The Life and Scientific Work of Hiroshi Kunita

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Dedicated to the memory of Professor Hiroshi Kunita

ABSTRACT. We describe the life and mathematical work of Hiroshi Kunita and add a few personal recollections which show how admirable person he was.

1. Brief History

Hiroshi Kunita was born on January 31, 1937 in Higashi-Yodogawa, Osaka. He spent his elementary and junior high school days in this area where his father was running a business. This was a part of the area which was heavily attacked mainly with the fire-bombs by the U.S. Airforce in 1945, but he escaped unharmed.

He entered and graduated from Kitano High School which was in the neighborhood. After graduation, he studied at Kyoto University between 1955 and 1959. He took a courses and seminars by K. Itô. In 1959 he entered the master course in mathematics, and graduated in 1961.

He worked as an assistant (corresponding to research associate at present) at Kyushu Univ. from 1961. Then he moved to Nagoya Univ. in 1964, and was appointed as an assistant professor (corresponding to associate professor) at the Department of Mathematics, Faculty of Science in 1965. He obtained his Ph. D. degree from Kyushu Univ. in April, 1965. This is entitled : Applications of Martin boundaries to instantaneous return Markov processes over a denumerable space.

He was appointed as a full professor at the Department of Applied Science, Faculty of Engineering, in Kyushu Univ. in 1977 and worked there until retirement age in 2000. After that he continued to work at Nanzan Univ. (a private university in Nagoya) until 2008.

After retiring from Nanzan Univ. he lived in his later years in an apartment in Fukuoka with his wife Yoshiko Kunita.

2. Post Doctoral Days

We recall some memories as recounted by Takesi Watanabe (interviewed by A. Kohatsu-Higa in July 2020).

When Kunita was in the master course at Kyoto Univ., we started discussing that led to the articles " Markov processes and Martin boundaries, Part I", Illinois J. Math. 9 (1965), 485–526, (with H. Kunita). I remember we prepared drafts for Part II and Part III.

I was at Osaka City Univ. in those days and then moved to the Department of Applied Science, Faculty of Engineering, at Kyushu Univ. During that time, I visited Illinois Univ. at Urbana-Champaign from 1962 to 1964. In the mean-time Kunita got a job in Kyushu Univ. in 1961 (Faculty of Engineering) and moved in 1962 (Faculty of Science).

I returned to Kyushu Univ. and Kunita moved to Nagoya Univ. in 1964, and then he took a leave of absence to Illinois Univ., invited by J.L. Doob. Later, Ken-iti Sato (68-69), Masatoshi Fukushima (69-70) also visited Illinois Univ.

I have delivered a talk at 5th Berkeley Symposium on Mathematical Statistics and Probability (1965) based on my cooperative work with Kunita. (adapted)

Kunita stayed at the University of Illinois, Urbana-Champagin, during August 1965 - July 1967 with his family. This was by invitation by Doob. (Kunita returned to Japan on 25 July 1967 exactly, by the memory of his wife.) At that time S. Watanabe stayed at Stanford Univ., and Kunita visited him in the summer of 1966 with his family. It was a year after Fifth Berkeley Symposium on Mathematical Statistics and Probability where several Japanese probabilists came to deliver their talks.

The author tries below to describe his days of 'Sturm und Drang' concerning some probabilistic notions.

2.1. Carré du champ. A deep discovery in this period is that of operator carré du champ (square of vector field). The notion has evolved in the field of functional-analytic study of Markovian generators and semigroups mainly for diffusions. The feature of Kunita's theory is that it can be applied to jump-diffusions (jump processes with diffusion parts) as well as to diffusions.

Just before this time P.A.Meyer made a basis of the theory of Markov processes following the works of Loève, Doob and Dynkin :

J'ai donc passe une année universitaire (59/60), moitiè a Berkeley avec Loève, moitiè a Urbana (Illinois) chez Doob. ... (P. A. Meyer, Titres et Travaux: Postface [22], p.7)

The periods in which Meyer and Kunita stayed in Illinois did not overlap. Nevertheless, the heritage of Doob and Meyer on Markov processes and potential theory may have been transmitted to Kunita.

