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LEONARD GROSS'S WORK IN INFINITE-DIMENSIONAL ANALYSIS AND HEAT KERNEL ANALYSIS

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Abstract. This paper describes a certain part of Leonard Gross's work in infinite-dimensional analysis, connected to the Gross Ergodicity Theorem. I then look at way in which Gross's work helped to create a new subject within (mostly) finite-dimensional analysis, a subject which may be called "harmonic analysis with respect to heat kernel measure." This subject transfers to Lie groups certain constructions on $\mathbb{R}^n$ that involves a Gaussian measure. On the Lie group, the role of the Gaussian measure is played by a heat kernel measure.

1. The Gross Ergodicity Theorem and its Consequences

The purpose of this article is to outline one part of Leonard Gross's work in infinite-dimensional analysis and to show how that work lead to the development of a new discipline within (mostly) finite-dimensional analysis, a discipline which may be called "harmonic analysis with respect to heat kernel measure," or "heat kernel analysis" more briefly. The story begins with what is now called the Gross Ergodicity Theorem, established in [21]. The result may be described as follows. Let K be a connected compact Lie group equipped with a bi-invariant Riemannian metric. Let $W(K)$ denote the *continuous path group*, i.e., the group of continuous maps of $[0,1]$ into K sending 0 to the identity e in K. We consider on $W(K)$ the Wiener measure ρ. Now let $\mathcal{L}(K)$ denote the *finite-energy loop group*, i.e., the group of maps of $[0,1]$ into K sending both 0 and 1 to e and having one distributional derivative in L^2. Then the left action of $\mathcal{L}(K)$ on $W(K)$ leaves the Wiener measure quasi-invariant.

Theorem 1.1. *Given $f \in L^2(W(K),\rho)$, suppose that for all $l \in \mathcal{L}(K)$, $f(l \cdot g) = f(g)$ for almost every g in $W(K)$. Then there exists a measurable function ϕ on K such that $f(g) = \phi(g(1))$ for almost every g in $W(K)$.*

That is to say, if a function on the path group is invariant under the left action of the loop group, then that function depends only on the endpoint of the path. The difficulty in this theorem comes in the mismatch between the levels of smoothness: The paths in $W(K)$ have to be continuous rather than finite energy, because the Wiener measure does not live on finite-energy paths (they form a set of measure zero). Meanwhile, the loops in $\mathcal{L}(K)$ have to be finite energy rather

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than continuous, because only finite-energy loops act in a way that leaves the Wiener measure quasi-invariant. If only it were possible to use paths and loops of the same smoothness, the result would be easy. If two continuous paths g_1 and g_2 have the same endpoint, then there is a loop l such that $lg_1 = g_2$. However, the loop l will only be continuous rather than finite energy, so invariance of f under $\mathcal{L}(K)$ does not imply (at least in any obvious way) that $f(lg_1) = f(g_2)$.

From Theorem 1.1, Gross was able to deduce that the action of $\mathcal{L}(K)$ on the continuous loop group is ergodic (i.e., all invariant subsets have measure 0 or measure 1). This ergodicity theorem has been used to prove that the pinned Wiener measure on the continuous loop group is equivalent to the heat kernel measure of Malliavin. Absolute continuity in one direction was established by Driver and Srimurthy in [13]; absolute continuity in the other direction follows easily from the Gross Ergodicity Theorem, as shown by Aida and Driver [1].

Gross's proof of Theorem 1.1 begins by "linearizing" the problem, using the Itô map. Let $W(\mathfrak{k})$ denote the continuous path space (beginning at the origin) in the Lie algebra $\mathfrak{k}$ of K. Then to a path $X(\cdot)$ in $W(\mathfrak{k})$ we associate a path $\theta(X)(\cdot)$ in $W(K)$ by solving the Stratonovich differential equation

$$\frac{d\theta(t)}{dt} = \theta(t) \circ dX(t).$$

This stochastic differential equation gives a map (defined almost everywhere) from $W(\mathfrak{k})$ to $W(K)$, and the push-forward under θ of the Wiener measure on $W(\mathfrak{k})$ coincides with the Wiener measure on $W(K)$. Given, then, a function f on $W(K)$, we can turn it into a function on $W(\mathfrak{k})$ by composing f with θ. Under this map, the loop-invariant functions (those satisfying $f(l \cdot g) = f(g)$) corresponds to functions on $W(\mathfrak{k})$ invariant under a certain action of the loop group $\mathcal{L}(K)$. (The action of the loop group on $W(\mathfrak{k})$ has the form of a gauge transformations in Yang–Mills theory; more on this point below.)

