Index future pricing under imperfect market and stochastic volatility

Wei-Hsien Li
Louisiana State University and Agricultural and Mechanical College

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Abstract

Financial markets in emerging countries are volatile and imperfect, so pricing model under traditional perfect-market frameset may not give reliable price of financial derivatives. The most famous pricing model for stock index future is the cost of carry model. The mis-pricing of cost of carry model inspires lots of following researches. Even transaction costs, dividends, stochastic interest rate, stochastic volatility, market imperfection, and other factors are considered, we still do not obtain a model price consistently better than cost of carry model. But these researches offer important insights, for example, the market needs time to mature and the more complex model usually perform better than cost of carry model in relatively imperfect or volatile markets. Therefore, a model extended from these literatures should still be useful in particular markets. Here I will propose a two-factor stock-index future pricing model includes stochastic volatility of spot index with imperfect financial market framework. The pricing formula may not have a close form solution, so I would use the finite difference method to approximate the solution.

The thesis is organized as follows, after introduction I will review the pricing models for the index futures in Chapter 2. In Chapter 3 the two-factor model is derived, and the solution is proposed. The empirical issues about this model are then proposed in Chapter 4. The conclusion and suggestion are in Chapter 5.
Chapter 1. Introduction

A stock index tracks changes in the value of a hypothetical portfolio of stocks. The weight of a stock in the portfolio equals the proportion of the portfolio invested in the stock. Dividends are usually not included in the calculation so that the index tracks the capital gain/loss from investing in the portfolio. Even though the hypothetical portfolio is well diversified, stock indices investors would still face systematic risk. Index futures contracts are widely used in financial market as a relatively low cost instrument to hedge the systematic risk.

Financial market in emerging countries is volatile and imperfect, so pricing model under traditional perfect-market frameset may not give reliable price of financial derivatives. The most famous model for pricing stock index futures is undoubtedly the cost of carry model [Cornell and French (1983a, 1983b)]. But following studies suggested that the model consistently produces pricing error with respect to the actual price. There are many explanations for mis-pricing puzzle being proposed. Cornell and French (1983b) suggested that the mis-pricing is the timing option for the tax purpose. Cornell (1985) defied the timing option and suggested that transaction costs and other factors may cause the mis-pricing. Modest and Sundaresan (1983), Modest (1984), and Klemkosky and Lee (1991) took transaction costs into consideration and used arbitrage method to construct an arbitrage interval for stock index price. On the other hand, following the stochastic interest rate framework by Cox, Ingersoll and Ross (1981) and Ramaswamy & Sundaresan (1985), Helmer and Longstaff (1991) developed a closed-form model of stock index futures prices in an economy with both stochastic interest rate and stochastic market volatility. Considering the market imperfection and the limits of arbitrage, Hsu and Wang (2004) proposed another general equilibrium pricing model
of stock index futures in imperfect markets.

The cost of carry model is derived from the perfect market and other unrealistic assumptions. For example, the model treats the variables in stock market as exogenous variables. But Kawaller, Koch, Koch (1987) and Stoll and Whaley (1990) documented the inter-reaction between the spot index and the associated futures. Also, the cost of carry model assumes that the risk free rate is deterministic. According to Cox, Ingersoll and Ross (1981), this model is in fact a forward pricing model, and the forward price is not equivalent to future price if the interest rate is stochastic. Moreover, the cost of carry model does not consider the volatility of stock price. According to Helmer and Longstaff (1991), the pricing error of the cost of carry model is correlated to the volatility from stock price.

More importantly, the capital market is not frictionless as the cost of carry model assumed. The transaction cost including commissions and bid-ask spread would make the instantaneous hedging economically infeasible. As Shleifer and Vishny (1997) discussed, in reality, almost all arbitrage requires capital, and is typically risky. On the other hand, market regulations such as short-sell constraint would make the no arbitrage arguments in most financial model being violated in the real market. For instance, Figlewski (1989) and Ofek, Richardson, and Whitelaw (2004) found evidences against the put-call parity arbitrage due to the short-sell constraints. Hsu and Wang (2004) proposed a pricing model under the imperfect financial market assumption; however, their market imperfectness measure is focus on the futures market itself. This would make their model unable to capture the volatility in the spot index market well. In this research, I am going to combine the stochastic volatility and imperfect market into one model, and pave a way for future empirical studies.
Chapter 2. Literature Review

2.1. Cost of Carry Model and Mis-pricing in Stock Index Future

Cornell and French (1983a, 1983b) assumed a frictionless capital market, exist a constant risk-free rate and borrowing rate equals to lending rate, dividend is a known constant, and Consider an investment strategy A: at time t, buy stocks worth $S(t)$. The cash flow at maturity time $T$ is $S(T) + D(t, T)$ where $S(T)$ is the stock value at time $T$ and $D(t, T)$ is the cumulated dividend during time $t$ to $T$.

Consider another investment strategy B: hold one stock index future contract and buy bonds worth $S(t)$. This strategy should have $S(t)e^{r(T-t)} + S(T) - F(S, t)$ cash flow at time $T$ where $r$ is risk-free rate so $S(t)e^{r(T-t)}$ is the cash flow from the bond position and $S(T) - F(S, t)$ is the cash flow from future position.

Since dividend payment is a known constant, these two strategies have same level of risk, and also have same cash flow at time $T$ for no arbitrage opportunities. So the future price should be

$$F(S, t) = S(t)e^{r(T-t)} - D(t, T),$$

[2-1]

and also for continuous compound and with constant dividend yield rate $q$,

$$F(S, t) = S(t)e^{(r-q)(T-t)},$$

[2-2]

where $S(t)$ is the actual spot price of the index at time $t$, $T-t$ is the annualize interval between $t$ to maturity time $T$. Since the risk-free rate $r$ in [2-2] is non stochastic, according to Cox, Ingersoll, and Ross (1981), this model can be also viewed as a forward pricing model.

Using S&P 500 index and the NYSE composite Index, Cornell and French (1983a, 1983b) found that the actual prices observed in the market are, in general, lower than the theoretical prices obtained from the Cost of Carry model. Similar
empirical conclusion was made by Figlewski (1984) and Modest & Sundaresan (1983). Eytan & Harpaz (1986) focused on the Value Line Composite Index (VLSI) which was excluded in prior research but an important part in Stock Index Future history. They also found a discount relative to its theoretical price which made a well complement for thoroughly empirical investigation. On the other hand, Bhatt & Cakici (1990) used the futures and stock index data obtained from the Chicago Mercantile Exchange, daily closing values from April, 1982 to June, 1987, and found the mis-pricing is usually small but positive.

