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A Hop-by-Hop Relay Selection Strategy in Multi-Hop Cognitive Relay Networks

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\section*{ABSTRACT}
In this paper, a hop-by-hop relay selection strategy for multi-hop underlay cognitive relay networks (CRNs) is proposed. In each stage, relays that successfully decode the message from previous hop form a decoding set. Taking both maximum transmit power and maximum interference constraints into consideration, the relay in the decoding set which has the largest number of channels with an acceptable signal-to-noise ratio (SNR) level to the relays in the next stage is selected for retransmission. Therefore, relay selection in each stage only relies on channel state information (CSI) of the channels in that stage and does not require the CSI of any other stage. We analyze the performance of the proposed strategy in terms of end-to-end outage probability and throughput, and show that the results match those obtained from simulation closely. Moreover, we derive the asymptotic end-to-end outage probability of the proposed strategy when there is no upper bound on transmitters’ power. We compare this strategy to other hop-by-hop strategies that have appeared recently in the literature and show that this strategy has the best performance in terms of outage probability and throughput. Finally it is shown that the outage probability and throughput of the proposed strategy are very close to that of exhaustive strategy which provides a lower bound for outage probability and an upper bound for throughput of all path selection strategies.

\section*{INDEX TERMS}
Cognitive radio, decode-and-forward, multi-hop relay networks, outage probability, relay selection.

\section*{I. INTRODUCTION}
Cognitive radio networks are expected to mitigate the problem of spectrum overcrowding by allowing secondary (unlicensed) users to dynamically access a frequency band as long as they do not cause harmful interference to the primary (licensed) users [1]–[4]. In the underlay paradigm of cognitive radio networks, secondary users can share the spectrum with the primary users as long as the interference they cause to the primary users remains below a specified threshold [5]–[7]. This constraint results in limited transmit power in the cognitive networks, thereby reducing the coverage area [8], [9]. To resolve this issue, multi-hop underlay cognitive relay networks (CRNs) have been proposed to extend the coverage area and to allow the transmitter’s messages to reach a distant destination using relay nodes. CRNs are being considered for a number of wireless network scenarios including multi-hop underlay device-to-device (D2D) communication in cellular networks, multi-hop underlay wireless sensor networks, IoT systems, vehicle tracking and roadside facilities [10]–[12].

\section*{A. RELATED WORK}
A key requirement in multi-hop networks is an efficient strategy to select the relays which comprise the path between the source and destination. Relay selection in multi-hop networks has been the subject of several studies. In [13], an exhaustive relay selection strategy is proposed where a central controller is required to collect the channel state information (CSI) of all the links in the network, and the path which has the highest signal-to-noise ratio (SNR) bottleneck is selected for end-to-end transmission. Several hop-by-hop relay selection strategies have been investigated, in [13]–[18], which have much lower complexity compared to the exhaustive strategy. In all these works, a single relay is selected in each hop for relaying. Performance of path selection in
multi-hop parallel relay networks is studied in [19], [20] and the performance of multi-user multi-hop decode-and-forward (DF) relay networks with decentralized relay selection is investigated in [21].

To extend the coverage area of secondary networks, multi-hop underlay CRNs have been investigated. In [22], the outage probability of multi-hop underlay CRNs is derived. The outage probability, bit-error-rate (BER), symbol error rate and ergodic capacity of multi-hop underlay CRNs with multiple primary receivers in independent Nakagami-m fading channels are derived in [23]. In [24], the outage probability of multi-hop underlay CRNs under multiple primary users’ interference is studied, in which both non-identical fading parameters as well as signal-to-interference-plus-noise ratio statistics are considered. In [25], closed-form and asymptotic expressions for the outage probability of multi-hop underlay CRNs over Nakagami-m fading channels in the presence of multiple primary transmitters and receivers are derived. The exact outage probability and BER, and approximate expressions for ergodic capacity of multi-hop underlay DF CRNs in non-identical Rayleigh fading channels are derived in [26]. In [27], performance of multi-hop underlay CRNs with imperfect CSI of interference channels is analyzed. Performance of multi-hop underlay CRNs for Nakagami-m fading channels with additive white generalised Gaussian noise is investigated in [28]. In [29], performance of multi-hop underlay CRNs over cascaded Rayleigh fading channels with imperfect CSI is analyzed, and a secondary user selection scheme is proposed.

Recently, relay selection strategies in multi-hop underlay CRNs are investigated. In [30], performance of multi-hop underlay CRNs using max-link-selection strategy is analyzed. Arbitrary relay (AR) and best-last arbitrary rest (BLAR) strategies in multi-hop underlay CRNs are investigated in [31], which shows that BLAR strategy has the same outage performance as max-link-selection strategy in [30], while requiring fewer number of channel estimates. In [32], two strategies named highest transmit power relay selection (HTPRS) and improved HTPRS (IHTPRS) are proposed for multi-hop underlay CRNs. In HTPRS, the relay in each hop which has the highest instantaneous transmit power is selected for retransmission. In IHTPRS, relay selection procedure is similar to HTPRS except that in the last relay cluster, the relay with the highest SNR to destination is selected.