After using the Itô map to transfer the problem to the linear path space $W(\mathfrak{k})$, Gross uses the so-called *chaos expansion*, that is, the expansion of a function on $W(\mathfrak{k})$ as a sum of multiple Wiener integrals. This expansion is the infinite-dimensional counterpart to the expansion of a function on $L^2(\mathbb{R}^n,\text{Gauss})$ as a sum of Hermite polynomials. In general, the nth "coefficient" in the chaos expansion is function on the n-simplex with values in the tensor-product space $\mathfrak{k}^{\otimes n}$. As shown in Theorem 5.1 of [21], for a loop-invariant function on $W(\mathfrak{k})$, these tensor-valued functions are all constants. Furthermore, the constants fit together to form an element of the tensor algebra over $\mathfrak{k}$ that is orthogonal to a the ideal J generated by elements of the form $XY - YX - [X, Y]$. From this, Gross was able to then show that loop-invariant functions are actually endpoint functions.

A consequence of Gross's proof is a sort of "Hermite expansion" for the compact group K. In infinite-dimensional terms, one starts with a function ϕ on K, forms the "endpoint function" $f(X) = \phi(\theta(X)(1))$. That is to say, an endpoint function is a function f on $W(\mathfrak{k})$ such that the value of $f(X)$ depends only on the endpoint of the Itô map θ applied to X. If one performs the chaos expansion of the function f associated to a function ϕ on K, one obtains an element of the space $J^\perp$. The squared L^2 norm of the function ϕ on K can be expressed as a sum of squares of coefficients of the element of $J^\perp$. Furthermore, if K is simply connected, then Gross shows that every element of $J^\perp$ arises from some function ϕ on K. The space $J^\perp$ may be thought of as (a completion of) of the universal enveloping algebra of the Lie algebra $\mathfrak{k}$. The Hermite expansion on K should be thought of as a generalization of the identification of $L^2(\mathbb{R}^n,\text{Gauss})$ with the Fock space of symmetric tensors. When $\mathbb{R}^n$ is replaced by the Lie group K, the Fock space of symmetric tensors is replaced by the universal enveloping algebra.

Of course, once it is known that there is a map of this sort (from functions on K to elements of $J^\perp$), it is possible to describe this map in purely finite-dimensional terms. In Gross's original paper [21], the finite-dimensional description is given in terms of how the map intertwines certain "annihilation operators." (See also [22, 23].) O. Hijab then gave a more explicit finite-dimensional description of the map in terms of derivatives of heat kernels. This lead to purely finite-dimensional proofs of the properties of the $J^\perp$ expansion for K by Hijab [41, 42] and B. Driver [9]. The most difficult part of these proofs is establishing that the map is *onto* $J^\perp$ in the case that K is simply connected.

The $J^\perp$ expansion established by Gross is analogous to the expansion of a function in $L^2(\mathbb{R}^n,\text{Gauss})$ as a sum of Hermite polynomials. Under this analogy, the Gaussian measure on $\mathbb{R}^n$ is replaced by a *heat kernel measure* on the compact group K. The Hermite polynomials on $\mathbb{R}^n$ are then replaced by logarithmic-type derivatives of the heat kernel. Specifically, we may identify the $J^\perp$ with (a completion of) the universal enveloping algebra of the Lie algebra $\mathfrak{k}$ of K. We think of the enveloping algebra as the algebra of left-invariant differential operators on K. Given a left-invariant operator α on K (an element of $J^\perp$), we associate to it the function $(\alpha\rho_t)/\rho_t$, where ρ_t is the heat kernel (at the identity) on K. In the $\mathbb{R}^n$ case, the heat kernel is just a Gaussian and such logarithmic-type derivatives of the heat kernel are the usual Hermite polynomials.

The Hermite expansion that came out of Gross's work lead him to suggest to me that I look for an analog of the Segal–Bargmann transform on a compact Lie group. This work became my PhD thesis and resulted in the paper [25]. The classical Segal–Bargmann transform [5, 53] is a unitary map from $L^2(\mathbb{R}^n,\text{Gauss})$ onto the L^2-space of *holomorphic* functions on $\mathbb{C}^n$ with respect to a Gaussian measure. The transform itself can be described in terms of the *heat operator* for $\mathbb{R}^n$. In [25], I replace the $\mathbb{R}^n$ with a compact group K and replace $\mathbb{C}^n$ with the "complexification" $K_{\mathbb{C}}$ of K. (For example, if $K = SU(n)$, the $K_{\mathbb{C}} = SL(n,\mathbb{C})$.) The Gaussian measures on $\mathbb{R}^n$ and $\mathbb{C}^n$ are then replaced by heat kernel measures and the heat operator on $\mathbb{R}^n$ is replaced by the heat operator on K. The resulting map is unitary from $L^2(K)$ (with respect to a heat kernel measure) onto the space of L^2 *holomorphic* functions on $K_{\mathbb{C}}$ (with respect to another heat kernel measure).