No matter what direction the mis-pricing is, the cost of carry model for stock index future does not seem to be a satisfying solution for researchers. Cornell and French (1983b) take factors like taxes, stochastic interest rate, and seasonal fluctuating dividend into model and proposed a timing option argument for the mis-pricing.

In real market, the tax of capital gain is not levied until transaction is made, it seems that for those stock investors, they have a timing option for lower their tax by realize capital loss or postpone capital gain. On the other hand, stock index future contract holders do not have such timing option. So holding a stock can be viewed as a portfolio of two assets, the first asset is a truncated security and the second asset is a timing option which can postpone capital gain. The truncated security is the cash flow creating asset when investor buy stocks at time t and sold stocks at time T, besides, if stock price falls during time t to T, the tax on the truncated security is just like the tax on future contract that is, no timing option. The timing option is valuable only when stock price goes up during time t to T. So we know the value of the stock at time t is:

\[ S(t) = P(t) + C(t) \] \[ 2-3 \]
where $P(t)$ is the price of the truncated security at time $t$ and $C(t)$ is the value of the timing option at time $t$.

However, in Cornell (1985), the empirical results embarrassed the timing option explanation. Cornell argued that the timing option maybe worthless because the marginal investor is a tax-exempt institution. Even if floor traders and arbitrageurs are the marginal investors, the timing option may still be limited value since such active traders do not hold the cash security indefinitely and thus forego the timing option. Also Cornell suggested that transaction costs, limitation on capital loss deductions, and other tax related constraints reduce the value of the timing option.

### 2.2. Transaction Costs and Short Sell Constraints

Modest & Sundaresan (1983) argued that the transaction costs cannot be ignored for investors who want to short sell stock index. They took transaction costs and short sell constraint into future pricing model and used no arbitrage argument to develop a no-arbitrage pricing interval for stock index futures.

$$\frac{S_t + C_{PS} + C_{FL}}{B(t,T)} \leq F(t,T) \leq \frac{S_t + C_{PL} + C_{FS}}{B(t,T)}, \quad [2-4]$$

where the transaction cost is on a per index basis, and

- $C_{PL}$ is the cost of being long in the spot index,
- $C_{PS}$ is the cost of being short in the spot index,
- $C_{FL}$ is the cost of being long in the future contract,
- $C_{FS}$ is the cost of being short in the future contract,
- $S_t$ is the price of one unit of the underlying asset at time $t$,
- $F(t,T)$ is the contract price at time $t$ for the future delivery of one unit of the underlying asset at time $T$,
- $B(t,T)$ is the price of a riskless pure discount bond that pays $1$ at $T > t$. 
Taking account of non-stochastic varying dividend, [2-4] becomes,

\[
\frac{S_t + C_{PS} + C_{FL} - \sum_{\tau=1}^{\tau_0} B(t, t + \tau) d_{\tau}}{B(t, T)} \leq F(t, T) \leq \frac{S_t + C_{PL} + C_{FS} - \sum_{\tau=1}^{\tau_0} B(t, t + \tau) d_{\tau}}{B(t, T)}, [2-5]
\]

where \( d_{\tau} \) is the known dividends to be paid at \( \tau \).

In Modest & Sundaresan’s empirical tests, the historical experience revealed that arbitrage opportunities are consistently available only for traders who have full use of proceeds under the assumption the ability to sell short the index at a reasonable cost.

Modest (1984) extended Modest & Sundaresan (1983) research in two ways. First, Modest took account of lumpiness of actual dividend payment. Under assumptions like no tax, known interest rate, and future dividend and transaction cost is uncertain, Modest developed a similar no-arbitrage pricing interval for Stock Index Futures.

\[
\frac{S_t + C_{PS} + C_{FL} - \sum_{\tau=1}^{\tau_0} B(t, t + \tau) \bar{d}_{\tau}}{B(t, T)} \leq \bar{F}(t, T) \leq \frac{S_t + C_{PL} + C_{FS} - \sum_{\tau=1}^{\tau_0} B(t, t + \tau) \bar{d}_{\tau}}{B(t, T)}, [2-6]
\]

where \( \bar{d}_{\tau} \) and \( \bar{d}_{\tau} \) are minimum and maximum amount of dividends to be paid at time \( \tau \).

Similar to Modest & Sundaresan, Modest also found the arbitrage opportunities are consistently available only for traders who have full use of proceeds. The result implied these theoretical boundaries works good under the assumption and the common case where traders can use less than full proceeds. Second, Modest suggested that under their assumption, the interest rate uncertainty and mark to market are likely have minimal effect on equilibrium prices.

By analyzing the mis-pricing of Hong Kong Hang Seng Index future contracts and conducting tests over three distinct regulatory regimes relating to the short selling
of stocks in Hong Kong, Fung & Draper (1999) found that relaxing the constraints on short selling reduces the extent of futures mis-pricing. Also Gay & Jung’s (1999) study on KOPSI 200 futures and Korean stock market also suggested that a substantial portion of under-pricing can be explained by transaction cost and the high frequency of under-pricing especially during the periods of downward market trends can be attributed in part to short sell restrictions.

2.3. Seasonal Dividend Payouts and Taxes

Klemkosky & Lee (1991) took transaction costs, differential borrowing and lending rates, and seasonal dividend payouts into consideration. They decided the upper bound by “borrowing cash to long spot index and write index future contract” strategy, and the lower bound by “long index future and lend the fund obtain from short selling spot index” strategy. They obtained the no arbitrage pricing interval for stock index Futures:

\[
F_t - C_{gf}(1 + r)^{T-t} - C_{sf}(1 + r)^{T-t} < F_a < F_t + C_{gf}(1 + r)^{T-t} + C_{sf}(1 + r)^{T-t},
\]