**B. CONTRIBUTIONS OF THIS PAPER**

Inspired by the research on multi-hop underlay CRNs in recent years, a hop-by-hop relay selection strategy for multi-hop underlay CRNs is proposed in this paper. We refer to our proposed strategy as MaxDS-CRN, since it selects the relay which maximizes the size of the decoding set in the following cluster. The main contributions of the paper are listed in the following.

- We propose a hop-by-hop relay selection strategy for multi-hop underlay CRNs. In this strategy, relay selection in every hop is only based on the CSI of the channels in the following hop (and the channels to primary user receiver, PU-Rx). In other words the CSI of the other hops in the relay network is not required.
- The exact end-to-end outage probability and throughput are derived subject to two power constraints: 1) maximum transmit power of the secondary nodes and 2) maximum interference power at PU-Rx. Moreover, the asymptotic outage probability when there is no upper bound on the transmit powers of the relays and the source is derived.
- MaxDS-CRN is compared to other recently proposed relay selection strategies and show that MaxDS-CRN outperforms the others in terms of outage probability and throughput.
- Numerical results are presented which show that the performance of MaxDS-CRN in terms of outage probability and throughput is very close to that of exhaustive strategy, which provides a lower bound for outage probability and an upper bound for throughput of all relay selection strategies for multi-hop underlay CRNs.

The rest of this paper is organized as follows. In Section II, the system model is presented. The proposed relay selection strategy is described in Section III. In Section IV, the end-to-end outage probability of the proposed strategy is derived and the asymptotic outage probability is also evaluated. End-to-end throughput of the system is given in Section V. Numerical results are presented in Section VI and conclusions are drawn in Section VII.

**II. SYSTEM MODEL**

As shown in Fig. 1, we consider a multi-hop underlay CRN with the source SS, the destination SD, and \( M - 1 \) relay clusters \( \{RC_m, m = 1, \ldots, M - 1\} \) between SS and SD. Each relay cluster \( RC_m \) includes \( L_m \) single-antenna half-duplex relay nodes. The \( r \)-th relay in \( RC_m \) is denoted by \( R_i^{(m)} \). Message transmission from SS to SD is implemented indirectly with the help of the \( M - 1 \) relay clusters. Therefore, there are a total of \( M \) hops from SS to SD, and it takes \( M \) orthogonal time slots for end-to-end transmission.
One PU-Rx is also in the vicinity of the cooperative relay system and may experience interference from SS and/or secondary relays. This system model is similar to those in [30]–[32]. As in [23], [26], [30]–[32], we assume that interference from the primary user transmitter, PU-Tx, to secondary network is negligible, since PU-Tx is far from the secondary network. We define our notation in the following.

**Notations:** Let \( h_{i,j}^{(m)} \) denote the instantaneous CSI from SS to relay \( R_i^{(1)} \) in the first cluster. Similarly, \( h_{i,j}^{(m)} \) denotes the instantaneous CSI from \( R_i^{(m-1)} \) to \( R_i^{(m)} \). Moreover, \( h_{i,P}^{(m)} \) and \( h_{i,P}^{(m-1)} \) denote the CSI to PU-Rx from SS and \( R_i^{(m-1)} \), respectively. Finally, the CSI from \( R_i^{(M-1)} \) to SD is denoted as \( h_{i,D}^{(M)} \). Let \( \mathcal{CN}(\mu, \sigma^2) \) denote the circularly symmetric complex Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \). We assume that \( h_{i,P}^{(m)} \sim \mathcal{CN}(0, \lambda_{i,P}^{(m)}) \), where \( \lambda_{i,P}^{(m)} \) is exponentially distributed with mean \( \lambda_{i,P}^{(m)} \). We assume that the channels from SS to all the relays in \( \mathcal{R}_C \) in independent and identically distributed (i.i.d.), i.e., \( \lambda_{S_i}^{(1)} = \lambda_{S_i}^{(1)} \). Similarly, we assume \( \lambda_{S_i}^{(1)} = \lambda_{S_i}^{(1)} \). The noise random variable at all relays are assumed to be i.i.d. with distribution \( \sim \mathcal{CN}(0, \nu_0) \).

Transmissions of secondary network are allowed as long as the resulting interference at PU-Rx remains below a given threshold level. Let \( I_p \) denote the maximum interference power that PU-Rx can tolerate. Then \( P_S^{(0)} \), the transmit power at SS, is limited by \( P_s^{(0)} \leq I_p \). Similarly, \( P_i^{(m)} \), the transmit power at \( R_i^{(m)} \), is limited by \( P_i^{(m)} \leq I_p \). Furthermore, we assume that the maximum transmit power of each node is \( P_m \). Therefore, the transmit power at SS is given as

\[
P_S^{(0)} = \min \left\{ \frac{I_p}{g_{S,P}^{(1)}}, P_m \right\}
\]

and the transmit power at \( R_i^{(m)} \) is given as

\[
P_i^{(m)} = \min \left\{ \frac{I_p}{g_{i,P}^{(m+1)}}, P_m \right\}
\]