M. Yor explains : (M. Yor - in memoriam Paul-André Meyer - [31]; see also [4] Vol. IV, Chapter XVI, Section 1-11)

I should also indicate that this volume contains a deep discussion of the operateur carré du champ, a notion due to H. Kunita (Nagoya M. J., 36, 1969 [11]) and rediscovered by J.P. Roth in his thesis (1976). This notion then played an important role in the derivation of log-Sobolev inequalities of Bakry-Emery.

Apart from the general Itô-Tanaka formula, this chapter contains the characterization of semi-martingales as 'good integrators' taking values in L^0.

The Dellacherie and Bichteler Theorem and the notion of 'good integrator' are results by Dellacherie and Bichiteler ([3], [2], [6]). cf. [4] Vol. II (Chap. VIII).

2.2. Additive functionals (from [11]). Let $\langle X, Y \rangle$ denote the mutual variation (co-variation) of semi-martingale processes X and Y, which is derived from a continuous additive functional $\langle X \rangle$ so that $X_t^2 - \langle X \rangle_t$ is a local martingale.

Let $(X, Y)_\phi$ be a Radon-Nikodym derivative of the continuous additive functional $\langle X, Y \rangle$ with respect to a canonical additive functional $\phi : \frac{d\langle X, Y \rangle}{d\phi}$ such that $\langle X, Y \rangle$ is absolutely continuous with respect to ϕ for every local martingale X and Y.

It is written as

$$(X, Y)_\phi = \frac{1}{2}(Auv - uAv - vAu). \tag{1}$$

Here $X = X^u, Y = X^v$; we assume X^u, X^v to be of the form

$$X_t^u = u(x_t) - u(x_0) + \int_0^t f(x_s)d\phi,$$

$$X_t^v = v(x_t) - v(x_0) + \int_0^t g(x_s)d\phi,$$

and A is an operator such that $Au = -f, Av = -g$, which is the infinitesimal generator of the process x_t.

The right-hand side of (1) is called a carré du champ operator and is denoted by $\Gamma(u, v)$. The left-hand side of (1) is viewed as an inner product of X^u, X^v with respect to ϕ. In the above X_t^u can be viewed as an additive functional in t (that is, $X_t + X_s(\theta_t) = X_{t+s}$ for each ω ; cf. [4] Chap. XV, Sect. 2 (23.1)), and so is X_t^v.

The paper by Ledoux [21] explains the origin of the operator clearly (in the diffusion case) in that Γ measures how far A is from a derivation (p.312):

$$\Gamma(f, g) = \lim_{t \to 0} \frac{1}{2t}[P_t(fg) - P_t f P_t g].$$

Here $P_t = e^{tA}$. In the case of diffusion process on a manifold M and

$$Af(x) = \sum_{i,j=1}^n g^{ij}(x)\frac{\partial^2 f}{\partial x^i \partial x^j}(x), \ x \in M,$$

one has

$$\Gamma(f, h) = \sum_{i,j=1}^n g^{ij}\frac{\partial f}{\partial x^i}\frac{\partial h}{\partial x^j}.$$

This is the origin of the terminology carré du champ. Indeed, the matrix $(g^{ij}(x))$ (when nondegenerate) defines a symmetric tensor field on M. The inverse tensor $(g_{ij}(x))$ then defines on M a Riemannian metric, and $\Gamma(f, h)$ is the square of the length of the gradient ∇f in this metric. ([21] p.315.)

We cite from Memoir by Shinzo Watanabe : Section 1 'Probability theory in Japan before 1960' of [30] ; it is a bit long, but it well explains the atmosphere among probabilists of these days.

Itô returned to Kyoto from Princeton in 1956.

H. P. McKean came to Japan, staying in Kyoto, gave a series of lectures that stimulated much younger researchers (T.Hida, N.Ikeda, M.Motoo, M.Nisio, H.Tanaka, T.Ueno, T.Watanabe,...), as well as graduate students (M.Fukushima, H.Kunita, K.Sato, S.Watanabe, T.Yamada,...).

