophe and also called *stock effect,* (10) becomes

$G_1' = \frac{rV_1' + \lambda_1 V_1'}{V_1' - \lambda V_1}$. By dividing the numerator and denominator of the right hand side by V_1', this can also be written as

$$G_1' = \frac{r + \lambda_1}{1 - \lambda\left(\frac{V_1}{V_1'}\right)}. \tag{11}$$

Since this is the condition that needs to be satisfied in the steady state, intuitively, it is easy to see that the role of exogenous risk, *in the presence of endogenous risk*, when there is stock effect, is to increase precaution, as it increases the numerator. To see this, we note that the relationship between V_1 and V_1' at $s = s_1$, when $V_2 = 0$ is given by $\frac{V_1}{V_1'} = \frac{1}{\lambda}(1 + \frac{p}{V_1'})$, since at singular solution $p - V_1' + \lambda V_1 = 0$ when $V_2 = 0, V_1' > p > 0$. Hence $\frac{V_1}{V_1'} > 0$. In that case, (11) can be rewritten as $G_1' = \frac{r+\lambda_1}{(p/m_1(s_1))}$, where $m_1(s_1)$ is the shadow price evaluated at steady state. If the market price reflects the shadow price, i.e., in case of no market failure, we get $G_1' = r + \lambda_1$, the result often found in the literature. This is an unambiguous result.

This is an important distinction from (i), and from previous studies in the literature in general. This implies that when there are both endogenous and exogenous risks, the risk of catastrophe won't lead people to act recklessly, and people actually react by increasing their precaution, especially if the market is pricing the resources precisely. Ambiguity is possible if resources are not priced accurately. The result in (i) was unconditional, the result here is not.

(iii) When there is no stock effect, the impact of exogenous risk, *in the presence of endogenous risk*, is ambiguous if the sign of $V_1' - V_2'$ is not constant. It crucially depends on which state the system will be in after the change in

the dynamic system. Suppose that post-catastrophe, the system moves to the state $s < s_2$. If after the catastrophe, the value function is believed to grow fast, whereas at the current state, the growth is comparatively stagnant, a not implausible scenario, the planner may not exercise precaution, especially since (a) she believes that the risk is exogenous and (b) though the transition is to a less desirable state, the growth rate in the less desirable state is higher than the growth rate she is in. If the planner believes that the risk is purely exogenous and that she could possibly do nothing to mitigate it, she is likely to be unworried, but if she knows further that the catastrophe will lead to a growth friendly state, she will be more upbeat and throw the caution away. However, for that to happen, the post-transformation marginal value function must be higher than pre catastrophe marginal value function.

(iv) The impact of exogenous risk *in the absence of endogenous risk* can't be analyzed by using (10), since derivation of (10) relies on (7). In the absence of endogenous risk, $\mathcal{L}_1(s) = \lambda V_1' + \lambda' V_1 - \lambda V_2' + \lambda' V_2 = 0$, and this implies that (7) is not a valid expression. However, the concavity of value function implies that the right hand side of equation (5), namely $\mathcal{L}_2(s)$, must be negative when $s < s_1$, and positive when $s > s_1$. Therefore at the steady state, $s = s_1$, $\mathcal{L}_1(s) = 0$, which, using $\lambda = \lambda' = 0$, implies that, at $G_1' = \frac{rV_1'(s) + \lambda_1(V_1' - V_2')}{V_1} = r + \lambda_1(1 - \frac{V_2'}{V_1'})$

This gives the *increase in exogenous risk implies an increase in discount rate* type result, found in Reed (1984), Yaari (1965) and many other papers in the literature. However, the expression above implies that the results of Yaari (1965) are special case of our model, and is valid when $V_2' = 0$ (i.e., when the catastrophe implies $V_2 = 0$ in all states). This is the opposite of the precautionary principle, but is often justified by claiming that people respond to exogenous risk by overexploiting resources since there is nothing they can do to prevent the catastrophe. It is also clear that in general, the value inside the parenthesis depends on the specification of both V_1 and V_2.

3 | CONCLUSIONS

Our results show the importance of correctly framing the risk analysis model. Our formulation also enables us to analyze the subtleties of risk. Notice that in the absence of analysis in such detail it is possible to arrive at an ambiguous result, without understanding the source of the ambiguity.