Finally the instantaneous SNR of link \( A \rightarrow B \) in hop \( m + 1 \), \( \gamma_{i,A}^{(m+1)} \), is given by

\[
\gamma_{i,A}^{(m+1)} = \frac{P_{i}^{(m)} h_{i,A}^{(m+1)}}{N_0} = \frac{P_{i}^{(m)} h_{i,A}^{(m+1)}}{N_0}
\]

**III. MaxDS-CRN RELAY SELECTION STRATEGY**

In this section, we introduce our proposed relay selection strategy referred to as MaxDS-CRN. We consider reactive DF relaying where in each relay cluster a single relay is selected for retransmission. It is assumed that the source has accurate estimate of the CSI to PU-Rx, and all the relays have accurate estimates of the CSI to all the nodes in the next hop as well as the CSI to PU-Rx.\(^1\) The proposed path selection strategy is as follows. In the first hop, SS determines its transmit power \( P_S^{(0)} \) according to (1) and broadcasts its signal to relays in \( \mathcal{R}_C \). In subsequent hops, the relays in \( \mathcal{R}_C \) which in \( m = 1, 2, \cdots, M - 2 \) which are able to correctly decode the received signal from the previous stage form a decoding set denoted by \( D_{(m)} \). The decoding set, defined formally later in (5), consists of all those relays whose SNR exceeds a predefined threshold \( T \), which is the minimum required SNR for successful decoding of the message. Each relay in \( D_{(m)} \) determines its transmit power \( P_i^{(m)} \) according to (2), and the corresponding instantaneous SNR of the link to each relay in \( \mathcal{R}_C \) is calculated from (3). For a relay \( R_i^{(m)} \) in \( D_{(m)} \), let \( N_i^{(m)} \) denote the number of links to relays in \( \mathcal{R}_C \) for which the instantaneous SNR exceeds \( T \). \( R_i^{(m)} \) now starts a timer inversely proportional to \( N_i^{(m)} \). The relay whose timer expires first, denoted by \( R_i^{(m)} \in D_{(m)} \), will retransmit. This relay has the largest number of “good” channels, i.e., \( r^* = \arg \max N_i^{(m)} \). All the other relays in \( D_{(m)} \) hear this transmission and remain silent.

We define

\[
N_{(m)} = \max \left\{ N_i^{(m)}, R_i^{(m)} \in D_{(m)} \right\}
\]

We should point out that if \( N_{(m)} = 0 \), then outage is declared. Finally in the last hop, the relay in \( D_{(M-1)} \) which has the highest instantaneous SNR to SD is selected for transmission. Denote by \( R_t \) the required per-hop-rate in bps/Hz. Then for \( m = 1, 2, \cdots, M - 1 \), \( D_{(m)} \) consists of those relays whose link capacity from the previous stage exceeds \( R_t \), i.e.,

\[
D_{(m)} = \left\{ R_j^{(m)} : \log_2 \left( 1 + \gamma_{i,j}^{(m)} \right) \geq R_t \right\}
\]

where \( \gamma_{i,j}^{(1)} \) is the SNR from SS to \( R_j^{(1)} \), and for \( m = 2, 3, \cdots, M - 1 \), \( \gamma_{i,j}^{(m)} \) is the SNR from the selected relay \( R_i^{(m-1)} \) to \( R_j^{(m)} \). The SNR threshold \( T \) for successful decoding of the message is defined as \( T \triangleq 2R_t - 1 \). In the following section, we derive the outage probability of MaxDS-CRN strategy.

**Remark:** In the proposed strategy, at each relaying stage the CSI to all the nodes in the next hop are required for relay selection. Assuming there are \( l_m \) relays in \( D_{(m)} \), in the secondary network, the number of CSI required in the \((m + 1)\)-th hop is \( l_m + 1 \), and the number of CSI required in the last hop is \( l_{M-1} + 1 \). In contrast, in BLAR and IHT-PRS strategies, only the CSI of the last hop is required for relay selection. Therefore, compared to BLAR and IHTPRS, MaxDS-CRN requires more CSIs for relay selection. However, as discussed in Section VI, MaxDS-CRN significantly outperforms BLAR and IHTPRS strategies in both outage probability and throughput.

\(^1\)Similar to [23], [26], [30], [31], we assume that secondary nodes are able to obtain their channel coefficients to PU-Rx.

\(^2\)A small randomization can be introduced into the timer to avoid collisions in the case of ties.
IV. OUTAGE PROBABILITY

For \( m = 1, 2, \ldots, M - 1 \), let \( D_{l_m}^{(m)} \) denote the event that the decoding set \( D^{(m)} \) has \( l_m \) relays, and let \( O_m = D_0^{(m)} \) i.e., \( O_m \) is the event that no relay in the \( m \)-th cluster can decode the message, and \( O_M \) denotes the event that SD cannot decode the message. Let \( P_{\text{out}}^{(m)} \) denote the probability that outage occurs in the \( m \)-th hop. Then we can write the end-to-end outage probability as

\[
P_{\text{out}} = \sum_{m=1}^{M} P_{\text{out}}^{(m)}
\]

According to (1), the transmit power at SS is determined by the channel condition between SS and PU-Rx. When \( g_{S,P}^{(1)} \leq \frac{I_p}{P_m} \), \( P_S^{(0)} = P_m \). When \( g_{S,P}^{(1)} > \frac{I_p}{P_m} \), \( P_S^{(0)} = \frac{I_p}{g_{S,P}^{(1)}} \).