The paper [25] describes the Segal–Bargmann transform for K in purely finite-dimensional terms. Nevertheless, the origin of the Hermite expansion for K suggests that there could be a connection to the infinite-dimensional analysis in Gross's paper [21]. Indeed, this turns out to be the case, as Gross demonstrated in a paper with P. Malliavin [24]. Gross and Malliavin show something similar to what we have for the Hermite expansion: The Segal–Bargmann transform for K can be viewed as the Segal–Bargmann transform for an infinite-dimensional linear space (the path space $W(\mathfrak{k})$) applied to *endpoint functions*. That is, given a function ϕ on K, we consider as before the function f on $W(\mathfrak{k})$ given by $f(X) = \phi(\theta(X)(1))$. Then the Segal–Bargmann transform of f is the function F given by $F(Z) = \Phi(\theta_{\mathbb{C}}(Z)(1))$. Here $\theta_{\mathbb{C}}$ is the Itô map for $\mathfrak{k}_{\mathbb{C}}$ and Φ is the holomorphic function on $K_{\mathbb{C}}$ obtained by applying the Segal–Bargmann transform for K, in the sense of [25].

In [39], A. Sengupta and I extend the reasoning of [24] to more general functions of the Itô map. This eventually led to the development of a unitary Segal–Bargmann transform for the path group $W(K)$. (See also [6].) Furthermore, the paper [32] turns the reasoning in [24] around and uses the Segal–Bargmann transform for K to give a new proof (one of several by now) of the Gross Ergodicity Theorem.

Meanwhile, in [12], Driver and I exploit the similarity between the action of $\mathcal{L}(K)$ on $W(\mathfrak{k})$ and gauge transformations to study the quantization of $(1+1)$-dimensional Yang–Mills theory. We develop a new form of the Segal–Bargmann transform for K (see also [27]) and prove a generalization of the result of Gross and Malliavin in that setting. Our argument there draws on the analysis in [21], especially the use of the chaos expansion to understand endpoint functions. Our work in [12] was motivated by the work of Wren [60].

We have, then, three unitary maps that come out of Gross's proof of Theorem 1.1. The maps are (1) the Hermite expansion, which maps functions on a compact group K to the space $J^{\perp}$, (2) the generalized Segal–Bargmann transform, mapping functions on K to holomorphic functions on $K_{\mathbb{C}}$, and (3) the Taylor map, mapping a space of holomorphic functions on $K_{\mathbb{C}}$ to $J^{\perp}$. The Taylor map can defined as the composition of the Hermite expansion and the inverse Segal–Bargmann transform, but as shown in [9] it can be computed in a natural direct fashion in terms of the derivatives at the origin of a holomorphic function on $K_{\mathbb{C}}$.

2. Heat Kernel Analysis on Groups

The study of these maps and their generalizations has become a subject of its own, which may be called "heat kernel analysis on Lie groups." This phrase should be understood not as analysis *of* heat kernel (though of course there is some of that involved) but rather as analysis *with respect to* a heat kernel measure. More specifically, heat kernel analysis means generalizing to the setting of Lie groups various results in analysis on Euclidean space that involve a Gaussian measure. In the Lie group setting, the Gaussian measure on $\mathbb{R}^n$ is replaced by a heat kernel measure on the group. The remainder of this article is devoted to explaining some of the developments over the last 15 years in this part of analysis. See also the papers [28, 30, 33] for more detailed exposition.

A major development in the study of the Taylor map is the observation by Driver and Gross that the proof of the isometricity of the Taylor map does not require any invariance of the inner product on $\mathfrak{k}_{\mathbb{C}}$. This observation opened the door for a development (in [10]) of a unitary Taylor expansion map in the setting of an arbitrary simply connected complex Lie group G. (That is, G is no longer required to be the complexification of a compact group K.) It is worth noting that some vestige of infinite-dimensional analysis lurks in [10]. Driver and Gross (building on a similar analysis in [9]) use the concept of "Taylor expansion along paths" to obtain some key estimates on the pointwise behavior of L^2 holomorphic functions on G. These estimates can be understood as follows: One can turn a holomorphic function F on G into an "endpoint function" on the path space over the Lie algebra $\mathfrak{g}$ of G. This endpoint function belongs to a Segal–Bargmann space over an infinite-dimensional linear space. The bounds on F can be viewed as coming from applying the standard pointwise bounds on functions in that Segal–Bargmann space and minimizing over all paths with the same endpoint. Recently, Driver and Gross, along with L. Saloff-Coste, have extend the results of [10] to the case of a sub-Riemannian metric on G (see [11]).

