Memories of Professor Hiroshi Kunita

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Dedicated to the memory of Professor Hiroshi Kunita

The first time I heard Professor Kunita's name was when the paper by Kunita-Watanabe [3] was mentioned in a discussion with someone. In short, this paper treated an extension of Ito's formula to martingales, and Ito's formula is based on stochastic integral, which is a very useful concept. However the stochastic integral was based on Brownian motion. The concept of stochastic integrals was generalized in the framework of martingale by using the martingale decomposition theorem due to P. -A. Meyer [4, 5]. To develop it further, Kunita-Watanabe showed that Ito's formula also holds in the framework of martingale. It was a very popular paper at that time, and it was an important one that marked an era. I learned the name of Professor Kunita through the paper, and I've always thought that he would be a tough person as an author of a famous paper. He might be a little scary. It was when I was a master course student at Kyoto University. At that time, a series of lectures of Professor Kunita was held at Kyoto University. I had a chance to see him. Moreover, there was an opportunity to exchange words directly with him. Professor Kunita asked me what I was studying, and I answered that I was studying stochastic differential equations. When I talked with him, I noticed that he is a very gentle and soft person, contrary to my image I had before. I'd had a completely wrong impression. I was very surprised. Still, Professor Kunita's mathematics is strict, and I wish I could become such a mathematician.

After that, I got a job as an assistant at Osaka University, and there was a series of lectures of Professor Kunita. Again, I had an opportunity to attend to his lectures. At that time, Professor Kunita was talking about the decomposition of stochastic processes on Lie groups. I was also interested in stochastic processes on Lie groups, so I was able to listen to it interestingly. That time, he was talking about the stochastic process on Lie groups, but after that, Professor Kunita's research was moved to the topic of stochastic flow. In a sense, it can be said that it is a natural development arising from the study of the stochastic process on the Lie group. His work on stochastic flow was resulted in the book [2] and it is now a basic and frequently quoted literature in this direction.

Later, I got interested in the Markov semigroup, and I was looking for a criterion of the Markov property of the semigroup in L^p-space. Then, I found Professor Kunita's paper [1]. This paper treated a formulation of the concept equivalent to

Markov property within the framework of Banach lattices. A linear operator T is sub-Markov if

$$0 \le f \le e, \quad 0 \le Tf \le e$$

for all f in a Banach space. Here e is a positive element called the identity element, which corresponds to the identity function 1 in a framework of real valued functions. However, these are all defined in the lattice theory setting. Discussions are held in an abstract framework. I was surprised that Professor Kunita did such a very abstract mathematics. I thought that he was a very flexible person. Such kind of flexibility may have supported Professor Kunita's wide range of works.

From time to time, my interests have been changed, but sometimes I got involved in the works that Professor Kunita was doing. And I think I was lucky to be inspired by his works each time. I strongly believe that in the future, the works by Professor Kunita will continue to be cited as important results.

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