In the theory of Markov processes, the most advanced countries around 1960 were the United States and the Soviet Union.

Gradually it began to be understood that there is a deep interplay between Markov process theory and the martingale theory:

J.L.Doob([5]) pointed out the martingale character of stochastic integrals, and suggested that a unified theory of stochastic integrals should be established in a framework of martingale theory. His program was accomplished by H.Kunita and S.Watanabe ([19]) and P.A.Meyer ([23]), among others.

In this study, a fundamental role is played by a random inner product $\langle M, N \rangle$, $M, N \in \mathcal{M}$, which is defined to be a continuous AF (additive functionals) with almost all sample paths locally of bounded variation.

During a period around 1963, H.Kunita and I (Watanabe) conceived the idea of extending results in [24] and [29] to a more general and abstract situation in which the natural filtration associated with the Hunt process is replaced by a general filtration.

Thus, the works [19] and [23], which finally appeared in the same year, are very much related; indeed, as Meyer kindly stated in [23], his work was motivated by the work [19].

A standard terminology (for $\langle M, N \rangle$) now is a predictable quadratic co-variation of M and N. Meyer [23] introduced another random inner product $[M, N]$, called the quadratic co-variation of M and N, which plays important role in the study of discontinuous semimartingales.

(Martingale representation theorems) The martingale representation theorem states that every local martingale with respect to the natural filtration of a Wiener process can be expressed as the sum of a constant and a stochastic integral of a predictable integrand $f(s)$ with $\int_0^t f(s)^2 ds < \infty$ for every t, a.s.

In other fields the inner product is called correlation. Correlation was used since the 1960s in analyzing the system using Wiener kernel, Gaussian channel and Wiener-Khintchine theorem; in statistics, neural science, ..., for example ([27]).

S. Watababe seems to have put emphasis on the notation $\langle M, N \rangle$ for general square-integrable martingales. Kunita seems to have been much influenced by P. A. Meyer and his school, and was used to use the notation $[M, N]$ and paid more attention to the role in the study of discontinuous semimartingales. Nowadays the angle bracket $\langle M, N \rangle$ is used to denote the process that arises in the Doob-Meyer decomposition and $[M, N]$ is always used for quadratic variation. The two processes agree for continuous martingales, but not in the jump case. See also [28] p.180.

3. Applications to Filtering Theory

Kunita's relatively long stay in the U.S. may have influenced him in the study of filtering theory. It may be related with his course in the Department of Applied

Science, Faculty of Engineering, Kyushu Univ. It was a routine for a Chair Professor in the department to provide a course for the quality control of products. Also he was keeping a close relation with G. Kallianpur (University of North Carolina, U.S.), a specialist in control theory.

A modern stochastic control system theory may be characterized as a nonlinear feed-back system described by a (backward or forward) SDE. Kunita wrote several papers with M. Fujisaki and G. Kallianpur in the 1970s. Later, he also wrote an article on filtering theory [17]. (M. Nisio was intent on the study of the stochastic control theory with her teacher, K. Itô.)

On the other hand, one deals with vector fields and integral curves in high-dimensional Euclidean space, to state results of filtering theory in multi dimensional stochastic control theory. Kunita seems to have been interested in stochastic geometry and (diffusion and jump-diffusion) processes in various geometric spaces.

According to his book [13] in Japanese, the framework of Kalman filter theory is as follows: The algorithm of the Kalman filter consists of the SDE (which the filter satisfies), and a matrix-valued differential equation (for the coefficients of the SDE). This differential equation is of Wiener-Hopf type. Later, as he moved his study to Malliavin calculus of jump type, his understanding framework (gestalt) of viewing stochastic systems as [(nondegenerate driving core part) plus (variable degenerate coefficients)] was unchanged. [1] He summarized his study in the book that appeared in 1977.

The contents of the book is as follows: 1 Probability space and random variables; 2 Stochastic processes and filters; 3 Gaussian processes; 4 Linear filters; 5 Martingales; 6 Stochastic integrals; 7 Nonlinear filters.