Therefore, the probability that there are \( l_1 \) relays in \( D^{(1)} \) can be expressed as

\[
\Pr[D^{(1)}_{l_1}] = \int_{x=0}^{\infty} \Pr[D^{(1)}_{l_1} | g_{S,P}^{(1)} = x] f_{g_{S,P}^{(1)}}(x) dx \\
= \int_{x=0}^{\infty} \Pr[D_{l_1}^{(1)} | P_{S}^{(0)} = P_m] f_{g_{S,P}^{(1)}}(x) dx \\
+ \int_{x=0}^{\infty} \Pr[D_{l_1}^{(1)} | P_{S}^{(0)} = \frac{I_p}{g_{S,P}^{(1)}}] f_{g_{S,P}^{(1)}}(x) dx \\
= \mathcal{I}_1 + \mathcal{I}_2
\]

in which

\[
\mathcal{I}_1 = \Pr[D^{(1)}_{l_1} | P_{S}^{(0)} = P_m] \frac{P_m}{f_{g_{S,P}^{(1)}}(x)} \\
= \frac{(L_1)}{(l_1)} \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \right) L_1 - l_1 \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \right)
\]

By using the Binomial theorem, we can get

\[
\mathcal{I}_2 = \int_{x=0}^{\infty} \left(\frac{(L_1)}{(l_1)} \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \right) L_1 - l_1 \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \right) \frac{1}{g_{S,P}^{(1)}} \right) \left(\frac{1}{g_{S,P}^{(1)}} \right) dx \\
= \frac{(L_1)}{(l_1)} \sum_{k=0}^{L_1 - l_1} \binom{L_1 - l_1}{k} (-1)^k e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \frac{1}{g_{S,P}^{(1)}}
\]

Now putting (8), (9) into (7), we get

\[
\Pr[D^{(1)}_{l_1}] \\
= \left(\frac{(L_1)}{(l_1)} \right) \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \right) L_1 - l_1 \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} S_{R}}} \right) \frac{1}{g_{S,P}^{(1)}} + 1
\]

The probability that outage occurs in the first hop is the probability that no relays in RC1 can successfully decode the message transmitted from SS, and can be calculated by putting \( l_1 = 0 \) into (10) as

\[
P_{\text{out}}^{(1)} = \Pr[O_1] = \Pr[D_0^{(1)}]
\]

For \( m = 2, \ldots, M \), the probability that outage occurs in the \( m \)-th hop can be expressed as

\[
P_{\text{out}}^{(m)} = \sum_{l_m=1}^{L_m} \sum_{l_{m-1}=1}^{L_{m-1}} \Pr[O_m \cap \{\gamma_{m-1}^{(m-1)} \leq T\}] \\
= \sum_{l_m=1}^{L_m} \sum_{l_{m-1}=1}^{L_{m-1}} \Pr[O_m | D_{l_{m-1}}^{(m-1)}] \Pr[D_{l_{m-1}}^{(m-1)}]
\]

In the following we evaluate the two terms in (12).

A. CALCULATION OF \( \Pr[O_m | D_{l_{m-1}}^{(m-1)}] \)

When \( 2 \leq m \leq M - 1 \), \( \Pr[O_m | D_{l_{m-1}}^{(m-1)}] \) is the probability that from any of the \( l_{m-1} \) relays in \( D^{(m-1)} \), the SNRs of all \( L_m \) links to the relays in RCm are below the threshold \( T \). Similar to (11), for any \( l_1^{(m-1)} \in D^{(m-1)} \), the probability that the SNRs of all the \( L_m \) links are below the threshold \( T \) is given by

\[
\Pr \left[ \max_{R_{i}^{(m-1)} \in \text{RC}_{m}} \{\gamma_{i}^{(m)}\} < T \right] = \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} R_{i}^{(m-1)}}} \right) \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} R_{i}^{(m-1)}}} \right) \\
+ \sum_{k=0}^{L_m} \binom{L_m}{k} (-1)^k e^{-\frac{N_0 T}{P_m^{(1)} R_{i}^{(m-1)}}} \frac{1}{g_{S,P}^{(1)}} \frac{1}{g_{S,P}^{(1)}} + 1
\]

