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Preface

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This special issue is dedicated to
Professor Leonard Gross
on the occasion of his 88th birthday.
(Beiju in Japanese)

PREFACE

This special issue brings together papers in honor of Leonard Gross. Many of the authors spoke at the the 40th International Conference on Quantum Probability and Infinite Dimensional Analysis, in honor of Professor Leonard Gross, held at The Ohio State University in August 2019.

Leonard Gross was born in 1931 and grew up in New York City. He obtained his PhD in 1958, working under the guidance of Irving E. Segal. In 1960 he joined the Mathematics faculty of Cornell University. Much of Gross's work has centered on mathematically rigorous results for quantum field theories.

In a 1967 paper [2], Gross constructed abstract Wiener spaces, a framework that has since become standard for doing infinite dimensional analysis. Briefly, he introduced the triple (B, H, μ) , where H is a separable real Hilbert space, B is the completion of H with respect to a "measurable norm," and μ is Gaussian measure on B . The measure μ is supposed to represent the "standard Gaussian measure on H ," which does not actually exist as a measure on H , but it a well-defined measure on B . The special case where B is the space of continuous paths in \mathbb{R}^d , starting at the origin, leads to the classical Wiener measure.

In the late 1960s and early 1970s Gross proved logarithmic Sobolev inequalities [3]. The basic such inequality is

$$\int_{\mathbb{R}^n} |f(x)|^2 \log |f(x)| d\nu(x) \leq \int_{\mathbb{R}^n} |\nabla f(x)|^2 d\nu(x) + \|f\|_2^2 \log \|f\|_2,$$

where $\|f\|_2$ is the $L^2(\nu)$ -norm and ν is standard Gaussian measure on \mathbb{R}^n . The remarkable feature of this formula is that it does not involve any constants depending on the dimension, n . Consequently, such inequalities hold in infinite dimensions as well. Gross also established a fermionic version [4] of such inequalities. Gross proved that whenever a logarithmic Sobolev inequality holds, the semigroup generated by the associated Dirichlet form operator is hypercontractive, meaning that it maps the relevant L^p space into an L^q space with $q > p$. Since Gross's original paper, log-Sobolev inequalities have become an entire industry within analysis.

In the 1990s, Gross made major advances in analysis over loop groups. He proved an ergodicity theorem [7] that, despite being a result for the infinite dimensional space of paths on a Lie group, led to a plethora of results for heat kernel analysis on Lie groups. (See [9].) Briefly, Gross showed that if a function on the Wiener space of paths $g : [0, 1] \rightarrow K$ on a compact Lie group K is invariant under the left-translation by the much smaller subgroup of finite-energy loops, then that function depends only on the endpoint $g(1)$ of the path. In this context Gross also developed [8] a chaos expansion for functions on the group K , by using the corresponding expansion for the Gaussian L^2 -space over the path space on K .

Key words and phrases. Leonard Gross, Quantum Probability, Gaussian Measure, Loop Spaces, Path Spaces, Yang-Mills Theory.

Gross has made contributions to gauge theories in papers over several decades. In the 1980s he proved [5] geometrical results, such as the Poincaré Lemma, on spaces of paths, and proved a formulation of the Yang–Mills equations using path space geometry. He also initiated work [6] on two-dimensional quantum Yang–Mills theory that eventually led to work by many others, including ongoing work by many researchers. Over the past ten years or so, Gross has focused on Yang–Mills theory, working in part with Nelia Charalambous, on Yang–Mills theory. Her paper in this issue described some of this work.

The conference QP40 brought together experts and new researchers from around the world. The talks spanned a very wide range of topics, many, though not all, influenced by ideas that originated with Leonard Gross’s works. The present special issue reflects this variety.

The QP series of conferences started in 1982 and has been held in many countries. The 40th conference was the first to be held in the United States. The conference was supported by the National Science Foundation, the US Army Research Office, and by a research fund of The Ohio State University. Our thanks to these agencies, and to the contributors at the conference, the authors who submitted papers for this issue, and the dedicated staff at The Ohio State University whose work helped make the conference and this issue a success.

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References

1. Charalambous, Nelia, Gross, Leonard, The Yang-Mills heat semigroup on three-manifolds with boundary, *Comm. Math. Phys.* **317** (2013), no. 3, 727–785.
2. Gross, L.: Abstract Wiener spaces, in: *Proc. 5th Berkeley Symp. Math. Stat. and Probab.* **2**, part 1 (1967) 31–42, University of California Press, Berkeley.
3. Gross, Leonard, Hypercontractivity and logarithmic Sobolev inequalities for the Clifford Dirichlet form, *Duke Math. J.* **42** (1975), no. 3, 383–396.
4. Gross, Leonard, Logarithmic Sobolev inequalities, *Amer. J. Math.* **97** (1975), no. 4, 1061–1083.
5. Gross, Leonard, A Poincaré lemma for connection forms, *J. Funct. Anal.* **63** (1985), no. 1, 1–46.
6. Gross, Leonard, King, Christopher, Sengupta, Ambar, Two-dimensional Yang-Mills theory via stochastic differential equations, *Ann. Physics* **194** (1989), no. 1, 65–112.
7. Gross, Leonard, Uniqueness of ground states for Schrödinger operators over loop groups, *J. Funct. Anal.* **112** (1993), no. 2, 373–441.
8. Gross, Leonard, The homogeneous chaos over compact Lie groups, in: *Stochastic processes*, Springer, New York, (1993) 117–123.
9. Hall, Brian C.: Leonard Gross’s work in infinite-dimensional analysis and heat kernel analysis, *Commun. Stoch. Anal.* **2** (2008), no. 1, 1–94.