where \( F_t = S(1 + r)^{T-t} - \sum_{\tau=t}^{T} d_{\tau}(1 + r)^{T-\tau} \) is the theoretical future price at short hedge at time \( t \); \( S \) is the spot index value at time \( t \); \( r' \) is the borrowing interest rate; \( r \) is the lending interest rate; \( d_{\tau} \) is the certain daily dollar dividend payout on index stocks at time \( \tau \); \( F_t = S(1 + r)^{T-t} - \sum_{\tau=t}^{T} d_{\tau}(1 + r)^{T-\tau} \) is the theoretical future price at short hedge at time \( t \); \( C_{gf} \) is the cost of long future; \( C_{ss} \) is the cost of short spot index; \( C_{sf} \) is the cost of short future; \( C_{ss} \) is the cost of short spot index.
where \( F_t = \frac{1}{1-g} \left[ S(1 + R)^{T-t} - gS - \sum_{\tau=t}^{T} D_{\tau}(1 + R)^{T-\tau} \right] \); \( g \) is the capital gain tax rate; \( R' = (1 - i) r' \) is the after-tax borrowing rate; \( R = (1 - i) r \) is the after-tax lending rate; \( i \) is the ordinary income tax rate; \( D_{\tau} = (1 - i) d_{\tau} \) is the after-tax daily dividend payout at time \( \tau \); \( f \) is the future tax rate; \( F_t = \frac{1}{1-g} \left[ S(1 + R)^{T-t} - gS - \sum_{\tau=t}^{T} D_{\tau}(1 + R)^{T-\tau} \right] \); \( R = (1 - i) r \) is the after-tax lending rate.

Klemkosky & Lee found that taxes dramatically reduce the frequency of mis-pricing of their model by enlarging the no arbitrage interval. In addition, the frequency and degree of mis-pricing diminishes as expiration of the futures contract approaches.

The reports about the relation between dividend and mis-pricing are mixed. Bhatt & Cakici (1990) showed the mis-pricing is positively and significantly related to time-to-maturity and dividend yield for both near and longer maturity contracts, rather than being stochastic over time. Yadav & Pope (1990) examined the data relating to the UK FTSE-100 stock index futures contract traded on the London International Financial Futures Exchanges (LIFFE). They reported that after taking account of transaction costs, dividend uncertainty does not appear to be an explanation of systematic mis-pricing.

2.4. Stochastic Interest Rate

According to Cox, Ingersoll, and Ross (1981), if interest rate is stochastic, future prices and forward prices need not to be equilavent. The cost of carry model assumes that risk-free rate is constant, which makes the model is indeed a forward pricing model. So there is an interest to look at what stochastic interest rate could
effect pricing on stock index futures.

Ramaswamy & Sundaresan (1985) assumed the spot index (S) follow the diffusion process:

\[ dS = (\alpha - \delta)Sdt + \sigma Sdz_1, \]  \[ 2-9 \]

where \( \alpha \), \( \sigma \) are the drift and volatility of the stochastic process; \( \delta \) is the dividend yield of the index; \( dz_1 \) is standard Wiener process. Following the Cox, Ingersoll, and Ross (1981, 1985) assumptions, Ramaswamy & Sundaresan also assumed that instantaneous riskless interest rate follows mean reverting square root process:

\[ dr = \kappa(\mu - r)dt + \sigma_2 \sqrt{r}dz_2, \]  \[ 2-10 \]

where \( \kappa \), \( \mu \), \( \sigma_2 \) are the speed of adjustment, long-run mean, and volatility of the stochastic process; \( dz_2 \) is standard Wiener process. The covariance between \( dz_1 \) and \( dz_2 \) is \( \rho dt \), \( \rho \) is the coefficient of correlation.

If the Local Expectations Hypothesis in Cox, Ingersoll, & Ross (1981) holds, following the pricing of option on index future in Ramaswamy and Sundaresan, the index future price is governed by this partial differential equation:

\[ \frac{1}{2} \sigma_2^2 rF_r + \frac{1}{2} \sigma_1^2 S^2 F_S + \rho \sigma_1 \sigma_2 S \sqrt{r}F_S + \kappa(\mu - r)F_r + (r - \delta)SF_r = F, \]  \[ 2-11 \]

In general, Equation [2-11] doesn’t have a closed-form solution and need to numerically solve with appropriate boundary conditions. However, if \( \rho = 0 \), Ramaswary and Sundaresan obtained a closed-form solution:

\[ F = Sd(\tau) e^{[b(r)\tau]}, \]  \[ 2-12 \]

where \( a(\tau) = \left\{ \frac{2\gamma \exp[(\gamma + \kappa)\tau / 2]}{2\gamma + (\gamma + \kappa)(\exp(\gamma \tau) - 1)} \right\}^{2\exp(\delta \tau) / \sigma_2^2} \left[ \exp(-\delta \tau) \right] \)
\[ b(\tau) = \frac{2 \exp(\gamma \tau) - 1}{2\gamma + (\gamma + \kappa)(\exp(\gamma \tau) - 1)} \]

\[ \gamma = \sqrt{\kappa^2 - 2\sigma_i^2} > 0 \quad \text{by assumption.} \]

In some other financial derivatives, models including stochastic interest rate usually offer better predictability, but the result in stock index future is not clear. Bailey (1989) studied Nikkei 225 index future and Stock 50 contract in Japanese stock index futures showed there is no substantially different in pricing error between cost of carry model and the above model. Although Bailey mentioned the possible explanation like low interest rate in Japan, his result did not show pricing improvement from cost of carry model to stochastic interest rate model.

Cakici & Chatterjee (1991) studied the S&P 500 index futures from April 21, 1982 to June 19 1987. They followed Marsh & Rosenfeld (1983) suggestion that the “lognormal” model gives a better statistical fit than the “square root” model for the interest rate sample. In their comparison among cost of carry model, [2-12], and their log-normal interest rate process, they found in 1986-1987 and the whole data set, the stochastic interest rate models are better than cost of carry model, but no significant difference in 1982-1985 periods. If the spot interest rate is significantly different from the long-run mean and the speed of adjustment is very high, the stochastic interest rate models give significantly better pricing than the cost of carry model. The correlation between the interest rate process and spot index process does not appear to have any significant impact on index future pricing. Moreover, their result also indicates that the superiority of the stochastic model is not sensitive to the exact specification of the model: virtually identical results are obtained for the square root model and the log-normal model.
2.5. Stochastic Volatility

In addition to Ramaswamy and Sundaresan (1985) ‘s model which treating risk-free rate as stochastic, Helmer and Longstaff used the general equilibrium framework of Cox, Ingersoll and Ross (1985) to develop a closed-form model of stock index futures prices in an economy with both stochastic interest rate and market volatility.