Therefore, given \( D^{(m-1)}_{l_{m-1}} \), we have

\[
\Pr[O_m | D_{l_{m-1}}^{(m-1)}] \\
= \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} R_{i}^{(m-1)}}} \right) \left(1 - e^{-\frac{N_0 T}{P_m^{(1)} R_{i}^{(m-1)}}} \right) \\
+ \sum_{k=0}^{L_m} \binom{L_m}{k} (-1)^k e^{-\frac{N_0 T}{P_m^{(1)} R_{i}^{(m-1)}}} \frac{1}{g_{S,P}^{(1)}} \frac{1}{g_{S,P}^{(1)}} + 1
\]
For the last hop $M$, the probability that a link from $R_i^{(M-1)}$ to SD is in outage is given by
\[
Pr\left[ y_{i,D}^{(M)} < T \right] = \left( 1 - e^{-\frac{N_0 T}{I_p R_{DM}}} \right) \left( 1 - e^{-\frac{I_p}{P_{M,R}}} \right) + \sum_{k=0}^{1} \binom{1}{k} \left( -1 \right)^k e^{-\left( \frac{N_0 T}{I_p R_{DM}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right)
\] (15)

Since in the last hop the relay with the highest SNR among all the $I_{M-1}$ relays in $D^{(M-1)}$ is selected for retransmission, outage occurs when the SNRs of all these $I_{M-1}$ links to SD are below the threshold $T$. Therefore we have
\[
Pr\left[ O_{M}\left| D^{(M-1)} \right. \right] = \left[ Pr\left[ y_{i,D}^{(M)} < T \right] \right]^{I_{M-1}} = \left( 1 - e^{-\frac{N_0 T}{I_p R_{DM}}} \right) \left( 1 - e^{-\frac{I_p}{P_{M,R}}} \right) + \sum_{k=0}^{1} \binom{1}{k} \left( -1 \right)^k e^{-\left( \frac{N_0 T}{I_p R_{DM}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right)
\] (16)

**B. CALCULATION OF** $Pr\left[ \cap_{n=1}^{m-1} D_{l_n}^{(n)} \right]$

When $m = 2$, $Pr\left[ \cap_{n=1}^{m-1} D_{l_n}^{(n)} \right] = Pr\left[ D_{l_1}^{(1)} \right]$ is given in (10). When $3 \leq m \leq M$, we have
\[
Pr\left[ \cap_{n=1}^{m-1} D_{l_n}^{(n)} \right] = Pr\left[ D_{l_1}^{(1)} \right] \prod_{n=2}^{m-1} Pr\left[ D_{l_n}^{(n)} | D_{l_{n-1}}^{(n-1)} \right] \tag{17}
\]

Let $A_{l_n}^{(n)}$ denote the event that in the $n$-th hop, from a relay in $D_{l_{n-1}}^{(n-1)}$, there are $l_n$ channels to relays in RC$_n$ whose instantaneous SNRs are above the threshold $T$, and let $B_{l_n}^{(n)} = \cup_{w=0}^{l_n-1} A_{l_n}^{(n)}$. Then we have
\[
Pr\left[ A_{l_n}^{(n)} \right] = \binom{L_n}{l_n} \left( 1 - e^{-\frac{N_0 T}{I_p m S_{R,R}}} \right)^{L_n-l_n} e^{-\frac{N_0 P_{l_n}}{I_p m S_{R,R}}} \left( 1 - e^{-\frac{I_p}{P_{M,R}}} \right) + \sum_{k=0}^{L_n} \binom{L_n}{k} \left( L_n - l_n \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right)
\] (18)

We also have
\[
Pr\left[ B_{l_n}^{(n)} \right] = \sum_{w=0}^{l_n-1} Pr\left[ A_{l_n}^{(n)} \right] \tag{19}
\]

To calculate $Pr\left[ D_{l_n}^{(n)} | D_{l_{n-1}}^{(n-1)} \right]$, we note that it is the probability that $l$ ($1 \leq l \leq l_{n-1}$) relays in $D_{l_{n-1}}^{(n-1)}$ have $l_n$ “good” channels$^3$ to relays in RC$_n$, while the remaining $l_{n-1} - l$ relays in $D_{l_{n-1}}^{(n-1)}$ have fewer than $l_n$ “good” channels. Therefore we can write
\[
Pr\left[ D_{l_n}^{(n)} | D_{l_{n-1}}^{(n-1)} \right] = \sum_{l=1}^{l_{n-1}} \binom{l_{n-1}}{l} \left( 1 - e^{-\frac{N_0 T}{I_p m S_{R,R}}} \right)^{l_{n-1}-l} e^{-\frac{N_0 P_{l_n}}{I_p m S_{R,R}}} \left( 1 - e^{-\frac{I_p}{P_{M,R}}} \right) + \sum_{k=0}^{l_{n-1}} \binom{l_{n-1}}{k} \left( -1 \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right) \tag{20}
\]