Our results also extend and clarify previous results. Polasky et al. (2011), whose framework we have used in the analysis, find support for precaution only in the case of the system dynamics effect for endogenous probability in regime shift change. In our setting, the support for precaution also extends to the case of the stock effect when the market is correctly pricing the resources.

To summarize, our work provides a framework for analyzing the optimal decision making of a planner facing different sources of threats to the system. We also provide a new, composite formulation for such risk. The consequences of such formulations are important. First, we clarified the conditions under which some of the results found in the literature are valid, thus providing a general setting for the analysis of risk. Second, our formulation provides a clear portrayal of aggregate risk and allows analysts to focus on different (endogenous and exogenous) parts of such risks. Third, our formulation also allows different uncertainty assumptions about the hazard rate. Fourth, our work provides clear evidence that the response to exogenous risk crucially depends on the presence or absence of endogenous risk in the system.

In terms of application, we contribute to the age-old problem of how an increase in exogenous (as well as endogenous) risk affects the behavior of individuals. Increase in exogenous risk in the absence of endogenous risk is well understood in literature as Yaari (1965) showed long ago, and as has been agreed upon by economists since then. However, we also provide the characterization for the situation in which exogenous risk increases in the presence of endogenous risk. In such a situation, if the risk of the stock collapse increases, people respond by increasing precaution. Suppose, an Armageddon is about to hit the earth. This is purely exogenous risk, and people respond to it by overconsuming and merry making. On the other hand, suppose people recognize that there are two different ways in which atmosphere may collapse: one is by increased solar ultraviolet rays and another is by increased consumption that leads to greenhouse gas release. If solar ultraviolet rays increase, people respond by decreasing consumption so as to reduce greenhouse gases, even though the only risk that increased was exogenous one. This is one of the main insights from this paper.

ACKNOWLEDGMENT

The authors thank an anonymous reviewer for constructive comments. Paudel's time in this paper is supported by USDA Hatch project #94358 “Management and Policy Challenges in a Water-Scarce World” and LWRRI grant titled “Economic Impacts of Groundwater Salinity in Louisiana Agriculture.”

ENDNOTES

[1] Note that when $s < s_2$; $V_2'(s)G_2(s) - rV_2(s) = 0$, which is an ordinary differential equation for $V_2(s)$. Dividing both sides by $G_2(s)$ and multiplying by $e^{-\int_0^s \frac{r}{G_2(x)}dx}$, we get $e^{-\int_0^s \frac{r}{G_2(x)}dx} V_2'(s) - e^{-\int_0^s \frac{r}{G_2(x)}dx} \frac{r}{G_2(s)} V_2(s) = 0$. Since the left hand

side is $\frac{d}{ds}[V_2(s)\, e^{-\int_0^s \frac{r}{G_2(x)} dx}]$, taking integration of both sides from 0 to s, and noting that $V_2 \ = \ 0$; and $V_2(s_2) =$, which is the net present value of payoff pG_2S_2 for perpetuity, we get the solution.

[2] This is a standard result in the literature. One way to see this is as follows. Throughout this analysis, we assume the Benveniste-Scheinkman condition (Benveniste & Scheinkman, 1979) for the envelope theorem holds. Note that for $s < s_2$; $0 = -rV_2 + V_2'(s)G_2(s)$, and taking derivative with respect to s, we get, $V_2''(s) = (r - G_2'(s))\frac{rV_2(s)}{(G_2(s))^2}$. The concavity of value function implies $r - G_2'(s) \leq 0$. Repeating same for s > we get $r - G_2'(s) \geq 0$, since $\frac{rV_2(s) - ph_m}{(G_2(s) - h_m)^2}$ is negative for $h_m \to \infty$. The continuity of $V_2''(s)$ then implies that $= G_2'(s_2)$.

[3] We haven't proven the fact that the state variable s_t would be monotonic over time. The proof is straightforward and relies on the arguments similar to those made in papers such as in Long (1978), Tsur and Zemel (1994), and Spraker and Biles (1996). Our calculation here follows a method similar to Tsur and Zemel (1994).

[4] We assume V_1 is concave here.

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How to cite this article: Poudel BN, Paudel KP. An integrated approach to analyzing risk in bioeconomic models. *Natural Resource Modeling*. 2018;31:e12172. https://doi.org/10.1111/nrm.12172