\[ F(S, r, V, \tau) = S e^{-q\tau} A(\tau) \exp(B(\tau)r + C(\tau)V) \]  \[  \text{[2-13]} \]

where

\[ A(\tau) = \left( \frac{2\phi \exp(\phi + \beta)\tau}{2 - \phi (e^{\phi\tau} - 1)} + 2\phi \right)^{\frac{2(\alpha + \delta)}{\eta^2}} \times \left( \frac{2\psi \exp(\psi + \beta - \gamma)\tau}{2 - \phi (e^{\psi\tau} - 1) + 2\psi} \right)^{\frac{2\delta}{\tau^2}} \]

\[ B(\tau) = \frac{2(e^{\phi\tau} - 1)}{\phi (e^{\psi\tau} - 1) + 2\phi} \]

\[ C(\tau) = B(\tau) + \frac{2(1 - e^{\psi\tau})}{(\beta + \psi - \gamma)(e^{\psi\tau} - 1) + 2\psi} \]

\[ \phi = \sqrt{\beta^2 - 2\eta^2} \text{ and } \psi = \sqrt{(\gamma - \beta)^2 + 2\zeta^2} \]

\( \alpha, \beta, \gamma, \delta, \zeta,\eta,\xi \) are related parameters from the state variable governed by the stochastic differential equations, despite of the math induction detail, this model shows the equilibrium stock index future price is an explicit function of S, r, V, \( \tau \), where S is spot price, \( r \) is risk-free rate, V is variance of market return, q is dividend rate, \( \tau \)=the time interval T-t. We can find that the interest-rate and market volatility parameters only through the three terms A(\( \tau \)), B(\( \tau \)), and C(\( \tau \)). This property makes this model easy to test. Also this model has two basic properties, 1) when maturity future price = spot price: when \( \tau = 0 \), we get A(\( \tau \))=1, B(\( \tau \))=0, C(\( \tau \))=0, therefore F(S, r, V, \( \tau \))=S. 2) when spot price=0, future price =0: when S=0 we get F(S, r, V, \( \tau \))=0.

This model implies that general equilibrium stock index future price F is a positive and monotone increasing function of stock index level S. This follows
because $A(\tau)>0$ and nonnegative $S$. Similarly, the stock index future price is a decreasing function of the dividend yield; $S$ is “discounted” by the dividend yield factor $\exp(-q\tau)$ and $q>0$.

Consequently, the stock index future price is a uniformly increasing function of the risk-free interest rate because $B(\tau)>0$ for $\tau>0$. This result is consistent with the cost of carry model in which the interest rate can be viewed as a “carrying cost” that is added to the index value to obtain the future price.

Hemler and Longstaff applied their study on NYSE stock index futures data from 1983 to 1987 to test market volatility in pricing stock index futures and compare their general equilibrium model to cost of carry model in pricing efficiency. In their finding, the market volatility has significant impact on stock index futures pricing. When including Oct 1987, the equilibrium model is significantly better than cost of carry model, but when excluding Oct 1987, the predictability of cost of carry model improves and the difference on pricing efficiency between these two models becomes insignificant.

Empirical studies on comparison between cost of carry model and equilibrium model are mixed. In both Gay & Jung’s (1999) study using Korean market and Brailsford and Cusack’s (1997) study using the data of individual share futures traded on the Sydney Futures Exchanges, they found that cost of carry model and equilibrium model provide similar results – as none was clearly supported.

**2.6. Market Imperfection**

Despite of above pricing models under the equilibrium assumptions, Figlewski (1984) argued that the pricing error maybe prevalent, since the market needs time to mature. In other words, the pricing error (discount) represented a situation of disequilibrium – a transitory phenomenon caused by unfamiliarity with the new
markets and institutional inertia in developing systems to take advantage of the opportunities presented. In some other researches like Saunders & Mahajan (1988), MacKinlay & Ramaswamy (1988), Bhatt & Cakici (1990), and Cakici & Chatterjee (1991) all report that the pricing errors in the U.S. stock index futures have diminished over time. Similarly, Bailey (1989) and Brenner, Subrahmanyam, Uno (1989) found that the largest pricing errors in Japanese index futures markets are observed in the first year of listing.

Even though the price errors in the mature market are diminishing, it is still an issue in the emerging markets. Hsu and Wang (2004) tried to induct the stock index future pricing model under imperfect markets by using concepts of price expectation and imperfect arbitrage. Assuming during contract life the underlying stock index (S) paying continuous dividend rate q, the imperfectness of the market is constant, and the (S) is a random factor with geometric Wiener process. i.e.

\[ dS = (u - q)Sdt + \sigma Sdz \]  

where \( u \) is the instantaneous growing rate of \( S \), \( \sigma^2 \) is the instantaneous variance of \( S \), and \( dZ \) is the standard Wiener process with \( E(dz) = 0 \) and \( dZ^2 = dt \).

Let \( u_f \) and \( \sigma_f^2 \) are the instantaneous expected return rate of \( S \) and the instantaneous variance of return of \( S \). Also construct a hedge portfolio \( P \) by future contracts and spot index with weight \( w_f \) and \( w_s \).

In perfect markets, the arbitrage works perfectly; the hedge portfolio \( P \) can be risk-free. That is, because of no arbitrage, the instantaneous return rate of \( P \) is risk-free rate, and can pick an arbitrary \( w_f^* \) and \( w_s^* \) where \( w_f^* \sigma_f + w_s^* \sigma = 0 \). However, under imperfect market, since the arbitrage does not work perfectly and the arbitrage is risky, the hedge portfolio \( P \) is not a perfect hedge portfolio. Also, the instantaneous return rate of \( P \) should be a certain expected return rate or growth rate, not risk-free rate.

Let \( u_p \) and \( \sigma_p \) are the instantaneous expected return rate of \( S \) and the instantaneous standard deviation of return of \( S \) under imperfect market. After induction, Hsu and Wang introduce a one factor stock index future pricing model.
under imperfect markets with their measure for market imperfection is $\sigma_p/\sigma$.

$$F(S, t) = S_0e^{(r_p-q)(T-t)}$$  \[2-15\]

In some alternative models mentioned above, the pricing efficiency difference between those alternative models and cost of carry model decline also with market maturity. However, these results do not mean the pricing error would completely disappear with market getting mature.
Chapter 3. The Pricing Model

In emerging financial markets, not only the derivatives market is immature but also the underlying financial asset price and return are more volatile due to market imperfection. In Hsu and Wang’s (2004) model, the degree of market imperfection is \( \sigma_p/\sigma \), a measure focus on the market imperfection on the futures market. Since the volatility of the spot market is in the denominator in the measure, when the volatility of the spot market is high, the degree of market imperfection would be lower. Therefore, their model may not weight the volatility of spot market enough. A pricing model considers both market imperfection in derivatives market and volatility in the spot market would help investors in early stage of emerging financial market.