Later Kunita made a new version of this 'book' [17] in 2011. He seems to have been interested in filtering theory up to his later years.

4. Stochastic Geometry

After a period in Nagoya, Kunita began his full research life in Fukuoka in the 1970s-1990s. In his former half of Kyushu days, his research had a tendency to study stochastic geometry and stochastic control, which were closely related with each other. Kunita wrote the lecture note [12] and a book [14] on stochastic flows.

The lecture note [12] was prepared for a summer school in Saint-Flour (1982). Chapters: 1 Stochastic caculus for continuous semimartingales; 2 Stochastic differential equations and stochastic flows of homeomorphisms; 3 Differential geometric analysis of stochastic flows

The contents of the book [14] published in 1990, which extended and gave further details to [12], are as follows: 1. Stochastic processes and random fields; 2. Continuous semimartingales and stochastic integrals; 3. Semimartingales with spatial parameter and stochastic integrals; 4. Stochastic flows; 5. Convergence of stochastic flows; 6. Stochastic partial differential equations.

In this monograph [14] Kunita took a new perspective to stochastic processes (mainly diffusions) to view them as 'flows'. A background for this vision may be the stochastic control theory of particles described in part **3**.

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[1]It may somewhat reflects his hybrid mentality as a Japanese influenced by European sense.

This new framework of the theory seems to be influenced by his stay in the U.S. (i.e. Doob, Feller). Although the main theme of the book is stochastic flows and stochastic geometry, Chapter 1 is actually written for general Markov processes and Feller semigroups.

The following is from the back cover to his book [14].

The main purpose of this book is to give a systematic treatment of the theory of stochastic differential equations and stochastic flow of diffeomorphisms, and through the former to study the properties of stochastic flows. The classical theory was initiated by K. Itô and since then has been much developed.

Kunita's approach here is to regard the stochastic differential equation as a dynamical system driven by a random vector field, including thereby Itô's theory as a special case. The book can be used with advanced courses on probability theory or for self-study.

A further development by him in this field can also be tracked in [1].

5. Malliavin Calculus of Jump Type

Starting from the 1980s Kunita and Ishikawa met at some workshops and began to talk. On the other hand, in the 1990s Ishikawa visited Strassburg, Erlangen and Clermont-Ferrand to study Malliavin calculus of jump type with background of potential theory with R. Léandre, N. Jacob and J. Picard. Kunita showed a strong interest on this subject. Kunita also kept a cooperative relation with a Korean mathematician Jae-Pill Oh on this topic, see [18].

In the beginning of 2000s Kunita visited Ehime Univ. (Matsuyama) several times, and delivered two invited lectures there. This led to the first joint result which appeared in SPA, 2006 [9].

Inspired by Picard's result in [26] (1996), they constructed Sobolev type norms on the Wiener-Poisson space, with rules and properties for operations such as the difference operators and their adjoints, and gave their estimates in the Wiener-Poisson space. The last point is particularly important in showing the existence of the transition density of a functional F, such as $F = X_T$ where $(X_t)_{0 \leq t \leq T}$ is a solution of SDE in the Wiener-Poisson space.

They used techniques in [26] Lemma 1.5 (identity with respect to $\tilde{\delta}^{(n)}$). It was proved that Picard and Kunita were ingenious for exploiting analysis in the adjoint (co-vector) space, hence for adjoint operators. Precisely, they also clarified the nondegenerate conditions, and developed the conditions for the existence of the density.

In [9] they introduced and showed the following:

  (i) expression in the Wiener-Poisson setting,
 (ii) estimate of the adjoint term $\tilde{\delta}(u)$ in Wiener-Poisson - proof was long, however it was later simplified in [10],
(iii) estimate of the inverse $S = R^{-1}$ of a kind of carré du champ operator: extension of a result in [26],
 (iv) integration by parts formula (in [9]).

6. New Version of Malliavin Calculus of Jump Type [10]

This is a paper as a revised edition of [9] on estimate of $\tilde{\delta}(u)$ and it provides with a new formulation using Sobolev norms in the Wiener-Poisson setting. It goes along the line between the stochastic geometry and the Malliavin calculus.