Putting (10) and (20) into (17), we get
\[
Pr\left[ \cap_{n=1}^{m-1} D_{l_n}^{(n)} \right] = \left( \frac{L_1}{l_1} \right) \left( 1 - e^{-\frac{N_0 T}{I_p m S_{R,R}}} \right)^{L_1-l_1} e^{-\frac{N_0 P_{l_1}}{I_p m S_{R,R}}} \left( 1 - e^{-\frac{I_p}{P_{M,R}}} \right) + \sum_{k=0}^{L_1} \binom{L_1}{k} \left( -1 \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right)
\]

\[
\times \prod_{n=2}^{m-1} \left( \frac{l_{n-1}}{l_n} \right) \left( 1 - e^{-\frac{N_0 T}{I_p m S_{R,R}}} \right)^{l_{n-1}-l_n} e^{-\frac{N_0 P_{l_n}}{I_p m S_{R,R}}} \left( 1 - e^{-\frac{I_p}{P_{M,R}}} \right) + \sum_{k=0}^{l_{n-1}} \binom{l_{n-1}}{k} \left( -1 \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right) \tag{21}
\]

Now inserting (10), (14), (16), and (21) into (12), $P_{\text{out}}^{(1)}$ is derived. Finally, putting (11), and (12) into (6), we get the exact outage probability of the proposed strategy.

Due to the maximum interference constraint $I_p$ from PU-Rx, outage probability of multi-hop underlay CRNs exhibits a floor level as $P_m$ increases. Therefore, we can calculate the asymptotic outage probability by letting $P_m \to \infty$. In the following we use the notation $\tilde{P}$ and $\tilde{Pr}$ to denote the probabilities when $P_m \to \infty$. For the first hop, letting $P_m \to \infty$ into (11), (10), we get
\[
\tilde{P}_{1 \text{out}} = \sum_{k=0}^{L_1} \binom{L_1}{k} \left( \frac{(-1)^k}{\tilde{P}_{S,R}^{(1)}} \right) + 1 \tag{22}
\]

and
\[
\tilde{Pr}\left[ D_{l_1}^{(1)} \right] = \left( \frac{L_1}{l_1} \right) \sum_{k=0}^{L_1} \binom{L_1}{k} \left( L_1 - l_1 \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right) + 1 \tag{23}
\]

Similarly, letting $P_m \to \infty$ in (14), (18), we get
\[
\tilde{Pr}\left[ O_{M}\left| D_{l_{n-1}}^{(m-1)} \right. \right] = \left[ \sum_{k=0}^{L_m} \binom{L_m}{k} \left( -1 \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right) \right]^{l_{n-1}} \tag{24}
\]

and
\[
\tilde{Pr}\left[ A_{l_n}^{(n)} \right] = \left( \frac{L_n}{l_n} \right) \sum_{k=0}^{l_{n-1}} \binom{l_{n-1}}{k} \left( L_n - l_n \right)^k e^{-\left( \frac{N_0 T}{I_p m S_{R,R}} + \frac{I_p}{P_{M,R}} \right) \frac{j_k}{N_0 T_{S,R}}} \left( 1 + \frac{1}{N_0 T_{S,R}} \right) + 1 \tag{25}
\]

$^3$Channels whose SNR exceed the threshold $T$.  

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Similar to (19), (20) we have
\[
\Pr \left[ B_{(n)}^{(m)} \right] = \sum_{w=0}^{l_n-1} \Pr \left[ A_{(w)}^{(m)} \right]
\] (26)
and
\[
\Pr \left[ D_{(n)}^{(m)} \mid D_{(n-1)}^{(m)} \right] = \sum_{l=1}^{l_n-1} \left( \frac{l_n-1}{l} \right) \left[ \Pr \left[ A_{(n)}^{(m)} \right] \right] \left[ \Pr \left[ B_{(n)}^{(m)} \right] \right]^{l_n-1-l}
\] (27)
Finally for the last hop, we have
\[
\Pr \left[ C_M \mid D_{(M-1)}^{(m)} \right] = \frac{\left( \frac{l_M-1}{k} \right) \left( -1 \right)^k \frac{N_0 T \rho^{(M)}_{R,D}}{I_p^{\eta} \lambda S,R} + 1}{l_M-1}
\] (28)
Putting (23) and (27) into (17), we get
\[
\Pr \left[ \mathcal{C} \mid D_{(n)}^{(m)} \right] = \frac{\left( \frac{l_n-1}{k} \right) \left( \frac{l_n-1}{l} \right) \left( -1 \right)^k \frac{N_0 T \rho^{(M)}_{R,D}}{I_p^{\eta} \lambda S,R} + 1}{l_n-1-l}
\times \prod_{n=2}^{M-1} \left( \frac{l_n-1}{l} \right) \left[ \Pr \left[ A_{(n)}^{(m)} \right] \right] \left[ \Pr \left[ B_{(n)}^{(m)} \right] \right]^{l_n-1-l}
\] (29)
Then using (23), (24), (28) and (29) in (12), asymptotic outage probability of hop \( m \), \( P_{out}^{(m)} \), is derived. Since \( P_{out}^{(1)} \) is already given in (22), by using (6) we get the asymptotic end-to-end outage probability.