3.1. Derivation of the Pricing PDE

My pricing model is an extension of Hsu and Wang (2004). That is, a pricing model including stochastic volatility under the imperfect market setting. The assumptions are: 1) the underlying spot index (S) pays a continuous dividend rate (q) during the duration of the future contract. 2) the market incompleteness is constant during the duration of the future contract. 3) the spot index and the volatility (here I use the standard error (\( \sigma \)) of the instantaneous return for the volatility measure) are stochastic, and S and \( \sigma \) follow the joint stochastic process specified as:

\[
\begin{align*}
    dS &= (u-q)Sdt + \sigma SdZ_1, \\
    d\sigma &= \alpha \sigma dt + \beta \sigma dZ_2,
\end{align*}
\]

where \( u \) is the instantaneous growing rate of S, \( \alpha \) and \( \beta \) depend on \( \sigma \) and t, and \( dZ_1 \) and \( dZ_2 \) are standard Wiener Process with \( dZ_1 \cdot dZ_2 = \rho dt \) where \( \rho \) denotes the correlation coefficient between the two Brownian Motions.

Assuming the index future price \( F(S, \sigma, t) \) is a twice continuously
differentiable function of \( S, \sigma \), and \( t \), we can use Itô’s Lemma to define its instantaneous price change as follows:

\[
dF = \left( \frac{1}{2} \sigma^2 S^2 F_S + \rhoSF \sigma^2 F_{\sigma S} + \frac{1}{2} \beta^2 \sigma^2 \sigma_{\sigma \sigma} + (u - q)SF + \alpha F + F \right) dt + (\sigma SF) dZ_1 + (\beta \sigma F) dZ_2, \tag{3-3}
\]

Let \( u_f \) and \( \sigma^2_f \) be the instantaneous expected return rate of index future and the instantaneous variance of return of index future. Also we can construct a hedge portfolio \( P \) by future contracts and spot index with weight \( w_f \) and \( w_s \).

Under imperfect market assumption, since the arbitrage does not work perfectly and the arbitrage is risky, the hedge portfolio \( P \) is not a perfect hedge portfolio. Also, the instantaneous return rate of \( P \) should be a certain expected return rate or growth rate, not risk-free rate. Let \( u_p \) and \( \sigma_p \) be the instantaneous expected return rate of the hedge portfolio and the instantaneous standard deviation of return of the portfolio under imperfect market. Since the cash outflow for future contract on purchase is zero, we have:

\[
w_f u_f + w_s u = w_s u_p, \tag{3-4}
\]
\[
w_f \sigma_f + w_s \sigma = w_s \sigma_p, \tag{3-5}
\]

As Hsu and Wang (2004) pointed out, the \( \sigma_p \) is not the true instantaneous standard error of the portfolio, but depend on the coefficients of \( dZ_1 \) and \( dZ_2 \) in [3-3]. From [3-4] and [3-5], we can obtain the index future equilibrium condition as:

\[
\frac{u_f}{\sigma_f - \sigma_p} = \frac{u - u_p}{\sigma - \sigma_p}, \tag{3-6}
\]

When constructing the hedge portfolio in order to obtain the partial differential equation for future price, I follow the framework and concept of market price of convenience yield risk proposed by Brennan and Schwartz (1979) and Gibson and Schwartz (1990). Therefore I can eliminate the one of the stochastic factor under the
equilibrium (i.e. the portfolio has instantaneous return rate \( u_p \)). As a result, I consider the spot index price risk only when constructing the hedge portfolio, which means, here \( \sigma_p \) would be the coefficients of \( dZ_i \) in [3-3].

With the previous equilibrium condition, the price of the index future must satisfy the following partial differential equation:

\[
\frac{1}{2} \sigma^2 S^2 F_{SS} + \rho S \beta \sigma^2 F_{S\sigma} + u_p S F_S + \alpha \sigma^2 F_{\sigma} + F_i - \left( u_a - (u_p - q) \right) F = 0 , \tag{3-7}
\]

where \( u_a = \left[ (u_p - q) - (u_q - q) \frac{\sigma_p}{\sigma} \right] \left( 1 - \frac{\sigma_p}{\sigma} \right) \),

and this partial differential equation must satisfy the terminal condition:

\[
F(S, \sigma, T) = S_T, \tag{3-8}
\]

Follow the argument of Hsu and Wang (2004), the \( u_a \) term can be further simplified by finding a \( w_f \) that minimizes the volatility of the hedged portfolio. As a result, we have:

\[
w_f = -\rho^* \frac{\sigma}{\sigma_f}, \tag{3-9}
\]

\[
u_p = -\rho^* u_f \frac{\sigma}{\sigma_f} + u , \tag{3-10}\]

\[
\sigma_p = (1 - \rho^*) \sigma , \tag{3-11}\]

where \( \rho^* \) is the instantaneous correlation coefficient between the futures return and the index return in the hedged portfolio. Then substitute [3-10] and [3-11] into

\[
u_a = \left[ (u_p - q) - (u_q - q) \frac{\sigma_p}{\sigma} \right] \left( 1 - \frac{\sigma_p}{\sigma} \right) , \tag{3-12}
\]

the parameter \( u_a \) can be simplified as follows:
When solving the partial differential equation [3-7], my first attempt is to check if there is a close form solution for the PDE. After reviewing the Ramaswamy and Sundaresan (1985), Helmer and Longstaff (1991), and Hsu and Wang (2004) with associated closed form solutions for their models are [2-12], [2-13], and [2-15]. I guess the closed form solution for [3-7] should have the form:

\[ F(S, \sigma, \tau) = S e^{(\theta - q)\tau} A(\tau) e^{\theta(\tau)\sigma}, \]  

where \( \tau = T - t \); \( A(\tau) \) and \( B(\tau) \) are functions of \( \tau \).