The results of Driver and Gross have been extended to give Taylor maps for (certain classes of) infinite-dimensional Lie groups by M. Gordina [18, 19, 20]. Gordina's work allows for the development of a Cameron–Martin group along with a "skeleton map" or "restriction map," generalizing results for the linear case. Specifically, Gordina considers a space of L^2 holomorphic functions on a certain sort of infinite-dimensional group G. She then constructs a Cameron–Martin subgroup G_{CM} and shows that each L^2 holomorphic function on G has a well-defined "restriction" to G_{CM}. This holds even though G_{CM} is a set of measure zero within G. Further work in this direction has been done by M. Cecil and Driver [8, 7].

Meanwhile, concerning the Hermite expansion, J. Mitchell [48, 49, 50] has studied the asymptotics of the "Hermite functions" on a compact group K as the time parameter tends to zero. The Hermite functions are defined as logarithmic-type derivatives of the heat kernel, and the limit as the time parameter tends to zero is equivalent to the limit as the metric on K is scaled by a large constant, so that the curvature tends to zero. (Compare [35].) Mitchell shows that in this limit, the Hermite functions tend to the ordinary Hermite polynomials on $\mathbb{R}^n$. More generally, Mitchell develops an asymptotic expansion (in powers of t) of the Hermite function on K, the coefficients of which are polynomials on $\mathbb{R}^n$.

The results of [25, 26] concerning the Segal–Bargmann transform for a compact group K have been extended to the setting of compact symmetric spaces by M. Stenzel [54]. More recently, work has begun on understanding the Segal–Bargmann transform in the setting of *noncompact* symmetric spaces. Except in the Euclidean case, noncompact symmetric spaces present a major conceptual difficulty, because of singularities that do not occur in the compact or Euclidean cases. Any theorem in the noncompact setting has to provide a way of dealing with these singularities, typically by means of a "cancellation of singularities" result. Complementary approaches to this problem have been given B. Krötz, G. Ólafsson, and B. Stanton [44] on the one hand, and by Mitchell and me on the other hand [36, 37]. Comparing the two approaches, we may say that the approach of [44] is more general (it applies to arbitrary symmetric spaces of the noncompact type), whereas the approach of [36, 37] is more parallel to the compact and Euclidean cases, but applies (so far) only to noncompact symmetric spaces of the complex type. Both approaches draw on the work of Krötz and Stanton [45, 46] on analytic continuation of matrix entries of principal-series representations, as well as the Gutzmer-type formula of J. Faraut [14, 15]. See also [51, 52, 38, 55] for ongoing research in this direction.

Generalizing in a different way, Krötz, S. Thangavelu, and Y. Xu have developed a Segal–Bargmann transform (or "heat kernel transform") for the Heisenberg group. This case turns out to be quite different from the $\mathbb{R}^n$ case, with the image being identified as the direct sum of two spaces, each of which is an L^2 space of holomorphic functions with respect to a signed measure.

In another direction, the Segal–Bargmann transform for compact groups, along with the associated "coherent states" [35] have been applied in several ways in the setting of loop quantum gravity, which is an alternative to string theory in the attempt to unify gravity with quantum mechanics. In [4], A. Ashtekar, J. Lewandowski, D. Marolf, J. Mourão, and T. Thiemann use the generalized Segal–Bargmann transform to deal with certain "reality conditions" in the theory. Meanwhile, a series of papers by T. Thiemann and O. Winkler [56, 57, 58] have used the coherent states on K that come from the Segal–Bargmann transform to investigate the classical limit of Thiemann's "quantum spin dynamics" approach to loop quantum gravity.

Meanwhile, the paper [31] shows that the Segal–Bargmann transform for a compact group can be viewed as a unitary pairing map in the setting of geometric quantization. In light of [12], this can be seen as a "quantization commutes with reduction" result, as explained in [29]. Further understanding of the situation has been given by C. Florentino, P. Matias, J. Mourão, and J. Nunes [16, 17] and by J. Huebschmann [43]. Florentino and co-authors understand the heat equation on K as coming from parallel transport in a Hilbert bundle associated to rescalings of the complex structure on $K_{\mathbb{C}}$. Huebschmann, in turn, sees the heat equation as connected to a Peter–Weyl-type decomposition on $K_{\mathbb{C}}$ and to the Kirillov character formula.

In the paper [34], W. Lewkeeratiyutkul and I develop a theory of "holomorphic Sobolev spaces," that allows us to characterize the image of $C^\infty(K)$ under the generalized Segal–Bargmann transform. Thangavelu [59] has given a similar characterization of the image of distributions on K, and has extend both results to the setting of compact symmetric spaces. Thangavelu's work draws on the very sharp estimates for heat kernels on noncompact symmetric spaces developed by J.-P. Anker and P. Ostellari [2, 3]; see also [40].

In conclusion, heat kernel analysis on Lie groups has become an active subject of its own. Although it has by now evolved far from its origins, the subject owes its existence to the work of Leonard Gross.

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