V. AVERAGE END-TO-END THROUGHPUT
Let \( \bar{R} \) denote the average end-to-end throughput of the multi-hop underlay CRNs, which can be expressed as
\[
\bar{R} = \frac{(1 - P_{out}) R_S}{M}
\] (30)
It can be seen that the average end-to-end throughput is determined by the required per-hop-rate \( R_S \), the end-to-end outage probability \( P_{out} \), and the number of hops \( M \). An upper bound on the average end-to-end throughput can be calculated from (30) by using the asymptotic end-to-end outage probability instead of \( P_{out} \).

VI. NUMERICAL RESULTS
In this section, we present our numerical results from analysis and compare to those obtained from simulation. Similar to [30], [31], we consider a linear \( M \)-hop underlay CRN with SS, RC\( m \) (\( m = 1, \ldots, M-1 \)), SD located in a two-dimensional plane. SS and SD are assumed to be located at coordinates (0, 0) and \( (d_{e2e}, 0) \), respectively. That is, the distance between SS and SD is \( d_{e2e} \). RC\( m \) is located at \( \left( \frac{k}{M} d_{e2e}, 0 \right) \). If the distance between any two nodes A and B in hop \( m \) is \( d_{AB}^{(m)} \), then the channel gain between these two nodes is exponentially distributed with mean
\[
\lambda_{AB}^{(m)} = d_{AB}^{(m)} - \eta, \text{ where } \eta \text{ denotes the path loss exponent.}
\]
Throughout this section, we consider \( d_{e2e} = 5 \), PU-Rx is located in (2.5, 1.5), and \( \eta = 3 \).

In Fig. 2, we show the outage probability versus \( P_m/N_0 \). All relay clusters have the same number of relays. We can see that when \( P_m/N_0 \) is small, the outage probability of MaxDS-CRN strategy decreases as \( P_m/N_0 \) increases. The reason is that for small values of \( P_m/N_0 \), the transmit power is mainly limited by \( P_m/N_0 \). As \( P_m/N_0 \) increases, the transmit power becomes limited by the interference threshold \( I_p/N_0 \). Consequently the outage probability exhibits a floor level and gets close to asymptotic outage probability which is determined by \( I_p/N_0 \). We also compare the outage probability of MaxDS-CRN to other hop-by-hop relay selection strategies. As we can see, MaxDS-CRN strategy has a much lower outage probability than HTPRS and BLAR strategies.

As discussed in Section I, the exhaustive relay selection strategy in [13] is a centralized strategy for path selection in multi-hop non-cognitive networks. In order to compare MaxDS-CRN with this strategy we have extended this strategy to multi-hop underlay CRNs as follows. In addition to the CSI collection of all the links in the secondary network, the limits of the transmit power of SS and all relays are also calculated according to their channel coefficients to PU-Rx. Then using the CSI and the transmit power limits, the central controller computes the SNR of all the links and selects the end-to-end path which has the highest SNR bottleneck. Clearly this exhaustive strategy provides a lower bound for the outage probability of any relay selection strategy. We have simulated this strategy and show the results of its outage probability in Fig. 2. It can be seen that the performance of MaxDS-CRN is very close to this exhaustive strategy. However, the exhaustive strategy is not a hop-by-hop relay selection strategy, and its complexity is significantly higher than that of MaxDS-CRN. In addition, since the CSI of all the links must be collected before path selection and transmission, as the number of hops increases, the collected CSI
may be significantly outdated. This would not only degrade
the performance of the secondary user, but more importantly,
may cause interference to the primary user well beyond the
specified threshold. Finally, the figure shows a close match
between the results from our analysis and simulation.

In Fig. 3, we show the outage probability versus $I_p/N_0$.
We can see that outage probability decreases as the inter-
ference threshold $I_p/N_0$ increases, and reaches a floor level
for large values of $P_m/N_0$ where the transmit power is limited
by $P_m/N_0$. Clearly, lower outage probability can be reached
for larger values of $P_m/N_0$. Again, our proposed strategy
MaxDS-CRN outperforms IHTPRS and BLAR strategies
with respect to outage probability, and is very close to the
performance of the exhaustive relay selection strategy.

In Figs. 6 and 7, we show the throughput versus $P_m/N_0$.
By comparing the results for different $M$, it is easy to see
that for low values of $P_m/N_0$, a smaller $M$ leads to lower

**FIGURE 3.** Outage Probability vs. $I_p/N_0$ (dB) with $R_s = 2$ bps/Hz, $M = 4$, and $L_1 = L_2 = L_3 = 3$.

**FIGURE 4.** Outage Probability vs. $I_p/N_0$ (dB) for MaxDS-CRN strategy with $R_s = 2$ bps/Hz, $M = 4$, and $L_1 = L_2 = L_3 = L$.

Fig. 4 shows the outage probability vs. the maximum
interference power-to-noise ratio $I_p/N_0$. For MaxDS-CRN,
as number of relays $L$ increases, we get lower outage prob-
bility. At high $I_p/N_0$ region, outage probability reaches a floor
level which is determined by $P_m/N_0$. The asymptotic outage
probability and exact outage probability diverge at high $I_p/N_0$
region, since asymptotic outage probability is not limited
by $P_m/N_0$.