Authors extended the theory to general Lévy measures μ with support $A_0(\rho)$. A basic assumption is the order condition :

$$c\rho^\alpha |u|^2 \leq \int_{A_0(\rho)} (u,z)^2 \mu(dz) \leq C\rho^\alpha |u|^2$$

as $\rho \to 0$ for some $\alpha \in (0,2)$. Here $\{A_0(\rho)\}_{\rho>0}$ is an increasing family of star shaped open neighborhoods of the origin. This setting is motivated by the fact that the driving process of the SDE may have jumps, and the observation of them may depend on the current direction and the range of jumps indexed by ρ. The SDE has coefficients which may be degenerate, so that we would have an extended view of jump-diffusion processes on an Euclidean space.

They introduced as in part **5** a finite difference operator on the Poisson space, and its adjoint operator $\tilde{\delta}$. The continuity theorem ([10] Theorem 3.1) of $\tilde{\delta}$ was one of the main results. With respect to that of the adjoint operator (which could be regarded as a corollary to Meyer's inequality), they remark that in the diffusion case it had been given in Nualart's book [25] whereas it was written in [10] in the Poisson case.

As in [9], they had the upper bound of the characteristic function $E[e^{i(v,F)}G]$ of a Wiener-Poisson functional F (where G is a test function), and show the existence of the density for F as in the usual Malliavin calculus.

7. Stochastic Flows and Jump-diffusions [16]

This is the last monograph of Kunita. In the former half of it, he studied SDEs and stochastic flows, and discussed the relation between stochastic flows and heat equations. In the latter half of it, he studied the Malliavin calculus on the Wiener space and a space of Poisson random measure.

The main parts are as follows.

In Chapter 5, he discussed the Malliavin calculus on the Wiener space and that on the Poisson space. In Chapter 6, he studied the existence of smooth densities of the laws of nondegenerate diffusions and nondegenerate jump-diffusion processes defined on a Euclidean space. In Chapter 7, he studied SDEs on a manifold, and the stochastic flow generated by an SDE on a manifold. In [16] only the case of a compact manifold was treated.

Originally, Kunita and T. Fujiwara discussed SDE on a compact manifold, and then Kunita and Ishikawa discussed SDEs both on compact and non-compact manifolds. But, in summer 2016, Kunita decided to treat the compact case only.

In August 2018 Kunita wrote an e-mail (to M. Tsuchiya and Ishikawa) to say the draft of his book [16] was ready. The book was published in April, 2019. He was delighted for a prompt process of publishing by the publisher.

An e-mail was sent to Ishikawa in July, 2019, to inform that pancreas cancer had been diagnosed at the end of March and he was in hospital. He said that,

after his book had been appeared, he had almost nothing more he can do as a mathematician.

1937.1.31 (Osaka) – 2019.7.26 (Fukuoka)