From [3-9], we can obtain:

\[ F_S = \frac{F}{S}, \]  

\[ F_{SS} = 0, \]  

\[ F_\sigma = B(\tau)F, \]  

\[ F_{\sigma\sigma} = [B(\tau)]^2 F, \]  

\[ F_{S\sigma} = \frac{B(\tau)}{S} F, \]  

\[ F_\tau = \frac{F}{A(\tau)} \frac{dA(\tau)}{d\tau} + (u - q)F + \sigma F \frac{dB(\tau)}{d\tau}, \]  

Substitute [3-14] through [3-19] into [3-7] then we have:

\[ \rho \beta B(\tau) \sigma^2 + \frac{1}{2} \beta^2 [B(\tau)]^2 \sigma^2 + \alpha B(\tau) \sigma - \frac{dB(\tau)}{d\tau} \sigma - \frac{1}{A(\tau)} \frac{dA(\tau)}{d\tau} = 0, \]  

Also from [3-13], in order to satisfy the start condition, \( F(S, \sigma, 0) = S, A(\tau) \) must be 1, and \( B(\tau) \) must be 0 when \( \tau = 0 \). However, I cannot find \( A(\tau) \) and \( B(\tau) \) which satisfy conditions: [3-16], \( A(0) = 1, \) and \( B(0) = 0 \). Therefore, there is
no closed form solution for [3-7] as the form of [3-13].

3.3. Solving the Pricing PDE by Finite Difference Method

The finite-difference method places a grid of points on the space over which the desired function takes value and then approximates the function value at each of these points. The method solves the equation numerically by introducing difference equations to approximate derivatives. Although the implicit method is stable and more efficient than the explicit method, here I used the explicit finite-difference method to approximation the solution. The reason is here I got three variables $S, \sigma, T$ to divide into grids where the 3-D matrix has large size which would make the inverse computation of implicit method extremely complicated. Since I can tweak the explicit method to get a stable approximation, here I use the straightforward explicit method to approximate the solution.

First, I represent the each partial differential term by the difference term of explicit method.

\[
F_s = \frac{F_{i+1,j,k} - F_{i-1,j,k}}{2\Delta S}, \quad [3-17]
\]

\[
F_{ss} = \frac{F_{i+1,j,k} - 2F_{i,j,k} + F_{i-1,j,k}}{(\Delta S)^2}, \quad [3-18]
\]

\[
F_\sigma = \frac{F_{i+1,j,k+1} - F_{i+1,j,k-1}}{2\Delta \sigma}, \quad [3-19]
\]

\[
F_{\sigma\sigma} = \frac{F_{i+1,j,k+1} - 2F_{i+1,j,k} + F_{i+1,j,k+1}}{(\Delta \sigma)^2}, \quad [3-20]
\]

\[
F_{s\sigma} = \frac{F_{i+1,j,k+1} - F_{i-1,j,k+1} - F_{i+1,j-1,k+1} + F_{i+1,j-1,k-1}}{4\Delta S \Delta \sigma}, \quad [3-21]
\]

\[
F_t = \frac{F_{i,j,k} - F_{i,j,k}}{\Delta t}, \quad [3-22]
\]

where $\Delta S$ is the variation of the spot index price within each grid, on the other
words, if the maximum spot index is $S_{MAX}$, the minimum spot index is 0, and the number of spot index grid is $M$, then $\Delta S = S_{MAX} / M$. In general term, $S_j = j\Delta S$ ($j = 0, 1, 2, ..., M$). $\Delta \sigma$ is the variation of the volatility within each grid, on the other words, if the maximum volatility is $\sigma_{MAX}$, the minimum spot index is 0, and the number of volatility grid is $Q$, then $\Delta \sigma = \sigma_{MAX} / Q$. In general term, $\sigma_j = k\Delta \sigma$ ($k = 0, 1, 2, ..., Q$). $\Delta t$ is the interval of time within each grid, on the other words, if time to the maturity is $T$, the starting time is 0, and the number of time grid is $N$, then $\Delta t = T / N$. In general term, $t_i = i\Delta t$ ($i = 0, 1, 2, ..., N$).

Then substitute [3-17] through [3-22] into [3-7] and use $S_j = j\Delta S$ and $\sigma_j = k\Delta \sigma$ to represent $S$ and $\sigma$. After rearranging, we have the difference equation.

$$
\begin{align*}
    a_{j,k} F_{i+1,j+1,k+1} + b_{j,k} F_{i+1,j+1,k} + c_{j,k} F_{i+1,j+1,k-1} + d_{j,k} F_{i+1,j,k+1} + e_{j,k} F_{i+1,j,k} + f_{j,k} F_{i+1,j,k-1} + g_{j,k} F_{i+1,j-1,k+1} + h_{j,k} F_{i+1,j-1,k} + I_{j,k} F_{i+1,j-1,k-1} =& 1 + (u_a - (u_\rho - q)\Delta t)F_{i,j,k} \\
\end{align*}
$$

[3-23]

where

$$
\begin{align*}
    a_{j,k} &= \frac{\rho \beta j k^2 \Delta \sigma \Delta t}{4}, \\
    b_{j,k} &= \left( \frac{j^2 k^2 (\Delta \sigma)^2 \Delta t}{2} + \mu_a j \Delta t \right), \\
    c_{j,k} &= -\frac{\rho \beta j k^2 \Delta \sigma \Delta t}{4}, \\
    d_{j,k} &= \left( \frac{\beta^2 k^2 \Delta t}{2} + \alpha k \Delta t \right), \\
    e_{j,k} &= \left( 1 - j^2 k^2 (\Delta \sigma)^2 \Delta t - \beta^2 k^2 \Delta t \right),
\end{align*}
$$
\[ f_{j,k} = \left( \frac{\beta^2 k^2 \Delta t}{2} - \frac{\alpha k \Delta t}{2} \right), \]

\[ g_{j,k} = -\frac{\rho \beta j k^2 \Delta \sigma \Delta t}{4}, \]

\[ h_{j,k} = \left( \frac{j^2 k^2 (\Delta \sigma)^2 \Delta t}{2} - \frac{\mu_j \Delta t}{2} \right), \]

\[ l_{j,k} = -\frac{\rho \beta j k^2 \Delta \sigma \Delta t}{4}, \]

with \( a_{j,k} + b_{j,k} + c_{j,k} + d_{j,k} + e_{j,k} + f_{j,k} + g_{j,k} + h_{j,k} + l_{j,k} = 1. \)

We then can use [3-23] to approximate the price of index futures.
Chapter 4. Empirical Issues

Due to the data accessibility, I cannot perform the empirical study for the pricing error for my model. But here I will discuss some issues about parameters estimation and the programming to provide a guideline for future empirical studies.