In Fig. 5, we show the outage probability versus $P_m/N_0$, when the relays are not equally distributed among the relay clusters. We can see that for MaxDS-CRN, when $L_1 = 4$, $L_2 = 3$, $L_3 = 5$, and $L_1 = 4$, $L_2 = 2$, $L_3 = 6$, outage probabilities are lower than that of equal distribution case $L_1 = L_2 = L_3 = 4$. As can be seen, these results are also true for IHTPRS and BLAR albeit for a different distribution of relays among the clusters. This indicates that equal
distribution of relays is not always the optimal choice for
outage probability and should be considered in deployment
scenarios.

**FIGURE 5.** Outage Probability vs. $P_m/N_0$ (dB) with $R_s = 1$ bps/Hz, $M = 4$ and $I_p/N_0 = 5$ dB.

**FIGURE 6.** Throughput vs. $P_m/N_0$ (dB) for with $R_s = 1$ bps/Hz, $M = 3$ and 4, $L_1 = L_2 = L_3 = 3$ and $I_p/N_0 = 5$ dB.
throughput. This is due to the fact that for low values of $P_m/N_0$ with small $M$, outage can occur in each hop, thereby limiting the end-to-end throughput. As $P_m/N_0$ increases, the end-to-end outage probability tends to zero, and the end-to-end throughput is mainly constrained by $M$ and gets close to its upper bound of $\frac{R_s}{L}$ which is $1/3$ and $1/4$ for $M = 3$ and $M = 4$, respectively. Fig. 6 shows that MaxDS-CRN provides a higher throughput than IHTPRS and BLAR for all values of $P_m/N_0$. Moreover, the throughput of MaxDS-CRN is very close to that of exhaustive strategy, which provides an upper bound for throughput of any relay selection strategy. From Fig. 7 we can see that a larger number of relays per cluster provides a larger diversity and leads to a higher throughput.

In Fig. 8, we show the throughput versus the number of hops $M$. $M = 1$ is the case that there are no relays between SS and SD, and SS transmits directly to SD. When $M = 2$, there is only one relay cluster between SS and SD. In this case the three strategies MaxDS-CRN, IHTPRS and BLAR can be considered as reactive opportunistic relay selection strategies under maximum transmit power and maximum interference constraints\(^4\) [33]. As we can see, as $M$ increases, the end-to-end throughput initially increases and then decreases. The reason is that when $M$ is small, the distance between adjacent clusters is large, and consequently the probability that outage occurs at each hop is high, leading to high end-to-end outage probability and lower throughput. Increasing $M$ will substantially decrease the end-to-end outage probability, thus increasing the end-to-end throughput. For large values of $M$, the end-to-end outage probability becomes very small and does not affect the throughput significantly. In this case the end-to-end throughput is mainly constrained by $M$. Therefore, end-to-end throughput decreases and gets close to $\frac{R_s}{L}$. This indicates that throughput is significantly affected by selecting different number of hops. We note that with some values of $M$, the improvement of MaxDS-CRN in throughput is small, which is expected, even though the outage probability improves significantly. The reason for small improvements in throughput is that the changes in outage probability do not have a significant affect on the throughput. However, we should point out that improvement in outage performance is paramount especially for applications which cannot tolerate large outage probabilities.

In Fig. 9, we show the throughput versus $P_m/N_0$ for two different required per-hop-rates $R_s$. It is interesting to note that for low $P_m/N_0$ region, $R_s = 1$ bps/Hz results in higher throughput compared to $R_s = 2$ bps/Hz. The reason is that when $P_m/N_0$ is small, decreasing the required per-hop-rate leads to a lower SNR threshold, which would significantly decrease the outage probability, and therefore higher throughput is achieved. At high $P_m/N_0$ region when the outage probability is small, throughput is mainly determined by $R_s$ and $M$, and thus $R_s = 2$ bps/Hz results in higher throughput than $R_s = 1$ bps/Hz.

\(^4\)Note that MaxDS-CRN, IHTPRS and BLAR strategies have the same relay selection procedure in the first and last hops. Therefore, for a two-hop network, all these strategies are simplified to the same two-hop strategy.
VII. CONCLUSION

In this paper, a hop-by-hop relay selection strategy named MaxDS-CRN is proposed for multi-hop underlay cognitive radio networks (CRNs) which can be implemented in a distributed manner. In this strategy, relay selection in each decoding set only depends on the channel state information (CSI) of a single hop. The end-to-end outage probability and throughput of the proposed scheme under both maximum transmit power and maximum interference constraints are derived. The asymptotic outage probability when there is no upper bound on the transmit power of the nodes is also evaluated. Numerical results show that MaxDS-CRN has the best performance in terms of outage probability and throughput compared to some recently proposed hop-by-hop relay selection strategies. Moreover, the outage probability and throughput of MaxDS-CRN are very close to that of exhaustive strategy, which provides a lower bound for outage probability and an upper bound for throughput of any relay selection strategy for multi-hop underlay CRNs.

REFERENCES

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