References

1. Applebaum, D. and Kunita, H.: Lévy flows on manifolds and Lévy processes on Lie groups, J. Math . Kyoto Univ. 33 (1993), no. 4, 1103–1123.
2. Beiglbock, M., Schachermayer, W., and Veliyev B.: A direct proof of the Bichteler-Dellacherie theorem and connections to arbitrage, Ann. of Probab. 39 (2011), no. 6, 2424–2440.
3. Beiglbock, M. and Siorpaes, P.: Riemann-integration and a new proof of the Bichteler-Dellacherie theorem, Stochastic Process. Appl. 124 (2014), 1226–1235.
4. Dellacherie, C. and Meyer. P. A.: Probabilities and Potential C: Potential Theory for Discrete and Continuous Semigroups, Hermann, Paris, 2012.
5. Doob, J. L.: Stochastic Processes, John Wiley & Sons, New York, 1953; Wiley Classics Library Edition, 1990.
6. Harms, P.: Stochastic integration and financial mathematics, Preprint, June 6, 2019. (https://www.stochastik.uni-freiburg.de/professoren/harms)
7. Hayashi, M. and Ishikawa, Y.: Composition with distributions of Wiener–Poisson variables and its asymptotic expansion, Math Nach 285 (2012), 619–658.
8. Ikeda, N. and Watanabe, S.: Stochastic Differential Equations and Diffusion Processes, Second Edition, North-Holland/Kodansha, Amsterdam/Tokyo, 1988.
9. Ishikawa, Y. and Kunita, H.: Malliavin calculus on the Wiener-Poisson space and its application to canonical SDE with jumps, Stochastic Process. Appl. 116 (2006), 1743–1769.
10. Ishikawa, Y., H. Kunita, H., and Tsuchiya, M.: Smooth density and its short time estimate for jump process determined by SDE, Stochastic Process. Appl. 128 (2018), 3181–3219.
11. Kunita, H.: Absolute continuity of Markov processes and generators, Nagoya Math. J. 36 (1969), 1–26.
12. Kunita, H.: Stochastic differential equations and stochastic flow of diffeomorphisms, École d'Été de Probabilités de Saint-Flour XII (1984) 144–300, Springer, Berlin.
13. Kunita, H.: Estimates of stochastic processes (in Japanese), Sangyo-Tosho, Tokyo, 1977.
14. Kunita, H.: Stochastic Flows and Stochastic Differential Equations (Cambridge Studies in Advanced Mathematics) Springer, 1990.
15. Kunita, H.: Stochastic differential equations based on Lévy processes and stochastic flows of diffeomorphisms, in Real and Stochastic Analysis (2004), ed. by M. M. Rao, Birkhäuser.
16. Kunita, H.: Stochastic Flows and Jump-Diffusions, Springer, 2019.
17. Kunita, H.: Nonlinear filtering problems I and II, in Oxford handbook of Nonlinear Filtering (2011) 19–54, ed. by Dan Crisan and Boris Rozovskii, Oxford University Press, Oxford.
18. Kunita, H. and Oh, Jae-Pill: Asymptotic properties of Lévy flows, J. Korean Math. Soc. 27 (1990), 255–280.
19. Kunita, H. and Watanabe, S.: On square integrable martingales, Nagoya Math. J. 30 (1967), 13–30.
20. Kunita, H. and Watanabe, T.: Markov processes and Martin boundaries, Bull. Amer. Math. Soc. 69, 386–391, 1963.
21. Ledoux, M.: The geometry of Markov diffusion generators, Ann. de fac. des sciences de Toulouse 9 (2000), 305–366.
22. Meyer, P. A.: Titres et Travaux : Postface, in In memorian Paul-André Meyer, Séminaire de Probabités XXXIX (2006) 1–12, Lecture Notes in Math. 1874, Springer.
23. Meyer, P. A.: Integrales stochastiques (4 exposes), Séminaire de Probabites I (1967) 72–162, Lecture Notes in Math. 39, Springer.

24. Motoo, M. and Watanabe, S.: On a class of additive functionals of Markov processes, J. Math. Kyoto Univ. 4 (1965), 429–468.
25. Nualart, D.: The Malliavin calculus and related topics, 2nd ed., Springer, Berlin, 2006.
26. Picard, J.: On the existence of smooth densities for jump processes, Probab. Th. Relat. Fields 105 (1996), 481–511. Erratum: ibid 147 (2010), 711–713.
27. Rieke, F. et al.: Spikes, MIT Press, Cambridge, 1999.
28. Stroock, D. W.: Markov Processes from K.Itô's Perspective, Annals of Math. Studies, 155, Princeton University Press, Princeton, 2003.
29. Watanabe, S.: On discontinuous additive functionals and Lévy measures of a Markov process, Japanese Jour. Math. 34 (1964), 53–70.
30. Watanabe, S.: The Japanese contributions to martingales, in development of mathematics 1950-2000, (ed. Jean-Paul Pier), Electronic Journal for History of Probability and Statistics 5 (2009), 13 pages.
31. Yor, M.: The life and scientific work of Paul-André Meyer, In memoriam Paul-André Meyer (2006) 13–26, Séminaire de Probabités 1874, Springer.

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