4.1. Parameters Estimation

4.1.1. Directly Estimating the \( u_a \)

Recall from [3-12], we can estimate \( u_a \) by directly estimate the variables \( u \), \( u_f \), \( \sigma \), and \( \sigma_f \) using historical data. With these estimates in hand, the parameter \( u_a \) is then computed.

The variables \( u \) and \( u_f \) can be estimated using the following formulae, respectively

\[
\ln(u_{t-1}) = \ln\left(\frac{S_{t-1}}{S_{t-2}}\right), \tag{4-1}
\]

\[
\ln(u_{f,t-1}) = \ln\left(\frac{F_{t-1}}{F_{t-2}}\right), \tag{4-2}
\]

where \( S_{t-1} \) and \( F_{t-1} \) are the spot and future prices, respectively, at time \( t-1 \). \( u \) and \( u_f \) at time \( t-1 \) are used as the estimates of \( u \) and \( u_f \) at time \( t \), respectively.

Regarding the variables \( \sigma \) and \( \sigma_f \), they can be estimated by different time-series models, for instance the well-known GARCH framework or simple moving average method.

4.1.2. Estimate Parameters \( \alpha \), \( \beta \) and \( \rho \)

In Gibson and Schwartz (1990)s’ derivation of a two-factor pricing model for the oil contingent claims, they assume the oil spot price (S) and convenience yield (\( \delta \)) follow the stochastic processes:
$dS = \mu S dt + \sigma_1 S dZ_1,$

$d\delta = \kappa(\alpha - \delta)S dt + \sigma_2 dZ_2,$

where $dZ_1$ and $dZ_2$ are standard Wiener Process with $dZ_1 \cdot dZ_2 = \rho dt$ where $\rho$ denotes the correlation coefficient between the two Brownian Motions.

Gibson and Schwartz use the seemingly unrelated regression model to estimate the coefficients $\rho$, $\kappa$, $\alpha$, $\sigma_2$. Here I will use the same scheme to estimate parameters $\alpha$, $\beta$ and $\rho$. First, transform [3-2] into the discrete approximation:

$\sigma_t - \sigma_{t-1} = a\sigma_{t-1} + \epsilon_t,$ \hspace{1cm} [4-3]

in conjunction with the following unrestricted regression model for $\ln(S_t/S_{t-1})$,

$\ln(S_t/S_{t-1}) = a + b\ln(S_{t-1}/S_{t-2}) + \epsilon_t,$ \hspace{1cm} [4-4]

Then use the seemingly unrelated regression to estimate the coefficients of [4-3] and [4-4]. The standard error of the $\sigma_{t-1}$ would be the estimation of $\beta$ and the correlation coefficient between $\epsilon_t$ and $\epsilon_t$ would be the estimation of $\rho$.

### 4.2. The Boundary Conditions for the Finite Difference Method

At the maturity date, the future price should converge to the spot price, therefore, the terminal condition is:

$F(S, \sigma, T) = S_T,$ \hspace{1cm} [4-5]

When the spot price approaches to infinity, the futures’ value should also approach to infinity, which can be easily checked from the closed form solutions from different models. Similarly, when the spot price approaches to 0, the futures’ value should also approach to 0. Therefore, we can set the boundary condition for the maximum spot price and the minimum spot price (which is 0), respectively, as:

$F(S_{\text{MAX}}, \sigma, t) = S_{\text{MAX}},$ \hspace{1cm} [4-6]
When the volatility of the spot price approaches to infinity, the futures’ value should also approach to infinity (here is set as $S_{\text{MAX}}$), which can be easily checked from equation [2-13]. Similarly, when the volatility approaches to 0, the futures’ value should not be influenced by volatility of spot price. Therefore, we can set the boundary condition for the maximum spot price and the minimum spot price (which is 0), respectively, as:

$$F(S, \sigma_{\text{MAX}}, t) = S_{\text{MAX}},$$  \hspace{1cm} [4-8]

$$F(S, 0, T) = S_t,$$  \hspace{1cm} [4-9]

Also, at the grid for the maximum volatility and the spot index price equals to 0, I set the futures value as:

$$F(0, \sigma_{\text{MAX}}, t) = 0,$$  \hspace{1cm} [4-10]

### 4.3. Discussion of the Finite Difference Method Programming

The explicit finite difference method is not stable and highly vulnerable for the parameter change and grid division. After many tests, I found an empirical condition for the approximation to converge. That is, the $j^2k^2(\Delta \sigma)^2 \Delta t$ term in [3-23] must be less than 1. Therefore, when I divide $\sigma$ into 100 grids, with $\sigma_{\text{max}}$ sets to 1, the condition became the maximum $j^2 \Delta t$ less than 1 due to the maximum of $k^2(\Delta \sigma)^2$ is equal to 1. For example, if the spot index price is divided into 150 intervals, sets $\Delta t$ equals to 20 minutes $(20/(365*24*60)$ year) would satisfy the converge condition and won’t make the grid too wide.

Due to the data accessibility, I only have the daily S&P 500 index price from Yahoo! Finance, and the annual dividend rate estimated by Brennan (1998). Because lack of the actual futures price data, I use the price from the cost of carry model to be
the proxy of the historical futures price. Lots of literature evidence that the S&P 500 index futures are trade in relatively mature market and the mis-pricing is mitigated. Therefore, although my pricing model is not designed for this relatively mature market, the current data still can be used for testing the numerical programming.

Now I consider the $S_{MAX} = 2250$, $M = 150$, $\sigma_{MAX} = 1$, $Q = 100$, and the time interval $\Delta t = 20$ minutes ($20/(365*24*60)\text{year}$). I consider the S&P 500 index futures contract with expiration date is 6/16/06, the price of the index future on Mar 31 (when the spot index price is 1294.87) is calculated by the program (see appendix) $= 1289.436$.

On the other hand, I also try to use the explicit method on the logarithm transformation, that is $Z = \ln(S)$, but in order to utilize the better grid dividing suggested by literature (e.g. Hull and White (1990) Geske and Shastri (1985), and Brennan and Schwartz (1978)), the matrix would be too large to be computed in Matlab. Moreover, the efficiency didn’t improved much when applying logarithm transformation even I choose grids which can obtain stable approximation.

<table>
<thead>
<tr>
<th>Contract</th>
<th>Time of evaluation</th>
<th>Spot index price</th>
<th>Futures price by cost of carry.</th>
<th>Approx. futures price by the model</th>
<th>Average CPU time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P index Jun 06</td>
<td>3/31/06</td>
<td>1294.87</td>
<td>1301.42</td>
<td>1289.44</td>
<td>128.432</td>
</tr>
</tbody>
</table>

This particular Matlab 6.5 program is run on a PC equipped with Pentium-M 1.8GHZ, 2gb system memory.
Chapter 5. Conclusion and Suggestion

In this thesis, I propose a two-factor index future pricing model of incorporate with stochastic volatility of the spot market and the market imperfection of the futures market. I also show how to prepare empirical study to test this model by discussing the parameter estimation and programming issue. I believe the model would help us to capture factors in both the highly volatile spot index market and the immature futures market in the emerging financial markets.

Another string of related research is studying the interaction between spot index market and futures markets. Faff and McKenzie (2002) and Szakmary and Kiefer (2004) documented the introduction of futures market reduce the seasonality of mean return. Moreover, after Chan, Chan, and Karolyi (1991) found the volatility interdependence in the stock index and the stock index future market, there are many empirical evidence from main international stock index future markets supporting the lead-lag relationships between spot and future returns as well as spot and futures volatility. A next step on the index future pricing model could be to capture the volatility lead-lag relationships observed in the markets.
References


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Appendix:

The Matlab Code for the Finite Difference Method

%the program to price index future under two-factor model by explicit finite difference method.
start = cputime;
% setting parameters
SMAX = 2250; % max spot index price
SMIN = 0;    % min spot index price
M = 150;     % total intervals for spot price
SIGMAX = 1;  % max volatility
SIGMIN = 0;  % min volatility
Q = 100;     % total intervals for volatility
N = 5544;    % numbers of time intervals
T = 20*N;    % total time to maturity in minutes
RHO = 0.0197; % correlation between two stochastic process
Beta = 0.1294816294/sqrt(254); % the coefficent of sigma*dZ in violatility process
DIV = (1+0.021384)^(1/254) - 1; % annual dividend rate
alfa = -0.17797/sqrt(254); % the coefficent from the volatility process
ASIG = sqrt(0.0080494533); % standard error of spot index price
SIGF = sqrt(0.0082772731); % standard error of futures price
U = -0.00202832; % spot index return rate.
UF =-0.002103465; % futures return
UA = (U-DIV)-UF*ASIG/SIGF; % parameter u-alfa
S = 1294.87; % spot price
FSV = zeros([N+1 M+1 Q+1]);  TI = zeros(N+1,1);  SP = zeros(M+1,1);
SIG = zeros(Q+1,1);  A1 = zeros(M-1, Q-1);  A2 = zeros(M-1, Q-1);
A3 = zeros(N+1, M-1);  A4 = zeros(Q-1,1);  A5 = zeros(Q-1,1);
% setting Time, Spot index, and Volatility grids
for I = 1:N+1
    TI(I) = (T/(365*24*60))*(I-1)/N;
    A6 = 1 + (UA-(-RHO*UF*ASIG/SIGF+U-DIV))*T/(N*365*24*60);
end;
for J = 1:M+1
    SP(J) = SMAX*(J-1)/M;
end;
for K = 1:Q+1
    SIG(K) = SIGMAX*(K-1)/Q;
end;
% setting final condition
for J = 1:M+1
    for K = 1:Q+1
        FSV(N+1, J, K) = SP(J);
    end;
end;
% setting boundary condition
for I = 1:N
    for K = 1:Q+1

FSV(I,M+1,K) = SP(M+1);
FSV(I,1,K) = SP(1);
end;
end;

for I = 1:N
  for J = 1:M+1
    if J == 1
      FSV(I,J,Q+1) = SP(1);
      FSV(I,J,1) = SP(1);
    else
      FSV(I,J,Q+1) = SP(M+1);
      FSV(I,J,1) = SP(J);
    end;
  end;
end;

% Calculating the futures value
for I = N:-1:1
  for J = 2:M
    A3(I,J-1) = UA*J*T/(N*365*24*60);
    for K = 2:Q
      A1(J-1,K-1) = RHO*Beta*J*K*SIGMAX/Q * T/(N*365*24*60);
      A2(J-1,K-1) = K*K*J*J*SIGMAX/Q * T/(N*365*24*60);
      A4(K-1) = Beta*Beta*K*K*T/(N*365*24*60);
      A5(K-1) = alfa*K*K*T/(N*365*24*60);
      FSV(I,J,K) = (1/A6) * ( A1(J-1,K-1)*0.25*FSV(I+1,J+1,K+1) + (A2(J-1,K-1)*0.5+A3(I,J-1)*0.5)*FSV(I+1,J+1,K) + (-A1(J-1,K-1)*0.25)*FSV(I+1,J+1,K-1) + (A4(K-1)*0.5+A5(K-1)*0.5)*FSV(I+1,J,K+1) + (-A2(J-1,K-1) - A4(K-1) + 1)*FSV(I+1,J,K) + (-A1(J-1,K-1)*0.25)*FSV(I+1,J+1,K) + (A2(J-1,K-1)*0.5-A3(I,J-1)*0.5)*FSV(I+1,J-1,K) + A1(J-1,K-1)*0.25*FSV(I+1,J-1,K-1));
    end;
  end;
end;

TEST = FSV(1,:,:);
for J = 1:M+1
  if (SP(J) < S & SP(J+1) > S)
    for K = 1:Q+1
      if (SIG(K) < ASIG & SIG(K+1) > ASIG)
        FP = TEST(1,J,K);
        check1 = J;
        check2 = K;
      end;
    end;
  end;
end;

elap = cputime -start;
Vita

Wei-Hsien Li was born on February 5, 1979, in Taipei, Taiwan. He finished his undergraduate studies at National Taiwan University June 2001. After he finished the military duty, he served as a teaching assistant of statistics and economics for Department of Finance, National Taiwan University. In August 2003 he came to Louisiana State University to pursue graduate degrees in both finance and mathematics, which will be awarded in August 2006.